AFRICAN UNIVERSITY OF SCIENCE & TECHNOLOGY

MASTERS THESIS

Biomechanics of Surface Runoff and Soil Water Percolation

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Abuja

Knowledge is Freedom

Declaration of Authorship

I, James Makol Madut Deng, declare that this thesis titled, "Biomechanics of Surface Runoff and Soil Water Percolation" and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
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Signed:

Date:

"Let your light so shine before men, that they may see your good works, and glorify your Father which is in heaven."

Matthew 5:16 (KJV)

AFRICAN UNIVERSITY OF SCIENCE & TECHNOLOGY

Abstract

Department of
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Biomechanics of Surface Runoff and Soil Water Percolation

by James Makol Madut Deng

In this study, the complex interaction of surface runoff with the biomechanics of soil water transport and heat transfer rate is theoretically investigated using mathematical model that rely on the two phase flows of an incompressible Newtonian fluid (stormwater) within the soil (porous medium) and on the soil surface (runoff). The flow and heat transfer characteristics within the soil are determined numerically based on Darcy-Brinkman-Forchheimer model for porous medium coupled with appropriate energy equation while analytical approach is employed to tackle the model for interacting surface runoff stormwater. The effects of various embedded biophysical parameters on the temperature distribution and water transport in soils and across the surface runoff together with soil-runoff interface skin friction and Nusselt number are display graphically and discussed quantitatively. It is found that an increase in surface runoff over tightly packed soil lessens stormwater percolation rate but enhances both soil erosion and heat transfer rate.

Keywords: Surface runoff, soil water percolation, soil erosion rate, soil heat transfer rate, groundwater aquifer

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DEDICATION

I dedicate this work to Almighy God through His son Jesus Christ for saving my life.

Chapter 1

Background of Study

1.1 Introduction

Surface runoff and soil water percolation are closely associated with rainfall and melting of snow, or glaciers. Soil inability to absorb excess stormwater and meltwater due to heavy rainfall, high melt rate of snow and glacier, soil saturation, impervious resulting from surface sealing or pavement, etc., do lead to surface runoff [10]. Surface runoff is the major cause of soil erosion and surface water pollution. In urban areas, runoff is the main cause of flooding which may damage properties and infrastructures including loss of life [23]. In order to alleviate the unpleasant effects of surface runoff, several proactive measures are needed to boost soil absorption of stormwater and meltwater. These measures may include minimizing impervious surfaces in urban areas, adopting soil erosion and flood control programs, etc. Moreover, percolation describes the downward flow rate of the stormwater or meltwater within the soil [6]. Water percolation in the soil contributes to the formation of groundwater aquifers which serves as a freshwater storage that can be utilized during droughts when surface water supplies are reduced. Generally, soil is regarded as a porous media; the soil loose sediments like sand and gravel are porous and permeable. It can hold water and allows water to flow through [5]. While the amount of porosity in a soil depends on its mineral content and structure, the rate of water percolation depends on soil permeability (i.e. the size of the soil pore spaces and how the pores are connected). For instance, sandy soils have large well connected pores and higher permeability than the clay soils [17]. The use of mathematical models to tackle the menace of surface runoff and enhance the soil water percolation for the formation of groundwater aquifers has attracted the attention of several scientists and researchers [7, 22, 4, 21, 20, 15]. For soil with weak permeability, the relationship between the flow rates and the pressure gradient would be practically linear based on the Brinkman form of Darcy law, while this relationship may be nonlinear for soil with strong permeability (Darcy–Forchheimer law) [25, 2, 18]. Bristow and Horton [3] theoretically investigated the influence of surface mulch soil water flow and heat transfer. The effects of temperature gradient on the soil water flow were studied by Gurr et al. [9]. Numerical results on soil water flow and heat transfer rate together with soil-atmosphere interaction was reported by Fetzer et al. [8]. In all the above studies, it is observed that mathematical model of soil-runoff interface at the continuum scale where water and energy fluxes are highly dynamic are often magnified. This may lead to inaccuracy in the result obtained.

In this present study, the biomechanics of surface runoff and its interaction with soil water percolation is numerically examined. The Darcy-Brinkman-Forchheimer nonlinear model for porous medium coupled with appropriate energy equation is employed in order to analysis the soil percolation rate while the model representing interacting surface runoff is based on modified Blasius flow with heat transfer characteristics. The groundwater aquifer servers as the lower boundary of the porous medium domain while the soil surface represents the upper boundary to the porous medium domain and is dramatically influenced by changes in velocity and temperature gradients of both runoff and stormwater percolation. In chapter 3, the model and its mathematical equations are obtained, analysed and solved.In chapter 4 Pertinent results are graphically presented and discussed. The thesis provides a mathematical treatment of surface runoff menace and veritable platform to understand the complex interaction between the surface runoff and the soil water percolation rate.

1.2 Definition of Terms

In this section we are going to define the important terminologies

1.2.1 Fluid [26]

A fluid is a substance that deforms continuously when acted on by a shearing stress of any magnitude.
Fluid can be divided into three part:

1. Liquids such as water,oil, and gasoline
2. Gases such as propane, methen, co_2 etc.
3. Plasma such as blood plasma.

Fluid Properties

To study any fluid's behavior, it is necessary to know and discuss some fluid properties such as Density (ρ), Specific Weight (γ), Specific Gravity (SG), and Specific Volume (ν).
1. **Density** (ρ):
Density is the mass of a fluid per unit volume, and its SI unit is ($kilogram/meter^3$)

$$\text{Density}, \rho = \frac{Mass}{Volume}\frac{kg}{m^3} \tag{1.1}$$

In general, density of a fluid decreases with increase in temperature. It increases with increase in pressure. The ideal gas equation is given by:

$$\begin{aligned} &PV = mRT \quad [\text{where} R \to \text{Universal Gas Constan}], \\ &P = \left(\frac{m}{V}\right) RT \\ &P = \rho RT \left[\text{since}, \rho = \frac{m}{V}\right]. \end{aligned} \tag{1.2}$$

Equation (1.2) is used to find the density of any fluid, if the pressure (P) and temperature (T) are known.
Note: The density of standard liquid (water) is 1000 kg/m^3
2. **Specific Weight** (γ):
Specific weigth is the weight of a fluid per unit volume, and its SI unit is N/m^3 (Newton per meter cubed)

$$\gamma = \frac{weight}{volume}\frac{N}{m^3} \tag{1.3}$$

Also, specific weight is a function of density as described by the following relationship:

$$\gamma = \rho g \tag{1.4}$$

where: g is the acceleration due to gravity, and $g = 9.81\ (m/s^2)$
Specific weight varies from place to place due to the change of acceleration due to gravity (g).
3. **Specific Gravity** (SG):
Specific gravity is ratio of the density of a fluid to the density of water at ($4°C$), at this temperature the density of water is 1000 kg/m^3 in SI unit.

$$SG = \frac{\rho_{Fluid}}{\rho_{H_2O}\, at\, 4°C} \tag{1.5}$$

We can also use the ratio of specific weight of a fluid to the specific weight of water at $4C$, at this temperature the specific weight of water is 9.81 KN/m^3 (kilo newton per meter cubed) in SI units.
Specific gravity may also be defined as the ratio of specific weight of the given fluid to the specific weight of standard fluid.

$$SG = \frac{\text{Specific Weight of Given Fluid}}{\text{Specific Weight of Standard Fluid}} = \frac{\gamma_{Fluid}}{\gamma_{H_2O}\, at\, 4°C} \tag{1.6}$$

4. **Specific Volume** (ν):
Specific volume is the volume of a fluid per unit mass. It is the reciprocal of density.

$$\nu = \frac{1}{\rho} \frac{m^3}{kg} \tag{1.7}$$

Viscosity

Is the term used to describe the fluidity of a fluid.
Dynamic Viscosity
Is the shear stress required to cause a unit change in the rate of angular deformation of the fluid.
Dynamic viscosity is a fluid property that relates shearing stress (τ) to fluid motion ($\frac{du}{dy}$).
By Newton's law of viscosity.

$$\tau = \mu \frac{du}{dy} \quad \text{Thus} \quad \mu = \tau \frac{dy}{du} \tag{1.8}$$

$\tau =$ shear stress or $\frac{F}{A}$ force per unit area $\frac{N}{m^2}$
$\mu =$ dynamic viscosity $\frac{N.S}{m^2}$
$\frac{du}{dy} =$ velocity gradiant or rate of angular deformation ($\frac{1}{S}$).
Notice that the dynamic viscosity (μ) is the most important factor in the equation which is able to control shear stress (τ) and gradient ($\frac{du}{dy}$).
The relationship between τ and $\frac{du}{dy}$ is linear with the slope equal to viscosity (μ).

Kinematic Viscosity
Is the relationship between dynamic viscosity (μ) and density (ρ) when the force dimension cancels.

$$\nu = \frac{\mu}{\rho} \tag{1.9}$$

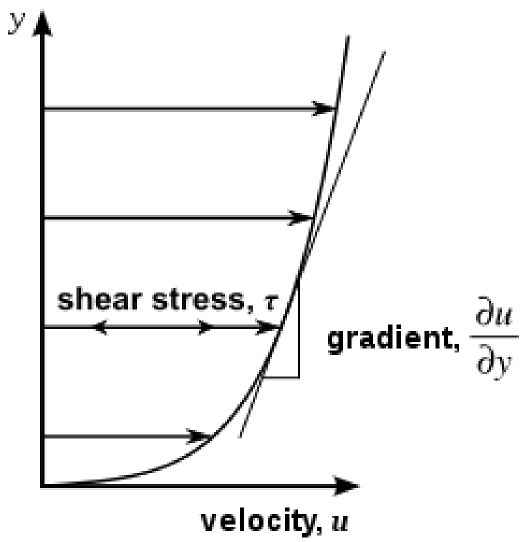

FIGURE 1.1: Retionship between τ and $\frac{du}{dy}$

Where:
ν = Kinematic Viscosity ($\frac{m^2}{s}$)
μ = Dynamic Viscosity ($\frac{N.s}{m^2}$)
ρ = Density ($\frac{kg}{m^3}$).

