DEVELOPMENT OF BREAKTHROUGH TIME CORRELATIONS FOR CONING IN BOTTOM WATER SUPPORTED RESERVOIRS

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DEVELOPMENT OF BREAKTHROUGH TIME CORRELATIONS FOR CONING IN BOTTOM WATER SUPPORTED RESERVOIRS

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To my Mum and Dad, Florence Adjei and Silvanus Amarfio (late)

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ABSTRACT

This research work mainly investigates the development and the behavior of cones (both water and gas cones) in oil reservoirs supported by strong aquifer, and from which analytical correlations are developed for quick engineering estimates of the time for water/gas cones to break into the perforations of the producing wells. The studies treated the cone development and breakthrough times in both horizontal and vertical well producing reservoirs and made analysis on them. The Ozkan and Raghavan (1990) method was employed as the base approach in the modeling of the cones; as well as their breakthrough times and then compare with that of Chaperon's approach(1986) with both the horizontal and vertical well applied. The developed models were then run on field data, the results were graphically represented in both the horizontal and vertical well cases. Analytical correlations were then developed from the results obtained for breakthrough time estimation and compared with literature on example case. This work actually employs the dimensionless (or the normalized approach) system to curtail the units complexities and represent the results in a more generalized form. These analytical correlations can be leveraged on to plan better future recompletion strategy as they provide an engineering estimate of when water breaks into the production wells.

CHAPTER ONE

1.1INTRODUCTION

Coning is the mechanism describing the movement of water/gas into the perforations of producing wells. For water coning the movement is upwards for the case of bottom water, side wards for edge water, but it is downwards for gas coning. The production of water from oil wells is a common occurrence which increases the cost of producing operations and may reduce the efficiency of the depletion mechanism and the recovery of reserves. The objective of this research work is to model the behaviour of this coning(mainly water coning, from bottom water) and then use it to evaluate the time it would take a cone to break into the producing well in reservoir of well-defined boundary conditions.

The coning of water into production wells is caused by pressure gradients established around the wellbore by the production of fluids from the well. These pressure gradients can raise the water-oil contact near the well where the gradients are dominant. The gravity forces that arise from fluid density differences counterbalance the flowing pressure gradients and tend to keep the water out of the oil zone. Hence, at any given time, there is a balance between the gravitational and the viscous forces at any point on and away from the completion interval. The water cone formed will break eventually into the well to produce water along with the oil when the viscous forces exceed that of the gravitational forces. This basic visualization of coning can be expanded further by introduction of the concept of stable cone, unstable cone and critical production rate.

STABLE CONE:

If a well is produced at a constant rate and the pressure gradient in the drainage system have constant, a steady state condition is reached, if at this condition, the dynamic forces (viscous forces) at the well are less than the gravity forces then the water or gas cone that has formed will not extend to the well .Moreover, the cone will not advance nor recede, thus establishing what is known as stable cone.

UNSTABLE CONE:

Conversely, if the pressure in the system is in an unsteady-state condition, then the cone that will be formed is unstable and it will continue to advance until the steady-state condition takes over. If the flowing pressure drop is sufficient to overcome the gravity forces, the unstable cone will mushroom and ultimately break into the well. In actual sense therefore, stable cones may only be 'pseudo-stable' because the drainage system and the pressure distribution generally change. For a example, during reservoir depletion, the water- oil contact may advance toward the completion interval, thereby increasing coning tendencies. Another one is reduction in productivity due to well damage requires a corresponding increase in the flowing pressure drop to maintain a given production rate. This increase in pressure drop may force an otherwise stable cone into the well.

CRITICAL PRODUCTION:

The critical production rate is the rate beyond and above which the flowing pressure gradient at the well causes water (or gas) to cone into the well. It is therefore, the maximum rate of oil production without concurrent production of water by coning. A build-up is stable at the critical rate but is at a position of incipient breakthrough.

One assumption in critical production rate is that the cone has built-up to just before the breakthrough into the well. But, these analyses reveal nothing directly about the time it takes for the cone to build-up to this incipient breakthrough position. Thus, water-free oil can be produced from a well for a prolonged period at rates above critical rate before the well reaches the condition to which the critical applies. Theoretically, the basic coning equation for a water-oil system can be developed by applying the conservation of mass to each of the phases relating flow velocities with pressure by Darcy's law, and relating pressure across water-oil contact interfaces by capillary pressure. With the usual boundaries at the well and reservoir the solutions of the resulting equations for the time behaviour of a water-oil interface constitutes a free-surface, boundary value problem.

1.2 STUDY APPROACH

The main study of this work is focused on engineering cone breakthrough time prediction in bottom water supported reservoirs either produced by horizontal well or vertical well with the application of Ozkan and Raghavan method(1990). This application would be compared with the chaperon(1986) model to investigate their behaviour on this study model.

1.3 OBJECTIVES

The objectives of this study are:

1. To model the water/gas behaviour in bottom water/gas supported reservoir

2. To develop simplified analytical correlations for engineering estimates of breakthrough time to enable better future recompletion strategy or prior to detailed simulation study.
3. To compare the breakthrough times in horizontal and vertical wells.

1.4 THE WORK OUTLINE

This research study comprises of six chapters. Chapter one introduces the problem of this study, the objectives and the approach of this study employs in tackling this problem. Chapter two is essentially, the literature reviews of related studies previously done on this problem. Chapter three describes the mechanics and the behaviours of cones. Chapter four tackles the model assumptions and mathematical formulations. Chapter five shows some field applications and the results and discussion of the study. And finally, the conclusion and recommendations are covered in chapter six.

CHAPTER TWO

2.1 LITERATURE REVIEW

Water coning has been a major hitch that worries reservoir Engineers since the inception of the petroleum industry. It seriously impacts the well productivity and influences the overall recovery of the oil reservoirs. It also increases the cost of water handling and disposal, promotes corrosive problems in production facilities and makes the early shut down of the affected wells. Although many methods have been used to control it, water coning, is still a major issue in many oil fields all over the world. Research shows that a lot of work has been done in order to gain understanding and better management procedures of this problem. Most of these works were done in these following bulleted areas:

- Experimental Studies of Water Coning
- Numerical Simulation Studies of Water Coning
- Water Coning Control Methods

Studies of water coning can be more or less grouped into three time stages:

- physical modeling and experimental study stage (before 1970),
- theoretical modeling and simulation stage (after 1970)
- Control technology development stage (after 1980).

Many correlations have been developed to calculate the critical oil production rate to control water breakthrough, predict water breakthrough time and post breakthrough behaviours such as water cut development.

2.1.1 Experimental Studies of Water Coning

Before 1970, most of the work on water coning problem focused on coning mechanisms and experimental studies. Many basic questions about water coning were solved in this stage. Muskat and Wyckoff (1935) published the first paper to analyze water coning in oil production theoretically. They presented the fundamental physical principles underlying the behaviour of the water oil contact (OWC) when oil was produced from a partial penetrated well in the oil zone before water breakthrough to the well. They suggested that, water coning was induced by the pressure differential existing in the well and the reservoir, and the advance of the OWC was directly proportional to this pressure differential. They also pointed out that, it was impossible to eliminate bottom water when producing from a thin oil zone unless the production rate of the well was reduced to uneconomically low values. Their calculation results showed that, water-free oil production rate could be maintained for a short-penetrated well, and this rate decreased with the increasing of well penetration. They defined the critical oil production rate (q) as the maximum allowable oil flow rate that can be imposed on the well to avoid a cone breakthrough. The critical rate would correspond to the development of a stable cone to an elevation just below the bottom of the perforated interval in an oil-water reservoir. By analytically solving the Laplace equation for single phase flow, they developed a correlation to calculate the critical rate for partial completed wells as:

$$q_{oc} = \frac{(4\pi kh\Delta\rho_o \sum a_n b_n)}{\mu_o\{-\frac{4}{h_o}\sum a_n b_n log\frac{4h_o}{r_e}\}} \qquad 2.1$$

In 1946, Muskat presented the way to determine the shape of water cones for various pressure drops in homogeneous reservoirs; he concluded that the critical pressure drop at the beginning of water coning was a function of well penetration and oil-zone thickness, and the critical oil production rate was controlled by the pressure gradient caused by the oil production.

Meyer and Garder (1954) derived a correlation for the critical oil rate required to achieve a stable water cone. They found that the critical rate for a well was determined by: the length of well penetration, density difference of oil and water and the oil zone thickness. Their correlation for critical oil rate is expressed as:

$$q_{0c} = 0.246 \times 10^{-4} \frac{\triangle\rho}{in\frac{r_e}{r_w}} \frac{k_o}{\mu_o B_o} (h_o^2 - h_{op}^2) \qquad 2.2$$

Chaney et al. (1956) developed a set of working curves to determine the critical oil rate. Their curves were generated by using a potentiometric analyser study and applying the water coning mathematical theory as developed by Muskat and Wyckoff (1935).

Chierici et al. (1964) used a potentiometric model to study the coning behaviour in vertical oil wells. They developed dimensionless graphs to address the water and gas coning problems and considered the vertical and horizontal permeability in their dimensionless graphs. With given reservoir and fluid properties, position and length of the perforated interval, the graphs could be used to determine the maximum oil production rate without gas/water coning. The graphs could also be used to determine the optimum position of the perforated interval with only reservoir and fluid properties. They also developed a correlation to calculate critical oil rate in oil/water reservoir given as;

$$q_{oc} = o.492 \times 10^{-4} \frac{\triangle\rho h_o^2}{\mu_o B_o} k_{ro} k_h \psi_w (r_{De}, \xi. \delta_W) \qquad 2.3$$

$$r_{De} = \frac{r_e}{h_o}\sqrt{\frac{k_h}{k_v}} \tag{2.4}$$

$$\xi = \frac{h_{op}}{h_o} \tag{2.5}$$

$$\delta W = \frac{D_b}{h_o} \tag{2.6}$$

Where, all the parameters are in field units.

Sobocinski and Cornelius (1965) presented a correlation to calculate the breakthrough time, which is the time needed for a water cone to enter the perforation after the beginning of oil production. Based on the experimental and modelling data, they developed a correlation to estimate the breakthrough time by using dimensionless cone height and dimensionless breakthrough time.

Bournazel and Jeanson (1971) simplified Sobocinski and Cornelius's correlation of breakthrough time by only using dimensionless breakthrough time. They also provide fast water coning evaluation relation for the critical oil rate in isotropic reservoir as:

$$q_{oc} = 5.14 \times 10^{-5} \frac{k_h h_o^2 \, \Delta\rho g \left(1 - \frac{h_{op}}{h_o}\right)}{\mu_o} \tag{2.7}$$

And for anisotropic reservoir, they used the following correlation to calculate the critical oil rate:

$$q_{oc} = 5.14 \times 10^{-5} \frac{k_h^2 h_o^2 \, \Delta\rho g \left(1 - \frac{h_{op}}{h_o}\right)}{\mu_o k_v} \qquad 2.8$$

Where, all the parameters are in field units.

Khan (1970) used a three-dimensional scaled laboratory model to observe the coning behaviour in a reservoir with natural water drive. Their results indicated that the degree of water coning and the value of the water cut increase with production rate, the mobility ratio and the ratio of aquifer to oil-sand thickness. They found that the mobility ration had great influence on the value of water cut and the degree of water coning at a given total production value: the higher the mobility ratio, the faster water coning develops.

2.1.2. Numerical Simulation Studies of Water Coning

With the increase of computing power and improvement of simulation technology, several computer simulators were made available after 1970. This makes it possible to simulate more complex coning problems in computer, while it saves much time comparing to physical experiments. Although several people used numerical simulation to study water coning problem before 1970, the major coning simulation publications turned up after 1970. People began to investigate complicated coning behaviour after water breakthrough in this stage, however, critical oil rate was still an important topic.

Welge and Weber carried out the first numerical simulation research on coning problem in 1964. They applied two-phase, two-dimensional model using the alternating direction implicit procedure (ADIP) in the gas and water coning simulation. They found that special computational techniques must be used after cone breakthrough to achieve reliable results and keep calculation costs within reasonable limits. Their simulation results matched the producing histories of laboratory sand packed model and of several producing wells experiencing water or free gas production by coning. They suggested that the average horizontal and vertical permeability and the k h / k v ratio are critical parameters in the coning study.

Pirson and Mehta (1967) developed a computer program to simulate water coning based on the Welge and Weber's mathematical model. They studied the effects of various factors such as vertical to horizontal permeability ratio, mobility ratio between oil and water, specific gravity differential between the two phases and flow rate on the advance of a water cone. The cone shapes and positions were drawn for each case, and the results were found to agree with known phenomena. Comparing their results to Muskat's approximate method, they found that Muskat's method gave higher critical rate because of ignoring the water-oil transition zone.

MacDonald and Coats (1970) described and evaluated three methods for the simulation of well coning behaviour. They improved upon the small time step restriction of coning problems by making the production and transmissibility terms implicit, and it could increase the simulation speed much more than the traditional IMPES (Implicit Pressure Explicit Saturation) method.

Letkeman and Ridings (1970) presented a numerical coning model based on implicit transmissibilities and linear interpolation which could use much larger time steps than those in IMPES simulators. The model exhibited stable saturation and production behaviour during cone formation and after breakthrough. Their work made coning simulation became practical and economical using modified equations.

Byrne and Morse (1973) presented a systematic numerical coning simulation study which included the effects of reservoir and well parameters. Their results showed that the critical oil rate decreased with the increase of well penetration depth, water breakthrough time decreased and WOR (water oil ratio) increased significantly when the production rate increased, however, the ultimate recovery was independent of production rate. The wellbore radius was not so important on water breakthrough time and WOR.

Schols (1972) developed an empirical formula for critical oil rate based on results obtained from numerical simulator and laboratory experiments as:

$$q_{oc} = 0.0783 \times 10^{-4} \left[\frac{\Delta\rho k_o\left(h_o^2 - h_{op}^2\right)}{\mu_o B_o}\right]\left[0.432 + \frac{\pi}{ln\frac{r_e}{r_w}}\right]\left(\frac{h_o}{r_e}\right)^{0.14} \qquad 2.9$$

Where, all the parameters are in field units.

Miller and Rogers (1973) presented detailed coning simulation which was suitable to evaluate water coning problem for a single well in a reservoir with bottom water. They simulated a single well using radial coordinates and a grid system which could be used to determine the most important parameters in water coning on both short term and long term production. Their results for critical oil rate matched well with Schols' critical rate correlation.

Mungan (1975) presented experimental and numerical studies on water coning in oil producing well under two-phase, immiscible and incompressible flow conditions. His results indicated that the numerical model simulated the experiments adequately. Increasing the production rate or the wellbore penetration led to earlier water breakthrough, however, oil recovery was independent of production rate. The oil recovery at any given WOR became greater when the ratio of gravity to viscous forces increased. High vertical permeability decreased the oil recovery, while the opposite was true for horizontal permeability.

Chappelear and Hirasaki (1976) developed a correlation to evaluate the critical oil production rate for a partially perforated well in a reservoir with bottom water. Their coning model was derived by assuming vertical equilibrium and segregated flow in two-phase, two-dimensional reservoir. Their critical oil rate correlation expressed as:

$$q_{oc} = \frac{2\pi h_t k_h k_{ro} \Delta\rho g (h_o - h_{cb})}{887.2 \mu_o B_o ln r'} \qquad 2.10$$

$$r' = 4h\sqrt{k_h/k_v}\left(\| \frac{h_0 - h_{cb}}{h_0 - h_{ct}} \|\right) \qquad 2.11$$

$$ln r' = \frac{\ln[r_e/(r_w + r')]}{[1 - (r_w + r\prime)^2/r_e]} - \frac{1}{2} \qquad 2.12$$

All parameters are in field units.

Kuo and Des Brisay (1983) used a numerical simulation to determine the sensitivity of water coning behaviour to various reservoir parameters. Based on the simulation results, they developed a simplified correlation to predict the water cut in bottom water drive reservoirs.

Chaperon (1986) proposed a simple correlation to estimate the critical rate of a vertical well in an anisotropic formation. The correlation accounted for the distance between the producing well and boundary. Comparing to other works, his correlation was more sensitive to anisotropy: critical rate slightly increased when vertical permeability decreased, but critical cone elevation did not change significantly. The proposed correlation has the following form:

$$q_{oc} = 0.0783 \times 10^{-4} \left[\frac{\Delta\rho k_h\left(h_o^2 - h_{op}^2\right)}{\mu_o B_o}\right]\left[0.7311 + \frac{1.943}{\frac{r_e}{r_w}\sqrt{\frac{k_v}{k_h}}}\right] \qquad 2.13$$

Where, all parameters are in field units.

Hoyland et al. (1989) presented two methods to predict critical oil rate for a partially penetrated well in an anisotropic bottom water drive reservoirs. The first method was an analytical solution, and the second was a numerical solution to the coning problem. Based on Muskat and Wyckoff's theory, they used the method of images and superimpose to address boundary conditions, and they developed a correlation to predict the critical rate in steady state condition as;

$$q_{oc} = 0.246 \times 10^{-4} \left[\frac{\Delta\rho k_h (h_o^2)}{\mu_o B_o}\right] q_{cD} \quad 2.14$$

Where, q_{cD} , is the dimensionless production rate which can be checked from their dimensionless plot.

Based on a large number of simulation runs with more than 50 critical rate values, the authors used a regression analysis routine to develop the critical oil rate correlations for isotropic and anisotropic reservoirs. For isotropic reservoirs, the correlation expressed as:

$$q_{oc} = \left[\frac{k_O(\rho_W - \rho_0)}{173.35 \mu_o B_o}\right] \left[1 - \frac{h_0}{h_0}\right] [\ln(r_e)]^{-1.990} h_o^{2.238} \quad 2.15$$

Where, all the parameters are in field units.

For anisotropic reservoirs, the authors correlated the dimensionless critical rate with the dimensionless radius and five different fractional well penetrations. The correlation was presented in a graphical form.

Guo and Lee (1993) demonstrated that the existence of the unstable water cone which depended on the vertical pressure gradient beneath the wellbore. They found that when the vertical pressure gradient was higher than the hydrostatic pressure gradient of the water, an unstable water cone happened. Based on the simulation data, they developed a correlation to calculate the critical oil rate and determine the optimized well penetration length as:

$$q_{oc} = \frac{\Delta\rho k_v}{\mu_o}\left[r_e - \sqrt{{r_e}^2 - r_e(h_o} - h_{op})\right]^2 \left[\frac{k_v}{\sqrt{{k_h}^2 + {k_v}^2}} + \frac{h_{op}\left[\frac{1}{r_w} - \frac{1}{r_e}\right]}{ln\frac{r_e}{r_w}}\right] \qquad 2.16$$

Where, all the parameters are in field units.

