

**OPTIMIZING FIELD PERFORMANCE USING INJECTION EFFICIENCY
FROM STREAMLINE-BASED WORKFLOW**

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FROM STREAMLINE-BASED WORKFLOW**

A

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ABSTRACT

Waterflooding is one of the most common secondary recovery mechanisms available for improving oil recovery from known accumulations. Due to the limitation of available water for injection, there's a need to optimize the use of this limited resource. To this end, a methodology has been developed to allocate injection water to more efficient wells than to those of lower efficiency.

Streamline simulation is a complementary tool to conventional finite difference simulation, provides a quick and efficient way for water re-allocation to optimize the waterflood. This tool has the ability to generate the contribution of a particular injector to offset producers. The producer-injector relationship yields what is termed Well allocation factors (WAF) to be used in computing injection efficiency.

This research seeks to optimize water allocation in an oilfield with the constraint of limited total volume of injection water. The proposed methodology is illustrated using a case study. Two reservoir models were built for 5-spot and 9-spot patterns to study maximization of oil recovery by optimizing water re-allocation during a waterflood. Injection wells were monitored until injection efficiency degraded to 60%; indicating unbalanced injection program. Water-reallocation program was initiated at this point using 6 months, 12 months and 18 months re-allocation cycles. The performance of the waterflood was monitored for a period of 5years.

Results show marked decrease in water-cut between unoptimised and optimised floods. No significant difference in flood performance was observed for the various re-allocation cycles. This may be due to the degree of heterogeneity in the model and /or the assumption of no workovers of injector and producer wells allowed during the 5-year flood re-allocation period. It is concluded that the proposed methodology can be applied to determine the optimum reallocation frequency to maximize oil recovery from the flood.

DEDICATION

To My most faithful Father, thank You for inspiration all the way.

For the life You gave I say Thank You

For the love shown, I say Thank you more.

You are indeed Faithful.

Special dedication to all those who seek after knowledge, we have come this far

Again. The sky is our starting point!!!

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CHAPTER 1.0

INTRODUCTION

1.1 Motivation

As the primary drive energy of a reservoir depletes, there is a need to seek alternative methods of oil recovery from existing known accumulations. Waterflooding is the most widely used secondary recovery process because water is widely available and inexpensive relative to other fluids, easy to inject and highly efficient in displacing oil.

In most situations a significant portion of the oil in the reservoir can be recovered by waterflooding. Capital costs, mainly for surface facilities to handle the injection and production water, are relatively inexpensive compared with those of most other enhanced oil recovery (EOR) methods. Operating costs for a waterflood are typically lower than for other EOR techniques. To evaluate the performance of a waterflood project, it should be clearly proven that the natural recovery processes are insufficient; otherwise there's risk that the substantial capital investment required for a secondary recovery project may be wasted (Ahmed, 2006).

Recovery optimisation is the most important factor to be considered before embarking on a waterflood project. This is achieved by maximising oil recovery at the surface per barrel of injected water while minimising formation damage and maintaining reservoir pressure. One of the first steps to be considered before applying a secondary recovery process is the project design. Waterflood design is a multidisciplinary project. Well placement in a waterflood may be done evenly or unevenly from each other depending on topology, lease boundaries, regulations or other factors. Many older fields were

developed on irregular spacing but in more recent times the trend is to use more uniform drilling patterns and well spacing. In pattern flooding, the injectors are distributed among the producers in some repeating patterns. Injectors are either newly drilled wells or producers converted to injectors.

The success of a waterflood project can be predicted from proper selection of waterflood patterns. The primary objective is to attain a balance between injection and producer wells within a pattern and minimize oil migration to adjacent patterns and loss into the formation. Balancing patterns essentially means that for every barrel of water injected, a barrel of hydrocarbon is recovered from the production wells surrounding the injector. Unbalanced pattern leads to poor sweep, premature breakthrough and high water cycling. An effective pattern balancing leads to better areal sweep and higher oil recovery.

In the past, analytical methods have been employed to optimise oil recovery from effective pattern balancing. Recent developments in reservoir management technology have fostered the use of reservoir simulation; especially streamline simulation, based on reservoir pressures and well rates in pattern balancing.

In this research, well allocation factors (WAF) are used to compute injection efficiency which forms the basis for the re-allocation of injection water to achieve a balanced pattern for optimum oil production. The use of streamline simulation as a tool for obtaining WAF directly shall be exploited.

Streamline simulation is a method for the visual representation of the amount of flow between the injection and production wells, i.e., the well rate allocation factor (WAF) for each well in an oil field. This method has been proven to help engineers balance flood patterns, determine visually and quickly the effectiveness of injection techniques thereby helping to avoid devoting economic resources to those wells or patterns from which there's ineffective production.

1.2 Objectives

The main objective of this study is to use streamline simulation to compute the water allocation factors and injection efficiency required to re-allocate water injection in order to optimize oil recovery from a waterflood.

The specific objectives of this research are as follows:

1. Develop a streamline simulation based workflow for computing water allocation factors and injection efficiency
2. Validate the workflow using a case study
3. Demonstrate pattern balancing as a flood optimisation tool by re-allocating injected water to production wells with higher efficiency in 5-spot and 9-spot patterns, thereby optimising oil production for each barrel of water injected.
4. Observe the trends of oil production versus time and water-cut versus time from the reservoir model and determine the frequency of computing and updating injection

efficiency to effect the optimum re-allocation schedule of injection water to maximize oil recovery.

1.3 Scope and Limitation of Work.

This research work shall be limited to repeated patterns in an oilfield where the use of pattern waterflooding has been recommended/proposed, Due to time limitation, the work will not be extended to peripheral pattern balancing technique. Only two regular pattern balancing techniques shall be addressed: 5-spot and 9-spot patterns.

1.4 Organization of Work

This work is organised in five chapters. A brief definition/statement of the problem, objectives, methodology, scope and limitation of work is presented in Chapter 1. Chapter 2 contains a review of the published literature and summary of related previous studies. This chapter also focuses on the advantages of streamline simulation over conventional simulation as it relates to recovery optimisation. In Chapter 3 the research methodology is presented. The chapter also discusses the use of streamline simulation to obtain WAF for computing injection efficiency in a waterflood. Chapter 4 discusses the results obtained from the simulations. Plots of oil production rate, water cut, and observations of the results and improvement in oil recovery by re-allocating water are discussed. Finally, the summary of analysis of results, conclusions, and recommendations are presented in Chapter 5.

CHAPTER 2.0

LITERATURE REVIEW

This chapter describes the fundamental concepts of waterflooding including arrangement of producer-injector wells, flood patterns and factors affecting pattern balance. Also discussed are the methods previously used to optimise oil recovery in waterflood projects.

2.1 What is Waterflooding?

Waterflooding is defined as a method of improved recovery in which water is injected into a reservoir to remove additional quantities of oil that have been left behind after primary recovery.

It is also defined water flooding as the injection of water into an oil-bearing reservoir to recover more petroleum from it. Reservoir analysis is carried out to predict the cumulative oil recovery with time, and the cumulative water injection with time. It is often more convenient to determine the cumulative oil recovery as a function of cumulative water injected. Cumulative recoveries by primary and secondary production, where the secondary production is waterflooding, average between 38 and 43 percent of the original oil in place (Doug Soveran, August 2010) .

Waterflooding is usually the first secondary recovery method applied to a reservoir. In most situations it will help recover a significant portion of the oil in the reservoir. Capital costs, mainly for surface facilities to handle the injection and production water, are relatively inexpensive compared with those of most other enhanced oil recovery (EOR)

methods. Operating costs for a waterflood are typically lower than for other EOR techniques (oil and gas glossary, August 2010).

Waterflood design is a multidisciplinary project. The success of a waterflood project can be predicted from proper selection of waterflood patterns. The primary objective is to attain a balance between injection and producer wells within a pattern and minimize oil migration to adjacent patterns and loss into the formation. Unbalanced pattern leads to poor sweep, premature breakthrough and high water cycling (Schlumberger, 2010). An effective pattern balancing leads to better areal sweep and along with well alignment leads to better oil recovery. Although analytical methods have been employed in the past to optimise oil recovery from effective pattern balancing, recent developments in reservoir management technology have favoured the use of reservoir simulation based on reservoir pressures and well rates in pattern balancing. This research seeks to review the methods previously adopted and the application of the most recent technique – streamline simulation in optimising oil recovery from a balanced waterflood pattern

2.2 Waterflood Patterns

Well placement in a waterflood may be done evenly or unevenly from each other depending on topology, lease boundaries, regulations or other factors. Many older fields were developed on irregular spacing, in more recent times, more uniform drilling patterns and well spacing have been used. In pattern flooding, the injectors are distributed among the producers in some repeating patterns. Injectors are either newly drilled wells or producers converted to injectors. Either way, the injector wells must be compatible with existing producers and should

1. take advantage of known reservoir uniformities or non-uniformities (fractures, directional permeability, regional permeability)
 2. provide sufficient fluid injection rate to yield desired production rate
 3. require minimum number of wells due to cost implication

Two general types of well locations are common; peripheral or central flooding where the injectors are grouped together and pattern flooding where certain patterns are repeated throughout the field as shown in Figure 2.1.

When possible, the injector scheme should take advantage of gravity i.e. dipping or inclined reservoir or gas caps or underlying aquifers.

Analytical methods, proper reservoir surveillance and reservoir simulation based on reservoir pressure and well rates are usually employed in pattern-balancing studies. However, newer tools have been developed to complement the conventional finite difference simulation technique. This is streamline simulation which is sometimes considered as an alternative which allows rapid evaluation of multiple geological and engineering scenarios for quick decision making.

Streamline simulation has been proven as an effective tool in pattern balancing and optimization of oil recovery. These are some of its benefits (Schlumberger,2010):

- Track the oil-water front movement along the streamlines to accurately predict water breakthrough
- Accurately estimate injector-to-producer relationship on a well by well basis

- Identify injectors not contributing to production or producers that are cycling injected water
- Assists to trace and modify cells with the maximum effect on flow behaviour, this invariably reduces trial and error and achieves history match with fewer iterations

Balancing water injection with fluid production on a gross basis is critical to maintain reservoir energy. This process is essentially a material balance problem and can be generally evaluated using Voidage Replacement Ratio (VRR) analysis wherein the volume of oil, water and gas produced in reservoir units is divided by the volume of water injected, converted to reservoir barrels. A balance VRR of 1 would result in pressure stability in the reservoir on a macro basis.

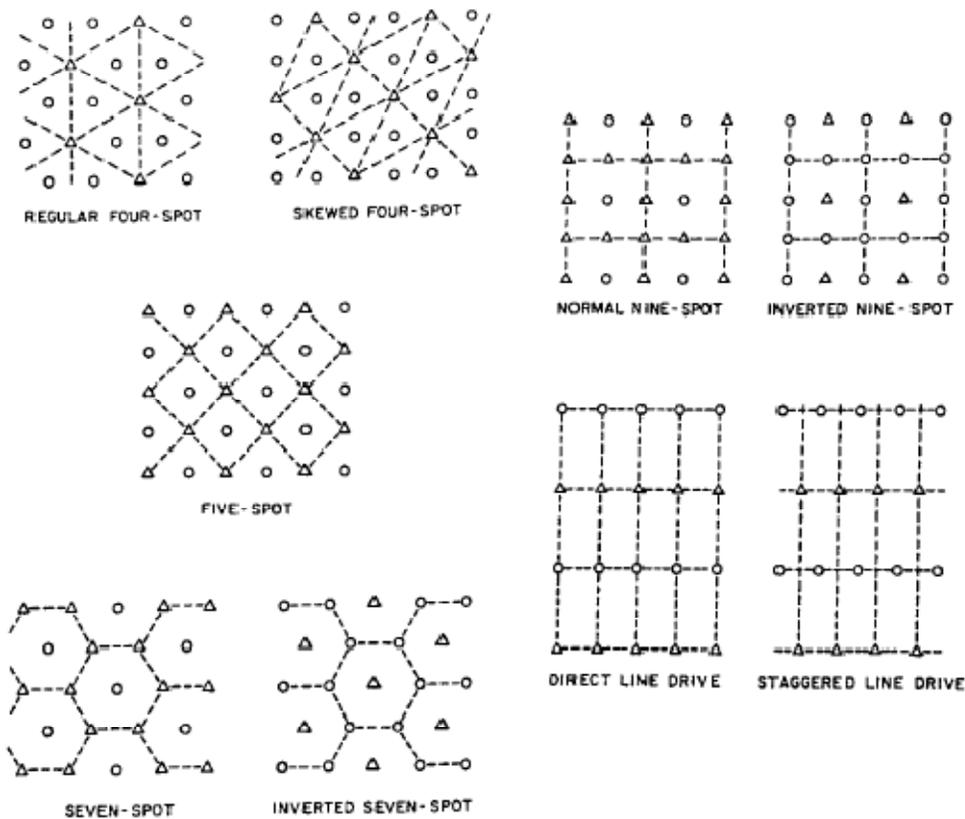


Figure 2.1 Various Types of Regular Flood Patterns (culled from Tarek, 2006)

However individual patterns may be out of balance or in balance depending on the volume of water injected in each pattern. This balance on a smaller scale is important in optimising:

1. Sweep Efficiency
2. Flux of reservoir energy across the field
3. Maximising the efficiency of each barrel of water injected.

An optimal water injection policy maximises oil recover per barrel of injected water while minimizing reservoir pressure (Zangl, 2004).

2.3 Techniques for Pattern Balancing

According to literature, research has been carried out in oil recovery optimisation and techniques for waterflood pattern balancing. Such techniques previously studied include:

- Analytical Solution
- Static Allocation
- Reservoir management and surveillance
- Conventional Simulation
- Streamline Simulation

2.3.1 Analytical Solution

In the 1930s, Muskat (1937) presented analytical solutions for several patterns using “ideal” flow assumptions which include single phase, incompressible fluid of constant viscosity, flow at a steady state in horizontal, homogeneous and isotropic reservoirs without free gas saturation. Deppe (1961) in his work also presented flow equations for ideal nine spot pattern. The single phase assumption also has been equivalent to unit mobility ratio condition for two-phase systems (i.e. when the end point mobility ratio is 1). Deppe showed that this criterion is not sufficient to apply pattern equations to real reservoir systems. Since the set of analytical equation is only applicable when $M = 1$, thus the applicability of the single phase equation is limited to a small subset of reservoirs.

Crawford (1960), in his laboratory experiments, showed that the sweep efficiency and steady state injection rates for various types of patterns may be used for waterflood programs. Assuming unit mobility ratio (mobility ratio here is taken to be before water breakthrough unless otherwise stated), homogeneous and uniform reservoir, steady state condition and ignoring gravity and capillary effects, he obtained efficiency of various sweep patterns as follows; 72% for 5-spot pattern, 56% for direct, line-drive square pattern and 45% to 90% for 9-spot pattern. Crawford (1960) further observed that these sweep efficiency values when water breakthrough occurs were dependent on initial oil, water and gas saturations. If initial oil saturation remain constant and connate water and free gas saturation were varied, sweep efficiency increases as the connate water decreased. If the connate water was maintained constant and initial oil and free gas

saturation varied, it was found that the sweep efficiency increased as initial oil saturation decreased. These values are in agreement with the solutions developed by Muskat in the 1930s based on theoretical assumptions.

Deppe, further research presents an approximate method for calculating water injectivity for unequal mobility (Deppe, 1961). He made reference to Muskat's analytical solutions and subsequently formulated an approximate solution which is applicable to regular, irregular and boundary patterns.

Hansen et al (2003) recently reported that other methods that can be used to predict water flood flow behaviour for optimal recovery in non-unit total mobility ratio in 2D systems include numerical simulation and analytical streamtube methods. These methods account for the complete fractional flow behaviour without full analytical treatment. The only limitation of these methods is that they do not provide insight into the parameters governing the flow process that an analytical solution can provide. Hansel et al in their research concluded that using analytical methods reveals how producer-injector ratio and a newly defined mobility ratio are the key mechanisms controlling pattern rates and pressure. They reported that the economically optimum pattern can be determined analytically using these relationships and other pertinent economic variables.

2.3.2 Static Allocation

Ideally, a waterflood operation is considered to be the most effective when all the wells recover the remaining amount of oil simultaneously and reach their economic limits. This suggests that producers located in drainage areas having large pore volumes should be produced at relatively high rates. Hence wells are allocated injection and production rates

according to the pore volumes, with an objective of minimizing project duration and operating costs. Well allocation factors (WAF) play a crucial role in determining the fraction of the injected water supporting the volume of oil produced at offset wells (Schlumberger, 2010).

Static allocation refers to the use of reservoir properties and neighbouring well distance as criterion for pattern balancing and optimisation of oil recovery. By describing injectors by patterns, production is allocated to injectors using angle open to flow and distance weighting methodology. This method poorly represents the physics involved in determining flow paths in reservoir between wells. Injection well profiles, water influx, and cross flow are typically not addressed, as they change with time (Grinestaff, 1999).

Conventionally, allocation factors of producer wells are assumed to be static. An operator generally assigns static allocation factors to producer wells, based on empirical data and/or expert knowledge. However, the assumption that allocation factors of producer wells are static is often an incorrect assumption, since at least one characteristic (e.g., fluid injection rate) associated with an injector well that determines the value of an allocation factor may dynamically change. The allocation factors of producer wells can be used to determine a performance parameter of a waterflooding operation. One such performance parameter is referred to as a voidage replacement ratio (VRR). By statically assigning allocation factors, the VRR value may be incorrectly computed, such that an operator may assume that a water flooding operation may be proceeding in the direction, when in fact the operation can be quite different from what is being indicated by the VRR value. As a result, the operator may make adjustments to control parameters associated with the wells that may cause sub-optimal performance of hydrocarbon production.

An application of static allocation is reported in the North West Fault Block Area of Prudhoe Bay, Alaska (Grinestaff, 2000) where static allocations are being used to set injector volumes. Injected fluids are fluxing to producers several patterns away and large amount of fluids actually leaving the planned waterflood area, leading to an unbalanced pattern. Due to the inability of static allocation to account for reservoir heterogeneity which occurs in reality, dynamic allocation factors are now used instead for pattern balancing.

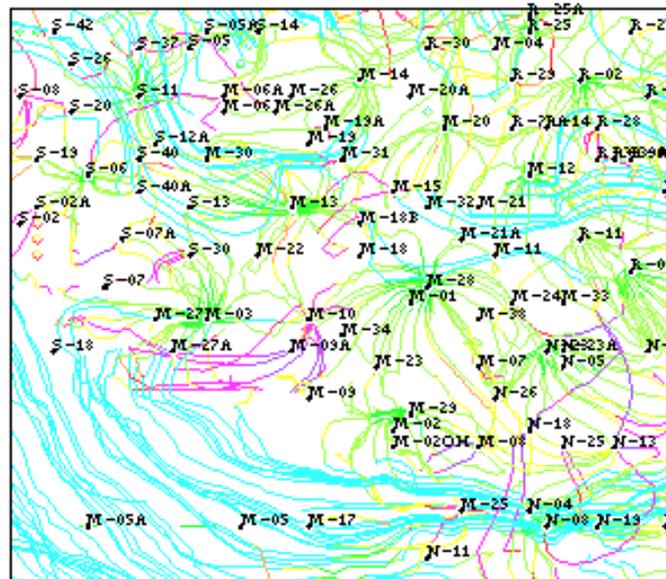


Figure 2.3 Static Allocation of NWFB Prudhoe Bay (culled from Grinestaff, 2000)

2.3.3 Reservoir Management and Surveillance

Reservoir management techniques are not actually to be treated in isolation for optimum oil recovery. There is a need to integrate all available data into a reservoir management program to achieve the desired goal. Clark and Robert (2007) used an innovative, unified information system to monitor voidage replacement ratio (VRR) to

provide a basis for pattern balancing in a case study of the Sabiriyah Field of North Kuwait. They observed that the water flood was successful in recharging the reservoir energy with an equal volume of water to replace the voidage created by withdrawn volume of oil. The successful integration of subsurface, water handling, well surveillance and production operation teams across the North Kuwait asset significantly improved the operating procedure for waterflooding the Sabiriyah Mauddud Field.

Clark and Robert (2007) reported that in 1997, a pilot waterflood was initiated in a quiescent part of the reservoir on a 40-acre 5-spot pattern. Four Producers were drilled around a central test water injection well to evaluate the efficacy of waterflooding the field. Kuwait Oil Company then initiated an inverted 9-spot pattern waterflood to develop the major extent of the field in the most rapid way possible as this entailed the least number of injection lines.

The reservoir was discovered at an under-saturated condition; thus, the drive mechanism is almost a pure depletion drive and this provided the impetus to initiate the waterflood operation. Theoretical sweep efficiency from an inverted 9-spot is inferior to a 5-spot pattern by approximately 50% versus 70%. However, to rapidly restore reservoir energy and initiate a waterflooding operations, a 9-spot pattern was adopted with the eventual goal of down spacing the reservoir to a 5-spot pattern.

Conclusively, Clark and Robert (2007) showed that the Kuwait Oil Company was able to optimise waterflooding using reservoir management team effective in achieving target reservoir pressures in the major part of the reservoir with the active participation of all team members, subsurface and surface alike. Using this approach, water-cut was flattened with effective decline in wet oil production. According to Moudi (2007) further research

on the same field showed that after applying the new management approach, individual waterflood pattern balance was significantly improved and the field wide Voidage Replacement Ratio (VRR) became about 1.2

2.3.4 Conventional Simulation

Reservoir simulation or modelling is one of the most powerful tools currently available to the reservoir engineer. In conventional simulation, the model requires that the field under study is described by grid cells. Each cell is assigned properties to describe the reservoir (Koederitz, 2004). The simulator will allow us describe a fully heterogeneous reservoir to include varied reservoir performance and to study different types of recovery mechanisms. Conventional simulation will help provide general answers, however time constraints prohibit developing and using a detailed model to capture complexities for each well. In cases where detailed finite difference simulation has been developed, model run times and data extraction prohibit its use to make well level decisions on a daily basis (Grinestaff, 1999). Finite difference (or finite element) formulations to the fluid flow equations require spatial discretization of reservoir model into a pattern of grid blocks and division of the total simulation time into smaller time steps.

