AN APPROACH TO IDENTIFYING THE OPTIMAL TIME TO INITIATE STIMULATION OPERATIONS.

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ABSTRACT

This research describes the application of a production economic approach which attempts to quantity stimulation treatments performed on different wells undergoing a decline phase in their production and to obtain based on the output from the model the best time during the declining phase of the wells to initiate stimulation treatments.

Two different production decline models were considered in this work i.e., the exponential model and the hyperbolic model. The optimization models which are in the form of non-linear programming models combine production decline curve analysis with economic discounting concepts. The objective function of the non-linear programming problem is subject to the constraints imposed by the production facilities, reservoir productivity and the stimulation budget approved by management.

The results from both optimization models show that the best time to initiate stimulation during the decline phase of a wells production should be during the periods when the current production rate is at least fifty percent of the initial production rate. The optimal discounted production for the exponential model shows that the best time to initiate stimulation is when the wells current rate is about seventy percent of the initial production rate.

The results from this work can thus serve as guide for reservoir management decision making and also as a tool for communication between the economists and the reservoir engineer.

DEDICATION

To the keeper and shepherd of my soul who watches over me

He

Neither sleeps Nor slumbers

To as many as had to go through one *trial or another* and are still standing *strong*

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Chapter One

Introduction

Well stimulation is being utilized throughout the world in an attempt to increase oil and gas production rates by maximizing reservoir energy, increasing the effective wellbore radius, increasing permeability and conductivity around the near wellbore, and recently in Enhanced Oil Recovery (EOR) processes.

It is one aspect of well construction operations that has attracted a lot of research and publications over the years because of its scope and the fact that the operation increases the production of petroleum from the reservoir. This increase in production is achieved by removing the damage around the wellbore or by superimposing a highly conductive structure onto the formation. Several well stimulation techniques have been used; the commonly used processes include hydraulic fracturing, recompletion, matrix acidizing and fracture acidizing.

As with many engineering processes, stimulation must culminate in the design, selection of the specific treatment and, of course, selection of candidate wells. To choose among the various options, of which one is to do nothing, a means for an economic comparison of the incremental benefits weighted against the costs is necessary [1]

Hence it is important to select stimulation candidate wells that have potentials for maximum benefit. Candidate Selection is the process of identifying and selecting wells for treatment which have the capacity for higher production and better economic return. Thus, in stimulation candidate well selection, the treatment yielding the highest discounted rate of return is the treatment which, in principle, should be carried out first.

To optimize a stimulation treatment, it is necessary that the expected incremental revenue is related to the cost of performing the job. Since the revenue is to be realized over time, it must be discounted to time zero which is taken as the time of the job execution. This time can lie anywhere between the start of decline and the point at which the well reaches its economic-limit. It is therefore important to investigate how the time to initiate stimulation affects the total revenue to be realized from the stimulation and thus the best time to initiate such stimulation decision.

For proper optimization, all of the factors that affect stimulation that is everything that supports, links to or can, in any way, affect stimulation, has to be fully integrated and optimized to maximize oil and gas production.

1.1 Economic Impact of Well Stimulation

Many factors have to be considered for a proper evaluation of the economics of any stimulation treatment. These factors may among others include: treatment cost, initial increase in production rate, additional reserves that may be produced before the well reaches its economic limit, rate of production decline before and after stimulation, and reservoir and mechanical problems that could cause the treatment to be unsuccessful.

Selection of the optimum size of a stimulation treatment is based primarily on economics. The most commonly used measure of economic effectiveness is the net present value (NPV). The NPV is the difference between the present value of all receipts and costs, both current and future, generated as a result of the stimulation treatment. Future receipts and costs are converted into present value using a discount rate and taking into account the year in which they will appear. Another measure of the economic effectiveness is the payout period (PO); that is, the time it takes for the cumulative present value of the net well revenue to equal the treatment costs. Other indicators include internal rate of return (IRR), profit-to-investment ratio (PIR) and growth rate of return (GRR). The NPV (as other equivalent indicators) is sensitive to the discount rate and to the predicted future hydrocarbon prices.[2]

As with almost any other engineering activities, costs increase almost linearly with the size of the stimulation treatment but (after a certain point) the revenues increase only marginally or may even decrease. This suggests that there is an optimum size of the treatment that will maximize the NPV (Balen et.al., 1988)

1.2 Objectives and Scope of the Study

The objectives of this proposed work include:

1. Reproduce in a spreadsheet environment the Optimization Model for stimulation candidate selection which was developed by Ugbenyen (2010).
2. Use the model to select the best timing to initiate stimulation treatment (i.e. the time that gives the best post stimulation output)
3. Use the model to study the impact of stimulation parameters on the optimal time to initiate stimulation.

Hence this research will attempt to answer the questions: “given the need to stimulate several wells in a field, what is the best time to initiate the stimulation treatments and what impact do the stimulation parameters have on the optimal time to initiate stimulation?” To answer these questions, an optimization model is developed with the objective function based on production decline curve analysis and economic discounting concepts. The model includes stimulation treatment design module to be used for determining the best time to initiate stimulation.

The research procedure begins in Chapter One with an introduction to the concept of well stimulation and the need to optimize stimulation decisions. Several papers published on formation damage and stimulation methods in the technical literature are reviewed in Chapter Two. Chapter Three contains an extension of the model developed by Benson Ugbenyen (2010). The Optimization model is used to study the effect of initiating stimulation at different times during production rate decline and finding the best time that gives the maximum stimulation benefits. The model presented in Chapter Three is used to solve for the objective function and for determining the optimal time for each set of well data. In Chapter Four, the results obtained from this work are presented and discussed. The conclusions and recommendations for future work are presented in Chapter Five.

1.3 Limitations of the Study

This research is intended for use as a first pass guide in deciding the best time to initiate stimulation treatments on wells that are undergoing decline in their production profiles. The data used to validate the methodology are from wells producing from the Niger Delta formations which are known for their good permeability. The available stimulation options that can be considered in the Niger Delta include matrix acidizing, recompletion and gravel packing. The matrix acidizing option was chosen for the model development. Acid fracturing and hydraulic fracturing are not considered in this work.

Chapter Two

Literature Review

Formation damage especially within the near well-bore area is often the basis for carrying out stimulation operations. To properly select candidate wells for stimulation, a good understanding of the concept of formation damage and well stimulation is needed. Many publications have been made on the subjects and the goal of this chapter is to give a brief review of formation damage and well stimulation methods

2.1 Formation Damage

2.1.1 Definition

Formation damage is an undesirable operational and economic problem that can occur during the various phases of oil and gas recovery from subsurface reservoirs including drilling, production, hydraulic fracturing, and work-over operations. It is a term used in referring to the impairment of the permeability of petroleum bearing formations by some of the processes mentioned above[4]. Bennion[5] viewed formation damage as any process that causes a reduction in the natural inherent productivity of an oil and gas producing formation, or a reduction in the injectivity of a water or gas injection well. The consequences of formation damage are the reduction of the oil and gas productivity of reservoirs and noneconomic operation. It is therefore essential, to develop experimental and analytical methods for understanding and preventing and/or controlling formation damage in oil and gas bearing formations [6].

2.1.2 Formation Damage Mechanisms

According to Bennion et al. (2000) formation damage mechanisms which are responsible for low productivity of reservoirs predominantly fall into the following three major classifications:

- Mechanical formation damage mechanisms
- Chemical formation damage mechanisms

- Biological formation damage mechanisms

2.1.2.1 Mechanical Formation Damage Mechanisms

The following problems tend to be the most significant for low permeability gas reservoirs

Fines Migration: This refers to the motion of naturally occurring in-situ particulates within the pore system caused by high interstitial fluid velocities induced by differential pressure gradients.

External Solids Entrainment: This damage mechanism refers to the invasion of solid particulate material into the matrix surrounding the wellbore or fracture face during overbalanced drilling or completion operations.

Glazing and Mashing: Refers to mechanical damage induced by the drill bit or rotating drill string and represents a layer of very thin damage immediately at the wellbore formation interface.

2.1.2.2 Chemical Formation Damage Mechanisms

Adverse Clay Interactions: Certain clay structures may either be susceptible to hydration by fresh or low salinity water contact (such as smectite or mixed layer clays), or deflocculating or dispersion (kaolinite and other fines) caused by abrupt salinity transitions or caustic pH.

Various Precipitates and Solids: Compatibility of introduced fluids with in-situ formation fluids is always of importance in an effective drilling, completion and stimulation program. Injection of fluids incompatible with the in-situ formation fluids can lead to precipitation of solids that can impair formation productivity.

2.1.2.3 Biological Damage Mechanisms

These are associated with the introduction of viable bacteria to the formation which may colonize and propagate in either an aerobic or anaerobic fashion. This may create issues with corrosion, souring of reservoir fluids (sulphate-reducing bacteria), or production of viscous polysaccharide bio-slimes which may occlude permeability. Most bacteria cannot thrive at temperatures above 100-110 ^{o}C which, in deeper tight gas formations, may reduce the severity of this problem.

2.1.3Formation Damage in Horizontal Wells

Horizontal wells are more susceptible to formation damage than vertical wells for the following reasons. [8, 9]

- The pay zone in a horizontal well comes into contact with a drilling fluid for a much longer period than a vertical pay zone (days compared with hours).
- Most horizontal wells are open-hole completions, which mean that even shallow damage that in a cased perforated completion would be bypassed by the perforations becomes significant.
- Because the fluid velocity and pressure gradient during flow back are usually small, cleanup of internal and external cakes is not as effective as in vertical wellbores. Thus, only a fraction of the wellbore contributes to flow when the well is returned to production.
- Removing mud-induced formation damage by acidizing horizontal wells is often very difficult and expensive because of the large volumes of acid required and the difficulty in placing the acid in the appropriate wellbore locations.

The damaged zone around the horizontal wellbore can therefore be modeled as an eccentric cone around the wellbore with a significantly larger depth of penetration at the heel and a shallower depth of penetration at the toe [9]

Because the drilling fluid is in contact with the producing zone for an extended period of time, drill-in fluids have been devised to minimize the potential for formation damage.

2.1.4 Formation Damage Quantification

The objective of quantifying formation damage is to accurately assess the amount, location, extent, and impact of formation damage on production. This is critical in designing the most effective remediation and preventive measures. Without quantifying formation damage, it is almost impossible to determine which damage mechanism is the most detrimental and therefore most important in well optimization. Formation damage can be quantified by using comparative analysis of production data, pressure transient analysis, laboratory analysis, and field analysis[10].

Some of the terms used in quantifying formation damage as presented by various authors include:

2.1.4.1 Skin Factor

Pressure drop usually expressed as the difference between the average pressure in the bulk of a reservoir and the flowing well pressure consists of two factors. The first being the pressure drop normally associated with more or less radial fluid flow and the second is abnormal pressure drop associated with non radial flow patterns near the well or with changes in permeability near the well. The skin concept was developed to allow mathematical treatment of the abnormal pressure drop. The magnitude of skin effect is directly proportional to the difference between actual pressure drop and that predicted by steady-state radial flow theory. Skin effect is used quantitatively in predicting well performance under various operating conditions, and is particularly valuable for predicting the probable results of well stimulation.

As originally conceived by van Everdingen and Hurst',[11] skin effect was thought to be due to an infinitesimally thin zone of formation damage around a producing well. Hawkins[12] extended the concept to a zone of finite thickness by developing an algebraic equation relating reservoir permeability k, skin permeability k_d, well radius r_w, and skin radius r_d to skin magnitude *s*.This can be expressed as:

$$s = \left(\frac{k}{k_d} - 1\right) ln\left(\frac{r_d}{r_w}\right) \quad \text{……………………………………………………………………………………2.1}$$

From equation 2.1 it can be deduced that if $k_d < k$, then $s > 0$, meaning the well is damaged; conversely, if $k_d > k$, then $s < 0$, meaning the well is stimulated. Although rare, for $s = 0$, it means the near-wellbore permeability is equal to the original reservoir permeability.

The most common method for determining skin is a pressure buildup test[13]. A Horner plot is constructed using the pressure-buildup data from which we can compute the skin and the product of the permeability and formation thickness, *kh,* of the reservoir (in field units).

$$S = 1.151\left[\frac{(p_{wf} - p_{ws,1hr})}{m} - log\left(\frac{k}{\mu c r_w^2}\right) + 3.23\right] \quad 2.2$$

$$kh = \frac{162.6q\mu B}{m} \quad 2.3$$

Here, m is the slope of the straight-line portion of the Horner plot, and $p_{ws,1hr}$ is the extrapolated shut-in pressure at a shut-in time of 1 hour.

In the absence of production data, Frick and Economides[14] postulated that, an elliptical cone is a more plausible shape of damage distribution along a horizontal well. Thus they developed a skin effect expression, analogous to the Hawkins formula:

$$S_{eq} = \left(\frac{k}{k_d} - 1\right) ln\left[\frac{1}{I_{ani}+1}\right]\sqrt{\frac{4}{3}\left(\frac{a_{SH,max}^2}{r_w^2} + \frac{a_{SH,max}}{r_w} + 1\right)} \quad 2.4$$

Where S_{eq} is the equivalent skin effect, I_{ani} is the index of anisotropy and $a_{SH,max}$ is the horizontal axis of the maximum ellipse, normal to the well trajectory. They stated that the permeability anisotropy would generate an elliptical shape normal to the well and that the shape of damage depends on the permeability anisotropy index I_{ani} given by equation 2.5

$$I_{ani} = \sqrt{\frac{K_H}{K_V}} \quad 2.5$$

where K_H is the horizontal permeability and K_V the vertical permeability.

Economides and Nolte[15] have shown that the total skin effect is a composite of a number of factors, most of which usually cannot be altered by conventional matrix treatments.

The total skin effect may be expressed as:

$$s_t = s_{c+\Theta} + s_p + s_d + \sum pseudoskins \quad 2.6$$

The last term in the right-hand side of Eq. 2.6 represents an array of pseudoskin factors, such as phase-dependent and rate-dependent effects that could be altered by hydraulic fracture treatments. The other three terms are the common skin factors. The third term s_d refers to the damage skin effect as defined by equation 2.1. The first term $s_{c+\Theta}$ is the skin effect caused by partial completion and slant. The second term s_p represents the skin effect resulting from perforations. It is described by Harris[16] and also expanding the concept, Karakas and Tariq[17] have shown that:

$$s_P = s_H + s_V + s_W \quad \text{...}2.7$$

In equation 2.7, the horizontal pseudoskin factor, s_H is a function of the perforation phasing angle and the wellbore radius. The vertical pseudoskin factor s_V and the wellbore skin effect s_W are functions of some dimensionless variables. A useful definition of these variables and the application of equation 2.7 are also documented by Economides and Nolte[18].