Menouar and Hakim (1995) studied the effects of various reservoir parameters such as anisotropy ratio and mobility ratio on water coning behaviour. They estimated the critical oil rate based on the large number of simulation data and their results were similar with the other published work.

2.2. Water Coning Control Methods

People began to seek ways to control water coning problem shortly after knowing the coning phenomenon. However, literature survey shows that most coning control work was published after 1980. Several practical solutions have been developed to delay the water breakthrough time and minimize the problems of water coning in vertical wells. The basic methods included increasing the distance between the bottom perforation and the original OWC, separating oil and water in the OWC using horizontal impermeable barriers, controlling the fluids mobility in the reservoir, producing oil and water separately by Down hole water sink (DWS) wells and so on. Some of the methods are briefly described as followings:

Karp et al. (1962) considered several factors involved in creating, designing and locating horizontal barriers for controlling water coning. They studied different designs of the horizontal barriers, such barrier radius, thickness, permeability and position. They established an experimental apparatus to test the effects of different cement barriers and choose the right materials for different reservoirs. They found that reservoirs with high-density or high viscosity crude oils, low permeability or thin oil-zone thickness were not suitable to use this technology. On the other hand, this technology might impede the water drive from the field point of view.

Pirson and Mehta (1967) found that the horizontal barrier just delayed the water breakthrough time while it did not provide absolute remedy to the water-coning problem. Water would overpass the barrier and breakthrough to the production interval when the cone radius became greater than the barrier. It is useful only where the horizontal fracture is available to form such an impermeable barrier.

Smith and Pirson (1963) investigated the effect of fluid injection to control water coning in oil and gas wells. They considered position and length of the completion interval, point of fluid injection, the viscosity of the injected fluid and thicknesses of the oil and water sections. They found that the WOR was reduced by injecting oil at a point below the producing interval and the reduction was improved if the injected fluid was more viscous than the reservoir oil. From the experimental results, they concluded that: the optimum point of fluid injection was the point closest to the bottom of the producing interval that did not interfere with the oil production when the production rate is normal; the injection point moved down with the increase of production rate for maximizing the coning control efficiency. However, more and more fluid should be injected back to the reservoir with the increase of time. Mobility control means injecting chemical additives such as surfactants and polymers or other gelling agents into the water phase to control its mobility.

Paul and Strom (1988) proposed to inject water-soluble polymeric gels to control the bottom water mobility. They designed different polymeric gels for various water properties and carried out a serial of experiments in the lab.

Kisman et al (1991, 1992) proposed two methods to reduce the water cut in the well by injecting a composite slug comprising water wetting agent into the reservoir to modify the reservoir matrix to increase its water-wetted character, and non-condensable gas for further laterally extending the matrix surface modification. The slug of a water-wetting agent ensured the main path of the following gas slag through the water zone where it would increase gas saturation area. Thus relative permeability to water would be reduced. The methods could delay the water breakthrough time and reduce water cut in the produced fluids. Comparing to the vertical wells, horizontal wells can be drilled along the top of the formation, and the distance between the producing interval and oil-water contact can be optimized. Thus, they can achieve a higher flow rate with the same pressure drawdown and have greater areal sweep efficiency (Joshi, 1991; Chen, 1993; Permadi et al. 1997). Gilman et al. (1995) gave a good field example of the application of horizontal wells in the Yates Field Unit in West Texas where the horizontal wells successfully reduced the gas and water coning in thin oil columns. Wu et al. (1995) described Texaco's efforts to evaluate, justify and drill a horizontal well in Amber Field to suppress gas and water coning problems. The target reservoir contained a very thin pancake oil zone sandwiched between a large gas cap and a strong bottom water aquifer. They numerically evaluated the possibility of using horizontal wells to reduce water coning and improve oil recovery in the Amber Field. The simulation results and field histories indicated that horizontal wells completed in the gas cap could significantly reduce water coning and improve the ultimate oil recovery in a thin oil reservoir with a moderate sized gas cap. However, water cresting problem in horizontal wells is also a big challenge to engineers which is very hard to control as well as water coning in vertical wells. In recent years, down hole oil-water separation (DOWS) technology as a technique of separating water down hole to reduce surface water production has been developed. This technique allows water to be separated in the wellbore and injected into a suitable injection zone down hole while oil is produced to the surface. Shortly after the introduction of the DOWS technology to the oil industry in the 1990's, considerable research work has been done and several trial applications have been undertaken to test the technology. Matthews et al. (1996) reported the successful installation of DOWS in Alliance Field, Canada. They concluded that the system held considerable promise to be both technically and economically feasible for mature fields producing at high water oil ratio. However, this technology does not solve the oil-bypass problem caused by the water development.

Ehlig-Economides et al. (1996) studied the effects of well's total penetration well on water coning control. The perforation interval of the well is extended to cover the entire oil zone and into the bottom water zone. In this way, the radial flow of fluid can be maintained to avoid the water cone development and the oil-bypass problem can be solved. This completion method can get higher oil production rate and ultimate recovery comparing with the partial penetration method. However, it produces much more water to the surface than other methods, so the cost of water handling is increased and large amount of water disposal may cause environmental problems.

2.2.1. Down Hole Water Sink (DWS) Technology

In 1955, Widmyer introduced and patented a novel coning control idea to the petroleum industry- down hole water sink (DWS) technology. In his patent, he used two separated completions in one well to control water coning: one produced oil from the oil zone and the other drained water in the aquifer. Thus, the water coning could be controlled by the two opposite pressure drawdown. Pirson and Mehta (1967) numerically tested this technology and concluded that, DWS might reduce the growth of water cone. Driscoll (1972) refined the idea by having multiple completions with the lowermost completion below the oil/water contact. However, little attention was paid to this technology at that time, the reasons might be that, industry had low confidence to install it and water coning problem was not as serious as nowadays at that time. The interest of the oil industry returned to the DWS technology after

Wojtanowicz et al. improved it even further into a more workable and successful method when they simulated a dual completion using a “tailpipe water sink” in 1991. First, an oil well is drilled through the oil-bearing zone to the underlying aquifer. Then, the well is dually completed both in the oil and water zones. A packer separates the oil and water perforations. During production, oil flows into the upper completion being produced up the annulus between the tubing and the casing, while water is drained through the lowermost completion through perforations in the casing and then lifted up through the open tubing below the initial OWC. As a result, the produced oil is water free and the drained water is oil free.

Swisher and Wojtanowicz (1995a, 1995b) described the first field application of

DWS wells in the Nebo-Hemphill Field, LaSalle Parish, Louisiana. The DWS well could not only prevent water coning, but also reverse the water cone after breakthrough. The well greatly increased oil production rate compared with conventional wells. Bowlin et al. (1997) reported another field application of the technology by Texaco Inc. with the name of in-situ gravity segregation in Kern County, California. The well was installed in a location with 10 years of prior water coning problems. The results showed that this installation successfully controlled the water coning problem, and the oil production rate was doubled. After that, considerable efforts have been put on the research of DWS worldwide (Shirman and Wojtanowicz, 1997; Gunning et al, 1999; Ould-amer et al, 2004; Siemek and Stopa, 2002, Utama, 2008 and so on). Until now, DWS completion has been field tested in numerous reservoirs all over the world with good results. Results also indicate that water coning develops fast while the oil reversing is a slow process. The drawback of this technology is that, it brings large amount of water to the surface which requires more water processing facility and adds the production costs. In order to conquer this disadvantage of DWS system, Wojtanowicz and Xu (1992) proposed a concept of Downhole Water Loop (DWL) technique to cut back the volume of formation water produced by an oil well from a hydrocarbon reservoir underlain by a water zone. The method employed dual completion of the well inside the water zone, below the OWC to install the water loop equipment (separated by a packer) in addition to the conventional completion in the oil zone (above the OWC). The water loop installation included a submersible pump, the upper (water sink) perforations and the lower (water source) perforations. A submersible pump would drain the formation water around the well from the water sink, and then would re-inject the same water back to the water zone through the water source perforations. A simulation study was conducted to investigate hydrodynamic performance of the method to restrain water movement towards oil-producing perforations. The down hole water loop was mathematically modelled by computing flow potential distribution generated by two constant-rate sinks (oil and water) and one constant-rate source (water) located between the three linear boundaries and the constant-pressure outer radial boundary. The study revealed that the shape of the dynamic OWC in the well's vicinity could be effectively controlled by the method so that the oil production rates could be 2-4 times higher than the critical rates obtained when using conventional completion. Also, the method had the advantage to become a solution to the environmental compliance problem associated with disposal of produced water. From the standpoint of the reservoir engineering theory presented in their study, formation water could be kept away from oil-production perforations so that the oil recovery per well could also be improved.

In another DWL study, Wojtanowicz and Shirman (1996) determined the effect of hydraulic communication (cement leak) between the water drainage and injection zones. The leak would reduce the size of the water drainage zone under the oil-producing perforations and make the system inefficient. In that study, the down hole drainage injection was mathematically modelled as a system of three sinks operating under steady-state flow conditions in a multi-layered porous medium. The results of simulation runs revealed principal relationships between the reservoir engineering factors (fluid mobility, configuration of geological strata, and the degree of zonal isolation) and the production design factors (the position of well completions, and the oil production and water injection rates). Also, the study showed that for a determined geological conditions and well completion geometry, there was a unique relationship between the water injection and oil production rates, which ensured stability of OWC resulting in the continuous production of oil with minimum amount of water.

In 2011, L. Jin and A. K. Wojtanowicz, came out with an analytical model for the assessment of water-free production in oil wells with down hole water loops (DWL) for coning control. In their research, they;

1. Develop an analytical model considering penetration length and reservoir anisotropy to assess the critical values for

   DWL: water-free oil production rate in the top completion, oil-free water loop rate between the middle and bottom completions, minimum spacing between middle and bottom completions needed for the well reservoir system;
2. Identify most significant factors that control DWL using sensitivity analysis;
3. Statistically assess which reservoirs are potentially suitable for installing DWL.

CHAPTER THREE

3.1 MODELING OF CONE BEHAVIOUR IN HORZONTAL WELLS AND BREAKTHROUGH TIME

The development and behavior of cone (water or gas) into oil reservoir is triggered by production rate and influenced by the fluid properties of the water or the gas and the oil involved, as well as, the acting gravitational force. Every production rate advances its own cone height which is stabilized by the force of gravity in action. In this study, Ozkan(1990) method for modeling the break through time of water cone into horizontal well in oil producing reservoir was adapted for the development of the cone behavior and the breakthrough time for every cone that will be advanced by a particular production rate.

3.1.1 MECHANISM OF WATER CONING

Oil reservoirs with bottom water drive have high oil recovery due to supplemental energy from the aquifer. As shown in Figure 3.1, wells are often penetrated in the top section of the oil formation to minimize or delay water coning when there is no gas cap in the reservoir. The main reason of water coning is that, water moves to the direction of least resistance in the reservoir while balanced by its gravity to keep equilibrium. It is obvious in Figure 3.1 that, oil production in the well creates a pressure drawdown which elevates the oil water contact (OWC) in the immediate vicinity of the well. Water has the tendency to remain below the oil because of its higher density, which counterbalances the pressure drawdown caused by the oil production. These counterbalancing forces deform the OWC into a cone (or crest in horizontal wells) shape as we see in Figure 3.1. Because the production rate of a well is directly proportional to both the pressure drawdown and the reservoir permeability, one has to impose a larger pressure drawdown in a low permeability reservoir to achieve given production rate than in a high permeability reservoir. As a result, water coning is easier to occur in a low permeability reservoirs.

Essentially, there are three forces that influence the fluid flow distribution around the wellbore. These are, as mentioned earlier, capillary force, gravity force and viscous force. Capillary force is quite small in almost most cases and is ignored in this water coning studies. Gravity force is downward in the vertical direction and arises from fluid density differences. Viscous force is the pressure drawdown causing fluids flowing in the reservoir as described by Darcy’s Law. At a given time, there is a balance of these forces at any points in the reservoir. When the oil production rate is constant in a well and the viscous force acting in the vertical direction is less than the gravity force at the wellbore, then the water cone is stable and will not break into the well. If the oil production rate is increased and makes the vertical-acting viscous force exceed the gravity force at the wellbore, the cone will rise and eventually break into the oil well, and is called water breakthrough. The minimum oil production rate at which breakthrough occurs is called the critical production rate. From the above discussion, it is clear to see that water coning can be minimized by reducing the pressure drawdown in the vicinity of the well. However, it is difficult to do in practice. Because the pressure drawdown is directly proportional to the oil production rate, reduction of pressure drawdown means the reduction of oil production rate at the same time. It is almost impossible to practice in the real fields. Engineers always reduce the well penetration or stimulate the horizontal permeability to reduce water coning. However, significantly additional pressure drawdown will be caused by short penetration skin which will accelerate the water coning. It is hard to lessen the vertical permeability, although the ratio of horizontal to vertical permeability can be increased by acidizing or hydraulically fracturing the formation.

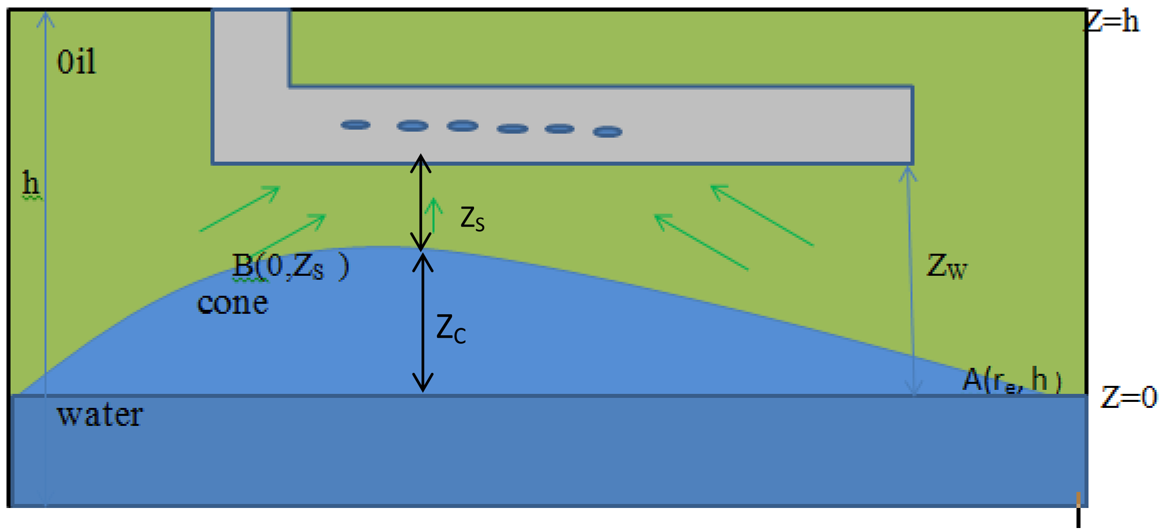

Figure 3.1 Schematic of water coning in reservoir under aquifer support in horizontal well.

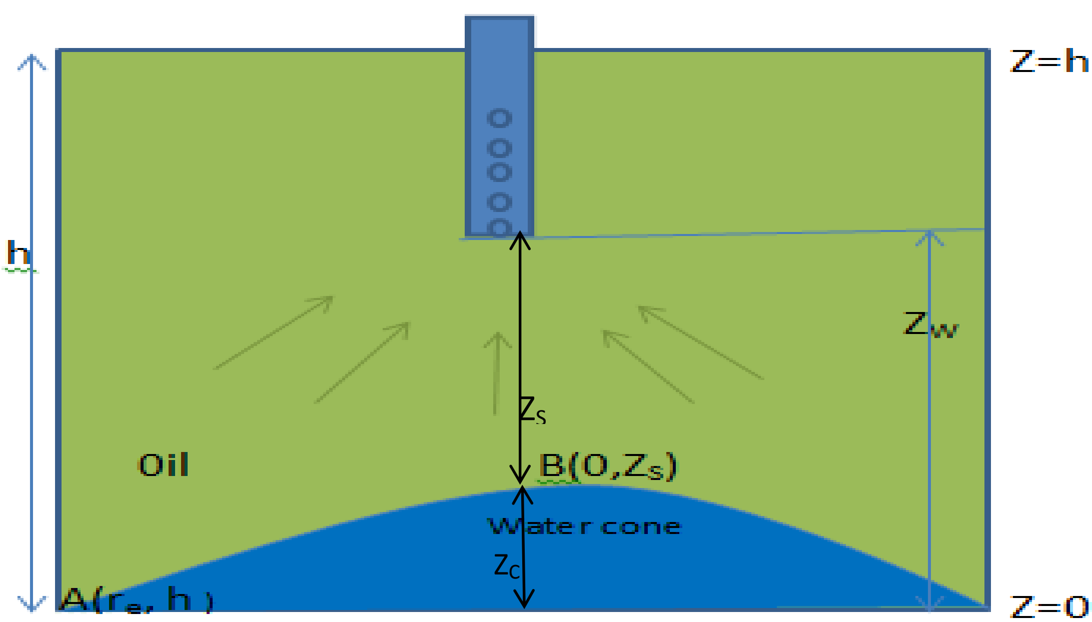

Figure3.2. Schematic of water coning in vertical wells

3.2 EQUILIBRIUM CONDITIONS

The establishment of equilibrium conditions of cone development at certain rate in reservoir forms the basis for theoretical derivation of rate- cone height relations under various conditions in producing reservoirs. In a coning prone reservoirs (i.e., aquifer supported or gas capped reservoirs), both static and dynamic equilibrium can be observed. For instance, if we consider a point A, away from the well on the water-oil interface with coordinates (r_e, h), and B on the apex of the water cone in equilibrium with the coordinate $(0, Zs)$ below an horizontal or vertical well producing at a rate, the static equilibrium condition may be expressed in terms of the flow potential difference and the gravitational force difference established. This is given as;

$$\Phi_A - \Phi_B = \Delta\rho g(h - Zs) \qquad 3.1$$

Thus a cone in equilibrium at elevation, Zs correspond to one rate as prescribed in the above equation.

And for dynamic equilibrium (implying that if disturbance of the interface occurs, buoyancy or gravitational forces will be stronger than the viscous forces.) the following expression holds;

$$\Delta\rho g \geq \frac{d}{dZs}(\Phi_A - \Phi_B) \qquad 3.2$$

The maximum and the critical rate correspond to the equal sign of the equation 3.2 above.