Temperature

It is the property that determines the degree of hotness or coldness or the level of heat intensity of a fluid. Temperature is measured by using temperature scales.There are 3 commonly used temperature scales. They are
1. Celsius (or centigrade) scale
2. Fahrenheit scale
3. Kelvin scale (or absolute temperature scale)
Kelvin scale is widely used in engineering. This is because, this scale is independent of properties of a substance.

Pressure

Pressure of a fluid is the force per unit area of the fluid. In other words, it is the ratio of force on a fluid to the area of the fluid held perpendicular to the direction of the force.
Pressure is denoted by the letter 'P'. Its unit is N/m^2.

1.2.2 Heat Transfer [19]

Heat transfer which is defined as the transmission of energy from one region to another as a result of temperature gradient takes place by the following three modes :
(i) Conduction ;
(ii) Convection ;
(iii) Radiation.
Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. Example : The water in a boiler shell receives its heat from the fire-bed by conducted, convected and radiated heat from the fire to the shell, conducted heat through the shell and conducted and convected heat from the inner shell wall, to the water. Heat always flows in the direction of lower temperature.

The above three modes are similar in that a temperature differential must exist and the heat exchange is in the direction of decreasing temperature ; each method, however, has different controlling laws.

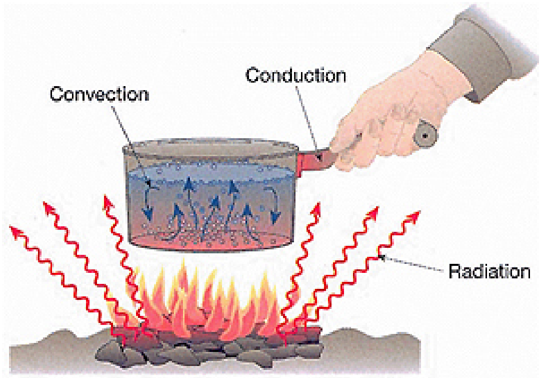

FIGURE 1.2: Illustration of conduction, convection and radiation heat transfer

Conduction

Conduction is the transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.
In solids, the heat is conducted by the following two mechanisms :
(i) By lattice vibration (The faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).
(ii) By transport of free electrons (Free electrons provide an energy flux in the direction of decreasing temperature—For metals, especially good electrical conductors, the electronic mechanism is responsible for the major portion of the heat flux except at low temperature).
In case of gases, the mechanisam of heat conduction is simple. The kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion ex-changing energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it loses energy by collisions.
In liquids, the mechanism of heat is nearer to that of gases. However, the molecules are more closely spaced and intermolecular forces come into play.

Convection

Convection is the transfer of heat within a fluid by mixing of one portion of the fluid with another.

1 Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.

2 Convection constitutes the macroform of the heat transfer since macroscopic particles of a fluid moving in space cause the heat exchange.

3 The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.

This mode of heat transfer is met with in situations where energy is transferred as heat to a flowing fluid at any surface over which flow occurs. This mode is basically conduction in a very thin fluid layer at the surface and then mixing caused by the flow. The heat flow depends on the properties of fluid and is independent of the properties of the material of the surface. However, the shape of the surface will influence the flow and hence the heat transfer.
Free or natural convection: Free or natural convection occurs where the fluid circulates by virtue of the natural differences in densities of hot and cold fluids ; the denser portions of the fluid move downward because of the greater force of gravity, as compared with the force on the less dense.
Forced convection: When the work is done to blow or pump the fluid, it is said to be forced convection.

Radiation

Radiation is the transfer of heat through space or matter by means other than conduction or convection. Radiation heat is thought of as electromagnetic waves or quanta (as convenient) an emanation of the same nature as light and radio waves. All bodies radiate heat ; so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits. Radiant energy (being electromagnetic radiation) requires no medium for propagation and will pass through a vacuum.
Note. The rapidly oscillating molecules of the hot body produce electromagnetic waves in hypothetical medium called ether. These waves are identical with light waves, radio waves and X-rays, differ from them only in wavelength and travel with an approximate velocity of $3 \times 10^8\ m/s$. These waves carry energy with them and transfer it to the relatively slow-moving molecules of the cold body on which they happen to fall. The molecular energy of the later increases and results in a rise of its temperature. Heat travelling by radiation is known as radiant heat.
The properties of radiant heat in general, are similar to those of light. Some of the properties are :
(i) It does not require the presence of a material medium for its transmission.
(ii) Radiant heat can be reflected from the surfaces and obeys the ordinary laws of reflection.
(iii) It travels with velocity of light.
(iv) Like light, it shows interference, diffraction and polarisation etc.
(v) It follows the law of inverse square.
The wavelength of heat radiations is longer than that of light waves, hence they are invisible to the eye.

1.2.3 Groundwater

Groundwater is fresh water in the rock and soil layers beneath Earth's land surface. Some of the precipitation (rain, snow, sleet, and hail) that falls on the land soaks into Earth's surface and becomes groundwater. Water-bearing rock layers called aquifers are saturated (soaked) with groundwater that moves, often very slowly, through small openings and spaces. This groundwater then returns to lakes, streams, and marshes (wet, low-lying land with grassy plants) on the land surface via springs and seeps (small springs or pools where groundwater slowly oozes to the surface).
Groundwater makes up more than one-fifth (22%) of Earth's total fresh water supply,

and it plays a number of critical hydro-logical (water-related), geological and biological roles on the continents. Soil and rock layers in groundwater recharge zones (a entry point where water enters an aquifer) reduce flooding by absorbing excess runoff after heavy rains and spring snowmelts. Aquifers store water through dry seasons and dry weather, and groundwater flow carries water beneath arid (dry) deserts and semi-arid grasslands. Groundwater discharge replenishes streams, lakes, and wetlands on the land surface and is especially important in arid regions that receive limited rainfall. Flowing groundwater interacts with rocks and minerals in aquifers, and carries dissolved rock-building chemicals and biological nutrients. Vibrant communities of plants and animals (ecosystems) live in and around groundwater springs and seeps.
Almost all of the fresh liquid water that is readily available for human use comes from underground. (The bulk of Earth's fresh water is frozen in ice in the North and South Pole regions. Water in streams, rivers, lakes, wetlands, the atmosphere, and within living organisms makes up only a tiny portion of Earth's fresh water.) For thousands of years, humans have used groundwater from springs and shallow wells to fill drinking water reservoirs, and water livestock and crops. Today, human water needs far exceed surface water supplies in many regions, and Earth's rapidly-growing human population relies heavily upon groundwater to meet its ever larger demand for clean, fresh water.

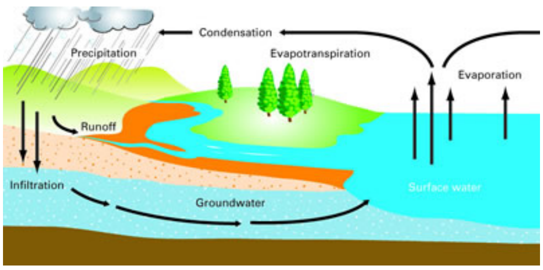

FIGURE 1.3: Groundwater

Aquifers

An aquifer is a body of rock or soil that yields water for human use. Most aquifers are water-saturated layers of rock or loose sediment. With the exception of a few aquifers that have water-filled caves within them, aquifers are not underground lakes or holding tanks, but rather rock "sponges" that hold groundwater in tiny cracks, cavities, and pores (tiny openings in which a liquid can pass) between mineral grains (rocks are made of minerals). The total amount of empty pore space in the rock material, called its porosity, determines the amount of groundwater the aquifer can hold. Materials like sand and gravel have high porosity, meaning that they can absorb a high amount of water. Rocks like granite, marble, and limestone have low porosity, and make poor groundwater reservoirs.
Aquifers must have high permeability in addition to high porosity. Permeability is the ability of the rock or other material to allow water to pass through it. The pore space in permeable materials is interconnected throughout the rock or sediment, allowing groundwater to move freely through it. Some high-porosity materials, like mud and clay, have very low permeability. They soak up and hold water, but don't release it easily to wells or other groundwater discharge points, so they are not good aquifer materials. Sandstone, limestone, fractured granite, glacial sediment, loose sand, and gravel are examples of materials that make good aquifers.

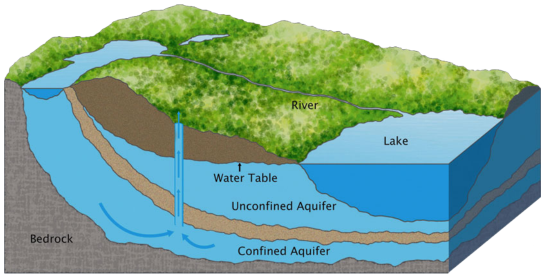

FIGURE 1.4: Aquifer

1.2.4 infiltration

Infiltration is the process by which water on the ground surface enters the soil.
Infiltration is governed by two forces, gravity, and capillary action.
While smaller pores offer greater resistance to gravity, very small pores pull water through capillary action in addition to and even against the force of gravity.
Infiltration rate in soil science is a measure of the rate at which a particular soil is able to absorb rainfall or irrigation.
It is measured in inches per hour or millimeters per hour.
The rate decreases as the soil becomes saturated.
If the precipitation rate exceeds the infiltration rate, runoff will usually occur unless there is some physical barrier.
It is related to the saturated hydraulic conductivity of the near-surface soil.

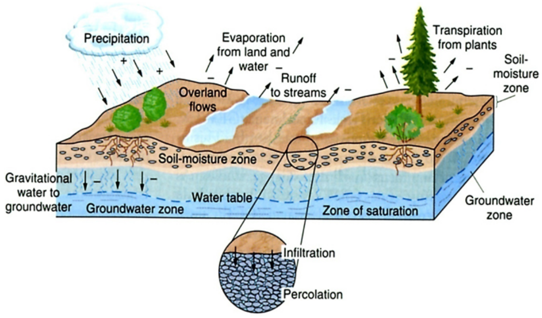

FIGURE 1.5: Infiltration

1.2.5 Percolation Rate

When we sprinkle water on the ground, it is soon absorbed by the soil. This is because water percolates through the soil. The process in which water passes down slowly through the soil is called percolation of water. But water does not percolate at the same rate in all types of soils.
Sandy soil allows maximum percolation of water and clay soil allows minimum percolation of water. Rainwater percolates through the soil and collects above the bedrock. This level of groundwater is called water table. Sandy soil is quite loose, so the percolation rate of water is highest in sandy soil but lowest in the clay soil because it is very compact.
Paddy (rice crops) is planted in standing water in the fields. Hence, the soil with a low percolation rate of water would be the most suitable for growing paddy because it will allow the water to remain in the fields for a much longer time.