Karakas and Tariq[19] also shown that a combination of the damage and perforation skin effects (s_d) can be approximated, for a case where the perforations terminate inside the damaged zone, by:

$$(S_d)p = \left(\frac{k}{k_d} - 1\right)\left[ln\frac{r_d}{r_w} + S_p\right] = (S_d)o + \frac{k}{k_d}S_p \quad \text{..................................} 2.8$$

r_d is the damaged zone radius, and $(s_d)o$ is the equivalent open hole skin effect (Eq. 2.1)

Other methods of quantifying formation damage include depth of damage, damage ratio, flow efficiency and Permeability Variation Index. A detailed literature on these methods can be found in Civan[20]

2.2 Economic Impact of Formation Damage on Reservoir Productivity

Amaefule *et al.*[21] presented a model that can estimate the economic impact of formation damage on reservoir productivity, q,in terms of the annual revenue loss by formation damage per well (*FD$L*) at a given price of oil, *p,* as:

$$FD\$L = \left(365\,\frac{days}{years}\right)\left(q\,\frac{bbl}{day}\right)\left(p\,\frac{\$}{bbl}\right)\left(DR\,\frac{bblunproduced}{bbltheoretical}\right) \text{...}2.9$$

Note: the term *DR* in Equation 2.9 represents the formation damage ratio.

Li *et al.*[22] and also Lee and Kasap[23] stated that because the degree of damage variation in the near-wellbore region, it is more appropriate to express the total skin, s used in any of the equations above as a sum of the individual skins over consecutive cylindrical segments of the formation as:

$$s = \sum_{i-1}^{N} s_i = \sum_{i-1}^{N} \left(\frac{k}{k_{di}} - 1\right) ln \left(\frac{r_i}{r_{i-1}}\right) \quad \text{.........} 2.10$$

where N is the number of cylindrical segments considered.

2.3 Well Stimulation

2.3.1 Definition

Well stimulation is any method used for optimally increasing well productivity by removing (or by-passing) formation damage in the near-wellbore region or by superimposing a highly conductive structure onto the formation.

2.3.2 Well Stimulation Objectives

The objectives of well stimulation can be divided into technical and economic objectives[24].

- **Technical Objectives**

  Completely remove, reduce or bypass the formation damage, reduce sand production and cleaning-up the perforations.

- **Economic Objectives**

  Increase flow rate and optimize production from the reservoir.

2.3.3 Well Stimulation Methods

There are several well stimulation techniques in use today, the commonly used ones include matrix acidizing , fracture acidizing , hydraulic fracturing , extreme overbalance perforating and *fracpack*[24].

Some of the commonly used stimulation methods are discussed in the following sections.

2.3.3.1 Matrix Stimulation

Also called matrix acidizing. It is a method of stimulation that involves injecting a fluid (e.g., acid or solvent) into the formation to dissolve and/or disperse materials that impair well production in sandstones or to create new, unimpaired flow channels between the wellbore and a carbonate formation. The injected fluid pressure is usually less than the fracture pressure of the formation. The matrix acidizing is effective in wells with a near wellbore damage where the zone of reduced permeability extends just a few feet of the wellbore radius beyond which the formation has a constant permeability[15].

Common matrix acidizing treatments are often categorized by formation rock type (carbonates and sandstones) and acid type.

The successful matrix stimulation of carbonate reservoirs requires that live acid penetrate past the formation damage. Effective acid penetration is facilitated either by transport through natural fractures or the formation of dissolution channels (wormholes) during the acidizing process. For a carbonate matrix stimulation treatment to be successful, it is important to acidize under conditions that will lead to the formation of deep penetrating wormholes using minimal acid volumes.

2.3.3.1.1 Types of wormhole models.

According to Fredd and Miller[25]modeling acid stimulation treatments in carbonate formations involves capturing the fundamentals of a microscopic dissolution phenomenon and then scaling the fundamental observations up to a massive scale. The challenge however is in capturing the effect of operating parameters and material properties on the efficiency of the acidizing process. The challenge of scaling the fundamental mechanics of carbonate dissolution to the field scale is the difficulty in translating the acid efficiency from one geometry and size to a different geometry and size in a manner that lends itself to easy, unencumbered utilization by those requiring a carbonate stimulation model[25].

Models for the acid stimulation of carbonates can be classified into five approaches. Each model based on these approaches applies some rule for capturing the acidizing fundamentals (efficiency) and/or translating laboratory models to field scale. Each approach has provided insight into some aspect of the problem, but none of the existing models combine a simple to apply, true scaling model that captures the full range of acid efficiency (or dissolution channel structure).

The five model approaches are as follows:

1. Transition pore theories
2. Capillary tube models (based on convection and diffusion of reactants)
3. Peclet number theories
4. Damköhler number theories
5. Network models (combination of stochastic and capillary tube theories)

2.3.3.1.2 Matrix Acidizing Selection of Acid Types and Additives.

Once a well has been determined as a good candidate for matrix acidizing the next stage is to select the appropriate acids for the acidizing job. The choice of acids is a function of the nature of the damage. The commonest acid used is HCl acid which is suitable for carbonate formations. Mud acid (HF/HCl) is the conventional treatment for sandstones[27].

Additives are materials blended with the acid to modify its behavior. Acids are naturally corrosive hence the need for these additives to reduce acid attack on steel pipes. The most necessary additives are usually corrosion inhibitors, surfactants, iron control agents, scale inhibitors, and clay stabilizers.

These additives are also used in preflushes and overflushes to stabilize clays and disperse paraffins and asphaltenes.

2.3.3.2 Fracture Acidizing.

In fracture acidizing acid is required to be continuously injected into the formation at a rate high enough to generate the pressure required to fracture the formation. This continuous injection of acid creates a pressure buildup around the wellbore until it is large enough to exceed the break down pressure of the formation. Once the formation “breaks down”, a crack is formed, and the injected fluid begins moving down the fracture increasing the fracture width and length as injection continues. Differential etching occurs as the acid reacts with the formation face, resulting in the formation of highly conductive etched channels which remain open after the fracture closes. A conventional fracture acidizing technique involves pumping an acid system after fracturing. It may be preceded by a nonacid preflush and usually is overflushed with a nonacid fluid[1].

The solubility of the formation is a key factor that influences whether fracture acidizing proppant treatments should be used. If the formation is less than 75% acid soluble, proppant treatments should be used. For acid solubilities between 75 and 85%, special lab work can help define which approach should be used. Above 85% acid solubility, fracture acidizing would be the most effective approach.

The success of an acid fracture treatment is tied to two characteristics of the etched fracture, i.e., the effective fracture length (which is a function of the rate of acid consumption, acid fluid loss (wormhole formation) and acid convection along the fracture) and effective fracture conductivity (a function of the etched pattern, volume of rock dissolved, roughness of etched surface, rock strength and closure stress). The acidized fracture length and fracture conductivity are therefore controlled largely by the treatment design and formation strength.

2.3.3.3 Hydraulic Fracturing.

Hydraulic fracturing consists of pumping a viscous fluid into the formation with such pressure that it induces the parting of the formation. Hydraulic fracturing differs from fracture acidizing in that hydraulic fracturing fluids usually are not chemically reactive, and a proppant is placed in the fracture to keep the fracture open and provide conductivity. In almost all cases, the overwhelming part of the production comes into the wellbore through the fracture; therefore, the originally present near-wellbore damage is ‘‘bypassed,’’ and the pretreatment positive skin does not affect the performance of the fractured well[27].

Because hydraulic fracturing is a mechanical process, it is only necessary to know that formation damage is present when designing such a treatment. When a well is hydraulically fractured, most pre-treatment skin effects such as formation damage, perforation skins and skins due to completion and partial penetrations are bypassed and have no effect on the post-treatment well performance. Phase-and rate-dependent skins effects are either eliminated or contribute in the calculation of the fracture skin effects. Generally pre-treatment skin effects are not added to post-fracture skin effects.

The objectives of hydraulic fracture treatments are to increase the productivity index of a producing well or the injectivity index of an injection well. The performance of such fracture treatments is controlled by a quantity known as the dimensionless fracture conductivity which depends on the fracture permeability, conductive fracture width, formation permeability and the conductive fracture single wing length. The fracture conductivity is increased by an increased fracture width, an increased proppant permeability (large, more spherical proppant grains have higher permeability), and minimizing the permeability damage to the proppant pack from the fracturing fluid.

Propped hydraulic fracture should only be considered when the well is connected to adequate producible reserves, reservoir pressure is high enough to maintain flow when producing these reserves**,** production system can process the extra production, experienced personnel are available for treatment design, execution and supervision together with high quality pumping, mixing and blending equipment.

Another important factor that controls fracture propagation is the fluid leak-off. Fluids leaks off usually during the pumping stage. The fluid leak-off depends on the rheological properties of the fluid and the permeability of the formation.For good fracture propagation there is need for a fluid with low fluid leak-off coefficient[28].

2.3.3.4 Recompletion

For wells with certain types of damage such as partially or totally plugged perforations, insufficient perforation density or low depth of perforation, it may be sufficient to recommend recompletion technique. Hence the idea of recompletion is to increase the perforation density or to increase the depth of perforations. The overall aim of this method is to increase production by bypassing the damage. Recompletion is also used effectively in reducing water production. In this approach the well is re-perforated at a new higher zone while the perforations in the water zone are plugged off[24].

2.3.3.5 Gravel Packing

Gravel packing is an expensive but effective sand control measure used in weak formations that have been producing sand or have the tendency to do so. The screen assembly is set and sized gravel mixed in a base fluid is pumped as slurry and placed behind screen and the sand face to stop movement of the formation grains. The Gravel quality and shape/size is important and it must be insoluble in brines and HCL and less than 1% soluble in 12/3 HCL/HF.

The productivity and life of the gravel pack depends on whether the perforations have been properly packed with gravel. If not packed, formation fines can invade the tunnels and impair productivity. Re-completions in low pressure reservoirs where formation sand has been produced, can accept large volumes of additional sand[28].

Chapter Three

Methodology

The methodology presented below is a modification of the modular approach to stimulation decisions as presented by Sinson, *et al*.[26]and Ugbenyen[24].

3.1 Well Screening Model

Before using the optimization model it is necessary for the engineer to match any well showing declining productivity with any of the stimulation treatment options. These include matrix acidizing, recompletion or gravel packing. For wells diagnosed with skin values showing formation damage problems, acidizing is the recommended treatment. Wells with mechanical problems such as partially or totally plugged perforations, insufficient perforations density, and low depth of perforation, or water production, recompletion is recommended. If the problem is sand production then gravel packing is recommended[24].

3.2 Stimulation Treatment Models

The treatment models that are used for the stimulation treatment design are the ones presented by Ogbenyen[24] and Sinson, *et.al*[26]. These include matrix acidizing design model, recompletion design model and gravel-pack design model.

3.2.1 Matrix Acidizing Design Model

The extent to which acid will penetrate a rock is dependent on both the rock properties and the local acid reaction rate. The reaction rate in turn depends on matrix properties and other variables like temperature, pressure, and composition of the reacting fluids. Since the Niger Delta formation is chiefly made up of sandstones, treatments are often carried out with a mud acid as this can react with most constituents of naturally occurring sandstones.

The following steps are presented for sandstone acidizing design:

1. Determine the present fracture gradient for the well.

2. Predict the maximum possible injection rate that does not fracture the formation.
3. Estimate the pipe or coil tubing friction pressure gradient.
4. Predict maximum surface pressure.
5. Determine the volume of mud acid to use.
6. Specify the acid treatment.
7. Calculate cost of sandstone matrix acidizing : Steps 1 through to 6 are required to arrive at the following equation used to calculate the cost of sandstone matrix acidizing.

   $$C_S = c_{sm} \times V_h \quad \text{......} 3.1$$

   where:

   C_s = cost of sandstone matrix acidizing, \$/ft

   c_{sm} = cost of acid used per unit volume, \$/gal

   V_h = volume of acid used, gal/ft

8. Calculate the maximum productivity ratio.

The maximum formation productivity ratio for sandstone acidizing is defined by the reciprocal of the flow efficiency, using the semi-steady state definition:

$$F_{max} = \frac{ln\left(\frac{0.472 r_e}{r_w}\right)+s}{ln\left(\frac{0.472 r_e}{r_w}\right)} \quad \text{......} 3.2$$

where:

F_{max}= maximum productivity ratio, *dimensionless*

s= skin factor (as defined in equation 2.1) *dimensionless*

r_w = wellbore radius, *ft*

r_e = drainage radius, *ft*

3.2.2Recompletion Design Model

The approach considered in this model assumes that the well is already completed. The concept of recompletion is either to increase the perforation density or increase the depth of perforation penetration in order to increase production.

The following steps based on the works of Strubhar *et al.*[24]are presented for the recompletion design model:

1. Calculate the skin due to perforation geometry

$$S_p = \left(\frac{h}{h_p} - 1\right)\left(ln\frac{h}{r_w} - 2\right) \quad \text{.......... 3.3}$$

where:

h = total formation thickness, *ft*

h_p= perforated interval length, *ft*

r_w= wellbore radius, *ft*

2. Calculate the perforation damage

$$S_{dp}=\left(\frac{h}{L_p n}\right)\left(ln\frac{r_{dp}}{r_p}\right)\left(\frac{k_r}{k_{dp}} - \frac{k_r}{k_d}\right) \quad \text{.......... 3.4}$$

where:

$$r_{dp} = r_p + 0.5 \quad \text{.......... 3.5}$$

$$\frac{k_r}{k_{dp}} = \frac{1}{0.03} \quad \text{.......... 3.6}$$

and:

L_p = depth of penetration in rock, *ft*

n = number of perforations

r_{dp}= radius of compacted zone around the perforations, *ft*

r_p = radius of perforation in rock, *ft*

k_r = reservoir permeability, md

k_d = damaged zone permeability, md

k_{dp}= permeability of compacted zone around perforation in rock, md

3. Calculate the total skin

$$s = s_p + s_{dp} \quad \text{.......... 3.7}$$

4. Calculate cost of recompletion

$$C_R = c_{perf} \times n_{perf} \quad \text{.......... 3.8}$$

where:

c_{perf}= cost per perforation, $

n_{perf}= number of perforations

5. Calculate the maximum productivity ratio

The productivity ratio is defined as the reciprocal of the flow efficiency

$$F_{max} = \frac{ln\left(\frac{0.472r_e}{r_w}\right)+s}{ln\left(\frac{0.472r_e}{r_w}\right)} \quad \text{.......}3.9$$

3.2.3 Gravel-Pack Design Model

The following gravel pack design module is modified from Schlumberger's gravel pack design and calculation manual. The volume of gravel required is dependent on the formation permeability, total length of the interval and the condition of the well (i.e. whether it is a new well or an old well). The ideal situation is that all perforation tunnels and screen casing annulus be filled with gravel. The gravel pack design considered is for re-completion of zones that have produced sands. The following steps are considered in the design[24].

