# DIRECTIONAL DRILLING HYDRAULICS OPTIMIZATION USING CONVENTIONAL AND NANOBASED DRILLING FLUIDS

BY

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A THESIS SUBMITTED TO THE AFRICAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

ABUJA – NIGERIA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE

AWARD OF MASTER OF SCIENCE IN (PETROLEUM ENGINEERING)

ADVISOR: PROFESSOR OSISANYA, O. SAMUEL

MAY 2013

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DEDICATION

Have you not known?

Have you not heard? The everlasting God, the Lord,

The Creator of the ends of the earth, Neither faints nor is weary.

His understanding is unsearchable. He gives power to the weak,

And to those who have no might He increases strength.

Even the youths shall faint and be weary, And the young men shall utterly fall,

But those who wait on the Lord, Shall renew their strength;

Isa 40:28-31a (NKJV)

Thank you, LORD for being my guiding light, strength, and shield.

## ACKNOWLEDGEMENT

I express my sincere gratitude to the Petroleum Training Development Fund for sponsoring my graduate studies at the African University of Science and Technology (AUST), Abuja. The knowledge acquired will, without doubt, add to the improvement of local expertise in the Nigerian oil and gas industry.

I appreciate all the members of faculty under whose tutelage I studied, especially the head of the Petroleum Engineering Department at the African University of Science and Technology, Professor Godwin A. Chukwu, my supervisor, Professor Samuel O. Osisanya of the Mewbourne School of Petroleum, University of Oklahoma, and Professor Wumi Iledare of the Center for Energy Studies, Louisiana State University.

I thank the members of my committee, Professor D.K Das and Engr. Franck Egbon for their contributions to the successful completion of this work.

I must appreciate all my colleagues especially the Petroleum Engineering students for the support I enjoyed in my capacity as the President of the Society of Petroleum Engineers (SPE) for the 2012/13 session.

Finally, I thank the members of my family: my Dad, Mum, sisters and brother; your enduring and renewable love is my energy for excellence.

## ABSTRACT


*During drilling operations, special attention must be given to wellbore hydraulics to ensure functional, safe, economic, and environmentally responsible delivery of the well. The Hydraulics system shares common purposes with the drilling fluid, it helps to control subsurface pressures, remove cuttings from the well, clean the bit, size the pump, increase the rate of penetration, and minimize surge and swab pressures. Hydraulics optimization is an attempt to maximize the pressure drop across the bit by minimizing the parasitic pressure losses. A properly designed hydraulic system will help to improve drilling efficiency and lower drilling time and cost.*

*This thesis examines hydraulics optimization using conventional drilling fluids and nano-based drilling fluids. Nanofluids are specialized fluids obtained by careful combination of nanoparticles and a base fluid. The nanoparticles are particles with an average diameter of less than 100nm. They possess unique characteristics that differentiate them from microparticles, and make them adaptable to a wide range of applications. The impacts of rheological models and equivalent annular diameter definitions on annular pressure loss and ECD were examined for the conventional drilling fluids while aluminium oxide nanoparticles were used to examine the influence of nanofluids on the annular pressure gradient and ECD. A user-friendly computer program was written to facilitate repeated analyses using any combination of rheological model and equivalent annular diameter definitions for the conventional drilling fluids for all ranges of inclination using the Rudi Rubiandini's cuttings transport model.*

*The conclusions drawn are:*

1. *The Hydraulic Radius Concept, Slot Approximation, and Lamb's Method give almost the same pressure gradients for various rheological models.*

2. *The Crittendon correlation overestimates the values of the annular pressure gradient and ECD for various rheological models. This phenomenon is compounded when the fluid is not in the laminar flow regime.*

3. *The increase in density and viscosity of the nanofluid compared to the base fluid leads to the need for a higher capacity pump to flow the system. However, using a low density and low viscosity base fluid, with lower density and concentration of nanoparticles would reduce the effect of increase in the density and viscosity of the mixture.*

*Hole related problems associated with conventional drilling fluids can be eliminated by the use of nano-engineered fluids. Nanoparticles can be deplored in high pressure, high temperature (HPHT) locations. The user-friendly program can be used for a quick evaluation of the optimization process.*

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## NOMENCLATURE

$\frac{dp}{dl} = Pressure\ gradient(psi / ft)$

$\rho = density\ of\ drilling\ mud(lb / gal)$

$g = acceleration\ due\ to\ gravity(32.2\ ft / s^2)$

$\theta = angle\ of\ inclination\ from\ the\ vertical$(degrees)

$\mu = viscosity(cp)$

$\bar{V} = average\ velocity\ (ft / s),\ and$

$d_e = effective\ diameter\ (inches)$

$n_{ann} = Annular\ flow$ behaviour index(dimensionless)