1.2.6 Soil Erosion

t is a process in which the top fertile layer of soil is lost. Due to soil erosion, the soil becomes less fertile. The top layer of soil is very light which is easily carried away by wind and water. The removal of topsoil by the natural forces is known as soil erosion.

Causes of Soil Erosion

Various agents, like wind, water, deforestation, overgrazing by cattle, etc., cause soil erosion. The various factors of soil erosion are:
1. Wind
When strong winds blow, the topsoil along with the organic matter is carried away by the wind. This happens more often when the land is not covered with grass or plants. Such conditions are very common in desert and semi-desert regions where strong winds blow very frequently.
2. Water
When it rains in the hilly areas, the soil gets washed away towards the plains. The running water deposits the mineral-rich soil in the riverbed and over the years this deposition of soil can change the course of the river. This can lead to floods which cause the destruction of life and property.

FIGURE 1.6: Water erosion leads to loss of agricultur potential

3. Overgrazing
When cattle are allowed to graze on the same field repeatedly, all the available grass, including the roots are eaten by them. This makes the topsoil vulnerable to wind and flowing water, leading to soil erosion.
4. Deforestation
Humans have taken land from the forest to cultivate in order to feed the ever-increasing population and to build houses, industries, etc. Cutting down of trees on a large scale for these purposes is deforestation. The roots of trees hold the soil together, thus preventing the soil from getting uprooted. When large areas of the forest are cleared, the topsoil gets eroded by wind and flowing water.

1.2.7 Stormwater

Stormwater is a term used to describe water that originates during precipitation events. It may also be used to apply to water that originates with snowmelt or runoff water from overwatering that enters the stormwater system. Stormwater that does not soak into the ground becomes surface runoff, which either flows directly into surface waterways or is channeled into storm sewers, which eventually discharge to surface waters and the ocean.

Stormwater is of concern for two main issues: one related to the volume and timing of runoff water (flood control and water supplies) and the other related to potential contaminants that the water is carrying, i.e. water pollution. Stormwater outfalls that discharge directly on a beach often create shallow streams or pools of water that are an "attractive nuisance." This water often has high concentrations of fecal indicator bacteria and may contain human pathogens. Contact with such water, especially by small children, should be avoided.

FIGURE 1.7: South Carolina stormdrain outfall and warning sign

1.2.8 Flow in Porous Media

A porous medium can be defined as a material consisting of solid matrix with an interconnected void. The interconnected pores are very important because they are the ones that affect the flow. The definition of the porosity of the porous medium can be given as the ratio of pore volume to the total volume of a given sample of material. Darcy's law [18] expressed as

$$\nabla P = -\frac{\mu}{K} Q \tag{1.10}$$

is used for flow in porous media, where K is the porous medium permeability, is the fluid dynamic viscosity, P is the fluid pressure and Q is the volumetric flow rate. It works with variables averaged over several pore widths. Darcy's law may be extended to include transitional flow between boundaries (i.e. Darcy –Brinkman model),

$$\mu \nabla^2 q - \nabla P - \frac{\mu_f}{K} q = 0, \tag{1.11}$$

where μ_f is known as the effective viscosity and q is the velocity vector. In order to account for the nonlinear behaviour of the pressure difference, the inertial term due to Forchheimer [18] may be add (i.e Darcy –Brinkman- Forchheimer model)

$$\mu \nabla^2 q - \nabla P - \frac{\mu_f}{K} q - \frac{c\rho}{\sqrt{K}} q^2 = 0 \tag{1.12}$$

where ρ is the fluid density and c is the porous medium inertial parameter. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. Also the flow through porous media is of interest in chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil physics, bio-physics and geophysics.

1.3 Aims and Objectives of Study

The main objectives of the study in this thesis are as follows:

- To derive a nonlinear mathematical model for porous medium coupled with appropriate energy equation.
- To solve the two points boundary value problem of soil water percolation numerically using Runge-Kutta-Fehlberg integration scheme coupled with shooting method.
- To illustrate the impact of embedded biophysical parameters on stormwater velocity profiles both within the soil in the region $0 \leq \eta \leq 1$ and the runoff in the region $\eta > 1$.
- To show the effects of various biophysical parameters on the stormwater temperature profiles both within the soil in the region $0 \leq \eta \leq 1$ and the runoff in the region $\eta > 1$.
- To demonstrate the effects of various biophysical parameters on the coefficient of skin friction which invariably lead to soil erosion and the heat transfer rate at the soil surface ($\eta = 1$) due to interaction between the runoff and the soil water percolation.

1.4 Significance of Study

Surface runoff is a water from rain, snowmelt, or other sources, that flows over the land surface, and is a major component of the water cycle. When runoff flows along the ground, it can pick up soil contaminants such as petroleum, pesticides, or fertilizers that become discharge or overland flow. Urbanization increases surface runoff, by creating more impervious surfaces such as pavement and buildings do not allow percolation of the water down through the soil to the aquifer. Increased runoff reduces groundwater recharge, thus lowering the water table and making droughts worse, especially for farmers and others who depend on water wells. Runoff is an economic threat, as well as an environmental one. Agribusiness loses millions of dollars to runoff every year. In the process of erosion, runoff can carry away the fertile layer of topsoil. Farmers rely on topsoil to grow crops. Tons of topsoil are lost to runoff every year. People can limit runoff pollution in many ways. Farmers and gardeners can reduce the amount of fertilizer they use. Urban areas can reduce the number of impervious surfaces. Soil acts as a natural sponge, filtering and absorbing many harmful chemicals. Communities can plant native vegetation. Shrubs and other plants prevent erosion and runoff from going into waterways.

1.5 Research Methodology

1.5.1 Continuity Equation

The system is a fixed quantity of mass, denoted by m.Thus the mass of the system is conserved and does not change.

$$\begin{aligned} m_{syst} &= \text{const} \\ \text{or} \quad \frac{dm}{dt} &= 0. \end{aligned} \tag{1.13}$$

The total mass of the fluid is:

$$dm = \rho \, dV \tag{1.14}$$

equation 1.14 implies

$$\begin{aligned} m &= \int dm \\ &= \int\limits_V \rho \, dV \end{aligned} \tag{1.15}$$

By substituting equation 1.15 into equation 1.13 we obtian:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int\limits_V \rho \, dV = 0 \tag{1.16}$$

By using Reynold transport theorem:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int\limits_V \rho \, dV = \int\limits_V \frac{\partial \rho}{\partial t} \, dV + \int\limits_S \rho \, (q.\hat{n}) dS = 0 \tag{1.17}$$

Now by using Gauss divergence theorem onto $\int\limits_S \rho(q.\hat{n})dS$ we have:

$$\int\limits_V \frac{\partial \rho}{\partial t} \, dV + \int\limits_V \nabla.(\rho q) \, dV = 0 \tag{1.18}$$

$$\Rightarrow \quad \int\limits_V \left(\frac{\partial \rho}{\partial t} + \nabla.(\rho q) \right) \, dV = 0 \tag{1.19}$$

$$\Rightarrow \quad \boxed{\frac{\partial \rho}{\partial t} + \nabla.(\rho q) = 0} \tag{1.20}$$

Equation 1.20 is called the continuity equation.
Where:
ρ is the mass density
q is the velocity vector.
when the density (ρ) is constant equation 1.20 becomes $\boxed{\nabla.q = 0}$ and this is the case for incompressible flow.

1.5.2 Momentum Equation

Momentum (p) is the product of mass (m) and velocity (q):

$$p = mq \tag{1.21}$$

Note that $q(u, v, w)$ is velocity vector and recall that:

$$\rho = \frac{m}{V} \quad \Rightarrow \quad m = \rho V \tag{1.22}$$

Where: ρ is the density, m is the mass, and V is the volume.
By substituting equation 1.22 into equation 1.21 we obtain:

$$\begin{aligned} & p = mq = (\rho q)V = q\rho V \\ \Rightarrow \quad & dp = \rho q \, dV \end{aligned} \tag{1.23}$$

By taking integral of equation 1.23 in both sides we obtain:

$$\begin{aligned} & \int dp = \int \rho q \, dV \\ \Rightarrow \quad & p = \int_V \rho q \, dV \end{aligned} \tag{1.24}$$

By differentaiting equation 1.24 with respect to time (t), we have:

$$\frac{dp}{dt} = \frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho q \, dV = F$$

Note that: Force (F) is the rate of change of momentum with respect to time (t).
let us assume an incompressible fluid (i.e. ρ is constant):

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho q \, dV = F_s + F_b \tag{1.25}$$

Where: F_s is the surface force, and F_b is the body force.

$$\begin{aligned} F_s &= \int_S \tau.\hat{n} \, dS \\ F_b &= \int_V \rho f_m \, dV \end{aligned} \tag{1.26}$$

Then substituting equation 1.26 into equation 1.25 we have:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \rho q \, dV = \int_S \tau.\hat{n} \, dS + \int_V \rho f_m \, dV \tag{1.27}$$

By using Reynold transport theorem equation 1.27 becomes:

$$\int_V \frac{\partial}{\partial t} (\rho q) \, dV + \int_S \rho q (q.\hat{n}) \, dS = \int_S \tau.\hat{n} \, dS + \int_V \rho f_m \, dV \tag{1.28}$$