1. Calculate the blank/casing annular volume
2. Calculate the volume of gravel to be injected into perforations
3. Calculate total volume of gravel needed
4. Calculate the weight of gravel needed
5. Calculate the carrier fluid volume
6. Calculate the slurry volume
7. Calculate the slurry density
8. Calculate the gravel-pack skin factor
9. Calculate cost of gravel packing
10. Calculate the maximum productivity ratio :

$$F_{max} = \frac{ln\left(\frac{0.472r_e}{r_w}\right)+s_{gp}}{ln\left(\frac{0.472r_e}{r_w}\right)} \quad \text{.......}3.10$$

where:

s_{gp}= skin factor due to Darcy flow through the gravel-pack, d*imensionless*

3.3 Model for Identifying the Optimal Time to Initiate Stimulation

Figure 3.1 shows the production profile (production rate vs. time) of a well that at some point during its producing life was profitably stimulated. This figure shall serve as the theoretical basis for the model described in the following sections.

From figure 3.1, the line OA is the well's pre-stimulation maximum production rate plateau at which the initial production rate is q_i. From the point A, the production rate starts declining and curve ABG represents the pre-stimulation decline curve profile. Point B represents any point along the curve below A and above G at which the well is considered for stimulation. The curve DEF is the resulting post-stimulation production profile. The production rate q_a is the abandonment rate of the well. The time t_a is the abandonment time of the well if it is not considered for stimulation, the treatment is initiated at time t_o at which production rate is q_o. At time t_s the stimulated well is open for production. Thus, the difference between the times t_o and t_s is the duration of the stimulation job. The production loss due to the duration of the stimulation job is represented by the shaded area BCHI. The initial production rate after stimulation is represented by q_s which corresponds to point D in Figure 3.1. The well is now produced along the curve DEF until the abandonment rate q_a is reached at time t_{as} corresponding to point F in the figure shown. The area DEFH represents the incremental production due to stimulation treatment.

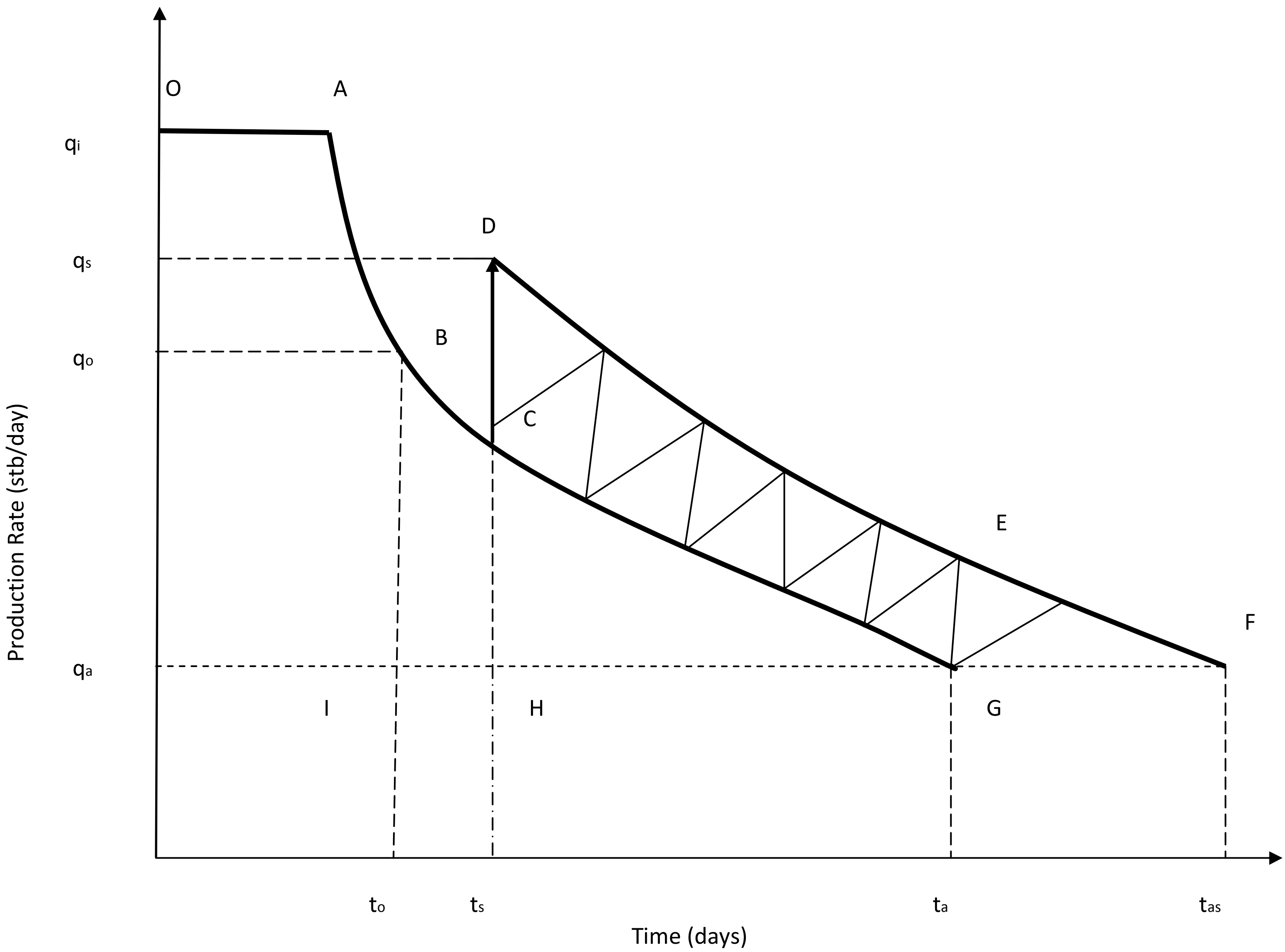

Fig 3.1 Production decline profile for a stimulated well (after Ugbenyen, 2010)

The optimization model used for this work was developed by Ugbenyen (2010) and it uses the production profile described above (Figure 3.1) and an exponential decline curve analysis with economic concept of continuous discounting. A brief description of the model is given in the sections below.

3.3.1 Model Assumptions

The following assumptions were made in the development of the model.

1. The stimulation will result in improved productivity.
2. The well could be operated profitably if stimulated.
3. The productivity ratio is not the same for all the points on the curve at which stimulation can be initiated
4. The factors that control production in the past will continue to control production in the future.
5. The well pre-stimulation decline profile will be the same as the post-stimulation profile.
6. The abandonment rate of the well is the same for both the pre-stimulation and post-stimulation profile.

3.3.2 Stimulation Productivity Ratio

The stimulation productivity ratio F is defined as the ratio of the initial (maximum) production rate obtained after stimulation to the production rate at which the well was considered for stimulation. From Figure 3.1,

$$F = \frac{q_s}{q_o} \qquad 3.11$$

3.3.3 The Present-Value Discount Factor

The present value interest factor (*PVIF*) for continuous or daily compounding is used in the following derivations and it is defined as:

$$PVIF = e^{It} \qquad 3.12$$

where I is the effective interest (discount) rate per day, and $t$is the time period considered in days.

3.3.4 Defining the Objective Function, Q_D

Well stimulation is only justified when the net (discounted) monetary benefit of the resulting extra oil or gas production is greater than the costs of the stimulation treatments all subject to the limitations imposed by the system. Hence, the objective function will be defined to maximize the net post-stimulation production. The objective function is defined as:

$$Q_D = Q_{DS} - Q_{DPL} - Q_{DC} \quad \text{3.13}$$

WhereQ_{DS} is the discounted production from stimulation, Q_{DPL} is the discounted production loss from stimulation, and Q_{DC} is the discounted production equivalent to total stimulation cost.

Both exponential and hyperbolic decline curve analysis shall be used to derive expressions for each of the components of Equation 3.3

Before proceeding with the derivation, it is necessary to define some of the variables in Figure 3.1. First, let us shift the time axis such that the time at the start of the stimulation job t_o is set to zero. Then let t_s be the duration of the stimulation job and t_{as} is the abandonment time of the post-stimulation production profile.

A. Discounted Incremental Post-Stimulation Production, Q_D

The discounted incremental production resulting from the stimulation process is derived from the area enclosed by DEFH in Figure 3.1 by:

$$Q_{DS} = \int_{t_s}^{t_{as}} q_s e^{-D(t-t_s)} * (PVIF) dt \quad \text{3.14}$$

Where, D is the exponential decline rate per day.

B. Discounted Production Loss Due to Stimulation, Q_{DPL}

The concept of production loss is similar to the idea of opportunity cost. The production loss is an essential component of the objective function that takes care of the well which is shut-in (zero production) during stimulation.

The discounted production loss during the stimulation process is derived from the area enclosed by BCHI in Figure 3.1 by:

$$Q_{DPL} = \int_0^{t_s} q_o e^{-D(t-t_s)} * (PVIF)dt \quad \text{......3.15}$$

C. Discounted Stimulation Cost, Q_{DC}

The total stimulation cost, which includes site preparation cost, equipment mobilization & demobilization cost and cost of consumables, e.g., chemicals and additives, is related to the size of the stimulation treatment, and can be converted to its equivalent discounted production as:

$$Q_{DC} = \frac{C}{P} * e^{-It_s} \quad \text{......3.16}$$

Where$C$ is the total cost of the stimulation treatment in dollars, and P is the price (in dollars) per barrel of oil.

3.4 Optimization Model Constraints

To obtain a practical solution to the objective function, the formulation must include some constraints. In this study, a budgetary constraint is imposed such that the cost of the stimulation does not exceed the budget as determined by top management. Also, a break-even condition is imposed such that the revenue obtained from the stimulation is at least equal to the stimulation cost. The reservoir sets a limit on the maximum cumulative production. Existing facilities, both in the sub-surface and surface, limit production rates that can be obtained from the choice of the stimulation treatment. These constraints are described below.

3.4.1 Constraint 1: Break-even Requirement

The discounted revenue from any stimulation treatment should be greater than or at least equal to the discounted cost of the project. That is:

$$Q_{DS} \geq Q_{DPL} + Q_{DC} \quad \text{......3.17}$$

3.4.2 Constraint 2: Remaining Reserves Limitation

The recovery from the stimulation should not exceed the remaining producible oil in place (reserves). Mathematically, this constraint can be expressed as:

$$\int_{t_s}^{t_{as}} F q_o e^{-D(t-t_s)}\ dt \le ROIP \quad \text{.......} 3.18$$

where ROIP is the remaining oil reserves in place during stimulation.

3.4.3 Constraint 3: Flow String Capacity

The production rate after stimulation should not exceed the maximum design capacity of the flow string. In the case of gas wells, this constraint is imposed by the gas pipeline capacity.

The exponential decline equation for the post-stimulation production rate q_t can be expressed as:

$$q_t\ = q_s e^{-D(t-t_s)}\ = F q_o e^{-D(t-t_s)} \quad \text{.......} 3.19$$

The maximum production rate is obtained when the well is opened for production just after stimulation, i.e. at time *t*=t_s (*see Fig. 3.1*). Using this substitution in Equation 3.19, constraint 3 can then be formulated as:

$$F q_o\ \le q_{max}$$

therefore:

$$F\ \le \frac{q_{max}}{q_o} \quad \text{.......} 3.20$$

where q_{max} is the maximum design capacity (flow rate) for the well tubing string.

3.4.4 Constraint 4: Budget Allocation

The total cost of stimulation should not exceed the maximum budget allocated by management for the job. This constraint is formulated mathematically as:

$$C \le C_{max} \quad \text{.......} 3.21$$

where C_{max} is the maximum budget allocated by management for stimulation.

3.4.5 Constraint 5: Maximum Formation Productivity Ratio

Given a set of reservoir and treatment parameters, the reservoir could only be stimulated to a certain maximum extent.

$$F \leq F_{max} \quad \text{……………………………………………………………}3.22$$

where $F_{\max}$ is the maximum productivity ratio that can be obtained given the reservoir and treatment parameters. It is the productivity ratio obtained from the design module.

3.4.6 Constraint 6: Productivity Improvement

The stimulation must, at least, result in an improvement in the productivity ratio and must not itself cause more damage to the formation. This constraint is imposed on the productivity ratio such that it must not be less than one or negative. It can be formulated mathematically as:

$$F \geq 1 \quad \text{………………………………………………………………………………}3.23$$

3.5 Stimulation Cost & Productivity Ratio Relationship

It can be observed from the design module that the input design parameters determine the stimulation cost(C) and the maximum productivity ratio (F). It follows then that a relationship can be formulated between the stimulation cost and the productivity ratio based on the design module. Hence in order to use the model presented in section 3.3 as an optimization model, it is necessary to develop a stimulation cost versus productivity ratio relationship for the stimulation treatment under consideration.

The combined effects of the treatment and reservoir variables are lumped into a stimulation cost versus productivity ratio equation of the form:

$$C = aF^b \quad \text{……………………………………………………………}3.24$$

where a and b are obtained from the power equation of the trend line of the log-log plot of stimulation cost versus productivity ratio.

The data for stimulation cost and productivity ratio required to calculate the parameters a and b from Equation 3.13 are to be derived from previous jobs in a given field or from analogous fields. It is this equation that incorporates the stimulation option into the optimization model.