All fluid flow problems in reservoir engineering are governed by certain constitutive equations which include the continuity equation, equation of state and equation of flow. In the standard form, the conservation equation for each phase is given as (Datta-Gupta 1998):

$$\phi \frac{\partial S_p}{\partial t} + \nabla \cdot \overline{V_p} = Q_p \quad p = \text{water, oil}, \quad (2.1)$$

$$\text{Where } \mathbf{V} = -k \frac{K_{rF}}{\mu F} \nabla (P_p + \rho_p g z) \quad (2.2)$$

Treatment of the model equations yields an IMPES (Implicit Pressure, Explicit Saturation) formulation, a fully implicit formulation or some combination of both (Koederitz, 2004).

Conventional simulation process as described above has been used extensively over the years. However, certain shortcomings have made the development of certain sophisticated process evolve. These shortcomings include the inability to quantify the efficiency of injectors, numerical smearing and computational efficiency for models with large number of grid cells (Lolomari, 2000). These limitations are being addressed with the use of streamline simulation.

2.3.5 Streamline Simulation

Improved recovery methods require improved field management needs. For example, the Alberta Energy Resources Conservation Board (ERCB) requires oil companies to balance their production patterns in order to maximize recovery from existing well patterns. Balancing patterns essentially means that for every barrel of water injected a barrel of fluid is recovered from the production wells surrounding the injector.

Streamline Simulation relates to a method for the visual representation of the amount of flow between injection and production wells, i.e., the well rate allocation factor (WAF), for each well in an oil field (www.patentstorm.us,2010)The model displays the percentage of support each injector well is giving to each producer well or the percentage of support each production well receives from each injection well. This, for example, will help engineers balance patterns, or to determine visually and quickly the effectiveness of

injection techniques thereby helping to avoid devoting economic resources to those wells or patterns from which there is ineffective production. By visualizing the interaction between well pairs, streamline simulation allows engineers to design more efficient recovery schemes and thereby increase the ultimate recovery from current hydrocarbon reservoirs.

In recent years there has been an increased interest in the streamline simulation technology. This is primarily due to two factors: One is with the developments in reservoir characterisation, we can routinely generate high resolution reservoir models consisting of multimillion cells. This has resulted in a gap between geologic modelling and flow simulation. Second, with increased model resolution, there is an increased acknowledgement of uncertainty. Conventional numerical simulators often are inadequate to satisfy these needs in a timely fashion.

Datta-Gupta (1998) in his paper gave the following explanation of the use of streamline simulation; the underlying principle and physics is explained below. Streamline simulators approximate 3D fluid-flow calculations by a sum of 1D solution along streamlines. Instead of moving fluids from cell to cell, streamline simulation breaks up the reservoir into 1D system or tubes. The transport equations are then solved along the 1D space defined by the streamlines using the concept of time of flight(Akhil,2000). By decoupling the transport problem from the underlying 3D geological model, fluids can be transported more efficiently. Large time-steps can be taken, numerical diffusion minimized and the CPU time varies nearly with model size (Datta-Gupta, 2000). The choice of streamline directions for the 1D calculation makes the approach extremely effective for modelling convection-dominated flows in the reservoir. This is typically the

case when heterogeneity is the predominant factor governing the flow behaviour. An important concept in streamline simulation which makes it highly practicable is the isolation of heterogeneity effect from the physics of flow calculations.

Mathematically this is achieved by using streamline **time of flight as a co-ordinate variable**. A co-ordinate system where all streamlines are straight lines and distance is replaced by time is used. The impact of heterogeneity is embedded in the time of flight and trajectory of the streamlines. The streamlines are generally distributed in space with higher resolution than the underlying spatial grid, thus providing excellent resolution than the underlying spatial grid system, thus providing excellent transverse resolution. Saturation calculations are decoupled from the underlying grid and can be carried out with little or no intrinsic time-step limitations (Datta-Gupta, 2000).

2.3.5.1 Application of Streamline Simulation

Streamline simulation has found applications in various reservoir modelling projects. According to Datta-Gupta (2000), the following are the major uses:

- Swept volume Calculations
- Rate Allocation and Pattern Balancing
- Modelling Tracer flow and waterflooding
- Ranking geostatistical Models
- Contaminant arrival/breakthrough time
- Upgrading and Upscaling

- History Matching and Production-Data Integration
- Primary recovery and Compressible flow
- Solvent flooding and Compositional Simulation

For the purpose of this research work, rate allocation and pattern balancing shall be addressed. The streamline approach can aid in reservoir management by providing important information, such as injector/producer relationships and allocation factors for injectors (Datta-Gupta, 2000). This information comes very naturally from streamlines models but not from conventional numerical simulators. Because each streamline is associated with a flow rate, allocation factors for the injectors can be easily computed (i.e., what fraction of the injected fluid is going into the individual producers). This information has been reported to be very useful in pattern balancing and flood front management

2.3.5.2 Field Application of Streamline Simulation In Pattern Balancing and Oil recovery optimisation.

One of simplest application of streamline simulation is areal pattern balancing which is amenable to two-dimensional calculations and to the steady state approximation. Assuming uniform initial fluid saturation and constant injection flux, then the flow equations leads to a self-similar solution in terms of (τ/t):

$$\frac{\tau}{t} = \frac{dF_w(s_w)}{S_w} \quad 2.3$$

The analytic solution of the above equation shows that $\tau(x,y,z)$ can be used to predict waterflood performance.

Streamline simulation has been applied in different fields of the world. One of such is the work reported by Ghori et al (2006,2007) of a Saudi Aramco reservoir. They demonstrated the use of streamline simulation as a complementary tool to finite difference simulation. They reported that this new and more robust technique is more effective in solving geologically complex heterogeneous systems.

Simulation results were used to determine optimum placement of injectors and visualise flow patterns. They used well allocation factors to estimate: distribution of injected water to allocated producer, water loss to the aquifer, percentage of oil produced due to each supporting injector and amount of water attributed to each supporting aquifer.

Batycky et al (2003) also demonstrated the application of streamlines in optimising waterflood performance using injection efficiency as a criteria. The injection efficiency for an injector was defined as follows:

$$IE = \frac{\text{offset oil production} \left(\frac{rb}{day} \right)}{\text{water injected} \left(\frac{rb}{day} \right)} \quad 2.4$$

Since the expression above is a ratio of rates, its represents an instantaneous quantity. The water injection rate is usually known while the offset oil production at the producers is estimated from the well allocation factors (WAF) determined by the streamlines. The injection efficiency cross-plot of water injected versus offset oil production is an effective way of assessing waterflooding project. For low values of injection efficiency (IE), such pairs are associated with high injection rates which are a clear indication of excessive water cycling and reservoir in need of active flood management. However, higher values

of IE indicate an efficient flood at that particular moment in time. They showed that streamline simulation is an excellent tool used to generate the data needed to construct an IE plot.

By moving injected volume from inefficient to efficient injector pairs, it is possible to increase oil production with increased volume of injected water or alternatively decreasing the amount of injected water to maintain desired oil target. This approach was demonstrated to be a quick and simple yet effective way to drive reservoir management practice. The ability of streamlines to offer a snapshot of how the reservoir is connected and how fluid is allocated between well pairs is the key for pattern balancing and optimal waterflood management.

2.4 Summary of Chapter on Literature Review

Waterflooding usually involves the injection of water through wells specially set up for water injection and the removal of water and oil from production wells drilled adjacent to the injection wells (oil and gas glossary, August 2010). It further referred it to a secondary recovery mechanism which involves the injection of water into the reservoir through injector well(s) to maintain reservoir pressure and drive the oil towards the producing well(s). Balancing water injection with fluid production is critical to maintaining reservoir energy. Harnessing the reservoir energy by pattern balancing is one of the ways to optimize oil recovery in a waterflood.

It has been seen from literature that streamline simulation is a viable tool for flood balancing and reservoir management for the following reasons:

1. It is computationally efficient and fast
2. Enables flow visualisation between injectors and producers making it easy to relate with by all team members of a project, irrespective of discipline.
3. Provide rate allocation factors for injectors.
4. Used to improve location for new injectors/producers to improve sweep efficiency.

Streamline simulation enables us to obtain rate allocation factors directly because the streamlines and the time of flight (which are essential for optimisation) are readily available, compared to finite difference where we need to perform the streamline tracing and time of flight computation using the total phase flux from the simulator.

In the following chapter, streamline simulation shall be used as an effective tool in optimising oil recovery by pattern balancing. The concept of injection efficiency shall be used as a criterion for decision making as was reported by Batycky et al (2007). The novel contribution of this work is that we shall go a step further in analyzing the trends of the WAF and injection efficiency to ensure that the frequency of optimisation of re-allocation is ascertained to support decision making for effective reservoir management.

CHAPTER 3.0

METHODOLOGY

The main objective of this study is to use a streamline reservoir simulator to optimise oil recovery by computing the well allocation factors to effectively re-allocate water to existing injection wells. A secondary objective is to establish the optimum frequency of updating injection efficiency during waterflood operations management. This chapter presents a description of the major steps followed in this project to optimise oil recovery in a 5-spot and 9-spot waterflood systems using streamline simulation. The results and discussion of results are presented in the next chapter.

3.1 Reservoir Modeling Case Study

The reservoir model built in this case study is loosely patterned after the water injection program operated in the Stevens Formation of Elk Hills Field, California. Two waterflood patterns, 5-spot and 9-spot were studied in this work. The emphasis of this reservoir modeling case study is to illustrate the proposed methodology and workflow of using injection efficiency and pattern balancing as a tool for oil recovery optimisation.

3.1.1 Data Collection

The data required for building the reservoir model for this case study was obtained from a typical reservoir in the Stevens Formation of Elk Hills Field (Ezekwe, 2010). The PVT data and Rock Properties are listed in Table 3.1.

Table 3.1: Summary of Reservoir Rock and Fluid Properties (Ezekwe, 2010)

Porosity Range	11- 26%
Air permeability	10-250md
Initial Water Saturation	30 – 45 %
Initial Reservoir Pressure	3150 psia
Initial Bubble Point Pressure	2965psia
Reservoir Temperature	210°F
Oil Viscosity	0.4cp
Oil Gravity	36° API
Mobility Ratio	0.6
Residual Oil Saturation to Water	25%

3.1.2 Streamline Simulator - S3D

The simulator used in this project is S3D version 3.2, a three dimensional streamline simulator (Datta-Gupta and King, 2007). This is used to model a two-phase waterflooding and also a single phase tracer transport under incompressibility assumptions. The streamline approach consist essentially of two steps: generating fluid flow equations in 3D space and then solving the 1D transport equations analytically or

numerically along the streamlines. The following are important features of the S3D software (Datta-Gupta and King, 2007):

1. Waterflood/tracer simulations.
2. Pressure updating.
3. Analytic/Numerical solution for 1D solution.
4. Infill drilling/changing well configuration.

The use of streamlines is unique in that it is possible to exactly quantify the flow between injector and producer wells. This means it is possible to calculate the fraction of fluid flowing to a particular producer supplied by the surrounding injectors in the field. These allocations are referred to as Well Allocation factors (WAF). These well allocation factors are essential data for balancing patterns; up till now they have been estimated using rough approximations. With the advent of streamline simulations, it is now possible to get an exact solution to WAF, far superior to other methods available. However, it worth noting that streamline simulation is complementary to the existing conventional simulators, because streamlines can provide reservoir flow information that conventional simulators cannot such as well allocation factors.

3.1.3 Description of Reservoir Model

The reservoir is modelled using 45x45x10 grid cells with nine (9) injectors and four (4) producers arranged in a 5-spot pattern (Figure 3.1). The rock and fluid data used is as reported in Table 3.1. All the wells are fully completed, penetrating the nine layers of the reservoir model. All the injectors are injecting water at different rates depending on their location. However, the total injected water is kept constant at 20,000stb/day throughout

the period of simulation. This same procedure is repeated for a 9-spot pattern but the reservoir model in this case contained 21 injector wells and four (4) production wells.

Figure 3.2 shows the well arrangement for the 9-spot pattern.

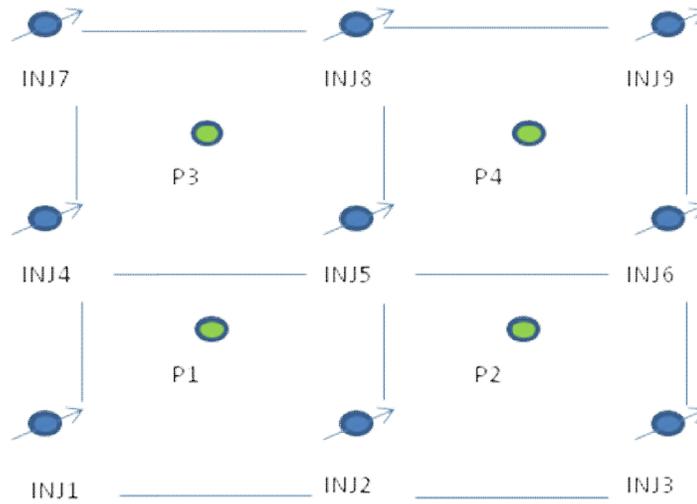


Figure 3.1 Arrangement of wells in 5-spot pattern

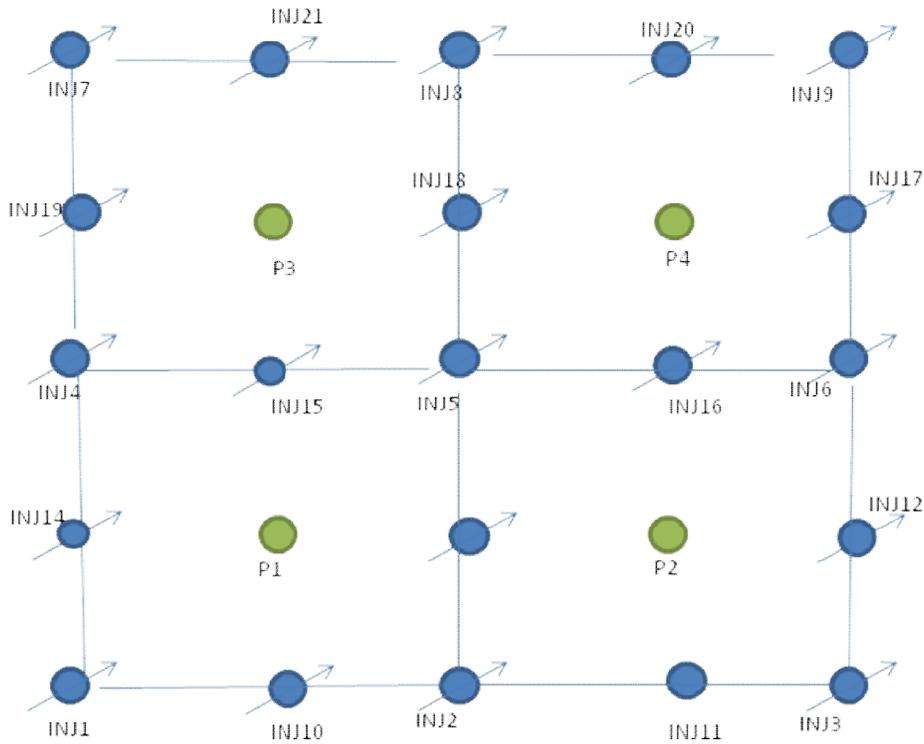


Figure 3.1: Streamline-Based Workflow

The reservoir model will be used to illustrate the basic steps in the injection optimisation workflow. The following should be noted:

- Production for 12 years of water injection is used to determine the start of re-allocation. This was done to monitor the production and injection performance of the waterflood from the beginning of water injection until the average injection efficiency had declined to 40%. Table 3.2 shows the production and injection data after 12 years of water injection—at the start of the re-allocation cycle.

- Our main objective is to increase oil production by re-allocating water injection at defined time steps or time intervals also called reallocation cycles.
- Three re-allocation cycles were considered with duration of 6 months, 12 months, and 18 months.
- At the beginning of the cycle, the WAF and injection efficiency are computed and the total volume of available injection water is re-allocated to the producers. Each injector receives a fraction of available injection water in accordance with its injection efficiency.
- Total water injection is held at 20,000 barrels per day for both types of patterns.
- The oil production rate, and water-cut from the optimised waterflood (i.e., the one with periodic update of WAF and re-allocation schedule) shall be compared with the un-optimised flood to provide guidelines for making reservoir management decisions.

3.2 Determination of Injection Well Efficiency

The criterion used for injection water re-allocation and flood optimisation is termed injection efficiency. This is an instantaneous ratio which shows the rate of oil produced at offset wells to rate of water injection. This is expressed as follows:

$$IE = \frac{\text{offset production, stb/day}}{\text{water injection, stb/day}} \quad 3.1$$

The water injection rate is known while the offset oil production is obtained from the well allocation factors (WAF) obtained from the simulator. This expression is applicable

to well pairs i.e. producer-injector pairs as well as cumulative volume, in this case it is termed an average value.

For the purpose of this work, oil production per well at each time step is calculated as shown in Table 3.2. To obtain the oil produced at the corresponding offset producers to any injector, the well allocation factors obtained shall be used. Figure 3.3 shows the well allocation factors of injector well INJ5 to its corresponding offset producer wells. Figure 3.3 shows how the water injected into injector well INJ5 is distributed to the offset producers; i.e. 17% goes to producer well P3, 27% to P4, 23% to P2 and 33% to P1. Figure 3.3 is called a Flux Pattern map (FPmap, Thiele et al, 2007) with the different colours and thickness of the arrows representing the amount of flux between well pairs. Production Well P1 receiving 37% water from injector INJ5 for instance has the widest arrow and well P3 (receives only 17%) having the arrow with the least width. By obtaining the associated well allocation factors to each injector, oil production at the offset can be obtained as shown in Table 3.3.

The Table 3.3 shows an average injection efficiency of 41%, hence injectors with efficiency below the average are termed “low efficiency” injectors while those above are termed “high efficiency injectors.” Hence, during re-allocation more water will be injected in high efficiency wells than in low efficiency wells.

Table 3.2: Summary of water injection and production rate at the individual wells for 5-spot Pattern at the Start of re-allocation (12 years of water Injection)

Injector	Injection Rate, STB/DAY @start of re-allocation	Producer	Oil Production Rate, STB/DAY
INJ1	1250	P1	2131
INJ2	2500	P2	1902
INJ3	1250	P3	2016
INJ4	2500	P4	2128
INJ5	5000	TOTAL	8177
INJ6	2500		
INJ7	1250		
INJ8	2500		
INJ9	1250		
TOTAL	20,000		

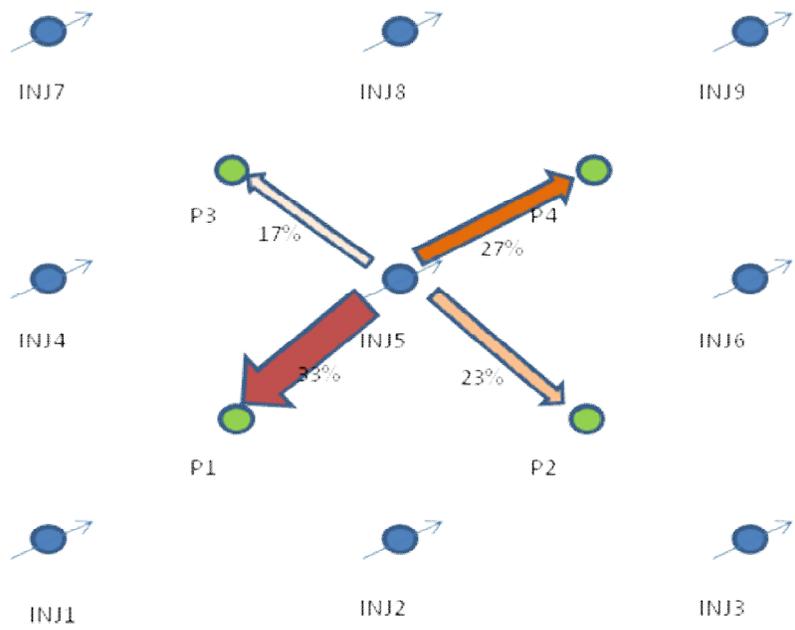


Figure 3.3 Flux Map showing connection between injector-producer well pairs with associated well allocation factors for Wel INJ5.

Table 3.3: Injection Efficiency as computed for individual injection well for 5-spot pattern

INJ. WELL	INJ. RATE, STB/DAY	OIL PRODUCTION RATE,STB/DAY	INJ. WELL IE, %
1	1250	434	35
2	2500	947	38
3	1250	526	42
4	2500	1011	40
5	5000	1990	40
6	2500	1031	41
7	1250	633	51
8	2500	1105	44
9	1250	489	39
	20000	8165	41

A cross-plot of injection rate versus oil production rate is generated to obtain the injection efficiency at any time step. Figure 3.4 shows such plot, with unit slope representing 100% efficiency, the situation whereby one barrel of oil is produced at the offset well for every barrel of water injected. We intend to optimize oil production per barrel of water injected; hence, water will be re-allocated to wells with higher efficiency which will invariably give higher oil production rate with less water cut at their offset producers.