3.3 TRANSFORMATION OF ANISOTROPIC TO ISOTROPIC SYSTEMS.

In order to make the analyzing of coning problem in anisotropic reservoirs simpler, it is necessary to transform the anisotropy to isotropy first. For isotropic system, where the horizontal and the vertical permeabilities are equal, the system is innately is simple to solve but where the $k_v \neq k_h$, then it must first be transformed into an isotropic system before analyzing the coning problem of the system.

Horizontal permeability is usually higher than vertical permeability, except in fractured formations. However, the vertical permeability effect is usually neglected when the well is fully penetrated and the horizontal permeability is much higher than the vertical permeability. Because at this situation, fluid moves in a horizontally radial flow which can be expressed as follows in steady state;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = 0 \qquad 3.3$$

But when the well is partially penetrated and the water drive of aquifer is strong, the vertical permeability effect cannot be ignored anymore (Kucuk and Brigham, 1979; Sheng, et al, 2006). The diffusivity equation then becomes;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{\partial^2 p}{\partial z} = 0 \qquad 3.4$$

For horizontal wells, except at the ends of the well flow is bi-dimensional in x-z or y-z plane depending on the direction the horizontal well is laid. Therefore, the changed in variables to be used to transform anisotropic case to isotropic form affects x(or y) and z co-ordinates only if the ends effects are not considered as in Chaperon case(as he treated his case as 2-D model)(Chaperon, 1986).

But for the vertical well, flow takes place in all three dimensions, the change in variables affect x, y and z coordinates. The transformation is such that;

$$K_x\frac{\partial}{\partial x} + k_z\frac{\partial p}{\partial z} = k'\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial z}\right) \quad \text{(For horizontal well)} \qquad 3.5$$

$$K_x\frac{\partial}{\partial x} + K_y\frac{\partial}{\partial y} + k_z\frac{\partial p}{\partial z} = k'\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z}\right) \quad \text{(For vertical system)} \qquad 3.6$$

In order to transform the original anisotropic system to equivalent isotropic system, the following relationship should be satisfied:

$$k' = k'_x = k'_y = k'_z \qquad 3.7$$

$$\Rightarrow \bar{k} = k'_x = k'_y = k'_z = \sqrt[3]{k_x k_y k_z} \qquad 3.8$$

$$\begin{cases} x' = \sqrt{\frac{\bar{k}}{k'_x}}\, x \\ y' = \sqrt{\frac{\bar{k}}{k'_y}}\, y \\ z' = \sqrt{\frac{\bar{k}}{k'_z}}\, y \\ \Delta\rho' = \sqrt{\frac{k_z}{\bar{k}}}\, \Delta\rho \end{cases} \qquad 3.9$$

With the same process, diffusivity equation in 2-D Cylinder coordinates is:

$$\frac{k_r}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{k_z}{\mu}\frac{\partial^2 p}{\partial z} = 0 \qquad 3.4$$

Define;

$$\begin{cases} r' = a_r r \\ z' = a_z z \end{cases} \qquad 3.5$$

Assume

$$a_r a_z = 1 \qquad 3.6$$

Then by substituting equation 3.5 into equation 3.4 gives;

$$\frac{a_r k_r}{r'} \frac{1}{r} \frac{\partial}{\partial r'}\left(r \frac{\partial p}{\partial r'}\right) + \frac{a_z k_z}{z'} \frac{\partial^2 p}{\partial z'} = 0 \qquad 3.7$$

$$\begin{cases} k_r' = a_r{}^2 k_r \\ k_z' = a_r{}^2 k_z \end{cases} \qquad 3.8$$

In order to transform the original anisotropic system to equivalent isotropic system, the following relationship should be satisfied;

$$k_r' = k_z' \qquad 3.9$$

$$\Rightarrow \; a_r{}^2 k_r = a_r{}^2 k_z \qquad 3.10$$

When we combine equation 3.6 and equation 3.10 the following can formed;

$$\begin{cases} a_r = \sqrt[4]{\frac{k_z}{k_r}} \\ a_z = \sqrt[4]{\frac{k_r}{k_z}} \end{cases} \qquad 3.11$$

$$\bar{k} = k_r' = k_z' = \sqrt[2]{k_r k_z} \qquad 3.12$$

$$\begin{cases} r' = \sqrt{\frac{\bar{k}}{k_r}} r \\ z' = \sqrt{\frac{\bar{k}}{k_z}} z \\ \Delta\rho' = \sqrt{\frac{k_z}{\bar{k}} \Delta\rho} \end{cases} \qquad 3.13$$

After treating the anisotropic system, an equivalent isotropic system can be obtained. Equation 3.3 can be used instead of Equation 3.4 in water coning problem, which will make the task much easier to carry out.

Note that, the above derivation is based on the method presented by Spivey and Lee (1999).

3.4 MODEL ASSUMPTIONS:

1. An oil reservoir with active aquifer at bottom or gas at the top was considered.(As water and gas coning into an oil bearing formation is studied with the same model, oil-water system is considered first for this study since the modeling of gas coning only needs the properties of the water phase to be replaced with those of the gas phase)
2. The model oil reservoir is homogeneous and is bounded at the radial direction i.e., ($r = \sqrt{x^2 + y^2}$)
3. The horizontal permeability, K_H and vertical permeability, K_V are assumed not to be the same, i.e., anisotropic.
4. The initial or the original oil-water (or gas-oil) contact is assumed flat and the oil zone has uniform thickness, h.
5. Capillary gradient and oil stripping are neglected,
6. Mobility of water and oil are assumed to the same
7. The density difference between oil and water is accounted for when computing the interfacial velocities
8. The water is assumed to be in gravity equilibrium at all times and initial pressure is preserved at the original oil-water contact(WOC)
9. It is assumed that, at high production rates, the density difference would have insignificant influence on the behavior of the cone, therefore at high production rate constant pressure prevails at all times.

3.5 DIFFUSSIVITY EQUATION FORMULATION

With the above stated assumptions, the reservoir model under this study is governed by steady-state condition with the following equation;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda_l\frac{\partial \Delta p_l}{\partial r}\right)+\left(\lambda_l\frac{\partial \Delta p_l}{\partial z}\right)=0 \qquad 3.14$$

Here:

$\Delta p = p_i - p_m$ Where p_i is the initial oil pressure?

l= oil, o or water, w

And

$$\lambda_l = \frac{k_l}{\mu_l} \qquad 3.15$$

3.6 MATHEMATICAL FORMULATION

The flow equation governing the movement of the water cone in the oil is given by subtracting the water flow equation from the oil flow equation shown as follows;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda or\frac{\partial(\Delta p_o - \Delta p_{w)}}{\partial r}\right)+\left(\lambda oz\frac{(\partial \Delta p_o - \Delta p_{w)}}{\partial z}\right)=0 \qquad 3.16$$

Where

$$\mathrm{M} = \frac{\lambda wr}{\lambda or} = \frac{\lambda wz}{\lambda oz} \text{ (assumed)} \qquad 3.17$$

Before breakthrough, there is no water production and water is at gravity equilibrium hence;

$$\Delta p_w = \rho_w \; gz \qquad 3.18$$

The velocity potential at oil phase is;

$$\Phi = \Delta p - p_o gz \qquad 3.19$$

The interfacial flow potential is given as;

$$\varphi = \frac{1}{\mu_0}[\Phi - (\rho_w \;\; - \rho_o \;\;)gz] \qquad 3.20$$

By substitution of equation (3.4), (3.5), (3.6) and (3.7) into (3.3), it becomes;

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_r}{k_z}\frac{\partial \varphi}{\partial r}\right) + \frac{\partial^2 \varphi}{\partial z^2} = 0 \qquad 3.21$$

3.6.1 DIMENSIONLESS TERMS DEFINITION

$$\varphi_D \; = \frac{\mu_O}{\Delta \rho g h}\varphi = \Phi_D - Z_D \qquad 3.22$$

Where

$$\Delta \rho = \rho_w - \rho_o \qquad 3.23$$

$$\Phi_D \; = \frac{\Phi}{\Delta \rho g h} \qquad 3.24$$

$$Z_D \; = \frac{z}{h} \qquad 3.25$$

And

$$r_D \; = \frac{r}{h}\sqrt{\frac{K_z}{K_r}} = \sqrt{{x_D}^2 + {y_D}^2} = \frac{\sqrt{x^2+y^2}}{h} \; \sqrt{\frac{K_z}{K_r}} \qquad 3.26$$

Therefore the dimensionless form of the resulting flow equation at the interface of the oil and water becomes;

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial\varphi_D}{\partial r_D}\right)+\left(\frac{\partial^2\varphi_D}{\partial {z_D}^2}\right)=0 \qquad 3.27$$

3.7 CONE BREAKTHROUGH TIME AND BUILD UP TIME DEVELOPMENT

For the advancing cone, distance or height attained by an oil particle moving at the apex is defined by this equation;

$$dz=V_Z dt=\frac{K_Z}{f}\frac{\partial\varphi}{\partial z}dt \qquad 3.28$$

Where $f=\emptyset(1-S_{wc}-S_{or})$ 3.29

$$t_D=\frac{K_z\Delta\rho g}{\mathrm{f}\mu_o h} \qquad 3.30a$$

$$t_D=\frac{K_z\Delta\rho g}{364.6\mathrm{f}\mu_o h} \text{ (in field units)} \qquad 3.30b$$

Integrating equation (3.15) and substituting equation (3.9), (3.12), (3.17), it becomes;

$$t_D=\int_0^{Z_D}\frac{dz_D}{(\partial\varphi_D/\partial z_D)r_{D=0}} \qquad 3.31$$

The breakthrough time is evaluated by;

$$t_{bD}=\int_{z_D}^{Z_{WD}}\frac{dz_D}{(\partial\varphi_D/\partial z_D)r_{D=0}} \qquad 3.32$$

CHAPTER FOUR

4.1 FLOW RATE AND THE CORRESPONDING CONE CORRELATION, AND BREAKTHROUGH TIME EVALUATION

4.1a. HORIZONTAL WELLS

The development of the diffusivity equation governing the movement of the oil at the oil-water interface resolves the major step in the estimation of the breakthrough time of a water cone or crest into horizontal well. Solving the velocity potential at the interface with the given reservoir and the horizontal well boundary conditions and then evaluating how it behaves with the developing cones(by differentiating the interface flow potential with respect to the cone height), the breakthrough time can be evaluated using the dimensionless time relation developed in chapter 3, equation (3.32)

4.1a.1 METHODOLOGY:

The development of the correlation for the breakthrough time of water/gas coning involves the following five steps:

1. Development of the solution of the fluid velocity potential at the interface, oil- water or gas-oil interface (which in this study diffusivity equation governing the reservoir condition stated in chapter three and green function were used).
2. Finding the differential of the developed flow potential with respect to z_D
3. Development of the flow rate and cone height correlation. This correlation determines the flow rate for every static cone height and can be used to

determine the critical rate of production for a particular well length and well location in the reservoir.

4. Estimation of the breakthrough time using equation(3.19 in chapter three)

5. Development of correlation for water cone breakthrough time, using Microsoft excels non-linear regression analysis technique for the breakthrough time-flow rates plot and breakthrough time-cone heights plot.

4.1a.2 INTERFACE VELOCITY POTENTIAL SOLUTION DEVELOPMENT

From the solution of the governing flow equation developed. i.e.,

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial \varphi_D}{\partial r_D}\right)+\left(\frac{\partial^2 \varphi_D}{\partial {z_D}^2}\right)=0 \qquad 4.1$$

Boundary condition:

$$\varphi_D(r_D, z_D=0)=0 \qquad 4.2$$

$$\frac{\partial \varphi_D}{\partial z_D}(r_D, z_D=1)=0 \qquad 4.3$$

Flux condition:

Assuming a point source at well location of $r_D = 0\ and\ z_D = z_{wD}$, and

The dimensionless rate defined by;

$$q_D=\frac{q\mu_{oB_O}}{2\pi k_r h^2 \Delta\rho g} \qquad 4.4a$$

In field units it becomes;

$$q_D=\frac{325.7q\mu_{oB_O}}{k_r h^2 \Delta\rho} \qquad 4.4b$$

The following flux condition is applied;

$$\text{Lim}\left(\lim \frac{1}{e}\int_{z_{D-\frac{e}{2}}}^{z_{D+\frac{e}{2}}} r_D \frac{\partial \varphi_D}{\partial r_D} z_{wD}\right) = -q_D$$

$$e \to 0 \; and \; r_D \to 0 \qquad 4.5$$

Where z_{wD} *is the well location*

The following general point source solution of flow potential at the interface is obtained by using the procedure given in Muskat, 1947.

$$\varphi_D = q_D \sum_{n=1}^{\infty} sin\epsilon_n \, z_D sin\epsilon_n z_{wD} B(\epsilon_n, r_D, r_{eD}) - z_D \qquad 4.6$$

Where

$$\epsilon_n = (2n-1)\,\pi/2 \qquad 4.7$$

And

$$B(\epsilon_n, r_D, r_{eD}) = K_0(\epsilon_n, r_D) + \frac{I_0(\epsilon_n, r_D)K_1(\epsilon_n, r_{eD})}{I_1(\epsilon_n, r_{eD})} \qquad 4.8$$

Horizontal solution:

The Horizontal well solution is obtained by the integration of the point source location i.e., equation 4.6, over the length given by;

$$\varphi_D = q_D \sum_{n=1}^{\infty} sin\epsilon_n \, z_D sin\epsilon_n z_{wD} \int_{-L_D}^{+L_D} B\,(\epsilon_n, r_D, r_{eD}) - z_D \qquad 4.9$$

By applying the relation in the Muskat (1947) and Ozkan and Raghavan(1990), the horizontal solution with the giving condition is obtained as;

$$\varphi_D(r_D = 0, z_D \neq z_{wD}) = \frac{q_D}{2L_D} In \frac{\tan(z_{wD+}z_D)\pi/4}{\tan(z_{wD-}z_D)\pi/4} - F_L + F_{BH} - z_D \qquad 4.10$$

Where

$$F_L = \frac{2q_D}{L_D}\sum_{n=1}^{\infty}\frac{sin\epsilon_n z_D sin\epsilon_n z_{wD} K_{i1}(\epsilon_n, L_D)}{\epsilon_n} \qquad 4.11$$

$$K_{i1}(\epsilon_n, L_D) = \int_{(\epsilon_n, L_D)}^{\infty} K_0(u)du = \frac{\pi}{2} - \int_0^{(\epsilon_n, L_D)} K_0(u)du \qquad 4.12$$

$$F_{BH} = \frac{2q_D}{L_D}\sum_{n=1}^{\infty}\frac{sin\epsilon_n z_D sin\epsilon_n z_{wD}}{\epsilon_n}\frac{K_1(\epsilon_{n,} r_{eD})}{I_1(\epsilon_n, r_{eD})}\int_0^{\epsilon_n} I_0(u)\,du \qquad 4.13$$

$$L_D = \frac{L}{2h}\sqrt{\frac{K_z}{K_r}} \qquad 4.14$$

$$r_{eD} = \frac{1}{h}\sqrt{\frac{K_z}{K_r}}\sqrt{x_e + \frac{k_x}{k_y}y_e} \qquad 4.15$$

Where L, is the length of the horizontal well

4.1a.3. DIFFERENTIATING THE HORIZONTAL WELL SOLUTION W.R.T CONE APEX (DIMENSIONLESS).

For

$$\beta = (z_{wD} - z_D)\,\pi/4 \qquad 4.15$$

$$\alpha = (z_{wD} + z_D)\,\pi/4 \qquad 4.16$$

$$\frac{\partial F_L}{\partial z_D} = \frac{2q_D}{L_D}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD} K_{i1}(\epsilon_n, L_D) \qquad 4.17$$

$$\frac{\partial F_{BH}}{\partial z_D} = \frac{2q_D}{L_D}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD}\,\frac{K_1(\epsilon_{n,}r_{eD})}{I_1(\epsilon_n,r_{eD})}\int_0^{\epsilon_n L_D} I_0(u)\,d \qquad 4.18$$

$$\frac{d\varphi_D}{dz_D} = \frac{q_D}{L_D}\frac{\pi}{4}\frac{tan^2\alpha}{tan\beta}\left(\frac{tan\alpha sec^2\,\beta + tan\beta sec^2\,\alpha}{tan^2\alpha}\right) - \frac{\partial FL}{\partial z_D} + \frac{\partial FHB}{\partial z_D} - 1 \qquad 4.19$$

4.1a.4 FLOW RATE CONE APEX (CONE HEIGHT) CORRELATION

Equation 3.1 forms the basis for the development of the flow rate-cone height correlation in this work, as already emphasized; it relates the different rates to the corresponding cone height.