3.6 Relationship between Initial Rate q_i and Current Production Rate q_o

It can be observed from figure 3.1 that the value of the current production rate (q_o) at which stimulation is initiated lies between the initial production rate (q_i) and the abandonment rate (q_a). Therefore, all q_o's can be expressed as a percentage of the initial production rate q_i. This relationship can be written in the form

$$q_o = \boldsymbol{R} q_i \quad \text{......} 3.25$$

where $\boldsymbol{R}$ is the ratio of current production rate to initial production rate

3.7 Summary of the Optimization Model

Combining the objective function and the constraints, the optimization model formulated can be summarized as (see Ugbenyen, 2010):

maximize:

$$Q_D = \alpha_1\alpha_2\alpha_3\alpha_5F^n - \alpha_1\alpha_2\alpha_3F - \alpha_4C - \alpha_1(\alpha_3 - 1) \quad \text{......} 3.26$$

subject to:

1. **Break-even Requirement:**

$$\alpha_1\alpha_2\alpha_3\alpha_5F^n - \alpha_1\alpha_2\alpha_3F \geq \alpha_4C + \alpha_1(\alpha_3 - 1) \quad \text{......} 3.27$$

2. **Remaining Reserve Limitation:**

$$\emptyset_1F - \emptyset_2 \leq ROIP \quad \text{......} 3.28$$

3. **Flow String Capacity:**

$$F \leq \frac{q_{max}}{q_o} \quad \text{......} 3.29$$

4. **Budget Allocation:**

$$C \leq C_{max} \qquad 3.30$$

5. **Maximum Formation Productivity Ratio:**

$$F \leq F_{max} \qquad 3.31$$

6. **Productivity Improvement:**

$$F \geq 1 \qquad 3.32$$

for all:

$$q_o = \boldsymbol{R} q_i \qquad 3.33$$

It is important to note that the optimization model is a non-linear programming (NLP) problem. The objective function consists of two variables, namely productivity ratio F and total stimulation cost C. The relationship between total stimulation cost C and productivity ratio F was established using Microsoft Excel regression analysis of the data in Table 4.1 to obtain the cost versus productivity ratio constants, *a* and *b* in Equation 3.24.

3.8 Solution to the Optimization Model

The above model was solved using the Solver in Microsoft Excel and also *What's Best! 10.0* LINDO Systems optimization software. The Solver which is an add-in in MS Excel (*developed by Frontline Systems*) uses numerical iterative methods (generalized reduced gradient method) to solve equations and to optimize linear and nonlinear functions with either continuous or integer variables. Thus both the solver and the *What's Best! 10.0* were used to generate the values of the objective function for each value of the input parameter.

The above procedure was repeated for the case of a hyperbolic decline using the optimization models developed by Sinson*et.al.*[26]which can be found in Ugbenyen 2010 and also included in Appendix A of this work.

Chapter Four

Results and Discussion

Several runs were made to study the behavior of the Optimization model to changes in input parameters. The main parameter of interest in this case is the current production rate (q_o). The current production rate is varied between a value lower than the initial production rate and some value above the abandonment rate. Some published data from the Niger Delta which are shown in Table 4.1 were also used to study the behavior of the objective function to changes in the input parameter. The results obtained are discussed in the following section.

4.1 How Stimulation Time Affects the Discounted Production

Based on the data generated from the runs, a plot of Discounted Production versus ratio of current to Initial rate of production is made. This is shown in fig 4.1. The time to initiate stimulation is calculated at each chosen current production rate from the decline curve parameters. It is observed from the plot that at higher values of current production rate we get the maximum amount of benefits from the stimulation. Maximum values occur at lower times until about half way through the decline where the derivable benefits starts declining as the time increases. This plot shows that the best time to initiate stimulation is when the current rate has not gone below fifty percent of the initial decline rate. In all cases only values of ratio of current rate of production to initial rate of production above 0.55 will be considered.

From Figure 4.1, it is observed that the maximum stimulation benefit is obtained between the points where q_o/q_i equals 0.6 and 0.9. To estimate the potential benefit from each of the points at which stimulation is initiated, a plot of production rate versus time to initiate stimulation is made. The decline profile for the ratios q_o/q_i =0.3, 0.5, 0.7, and 0.9 is shown in Figure. 4.2. This same plot was made for all the four wells considered in this project and in all cases it is observed that the curve for the ratio q_o/q_i = 0.7 lies above all the other curves. To further illustrate this, a semi-log plot of net production rate versus time for all four wells is also made to show the expected production rate gain due to stimulation treatment. The area between the original decline curve and any other curve represents the expected extra net oil production as a result of stimulation. The input data used to analyze the four candidate wells are presented in Table 4.1. These results of this analysis are shown in Figure 4.3 through Figure 4.6. From these plots it can be seen that the benefit obtained for ratio q_o/q_i equal 0.7 is maximal.

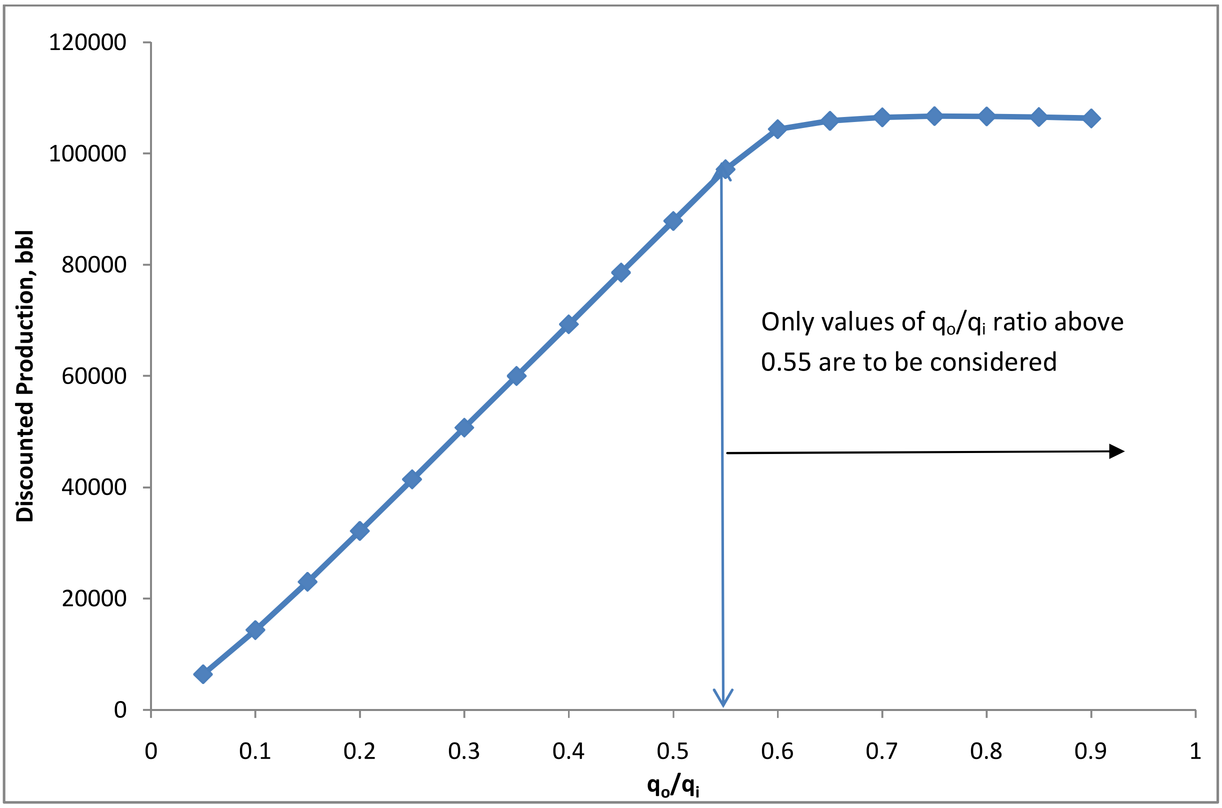

Figure 4.1 Discounted Production versus Ratio of Current to Initial Rate of Production