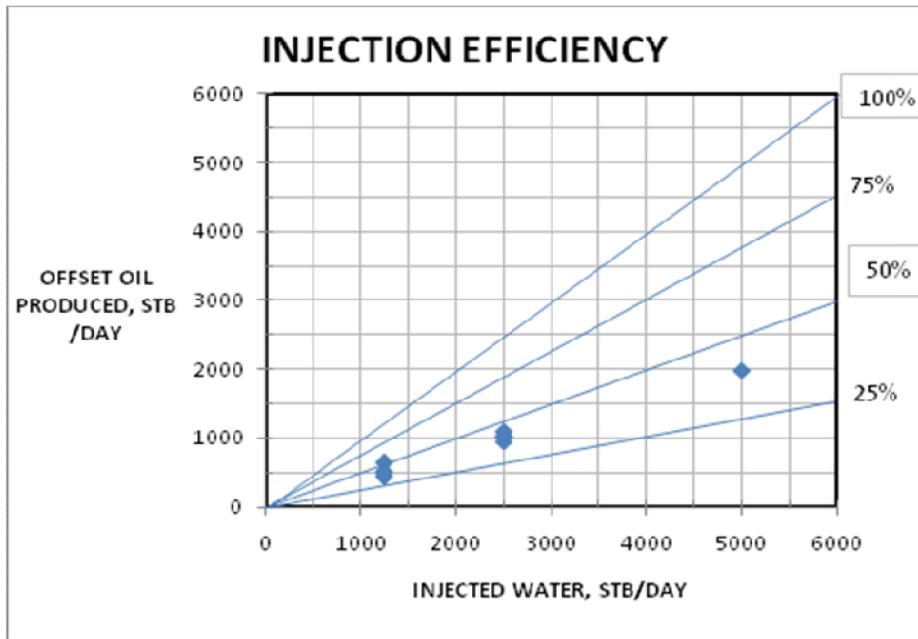


Figure 3.4 Injection Efficiency Plot

3.3 Reallocation of Water injection

Due to limited availability of water and the need to produce more oil from the reservoir, we shall be re-allocating the water to wells with higher efficiency. This discussion is illustrated using the 6-month cycle re-allocation for the 5-spot pattern. From Table 3.3, the wells with the higher efficiency are INJ7, INJ8, INJ3, and INJ6. The strategy is to reduce water injection rates in low efficiency wells and increase injection rates in the more efficient wells. A re-allocation scheme shall be adopted, which uses a weighting average to optimise rate allocation. This is given as follows (Thiele , et al, 2007):

$$e_i > \bar{e}, w_i = \min \left[w_{max}, w_{max} * \left(\frac{e_i - \bar{e}}{e_{max} - \bar{e}} \right)^a \right] \quad 3.2a$$

$$ei < \bar{e}, wi = \max \left[wmin, wmin * \left(\frac{e - ei}{\bar{e} - emin} \right)^a \right] \quad 3.2b$$

where,

e_i , injection efficiency of an injector well

\bar{e} , average injection efficiency

w_i , increase/ decrease weight

w_{min} , minimum weight

w_{max} , maximum weight

a , exponent = 2.0

By implementing the weighting scheme of Equation 3.2, the following new rates shown in Table 3.4 were obtained for the 5-spot pattern using the 6-month interval or 6-month re-cycle re-allocation scheme.

Table 3.4: Re-allocation of Water Injection using 6-month cycle re-allocation scheme for 5-spot reservoir model.

INJ. WELL	next 6months re-allocation			$(1+w_i)*q_i(\text{unconstrained})$	$q_i(\text{constrained})$
	INJ. WELL IE, %	INJ. RATE, STB/DAY	w_i		
1	0.35	1250	-0.0260	1218	1213
2	0.38	2500	-0.0060	2485	2476
3	0.42	1250	0.0012	1252	1247
4	0.40	2500	-0.0001	2500	2491
5	0.40	5000	-0.0007	4996	4979
6	0.41	2500	0.0001	2500	2492
7	0.51	1250	0.0782	1348	1343
8	0.44	2500	0.0094	2523	2515
9	0.39	1250	-0.0020	1248	1243
	0.41	20000		20069	20000

As shown in Table 3.4, it was observed that after re-allocation using the unconstrained rate, $q_i(\text{unconstrained})$, the total amount of water injected exceeded the available volume of water—a material balance error, and so an adjustment was done by introducing the factor r , which is expressed as:

$$r = \frac{\text{maximum water injection rate } \left(\frac{\text{stb}}{\text{day}}\right)}{\text{total unconstrained injection rate (stb/day)}} \quad 3.3$$

The adjustment factor, r , can be considered an injection rate normalization factor to enforce injection water material balance.

As clearly illustrated in Table 3.5, there's an improvement in oil production rate with the reallocation scheme compared with the old injection rate. The average injection well efficiency has increased from 41% to 71%.

3.4 Recurrent Input Data for Changing Re-allocation Cycles in the Simulator

The recurrent input data in the S3D simulator can be changed to vary the injection rates and the length of the re-allocation cycles. The recurrent data includes rate changes in the injector wells as a result of re-allocation of available water. The input data was updated using cycle length of 6months, 12months and 18 months. Note that for each re-allocation cycle, the simulations were run for 5 additional years, i.e., total simulation time of 17 years. The oil production rate and water-cut during the 5-year simulation run for each of the re-allocation cycles were analyzed. This was done to find the optimum schedule for updating the WAF and injection efficiency, and to effect the re-allocation. Several factors need to be considered in the selection of the best time to update the WAF, calculate the Injection efficiency and re-allocate the total volume of injection water available. The optimum re-allocation scheme must agree with project operating guidelines and resources (e.g., manpower and costs; simulation time to calculate the WAF, injection efficiency and rates; and time to execute the project) available for reservoir management.

Table 3.5 Oil Production after 6months optimization, 5-spot pattern

INJ. WELL	INJ. RATE, STB/DAY	OIL PRODUCTION RATE,STB/DAY	INJ. WELL IE, %
INJ1	1213	773.51	63.75
INJ 2	2476	1688.82	68.19
INJ 3	1247	934.75	74.95
INJ 4	2491	1752.81	70.36
INJ 5	4979	3475.48	69.80
INJ 6	2492	1793.05	71.96
INJ 7	1343	1093.80	81.44
INJ 8	2515	1889.94	75.15
INJ 9	1243	794.04	63.87
	20000	14196.18	70.98

3.5 Methodology Chapter Summary

This research has presented a methodology for balancing the amount of injected water distributed to the patterns of injectors to optimize oil recovery in a waterflood. It uses a case study to illustrate the work flow and methodology. Three re-allocation cycles 6-month, 12-month and 18-month duration were analyzed with a view to finding the optimum re-allocation schedule. The results obtained from the water injection optimization study are presented and discussed in the next chapter.

CHAPTER 4.0

RESULTS AND DISCUSSION

This Chapter presents the results of the simulation study of the reservoir models built during the case study. Using the 5-spot and 9-spot pattern reservoir models, flow simulation of a 5-year waterflooding was carried out to monitor the flood performance (oil rate and water cut flow). Waterflood injection rates were varied during the 5-year period by re-allocating more volumes of the injected water to the injectors with the higher efficiency and penalising the low efficient injectors. The injection efficiency was computed from water allocation factors determined at specific time intervals or cycles. Water re-allocation and injection rates were updated using 6months, 12 months and 18months intervals and the performance from the waterfloods observed. The need to re-allocate and recompute injection efficiency arises because the efficiency of the both producer and injector wells is expected to decrease as the oil is being swept from the reservoir. The results obtained from simulations of the two reservoir models (5-spot and 9-spot Pattern) are discussed in the following sections.

4.1 Performance of 5-spot Pattern

4.1.1 Comparison of Injection Efficiency Before and After Water Re-allocation

Figure 4.1 and 4.2 show the injection efficiency (IE) before and after the 6-month re-allocation cycle. A comparison of injection efficiency before and after re-allocation of water (considering the constraint of maximum available water for injection) shows a

sharp increase in the oil production rate and the injection efficiency obtained after the re-allocation.

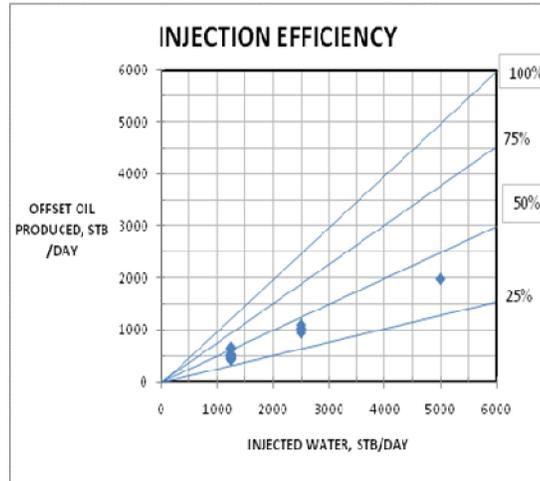


Figure 4.1 Injection Efficiency of 5-spot pattern before optimisation with 6month re-allocation cycles.

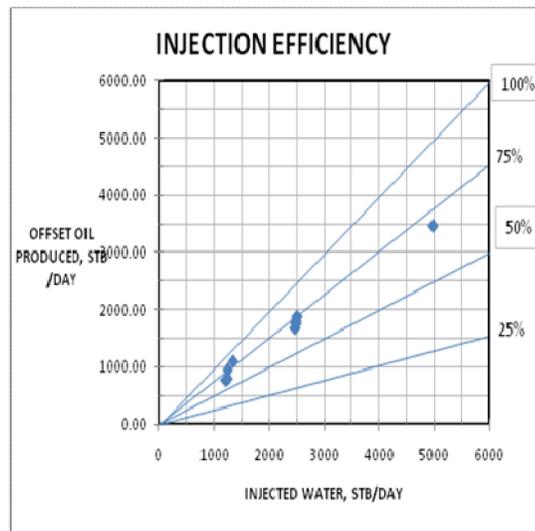


Figure 4.2 Injection Efficiency of 5-spot pattern after optimization using 6month re-allocation cycles.

Table 4.1 and 4.2 summarizes the results of before and after optimization using the 6-month re-allocation cycles.

From the plots of Figures 4.1 and 4.2, we observe that the injection efficiency increased remarkably to an average of 70% from about 41 % before optimization. This can be further seen in Table 4.1 and in Figures 4,3 and 4,4 which show injection well rates, the injection well efficiency and the oil production rates from offset wells before and after the 6-month re-allocation. The data clearly shows the injectors with high IEs as well as those with lower IEs.

Recall, the injection wells are classified as high or low injector after computing the IE.

The term “high” and “low” is in comparison to the average field Injection Efficiency. Hence, it follows that for optimisation, more water shall be injected into the wells with high efficiency than volume injected into the low injectors. As illustrated in Figure 4.5 the injection rate profile of injector well INJ7 with the highest IE value compared to that of well INJ1 (with the lowest IE) shows an increase in rate (Well INJ7 is “rewarded” with more injection water) and decrease in rate for well INJ1 (“penalized” through rate re-allocation) over the 5-year period of simulation with the 6-month cycles of re-allocation. Note that the water injection rate re-allocation scheme was based on the well injection efficiency as a measure of performance. Figures 4.3 through 4.6 show the improvement in field performance—oil rate, and water-cut of offset producers after 6-month injection water re-allocation in the 5-spot pattern.

Table 4.1 Comparison of Injection Efficiency before and after optimization (6-month re-allocation scheme for 5-spot pattern)

INJ. WELL	INJ. RATE, STB/DAY		OIL PRODUCTION RATE, STB/DAY		INJECTION EFFICIENCY, %	
	BEFORE	AFTER	BEFORE	AFTER	BEFORE	AFTER
INJ1	1250.00	1213.30	433.69	773.51	34.70	63.75
INJ2	2500.00	2476.49	947.16	1688.82	37.89	68.19
INJ3	1250.00	1247.24	525.75	934.75	42.06	74.95
INJ4	2500.00	2491.10	1010.55	1752.81	40.42	70.36
INJ5	5000.00	4979.15	1990.13	3475.48	39.80	69.80
INJ6	2500.00	2491.71	1030.76	1793.05	41.23	71.96
INJ7	1250.00	1343.04	632.65	1093.80	50.61	81.44
INJ8	2500.00	2514.76	1105.42	1889.94	44.22	75.15
INJ9	1250.00	1243.20	489.09	794.04	39.13	63.87
	20000.00	20000.00	8165.21	14196.18	40.83	70.98
			74% increase			

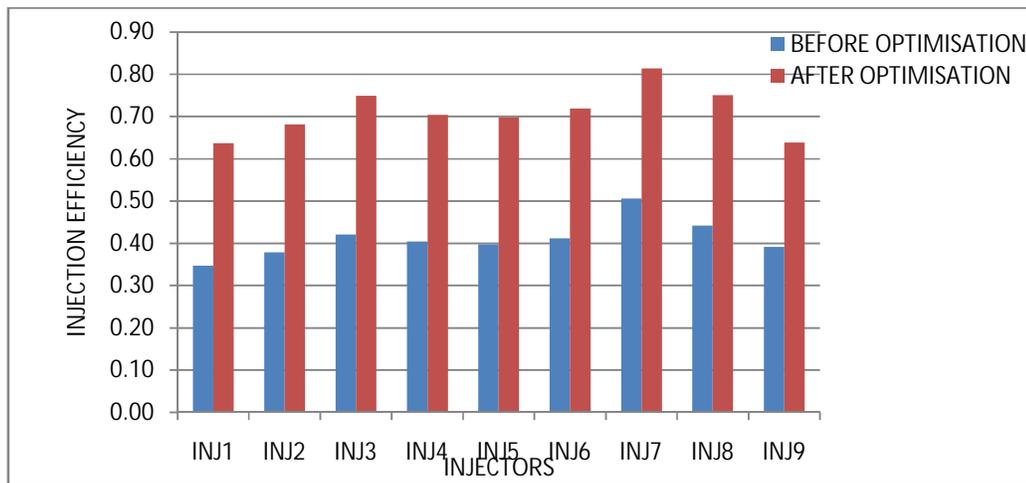


Figure 4.3 Injection Efficiency before and after Optimisation for first 6 months in 5-spot Pattern

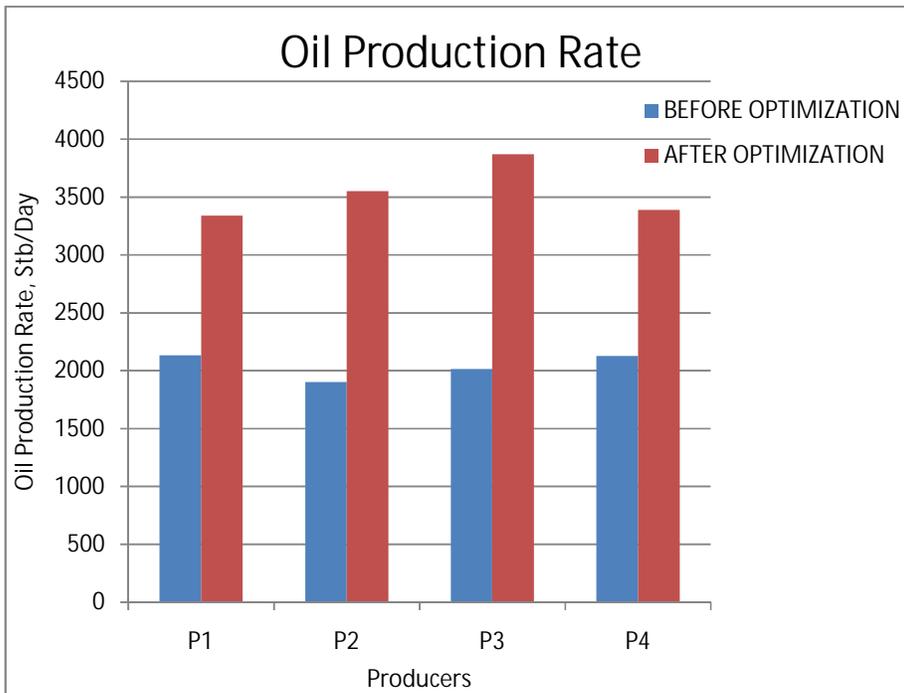


Figure 4.4 Oil Production Rate before and after Optimisation for first 6 months in 5-spot Pattern

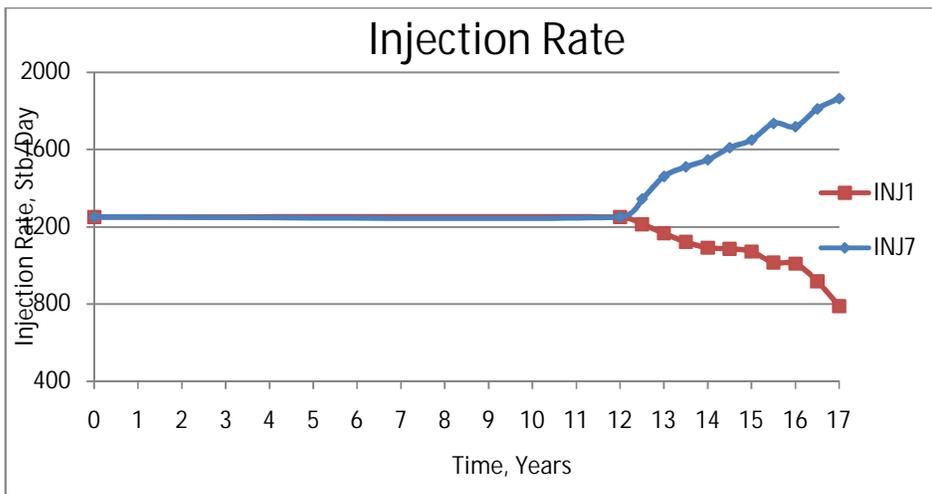


Figure 4.5 Shows increasing and decreasing trend in injection rate for well INJ7 and INJ1 with the highest and lowest injection efficiency respectively for 6-month re-allocation scheme.

4.1.2 Re-allocation in 5-Spot with 6months Cycles

The re-allocation of water for optimal performance of the water injection project was done every 6 months. The results (Figure 4.6) showed a significant increase in oil production rate in the 5-year period compared to the oil rate for the do-nothing scenario. Figure 4.7 is a plot to compare the field water-cut and it shows that the effect of re-allocation is to delay water production. As shown in Figure 4.4, the impacts of injection rate re-allocation on oil production rate for the different producer wells were different; probably as a result of inherent reservoir heterogeneity. Producer P3 showed the best improvement in terms of reduced water cut, hence higher oil production.

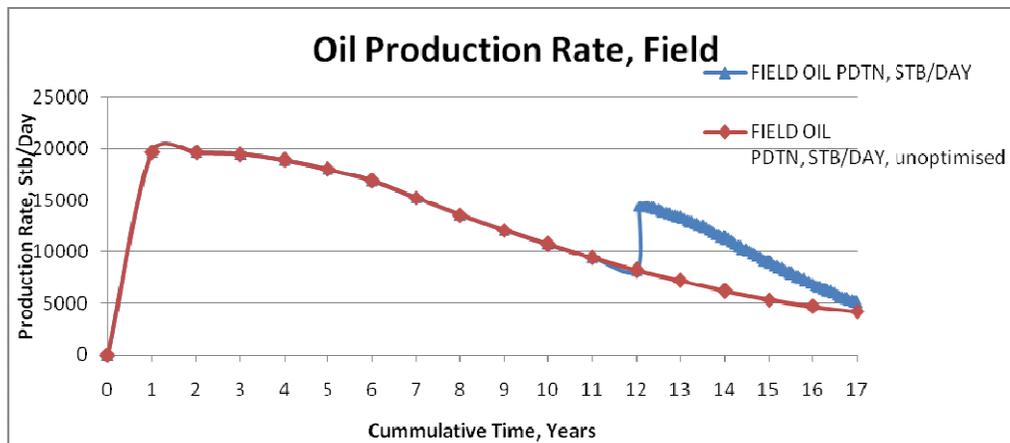


Figure 4.6 Oil Production Rate for 6 months re-allocation scheme, 5-spot

One of the objectives of this study is to find the optimum frequency of injection rate re-allocation in order to maximize oil recovery. To this end, three re-allocation cycles were considered with duration of 6 months, 12 months and 18 months. The results obtained from the simulations were compared with the “do-nothing” –no rate allocation case.

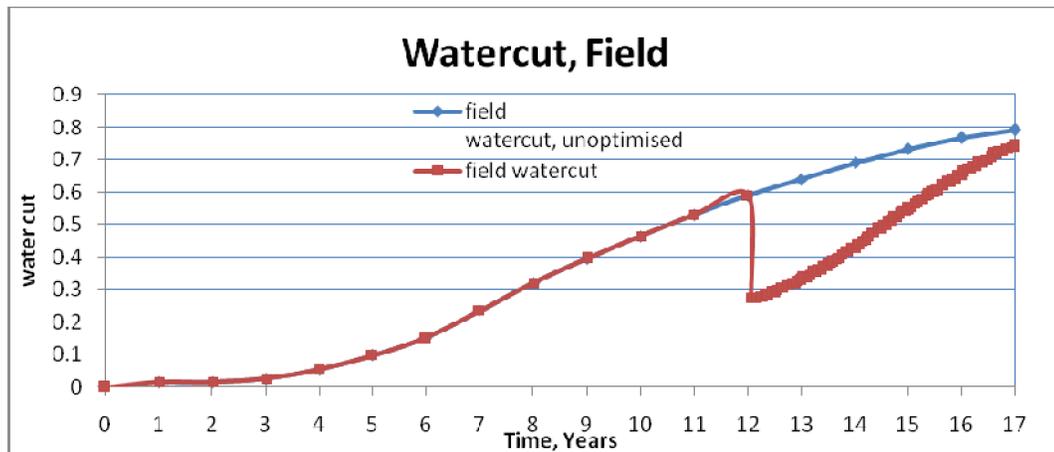


Figure 4.7 Water cut for 6 months re-allocation scheme, 5-spot pattern

4.1.3 Re-allocation in 5-Spot with 12 month Cycles

Further investigation was carried out to understand the impact of optimisation at later periods other than 6 months and the results are analysed. This is also compared with the do-nothing scenario where no optimisation is carried out. The results of the oil production and water cut after the 12-month re-allocation are shown in Figures 4.8 and 4.9. The results and trends are similar to those observed for the 6-month re-allocation cycle. That is, we observe a significant increase in oil production rate and remarkable decrease in water cut, immediately after rate re-allocation; followed by “gradual” decline in oil rate and slight increase in water cut. However, we observe at the end of the simulations that the final water-cut was delayed; it is about 10% lesser than the water-cut from the un-optimised or do-nothing case.