$n_p = Pipe\ flow$ behaviour index(dimensionless)

$f_{lam} = la\min ar\ flow\ friction\ factor$

$f_{trans} = transitional\ flow\ friction\ factor$

$f_{turb} = turbulent\ flow\ friction\ factor$

$P\_dp =$ Fluid Pressure in the drill pipe

$Pa\_dp =$ Fluid Pressure in the drill pipe annulus

$P\_dc =$ Fluid Pressur in the drill collar

$Pa\_dc =$ Fluid Pressure in the drill collar annulus

$V\_dp =$ Fluid velocity in the drill pipe

$Va\_dp =$ Fluid velocity in the annulus around drill pipe

$V\_dc =$ Fluid velocity in the drill collar

$f\_lam =$ Friction factor for laminar to transitional flow

$Va\_dc =$ Fluid Velocity in the annulus around the drill collar

$dpi =$ Drill pipe internal diameter

$dpo =$ Drill pipe external diameter

$dci =$ Drill collar internal diameter

$dco =$ Drill collar external diameter

$dhole\_casing =$ Diameter of hole or casing in the section

# CHAPTER 1

# FORMULATION OF THE PROBLEM

## 1.1 Introduction

Hydraulics system plays an important role during rotary drilling operations. Proper design and maintenance of this system increases drilling efficiency (high rate of penetration) and lowers the overall drilling cost. The hydraulic system is the drilling fluid system in the wellbore when the fluid is in static or dynamic state. The dynamic state deals with the fluid movement, pipe movement, and cutting transport. Drilling is the art and science of making boreholes for hydrocarbon production, in a manner that is safe, economic, and environmentally responsible. An efficient hydraulics system is a prerequisite to the success of any drilling and completions operation. It affects mud circulation, hole-cleaning efficiency, cementing, rate of penetration (ROP), and hence total drilling time and cost. Rotary drilling hydraulics is concerned with proper utilization of the drilling fluid pump horsepower. It is affected by the drilling fluid properties and geometry (configuration) of the circulating system.

Rotary drilling involves the circulation of formulated drilling fluids (called mud), to perform certain functions. The functions of the drilling fluid include:

1. To maintain well control by counteracting and suppressing the formation pressure
2. To clean the surface of the bit and transport cuttings to the surface
3. To lubricate and cool the drill bits.
4. To hold the cuttings in suspension when fluid circulation is stopped

5. To transmit hydraulic force from the surface through the drill string to the bottom hole assembly (BHA)
6. To enhance wellbore stability
7. To serve as a means of logging and formation evaluation

Hydraulics optimization is an attempt to minimize the parasitic pressure losses and thereby maximize the pressure drop across the bit. The pressure drop across the bit directly affects the rate of penetration and hole cleaning efficiency. The hydraulics system has many purposes while drilling a well. In general, it is centered around well geometry and fluid properties, and thus the purposes of the drilling fluids and hydraulics are common to each other. The hydraulics system has many effects on well drilling operations. The more common purposes are

- To control subsurface pressures
- To remove cuttings from the well and clean the bit
- To increase rate of penetration (ROP)
- To size the pump
- To minimize surge and swab pressures.

The parasitic losses are frictional losses in and around the drill string and pressure losses through surface connections. The pressure losses in and around the drill string depends on the flow regime (laminar, transitional, or turbulent), the geometry of the drill string and well bore, and the rheological properties of the drilling fluid. In addition, several equations exist for calculating the equivalent annular diameter for a given wellbore profile. These give different values and affect the value of the Reynolds number. Bit optimization uses three approaches, namely: jet bit velocity; bit hydraulic horsepower; and jet impact force.

The transportation of cuttings to the surface is a very important aspect of drilling engineering. Inadequate hole cleaning can lead to several drilling problems which include stuck pipe, premature bit wear, formation damage, reduced rate of penetration (ROP), increased torque and drag, and hole pack-off. These problems invariably increase the cost and time for drilling operations. The transport mechanisms involved in cuttings transport are rolling, saltation, and suspension. The complexity of the transport mechanisms increases as the well inclination increases. Three main velocities dictate the effectiveness of hole cleaning: the slip velocity, which is the threshold, settling velocity of the cuttings; cuttings velocity; and minimum transport velocity, which has been defined as the sum of the other two velocities.