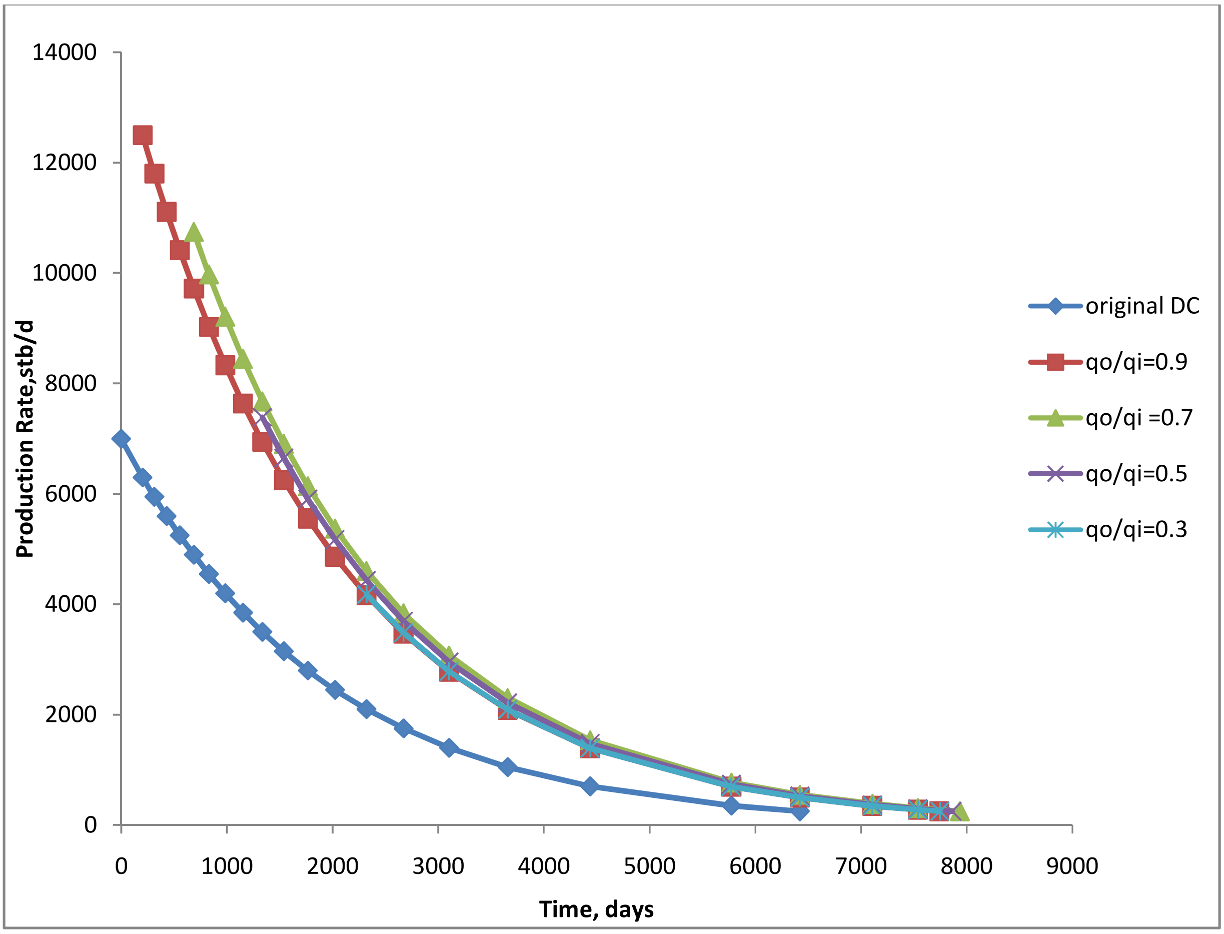

Figure 4.2 Production Rate versus Time to Initiate Stimulation Well NA1

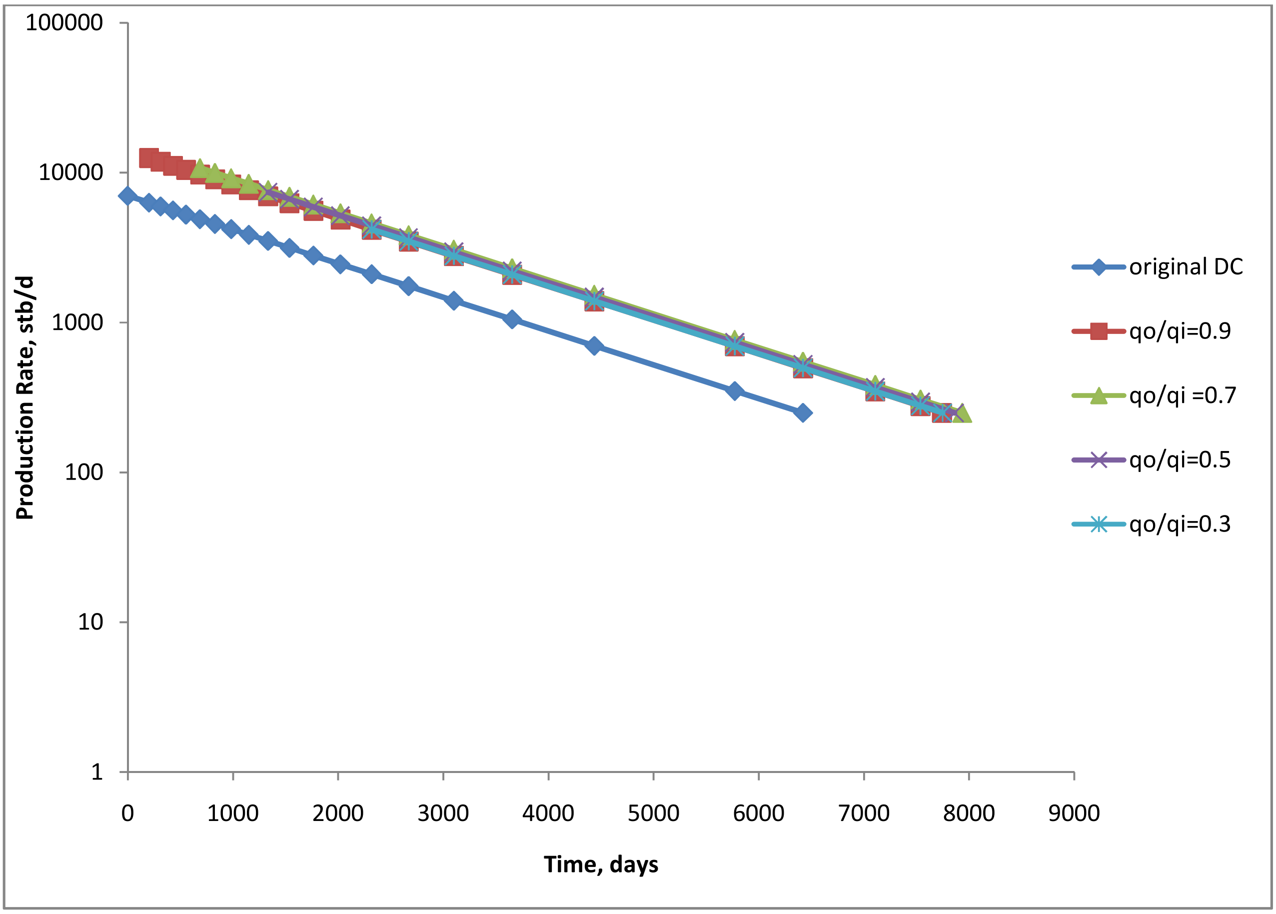

Figure 4.3 Semi-log plot of production rate versus time well NA1

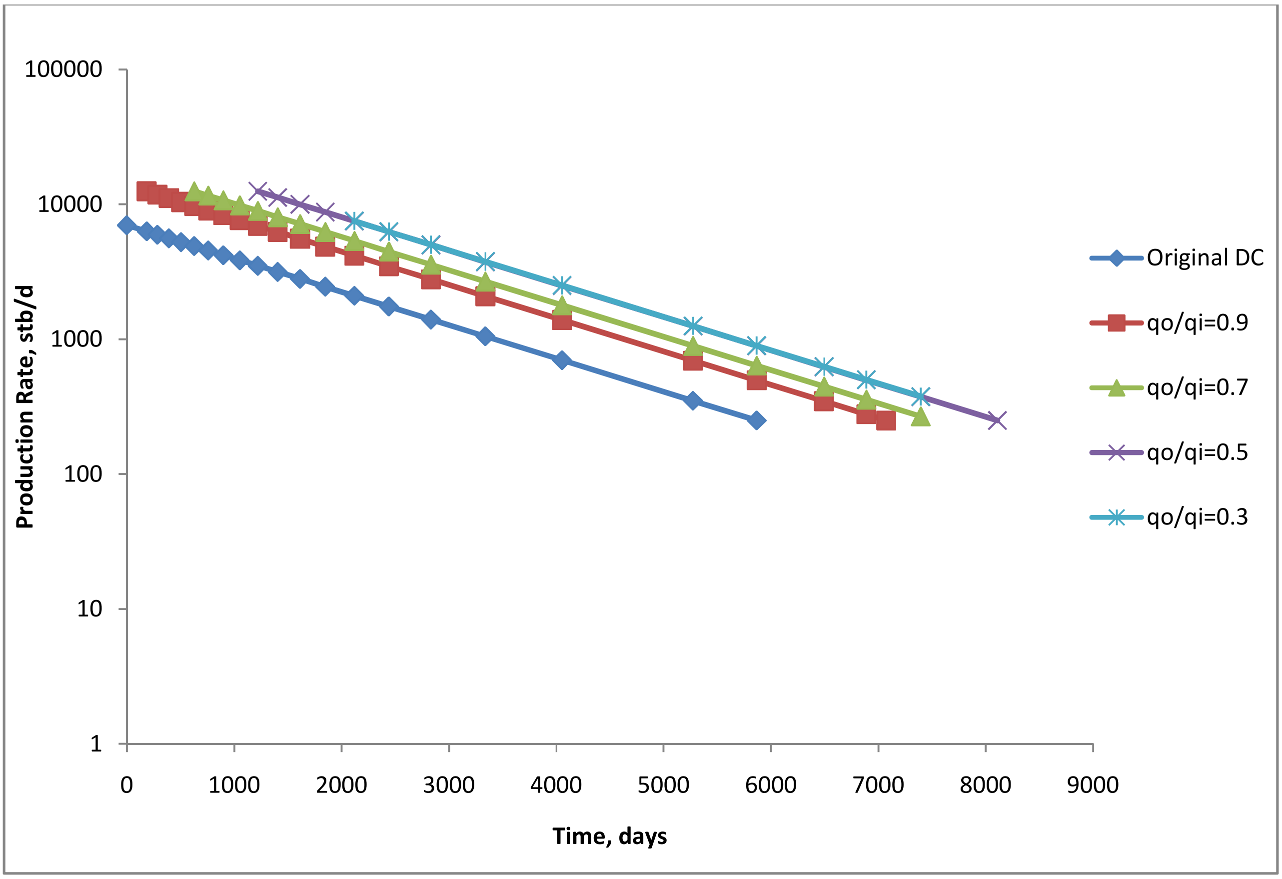

Figure 4.4 Semi-log plot of production rate versus time well NA2

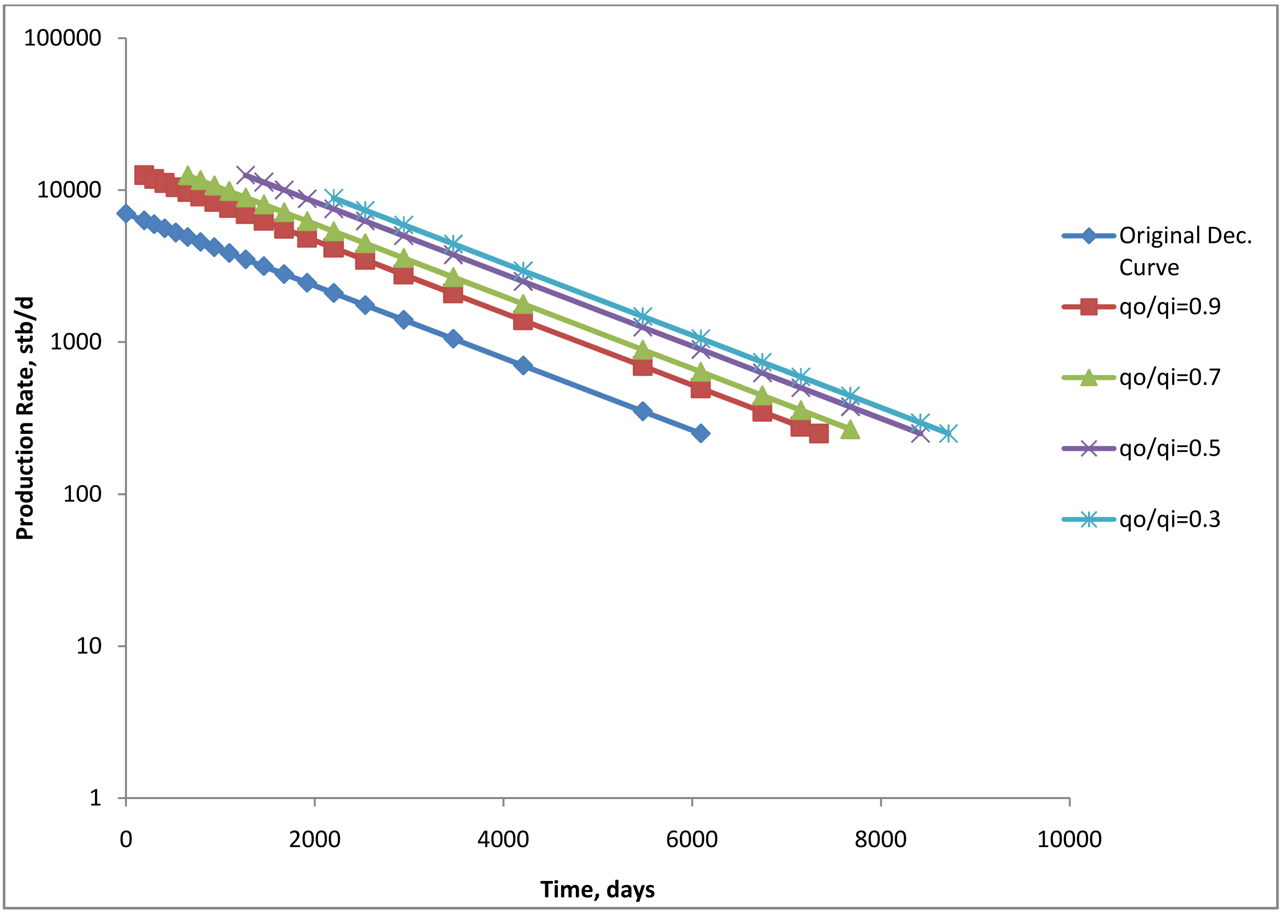

Figure 4.5 Semi-log plot of production rate versus time well NA3

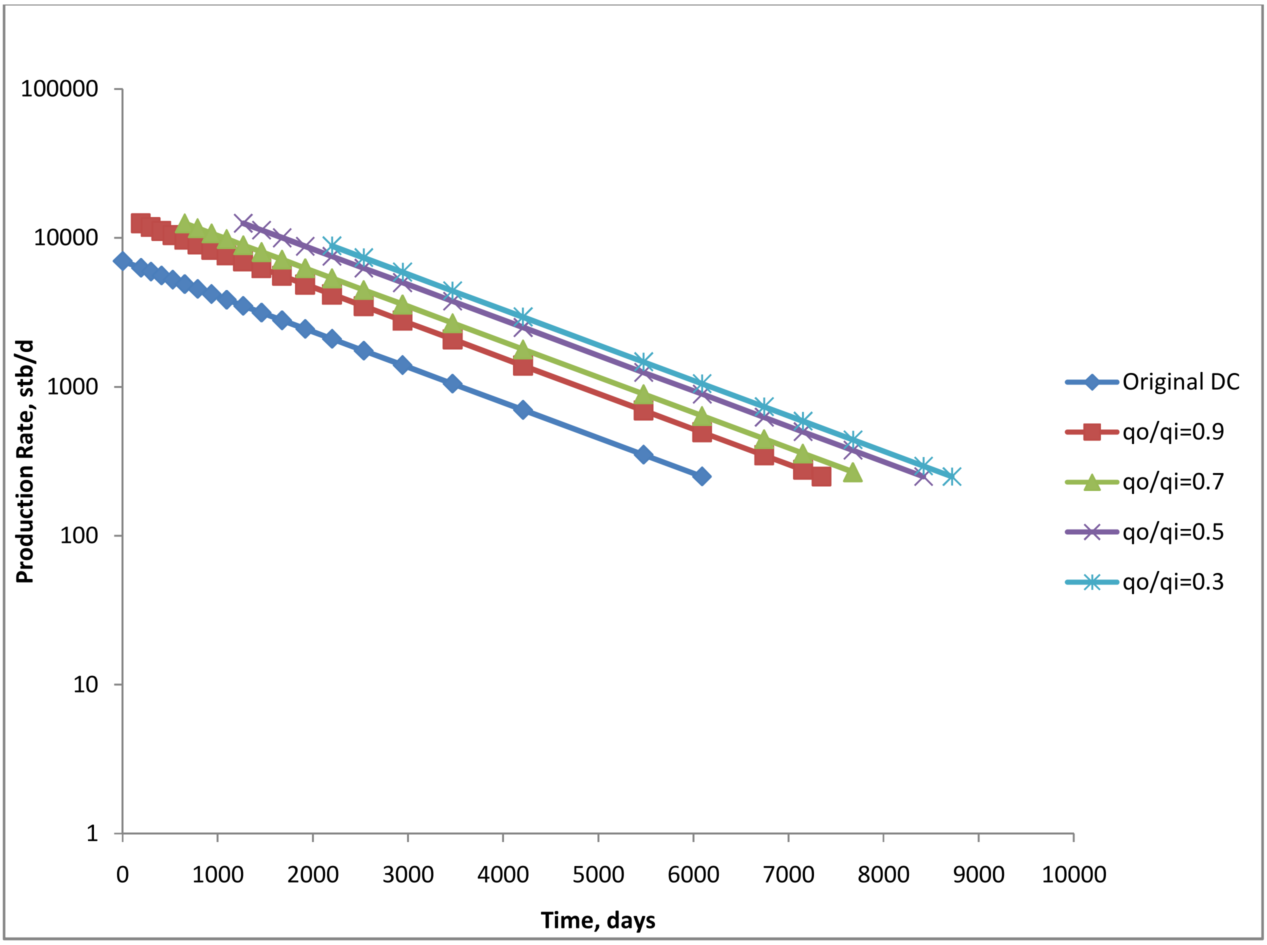

Figure 4.6 Semi-log plot of production rate versus time well NA4

Table 4.1 Model Input Data from Wells NA1, Well NA2, Well NA3, and Well NA4

	Well NA 1	*Well NA 2*	*Well NA 3*	*Well NA 4*
Average Reservoir Pressure, Pr (*psi*)	3200	3200	3200	3200
Drainage Radius, (*ft*)	1000	1000	1000	1000
Wellbore Radius, rw	0.3	0.3	0.3	0.3
Net Pay Thickness, h (*ft*)	120	85.6	148.29	68.2
Depth of Formation, D_f(*ft*)	9000	9060	8950	9000
Damaged Zone Radius, r_d (*ft)*	6.8	6.4	6	6.5
Undamaged Reservoir Permeability,*k(md)*	3500	3300	4600	3500
Damaged Zone Permeability, k_d(md)	900	450	510	580
Porosity *(%)*	25	25	25	25
Formation Volume Factor *(bbl/stb)*	1.159	1.159	1.159	1.159
Acid Hydrostatic Gradient *(psi/ft)*	0.45	0.45	0.45	0.45
Specific Gravity of Acid	1.04	1.04	1.04	1.04
Viscosity of Injected Acid *(cp)*	0.57	0.57	0.57	0.57
Pump Rate *(bbl/min)*	3.5	3.5	3.5	3.5
Safe Margin for Injection Pressure *(psi)*	200	200	200	200
Diameter of Coil Tubing *(inches)*	1.75	1.75	1.75	1.75
Cost of Acid Per Unit Volume*($ pergal)*	30	30	30	30
α (psi/ft)	0.4	0.4	0.4	0.4
Current Production Rate, q_o*(stb/d*)	4000	4100	3900	5200
Abandonment Rate, q_a(*stb/d*)	250	250	250	250
Exponential Decline Rate, *D (per day)*	0.000519	0.000568	0.000547	0.000533
Duration of Stimulation, t_s*(day)*	1	1	1	1
Effective Interest(Discount) Rate per day	0.10	0.10	0.10	0.10
Current Price of Oil *$/bbl*	80	80	80	80
Maximum Stimulation Budget, C_{max},MM$	1.2	1.2	1.2	1.2

Source: Ugbenyen, 2010

To study the changes in the optimal time to initiate stimulation operations with changes in some of the input parameters, a sensitivity (impact) analysis was carried out on the optimization model. The results obtained are discussed in the following section.