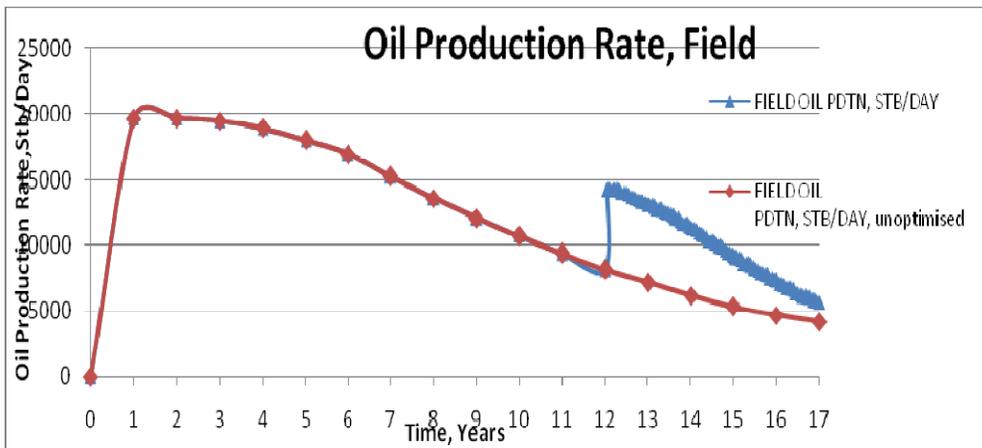


Figure 4.8 Oil Production Rate for 5_Spot with 12months re-allocation scheme

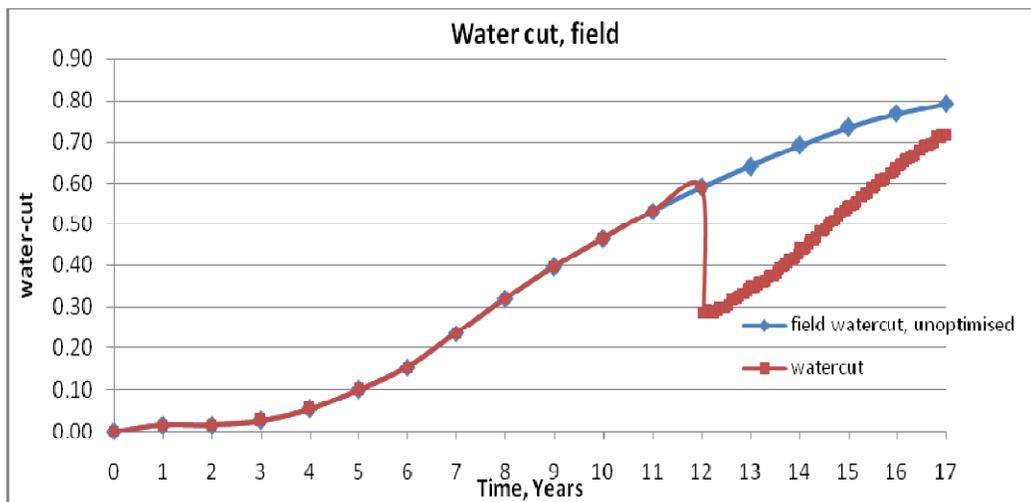


Figure 4.9 Water-cut for 5_Spot with 12months re-allocation scheme

4.1.4 Re-allocation in 5-Spot with 18 month cycles

As was done in the study of 6-month and 12-month cycles, we also investigated the impact of using 18-month re-allocation cycles on reservoir performance. The results shown in Figures 4.11 and 4.12 are similar to those obtained for the 6-month and 12-month re-allocation cycles. The results clearly demonstrate that injection water re-

allocation improves oil production and delays the water –cut when compared to the results from the do-nothing case.

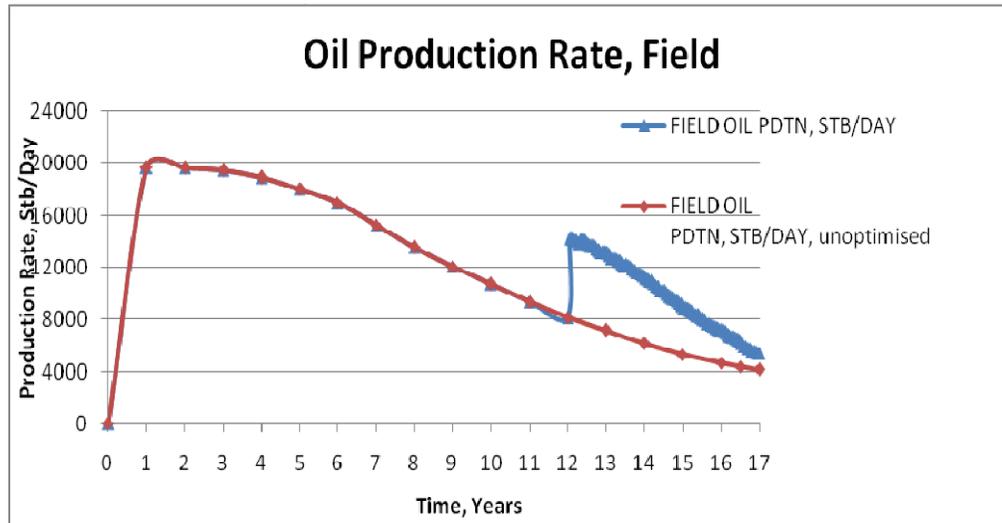


Figure 4.10 Oil Production Rate in 5_Spot with 18 months re-allocation scheme

The lack of clear distinction between 6-month and 12-month; and between 12-month and 18 month long cycles implies that for this case study, the frequency of re-allocation does not affect the results. This observation could be due to any one or combination of the following factors:

- Lack of areal heterogeneities in the model
- Minimum vertical variation in the model
- Completion of the injectors and producers in all the 8 layers of the reservoir.
- Assignment of initial production and injection rates based on pattern geometry.
- Assumption of no well workovers for both producers and injectors during the 5-year period of simulation. In practice it is possible that some of the wells would require remedial action to clean up the wellbore and tubing string. Some infill drilling may also have been carried out within the 5-year period.

- The length of the re-allocation cycles studied may be too short for us to see any difference in well performance. The use of longer cycles (for example, 24-month and 30-month cycles) is recommended for a future study.

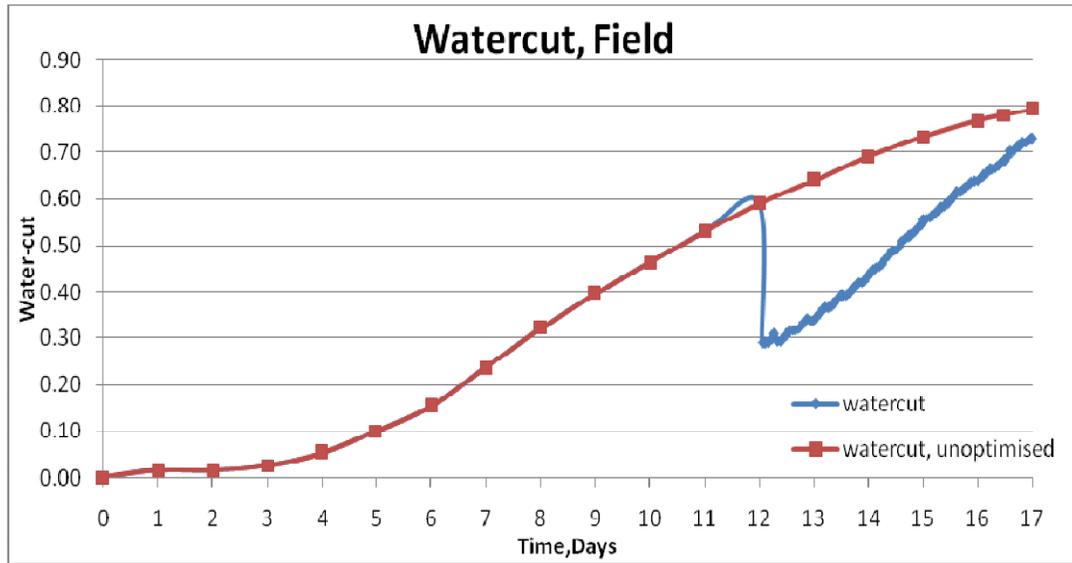


Figure 4.11 Water cut for in 5_Spot with 18 months re-allocation scheme

4.2 Performance of 9-Spot Pattern

The case study was repeated for the 9-Spot reservoir model. The 9-Spot model contains 21 injectors and 4 producers. The results of the simulations are similar to those obtained from the 5-Spot pattern, Additional discussion of the results from the 9-spot reservoir model are presented below.

4.2.1 Injection Efficiency

A comparison of injection efficiency before and after re-allocation of water using the 6-month cycles is shown in Figures 4,12 and 4,13. These plots show an improvement in injection efficiency; minimum efficiency is above 50% compared to the unoptimised scenario where minimum efficiency is about 25%.

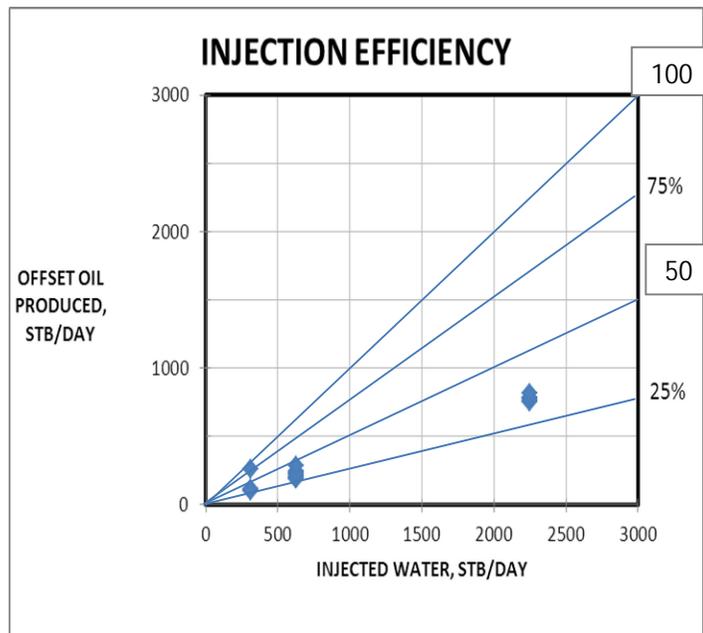


Figure 4.12 IE before optimisation in 6months, 9-spot pattern

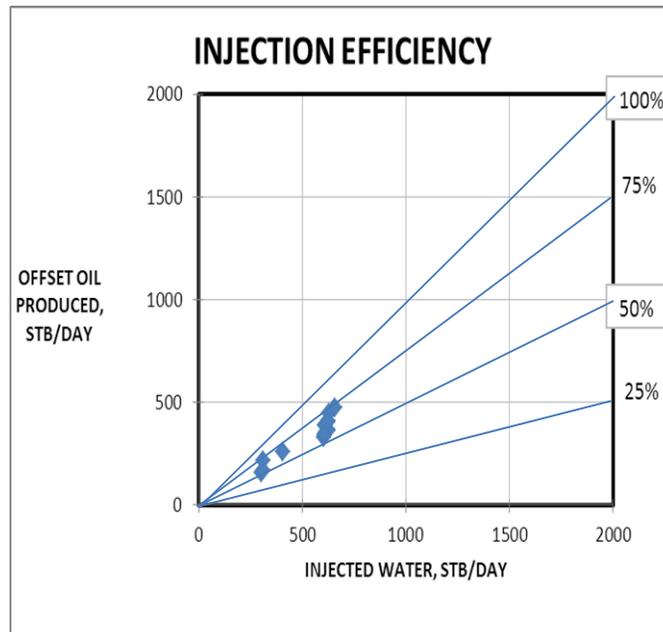


Figure 4.13 IE after optimisation 6months,9-spot pattern

Figure 4.14 shows the injection efficiency before and after for the 9-Spot reservoir model during the first 6 months of re-allocation. It can be concluded that injector well INJ21 is the most efficient injector and injector well INJ1 remains the least efficient. As can be seen from Figure 4.15, over the period of evaluation this injector will be allocated more water than other injectors and injector well INJ1 with least amount of water in order to maximize oil recovery. Injector well INJ1 might be a good candidate to be shut-in over time, as the injection profile shows that its contribution to the flood has declined rapidly to less than 50 STB/Day at the end of the 5-year evaluation period.

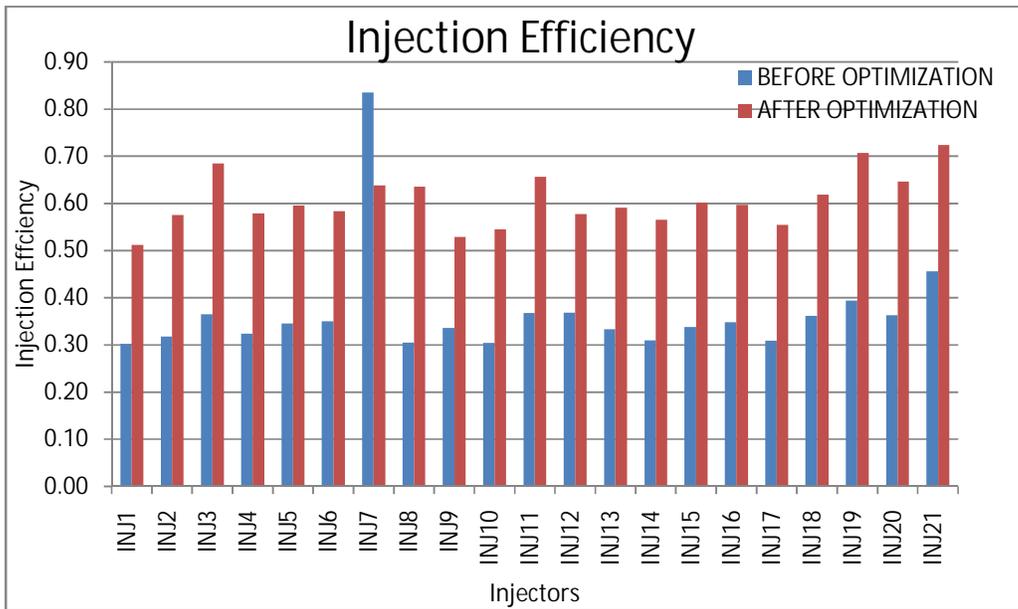


Figure 4.14 Injection Efficiency before and after Optimisation for first 6months in 9-spot Pattern

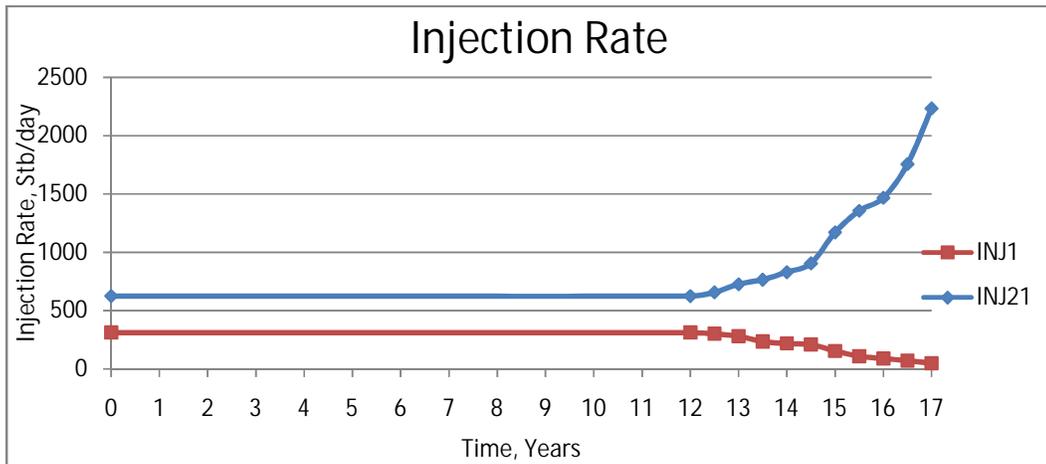


Figure 4.15 Shows increasing and decreasing trend in injection rate for well INJ21 and INJ1 with the highest and lowest injection efficiency respectively for 6months re-allocation scheme in 9spot pattern

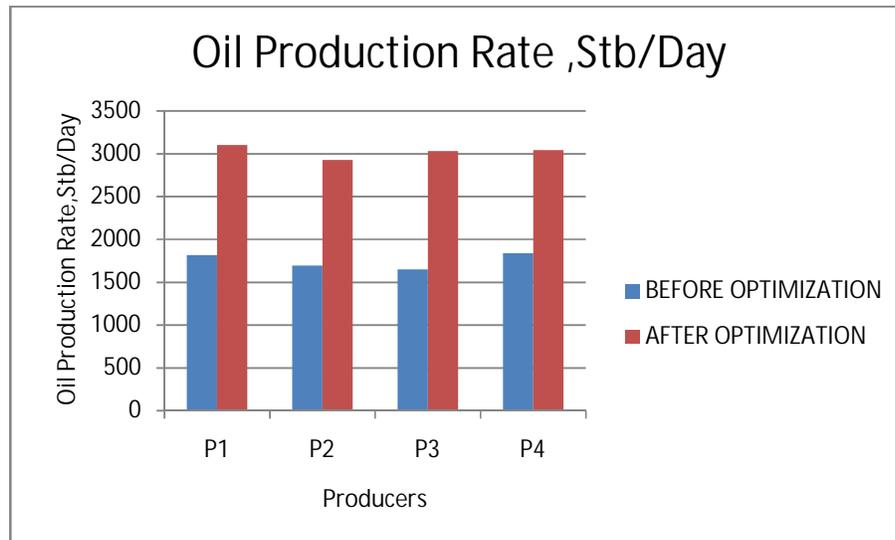


Figure 4.16 Oil Production Rate before and after Optimisation for first 6months in 9-spot Pattern

4.2.2. Oil Production Rate and Water Cut in the 9-Spot Pattern

Figures 4.16 through 4.18 show the trends of the oil production versus time, water-cut versus time obtained from the 5-year simulations of the 9-Spot reservoir model using a 6-month re-allocation. The results are similar to those obtained from the 5-Spot model; that is, the data show a significant increase in oil production rate and a delayed water-cut during the 5-year evaluation period compared to the results from the do-nothing scenario. As was done in the case of the 5-Spot reservoir model, we also analyzed the trends of the oil production rate versus time, and water-cut versus time using the 12-month and 18-month re-allocation cycles. It was found that the results and trends for the 12-month and 18-month cycles (shown in the Appendix) were similar to those from the 6-month re-allocation cycles.

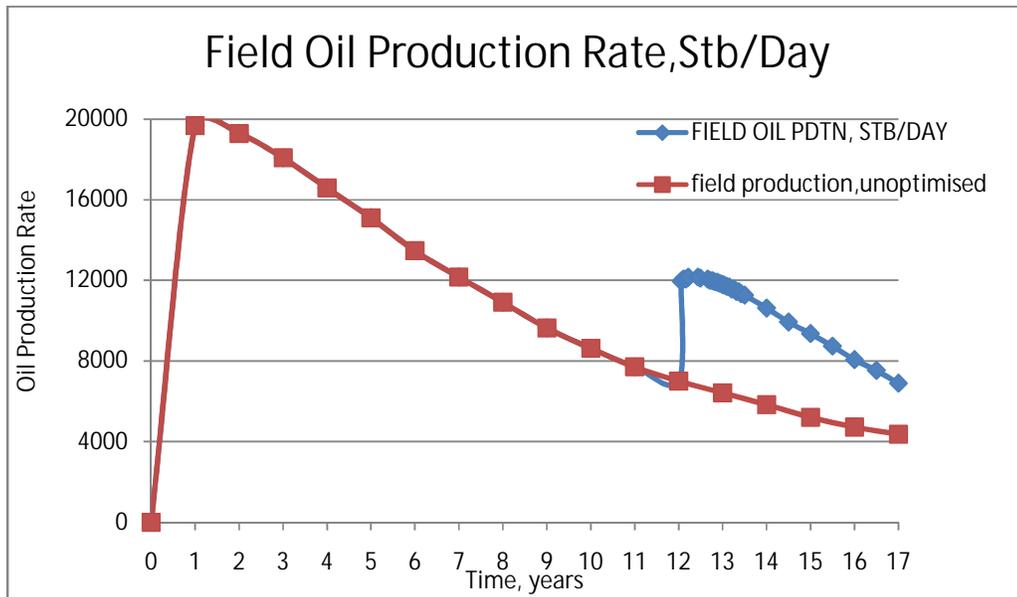


Figure 4.18 Oil Production Rate for 6months re-allocation scheme in 9-spot pattern

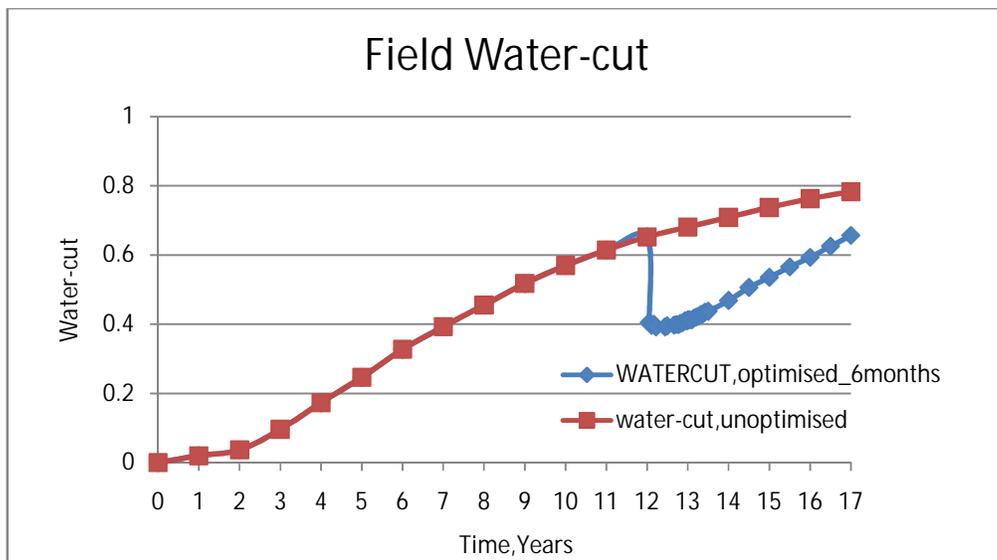


Figure 4.19 Water cut for 6 months re-allocation scheme, 9-spot Pattern

4.3 Pattern Selection and Frequency of Injection Water Re-Allocation

One of the objectives of this study is to determine the frequency for calculating the injection efficiency required for re-distribution of the total volume of injection water to the injectors in order to maximise oil recovery from the waterflood. To this end, we considered three re-allocation cycles of 6 months, 12 months and 18 months duration and analyzed the waterflood performance during the 5-year evaluation period. Figures 4.19 and 4.20 shows the plots of the oil production rate versus time for 5-Spot and 9-Spot models considering the three injection water re-allocation cycles. The results for the water-cut are shown in Figures 4.21 and 4.22. The result clearly demonstrates the following:

- (1) Re-allocation of injection water is a viable way to optimize oil recovery in a waterflood because it results in increased oil production rate and a delay in the water-cut compared to the do-nothing (un-optimized) case.
- (2) The performance trends for the 5-Spot pattern are similar to those from the 9-Spot pattern regardless of the re-allocation cycles considered in this work.
- (3) With the same injection rate constraint, results shows that in the same 5-year period of forecast, water-cut from 9-spot was still as low as 0.6 field wide as against 5-spot which gave water-cut of 0.75, almost matching the unoptimised scenario at the end of the period.
- (4) The 9-spot showed more delay in water cut after the 5 years of flood evaluation than the 5-spot. Compare the results from Figures 4.21 and 4.22. It is observed from Figure 4.21 that the trend of water-cut versus time from the 5-spot model shows a rapid decline from 0.6 to 0.3 immediately following the re-allocation (at about 12 years); and then it

increases to 0.75 at the end of the 5 years. Figure 4.22 shows that for the 9-spot pattern, the water cut declined from 0.65 to 0.4 immediately following the re-allocation before increasing to 0.65 after 5 years.