$\Phi_A - \Phi_B = \Delta\rho g(h - Zs)$ (Recall equation 3.1)

In dimensionless form; by dividing the R.H with $\Delta\rho gh$ it becomes;

$$\Phi_{DA} - \Phi_{DB} = 1 - z_D \qquad 4.20$$

Equation 4.20 assumes the completion is at vertical distance h ft from the oil water contact OWC. If the completion is at elevation Z_w ft from the OWC, it is assumed that the completion will not drain the oil above the completion. Therefore, equation 4.20 becomes:

$$\Phi_{DA} - \Phi_{DB} = z_{wD} - z_D \qquad 4.21$$

By applying equation 3.11, Φ_D turns;

$$\Phi_D = \frac{q_D}{2L_D} In\frac{\tan(z_{wD+}z_D)\pi/4}{\tan(z_{wD-}z_D)\pi/4} - F_L + F_{BH} - z_D \qquad 4.22$$

By applying equation 4.21 and resolving the coning potential difference at A and B using the figure 3.1, q_D is developed as follows:

$$\Phi_A = \frac{q_D}{2L_D} In \frac{\tan(z_{wD+}z_{wD} - R_{wD})\pi/4}{\tan(z_{wD-}z_{wD} - R_{wD})\pi/4} - F_L(z_{wD} - R_{wD}) + F_{BH}(z_{wD} - R_{wD}) - z_{wD} - R_{wD} \quad 4.23$$

$$\Phi_B = \frac{q_D}{2L_D} In \frac{\tan(z_{wD+}z_D)\pi/4}{\tan(z_{wD-}z_D)\pi/4} - F_L(z_D) + F_{BH}(z_D) - z_D \quad 4.24$$

$$\Phi_A - \Phi_B = \frac{q_D}{2L_D}\left[In \frac{\tan(z_{wD+}z_{wD} - R_{wD})\pi/4}{\tan(z_{wD-}z_{wD} - R_{wD})\pi/4} - In \frac{\tan(z_{wD+}z_D)\pi/4}{\tan(z_{wD-}z_D)\pi/4}\right] - F_L(z_{wD} - R_{wD}) + F_{BH}(z_{wD} - R_{wD}) - z_{wD} - R_{wD} + F_L(z_D) - F_{BH}(z_D) + z_D = z_{wD} - z_{DS} \quad 4.25$$

Where

$$z_{wD} - z_{DS} = z_D \text{ or } z_{wD} - z_{DS} = z_{Dc} \quad 4.26$$

$$\Phi_A - \Phi_B = \frac{q_D}{2L_D}\left[In\left\{\frac{\tan(2z_{wD} - R_{wD})\pi/4}{\tan(R_{wD})\pi/4} \frac{\tan(z_{wD-}z_D)\pi/4}{\tan(z_{wD+}z_D)\pi/4}\right\}\right] - F_L(z_{wD} - R_{wD}) + F_{BH}(z_{wD} - R_{wD}) + F_L(z_D) - F_{BH}(z_D) = z_{wD} - R_{wD} \quad 4.27$$

$$T = \frac{1}{2L_D}\left[In\left\{\frac{\tan(2z_{wD} - R_{wD})\pi/4}{\tan(R_{wD})\pi/4} \frac{\tan(z_{wD-}z_D)\pi/4}{\tan(z_{wD+}z_D)\pi/4}\right\}\right] - G_L(z_{wD} - R_{wD}) + G_{BH}(z_{wD} - R_{wD}) + G_L(z_D) - G_{BH}(z_D) \quad 4.28$$

$$q_D = \frac{z_{wD} - R_{wD}}{T} \quad 4.29$$

$$G_L(z_D) = \frac{2}{L_D}\sum_{n=1}^{\infty} \frac{sin\epsilon_n z_D sin\epsilon_n z_{wD} K_{i1}(\epsilon_n, L_D)}{\epsilon_n} \quad 4.30$$

$$G_{BH}(z_D) = \frac{2}{L_D}\sum_{n=1}^{\infty} \frac{sin\epsilon_n z_D sin\epsilon_n z_{wD}}{\epsilon_n} \frac{K_1(\epsilon_n, r_{eD})}{I_1(\epsilon_n, r_{eD})} \int_0^{\epsilon_n} I_0(u)\, du \quad 4.31$$

If F_L and F_{BH} are neglected then q_D becomes;

$$q_D = \frac{(z_{wD} - r_{wD})2L_D}{\left\{In\frac{\tan(2z_{wD} - r_{wD})\pi/4}{\tan(r_{wD})\pi/4} - In\frac{\tan(z_{wD} + z_D)\pi/4}{\tan(z_{wD} - z_D)\pi/4}\right\}} \qquad 4.32$$

Where

z_{Ds} is the cone height, z_{wD} is the well location and z_D is the cone apex from the well location.

4.1a.5 ESTIMATION OF BREAKTHROUGH TIME.

The estimation of the breakthrough time is done numerically in this studies using Matlab program with equation 3.32 as the base formula.

$$t_{bD} = \int_{z_D}^{Z_{WD}} \frac{dz_D}{(\partial\varphi_D/\partial z_D)r_{D=0}} \qquad \text{(Recall equation 3.32)}$$

The breakthrough times are computed from the following equation;

$$t_{bD} = \int_{z_D}^{Z_{WD}} \frac{dz_D}{\left\{\frac{q_D\pi tan^2\alpha}{L_D 4\ tan\beta}\left(\frac{tan\alpha sec^2\beta + tan\beta sec^2\alpha}{tan^2\alpha}\right) - \frac{\partial FL}{\partial z_D} + \frac{\partial FHB}{\partial z_D} - 1\right\}} \qquad 4.26$$

4.1b VERTICAL WELL

The development of the cone behavior in vertical wells follows the same procedures as the one applied to the horizontal well. It is only the velocity potential that changes here and rest of the procedures just follows

4.1b.1 INTERFACE VELOCITY POTENTIAL SOLUTION DEVELOPMENT

The Velocity potential of the vertical well under the same flux and boundary conditions becomes ;(Ozkan (1990))

$$\varphi_D = \frac{q_D}{b} In\left\{\frac{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)}{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}\right\} + \frac{q_D}{2b} In\left\{\frac{\tan(1_+z_{D-}b)\pi/4}{\tan(1_-z_{D+}b)\pi/4}\right\} + F_{BV} - z_D \qquad 4.27$$

Where

$$F_{BV} = \frac{2q_D}{b}\sum_{n=1}^{\infty}\frac{sin\epsilon_n z_D sin\epsilon_n b}{\epsilon_n}\frac{K_1(\epsilon_{n,}r_{eD})}{I_1(\epsilon_{n,}r_{eD})} \qquad 4.28$$

4.1b.2 DIFFERENTIATING THE VERTICAL WELL SOLUTION W.R.T CONE HEIGHT (DIMENSIONLESS)

$$\frac{d\varphi_D}{dz_D} = \frac{q_D}{b}\ \frac{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)(\ \Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right))^2}\{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)[$$

$$\Gamma\left(\frac{3-z_D+b}{4}\right)(\frac{-1}{4})\Gamma\ \left(\frac{1-z_D-b}{4}\right)\psi\left(\frac{1-z_D-b}{4}\right) +$$

$$\Gamma\left(\frac{1-z_D-b}{4}\right)(\frac{1}{4})\Gamma\ \left(\frac{3-z_D+b}{4}\right)\psi\left(\frac{3-z_D+b}{4}\right)] -$$

$$\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)[\Gamma\left(\frac{3-z_D-b}{4}\right)\left(\frac{-1}{4}\right)\Gamma\ \left(\frac{1-z_D+b}{4}\right)\psi\left(\frac{1-z_D+b}{4}\right) +$$

$$\Gamma\left(\frac{1-z_D+b}{4}\right)\left(\frac{-1}{4}\right)\Gamma\ \left(\frac{3-z_D-b}{4}\right)\psi\left(\frac{3-z_D-b}{4}\right)]\} \qquad +$$

$$\frac{q_{D\pi}}{8b}[(-1/tank) - tank) + (1/tany) + tany)] \qquad +$$

$$\frac{2q_D}{b}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD}\frac{K_1(\epsilon_{n,}r_{eD})}{I_1(\epsilon_{n,}r_{eD})} - 1 \qquad 4.29$$

Where;

$k = (1 - z_D - b)$ and $y = (1 - z_D + b)$

For simplification

$J =$

$$\frac{q_D}{b}\frac{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)(\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right))^2}\{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)[\Gamma\left(\frac{3-z_D+b}{4}\right)(\frac{-1}{4})\Gamma\left(\frac{1-z_D-b}{4}\right)\psi\left(\frac{1-z_D-b}{4}\right)+\Gamma\left(\frac{1-z_D-b}{4}\right)(\frac{1}{4})\Gamma\left(\frac{3-z_D+b}{4}\right)\psi\left(\frac{3-z_D+b}{4}\right)]-$$

$$\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)[\Gamma\left(\frac{3-z_D-b}{4}\right)\left(\frac{-1}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)\psi\left(\frac{1-z_D+b}{4}\right)+\Gamma\left(\frac{1-z_D+b}{4}\right)\left(\frac{-1}{4}\right)\Gamma\left(\frac{3-z_D-b}{4}\right)\psi\left(\frac{3-z_D-b}{4}\right)]\} \qquad 4.30$$

$$G = \frac{q_{D\pi}}{8b}[(-1/tank) - tank) + (1/tany) + tany)] \qquad 4.31$$

And

$$\frac{\partial FBV}{\partial z_D} = \frac{2q_D}{b}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD}\frac{K_1(\epsilon_{n,} r_{eD})}{I_1(\epsilon_{n,} r_{eD})} - 1 \qquad 4.32$$

Therefore the equation 4.25 becomes;

$$\frac{d\varphi_D}{dz_D} = J + G + \frac{\partial FBV}{\partial z_D} - 1 \qquad 4.33$$

4.1b.3 FLOW RATE -CONE APEX (CONE HEIGHT) RELATION

The production rate-cone height relation is developed here using the same principles as that of under 4.1a4.

$$\Phi_A = \frac{q_D}{b}In\left\{\frac{\Gamma\left(\frac{3-z_{wD}+b}{4}\right)\Gamma\left(\frac{1-z_{wD}-b}{4}\right)}{\Gamma\left(\frac{3-z_{wD}-b}{4}\right)\Gamma\left(\frac{1-z_{wD}+b}{4}\right)}\right\} + \frac{q_D}{2b}In\left\{\frac{\tan\{(1_+z_{wD-}b)\pi/4\}}{\tan\{(1_-z_{wD+}b)\pi/4\}}\right\} + F_{BV}(z_{wD}) - z_{wD}$$

4.34

$$\Phi_B = \frac{q_D}{b} In\left\{\frac{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)}{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}\right\} + \frac{q_D}{2b} In\left\{\frac{\tan\{(1_+z_{D-}b)\pi/4\}}{\tan\{(1_-z_{D+}b)\pi/4\}}\right\} + F_{BV}(z_D) - z_D \quad 4.35$$

$$\Phi_A - \Phi_B = \frac{q_D}{b}\left[In\frac{\Gamma\left(\frac{3-z_{wD}+b}{4}\right)\Gamma\left(\frac{1-z_{wD}-b}{4}\right)}{\Gamma\left(\frac{3-z_{wD}-b}{4}\right)\Gamma\left(\frac{1-z_{wD}+b}{4}\right)} - In\frac{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)}{\Gamma\left(\frac{3-z_D-b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}\right]$$

$$+\frac{q_D}{2b}\left[In\frac{\tan\{(1+z_{wD}-b)\pi/4\}}{\tan\{(1-z_{wD}+b)\pi/4\}} - In\frac{\tan\{(1+z_D-b)\pi/4\}}{\tan\{(1-z_D+b)\pi/4\}}\right] + F_{BV}(z_{wD}) - z_{wD} -$$

$$+F_{BV}(z_D) + z_D = z_{wD} - z_{Ds} \quad 4.36$$

Where

$z_{wD} - z_{Ds} = z_D$ or $z_{wD} - z_{Ds} = z_{Dc}$ 4.37

$$\Phi_A - \Phi_B = \frac{q_D}{b}\left[In\left\{\frac{\Gamma\left(\frac{3-z_{wD}+b}{4}\right)\Gamma\left(\frac{1-z_{wD}-b}{4}\right)}{\Gamma\left(\frac{3-z_{wD}-b}{4}\right)\Gamma\left(\frac{1-z_{wD}+b}{4}\right)}\frac{\Gamma\left(\frac{3-z_D\,b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)}\right\}\right]$$

$$+\frac{q_D}{2b}\left[In\left\{\frac{\tan\{(1+z_{wD-}b)\pi/4\}}{\tan\{(1-z_{wD+}b)\pi/4\}}\frac{\tan\{(1-z_D+b)\pi/4\}}{\tan\{(1+z_D-b)\pi/4\}}\right\}\right] + F_{BV}(z_{wD}) - F_{BV}(z_D) = z_{wD} \quad 4.38$$

$$T = \frac{1}{b}\left[In\left\{\frac{\Gamma\left(\frac{3-z_{wD}+b}{4}\right)\Gamma\left(\frac{1-z_{wD}-b}{4}\right)}{\Gamma\left(\frac{3-z_{wD}-b}{4}\right)\Gamma\left(\frac{1-z_{wD}+b}{4}\right)}\frac{\Gamma\left(\frac{3-z_D\,b}{4}\right)\Gamma\left(\frac{1-z_D+b}{4}\right)}{\Gamma\left(\frac{3-z_D+b}{4}\right)\Gamma\left(\frac{1-z_D-b}{4}\right)}\right\}\right]$$

$$+\frac{1}{2b}\left[In\left\{\frac{\tan\{(1+z_{wD-}b)\pi/4\}}{\tan\{(1-z_{wD+}b)\pi/4\}}\frac{\tan\{(1-z_D+b)\pi/4\}}{\tan\{(1+z_D-b)\pi/4\}}\right\}\right] + G_{BV}(z_{wD}) - G_{BV}(z_D) \quad 4.39$$

$$q_D = \frac{z_{wD} - G_{BV}(z_{wD}) + G_{BV}(z_D)}{T} \quad 4.40$$

$$G_{BV}(z_D) = \frac{2}{b}\sum_{n=1}^{\infty}\frac{sin\epsilon_n z_D sin\epsilon_n b}{\epsilon_n}\frac{K_1(\epsilon_{n,} r_{eD})}{I_1(\epsilon_{n,} r_{eD})} \quad 4.41$$

If we neglect second and the third terms in the numerator of the above equation then:

$$q_D = \frac{z_{wD}}{T} \quad 4.42$$

4.1b.4 BREAKTHROUGH TIME EVALUATION

This is done by substituting equation 4.25 into equation 3.32, which becomes;

$$t_{bD} = \int_{z_D}^{z_{wD}} \frac{dz_D}{\left\{J+G+\frac{dFBV}{dx}-1\right\}} \quad 4.43$$

4.2 CHAPERON APPROACH.

The chaperon (2-D model) approach for this work is strewed through the same methodology given under Ozkan approach.

4.2a HORIZONTAL WELLS

4.2a.1 INTERFACE VELOCITY POTENTIAL SOLUTION DEVELOPMENT

The interfacial flow potential is derived from the chaperon's fluid flow potential which is given as;

$$\Phi(\mathrm{x}, \mathrm{z}) = \frac{Q\mu}{2\pi Lk} \log\left(cosh\frac{\pi x}{h} - cos\frac{\pi z}{h}\right) \quad 4.44$$

Converting in to the dimensionless form gives;

$$\Phi_D = \frac{q_D}{2\pi L_D} \log(cosh\pi x_A - cos\pi z_D) \quad 4.45$$

Interfacial flow potential φ_D becomes ;

$$\varphi_D = \frac{q_D}{2\pi L_D}\log(cosh\pi x_A - cos\pi z_D) \qquad 4.46$$

4.2a. 2 DIFFERENTIATING THE WELL SOLUTION W.R.T CONE APEX

$$\frac{d\varphi_D}{dz_D} = \frac{q_D \pi sin\pi z_D}{2\pi L_D(cosh\pi x_A - cos\pi z_D)} \qquad 4.47$$

4.2a.3 FLOW RATE -CONE APEX (CONE HEIGHT) RELATION

This development follows the same procedure under the 4.1a.4 and 4.1b.3

By substituting the flow potential at point A and B into equation 4.20 using figure 3.1 gives

$$q_D = \frac{29.4L_D(z_{wD} - z_D)}{\log\left(\frac{\cosh\left(\frac{\pi x_A}{h}\right)}{1 - cos\pi z_D}\right)} \qquad 4.48a$$

$$q_D = \frac{29.4L_D(z_{wD} - z_D)}{\log\left(\frac{\cosh(\frac{\pi x_A}{h}\sqrt{\frac{k_v}{k_h}}}{1 - cos\pi z_D\sqrt{\frac{k_v}{k_h}}}\right)} \text{ (For anisotropic form)} \qquad 4.48b$$

4.2a.4 ESTIMATION OF BREAKTHROUGH TIME.

This is done using mat lab programming with the equation 3.21 as the base formula. This becomes;

$$t_{bD} = \int_{z_D}^{z_{wD}} \frac{dz_D}{\left\{\frac{q_D\ \pi sin\pi z_D}{2\pi L_D\ (cosh\pi x_A - cos\pi z_D)}\right\}} \qquad 4.49$$

4.2b VERTICAL WELLS

The same methodology for the horizontals is applied to the vertical well to enable the estimation of the breakthrough time conditions.

4.2b.1INTERFACE FLOW POTENTIAL DEVELOPMENT

Chaperon fluid flow potential is given as following, from which the interfacial flow potential is derived.

$$\Phi(z, x = 0) = \frac{Q\mu}{2\pi k} \sum_{-\infty}^{+\infty} \frac{1}{\|z_s + 2nh\|} \quad 4.50$$

In dimensionless unit it becomes;

$$\Phi_D = q_D \sum_{-\infty}^{+\infty} \frac{1}{\|z_D + 2n\|} \quad 4.51$$

Again, from the relation in equation 3.9, the interfacial flow potential, φ_D for vertical well becomes;

$$\varphi_D = q_D \sum_{-\infty}^{+\infty} \frac{1}{\|z_D + 2n\|} \quad 4.52$$

4.2b.2 DIFFERENTIATING THE WELL SOLUTION W.R.T CONE APEX

$$\frac{d\varphi_D}{dz_D} = q_D \sum_{-\infty}^{+\infty} \frac{1}{\|z_D + 2n\|^2} \quad 4.53$$

4.2b.3 FLOWRATE CONE HEIGHT RELATION

By applying the same principles under 4.1a.4 with the employment of equation 3.1 gives the rate-cone height relations as;

$$q(z,r_e) = \frac{0.0485\Delta\rho(h-z_s)k_z}{\|z_s+2nh\|-[r_e{}^2(z_s+2nh)^2]^{0.5}} \quad 4.54$$

The dimensionless form of the flow rate is computed using equation 4.4b

Which is, $q_{D=}\frac{325.7q\mu_{oB_O}}{k_r h^2\Delta\rho}$ (Recall equ. 4.4b)

4.2b.4 BREAKTHROUGH TIME EVALUATION OF VERTICAL WELL

The breakthrough time evaluation in the vertical well is also done numerically in mat lab program by the substituting equation 4.53 into equation 3.32.

This becomes;

$$t_{bD} = \int_{z_D}^{z_{wD}} \frac{dz_D}{\left\{q_D \sum_{-\infty}^{+\infty}\frac{1}{\|z_D+2n\|^2} - 1\right\}} \quad 4.55$$

4.3 GREEN'S SOURCE FUNCTION APPROACH

The source function is also another approach that can be used to model the water/gas coning behavior in reservoirs. It is treated in this work to show how it can be applied in the coning behavior modeling (that is how it is treated to evaluate rate –cone height relationship and breakthrough time) following the same laid down procedures outlined under the Ozkan and Chaperon approach. This method is only treated to horizontal well in this case, the details of which is given under appendix. But, the vertical well can also treated this approach applying the same principles.