4.2 Sensitivity (Impact) Analysis

As seen from the previous section, where the optimal time to initiate stimulation operations is when the well production rate has declined to about seventy percent of the initial production rate. The analysis in this section was carried out to see the behavior of the curve with changes in some input parameters. Graphs of this nature can be used as a tool for managerial decision because they give the management an idea of the profit they can make for stimulating the well at different points(times) during the decline phase of the well taking into account some input parameters as will be discussed below.

4.2.1 Effect of Price

The price of oil determines the amount of revenue that can be derived from the stimulation. Therefore as shown in Fig. 4.7 an increase in the oil price is accompanied by a corresponding increase in the profit derived from the stimulation. Thus higher price of oil will encourage management to carry out stimulation.

4.2.2 Effect of Discount Rate

The effect of the discount rate is shown is Fig. 4.8. The value of the discount rate was varied from 10% to 25%. As seen on the graph, the discounted production decreases with increase in the discount rate.

4.2.3 Effect of Decline Rate

The effect of the decline rate on the discounted production is shown in Fig. 4.9. The value of the decline constant was varied between 0.547/day and 0.000547/day. It is observed that the smaller the decline rate the higher the benefits derived from the stimulation job. Thus smaller decline rates are desirable for profitable stimulation decision.

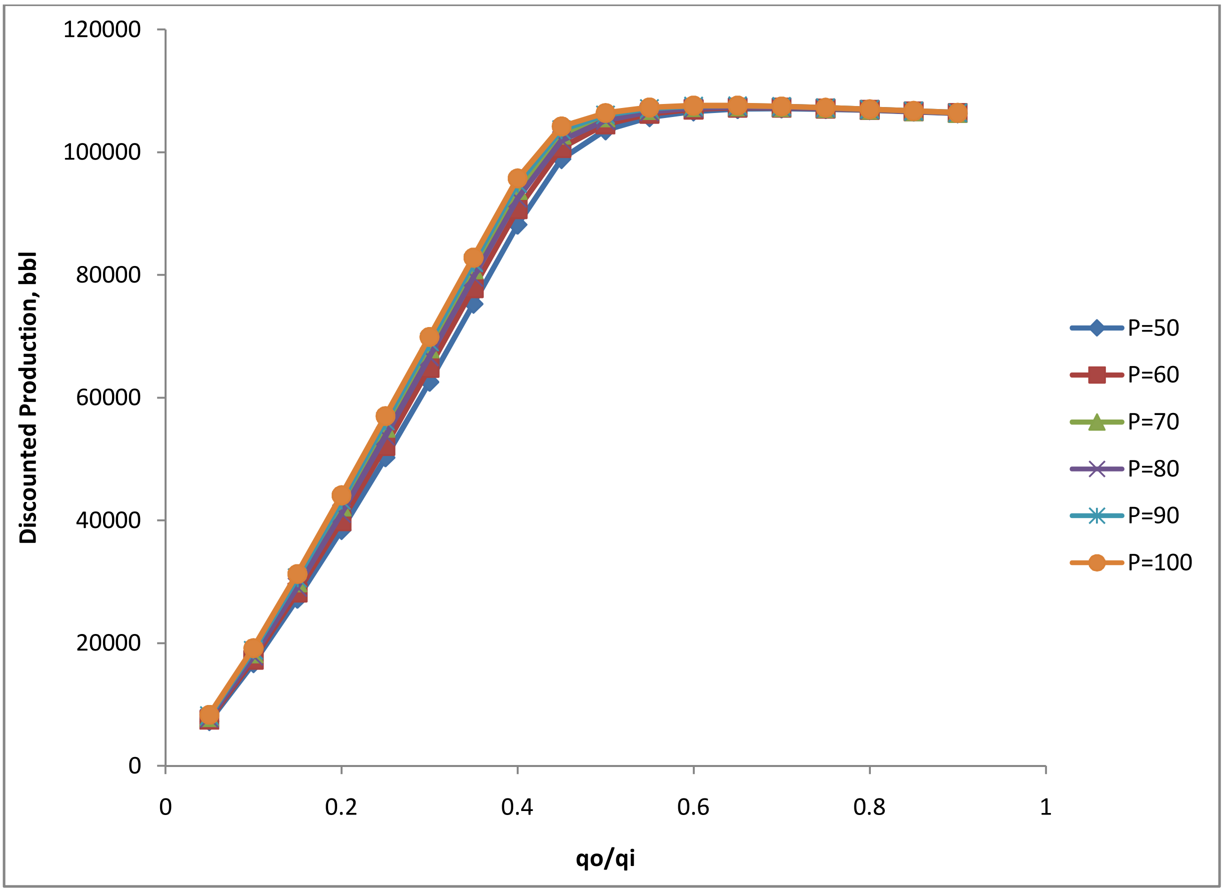

Figure 4.7 Effect of oil price on the objective function

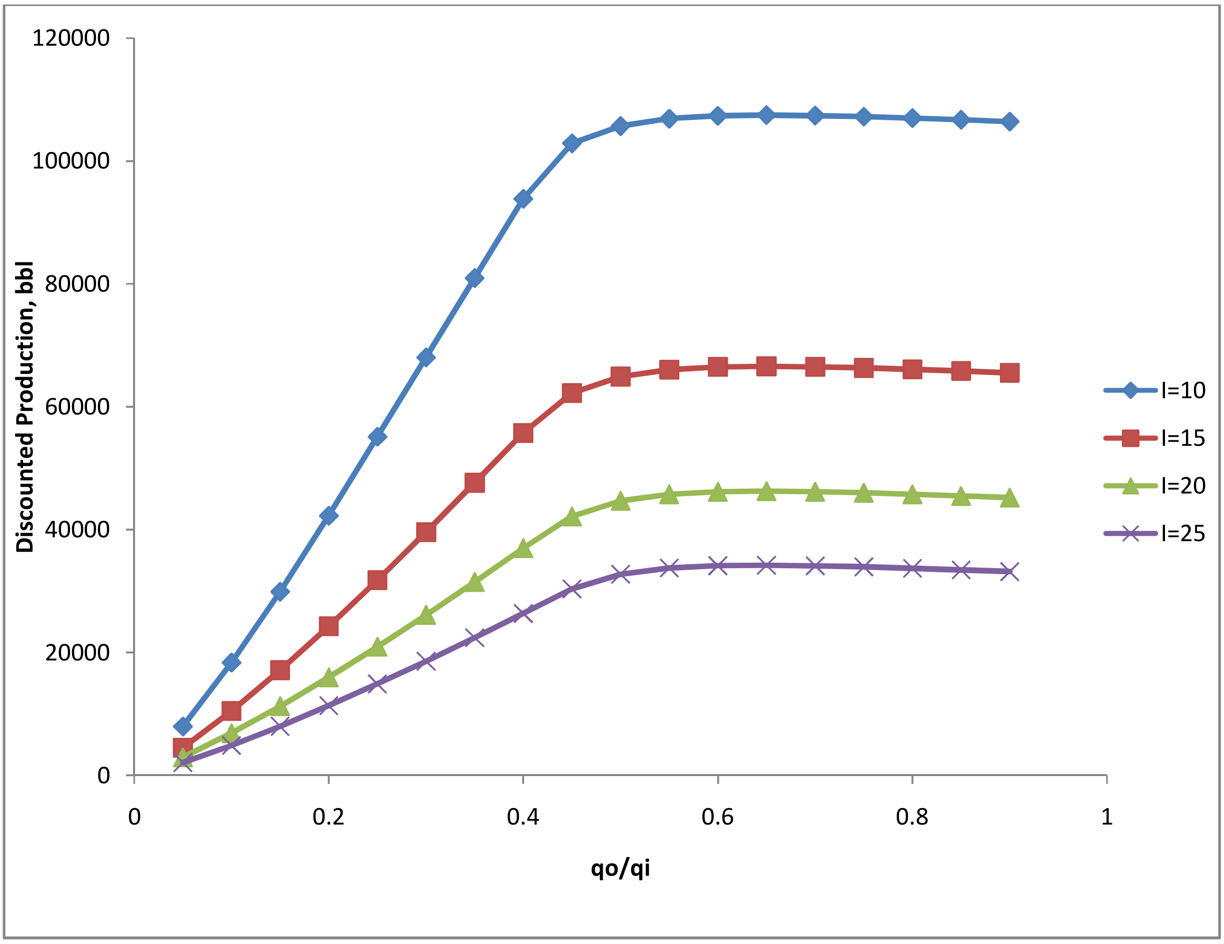

Figure 4.8 Effect of discount rate on the objective function

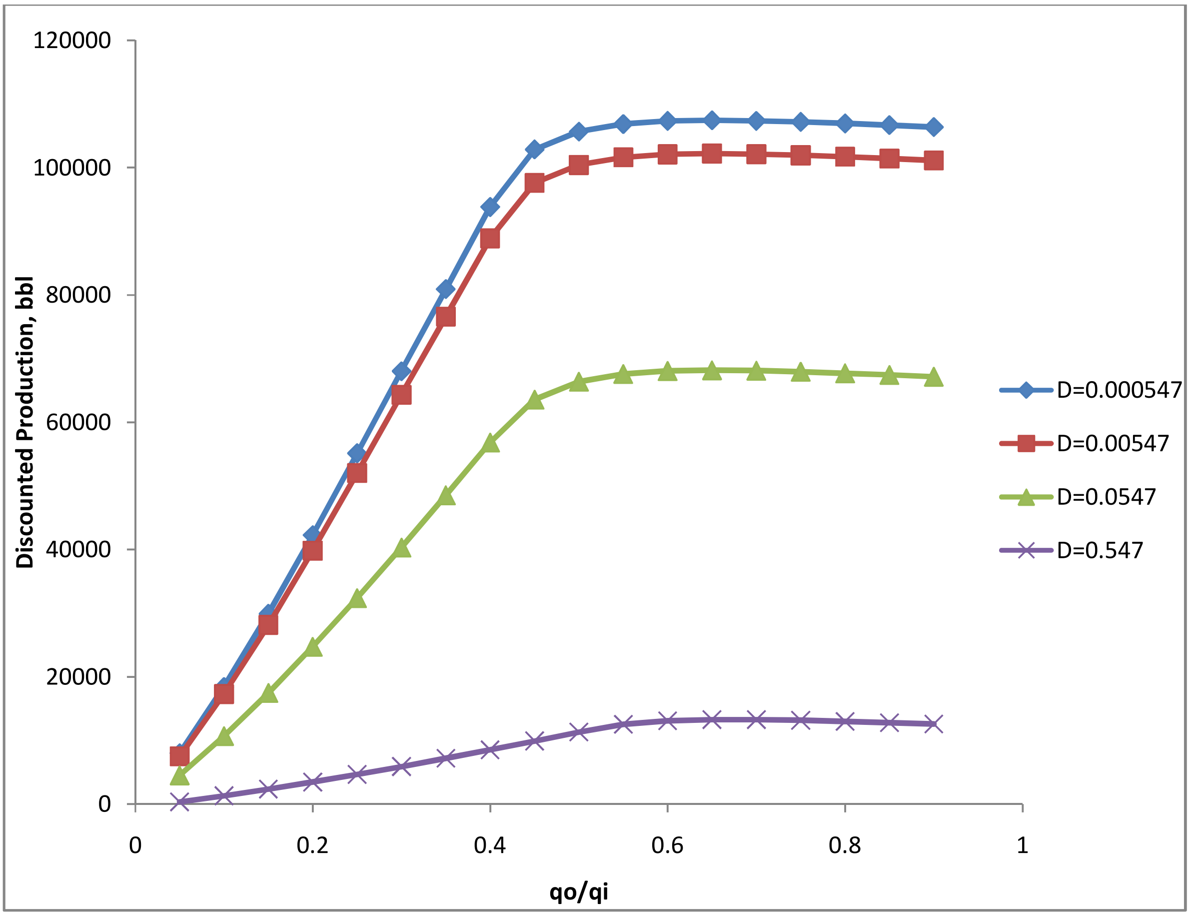

Figure 4.9 Effect of decline rate on the objective function

4.2.4 Effect of Stimulation Time

The effect time spent during stimulation on the discounted production is shown in Fig. 4.10. As observed from this plot, if more time is spent on stimulation, the production loss during the duration of stimulation will increase and hence lower the overall benefit derivable from the stimulation job. It is thus more beneficial if possible to reduce the duration of stimulation job to a minimum of a day.