Using Gauss divergence theorem equation 1.28 becomes:

$$\int_V \frac{\partial}{\partial t}(pq)\, dV + \int_V \nabla.(\rho q) q\, dV = \int_V \nabla.\tau\, dV + \int_V \rho f_m\, dV$$
$$\Rightarrow \int_V \left[\rho \frac{\partial q}{\partial t} + \nabla.(\rho q) q - \nabla.\tau - \rho f_m\right] dV = 0$$
$$\Rightarrow \rho \frac{\partial q}{\partial t} + \nabla.(\rho q) q = \nabla.\tau + \rho f_m \tag{1.29}$$

But $\nabla.(\rho q)q = (\rho q.\nabla)q + \rho q(\nabla.q)$, for incompressible fluid $\nabla.q = 0$, therefore,

$$\nabla.(\rho q)q = (\rho q.\nabla)q \tag{1.30}$$

By substituting equation 1.30 into equation 1.29 we obtain:

$$\rho \frac{\partial q}{\partial t} + (\rho q.\nabla)q = \nabla.\tau + \rho f_m \tag{1.31}$$

$$\tau = \begin{pmatrix} -p + \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -p + \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -p + \tau_{zz} \end{pmatrix} = -p \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

Where:
$\tau_{xx} = 2\mu \frac{\partial u}{\partial x}, \quad \tau_{yy} = 2\mu \frac{\partial v}{\partial y}, \quad \tau_{zz} = 2\mu \frac{\partial w}{\partial z}$
$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$
$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)$
$\tau_{xz} = \tau_{zx} = \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$

$$\nabla.\tau = \left(-\frac{\partial p}{\partial x} + \mu \nabla^2 u\right) i + \left(-\frac{\partial p}{\partial y} + \mu \nabla^2 v\right) j + \left(-\frac{\partial p}{\partial z} + \mu \nabla^2 w\right) k$$
$$= -\nabla p + \mu \nabla^2 q \tag{1.32}$$

By substituting equation 1.32 into equation 1.31 we get:

$$\rho \frac{\partial q}{\partial t} + (\rho q.\nabla)q = -\nabla p + \mu \nabla^2 q + \rho f_m \tag{1.33}$$

By dividing both sides of equation 1.33 by ρ we have:

$$\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 q + f_m \tag{1.34}$$

Let $\nu = \frac{\mu}{\rho}$ and substitute it into equation 1.34 to get:

$$\boxed{\frac{\partial q}{\partial t} + (q.\nabla)q = -\frac{1}{\rho}\nabla p + \nu \nabla^2 q + f_m} \tag{1.35}$$

Equation 1.35 is called Navier - Stokes equation for an incompressible viscous fluid.
Where:
q is the velocity vector of the flow.
p is the pressure.

f_m is the body force.
ν is the kinematic viscosity.

1.5.3 Energy Equation

From the first law of thermodynamics $\Delta E = W + Q$ where E is the total internal energy of the system, W is the work done, and Q is the heat transfer.
Note that: Q can be positive or negative.

$$\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt} \tag{1.36}$$

We want to use equation 1.36 to get the energy equation for fluid.
Total energy of the system = kinetic energy + potential energy.

$$dE = \rho e dV + \frac{\rho}{2} q^2 dV \quad \text{where} \quad e = c_v \tau = c_p T$$

The total energy is $E = \int \rho e \, dV + \int \frac{\rho q^2}{2} \, dV$ therefore,

$$\frac{dE}{dt} = \frac{\mathrm{d}}{\mathrm{d}t} \left[\int \rho e \, dV + \int \frac{\rho q^2}{2} \, dV \right]$$

$$\frac{dE}{dt} = \frac{\mathrm{d}}{\mathrm{d}t} \int_V \left(\rho e + \frac{\rho q^2}{2} \right) dV \tag{1.37}$$

$$\frac{dQ}{dt} = - \int_S (H.\hat{n}) \, dS \tag{1.38}$$

Where H is the heat transfer rate.
Fourer Heat Conduction Law gives us an expression for heat transfer which is given as $H = -k\nabla T$ where k is thermal conductivity, also from equation 1.36 $\frac{dW}{dt}$ = work done by surface force (qF_s) + work done by body force (qF_b)

$$\frac{dW}{dt} = \int_S q(\tau.\hat{n}) \, dS + \int_V \rho(q f_m) \, dV \tag{1.39}$$

Let's substitute equations (1.37, 1.38, and 1.39) into equation 1.36 to obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_V \left(\rho e + \frac{\rho q^2}{2} \right) dV = \int_S (k\nabla T.\hat{n}) \, dS + \int_S q(\tau.\hat{n}) \, dS + \int_V \rho(q f_m) \, dV$$

We applied Reynolds Transport theorem:

$$\int_V \frac{\partial}{\partial t} \left(\rho e + \frac{\rho q^2}{2} \right) dV + \int_S q \left[\left(\rho e + \frac{\rho q^2}{2} \right) .\hat{n} \right] dS = \int_S k\nabla T.\hat{n} \, dS + \int_S q(\tau.\hat{n}) \, dS + \int_V \rho(q f_m) \, dV$$

$\Rightarrow$

$$\int_V \frac{\partial \rho e}{\partial t} \, dV + \int_V \frac{\partial}{\partial t} \left(\frac{\rho q^2}{2} \right) dV + \int_S \rho e(q.\hat{n}) \, dS + \int_S \frac{\rho q^2}{2} (q.\hat{n}) \, dS = \int_S k\nabla T.\hat{n} \, dS + \int_S q(\tau.\hat{n}) \, dS + \int_V \rho(q f_m) \, dV$$

Therefore, we have

$$\int_V \frac{\partial \rho e}{\partial t}\, dV + \int_V \rho q \frac{\partial q}{\partial t}\, dV + \int_S \rho e(q.\hat{n})\, dS + \int_S \frac{\rho q^2}{2}(q.\hat{n})\, dS = \int_S k\nabla T.\hat{n}\, dS + \int_S q(\tau.\hat{n})\, dS + \int_V \rho(q f_m)\, dV$$

Now let's apply Gauss Divergence Theorem:

$$\int_V \rho \frac{\partial e}{\partial t}\, dV + \int_V \rho q \frac{\partial q}{\partial t}\, dV + \int_V \nabla.(\rho e q)\, dV + \int_V \nabla.(\frac{\rho q^2}{2} q)\, dV = \int_V \nabla.(k\nabla T)\, dV + \int_V q(\nabla.\tau)\, dV + \int_V \rho(q f_m)\, dV \tag{1.40}$$

Note $\nabla.\frac{\rho q^2}{2} q = q.\nabla(\frac{\rho q^2}{2}) + \frac{\rho q^2}{2}\nabla.q$ ($\nabla.q = 0$ for incompressible)
$\implies q.\nabla(\frac{\rho q^2}{2}) = q.\frac{\partial}{\partial q}(\frac{\rho q^2}{2})\frac{\partial q}{\partial x} = q.(\rho q)\nabla q$
In the same way $\nabla.q\tau = q(\nabla.\tau) + (\tau.\nabla)q$
Now equation 1.40 can be written as:

$$\int_V \rho \frac{\partial e}{\partial t}\, dV + \int_V \rho q \frac{\partial q}{\partial t}\, dV + \int_V \nabla.(\rho e q)\, dV + \int_V q(\rho q)\nabla q\, dV = \int_V \nabla.(k\nabla T)\, dV + \int_V q(\nabla.\tau)\, dV + \int_V (\tau.\nabla)q\, dV + \int_V \rho q f_m\, dV \tag{1.41}$$

Note that: By using Navier - Stokes Equation $\int_V \rho q \left(\frac{\partial q}{\partial t} + q.\nabla q\right)\, dV = \int_V q\,(\nabla.\tau + \rho f_m)\, dV$
then equation 1.41 becomes:
$\int_V \left(\rho\frac{\partial e}{\partial t} + \rho(\nabla.eq)\right)\, dV = \int_V (\nabla.k\nabla T + (\tau.\nabla)q)\, dV$
This implies that $\rho\left(\frac{\partial e}{\partial t} + \nabla.eq\right) = \nabla.k\nabla T + (\tau.\nabla)q$
Note: $\nabla.eq = e(\nabla.q) + (q.\nabla)e$
Therefore, $\rho\left(\frac{\partial e}{\partial t} + e(\nabla.q) + (q.\nabla)e = \nabla.k\nabla T + (\tau.\nabla)q\right)$, and $(\nabla.q = 0)$
But $e = c_p T$, therefore:

$$\rho c_p\left(\frac{\partial T}{\partial t} + q.\nabla T\right) = \nabla.k\nabla T + (\tau.\nabla)q$$

If k is constant then we have:

$$\rho c_p\left(\frac{\partial T}{\partial t} + q.\nabla T\right) = k\nabla^2 T + (\tau.\nabla)q$$

Note: $(\tau.q)q = \Phi$ (viscous dissipation function)

$$\rho c_p\left(\frac{\partial T}{\partial t} + q.\nabla T\right) = k\nabla^2 T + \Phi$$

$$\boxed{\frac{\partial T}{\partial t} + q.\nabla T = \frac{k}{\rho c_p}\nabla^2 T + \frac{1}{\rho c_p}\Phi} \tag{1.42}$$

Equation 1.42 is called Energy Equation.
Where:
$q.\nabla T$ is the heat convection term.
$\frac{k}{\rho c_p}\nabla^2 T$ is the heat conduction term.
$\frac{1}{\rho c_p}\Phi$ is the dissipation term.
Note that: In cartesian coordinates

$$\Phi = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right]$$

1.5.4 Shooting method [14]

Consider a 2nd order ordinary differential equation with two boundary conditions

$$\begin{aligned} &y'' = f(x, y, y'), \quad a < x < b \\ &y(a) = \alpha \\ &y(b) = \beta, \end{aligned}$$

where a, b, α, β are given constants, y is the unknown function of x, f is a given function that specifies the differential equation. This is a two-point boundary value problem. An initial value problem (IVP) would require that the two conditions be given at the same value of x. For example, $y(a) = \alpha$ and $y'(a) = \gamma$. Because the two separate boundary conditions, the above two-point boundary value problem (BVP) is more difficult to solve.
The basic idea of "shooting method" is to replace the above BVP by an IVP. But of course, we do not know the derivative of y at $x = a$. But we can guess and then further improve the guess iteratively. More precisely, we treat $y'(a)$ as the unknown, and use secant method or Newton's method (or other methods for solving nonlinear equations) to determine $y'(a)$.
We introduce a function u, which is a function of x, but it also depends on a parameter t. Namely, $u = u(x;t)$. We use u' and u'' to denote the partial derivative of u, with respect to x. We want u to be exactly y, if t is properly chosen. But u is defined for any t, by

$$\begin{aligned} &u'' = f(x, u, u') \\ &u(a;t) = \alpha \\ &u'(a;t) = t. \end{aligned}$$