CHAPTER FIVE

APPLICATION OF THE SOLUTIONS TO EXAMPLES AND FIELD CASES

Table 5.1 Ozkan example case and Field case example

Example Case 1.	Field Case
Initial oil zone thickness, h, ft =42	Initial oil zone thickness, h, ft =56
Oil density, ρ_o,gcc= 0.861	Oil density, ρ_o,gcc= 0.86
Formation volume factor, B_O=1.102	Formation volume factor, B_O=1.12
Oil viscosity,μ_o =1.44	Oil viscosity,μ_o =2.78
Horizontal permeability,k_h, md =37	Horizontal permeability,k_h, md =2000
Vertical permeability, $k_v,\ md = 3.7$	Vertical permeability, $k_v,\ md$ =1400
Well bore radius, $r_w, ft = 0.29$	Well bore radius, $r_w, ft = 0.29$
Drainage radius, r_e, ft =1053	Drainage radius, $r_e, ft = 785$
Porosity,$\phi = 0.164$	Porosity,$\phi = 0.164$
Residual oil saturation, $S_{or} = 0.337$	Residual oil saturation, $S_{or} = 0.337$
Connate water saturation,S_{wc}=0.288	Connate water saturation,S_{wc}=0.288
Horizontal well length, ft= 660	Horizontal well length, ft= 1365
Vertical well perforated interval, ft = 24	Vertical well perforated interval, ft =24

Table 5.2 Joshi Example case

Joshi Example case
Initial oil zone thickness, h, ft =160
Oil density, ρ_o,gcc= 0.86
Formation volume factor, B_O=1.1
Oil viscosity,μ_o =1.3
Horizontal permeability,k_h, md =200
Vertical permeability, k_v, md =20
Well bore radius, r_w, ft = 0.27
Drainage radius, r_e, ft =1500
Porosity,ϕ =0.165
Residual oil saturation, $S_{or} = 0.337$
Connate water saturation,S_{wc}=0.288
Horizontal well length, ft=1000
Vertical well perforated interval, ft = 24

5.0 RESULTS AND DISCUSSION

5.1 HORIZONTAL WELL:

The horizontal well is considered in this study as an infinite line source at a dimensionless well length of above or equal to 2.4($L_D \geq$ 2.5), and finite for $L_D < 2.4$, but FL and FBH terms in equations 4.10 and 4.19 were applied in both conditions contrary to the what Ozkan did in their work(Ozkan ,1990).

The Breakthrough time curves are captured in figure 5.1., which shows the times(in dimensionless form) each cone height developed from the water-oil contact(WOC) by their corresponding rates breaks into the horizontal well as the apex of the cones move steadily towards the well bore. The breakthrough curves were developed for three different well locations where the variations of each breakthrough time conditions for each well position is clearly depicted.

Figure 5.2 shows the breakthrough time conditions for different field data application. In the case of the Ozkan field data example where the well location, zwD= 1, the breakthrough time curve is the highest compared to the Field data curve and the Joshi field data curve. This indicates that the well location, zwD is a key parameter that influences breakthrough time of coning. Though, the two other cases curves (the Field data and the Joshi example case) have the same well location but different dimensionless horizontal well length their curves almost lie

on each other. This is due to the application of the same rate (i.e. qD /$2L_D$) in each of the three field case examples. A difference in their curves position would have been clearly shown by using different rate for each of the three field applications as shown in Figure 5.3 and 5.4. Here the Field data example with the highest dimensionless length gives the highest rate curve followed by the Ozkan field data case which in turn has a higher L_D of 2.5. The Joshi field case has L_D equal to 1.98 and hence the lowest rate curves in both figures 5.3 and 5.4.

GRAPHICAL RESULTS:

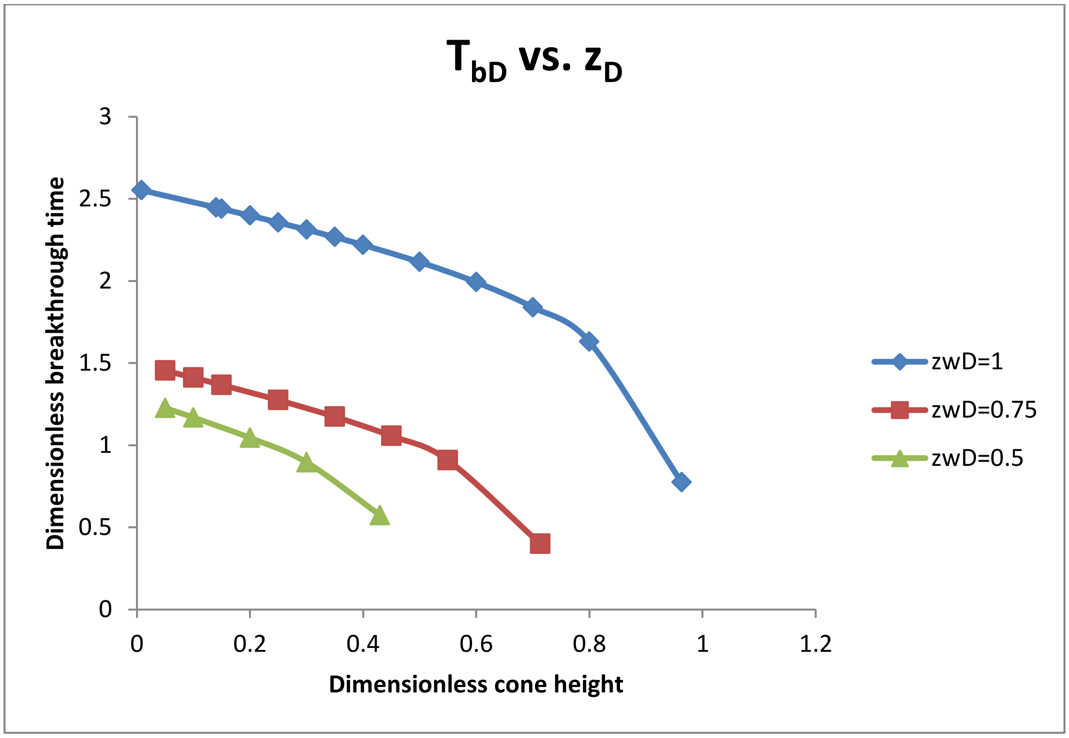

Figure 5.1 Dimensionless breakthrough time vs. D. cone height (Ozkan Approach, horizontal wells. Only Ozkan field example applied)

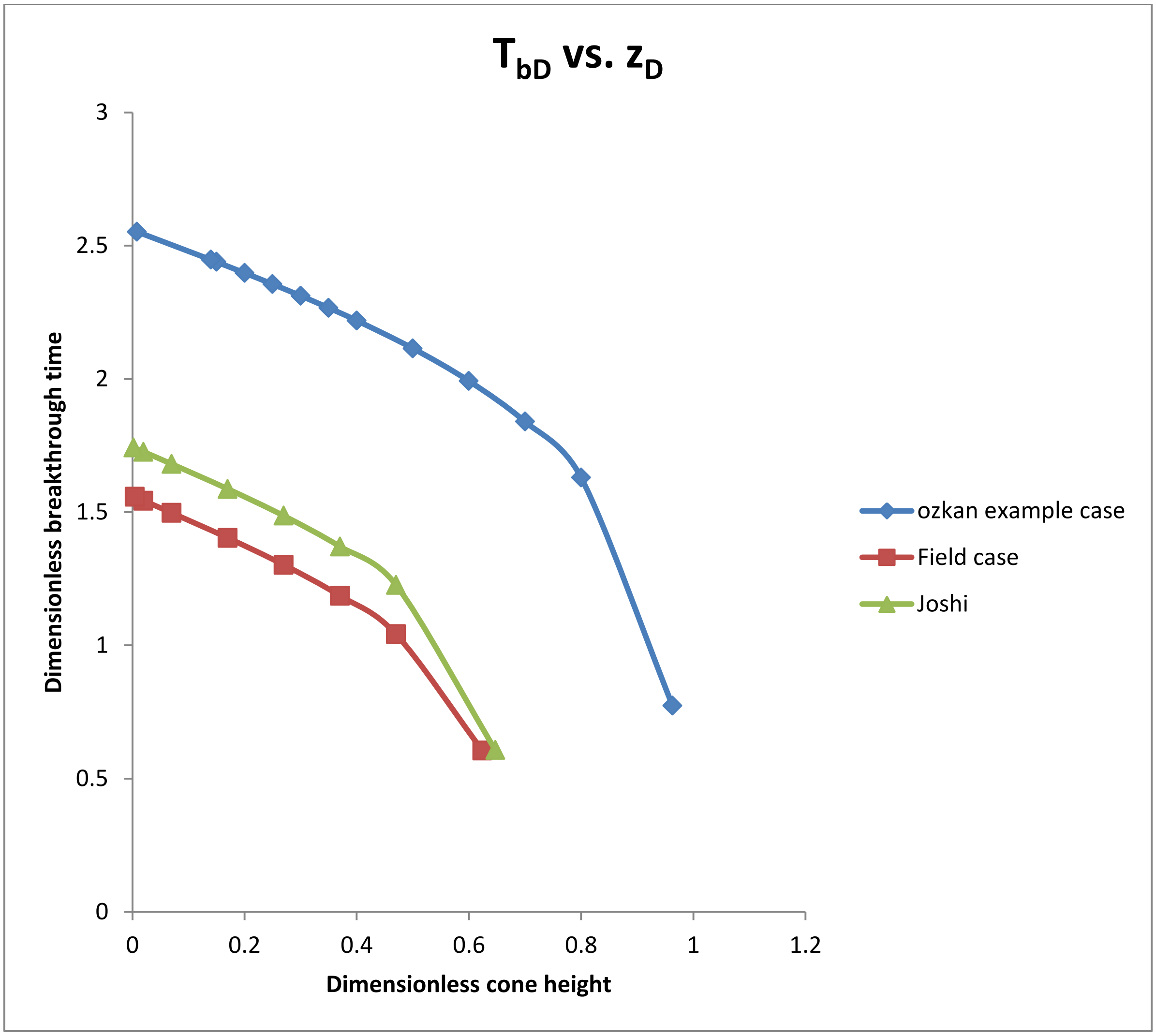

Figure 5.2 Dimensionless cone height vs. D. breakthrough time (Ozkan Approach, horizontal wells)

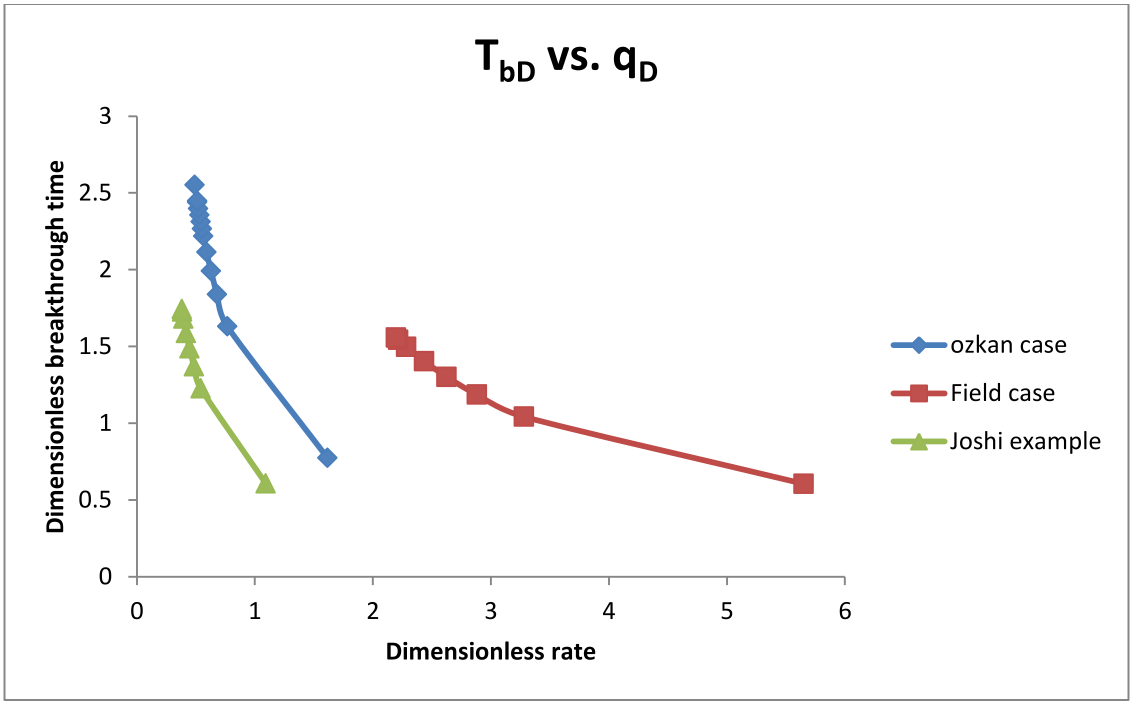

Figure 5.3 Dimensionless breakthrough time vs. Dimensionless rate (Ozkan Approach, horizontal wells. Only Ozkan field example applied)

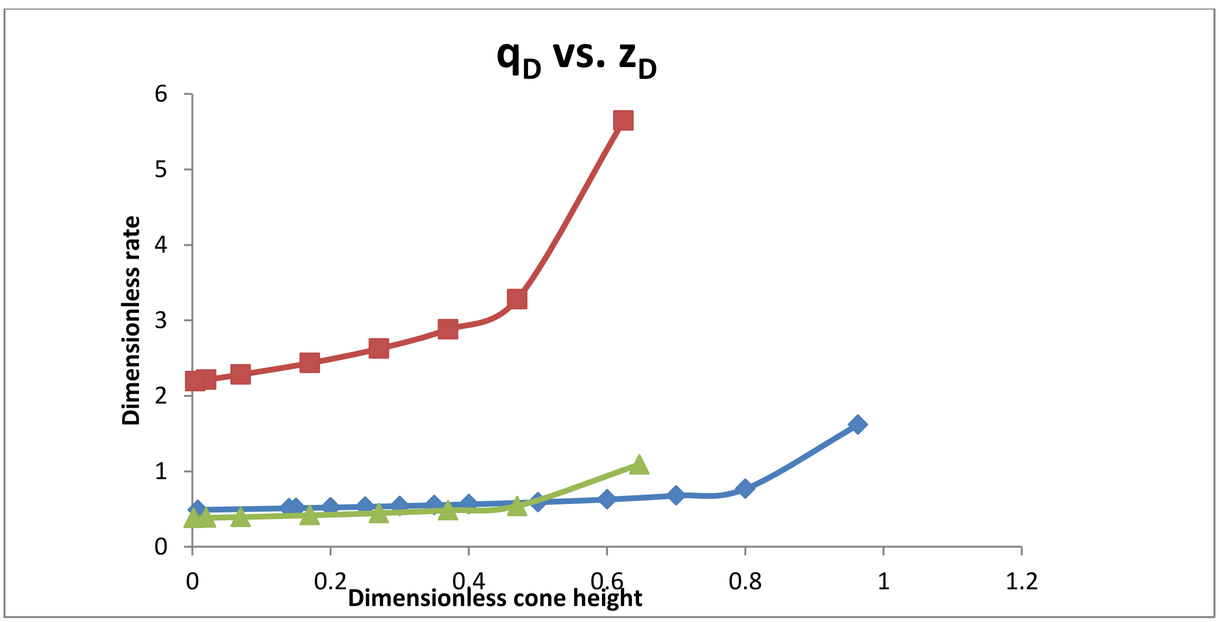

Figure 5.4 Dimensionless rate vs. D cone height (Ozkan Approach, horizontal wells)

5.2.CHAPERON APPROACH RESULTS DISCUSSION ON HORIZONTAL WELL

Chaperon method was applied to the three field data cases here, in which results are shown in figure 5.5, 5.6, and 5.7 In Figure 5.5 Chaperon method behaved almost similar to that of the Ozkan method, except that the chaperon model is two dimensional whiles the Ozkan applied the three dimensional model.

In Figure 5.6, the dimensionless rate-cone height curves showed an entirely different behavior from the normal trend, unlike the ozkan method, the dimensionless rate – cone curve shifts down as we reduce the well location.