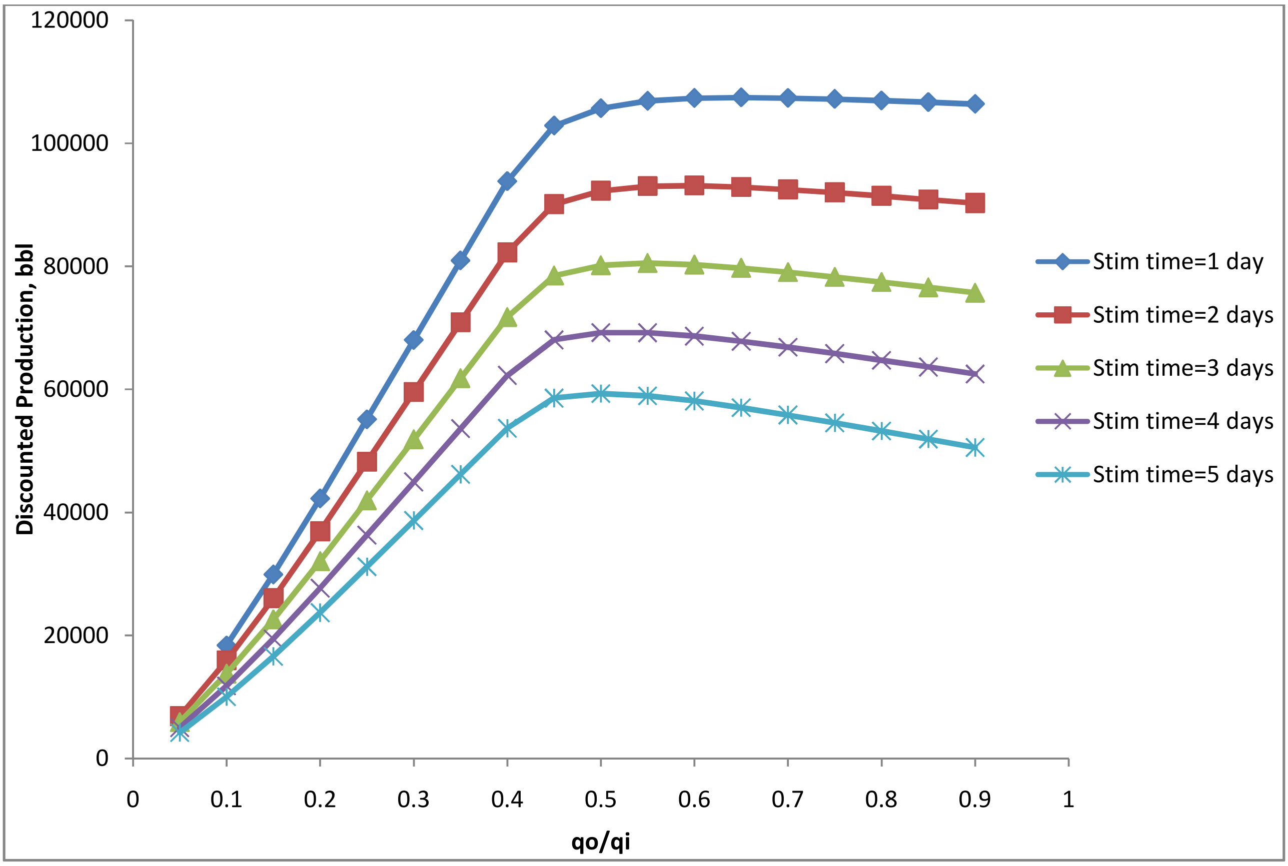

Figure 4.10 Effect of duration of stimulation on the objective function

Chapter Five

Conclusions and Recommendations

5.0 Conclusions

This research seeks a method to obtain the best time to initiate stimulation decisions by comparing the stimulation benefits derivable by initiating the treatment at different times from four candidate wells. To achieve this, both the exponential and hyperbolic models developed by Sinson *et al*[26] and modified by Ugbenyen (2010) are used in combination with any of the stimulation treatment modules.

The optimization model used in this research combines the output from the stimulation treatment design module with production decline curve analysis and economic continuous discounting concepts. The objective function (Q_D) is formulated in the form of a non-linear programming problem subjected to some economic and technical constraints. Thus a constrained optimization problem is presented. The solution of the objective function seeks a maximum discounted production that satisfies the constraints. The constraints considered include those imposed by the production facilities, remaining recoverable reserves and the stimulation budget approved by management.
To solve the objective function, a non-linear programming solver in Microsoft Excel and LINDO Systems' *What's Best 10* were used to get an optimum solution. In all similar cases considered, the same optimum solutions were obtained using either of the two solvers.

From all the model runs it can be seen that the greatest value of the discounted production is obtained at current production rates between 60 to 75 percent of the initial decline rate. The exponential model shows that the best time to initiate stimulation is when the current production rate is about seventy percent of the initial decline rate that is at qo/qi equals 0.7. it is noted that the analysis presented in this work is limited to matrix acidizing.

5.1 Recommendations

The following recommendations are presented to highlight areas of improvement of the model for future studies.

The effect of tubing performance should be incorporated into the model to account for decreasing vertical lift performance.

Another measure of profitability that takes into account the probability of success of a stimulation job will give results that are closer to what is obtainable in the field.

The current work can be extended to recompletion and gravel-packing stimulation designs in the Niger Delta.

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Nomenclature

C_{max}	maximum stimulation budget, $
C_s	cost of sandstone matrix acidizing, $/ft
C_{sm}	cost of acid per unit volume, $\$/gal$
C_{perf}	cost of perforation, $
d	diameter of pipe, $inches$
D	decline rate, $per\ day$
F_{max}	maximum productivity ratio, $dimensionless$
g_f	fracture gradient, psi/ft
q_{max}	tubing maximum design flow rate, stb/day
h	thickness of oil sand, ft
h_p	perforated interval length, ft
I	effective discount rate per day, %
I_{ani}	index of anisotropy
K_H	horizontal permeability, md
k_r	reservoir permeability, md
K_{dp}	permeability of compacted zone around perforation in rock, md
k_d	damaged zone permeability, md
k_{dp}	permeability of compacted zone around perforation in rock md
L_p	depth of penetration in rock, ft
n	number of perforations

n_{perf}	number of perforations open
$P_{s,max}$	maximum surface pressure, psi
P_r	average reservoir pressure, psi
P	price per barrel of oil, \$
q_o	current oil production rate, stb/day
q_i	initial oil production rate, stb/day
q_a	production rate at abandonment, stb/day
r_e	reservoir drainage radius, ft
r_w	well bore radius, ft
r_{dp}	radius of compacted zone around the perforations, ft
r_p	radius of perforation in rock, ft
$ROIP$	remaining recoverable reserve, bbl
$S_{C+\theta}$	skin effect caused by partial completion and slant, $dimensionless$
S	skin effect, $dimensionless$
S_{eq}	equivalent skin effect, $dimensionless$
S_{gp}	skin factor due to Darcy flow through gravel-pack, $dimensionless$
t_s	duration of the stimulation job, $days$
t_a	abandonment time, $days$
t_{as}	abandonment time of the post-stimulation production, $days$
V_m	volume of mud acid, gal/ft
V_{Hcl}	volume of HCl required, gal/ft

ϕ porosity, $fraction$

γ specific gravity of the acid (or density of the acid in g/cc), $dimensionless$

Appendix A

The derivation of the Objective Function for Other Decline Cases

The derivation of the optimization model presented in this section is modified from the published work of Sinson et al[26]. Let us start by deriving the optimization model for the general decline curve analysis.

A.1 General Decline Curve Optimization Model

The general decline curve equation is given by:

$$q(t) = \frac{q_i}{[1 + bD_i]^{\frac{1}{b}}} \qquad (A.1)$$

where:

b=hyperbolic constant

D_i= Intial decline rate

The general form the discounted production from stimulation, Q_{DS}, is expressed as:

$$Q_{DS} = \int_{t_s}^{t_{as}} \frac{F q_i e^{-It}}{[1 + bD_i(t - t_s)]^{\frac{1}{b}}} dt \qquad (\mathrm{A.2})$$

If we denote the denominator of $(A.2)$ by x, and solving for t:

$$x = 1 + bD_i(t - t_s) \qquad (A.3)$$

$$t = \frac{1}{bD_i}(x - 1 + bD_i t_s) \qquad (A.4)$$

$$dt = \frac{1}{bD_i} dx \qquad (A.5)$$

Substituting $(A.4)$ and $(A.5)$ into the original equation $(A.2)$:

$$Q_{DS} = Fq_i \int_{t_s}^{t_{as}} \frac{e^{-I\frac{1}{bD_i}(x-1+bD_i t_s)}}{x^{\frac{1}{b}}} \frac{1}{bD_i} dx \qquad (A.6)$$

Since, b, D_i and t_s are constants, we can express the equation as:

$$Q_{DS} = \frac{Fq_i}{bD_i} e^{-\frac{1}{bD_i}-It_s} \int_{t_s}^{t_{as}} \frac{e^{-\frac{1}{bD_i}x}}{x^{\frac{1}{b}}} dx \qquad (A.7)$$

Similarly the general form of the production loss component, Q_{DPL}, is:

$$Q_{DPL} = \int_0^{t_s} \frac{Fq_i e^{-It}}{[1 + bD_i(t - t_s)]^{\frac{1}{b}}} dt \qquad (A.8)$$

Using equation $(A.3)$, $(A.4)$, and $(A.5)$ and simplifying we obtain:

$$Q_{DPL} = \frac{q_i}{bD_i} e^{-\frac{1}{bD_i}-It_s} \int_0^{t_s} \frac{e^{-\frac{1}{bD_i}x}}{x^{\frac{1}{b}}} dx \qquad (A.9)$$

Note that the general form of the solution for Q_{DS} and Q_{DPL} using integral transformations is of the form:

$$\int \frac{e^{ax}}{x^m} dx = \frac{-1}{m} \frac{e^{ax}}{x^{m-1}} + \frac{a}{m-1} \int \frac{e^{ax}}{x^{m-1}} dx \; (A.10)$$

A closed form solution exists when:

$$m = 0 \qquad \int e^{ax}\, dx = \frac{e^{ax}}{a} \qquad (EXPONENTIAL\ DECLINE\ CASE) \;\; (A.11)$$

$$m = 1 \qquad \int e^{ax}\, dx = E_i(ax) \qquad (HARMONIC\ DECLINE\ CASE) \;\; (A.12)$$

$$m = 2 \qquad \int \frac{e^{ax}}{x^2} dx = \frac{e^{ax}}{x} + aE_i(ax) \qquad (HYPERBOLIC\ DECLINE\ CASE) \;\; (A.13)$$

Note that equation (A.13) is the most common form of hyperbolic decline curve.

Let:

$$\frac{1}{b} = a, \qquad (A.14)$$

$$D_i = b_i \qquad (A.15)$$

A.2 Harmonic Decline Optimization Model

A.2.1 Objective Function, Q_D

$$Q_D = Q_{Ds} - Q_{DPL} - Q_{DC} \qquad (A.16)$$

Where:

Q_{Ds} = discounted production from stimulation

Q_{DPL} = discounted production loss

Q_{DC} = discounted equivalent production cost

1. **The discounted production from stimulation** is:

$$Q_{Ds} = \int_{t_s}^{t_{as}} \frac{Fq_i}{1 + bD_i(t - t_s)} e^{-It}\, dt \qquad (A.17)$$

Where:

q_i = the production rate before stimulation

F = the productivity ratio

b_i = the nominal decline rate before stimulation

Changing variables, we get:

$$x = 1 - b_i t_s + b_i t \qquad (A.18)$$

Such that:

$$t = \frac{1}{b_i}(x - 1 + b_i t_s)(A.19)$$

And

$$dt = \frac{I}{b_i} dx \qquad (A.20)$$

Substituting (A.19) and (A.20) into the original equation, (A.17):

$$Q_{DS} = \frac{Fq_i}{b_i} e^{(\frac{I}{b_i} - It_s)} \int_s^{as} \frac{e^{-\frac{I}{b_i}x}}{x} dx \qquad (A.21)$$

Integrating:

$$Q_{DS} = \frac{Fq_i}{b_i} e^{(\frac{I}{b_i} - It_s)} \left[E_i \left(\left(\frac{-I}{b_i} \right) (1 - b_i t_s + b_i t \ ) \right) \right]_{t_s}^{t_{as}} \qquad (A.22)$$

Simplifying:

$$Q_{DS} = \frac{Fq_i}{b_i} e^{(\frac{I}{b_i} - It_s)} \left[E_i \left(\frac{-1}{b_i} + It_s - It \right) \right]_{t_s}^{t_{as}} \qquad (A.23)$$

and:

$$Q_{DS} = \frac{Fq_i}{b_i} e^{(\frac{I}{b_i} - It_s)} \left[E_i \left(\frac{-I}{b_i} + It_s - It_{as} \right) - \left(\frac{-I}{b_i} \right) \right] \qquad (A.24)$$

Note that the time to reach the economic limit is:

$$t_{as} = \frac{Fq_i}{q_a b_i} - \frac{1}{b_i} \qquad (A.25)$$

Where:

q_a = abandonment production rate

Therefore equation (A.24) becomes:

$$Q_{DS} = \frac{Fq_i}{b_i} e^{\left(\frac{1}{b_i} - It_s\right)} \left[E_i \left(It_s - \frac{IFq_i}{q_a b_i} \right) - E_i \left(\frac{-1}{b_i} \right) \right] (A.26)$$

2. The discounted production lost during stimulation is:

$$Q_{DPL} = \int_0^{t_s} \frac{q_i e^{-It}}{(1 + b_i t)} dt \qquad (A.27)$$

Using the same change of variable:

$$x = 1 - b_i t_s + b_i t \qquad (A.18)$$

Such that:

$$t = \frac{1}{b_i}(x - 1 + b_i t_s) \qquad (A.19)$$

and

$$dt = \frac{1}{b_i} dx \qquad (A.20)$$

$$Q_{DPL} = \frac{q_i}{b_i} e^{\frac{1}{b_i}} \int_0^{t_s} \frac{e^{-\frac{I}{b_i}x}}{x} dt \qquad (A.28)$$

$$Q_{DPL} = \frac{q_i}{b_i} e^{\frac{I}{b_i}} \left[E_i \left(-\frac{I}{b_i} \right) (1 + b_i t\ ) \right]_0^{t_s} \qquad (A.29)$$

$$Q_{DPL} = \frac{q_i}{b_i} e^{\frac{I}{b_i}} \left[E_i \left(\frac{-I}{b_i} - It_s \right) \right]_0^{t_s} \qquad (A.30)$$

$$Q_{DPL} = \frac{q_i}{b_i} e^{\frac{I}{b_i}} \left[E_i \left(\frac{-I}{b_i} - It_s \right) - E_i \left(\frac{-I}{b_i} \right) \right] \qquad (A.31)$$

3. The discounted equivalent production from stimulation cost is:

$$Q_{DC} = \frac{C}{P} e^{-It_s} (A.32)$$

With the above derivations, Q_D can now be written as:

$$Q_D = \beta_1 E_i(\beta_2 - \beta_3 F)F - \beta_1 E_i(\beta_4)F - \beta_6 C - \beta_5 \qquad (A.33)$$

Where:

$$\beta_1 = \frac{q_i}{b_i} e^{\left(\frac{I}{b_i} - It_s\right)} \qquad (A.34)$$

$$\beta_2 = It_s \qquad (A.35)$$

$$\beta_3 = \frac{Iq_i}{q_a b_i} \qquad (A.36)$$

$$\beta_4 = \frac{-I}{b_i} \qquad (A.37)$$

$$\beta_5 = \frac{q_i}{b_i} e^{\frac{I}{b_i}} \left[E_i\left(\frac{-I}{b_i} - It_s\right) - E_i\left(\frac{-I}{b_i}\right)\right] \qquad (A.38)$$

$$\beta_6 = \frac{e^{-It_s}}{P} \qquad (A.39)$$

A.2.2 Constraints

The same constraints formulated in the exponential case also applied here.