- (5) The 5-Spot pattern tends to give a higher initial post re-allocation oil production rate (oil production rate observed immediately after the carrying out the injection water re-allocation at about 12.1 years of water injection in Figures 4.19 and 4.20) than the 9-spot pattern. As shown in Figures 4.19 and 4.20 the initial post re-allocation oil rate is 15000 STB/DAY for the 5-spot compared to the 12500 STB/DAY for the 9-Spot pattern.

Pertinent Remarks.

It should be noted that significant improvement was observed in oil production rate for 5-spot at the initial stage of re-allocation in comparison to 9-spot pattern. We did not investigate the reasons for this difference in oil rate production in this study. However, it is possible that the slightly lower oil production rate obtained from the 9-spot may be due to interference from the additional injectors in the 9-spot model

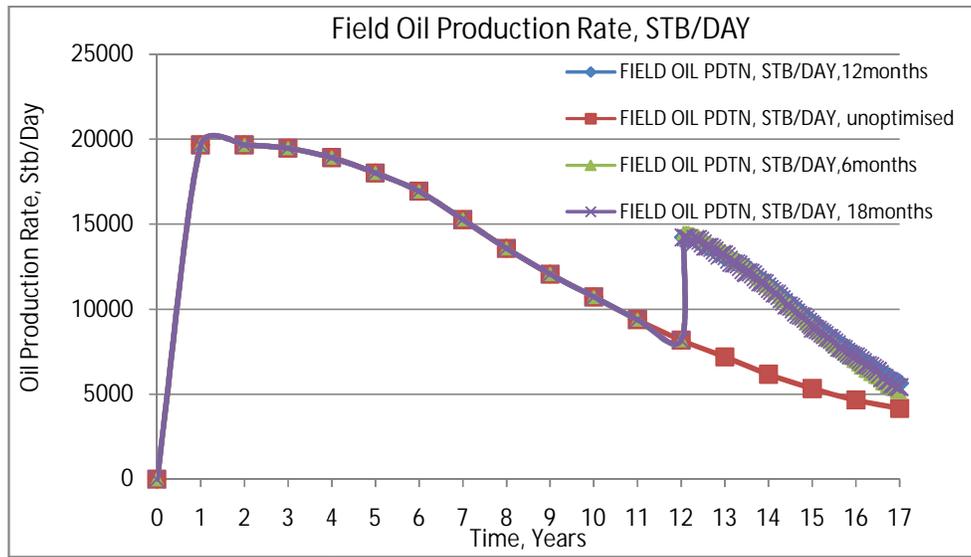


Figure 4.19 Comparison of Oil Production Rate for 6, 12 and 18 months re-allocation scheme, 5-spots Pattern

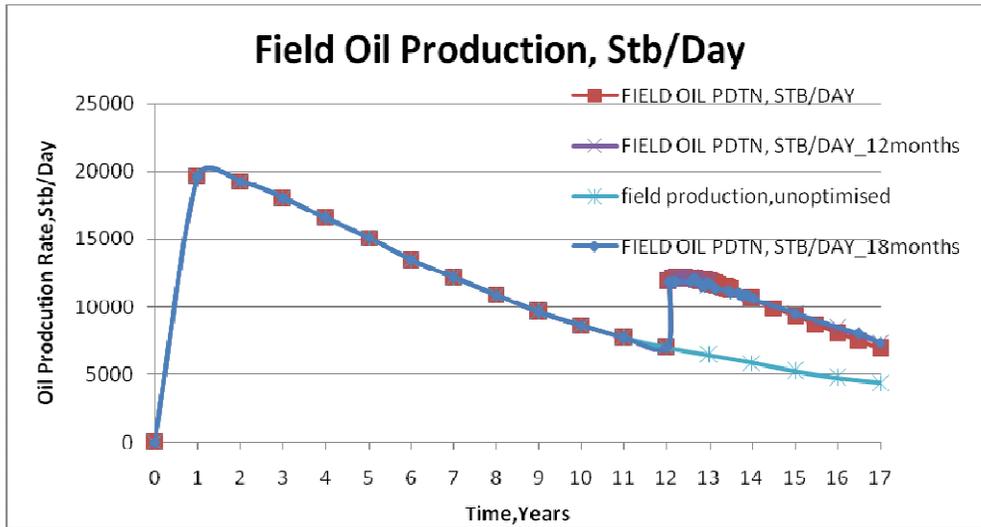


Figure 4.20 Comparison of Oil Production Rate for 6, 12 and 18 months re-allocation scheme, 9-spot Pattern

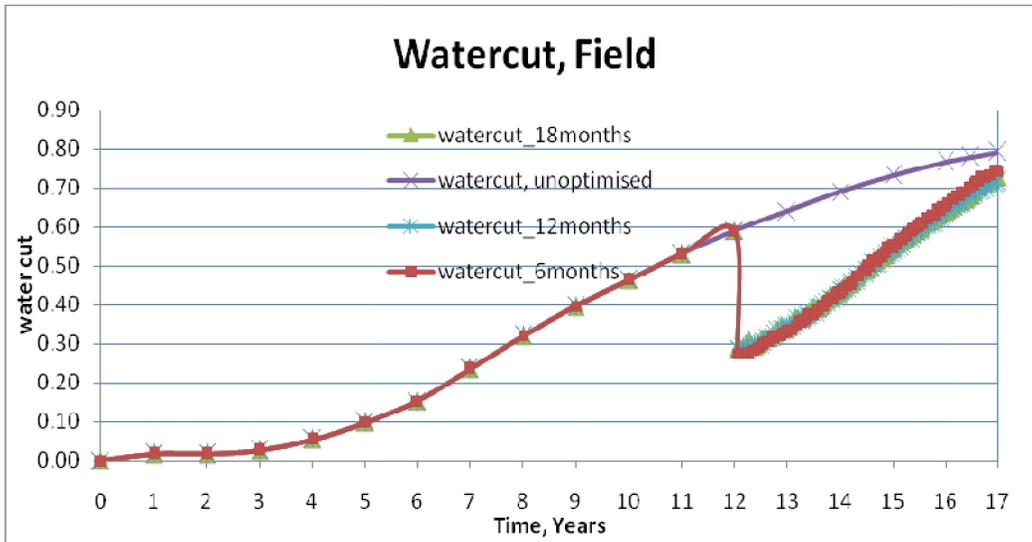


Figure 4.21 Field Water-cut for all re-allocation schemes in 5-spot pattern

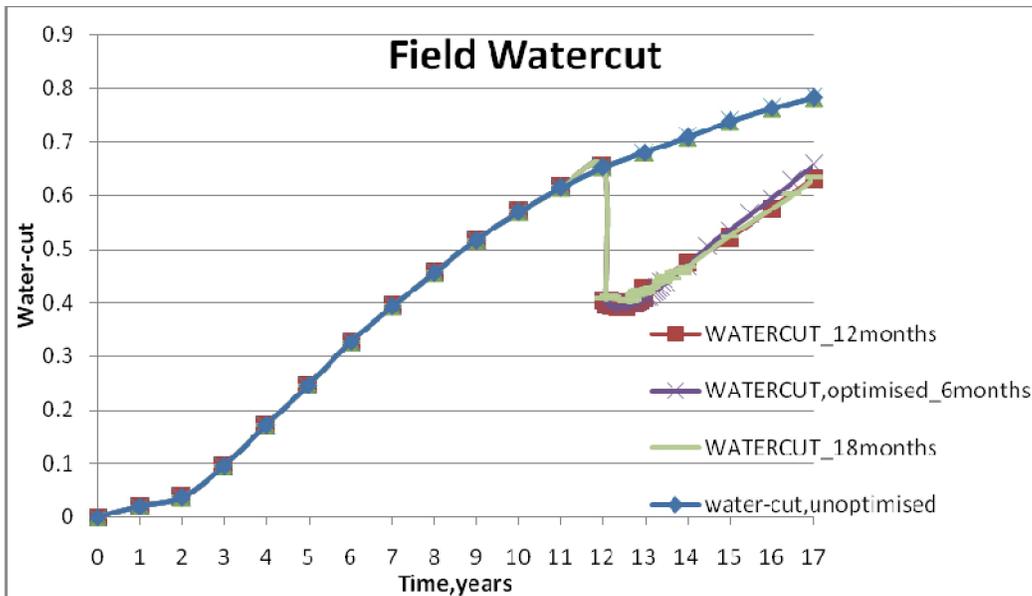


Figure 4.22 Field Water-cut for all re-allocation schemes in 9-spot pattern

4.4 Evaluation of Injector Performance

The performance of the injector wells can also be evaluated based on their individual injection efficiency. While certain wells in a particular pattern were observed to have very high performance depending on the degree of heterogeneity of rock and fluid properties at their individual well locations, other injectors showed poor performance. To this end, some qualitative reservoir management decisions can be made based on the injection efficiency of these wells. Overtime some of these low injectivity wells might be good candidates for shut-in if their contribution to the field-wide performance is insignificant. An example of an excellent well in the 5-spot pattern is injector well INJ7 (Figure 4.23) whose efficiency is quite high compared to the average field efficiency; whereas well INJ1 in the same pattern has relatively low efficiency--poor injection performance; such an injector (well INJ1) could be a candidate for shut-in at some time during the waterflood. It is noted that in practice several reservoir management conditions must be considered before deciding whether or not a well should be shut-in. These may include, determination of the extent of the wellbore formation damage from a well test; determination of injectivity and layer-contributions from production logging; carrying out a workover or some remedial treatment (e.g., acid clean-up, wax removal, recompletion and re-perforation to other layers, fracturing, etc) to restore injectivity or productivity of the well .

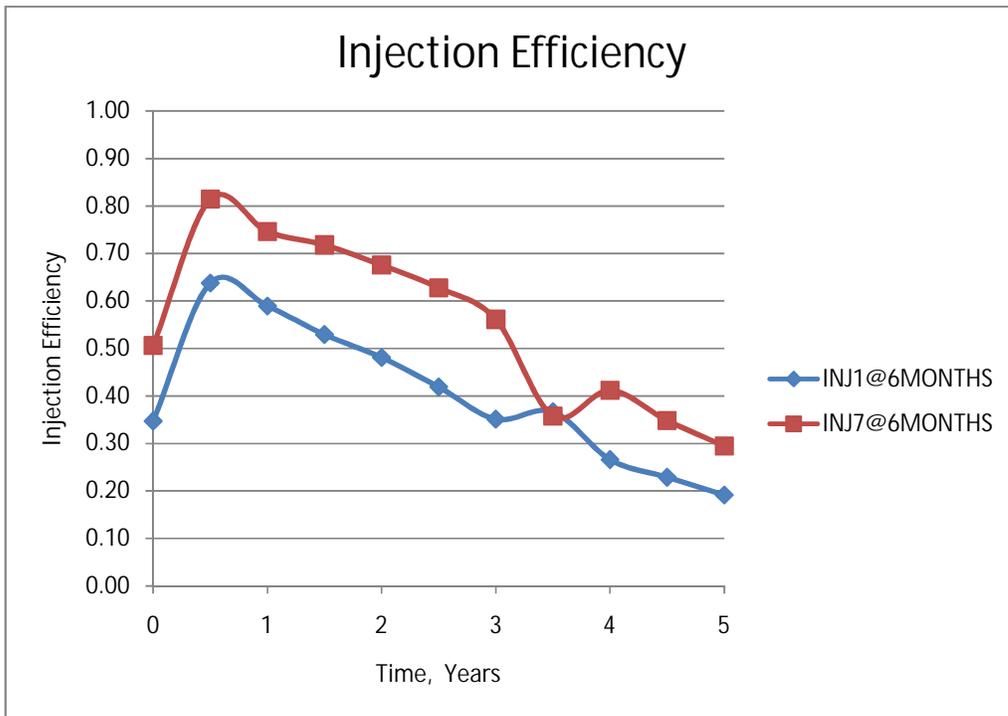


Figure 4.23: Comparison of Injection Efficiency for INJ7 and INJ1 in 5-spot pattern for 6months re-allocation cycle during the 5year re-allocation period

From the results of this study, we can conclude that injection efficiency as a criterion is a viable tool for field performance optimization. It can also be applicable to producers on the field; in that case we would consider well production efficiency. Streamline simulation offers a quick and effective way of obtaining IEs from well allocation factors.

CHAPTER 5.0

CONCLUSIONS AND RECOMMENDATIONS

This Chapter presents the various conclusions arrived at during the study designed to optimize the performance of a field from a streamline based workflow.

Recommendations shall be given to point out areas of further research to expand the scope of work and to improve the proposed methodology.

5.1 CONCLUSIONS

1. Streamline simulation has been used as an efficient and fast tool for obtaining Well allocation factors, which was used to obtain Injection efficiency.
2. The use of injection efficiency as a tool for re-allocating water injection rate was successfully demonstrated in case study using streamline-based simulation technique.
3. The case study demonstrated that the flood injection efficiency can be improved using the rate re-allocation schemes investigated in this work. For example, case study of the 5-spot model showed an average field-wide injection efficiency increasing from 41% before re-allocation to about 71% after optimisation.
4. By increasing injection rate of more efficient injectors, it is possible to increase oil production rate without increasing the total volume of injection water available for the project.
5. There was no significant difference in the oil production rate and water-cut for a given pattern in the case study when the duration of the re-allocation cycle was varied from 6 months to 12 month and to 18 months. This lack of difference may be attributed to

several factors including: lack of areal heterogeneity in the reservoir models; delayed time to initiate the re-allocation program, which was 12 years in the case study; and the assumption of no injection well or production well work over during the 5-year period of waterflood performance evaluation.

6. Re-allocation of water injection rate allows for delayed water production hence reduced water cycling in the waterflood management scheme.

7. Screening of wells for shut-in and other reservoir management decisions such as selecting injection well workover candidates exhibiting poor injectivity can be easily made with the injection efficiency data derived from the streamline simulation methodology proposed in this study.

5.2 RECOMMENDATIONS

The following recommendations are presented for further studies to improve the methodology and results discussed in this work,

1. More robust software with a more sophisticated user interface and computational engine should be used in a future study. It will reduce the time for computations, time for data analysis and for pre- and post-processing of results. Application of robust software with the proposed methodology will allow running fine-grid simulations to account for reservoir heterogeneity and estimation of uncertainty. It will also allow evaluating more re-allocation cycles to better understand the relationship between reservoir performance (oil production rate and water-cut) and the frequency of injection rate re-allocation. The advantages of streamline simulation can be better appreciated with the more robust tools.

2. This study considered both the injection and production wells as fully completed, with perforations in all 8 reservoir layers. Variable layer completions should be considered in a future study in order to identify the best zones for completion to minimize water production.

3. The model can be applied to other pattern types both regular and irregular to gain knowledge to support effective reservoir management decisions.

The time of intervention of this optimization tool in any known reservoir is critical in order to obtain best results. In this analysis the time to initiate rate re-allocation was 12 years. It is possible that by starting the optimisation earlier, the water-cut will be delayed and oil recovery maximised. The determination of the time to start the rate re-allocation is suggested as topic for future research.

NOMENCLATURE

K = Absolute Permeability

K_{rp} = relative permeability

P_p = phase pressure[M/(LT²)]

P_c = Capillary Pressure

Q_p = Injection/Production rate [L³/T]

S_w = Water Saturation

S_p = Phase saturation

IE = Injection Efficiency

e_i = Injection efficiency of an injector well

w_i = increase/ decrease weight

w_{min} =minimum weight

w_{max} =maximum weight

a = exponent 2.0

Greek Symbols

μ_p = phase mobility

μ_t = total mobility

μ_p = phase viscosity

ρ_p = phase density

t_{of} = time of flight

= porosity

= Bi-streamfunction[L]

= Bi-streamfunction [L₂/T]

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APPENDIX A – Mathematical Formulation of Streamline Simulation Software in Chapter 2

A1. Method of Streamline Simulation

The streamline simulation technique is governed by the IMPES method. By ignoring capillary and dispersion effects, the governing equation for pressure for incompressible, multiphase flow is given by Batycky (1997)

$$\Phi \frac{\partial s_p}{\partial t} + \nabla \cdot \vec{V} p = Q_p \quad p = \text{water, oil}, \quad (1)$$

$$\text{Where } \vec{V} = -k \frac{K_{rp}}{\mu p} \nabla (P_p + \rho_p g z) \quad p = \text{water, oil} \quad (2)$$

It should be noted that current streamline simulation includes compressibility (both fluid and rock) and multicomponent multiphase flow. However the essential nature of streamline based simulation can be discussed based on the above equation.

The phase saturation sum to unity, $S_w + S_o = 1$ and capillary pressure is conventionally defined with water as the wetting phase; $P_c = P_o - P_w$. The porosity and absolute permeability are in general, position dependent. The relative permeabilities and capillary pressure functions may also depend upon position, but are primarily functions of water saturation, the total mobilities, $\lambda = \lambda_w + \lambda_o$, the total source and sink term $Q = Q_w + Q_o$ and the fractional water flow $F_w = \lambda_w / \lambda$.

The streamline formulation is based on the total fluid velocity and the conservation equation for let's say the water phase:

$$\nabla \cdot \vec{V} = Q \quad (3)$$

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot \left\{ \frac{\lambda_w}{\lambda} \bar{V} + \frac{\lambda_w \lambda_D}{\lambda} \nabla (P_c + \Delta \rho g z) \right\} = Q_w \quad (4)$$

In the simplest streamline calculations, the effect of capillary and gravity are neglected, so that leading to the multidimensional non-linear convection equation away from the source terms:

$$\nabla \cdot \bar{V} = 0 \quad (5)$$

$$\phi \frac{\partial s_g}{\partial t} + \bar{V} \cdot \nabla F_w = 0 \quad (6)$$

A2. Co-ordinate Transformation

At a fundamental level all streamline techniques are based on a co-ordinate transformation from physical space to a coordinate system following the flow directions. This transformation is based upon the bi-streamfunctions and an additional time of flight coordinate. Introducing the bi-streamfunctions to construct an irrotational velocity field, we have:

$$\bar{V} = \nabla \Psi * \nabla \chi \quad (7)$$

A streamline is defined by the intersection of a constant value for Ψ with a constant value for χ . In two dimensional applications, the simplified functional forms, $\Psi = \Psi(x, y)$, $\chi = z$, leading to the more familiar expressions where Ψ is recognized to be the streamfunction.

The time of flight co-ordinate is then introduced to measure distance along a streamline.

$$\frac{d\tau}{ds} = \Phi \frac{\vec{V}}{|\vec{V}|^2} \quad (8)$$

For steady velocities, $\tau(x,y,z)$ can be defined by integrating the equation above from a position (x,y,z) within the domain back along the streamline, until the source for the flow is reached (either a well or a flow boundary)³⁰:

$$\tau(x,y,z) = \int_{\text{inlet}}^{(x,y,z)} \frac{ds}{|\vec{V}|} \quad (9)$$

Physically, $\tau(x,y,z)$ is the time at which a neutral tracer would appear at (x,y,z) , it provides important information about heterogeneity of the reservoir. For instance, the incompressibility of the flow is important as it ensures that streamlines will only begin and terminate at wells, not within the three dimensional domain. In this integral, s is the distance along the streamline and $|\vec{V}|$ is the local speed.

The coordinate transformation from (x,y,z) to (τ, Ψ, χ) has several important properties. The first is pore volume which shows a relationship between the physical space and time of flight coordinates following the flow direction^{20,25}:

$$\Phi dx dy dz = d\tau d\Psi d\chi \quad (10)$$

It follows that from evaluating the Jacobian of the co-ordinate transformation ^{20,25},

$$\left\| \frac{\delta(\tau, \Psi, \chi)}{\delta(x, y, z)} \right\| = \nabla_{\tau}(\nabla_{\Psi} * \nabla_{\chi}) = \nabla_{\tau} \vec{V} = \Phi$$

The second property is based on re-expressing the gradient in these co-ordinates. Spatial gradients along streamlines is transformed in the time of flight coordinates in the (τ, Ψ, χ) :

$$\vec{v} = \nabla \tau \frac{\partial}{\partial \tau} + \nabla \Psi \frac{\partial}{\partial \Psi} + \frac{\partial}{\partial x} \quad (11)$$

and recognizing that equation (6) now appears in a one-dimensional form:

$$\frac{\partial S_w}{\partial t} + \frac{\partial F_w}{\partial \tau} = 0$$

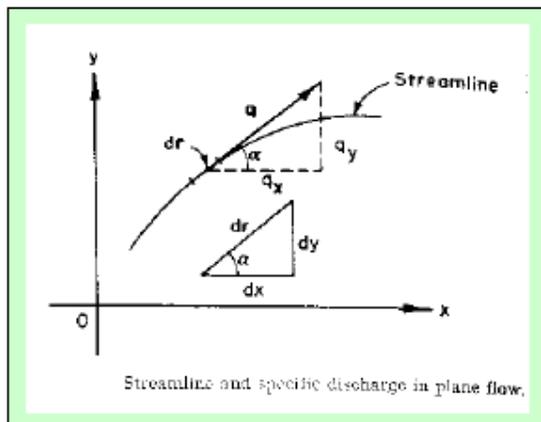
Equation 12 below shows that these coordinates are identical to a transformation to characteristic variables.