If you choose some t, you can then solve the above IVP of u. In general u is not the same as y, since $u'(a) = t \neq y'(a)$. But if t is $y'(a)$, then u is y. Since we do not know $y'(a)$, we determine it from the boundary condition at $x = b$. Namely, we solve t from:

$$\phi(t) = u(b;t) - \beta = 0.$$

If a solution t is found such that $\phi(t) = 0$, that means $u(b;t) = \beta$. Therefore, u satisfies the same two boundary conditions at $x = a$ and $x = b$, as y. In other words, $u = y$. Thus, the solution t of $\phi(t) = 0$ must be $t = y'(a)$.
If we can solve the IVP of u (for arbitrary t) analytically, we can write down a formula for $\phi(t) = u(b;t) - \beta$. Of course, this is not possible in general. However, without an analytic formula, we can still solve $\phi(t) = 0$ numerically. For any t, a numerical

method for IVP of u can be used to find an approximate value of $u(b;t)$ (thus $\phi(t)$). The simplest method is to use the secant method.

$$t_{j+1} = t_j - \frac{t_j - t_{j-1}}{\phi(t_j) - \phi(t_{j-1})}\phi(t_j), \quad j = 1, 2, 3, \ldots$$

For that purpose, we need two initial guesses: t_0 and t_1 . We can also use Newton's method:

$$t_{j+1} = t_j - \frac{\phi(t_j)}{\phi'(t_j)}, \quad j = 0, 1, 2, \ldots$$

We need a method to calculate the derivative $\phi(t)$. Since $\phi(t) = u(b;t) - \beta$, we have

$$\phi'(t) = \frac{\partial u}{\partial t}(b;t) - 0 = \frac{\partial u}{\partial t}(b;t).$$

If we define $v(x;t) = \frac{\partial u}{\partial t}$, we have the following IVP for v:

$$\begin{aligned} v'' &= f_u(x, u, u')v + f_{u'}(x, u, u')v' \\ v(a;t) &= 0 \\ v'(a;t) &= 1. \end{aligned}$$

Here v' and v'' are the first and 2nd order partial derivatives of v, with respect to x. The above set of equations are obtained from taking partial derivative with respect to x for the system for u. The chain rule is used to obtain the differential equation of v. Now, we have $\phi'(t) = v(b;t)$.

1.5.5 Runge - Kutta Method [12]

The Runge - Kutta method is the most widely used method of solving differential equations with numerical methods. It differs from the Taylor series method in that we use values of the first derivative of $f(x,y)$ at several points instead of the values of successive derivatives at a single point.
For a Runge - Kutta method of order 2, the following formulas are applicable.

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + h, y_n + h) \\ y_{n+1} &= y_n + \frac{1}{2}(k_1 + k_2) \end{aligned} \tag{1.43}$$

Equation 1.43 is for Runge - Kutta Method of order 2.
When higher accuracy is desired, we can use order 3 or order 4. The applicable formulas are as follows:

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\ k_3 &= hf(x_n + h, y_n + 2k_2 - k_1) \\ y_{n+1} &= y_n + \frac{1}{6}(k_1 + 4k_2 + k_3) \end{aligned} \tag{1.44}$$

Equation 1.44 is for Runge - Kutta Method of order 3

$$\begin{aligned}
k_1 &= hf(x_n, y_n) \\
k_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\
k_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \\
k_4 &= hf(x_n + h, y_n + k_3) \\
y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2K_2 + 2k_3 + k_4)
\end{aligned} \tag{1.45}$$

Equation 1.45 is for Runge - Kutta Method of order 4

1.5.6 Runge-Kutta-Fehlberg Method (RKF45) [16]

One way to guarantee accuracy in the solution of an I.V.P. is to solve the problem twice using step sizes h and $h/2$ and compare answers at the mesh points corresponding to the larger step size. But this requires a significant amount of computation for the smaller step size and must be repeated if it is determined that the agreement is not good enough.
The Runge-Kutta-Fehlberg method (denoted RKF45) is one way to try to resolve this problem. It has a procedure to determine if the proper step size h is being used. At each step, two different approximations for the solution are made and compared. If the two answers are in close agreement, the approximation is accepted. If the two answers do not agree to a specified accuracy, the step size is reduced. If the answers agree to more significant digits than required, the step size is increased.
Each step requires the use of the following six values:

$$\begin{aligned}
k_1 &= hf(x_n, y_n) \\
k_2 &= hf(x_n + \frac{1}{4}h, y_n + \frac{1}{4}k_1), \\
k_3 &= hf\left(x_n + \frac{3}{8}h, y_n + \frac{3}{32}k_1 + \frac{9}{32}k_2\right), \\
k_4 &= hf\left(x_n + \frac{12}{13}h, y_n + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right), \\
k_5 &= hf\left(x_n + h, y_n + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right), \\
k_6 &= hf\left(x_n + \frac{1}{2}h, y_n - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)
\end{aligned} \tag{1.46}$$

Then an approximation to the solution of the I.V.P. is made using a Runge-Kutta method of order 4:

$$y_{n+1} = y_n + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5 \tag{1.47}$$

where the four function values k_1 , k_3 , K_4 , and k_5 are used. Notice that k_2 is not used in formula (1.47). A better value for the solution is determined using a Runge-Kutta method of order 5:

$$z_{n+1} = y_n + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6. \tag{1.48}$$

1.6 Study Limitations

The work in the present was done with the help of mathematical tools and methods in which some assumption have been made for an accuracy of the results. For instance, in order to gain an insight into the complex interaction between the surface runoff and soil water percolation, some assumption were made for the values of biophysical parameters that were utilised for the numerical computations. This study only consider the problem on the two phase flows of an incompressible Newtonian fluid within the soil and on the soil surface. It could be extended to the three phase flows of an incompressible Newtonian fluid.

Chapter 2

Literature Review

Surface runoff and soil water percolation are closely associated with rainfall and melting of snow, or glaciers. Soil inability to absorb excess stormwater and meltwater due to heavy rainfall, high melt rate of snow and glacier, soil saturation, impervious resulting from surface sealing or pavement, etc., do lead to surface runoff [10].

Johnsson, H. and Lundin, L-C. (1991) in their paper titled "Surface runoff and soil water percolation as affected by snow and soil frost". They used a couple soil water and heat model to study the influence of soil frost and snow on infiltration and drainage flow in agricultural field in central Sweden. Knowledge of flow pathways of snowmelt in arable fields is crucial in studies of environmental problems in cold climates such as soil erosion and leaching of nutrients. Whereas surface runoff containing eroded soil is a major source of phosphorus (e.g. Burwell et al., 1975; Ul n, 1985), the percolation and drainage of snowmelt result in nitrate leaching. To calculate annual soil water balances in arable lands also requires knowledge of snowmelt flow pathways (e.g. Jansson et al., 1989). The partitioning of snowmelt into percolation and surface runoff is influenced by field drainage characteristics, the soil frost conditions and the snow-melting rate [11].

Tatiana Kaletova and Zuzana Nemetova (2017) have focused on a determination of a volume of surface water runoff and its velocity and their results have shown that the higher slope, the higher the runoff velocity and volume is. The surface runoff and its characteristics depend mainly on amount of precipitation, amount of infiltrated water, surface accumulation of precipitation and interception (Muchová and Antal 2013). Those conditions then create the surface runoff, flood wave, soil erosion, soil water and groundwater storage, change of surface roughness, etc. Qian et al. (2016) mentioned that flow velocity is a crucial indicator in the study of soil erosion. Eroded materials slowly decrement a capacity of water reservoirs (Kubinský et al. 2015); therefore, its initial purpose is progressively changed, eventually there is no more the water reservoir. The surface roughness highly impacts the soil erosion process, as well as hydrological processes in the landscape, e.g., velocity of surface runoff. The rough surface has a lot of depressions and barriers, which decrement a transport capacity of a water body by the decrease of runoff velocity (Šinka and Moravčík 2015) [24]

Yuyang Wu et al. (2017) highlighted that soil erosion was very closely and positively correlated with surface runoff. Soil loss was higher in snowmelt periods than in rainy periods due to the higher surface runoff in early spring, and the decreased soil infiltrability in snowmelt periods contributed much to this higher surface runoff. These findings are helpful for identification of critical soil erosion periods when making soil management before critical months, especially those before snowmelt periods. Surface runoff is always accompanied by soil erosion and sediment yield (Abrol et al., 2016), with the flow carrying soil materials away. In addition to surface flow velocity, surface runoff affects soil erosion mainly in the variations of runoff volume. In rainy seasons with high precipitation intensity, increasing runoff discharge was the dominant factor influencing soil erosion in a study reported by Vaezi et al. (2017). While in early spring, runoff led by snowmelt will also aggravate the erosion of surface soils (Zuzel et al., 1982). As mentioned above, although surface runoff exhibits a growing trend in both rainy and snowmelt periods due to concentrated rainfall and snowmelt water, the annual runoff feature has not always been the same as the seasonal runoff feature (Luo et al.,2017). For this reason, detection of runoff variations on the seasonal scale in response to precipitation and temperature variations is necessary [27].

Baoli Xu et al.(2017) Wrote that understanding soil water percolation in paddy fields is helpful to optimize irrigation schedule for rice production and improve water use efficiency under various irrigation practices and groundwater depths. Percolation is a vital component of water balance in hydrologic processes in which water moves downward to groundwater. As a pathway for water losses from the rice root zone, deep percolation (DP) reduces water use efficiency in paddy field and accounts for (50–80%) of water input (Belder et al., 2007; Cesari de Maria et al., 2016) [1].