In Figure 5.7, the curves behave partially normal. For instance, it shows normal trend in the latter region of their curves. (As in where, the highest well location correspond to highest breakthrough time)

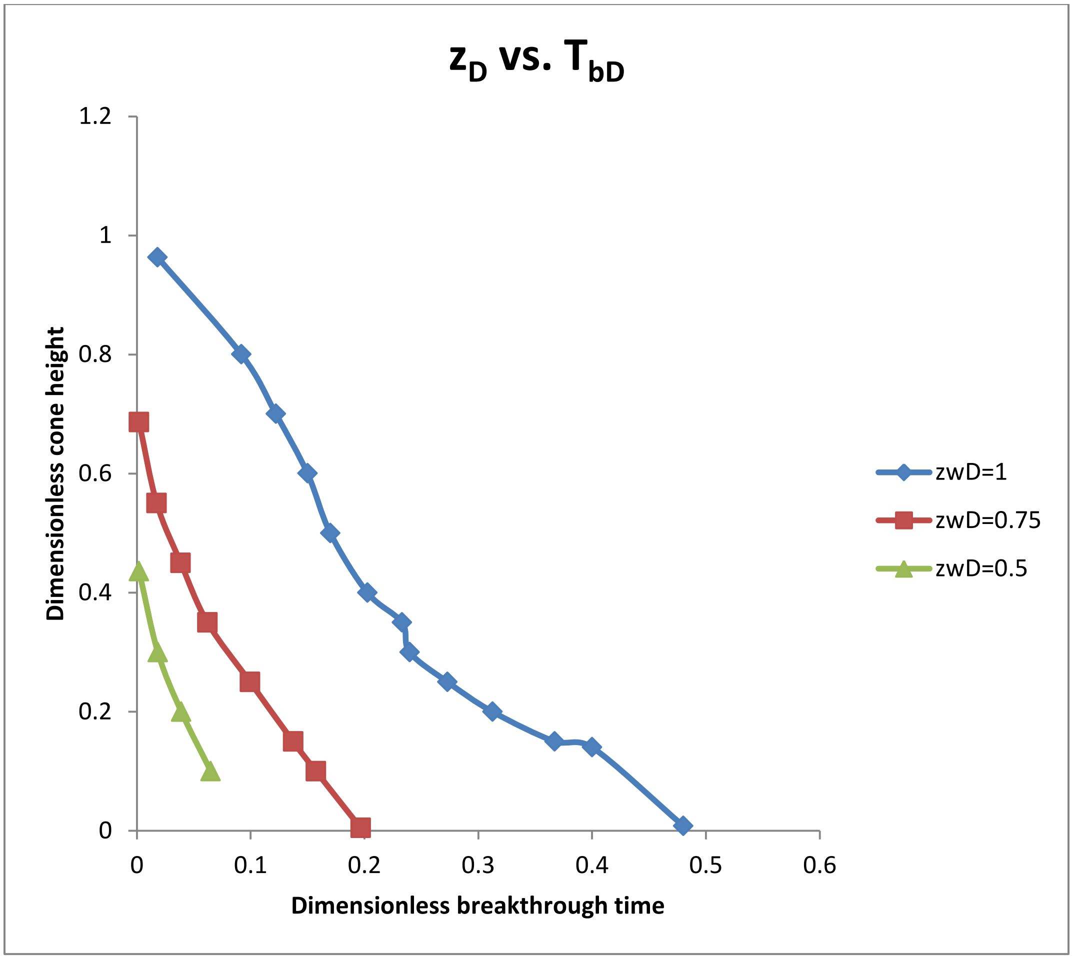

Figure 5.5 Dimensionless cone height vs. D. breakthrough time (Chaperon's Approach, horizontal wells)

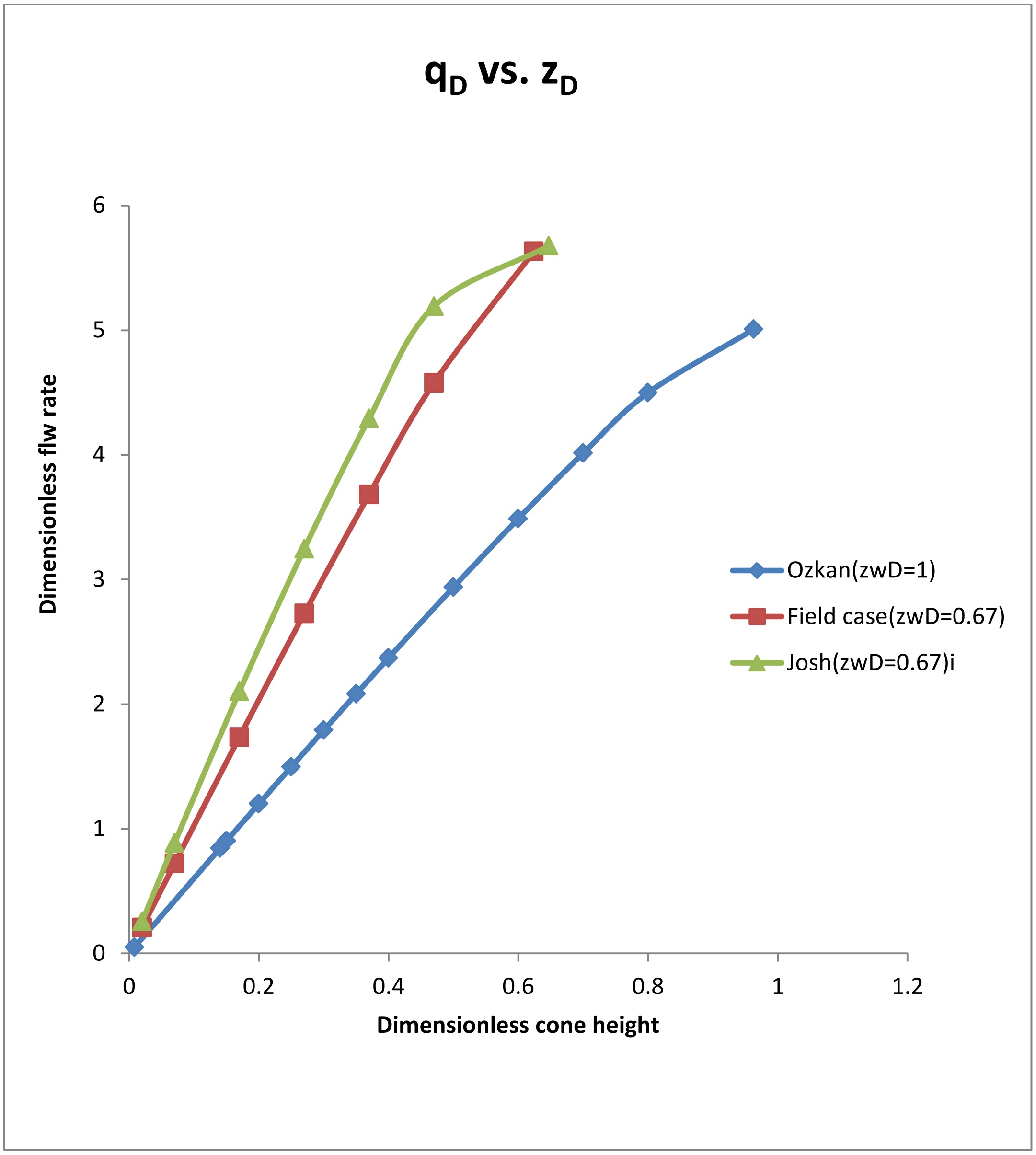

Figure 5.6 Dimensionless rate vs. D. cone height(Chaperon's Approach, horizontal wells)

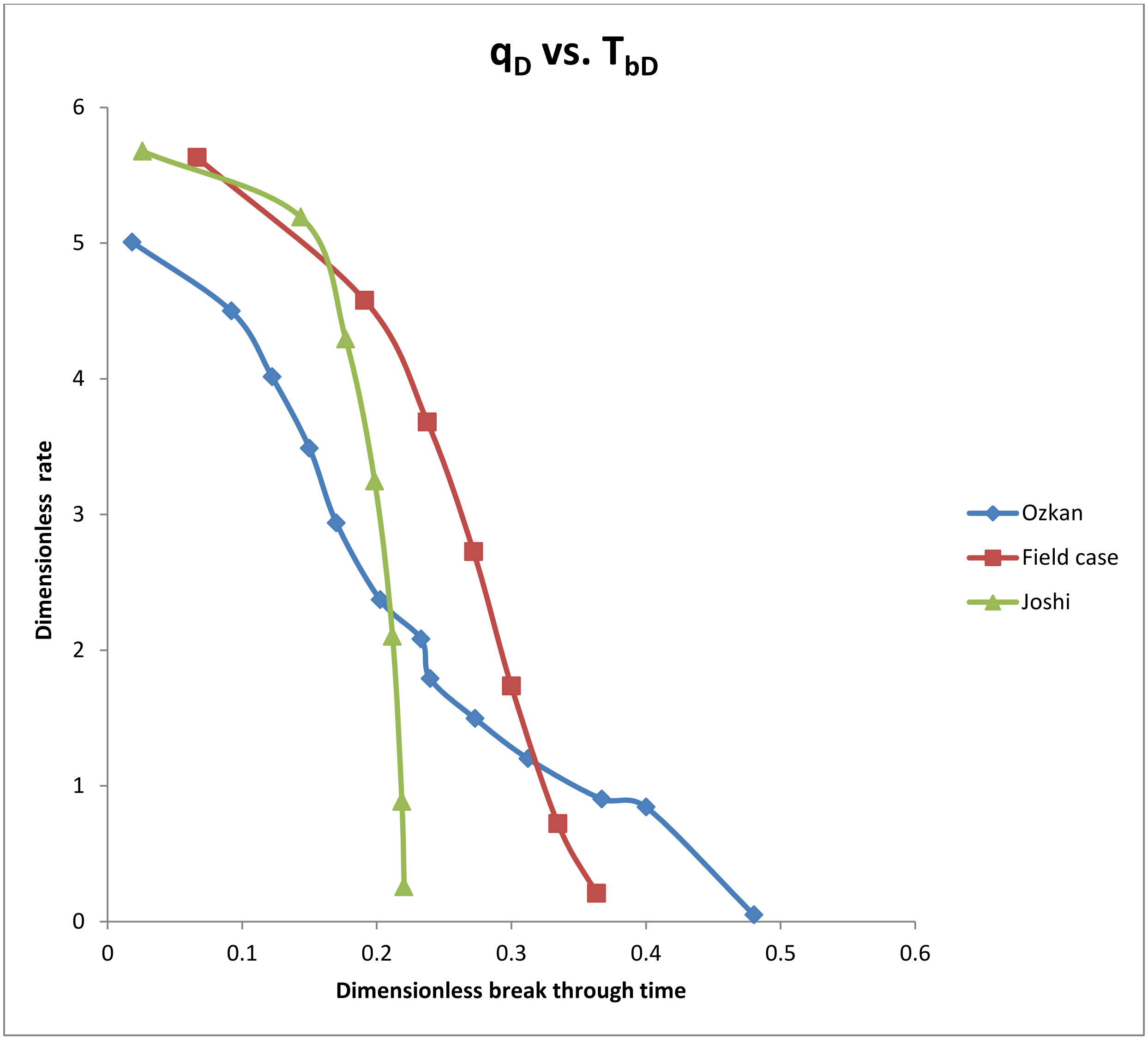

Figure 5.7 Dimensionless rate vs. D. breakthrough time (Chaperon's Approach, horizontal wells)

5.3 VERTICAL WELLS

Figure 5.8 and 5.9 represents the results of the influence of varying the penetration ratio, b on breakthrough time condition of vertical wells based our study's model (Ozkan's method). Figure 5.9 shows the results exhibited by Chaperon's 2-D model approach.

In both cases, the key role of the penetration ratio is clearly shown in the results and the breakthrough time, T_{bD}increases for a particular cone height, z_D as the penetration ration, b decreases. At the top of the reservoir where, b=0, the breakthrough time condition is the highest and it reduces as the well penetrate deeper into oil reservoir.

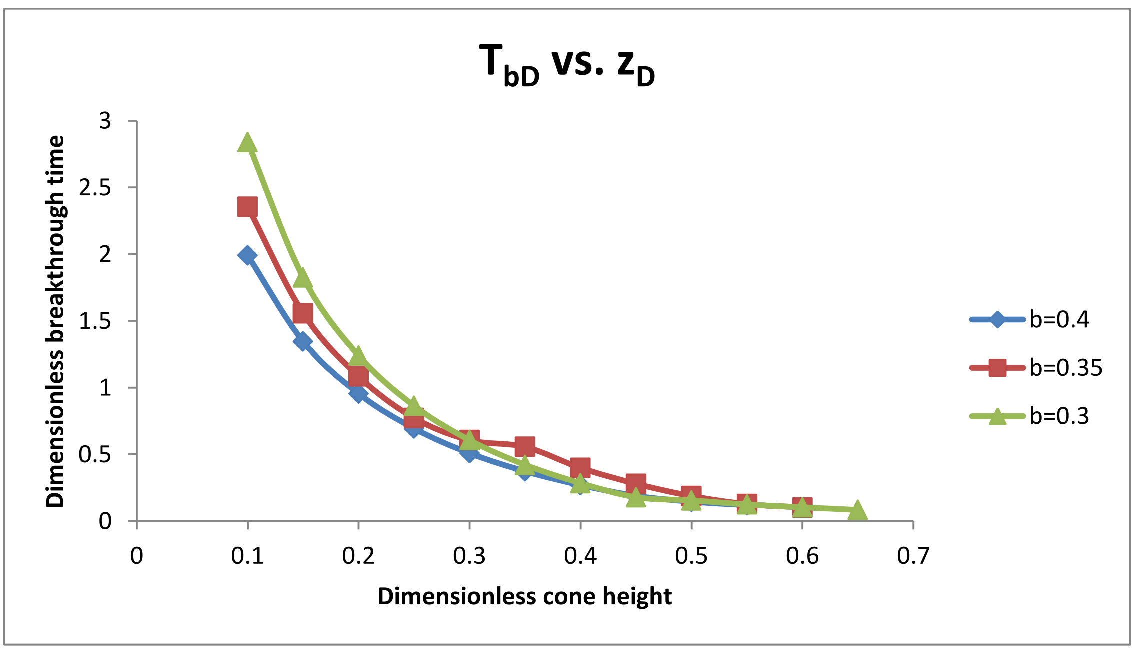

Figure 5.8 Dimensionless breakthrough time vs. D. cone height (Okan Approach, vertical wells)

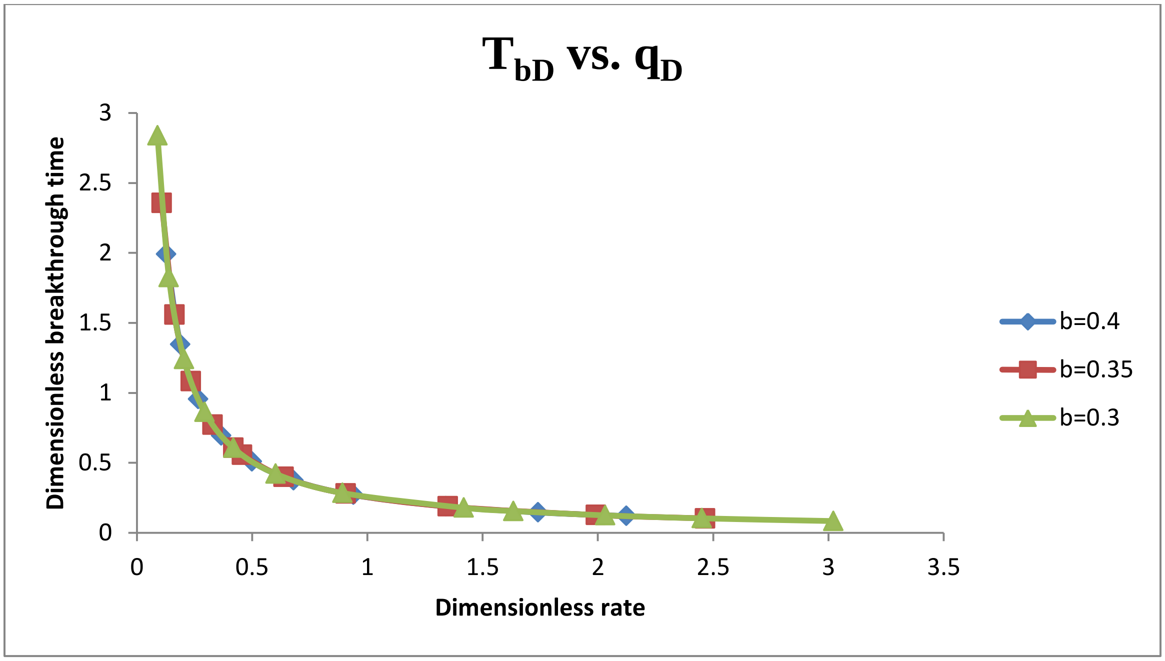

Figure 5.9 Dimensionless breakthrough time vs. D. rate (Ozkan Approach, vertical wells)

5.4 CHAPERON APPROACH RESULTS ON VERTICAL WELL

The major challenge with chaperon model for vertical wells is that it does not have the penetration ratio factored in the formulation. But as shown in figure 5.10, the breakthrough time increases as the penetration ratio of the wells are reduced.

In figure 5.11, it is clear that, the rate increases, as well as, the breakthrough time as the penetration ratio decreases.

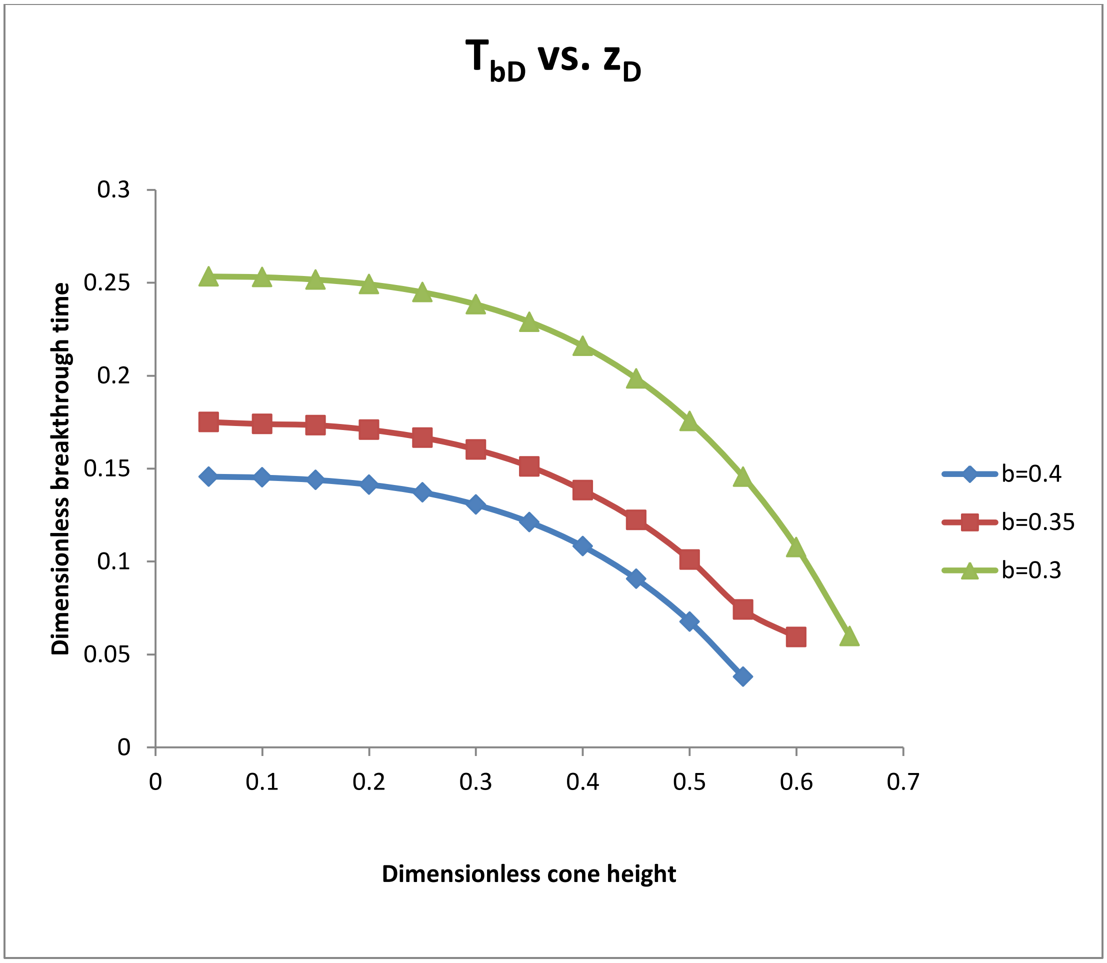

Figure 5.10 Dimensionless breakthrough time vs. D. cone height (Chaperon's Approach, vertical wells)