The API based on angle of inclination divided the wellbore profile into three regions: Vertical and near-vertical segment 0 - $30^0$; inclined segment 30 - $60^0$; and horizontal and near-horizontal segment 60 - $90^0$. Experimental (Peden et al. 1990; Ford et al. 1996; Larsen et al. 1997), and theoretical studies (Kamp et al. 1999; Masuda et al. 2000; Cho et al. 2000; Li et al. 2007) show that an inclination angle between 30 - $60^0$ poses the greatest problem in hole cleaning due to the formation of cuttings bed that may slide down the wellbore; packing off the bottom hole assembly. A sufficiently high annular flow rate will effectively clean the well. However, the maximum permissible annular flow rate is constrained by the characteristics of the formation; extremely high annular velocities will erode unconsolidated formations. The available mud weight window also places a limitation on the maximum allowable Equivalent Circulating Density (ECD) to avoid accidental fracturing of the formation. This is further complicated in angled wells due to the reduction in the vertical component of the fluid velocity.

Two main methods are used in the study of cuttings transport through the annulus of a wellbore: Mechanistic models and Empirical models. The mechanistic models consider the forces acting on the cuttings in the various segments of the well bore. By using the principles of conservation of mass and momentum, several authors(Kamp et al. 1999; Cho et al. 2000) proposed two-layer and three-layer models to characterize the formation of cuttings bed. By using dimensional analysis and other analytical techniques, the authors proposed equations to estimate the size and height of the cuttings bed, and the critical velocity necessary to ensure efficient hole cleaning. The empirical approach involves experimental setups and flow loops to simulate and study well geometry and drilling conditions. The conditions studied include the effect of pipe rotation, effect of inclination, effect of eccentricity; and the effect of flow rate.

### 1.2 Literature Review

This literature review is divided into several parts: hydraulic optimization, cuttings transport modelling, and nano-fluids

#### 1.2.1 Hydraulics optimization

Drilling hydraulics is essentially concerned with the circulation of fluid and other circulation materials effectively and efficiently. Hydraulics optimization entails attempts to minimize the parasitic pressure losses and thereby maximize the pressure drop across the bit – as this directly affects the rate of penetration and hole cleaning efficiency. Two main approaches are used in bit optimization: hydraulic horsepower and jet impact force. Several authors and researchers (Lummus 1970; Lim et al. 1996; Bailey et al. 2000; Cho et al. 2000) have studied and postulated various theories and principles for hydraulics optimization.

Bahari et al. 1967 performed optimization on Khangiran field by using what they called the "primary" and "secondary" optimization. The primary optimization involved cost per foot analysis and penetration rate analysis; the secondary optimization included fluid properties, optimum mechanical energy for each depth interval, optimum roller cone bit selection, and optimum bit weight and rotary speed. They recommended the use of optimization techniques in order to reduce drilling time and cost to the barest minimum. Reed et al. 1993 developed a generalized model for laminar, transitional, and turbulent flow of drilling muds by defining the concept of equivalent annular diameter, effective viscosity, and generalized Reynold's number. They showed that increasing the yield stress of a fluid, delays the flow transition. They also showed that increasing the ratio of inner to outer diameter, delays transition in an annulus to higher pump rates. Amaral 1994 developed a software for drilling hydraulics calculation using the six dial viscometer readings, five rheological models and using specific friction loss correlations to assist in the design and supervision of drilling activities for PETROBRAS. The rheological models studied were Bingham, Ostwald de Waele, Casson, Robertson & Stiff, and the Herschel-Bulkley models.

Bailey 2000 extended the work of Reed et al. 1993 to develop a generalized and consistent pressure drop and flow regime transition model for drilling hydraulics. Khataniar et al. 2003 showed that optimum hydraulics design ensures minimum drilling cost. They performed an economic comparative analysis of Hydraulics Horse Power, and Jet Impact force by determining the optimum sizes of the bit nozzles, and the energy cost for each method. They reported that Jet Impact Force was more cost effective than Hydraulic Horse Power. Bern et al. 2006 presented an update on the studies carried out by a task group to modernize the API Recommended Practice on Rheology and Hydraulics (API RP13D, 2003) to achieve a better description of fluid properties. They presented the methods of validating the new models in various reviewed sections.

### 1.2.2 Cuttings Transport Modelling

Peden et al. 1990 reported the results of a comprehensive experimental investigation of drilled cuttings in inclined well including the effects of rotation and eccentricity. By using the concept of minimum transport velocity (MTV), they investigated the relationship between cuttings transport efficiency and several factors such as hole angle, fluid rheology, cuttings size, drill pipe eccentricity, circulation rate, annular size and pipe rotation. They reported that fluid rheology and flow regime had the highest impact on the minimum transport velocity for a given eccentricity and annular size. They also reported that pipe rotation greatly improved the transport efficiency for various levels of eccentricity. Luo et al. 1992 conducted experiments at BP research centre and developed a physical model to predict the critical (minimum) flow rate requirements for cleaning deviated wells by using dimensional analysis to analyze the forces acting on the cuttings. They compared the results from the physical model with those obtained from the experiments and field data.