Xiaoming Lai et al. (2016) reported that the knowledge of soil water percolation below the rooting zone and its responses to the dynamic interactions of different factors are important for the control of non-point source pollution. (Ochoa et al., 2007; Zhang et al., 2007; He et al., 2012) wrote that soil water percolation below the rooting zone is a momentous link in the terrestrial hydrological cycle. As a primary pathway to recharge groundwater, deep percolation (DP) provides important hydrologic and ecosystem benefits in different regions [13].

The use of mathematical models to tackle the menace of surface runoff and enhance the soil water percolation for the formation of groundwater aquifers has attracted the attention of several scientists and researchers [7, 22, 4, 21, 20, 15]. For soil with high permeability like sandy soil, the relationship between the flow rates and the pressure gradient would be practically linear based on the Brinkman form of Darcy law, while this relationship may be nonlinear for soil with low permeability lay clay soil (Darcy–Forchheimer law) [25, 2, 18]. Bristow and Horton [3] theoretically investigated the influence of surface mulch soil water flow and heat transfer. The effects of temperature gradient on the soil water flow were studied by Gurr et al. [9]. Numerical results on soil water flow and heat transfer rate together with soil-atmosphere interaction was reported by Fetzer et al. [8]. In all the above studies, it is observed that mathematical model of soil-runoff interface at the continuum scale where water and energy fluxes are highly dynamic are often magnified. This may lead to inaccuracy in the result obtained.

Chapter 3

Model Problem and Solution Procedure

3.1 Model Problem

The soil is regarded as a permeable porous media due to its composition of compactly packed different particle sizes that form pore spaces which permit both infiltration and percolation of water to take place. It is bounded below by the groundwater accumulation or bedrock with temperature T_0 and above by the presence of runoff water at varying temperature T_2 and velocity u_2 caused by excessive rainfall that exceed permeability, soil saturation, snowmelt or other sources. The soil temperature and water velocity due to percolation within the soil is given by T_1 and u_1 respectively as shown in figure 1. It is assumed that the runoff water is incompressible with constant properties.

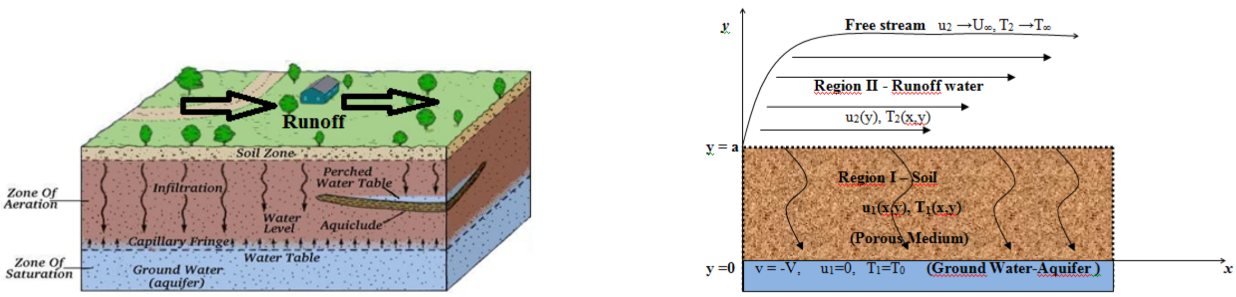

FIGURE 3.1: Schematic diagram of the problem

Under these assumptions, the model momentum and energy equations for both the soil region-I and the runoff water region-II can be written as:

Region I - Soil (Water Percolation)

$$-V\frac{du_1}{dy} = -\frac{1}{\rho_1}\frac{\partial P}{\partial x} + \frac{\mu_1}{\rho_1}\frac{d^2u_1}{dy^2} - \frac{\mu_1 u_1}{\rho_1 K} - \frac{cu_1^2}{\rho_1\sqrt{K}}, \tag{3.1}$$

$$-V\frac{dT_1}{dy} = \frac{k_1}{\rho_1 c_p}\frac{d^2T_1}{dy^2} + \frac{\mu_1}{\rho_1 c_p}\left(\frac{du_1}{dy}\right)^2 + \frac{\mu_1 u_1^2}{\rho_1 c_p K} + \frac{cu_1^3}{\rho_1 c_p\sqrt{K}} \tag{3.2}$$

Region II - Runoff Water

$$\mu_2 \frac{d^2 u_2}{dy^2} + \rho_2 V \frac{du_2}{dy} = 0, \tag{3.3}$$

$$k_2 \frac{d^2 T_2}{dy^2} + \rho_2 c_{pf} V \frac{dT_2}{dy} + \mu_2 \left(\frac{du}{dy}\right)^2 = 0, \tag{3.4}$$

The appropriate boundary conditions at the runoff water free surface as well as the interface between the soil and the runoff water including the underground soil bound are given as,

$$\begin{aligned} &u_1(0) = 0, u_1(a) = u_2(a), \mu_1 \frac{du_1}{dy}(a) = \mu_2 \frac{du_2}{dy}(a), u_2 \to U_\infty, \\ &T_1(0) = T_0, T_1(a) = T_2(a), k_1 \frac{dT_1}{dy}(a) = k_2 \frac{dT_2}{dy}(a), T_2(\infty) \to T_\infty, \end{aligned} \tag{3.5}$$

where u_1 is the soil water percolation velocity (m/s), u_2 is the runoff water velocity (m/s), T_1 is the soil temperature $(Kelvin)$, T_2 is the runoff water temperature$(Kelvin)$, T_0 is the groundwater or bedrock temperature $(Kelvin)$, T_∞ is the runoff water free stream temperature$(Kelvin)$, a is the soil depth (m), k_1 is the saturated soil thermal conductivity $(W/mKelvin)$, k_2 is the runoff water thermal conductivity $(W/mKelvin)$, K is the soil permeability rate (m^2), V is the soil water suction velocity(m/s), μ_1 is the soil water dynamic viscosity (kg/ms), mu_2 is the runoff water dynamic viscosity $(kg/m.s)$, ρ_1 is the soil water density (kg/m^3), ρ_2 is the runoff water density (kg/m^3), c_{pf} is the runoff water specific heat capacity $(J/kgKelvin)$ and c_p is the soil specific heat capacity $(J/kgKelvin)$. In order to render the model equations dimensionless, the following variables and parameters are introduce into equations (3.1)-(3.5);

$$\begin{aligned} &\theta_1 = \frac{T_1 - T_0}{T_\infty - T_0}, \theta_2 = \frac{T_2 - T_0}{T_\infty - T_0}, W_1 = \frac{u_1}{V}, W_2 = \frac{u_2}{V}, \eta = \frac{y}{a}, X = \frac{x}{a}, S = \frac{cVa^2}{\mu_1 \sqrt{K}}, L = \frac{U_\infty}{V}, \\ &\bar{P} = \frac{aP}{\mu_1 V}, Da = \frac{K}{a^2}, Re = \frac{Va}{\nu_1}, \nu_1 = \frac{\mu_1}{\rho_1}, \nu_2 = \frac{\mu_2}{\rho_2}, Pr_1 = \frac{\mu_1 c_p}{k_1}, Pr_2 = \frac{\mu_2 c_{pf}}{k_2}, m = \frac{k_1}{k_2}, \\ &\gamma = \frac{\mu_1}{\mu_2}, Ec = \frac{V^2}{c_{pf}(T_\infty - T_0)}, \delta = \frac{c_{pf}}{c_p}, A = -\frac{d\bar{P}}{dX}, n = \frac{\nu_1}{\nu_2}, \end{aligned} \tag{3.6}$$

and we obtian

Region I - Soil (Water Percolation)

$$\frac{d^2 W_1}{d\eta^2} + Re \frac{dW_1}{d\eta} - \frac{W_1}{Da} - SW_1^2 + A = 0, \tag{3.7}$$

$$\frac{d^2 \theta_1}{d\eta^2} + Pr_1 Re \frac{d\theta_1}{d\eta} = -\delta Ec Pr_1 \left[\left(\frac{dW_1}{d\eta}\right)^2 + \frac{W_1^2}{Da} + SW_1^3 \right], \tag{3.8}$$

Region II - Runoff Water

$$\frac{d^2 W_2}{d\eta^2} + nRe \frac{dW_2}{d\eta} = 0, \tag{3.9}$$

$$\frac{d^2 \theta_2}{d\eta^2} + nPr_2 Re \frac{d\theta_2}{d\eta} + Pr_2 Ec \left(\frac{dW_2}{d\eta}\right)^2 = 0, \tag{3.10}$$

with the appropriate boundary conditions in dimensionless form as

$$
\begin{aligned}
&W_1(0) = 0, \quad W_1(1) = W_2(1), \quad \gamma \frac{dW_1}{d\eta}(1) = \frac{dW_2}{d\eta}(1), \quad W_2 \to L, \\
&\theta_1(0) = 0, \quad \theta_1(1) = \theta_2(1), \quad m\frac{d\theta_1}{d\eta}(1) = \frac{d\theta_2}{d\eta}(1), \quad \theta_2(\infty) \to 1,
\end{aligned} \tag{3.11}
$$

where Re is the soil water suction Reynolds number, Da is the soil Darcy number, S is the Forchheimer parameter (i.e. soil nonlinear permeability parameter), Ec_2 is the runoff water Eckert number, δ is the surface runoff-soil water specific heat capacity ratio, Ec is the runoff and soil water Eckert number, Pr_1 is the soil water Prandtl number, Pr_2 (≈ 6.2) is the runoff Prandtl number, L is the free stream velocity parameter, m is soil water-runoff thermal conductivity ratio, γ is soil water-runoff dynamic viscosity ratio and n is the soil water-runoff kinematic viscosity ratio. It is important to note that Eckert number Ec represents the effects of internal heat generation due to soil water and surface runoff energy dissipation. The Darcy number Da and Forchhiemer number S show the both linear and nonlinear soil permeability rate due to its composition compactly packed particles which form a porous matrix that permit water percolation. Other parameters of interest are the skin friction coefficients (C_f) that is the leading cause of surface runoff soil erosion and the Nusselt number (Nu) at the soil interface with the runoff water, are given as:

$$
C_f = \frac{a^2\tau_w}{\rho_1 V^2} = \left.\frac{dW_1}{d\eta}\right|_{\eta=1}, \quad Nu = \frac{aq_w}{k_1(T_0 - T_\infty)} = -\left.\frac{d\theta_1}{d\eta}\right|_{\eta=1}, \tag{3.12}
$$

where the shear stress τ_w and the heat flux q_w at the soil surface are given as

$$
\tau_w = \mu_1 \left.\frac{du_1}{dy}\right|_{y=a}, \quad q_w = -k_1 \left.\frac{dT_1}{dy}\right|_{y=a}. \tag{3.13}
$$