From the derivation above, for a simple case of an incompressible waterflood it is thus possible to write as follows, the most important detail of the equation is the fact that the total velocity in the 3D problem has disappeared into the TOF of each individual streamline i.e. the decoupling of a 3D heterogeneous system into a series of 1D homogenous system in terms of time of flight, it is for this reason that streamline method is so attractive

$$\frac{\partial S_w}{\partial t} + \vec{v} \cdot \nabla F_w = 0 = \sum_{\text{streamlines}}^{\text{all}} \left(\frac{\partial S_w}{\partial \tau} \right) \quad (12)$$

Where \vec{v} is orthogonal to $\nabla \tau$ and $\nabla \Psi$ then, expanding $\vec{v} \cdot \nabla = \Phi \frac{\partial}{\partial \tau}$ and transforming using the τ coordinate in equation (6), the result is equation (12). The variable although measured in units of time, acts as a spatial co-ordinate. In this form, the solution of the multidimensional flow equation, may be represented as a sum of one-dimensional solutions along each streamline.

The transformation to τ coordinate resulted in decomposing the three dimensional fluid flow into a series of one dimensional evolution equation for Sw along streamlines, and for homogeneous and heterogeneous media. The τ coordinate incorporates all these effects. The only requirement is the velocity field and the calculation of the line integral in equation 1. The visual representation of a one dimensional stream line solution of the line integral is simply represented as follows. The streamline are represented as instantaneous curves that are at every point tangent to the direction of the fluid velocity.



FigureA1: Streamline and Specific discharge in plane flow(Ogbe, 2003)

The 1D streamline function could be solved numerically or analytically as shown above. Streamline simulations requires a new way of thinking with a different approach to conventional Simulation. The fluid moves along streamlines and not grid cells. The figure below shows the movement of fluid between two wells along streamlines contrary to the general belief in numerical simulation where fluids are confined to grid cells faces with numerical diffusion since it gives an approximation of the result.

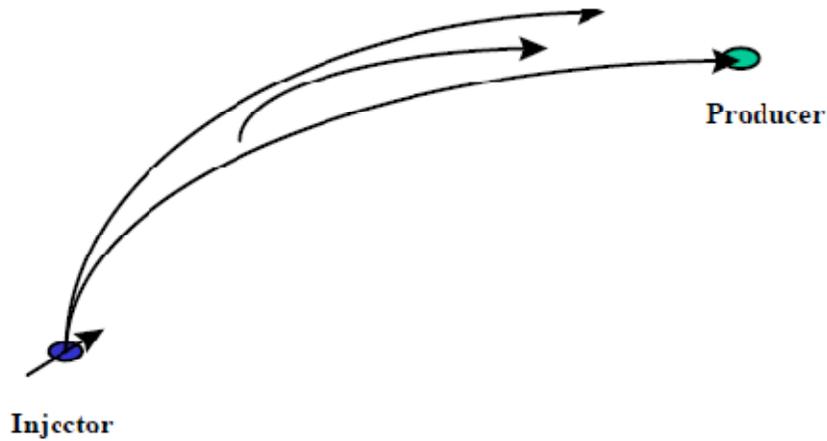


Figure A2: Streamlines Models have discrete flow paths(culled from Grinestaff 1999)

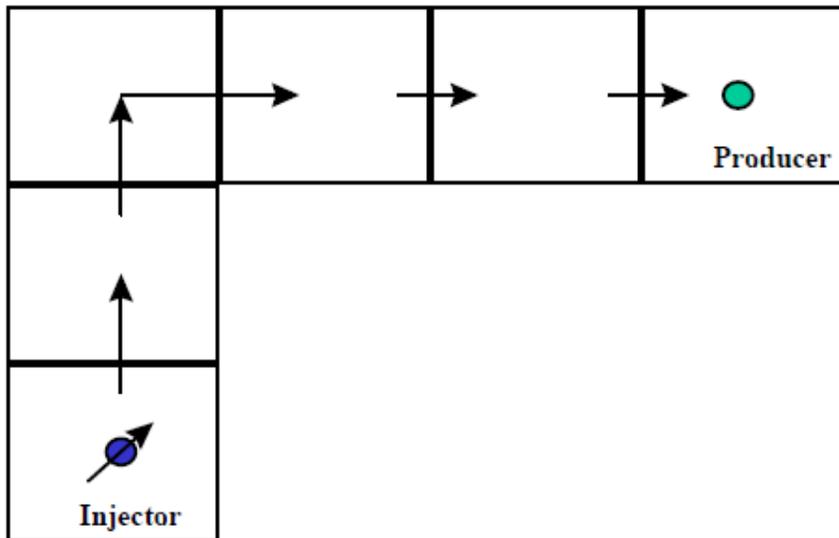


Figure A3 Finite Difference Models use grid cells (culled from Grinestaff 1999)

Decoupling the fluid transport from the underlying grid obtainable in 3D numerical simulation makes the process more computationally efficient and very large time steps can be taken without loss in solution accuracy.(Batycky 1997). This is illustrated in figure 2.5a and 2.5b above. The simulation method also allows for updating of streamline

paths to account for changing mobility field with time. Batycky et al(1997) applied this in House Mountain waterflood project in central Alberta.

APPENDIX B: Additional Tables and Figures for Chapter 4

Table B1: Summary of water injection and production rate at the individual wells for 9spot Pattern prior to re-allocation

INJ. WELL	INJ. RATE, STB/DAY	PRODUCERS	PROD. RATE AT PRODUCERS,STB/DAY
INJ1	312.5	P1	1814.85
INJ2	625	P2	1695.66
INJ3	312.5	P3	1651.19
INJ4	625	P4	1841.85
INJ5	2250		
INJ6	625		
INJ7	312.5		
INJ8	625		
INJ9	312.5		
INJ10	625		
INJ11	625		
INJ12	625		
INJ13	2250		
INJ14	625		
INJ15	2250		
INJ16	2250		
INJ17	625		
INJ18	2250		
INJ19	625		
INJ20	625		
INJ21	625		
	20000		7003.55

Table B2: Summary of oil production rate at the individual wells for 9spot Pattern prior to re-allocation

INJ. WELL	INJ. RATE, STB/DAY		OIL PDTN RATE,STB/DAY		INJ. WELL IE, %	
	BEFORE	AFTER	BEFORE	AFTER	BEFORE	AFTER
INJ1	313	302	95	155	30	51
INJ2	625	614	198	353	32	58
INJ3	313	312	114	214	36	68
INJ4	625	618	203	358	32	58
INJ5	2250	2246	778	1338	35	60
INJ6	625	624	219	364	35	58
INJ7	313	406	261	259	83	64
INJ8	625	606	191	385	30	64
INJ9	313	311	105	165	34	53
INJ10	625	606	190	330	30	55
INJ11	625	625	230	410	37	66
INJ12	625	625	230	361	37	58
INJ13	2250	2236	749	1321	33	59
INJ14	625	609	194	344	31	57
INJ15	2250	2242	761	1350	34	60
INJ16	2250	2247	783	1343	35	60
INJ17	625	609	193	338	31	55
INJ18	2250	2249	814	1391	36	62
INJ19	625	630	246	445	39	71
INJ20	625	625	227	404	36	65
INJ21	625	657	285	475	46	72
	20000	20000	7065	12103	35	61

Table B3: Summary of water injection rate at the individual wells for 9spot Pattern for 17years; 6months re-allocation scheme

INJECTION RATE,STB/DAY,9SPOTS PATTERN FOR 17YEARS

Time,Days	0	12	12.5	13	13.5	14	14.5	15	15.5	16	16.5	17
INJ1	312.5	312.5	302	282	236	219	209	155	109	89	71	48
INJ2	625	625	614	608	600	595	576	491	469	460	443	368
INJ3	312.5	312.5	312	327	333	342	345	366	375	388	417	437
INJ4	625	625	618	612	606	604	588	563	500	485	427	390
INJ5	2250	2250	2246	2235	2231	2224	2211	2190	2182	2165	2157	2049
INJ6	625	625	624	620	606	599	595	589	587	582	585	564
INJ7	312.5	312.5	406	408	425	443	487	587	638	694	787	1001
INJ8	625	625	606	608	686	694	714	714	713	704	597	560
INJ9	312.5	312.5	311	297	280	281	282	347	343	345	344	336
INJ10	625	625	606	588	581	570	548	449	416	399	351	325
INJ11	625	625	625	635	638	637	640	651	651	648	655	646
INJ12	625	625	625	619	617	613	610	604	604	601	608	595
INJ13	2250	2250	2236	2223	2187	2164	2121	1967	1915	1878	1802	1712
INJ14	625	625	609	600	594	577	547	456	427	405	287	253
INJ15	2250	2250	2242	2232	2218	2211	2201	2169	2157	2139	2131	2032
INJ16	2250	2250	2247	2236	2231	2224	2213	2186	2179	2167	2178	2112
INJ17	625	625	609	595	586	570	560	461	448	429	345	320
INJ18	2250	2250	2249	2242	2237	2231	2226	2248	2252	2248	2277	2231
INJ19	625	625	630	679	713	737	793	1025	1079	1111	1198	1243
INJ20	625	625	625	630	630	634	629	613	602	597	588	546
INJ21	625	625	657	726	765	830	905	1170	1355	1466	1755	2232

Table B4: Summary of water injection rate at the individual wells for 9spot Pattern for 17years;12months re-allocation scheme

Time,Days	INJECTION RATE, STB/DAY						
	0	12	13	14	15	16	17
INJ1	312.5	312.5	302	288	251	238	230
INJ2	625	625	614	615	617	610	603
INJ3	312.5	312.5	312	374	381	422	465
INJ4	625	625	618	606	592	573	558
INJ5	2250	2250	2246	2209	2204	2176	2153
INJ6	625	625	624	614	613	607	602
INJ7	625	625	406	492	501	531	541
INJ8	625	625	606	701	714	790	783
INJ9	312.5	312.5	311	334	354	354	403
INJ10	625	625	606	588	563	541	525
INJ11	625	625	625	631	641	649	653
INJ12	625	625	625	614	618	613	608
INJ13	2250	2250	2236	2196	2166	2130	2102
INJ14	625	625	609	598	580	548	532
INJ15	2250	2250	2242	2202	2181	2157	2131
INJ16	2250	2250	2247	2210	2206	2192	2173
INJ17	625	625	609	577	557	544	528
INJ18	2250	2250	2249	2215	2210	2211	2219
INJ19	625	625	630	668	720	759	771
INJ20	625	625	625	609	608	606	636
INJ21	625	625	657	660	724	750	783

**Table B5: Summary of water injection rate at the individual wells for
9spot Pattern for 17years;18months re-allocation scheme**

Time,years	INJECTION RATE, STB/DAY					
	0	12	13.5	15	16.5	18
INJ1	312.5	312.5	302	293	274	263
INJ2	625	625	614	597	579	568
INJ3	312.5	312.5	312	317	359	367
INJ4	625	625	618	601	582	570
INJ5	2250	2250	2246	2188	2153	2121
INJ6	625	625	624	608	595	587
INJ7	625	625	406	514	551	648
INJ8	625	625	606	584	640	630
INJ9	312.5	312.5	311	303	286	305
INJ10	625	625	606	588	564	550
INJ11	625	625	625	651	650	652
INJ12	625	625	625	656	652	653
INJ13	2250	2250	2236	2177	2137	2099
INJ14	625	625	609	592	533	512
INJ15	2250	2250	2242	2182	2149	2111
INJ16	2250	2250	2247	2189	2168	2139
INJ17	625	625	609	589	581	567
INJ18	2250	2250	2249	2194	2167	2144
INJ19	625	625	630	712	776	800
INJ20	625	625	625	635	627	618
INJ21	625	625	657	832	977	1097