<u>Constraint 1</u>. Break-Even Point

The incremental revenue from any stimulation decision should be greater than or at least equal to the cost of the project.

$$\int_{t_s}^{t_{as}} \frac{Fq_i}{(1 - b_i t + b_i t_s)} dt \geq \frac{C}{P} e^{-It_s} + \int_0^{t_s} \frac{q_i e^{-It}}{1 + b_i t} dt \qquad (A.40)$$

Performing the integration and using the definition of t_{as} given in equation $(A.25)$ and using the constants above, we get:

$$\beta_1 E_i(\beta_2 - \beta_3 F)F - \beta_1 E_i(\beta_4)F \geq \beta_6 C - \beta_5 \qquad (A.41)$$

Constraint 2:Recoverable Oil in Place

The recovery from the stimulation cannot exceed the remaining oil in place.

$$\int_{t_s}^{t_{as}} \frac{Fq_i}{(1 - b_i t + b_i t_s)} dt \leq ROIP \qquad (A.42)$$

The integral is equal to:

$$LHS = \frac{Fq_i}{b_i} [lnx]_{t_s}^{t_{as}} \qquad (A.43)$$

Using the definition of t_{as} and substituting in the equation:

$$\frac{Fq_i}{b_i} ln\left(\frac{Fq_i - q_a}{q_a b_i}\right) - \frac{Fq_i}{b_i} lnt_s \leq ROIP \qquad (A.44)$$

and:

$$\beta_7 \ln(\beta_8 - \beta_9)\, F - \beta_{10} \leq ROIP \qquad (A.45)$$

where:$\beta_7 = \frac{q_i}{b_i} (A.46)$

$$\beta_8 = \frac{q_i}{q_a b_i} \qquad (A.47)$$

$$\beta_9 = \frac{1}{b_i} \qquad (A.48)$$

$$\beta_{10} = \frac{q_i}{b_i}\ lnt_s \qquad (A.49)$$

Constraint 3: FlowString Capacity

The production rate after stimulation should not exceed the maximum design capacity of the flow string, i.e.,

$$F \leq \frac{q_{max}}{q_o} \qquad (A.50)$$

where q_{max} is the maximum design capacity of the flowing string.

<u>Constraint 4</u>: Budget Limitation.
The cost of stimulation should not exceed the budget.

$$C \leq C_{max} \qquad (A.51)$$

where C_{max} is the budget for the stimulation job.

<u>Constraint 5</u>: Reservoir Productivity Ratio Constraint
The maximum attainable productivity ratio from stimulation depends on the reservoir properties and treatment parameters.

$$F \leq F_{max} \qquad (A.52)$$

where F_{max} is the maximum attainable productivity ratio.

<u>Constraint 6</u>: Cost Productivity Ratio Equation
The cost and productivity ratio relationship can be formulated into the following equation.

$$C = 10^{b_0} F^{b_1} \qquad (A.53)$$

where b_0 and b_1 are the intercept and slope of a regression line through the data.

A.2.3 Form of the NLP
Equations (A.33) and (A.53) define the NLP model for the harmonic case.

Maximize:

$$Q_D = \beta_1 E_i(\beta_2 - \beta_3 F)F - \beta_1 E_i(\beta_4)F - \beta_6 C - \beta_5 \qquad (A.33)$$

Subject to:

$$\beta_1 E_i(\beta_2 - \beta_3 F)F - \beta_1 E_i(\beta_4)F \geq \beta_6 C - \beta_5 \qquad (A.41)$$

$$\beta_7 \ln(\beta_8 - \beta_9)\, F - \beta_{10} \leq ROIP \qquad (A.45)$$

$$F \leq \frac{q_{max}}{q_o} \qquad (A.50)$$

$$C \leq C_{max} \qquad (A.51)$$

$$F \leq F_{max} \qquad (A.52)$$

$$C = 10^{b_0} F^{b_1} \tag{A.53}$$

A.3 Hyperbolic Decline Optimization Model

To develop a closed form solution to the model, we shall consider only the case when m=2. The solution form to the general case is given in equation (A.13).

A.3.1 Objective Function, Q_D

The objective function is formulated as:

$$Q_D = Q_{Ds} - Q_{DPL} - Q_{DC} \tag{A.16}$$

where:

Q_{Ds} =discounted production from stimulation

Q_{DPL} = discounted production loss

Q_{DC} = discounted equivalent production cost

1. The discounted production from stimulation is:

$$Q_{DS} = \int_{t_s}^{t_{as}} \frac{F q_i}{\left[1 + \frac{b_i}{2}(t - t_s)\right]^2} e^{-It} dt \tag{A.54}$$

Changing variables, we get:

$$x = 1 - \frac{b_i}{2} t_s + \frac{b_i}{2} t \tag{A.55}$$

Such that:

$$t = \frac{2}{b_i}\left(x - 1 + \frac{b_i}{2} t_s\right) \tag{A.56}$$

and

$$dt = \frac{2}{b_i} dx \tag{A.57}$$

Substituting equations (A.55), (A.56) and (A.57) into (A.54), then simplifying:

$$Q_{DS} = \frac{2Fq_i}{b_i} e^{\left(\frac{2I}{b_i}x - It_s\right)} \int_{t_s}^{t_{as}} \frac{e^{-\frac{2I}{b_i}x}}{x^2} dx \qquad (A.58)$$

The economic life is:

$$t_{as} = \frac{2}{b_i}\left(\frac{Fq_i}{q_a}\right)^{0.5} - \frac{2}{b_i} \qquad (A.59)$$

Integrating equation (A.58), then substituting equation (A.59) and simplifying:

$$Q_{DS} = \frac{-\varepsilon_3 e^{It_s - \frac{2}{b_i}\left(\frac{Fq_i}{q_a}\right)^{0.5}} F}{-\frac{b_i}{2}t_s + \left(\frac{Fq_i}{q_a}\right)^{0.5}} + \varepsilon_1\varepsilon_3 F - \varepsilon_2\varepsilon_3 F \qquad (A.60)$$

where:

$$\varepsilon_1 = \left(\frac{-2I}{b_i}\right) E_i\left[It_s - \frac{2I}{b_i}\left(\frac{Fq_i}{q_a}\right)^{0.5}\right] \qquad (A.61)$$

$$\varepsilon_2 = e^{\frac{-2I}{b_i}} + \left(\frac{-2I}{b_i}\right) E_i\left[-\frac{2I}{b_i}\right] \qquad (A.62)$$

$$\varepsilon_3 = \frac{2q_i e^{\frac{2I}{b_i}} e^{It_s}}{b_i} \qquad (A.63)$$

2. The discounted production lost during stimulation is:

$$Q_{DPL} = \int_0^{t_s} \frac{q_i e^{-It}}{1 + \frac{b_i}{2}t} dt \; (A.64)$$

Using similar variable change and simplifying:

$$Q_{DPL} = \frac{2q_i e^{\frac{2I}{b_i}}}{b_i}\left[\frac{-e^{\left(\frac{-2I}{b_i}\right)\left(1+\frac{b_i}{2}t\right)}}{\left(1+\frac{b_i}{2}t\right)} + \left(\frac{-2I}{b_i}\right) E_i\left[\left(\frac{-2I}{b_i}\right)\left(1+\frac{b_i}{2}t\right)\right]\right]_0^{t_s} \qquad (A.65)$$

Simplifying further:

$$Q_{DPL} = \frac{2q_i e^{\frac{2I}{b_i}}}{b_i}\left[\frac{-e^{\left(\frac{-2I}{b_i}\right)\left(1+\frac{b_i}{2}t\right)}}{\left(1+\frac{b_i}{2}t_s\right)} + \left(\frac{-2I}{b_i}\right) E_i\left[\frac{-2I}{b_i} - It_s\right] + e^{\frac{-2I}{b_i}} + \left(\frac{-2I}{b_i}\right) E_i\left[\frac{-2I}{b_i}\right]\right] \qquad (A.66)$$

This expression for Q_{DPL} is constant.

We can define:

$$\varepsilon_4 = \frac{2q_i}{b_i} e^{\frac{2I}{b_i}}\left[\frac{-e^{-2I-It_s}}{\left(1+\frac{b_i}{2}t_s\right)} + \left(\frac{-2I}{b_i}\right) E_i\left[\frac{-2I}{b_i} - It_s\right] + e^{\frac{-2I}{b_i}} + \left(\frac{-2I}{b_i}\right) E_i\left[\frac{-2I}{b_i}\right]\right] \qquad (A.67)$$

1. The discounted equivalent production from stimulation cost is:

$$Q_{DC} = \frac{C}{P} e^{-It_s} (A.32)$$

We can define:

$$\varepsilon_5 = \frac{Ce^{-It_s}}{P} \qquad (A.69)$$

Therefore:

$$Q_{DC} = \varepsilon_5 C \qquad (A.70)$$

The objective function can now be written as:

$$Q_D = \frac{-\varepsilon_3 e^{\varepsilon_8} e^{\varepsilon_6} F^{0.5} F}{\varepsilon_9 + \varepsilon_7 F^{0.5}} + \varepsilon_1 \varepsilon_3 F - \varepsilon_2 \varepsilon_3 F - \varepsilon_4 - \varepsilon_5 C \qquad (A.71)$$

Where:

$$\varepsilon_6 = \frac{-2I}{b_i} \left(\frac{q_i}{q_a}\right)^{0.5} \qquad (A.72)$$

$$\varepsilon_7 = \left(\frac{q_i}{q_a}\right)^{0.5} \qquad (A.73)$$

$$\varepsilon_8 = I t_s \qquad (A.74)$$

$$\varepsilon_9 = \frac{-b_i t_s}{2} \qquad (A.75)$$

A.3.2 Constraints

The same constraints formulation as in previous cases applies here.

<u>Constraint :</u> Break-Even Point

The incremental revenue from any stimulation decision should be greater than or at least equal to the cost of the project.

$$\int_{t_s}^{t_{as}} \frac{F q_i}{\left[1 + \frac{b_i}{2}(t - t_s)\right]^2} e^{-It}\, dt \geq \frac{C}{P} e^{-It_s} + \int_0^{t_s} \frac{q_i e^{-It}}{1 + \frac{b_i}{2} t} dt \qquad (A.76)$$

Evaluating:

$$\frac{-\varepsilon_3 e^{It_s - \frac{2I}{b_i}\left(\frac{Fq_i}{q_a}\right)^{0.5}} F}{-\frac{b_i t_s}{2} + \left(\frac{Fq_i}{q_a}\right)^{0.5}} + \varepsilon_1 \varepsilon_3 F - \varepsilon_2 \varepsilon_3 F \geq \varepsilon_4 + \varepsilon_5 C \qquad (A.77)$$

<u>Constraint 2:</u> Recoverable Oil in Place

The recovery from the stimulation cannot exceed the remaining oil in place.

$$\int_{t_s}^{t_{as}} \frac{Fq_i}{\left[1+\frac{b_i}{2}(t-t_s)\right]^2} e^{-It}\, dt \le ROIP \qquad (A.78)$$

Evaluating the left hand side:

$$LHS = \frac{2Fq_i}{b_i}\left[\frac{-1}{1+\frac{b_i}{2}t-\frac{b_i}{2}t_s}\right]_{t_s}^{t_{as}} \qquad (A.79)$$

Simplifying using the definition of t_{as} and the defined constants:

$$\varepsilon_{10}F\,\frac{\varepsilon_7 F^{0.5}-\varepsilon_{11}}{\varepsilon_7 F^{0.5}-\varepsilon_9} \le ROIP \qquad (A.80)$$

Where

$$\varepsilon_{10} = \frac{2q_i}{b_i} \qquad (A.81)$$

<u>Constraint 3</u>: Flow String Capacity

The production rate after stimulation should not exceed the maximum design capacity of the flow string, i.e.,

$$F \le \frac{q_{max}}{q_i} \qquad (A.82)$$

where q_{max} is the maximum design capacity of the flowing string.

<u>Constraint 4</u>: Budget Limitation

The cost of the stimulation should not exceed the budget.

$$C \le C_{max} \qquad (A.51)$$

where C_{max} is the budget for the stimulation job.

<u>Constraint 5:</u> Reservoir Productivity Ratio Constraint

The maximum attainable productivity ration from stimulation depends on the reservoir properties and treatment parameters.

$$F \leq F_{max} \tag{A.52}$$

<u>Constraint 6</u>: Cost Productivity Ratio Equation

The cost and productivity ratio relationship can be formulated into the following equation.

$$C = 10^{b_0} F^{b_1} \tag{A.53}$$

where b_0 and b_1 are the intercept and slope of a regression line through the data.

A.3.3 Form of the NLP

The equation $(A.71)$, together with all the constraints considered, can be summarized as:

$$Maximize:$$

$$Q_D = \frac{-\varepsilon_3 e^{\varepsilon_8} e^{\varepsilon_6} F^{0.5} F}{\varepsilon_9 + \varepsilon_7 F^{0.5}} + \varepsilon_1 \varepsilon_3 F - \varepsilon_2 \varepsilon_3 F - \varepsilon_4 - \varepsilon_5 C \tag{A.71}$$

$$Subject\ to:$$

$$\frac{-\varepsilon_3 e^{It_s - \frac{2I}{b_i}\left(\frac{Fq_i}{q_a}\right)^{0.5}} F}{-\frac{b_i t_s}{2} + \left(\frac{Fq_i}{q_a}\right)^{0.5}} + \varepsilon_1 \varepsilon_3 F - \varepsilon_2 \varepsilon_3 F \geq \varepsilon_4 + \varepsilon_5 C \tag{A.77}$$

$$\varepsilon_{10} F \frac{\varepsilon_7 F^{0.5} - \varepsilon_{11}}{\varepsilon_7 F^{0.5} - \varepsilon_9} \leq ROIP \tag{A.80}$$

$$F \leq \frac{q_{max}}{q_i} \tag{A.82}$$

$$C \leq C_{max} \tag{A.51}$$

$$F \leq F_{max} \tag{A.52}$$

$$C = 10^{b_0} F^{b_1} \tag{A.53}$$

A.4 Summary

The stimulation optimization models developed in this section of the appendix can be used in place of the exponential model used in the thesis. The solution procedure is the same.