3.2 Solution Procedure

The dimensionless model equations (3.7)-(3.10) together with its boundary conditions (3.11)form a three points boundary value problem that seems intractable numerically, however, we reduce the problem to a two points boundary value problem by analytically determined the solutions for region-II (Surface runoff water) that satisfy the free stream conditions. The velocity and temperature profiles of the surface runoff water are given as:

$$
W_2(\eta) = L + A_1 e^{-nRe\eta}, \quad \theta_2(\eta) = 1 + \frac{EcPr_2 A_1^2}{2(Pr_2 - 2)} e^{-2nRe\eta}, \tag{3.14}
$$

where A_1 is to be numerical determined base on the prescribed interface conditions between the soil surface and the runoff water in equation (3.11). The two point's boundary value problem of region-I (soil water percolation) is numerically tackled using Runge-Kutta-Fehlberg integration scheme coupled with shooting method. From the numerical solution for velocity and temperature profiles, we compute the values for the skin friction (C_f) (soil erosion factor) and the Nusselt number (Nu) as given by equations (3.12).

Chapter 4

Discussion and Graphical Results

For numerical results, the following parameter values $A = 1$, $Da = 0..0.3$, $S = 10..700$, $Ec = 0..5$, $Re = 0..1$, $L = 0.5..1$, $Pr_1 = 20$, $Pr_2 = 6.2$,$\delta = 0.5$, $m = 2$, $n = 5$, $\gamma = 10$ are utilised.
Graphically results depicting the velocity and temperature profiles of surface runoff and soil water percolation as well as soil-runoff interface skin friction and Nusselt number are displayed.

4.1 Discussion

4.1.1 Runoff and Soil Percolation Velocity Profiles

Figures 4.1 to 4.4 illustrate the impact of embedded biophysical parameters on stormwater velocity profiles both within the soil in the region $0 \leq \eta \leq 1$ and the runoff in the region $\eta > 1$. The das lines correspond to water percolation rate within the soil while the thick lines indicate surface runoff velocity. It is interesting to note an escalation in the velocities of both soil water percolation and surface runoff with increasing values of soil suction parameter Re and runoff rate parameter L as shown in figures 4.1 and 4.2. This may be due to loose texture of soil composition which enhanced stormwater infiltration and percolation into the soil with increasing rate of surface runoff, leading to an increase in groundwater accumulation. Figure 4.3 shows that an increase in soil permeability boost the stormwater percolation. As Darcy number Da increases, the percolation rates and the soil pressure gradient would be practically linear, consequently, the stormwater infiltration and groundwater accumulation amplify. This scenario is evidence in runoff over a sandy or loosely packed soil composition. Moreover, as Forchheimer parameter S rises, the soil permeability diminishes, leading to a tightly packed soil composition like clay soil. Hence, the percolation rates and the soil pressure gradient becomes nonlinear, consequently, the stormwater percolation into the soil drastically reduced as shown in figure 4.4.

4.1.2 Surface Runoff and Soil Percolation Temperature Profiles

Figures 4.5 to 4.9 show the effects of various biophysical parameters variation on the stormwater temperature profiles both within the soil in the region $0 \leq \eta \leq 1$ and the runoff in the region $\eta > 1$. The das lines correspond to the temperature distribution within the soil water while the thick lines depict the temperature distribution within the surface runoff. Figure 4.5 revealed that a boost in suction Reynolds number causes the soil temperature to rise while the stormwater runoff temperature decreases. This may be due to exchange of energy between the surface runoff and the soil coupled with increasing rate of percolation. Meanwhile, an increase in the runoff rate parameter L and the Eckert number Ec enhance the soil temperature as shown in figures 4.6 and 4.7. These parameters enhance the percolation rate which invariably lead to an elevation in the rate of internal heat generation within the soil. In figure 4.8 we observed that an increase in soil permeability (with Da increases) decreases the soil temperature. This can be attributed to loose texture of soil composition which enhance heat loss. The trend is opposite with a rise in Forchheimer parameter S due to a decrease in the soil permeability as shown in figure 4.9, the soil composition is tightly packed like clay soil, consequently, the heat loss diminishes and soil temperature augments.

4.1.3 Soil Surface Erosion (Skin Friction) and Heat Transfer Rate (Nusselt Number)

Figures 4.10 to 4.13 demonstrate the effects of various biophysical parameters on the coefficient of skin friction which invariably lead to soil erosion and the heat transfer rate at the soil surface ($\eta = 1$) due to interaction between the runoff and the soil water percolation.
Interestingly, the soil surface erosion caused by skin friction escalates with increasing values of suction Reynolds number Re, Forchheimer parameter S and runoff rate parameter L as shown in figures 4.10 and 4.11. As these parameters increase, stormwater runoff velocity gradient at the soil surface amplifies and soil permeability diminishes, consequently, the rate of erosion of top soil surface escalates. This extreme scenario may damage tightly packed soil surface and cause surface water pollution. Moreover, a rise in soil permeability with increasing Darcy number Da, causes the skin friction to fall and lessens the effect soil erosion. This may be attributed to an increase in the soil permeability which enhance stormwater percolation and diminish the runoff velocity gradient at the soil surface. The effects of various parameters on heat transfer rate at the soil surface are depicted in figures 4.12 and 4.13. The Nusselt number is rises with an amplification in the parameter values of Re, L, Ec and S. As these parameters increase, the soil becomes tightly packed and the surface runoff rate rises, leading to an escalation in the temperature gradient at the soil surface and heat flux. The trend is opposite when the soil is loosely packed with high permeability (Da increasing), consequently, the Nusselt number deceases.