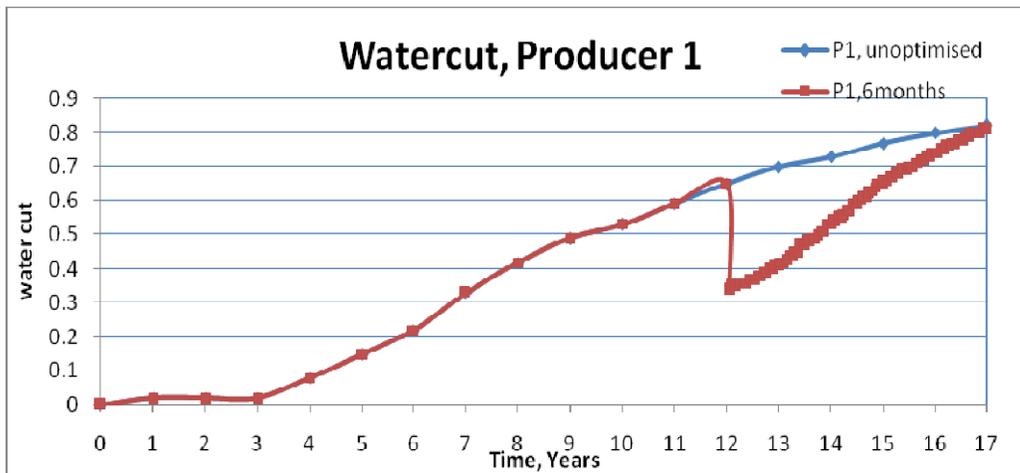


Figure B1 – Water-cut for Producer 1; 5-spot pattern

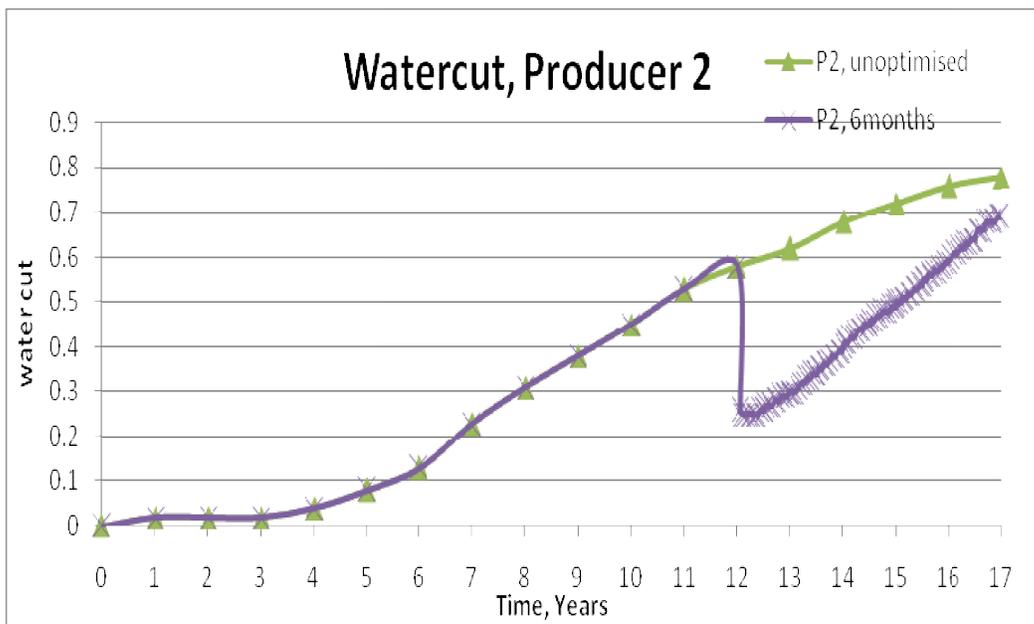


Figure B2 – Water-cut for Producer 2

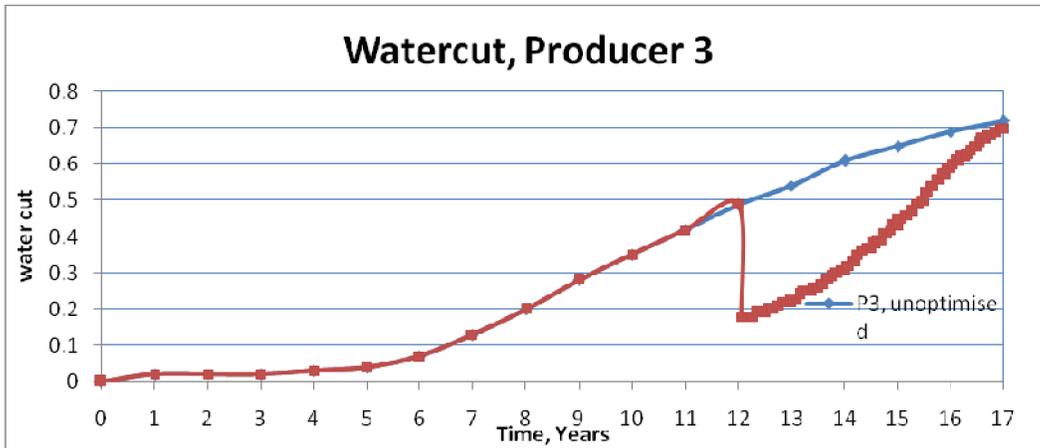


Figure B3 – Water-cut for Producer 3

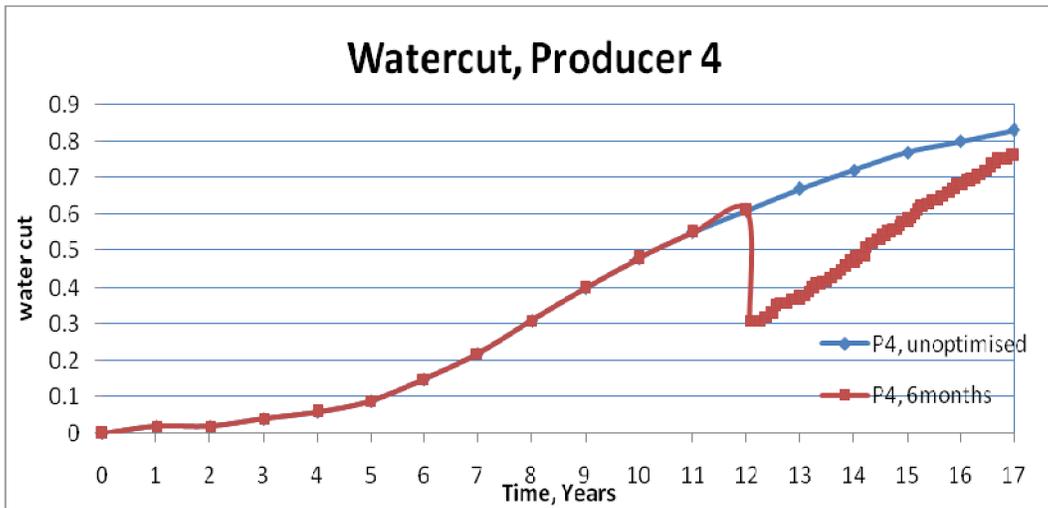


Figure B4 – Water-cut for Producer 4

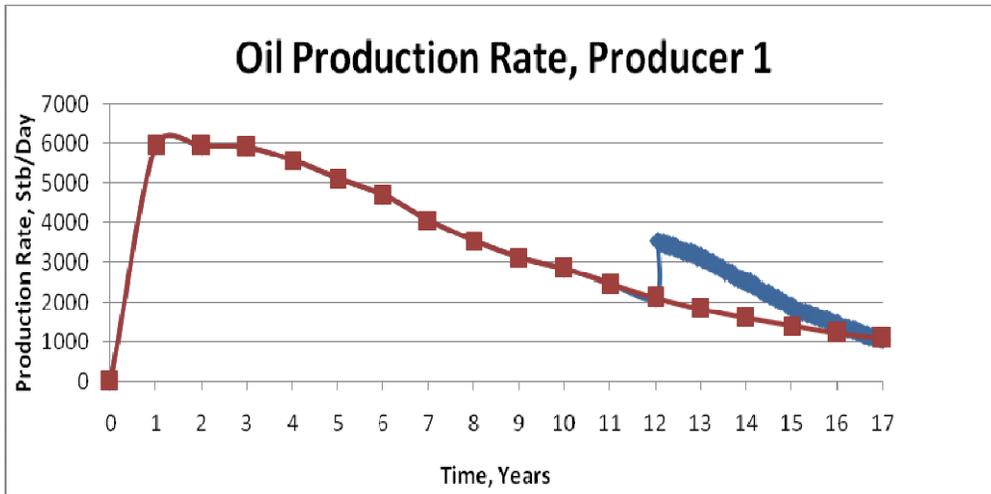


Figure B5 – Oil Production Rate for Producer 1

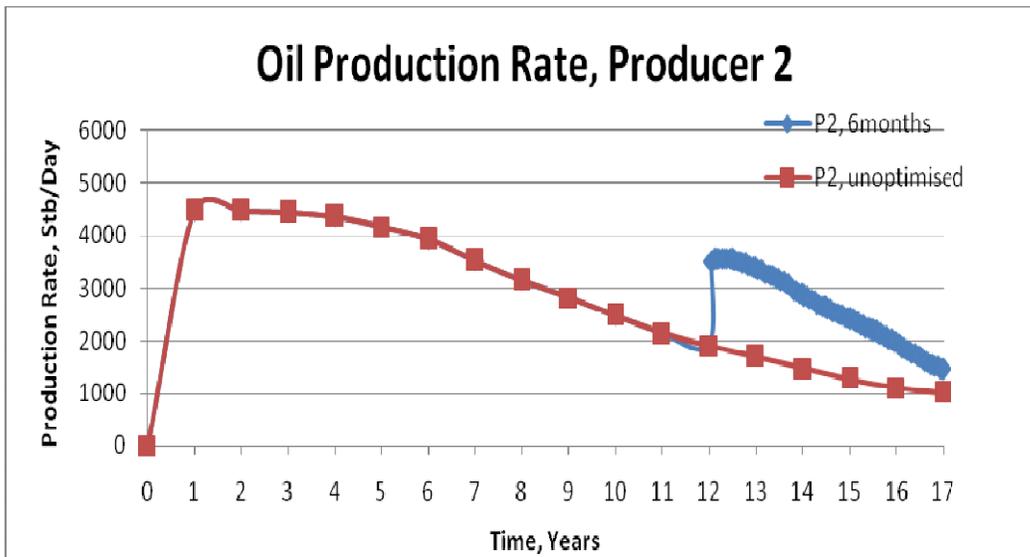


Figure B6 – Oil Production Rate for Producer 2

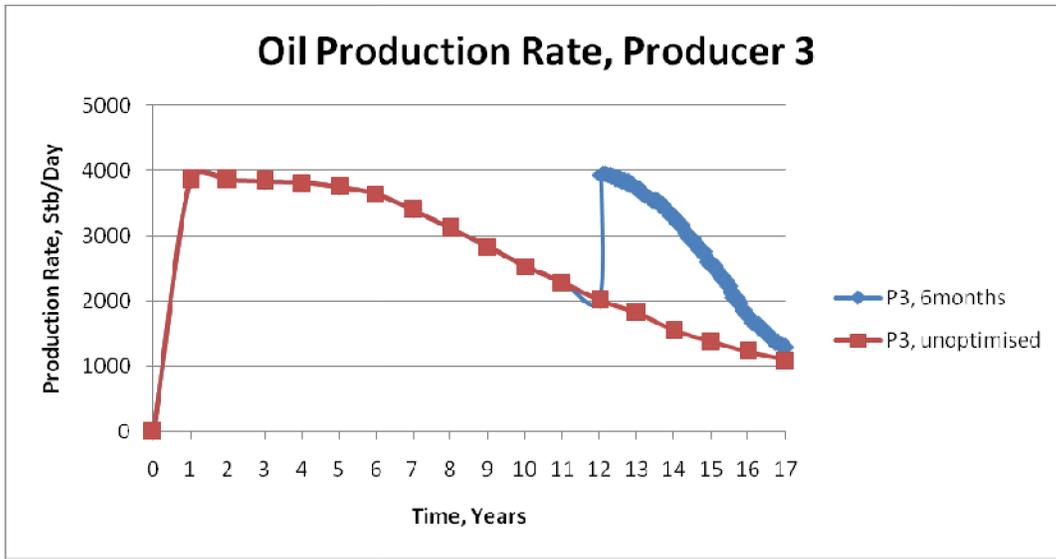


Figure B7 – Oil Production Rate for Producer 3

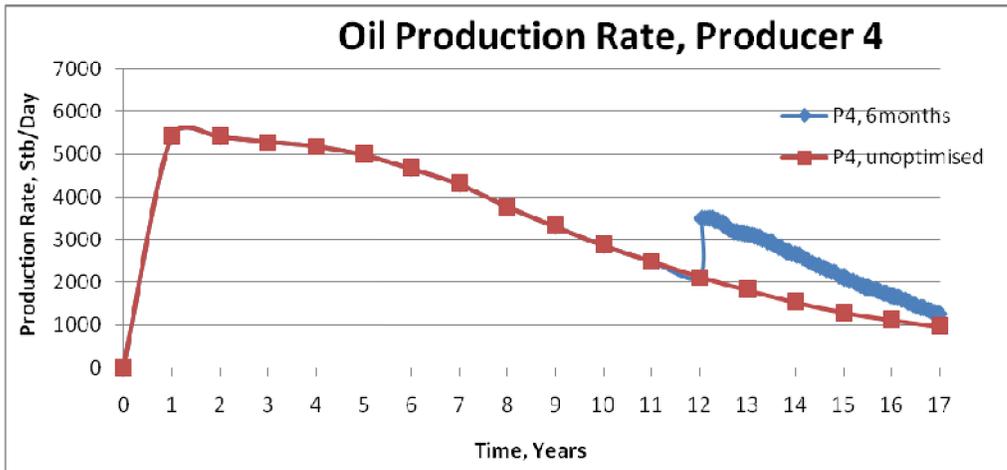


Figure B8 – Oil Production Rate for Producer 4,5spot pattern

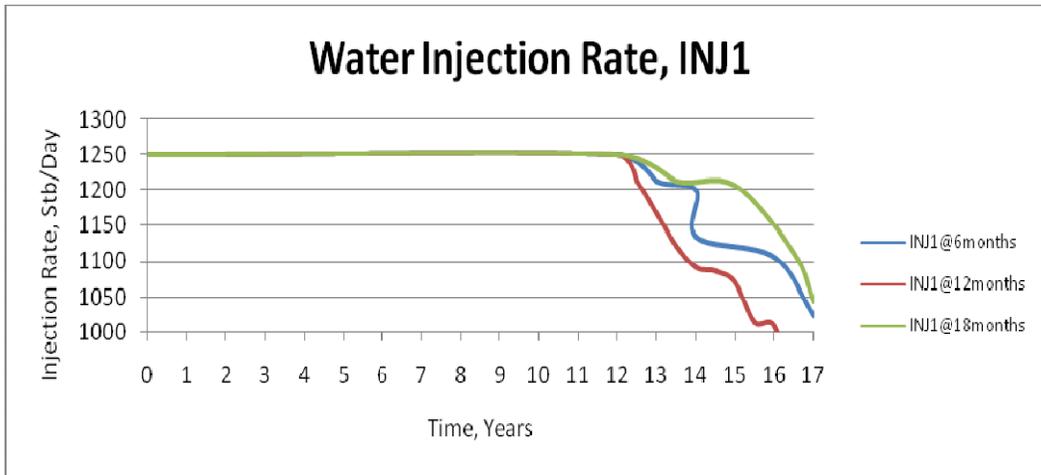


Figure B9 – Injection Rate Profile for Injector 1,5spot pattern

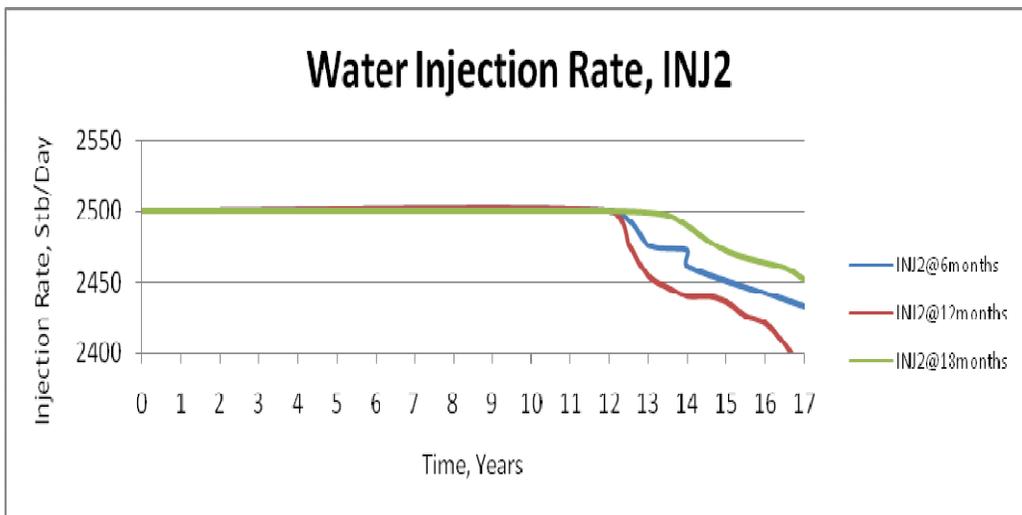


Figure B10 – Injection Rate Profile for Injector 2,5spot pattern

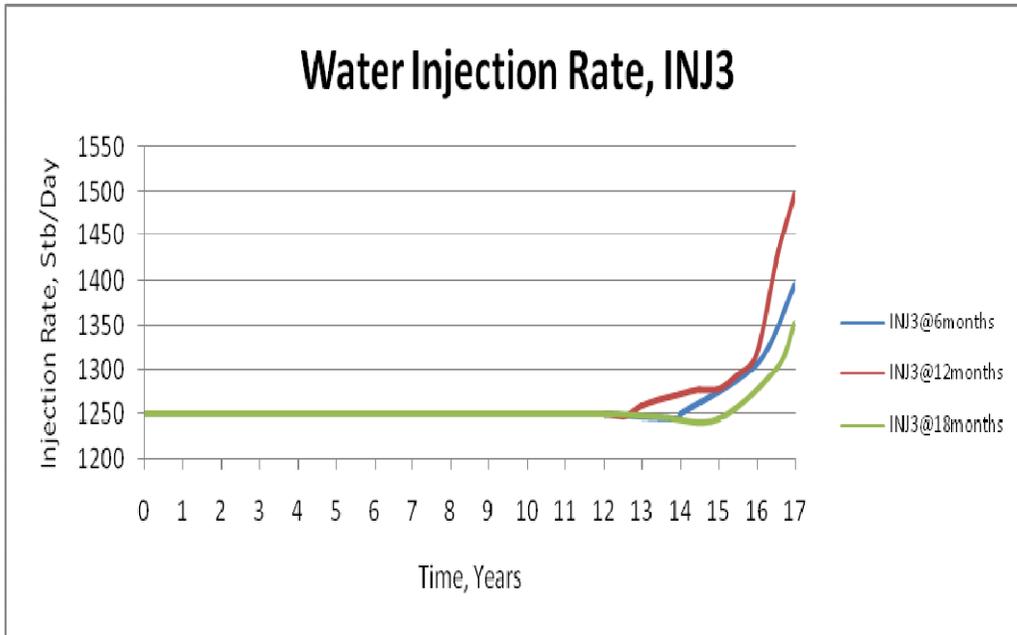


Figure B11 – Injection Rate Profile for Injector 3,5spot pattern

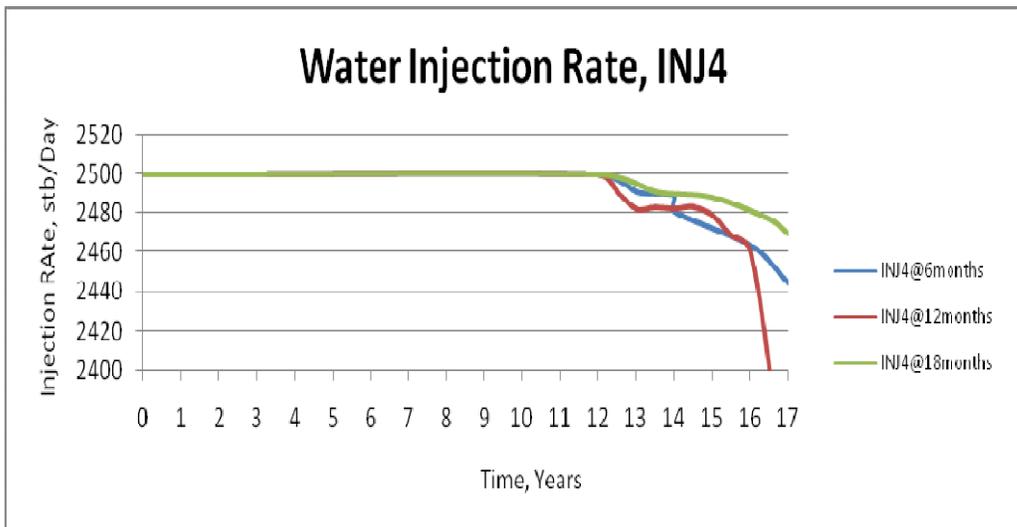


Figure B12 – Injection Rate Profile for Injector 4,5spot pattern

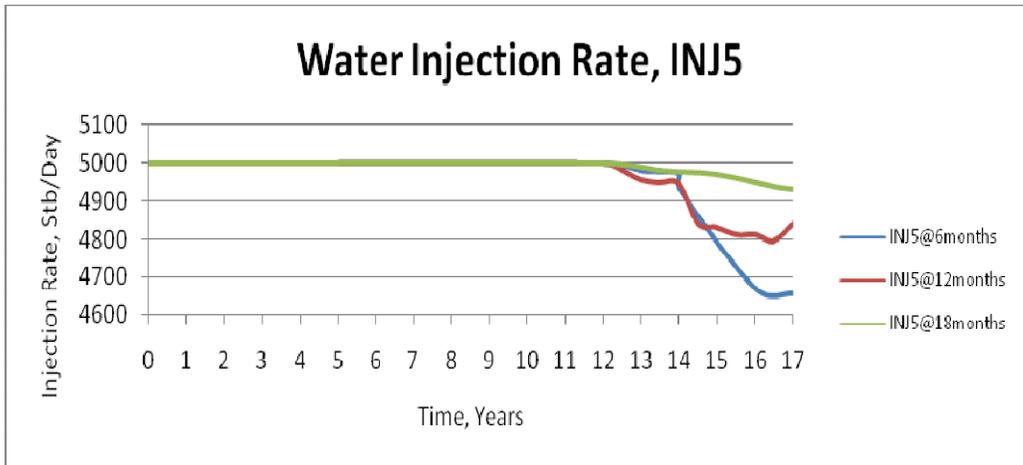


Figure B13 – Injection Rate Profile for Injector 5,5spot pattern

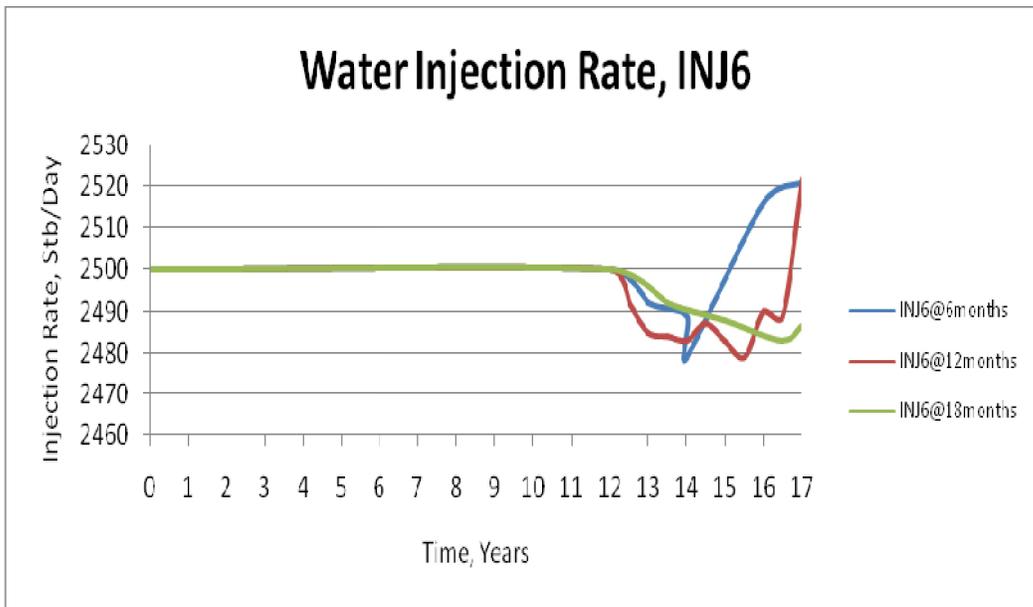


Figure B14 – Injection Rate Profile for Injector 6,5spot pattern

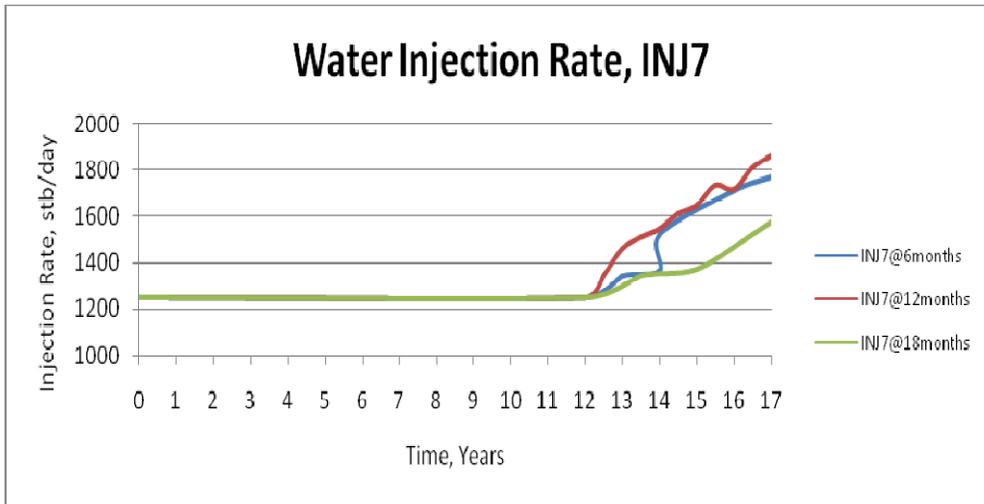


Figure B15 – Injection Rate Profile for Injector 7,5spot pattern

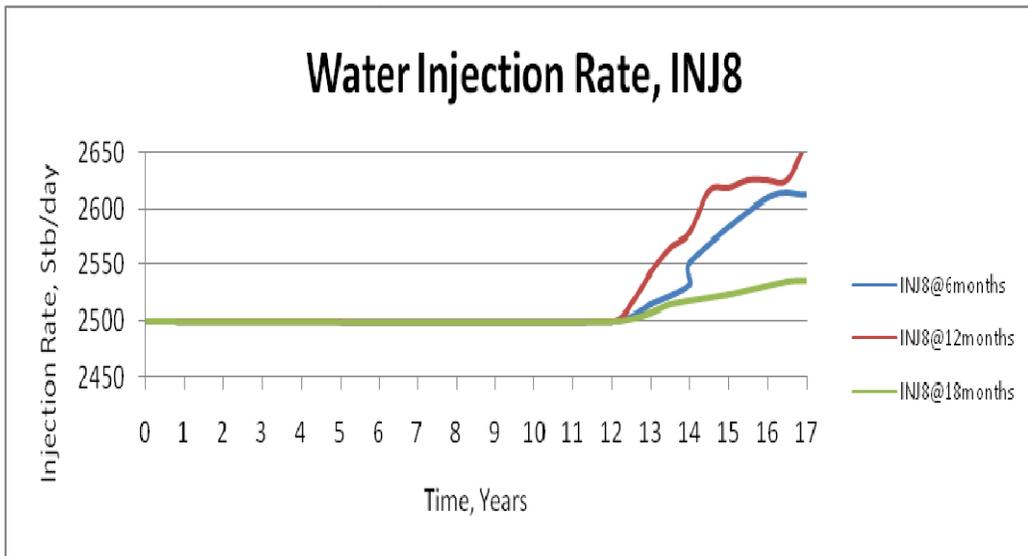


Figure B16 – Injection Rate Profile for Injector 8,5spot pattern

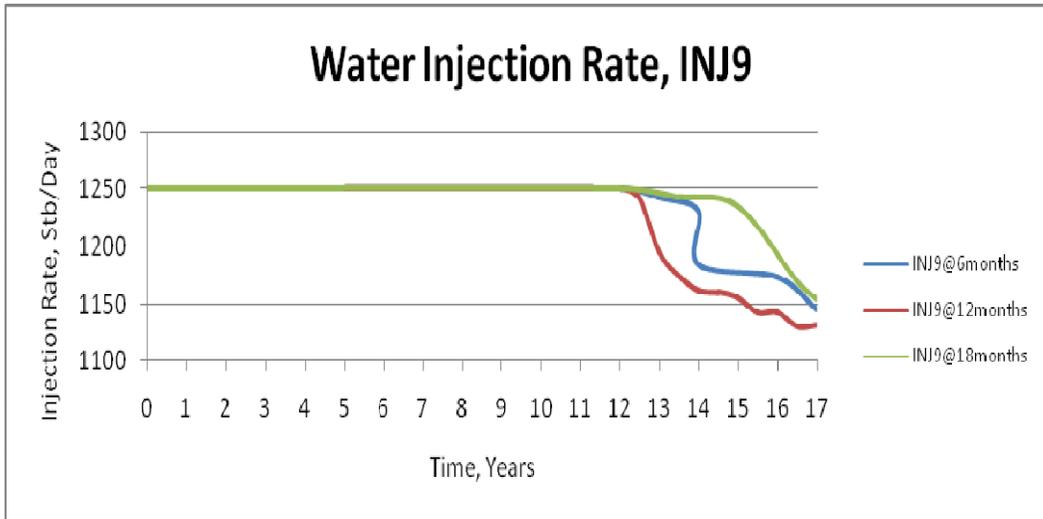


Figure B17 – Injection Rate Profile for Injector 9,5spot pattern

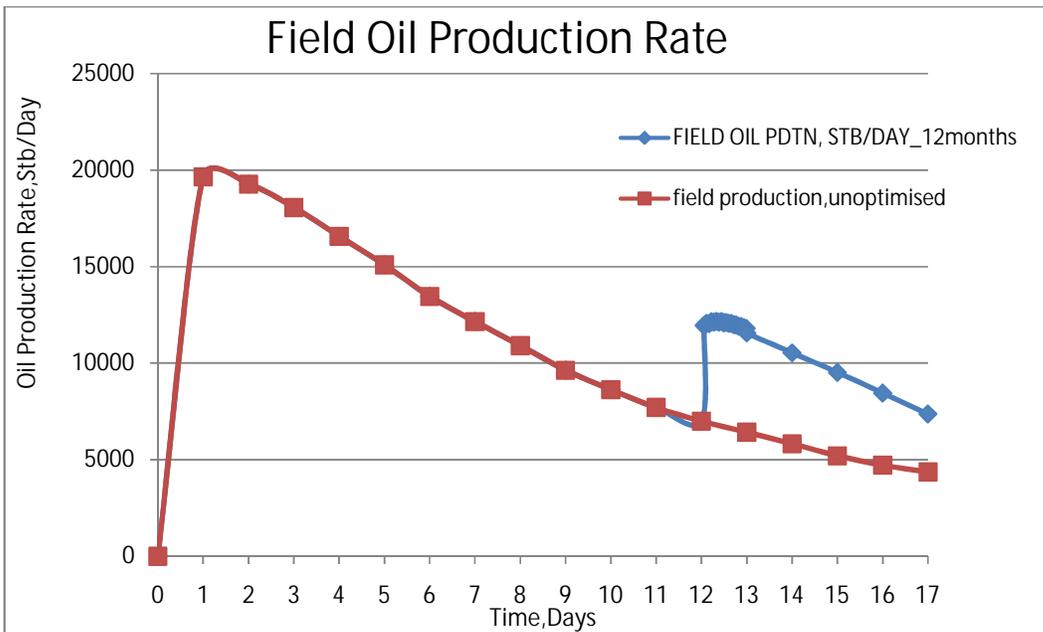


Figure B18 Oil Production Rate for 12months re-allocation scheme in 9spot pattern