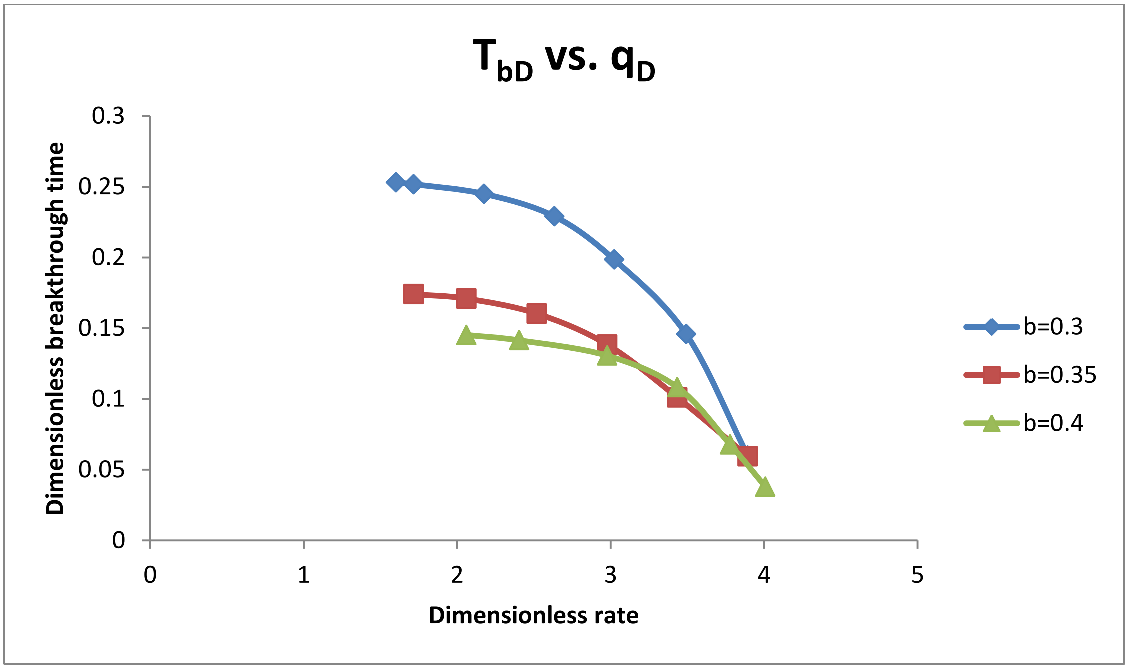

Figure 5.11 Dimensionless breakthrough time vs. D. rate (Chaperon's Approach, vertical wells)

5.5 GAS CONING IN HORIZONTAL WELL

As stated in the earlier chapters, the water coning model can be applied to the gas coning model by replacing the water parameters with that of gas phase.

Figures 5.12, 5.13 and 5.14, show the results of the water coning model application to gas coning. The behaviors of the curves in all the three charts are similar to the Ozkan field case example. Only Ozkan method was applied to this case.

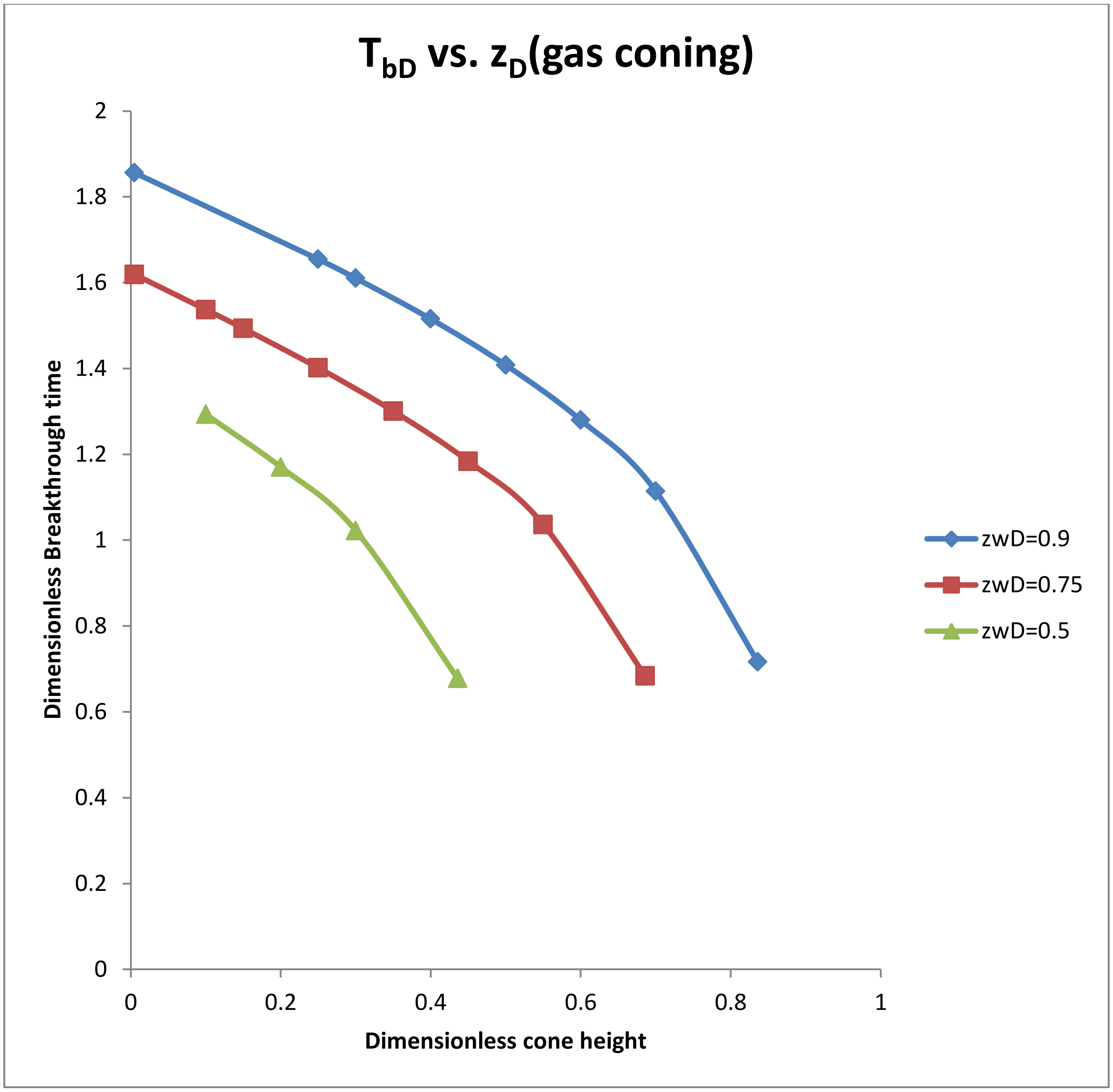

Figure 5.12 Dimensionless breakthrough time vs. D. cone height (Okan Approach, horizontal wells. Gas coning)

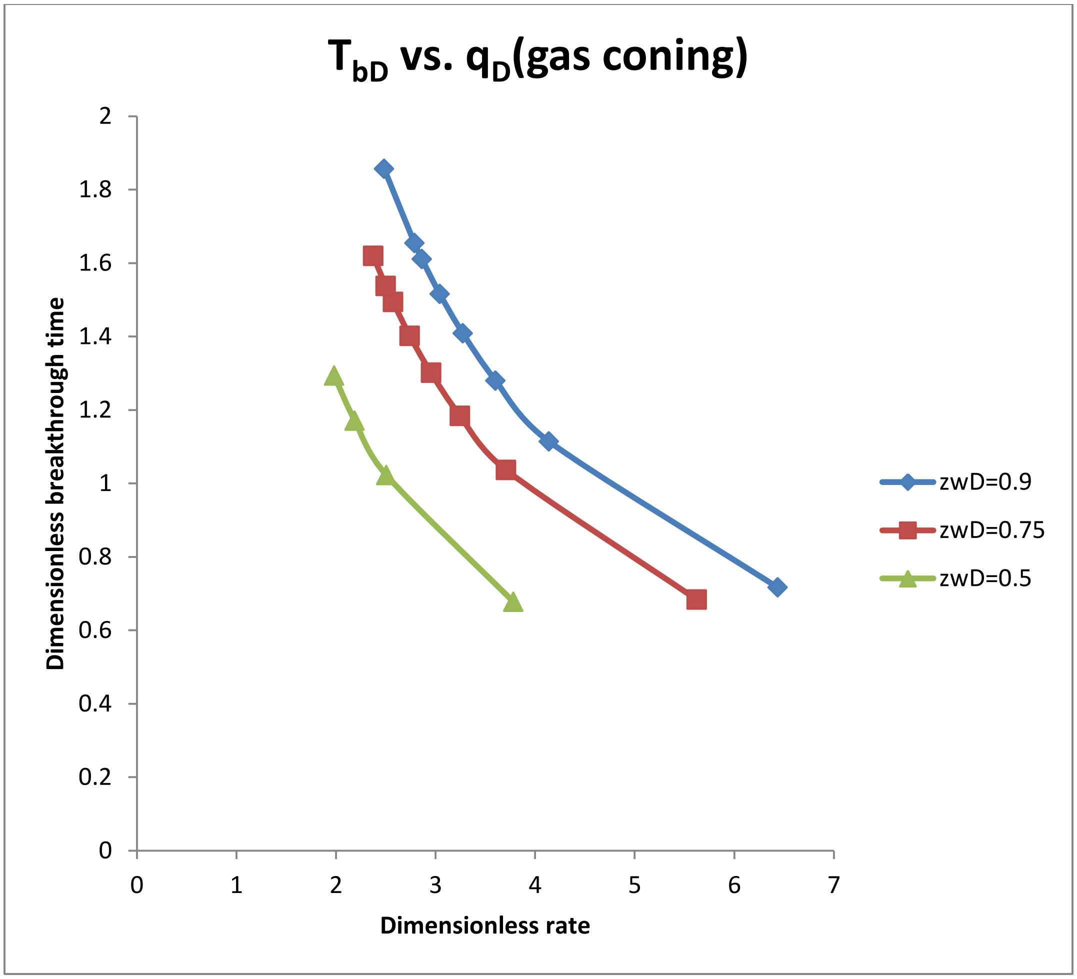

Figure 5.13 Dimensionless breakthrough time vs. D. rate (Okan Approach, horizontal wells. Gas coning)

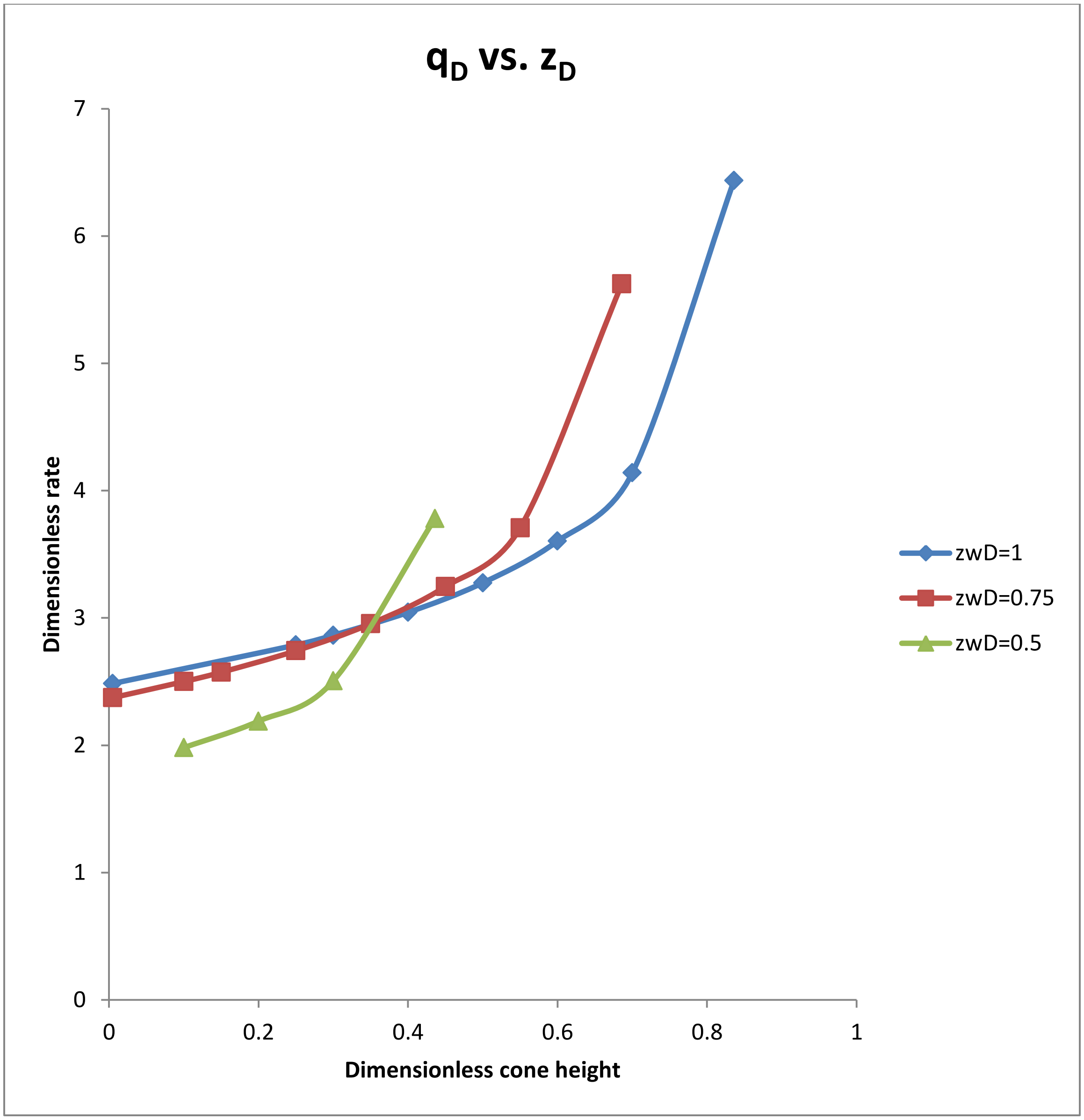

Figure 5.14 Dimensionless rate vs. D. cone height (Okan Approach, horizontal wells. Gas coning)

5.6 BREAKTHROUGH TIME CORRELATIONS DEVELOPED

Microsoft excel non-linear regression analysis technique was used in developing the following correlations.

Horizontal wells:

For the results in figure 5.3, for horizontal well, the breakthrough time equation can be approximated for z_{wD}=1 and for 0.1≤ q_D as;

$$T_{bD} = \frac{1.25}{q_D} \quad 5.1$$

For figure 5.1 of the horizontal well, the breakthrough time in terms of dimensionless cone height can be approximated as following for z_{wD}= 1;

$$T_{bD} = 2.4347 + 0.3955z_D - 2.0209{z_D}^2 \quad 5.2$$

Vertical wells:

For vertical well, we computed the following equation from the results in figure 5.10 as; for 0.01≤ $q_D \leq 5$

$$T_{bD} = \frac{0.2538}{q_D} \quad 5.3$$

For figure 5.9 and $0 \leq z_D \leq 1$ the breakthrough time is given as;

$$T_{bD} = 4.4936e^{-6.521z_D} \quad 5.5$$

Gas coning in horizontal well;

For gas coning represented by figures 5.14 for horizontal wells, the breakthrough time correlation is approximated by;

$$T_{bD} = 3.1205e^{-0.23q_D} \quad 5.6$$

5.7 COMPARISON OF RESULTS WITH LITERATURE

At the rate of 1000 stb/d given under the Ozkan example case the following results were obtained using the derived breakthrough time correlation against the obtained by Ozkan and Raghavan in their work.

Table 5.3 HORIZONTAL WELL RESULTS COMPARISON WITH LITERATURE

HORIZONTAL WELL RESULTS COMPARISON WITH LITERATURE		
	Derived correlation	LITERATURE(Ozkan)
	$t_{bD} = \frac{1.25}{q_D}$	$t_{bD} = \frac{z_{wD}L_D}{2q_D}$
tbD	0.036928	0.03
Tb(days)	57.7319	46.9

Table 5.4 VERTICAL WELL RESULTS COMPARISON WITH LITERATURE

VERTICAL WELL RESULTS COMPARISON WITH LITERATURE		
	Derived correlation	LITERATURE(Ozkan)
	$t_{bD} = \frac{0.2538}{q_D}$	$t_{bD} = \frac{1}{3.94q_D}$
tbD	0.007498	0.007498
Tb(days)	11.722	11.722

CHAPTER SIX

6.0 S CONCLUSION AND RECOMMENDATION

6.1 CONCLUSION

In conclusion, water coning is an unavoidable phenomenon in an aquifer supported reservoirs. Its development in producing reservoirs greatly affects the productivity of wells and eventually causes their shut-in after breakthrough.

Water coning behavior in both horizontal and vertical wells were modeled. Although, many cutting edge technology, such as horizontal well technology and down hole water sink (DWS) are employed, they do not eliminate completely the possibility of producing water or gas.

The Ozkan and Raghavan (1990) method was used to investigate the characteristics of water/gas coning in bottom water or gas drive supported reservoirs(mainly water coning in this study). It was also checked and compared with that of the Chaperon model (1986) where differing occurrences were analyzed. The application of the chaperon method in this work, showed a lot of discrepancy from Ozkan approach mainly because it's a 2-D model.

The horizontal wells shown from this studies, showed a higher breakthrough time than observed in vertical wells

The breakthrough times in horizontal wells have proportional relationships with the horizontal well length and the well location

Also showed by this study is the fact that completing at the top of reservoir increases the breakthrough time in both vertical and horizontal well

A simplified, analytical correlations were derived for the breakthrough time conditions in this study which can be used for quick estimation of breakthrough time prior to actual detailed simulation studies.

6.2 RECOMMENDATION

An approach, that involves most of the reservoir parameters, should be carried out in the studies of this problem to strengthen the understanding of the coning behaviors in wells.

More simplified approaches should be studied for the evaluations of coning problems rather than the complex roots most researchers try to follow which leaves a lot of nuances and subtleties understanding them.

For proper confirmation of the breakthrough time predictions made by this simplified correlations, a detailed simulation should at least be carried out once in the life of the reservoir to strengthen assurance of when to expect water production.

The approach to water coning employed in this study can also applied in the case where we both gas cap and aquifer are present.