4.2 Graphical Results

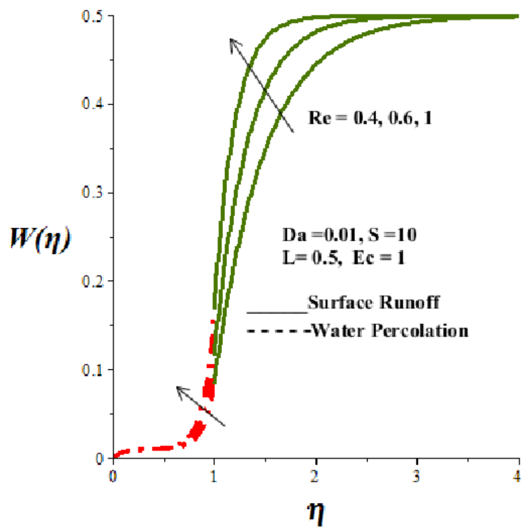

FIGURE 4.1: Velocity profile with increasing *Re*

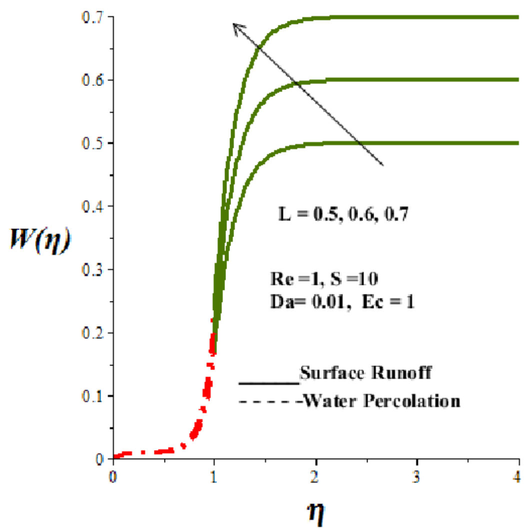

FIGURE 4.2: Velocity profile with increasing *L*

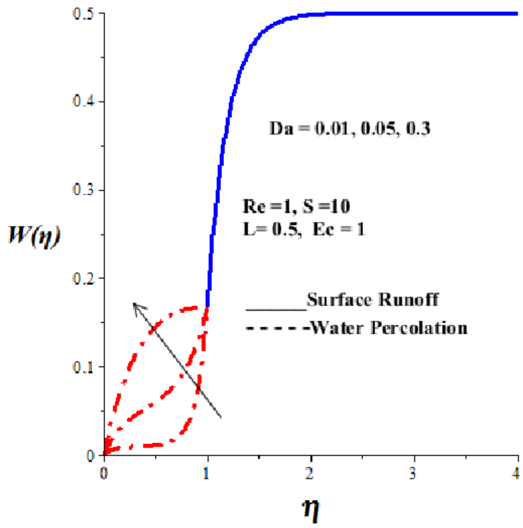

FIGURE 4.3: Velocity profile with increasing *Da*

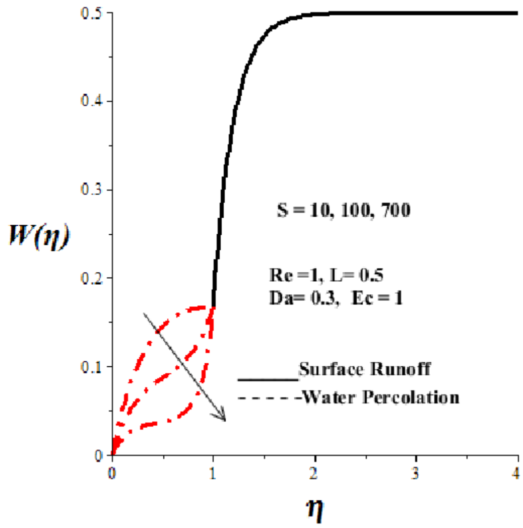

FIGURE 4.4: Velocity profile with increasing *S*

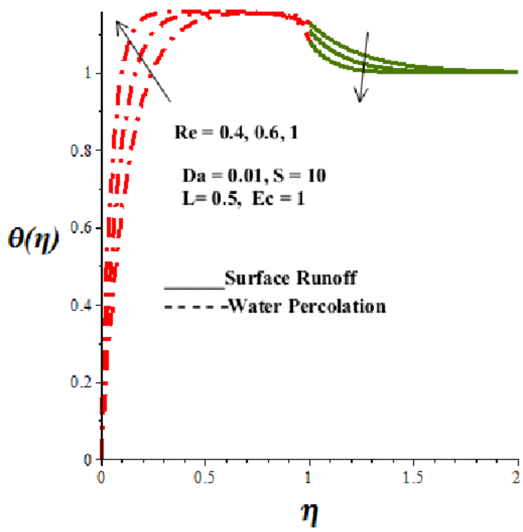

FIGURE 4.5: Temperature profile with increasing *Re*

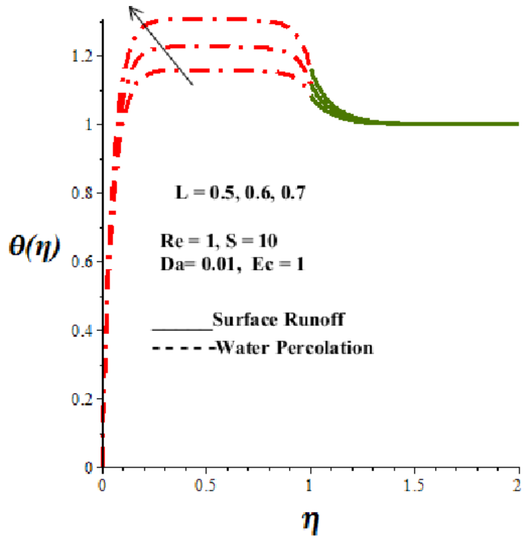

FIGURE 4.6: Temperature profile with increasing L

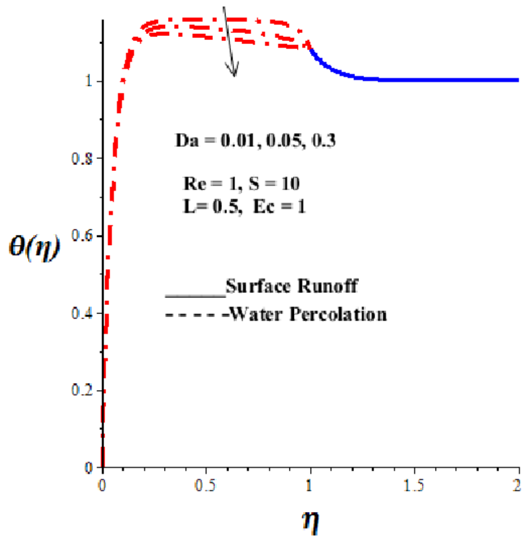

FIGURE 4.7: Temperature profile with increasing Da

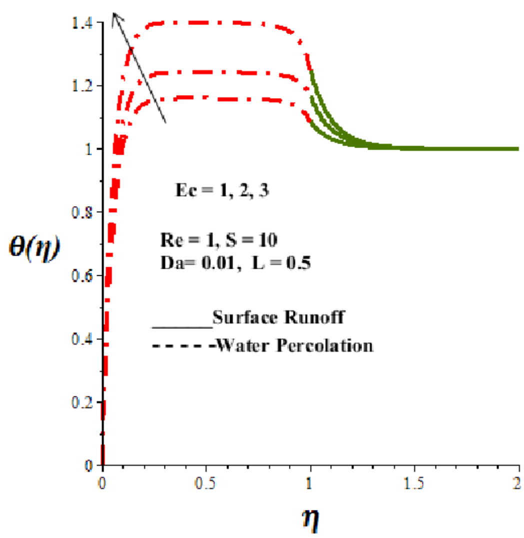

FIGURE 4.8: Temperature profile with increasing Ec

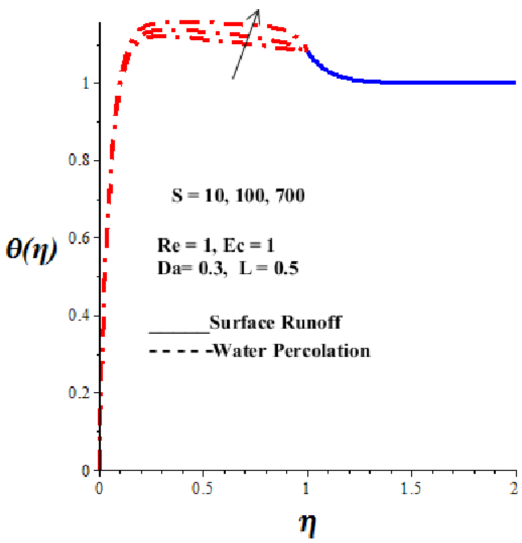

FIGURE 4.9: Temperature profile with increasing S

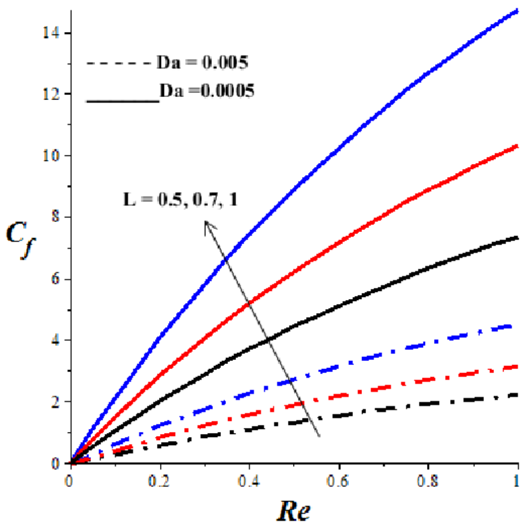

FIGURE 4.10: Skin friction with increasing Re, L Da

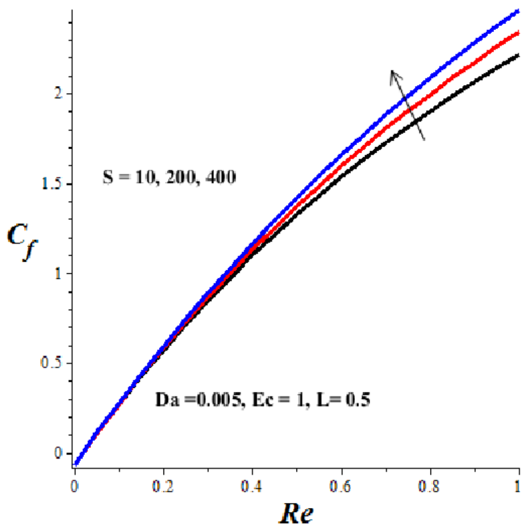

FIGURE 4.11: Skin friction with increasing S

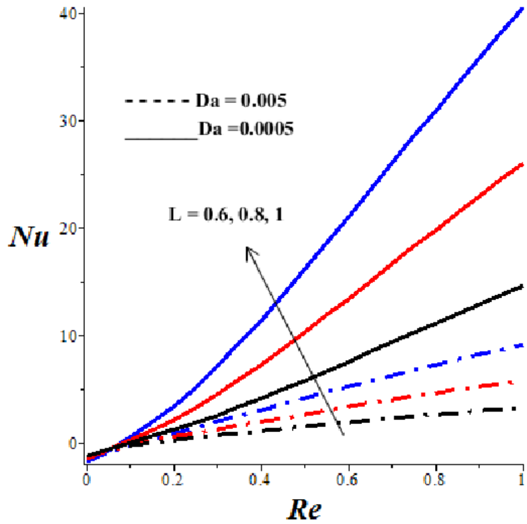

FIGURE 4.12: Nusselt number with increasing *Re*, *L*, *Da*

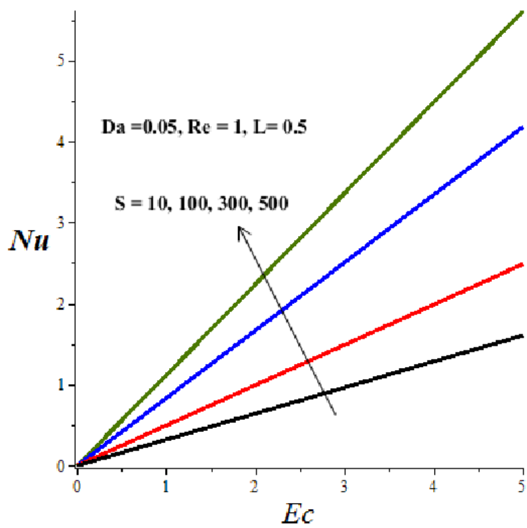

FIGURE 4.13: Nusselt number with increasing *Ec*, *S*

Chapter 5

Conclusion

Conclusion

The complex interaction between the surface runoff and the soil water percolation is theoretically investigated using two phase flow model that relies on the continuum mechanics principle of conservation laws of mass, momentum and energy. The model boundary value problem is numerically tackled. The impact of various embedded biophysical parameters on the velocity and temperature distribution of stormwater transport within the soil and across the surface runoff together with soil-runoff interface skin friction and Nusselt number are determined. Our results can be summarised as follows:

• Increase in Re, L, and Da boost the soil water percolation rate coupled enhanced rate of runoff while increase in S lessens it.

• Increase in Ec, S, L and Re boost the soil temperature while increase in Da lessens it.

• Increase in Re, L and S boost skin friction and promote soil surface erosion while increase in Da lessens it.

• Increase in Re, L, Ec and S enhances Nusselt number while increase in Da lessens it.

Finally, the biophysical conditions of the soil and its usefulness for agricultural and industrial purpose may be determined by its interaction with surface runoff. Adequate knowledge of complex interaction of surface runoff and stormwater percolation are essential in order to thwart flooding, soil erosion, surface water pollution and augment groundwater accumulation. Our results will no doubt be of agricultural and environmental interest.

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