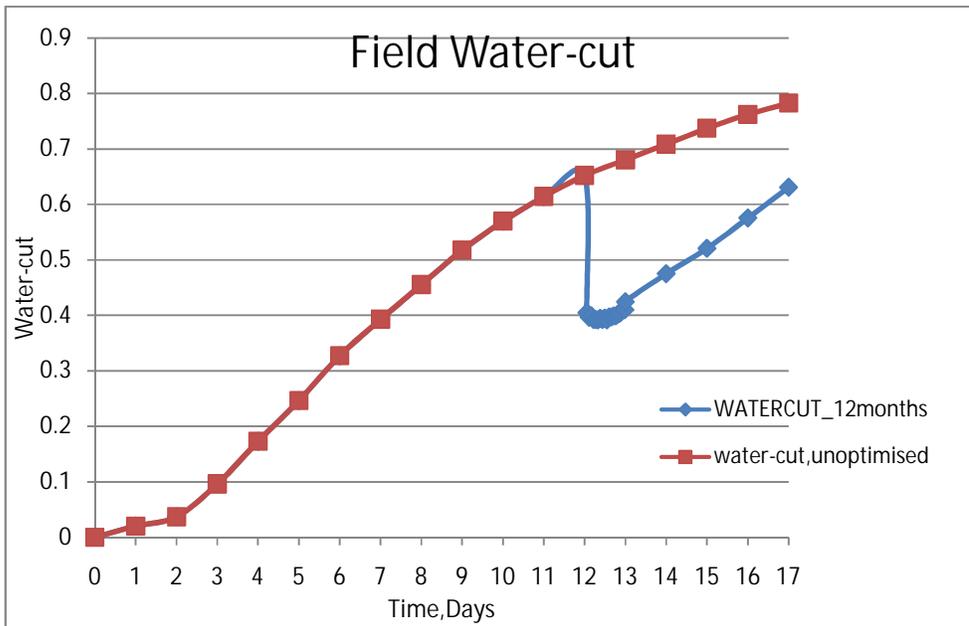


Figure B19 Water cut for 12 months re-allocation scheme, 9spot Pattern

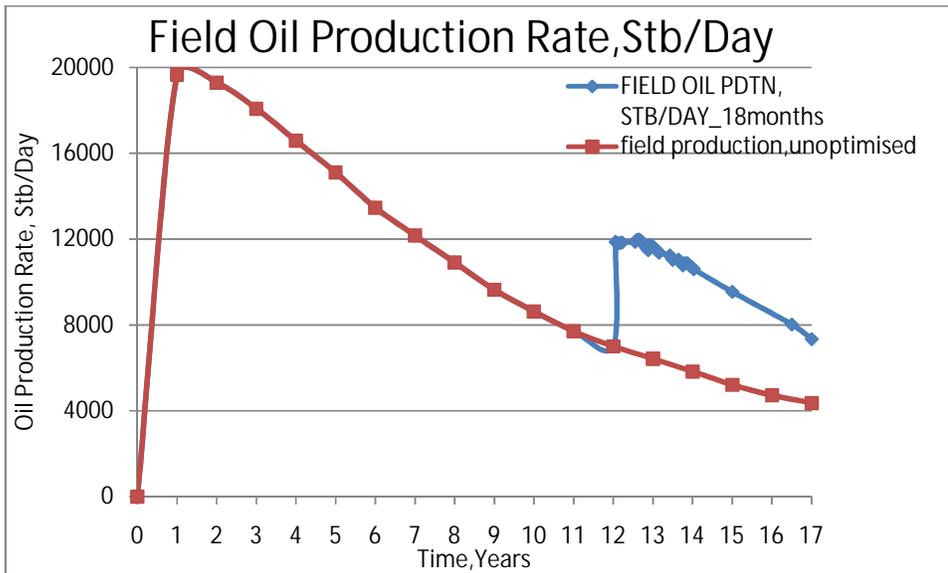


Figure B20 Oil Production Rate for 18months re-allocation scheme in 9spot pattern

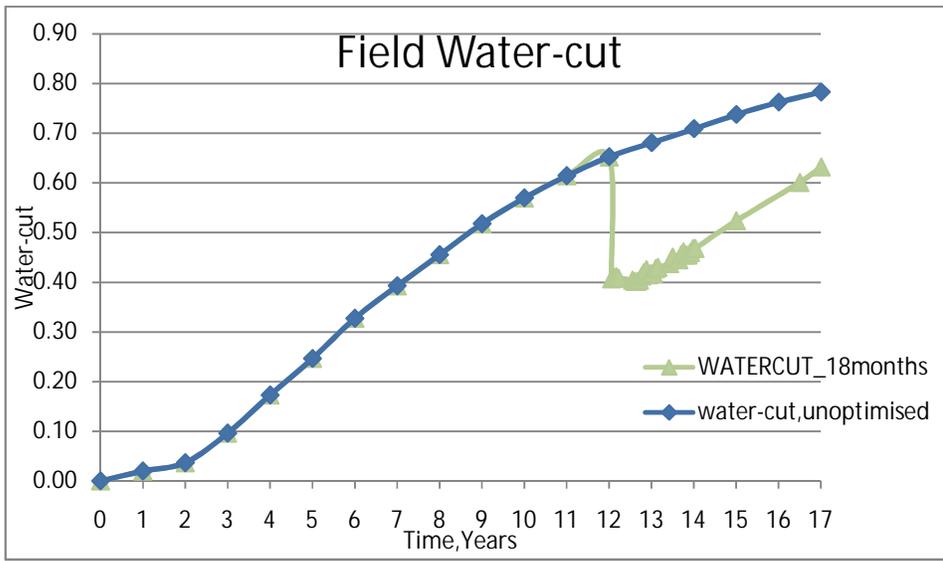


Figure B21 Water cut for 18 months re-allocation scheme, 9spot Pattern

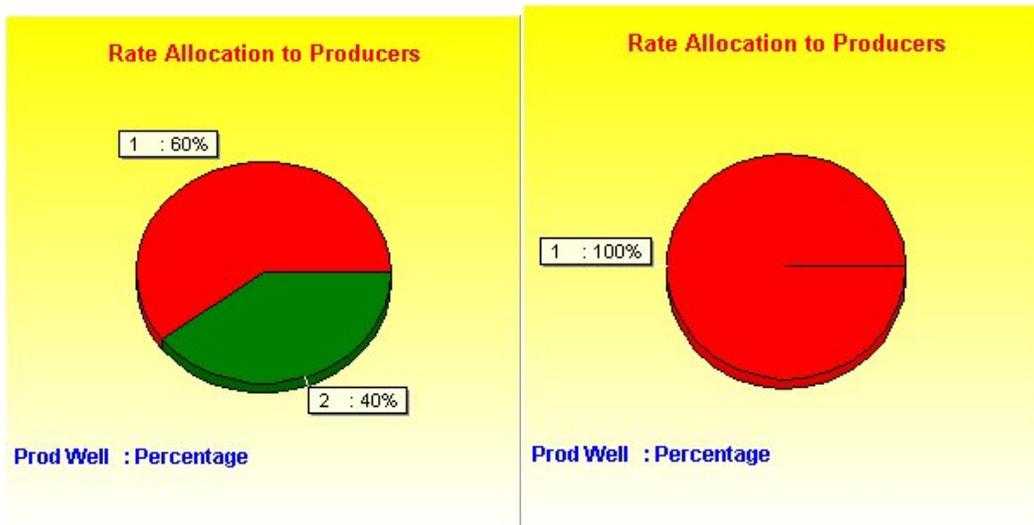


Figure B22 – Rate Allocation of INJ1and INJ2 to Offset Producers

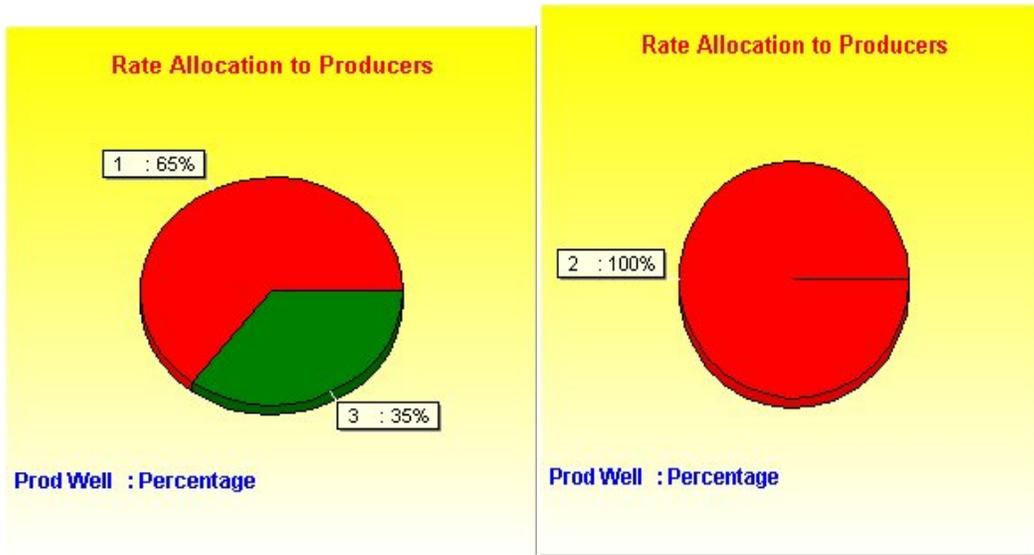


Figure B23 – Rate Allocation of INJ3 and INJ4 to Offset Producers

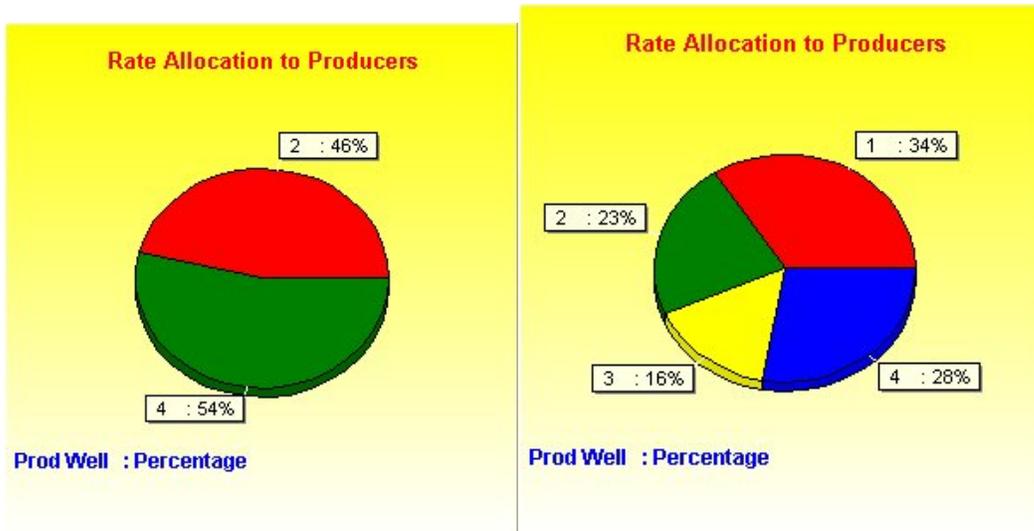


Figure B24 – Rate Allocation of INJ5 and INJ6 to Offset Producers

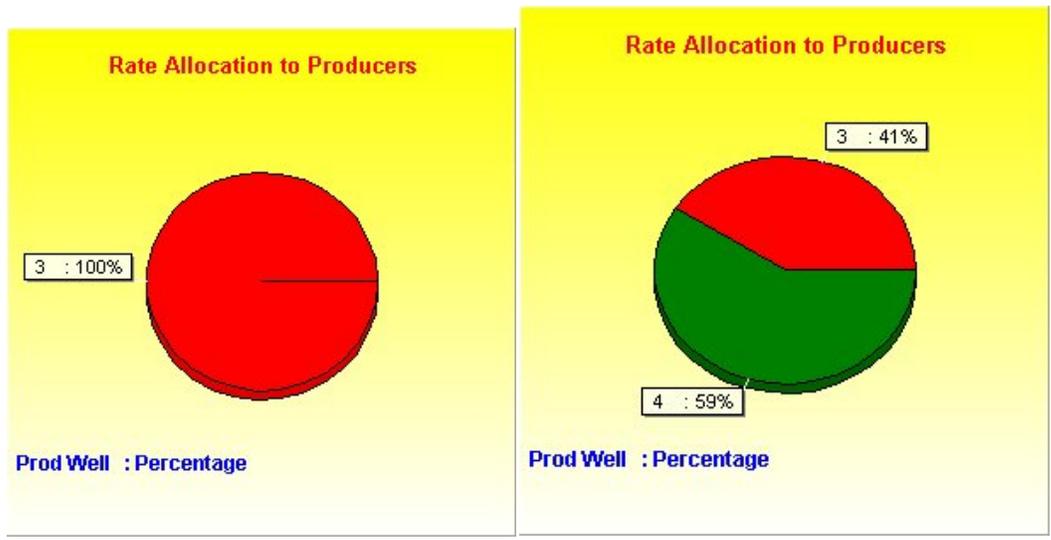


Figure B25 – Rate Allocation of INJ7 and INJ8 To Offset Producers

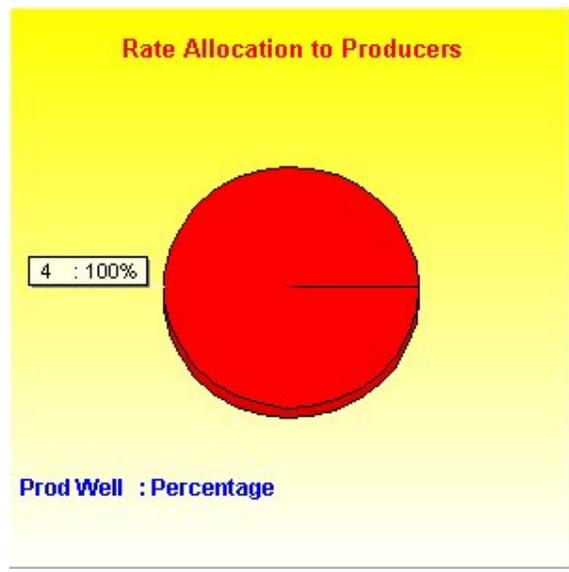


Figure B26 – Rate Allocation of INJ9 To Offset Producers

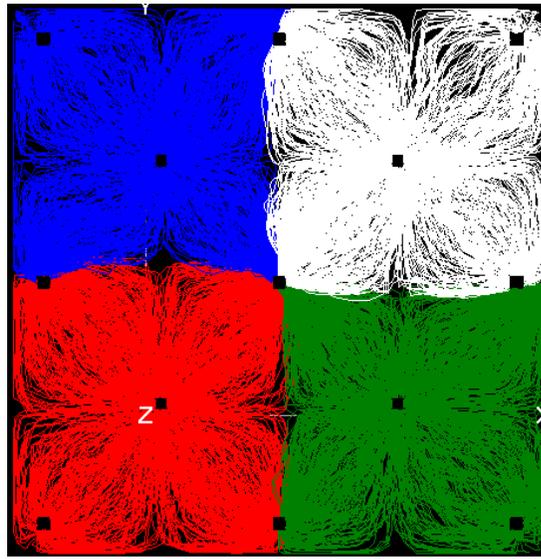


Figure B27 Streamlines showing producer-injector relationship, 5spot

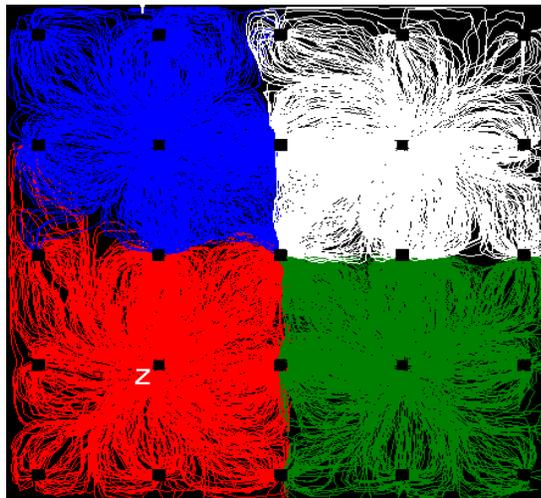


Figure B28: Streamlines showing producer-injector relationship, 9spot

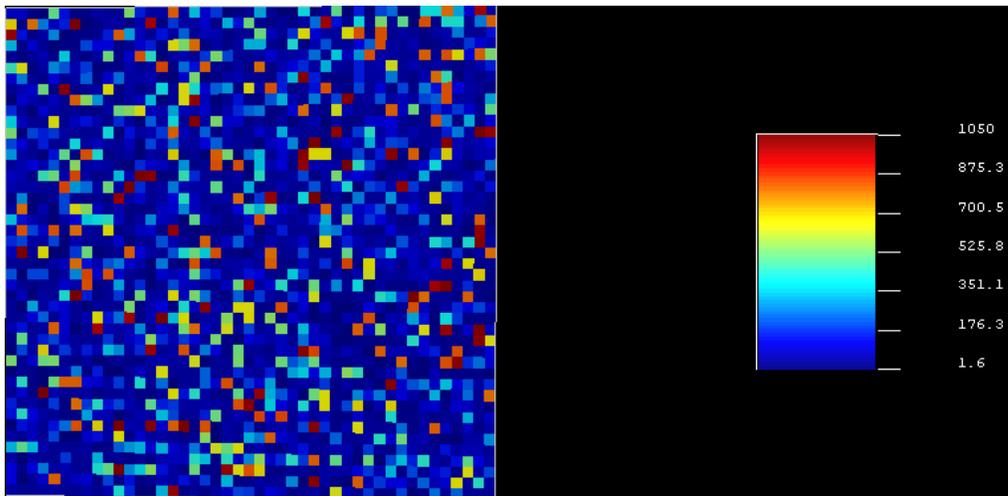


Figure B29 Reservoir Permeability Distribution

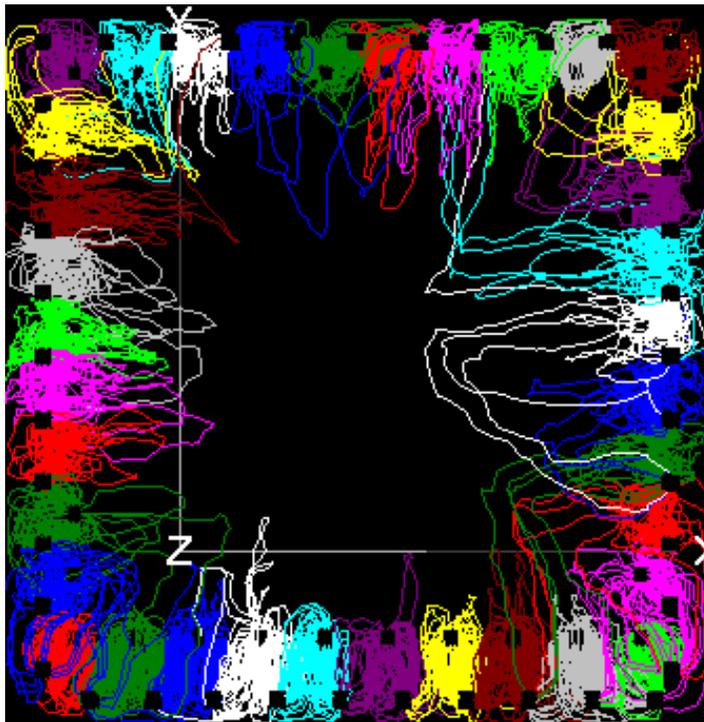


Figure B30: Streamlines showing producer-injector relationship, Peripheral

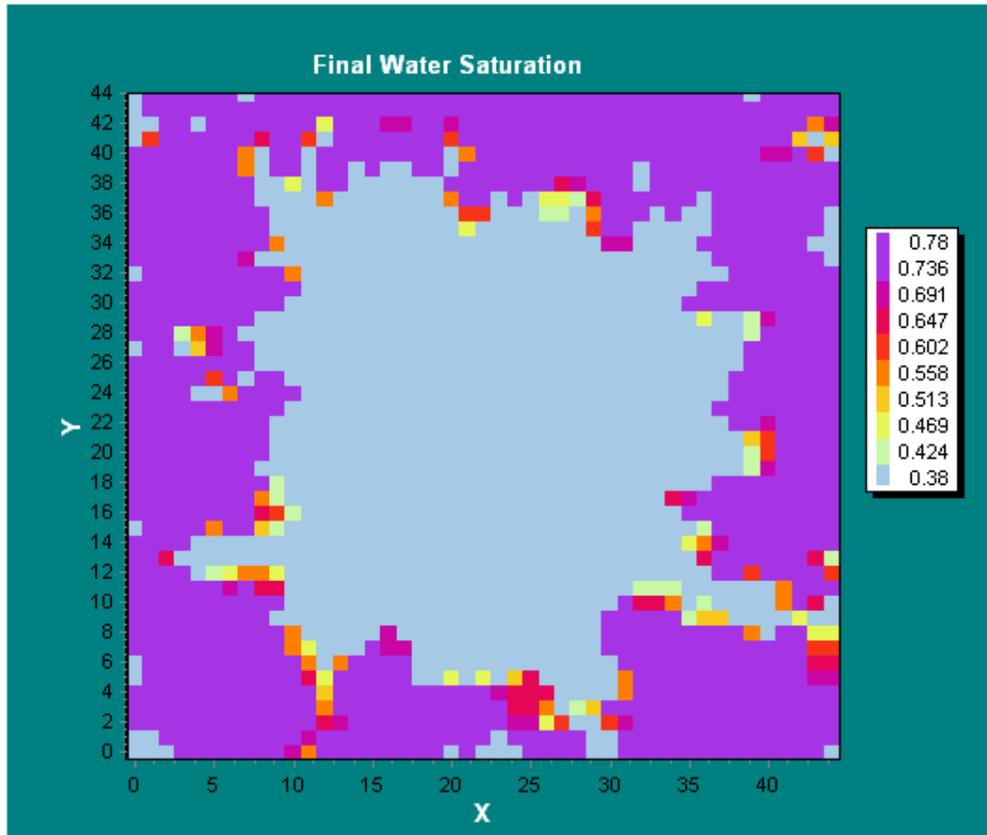


Figure B30: Final Water Saturation in Peripheral flooding showing unswept area with Field Injection Efficiency of 21% in the 12th year.

Table B1: Field Injection Efficiency and Water Cut, Peripheral Flooding

Years	FIELD IE,%	FIELD WATERCUT
0	0.00	0.00
1	0.86	0.19
2	0.64	0.39
3	0.51	0.52
4	0.42	0.60
5	0.37	0.65
6	0.33	0.69
7	0.30	0.72
8	0.28	0.74
9	0.26	0.76
10	0.24	0.77
11	0.23	0.78
12	0.21	0.80

From the results of peripheral flooding above, it is observed that the middle of the field has an oil bank in the middle of the field. Its optimization has been previously recommended for optimization in future research by re-allocating total water injected.