NOMENCLATURE

b = penetration ratio

B_O = Oil formation volume factor. Rb/stb

f = microscopic displacement efficiency, fraction

g = gravitational constant, 32.17 ft/s^2 [9.81m/ s^{2}]

h = height of the oil zone, ft [m]

k_r = horizontal permeability, md [m^2]

k_z = vertical permeability, md [m^2]

L = horizontal well length, ft [m]

P = pressure, psi [pa]

P_i = initial pressure, psi [pa]

q = production rate, stb/d [stm^3/s]

r = radial distance, ft [m]

S_{or} = residual oil saturation, fraction

S_{wc} = connate water saturation, fraction

t = time, d [s]

x = distance in the x-direction, ft [m]

y = distance in the y-direction, ft [m]

z = distance in the z-direction, ft [m]

Δ = difference operator

μ_o = oil viscosity, cp [pa.s]

ρ_o = oil density, g/cc [kg/m^3]

ρ_w = water density, g/cc [kg/m^3]

Φ = velocity potential of oil, psi [pa]

φ = interfacial velocity potential, d^{-1}[s^{-1}]

SUBSCRIPTS

b = breakthrough

D = dimensionless

e = external boundary

l = oil or water

w = wellbore

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APPENDIX

Table A.1 Results of Ozkan field example on horizontal well.

LD	Z_{wD}	Z_{Dh}	Z_D	q_D	Q stb/d		T_{bD}	t_b(days)
2.5	1	0.037	0.963	1.616035518	47.7535012	Our result	0.773498	121.156
2.5	1	0.2	0.8	0.767087237	22.66726249		1.629541	255.2415
2.5	1	0.3	0.7	0.679601348	20.08207334		1.839314	288.099
2.5	1	0.4	0.6	0.627539885	18.54366834		1.991905	312
2.5	1	0.5	0.5	0.591204308	17.46995986		2.114328	331.1757
2.5	1	0.6	0.4	0.563382119	16.64782017		2.218743	347.5305
2.5	1	0.65	0.35	0.551538189	16.2978346		2.266389	354.9935
2.5	1	0.7	0.3	0.54070637	15.97775668		2.311791	362.105
2.5	1	0.75	0.25	0.530693667	15.68188346		2.355408	368.9369
2.5	1	0.8	0.2	0.521349074	15.40575274		2.397626	375.5497
2.5	1	0.85	0.15	0.512551772	15.14579437		2.438778	381.9955
2.5	1	0.86	0.14	0.510848979	15.09547721		2.446907	383.2688
2.5	1	0.991919	0.00808	0.489758465	14.47225706		2.552279	399.7735
			max rate	1.578768	46.65225408	Ozkan result		
			max rate	4.966658705	46.41075512	Chaperon		

Table A.2 Results of field case example on horizontal well.

LD	Z_{wD}	Z_{Dh}	Z_D	q_D	Q stb/d		T_{bD}	t_b(days)
10.2	0.67	0.046	0.624	5.6495509	2669.517	our results	0.604827	13.81759
10.2	0.67	0.2	0.47	3.2808284	1550.252		1.041505	23.79374
10.2	0.67	0.3	0.37	2.8806911	1361.18		1.186174	27.09877
10.2	0.67	0.4	0.27	2.6245813	1240.163		1.301922	29.7431
10.2	0.67	0.5	0.17	2.435126	1150.642		1.403213	32.05714
10.2	0.67	0.6	0.07	2.2822886	1078.424		1.497181	34.2039
10.2	0.67	0.65	0.02	2.2146529	1046.464		1.542905	35.24849
10.2	0.67	0.67	0.004482	2.1945535	1036.967		1.557037	35.57133
			max	5.6379596	2664.04	Ozkan		
			max	5.6313876	2226.298	Chaperon		

Table A.3 Results of Joshi's case example on horizontal well.

LD	Z_{wD}	Z_D	q_D	Q stb/d	T_{bD}	t_b(days)
1.98	0.67	0.023	1.092301	3001.917	0.60725	56.64383
1.98	0.67	0.2	0.540798	1486.247	1.226522	114.4091
1.98	0.67	0.3	0.483921	1329.935	1.370679	127.8559
1.98	0.67	0.4	0.446361	1226.711	1.486018	138.6147
1.98	0.67	0.5	0.417971	1148.69	1.586951	148.0296
1.98	0.67	0.6	0.394683	1084.689	1.680587	156.7639
1.98	0.67	0.65	0.384266	1056.058	1.72615	161.014
1.98	0.67	0.667879	0.380688	1046.225	1.742372	162.5272
			1.094427	3007.76	Ozkan	
			5.676099	4932.943	Chaperon	

Table A.4 Results of Ozkan field example case on vertical well for b=0.3

b	Z_{wD}	Z_{Dh}	Z_D	q_D	Q(stb/d)	T_{bD}	t_b(days)
0.3	0.7	0.05	0.65	3.0212	89.27581	0.084009	13.15862
0.3	0.7	0.1	0.6	2.4512	72.43243	0.103544	16.21851
0.3	0.7	0.15	0.55	2.0321	60.04812	0.124899	19.56342
0.3	0.7	0.2	0.5	1.6332	48.26071	0.155405	24.34167
0.3	0.7	0.25	0.45	1.4178	41.89568	0.179015	28.03979
0.3	0.7	0.3	0.4	0.8925	26.37318	0.284378	44.54321
0.3	0.7	0.35	0.35	0.6016	17.77715	0.421887	66.08181
0.3	0.7	0.4	0.3	0.4184	12.36363	0.606614	95.01629
0.3	0.7	0.45	0.25	0.2938	8.681727	0.863877	135.3125
0.3	0.7	0.5	0.2	0.2047	6.048841	1.239898	194.2101
0.3	0.7	0.55	0.15	0.139	4.10742	1.82595	286.0059
0.3	0.7	0.6	0.1	0.0894	2.641751	2.839006	444.6848

Table A.5 Results of Ozkan field example case on vertical well for b=0.35

b	Z_{wD}	Z_{Dh}	Z_D	q_D	Q(stb/d)	T_{bD}	tb(days)
0.35	0.65	0.05	0.6	2.4654	72.85204	0.102948	16.1251
0.35	0.65	0.1	0.55	1.992	58.86317	0.127413	19.95724
0.35	0.65	0.15	0.5	1.3494	39.87448	0.188089	29.46111
0.35	0.65	0.2	0.45	0.9065	26.78688	0.279986	43.85529
0.35	0.65	0.25	0.4	0.6366	18.81139	0.398692	62.44866
0.35	0.65	0.3	0.35	0.4562	13.48061	0.556351	87.1434
0.35	0.65	0.35	0.3	0.4184	12.36363	0.606614	95.01629
0.35	0.65	0.4	0.25	0.3284	9.704149	0.77286	121.0561
0.35	0.65	0.45	0.2	0.2342	6.92056	1.083719	169.7473
0.35	0.65	0.5	0.15	0.1629	4.81366	1.558055	244.0443
0.35	0.65	0.55	0.1	0.1077	3.182512	2.356612	369.1255

Table A.6 Results of Ozkan field example case on vertical well for b=0.4

b	Z_{wD}	Z_{Dh}	Z_D	q_D	Q(stb/d)	T_{bD}	t_b(days)
0.4	0.6	0.05	0.55	2.1241	62.7667	0.119489	18.71608
0.4	0.6	0.1	0.5	1.7403	51.42549	0.145841	22.84366
0.4	0.6	0.15	0.45	1.3412	39.63217	0.189239	29.64123
0.4	0.6	0.2	0.4	0.9398	27.77089	0.270065	42.30136
0.4	0.6	0.25	0.35	0.6794	20.07612	0.373575	58.5146
0.4	0.6	0.3	0.3	0.4989	14.74239	0.508733	79.68494
0.4	0.6	0.35	0.25	0.3657	10.80636	0.694031	108.7088
0.4	0.6	0.4	0.2	0.2658	7.854333	0.95488	149.5667
0.4	0.6	0.45	0.15	0.1885	5.570134	1.346457	210.9009
0.4	0.6	0.5	0.1	0.1275	3.767598	1.990644	311.8025

A.1 GREEN'S SOURCE FUNCTION APPROACH

Under this method of consideration for the cone behavior (water coning and breakthrough time) evaluation in horizontal wells, the same methodology as used under Ozkan and Chaperon's approaches is applied after developing the flow velocity potential from the source functions.

For this case two mixed boundary conditions with a no- flow boundary at the East and West flange and no-flow boundary at the top with water movement from the bottom. The North and South flanks are mixed boundaries. The solution for these boundary conditions is;

$$\Delta P = Pi - P(x, y, z, t) = \left[\frac{5.615qB}{\phi C_t \mu abhL}\right] \int_0^t \int_{y_1}^{y_2} (S_1.S_2.S_3)\, d_{yw} d_\tau \qquad \text{A.1}$$

S_1, S_2 and S_3 are the instantaneous point sink functions (Green's Functions) located at x, y, and z respectively and satisfying the zero flux boundary conditions at x = 0, y = 0, and z = 0. These Green's Functions for this case can be expressed as:

$$S_1 = \frac{1}{a}\left(1 + 2\sum_{n=1}^{\infty} \cos\frac{n\pi x_w}{a} \cos\frac{n\pi x}{a}\, exp\left[-\frac{n^2\pi^2\eta_x t}{a^2}\right]\right) \qquad \text{A.2}$$

$$S_2 = \frac{1}{a}\left(1 + 2\sum_{m=1}^{\infty} \cos\frac{n\pi y_w}{b} \cos\frac{n\pi y}{b}\, exp\left[-\frac{n^2\pi^2\eta_y t}{b^2}\right]\right) \qquad A.3$$

$$S_3 = \frac{2}{h}\sum_{l=1}^{\infty} \cos\frac{(2n+1)\pi(h - z_w)}{h} \cos\frac{(2n+1)\pi(h - z)}{h}\, exp\left[-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2}\right] \qquad \text{A.4}$$

Where

$$\eta = \frac{0.006329k}{\phi C_t \mu} \qquad \text{A.5}$$

Multiplying the above source functions and carrying out the integrations with respect to y-direction from y_1 to y_2 and time from t=0 to t. The pressure drop (the velocity flow potential) at horizontal well line source is obtained as;

$$\Delta P =$$

$$\left[\frac{5.615qB}{abhL}\right]\left[\frac{1264h^2}{abh\pi^2 k_x}\sum_{n=1}^{\infty}\frac{\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{(2n+1)^2}\left(1-exp\left(-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2}\right)\right)+\right.$$

$$\frac{2528b}{abh\pi^2 k_y}\sum_{n,n=1}^{\infty}\frac{\cos\frac{n\pi y_w}{b}\cos\frac{n\pi y}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_y}{b^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}\left(1-exp\left(-\frac{n^2\pi^2\eta_y t}{b^2}-\right.\right.$$

$$\left.\left.\frac{(2n+1)^2\pi^2\eta_z t}{4h^2}\right)\right)+\frac{2528}{abh\pi^2 k_x}\sum_{n,n=1}^{\infty}\frac{\cos\frac{n\pi x_w}{b}\cos\frac{n\pi x}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_y}{a^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}\left(1-\right.$$

$$\left.exp\left(-\frac{n^2\pi^2\eta_x t}{a^2}-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2}\right)\right)+$$

$$\frac{5056b}{abh\pi^2 k_x}\sum_{n,n,n=1}^{\infty}\frac{\cos\frac{n\pi x_w}{b}\cos\frac{n\pi x}{b}\cos\frac{n\pi y_w}{b}\cos\frac{n\pi y}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_x}{a^2}+\frac{4n^2 k_y}{b^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}\left(1-\right.$$

$$\left.\left.exp\left(-\frac{n^2\pi^2\eta_x t}{a^2}-\frac{n^2\pi^2\eta_y t}{b^2}-\frac{(2n+1)^2\pi^2\eta_z t}{4h^2}\right)\right)\right] \quad \text{A.6}$$

The flow potential of the reservoir is evaluated at the late time of the function. This makes the exponential terms very small, and therefore they vanish as the time, t in the pressure drop function approaches infinity.

The flow potential therefore becomes;

$\Delta P =$

$$\left[\frac{5.615qB}{abhL}\right]\left[\frac{1264h^2}{abh\pi^2 k_x}\sum_{n=1}^{\infty}\frac{\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{(2n+1)^2}+\right.$$

$$\frac{2528b}{abh\pi^2 k_y}\sum_{n,n=1}^{\infty}\frac{\cos\frac{n\pi y_w}{b}\cos\frac{n\pi y}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_y}{b^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}+$$

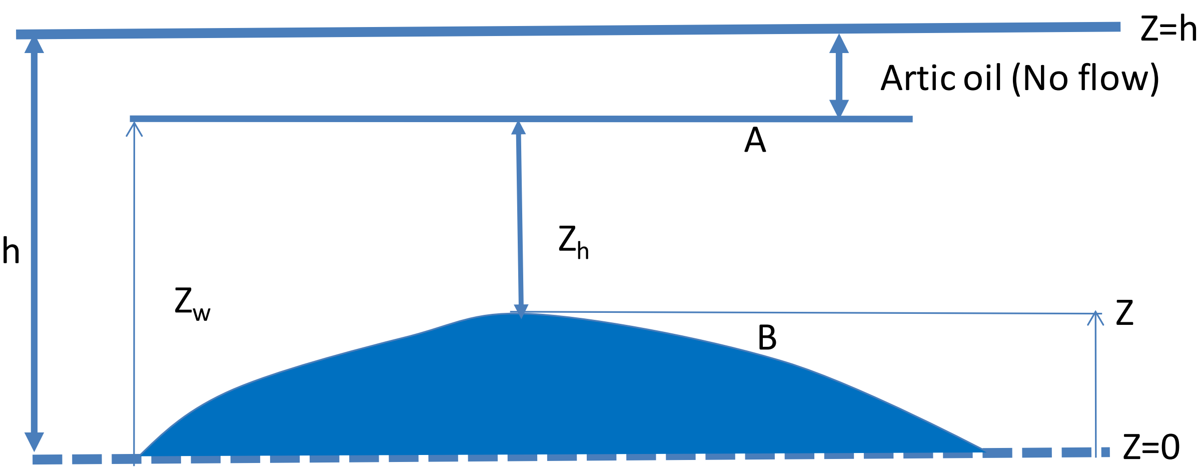
$$\frac{2528}{abh\pi^2 k_x}\sum_{n,n=1}^{\infty}\frac{\cos\frac{n\pi x_w}{b}\cos\frac{n\pi x}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_y}{a^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}+$$

$$\left.\frac{5056b}{abh\pi^2 k_x}\sum_{n,n,n=1}^{\infty}\frac{\cos\frac{n\pi x_w}{b}\cos\frac{n\pi x}{b}\cos\frac{n\pi y_w}{b}\cos\frac{n\pi y}{b}\cos\frac{(2n+1)\pi(h-z_w)}{h}\cos\frac{(2n+1)\pi(h-z)}{h}}{\left[\frac{4n^2 k_x}{a^2}+\frac{4n^2 k_y}{b^2}+\frac{(2n+1)^2 k_z}{h^2}\right]}\right] \qquad \text{A.7}$$

Flow rate -cone height correlation:

The flow rate cone height relationship is developed from equation 4.47

Figure A.1 Schematic of water coning in horizontal well



$$\Delta P_A - \Delta P_B = 0.433(Z_w - Z_h)(\rho_w - \rho_o) \qquad A.8$$

$$\Delta P_A - \Delta P_B = 0.433Z(\rho_w - \rho_o) \qquad A.9$$

Where at point A, $Z=Z_W$ as shown in figure 6.1

Hence;

$$q = \frac{0.4339(z_w - z_h)(\rho_w - \rho_o)}{(\Delta P_A - \Delta P_B)} \qquad A.10$$

$$q = \frac{0.4339Z(\rho_w - \rho_o)}{(\Delta P_A - \Delta P_B)} \qquad A.11$$

We put the flow potential, $\Delta P(\Phi)$, equation 4.47 into dimensionless form by dividing it by $\Delta\rho g h$; this becomes;

$$\Delta P_D = \Phi_D = \frac{\Delta P}{\Delta\rho g h} \qquad A.12$$

Now, the interfacial flow potential, φ_D becomes

$$\varphi_D = \Delta P_D \qquad A.13$$

Differentiating, φ_D w. r. t z_D turns;

$$\frac{d\,\varphi_D}{dz_D} =$$

$$\left[\frac{5.615qB}{\Delta\rho g a b h^2 L}\right]\left[\frac{1264}{ab\pi k_x}\sum_{n=1}^{\infty}\frac{cos(2n+1)\pi(1-z_{wD})\,sin(2n+1)\pi(h-z_D)}{(2n+1)} +\right.$$

$$\frac{2528(2n+1)}{ah^2\pi k_y}\sum_{n,n=1}^{\infty}\frac{cos\frac{n\pi y_w}{b}cos\frac{n\pi y}{b}cos(2n+1)\pi(1-z_{wD})\sin(2n+1)\pi(1-z_D)}{\left[\frac{4n^2k_y}{b^2}+\frac{(2n+1)^2k_z}{h^2}\right]}+$$

$$\frac{2528(2n+1)\pi}{abh^2\pi^2k_x}\sum_{n,n=1}^{\infty}\frac{cos\frac{n\pi x_w}{a}cos\frac{n\pi x}{a}cos(2n+1)\pi(1-z_{wD})\sin(2n+1)\pi(1-z_D)}{\left[\frac{4n^2k_y}{a^2}+\frac{(2n+1)^2k_z}{h^2}\right]}+$$

$$\left.\frac{5056(2n+1)}{ah^2\pi k_x}\sum_{n,n,n=1}^{\infty}\frac{cos\frac{n\pi x_w}{a}cos\frac{n\pi x}{a}cos\frac{n\pi y_w}{b}cos\frac{n\pi y}{b}cos(2n+1)\pi(1-z_{wD})\sin(2n+1)\pi(1-z_D).}{\left[\frac{4n^2k_x}{a^2}+\frac{4n^2k_y}{b^2}+\frac{(2n+1)^2k_z}{h^2}\right]}\right]$$

A.14

The breakthrough time is then evaluated using equation 3.32 which is;

$$t_{bD} = \int_{z_D}^{z_{WD}} \frac{dz_D}{(\partial\varphi_D/\partial z_D)r_{D=0}}$$

Where, $\frac{d\ \varphi_D}{dz_D}$ represent equation A.12 above.