Lim et al. 1996 conducted experiments at the University of Alaska, Fairbanks, to study the effect of pressure and flow parameters on efficient borehole cleaning in vertical and near-vertical wells. By considering, jet impact force and hydraulics horse power, they showed that hole cleaning is improved if an unbalanced hydraulic jet force is available. Ford et al. 1996 developed a MTV computer package for hole-cleaning design and analysis based on experimental and theoretical studies. The Herschel-Bulkley model was used in the development and the effects of several operational and geometric parameters on hole cleaning were studied. They reported about 20% variance between the experimental results and the computer-generated results. Several assumptions limited the robustness of the developed model. They assumed amongst others that the cuttings were spherical, represented by a single discrete particle and that the cuttings do not affect the annular velocity profile.

Rubiandini 1999 defined dimensionless slip velocity and developed a new set of equations for estimating the mud minimum rate for cuttings transport by combining the works of Larsen, Ford and Moore. He reported close results with the Larsen and Fords' methods for angles greater than $45^0$, but the equations predicted higher values for angles less than $45^0$. The equations gave the same results as Moore's' model for vertical wells. Cho et al. 2000 proposed a method to predict cuttings transport efficiency in deviated holes with inclination of 30 to $60^0$ by studying the effect of fluid flow in a porous cuttings-bed on cuttings transport efficiency and hydraulics. They proposed a mathematical model, based on the continuity and conservation of momentum principles, to identify the effects of fluid flow in porous cuttings-bed, relative velocities between carrier fluid and cuttings, and pressure losses on hydraulics. They concluded that due to the relatively small velocity of the cuttings-bed in a heterogeneous layer, the cuttings-bed had little effect on the pressure gradient; given the same nominal annular velocity, a highly viscous fluid will reduce the cuttings bed and increase the pressure drop. They recommended that the combined effects of cuttings-bed area, pressure gradient, fluid rheology and nominal annular velocity should be considered in hydraulics optimization especially in highly deviated wellbore with high cuttings-bed area.

Kelessidis et al. 2003 presented a paper based on a two layer mechanistic model to describe the flow patterns and minimum suspension velocity for efficient cuttings transport in horizontal and deviated wells using coiled tubing drilling. They reported that annular velocity is the most significant velocity in cleaning efficiency; turbulent annular flow regime enhanced hole cleaning; eccentricity had adverse effect on cleaning efficiency, the higher the level of eccentricity, the lower the cuttings transport efficiency; and rheology had little effect on cuttings transport. Ozbayoglu et al. 2007 developed empirical correlations for determining the critical velocity to prevent the formation of cuttings bed in horizontal and inclined wellbores based on experiments carried out at the Middle East Technical University (METU). They reported that the shear stress acting is the major factor in preventing cuttings bed formation. They reported a percentage error of 15% from the correlations.

Garcia-hernandez et al. 2007 also reported the results experiments conducted to determine the cuttings lag in horizontal and deviated wells. They highlighted the importance of cuttings investigation to obtaining accurate geological models. They presented a model to determine the source depth of the cuttings collected at the surface. They, also, reported that inclination affects cuttings transport when combined with fluid and pipe rotation. Nazari et al. 2010 presented a thorough review of the approaches used in cuttings transport studies in directional drilling. They categorized the studies into four groups: sensitivity analysis; cuttings concentration modelling; hole cleaning monitoring; and hole cleaning control. They recommended the need for a generalized systematic model for the description of cuttings transport. Mohammadsalehi et al. 2011 presented an approach for optimizing hole cleaning and drilling hydraulics within all ranges of hole inclination by combining Moore's correlation and Larsen's correlation.

### 1.2.3 Nanofluids Application in Drilling Engineering

Nanofluids are specialized fluids obtained by careful combination of nanoparticles and a base fluid. The nanoparticles are particles with an average diameter of less than 100nm. They possess unique characteristics that differentiate them from microparticles, and make them adaptable to a wide range of applications. Chief amongst these characteristics is their high surface-to-volume ratio. Nanofluids are formed by dispersing nanoparticles in base fluids. The effective density and viscosity of the nanofluids are functions of the respective properties of the base fluid and the volumetric concentration of the nanoparticles. The successful application of nanoparticles in the drilling industry dates back over 50 years (Krishnamoorti 2006; Matteo et al. 2012). The peculiar properties of nanofluids that enhance their application for drilling and completion purposes include lightness, corrosion resistance, and mechanical strength (Matteo et al. 2012). The benefits of such properties include the extension of the life of downhole equipment, improved cement integrity, hole quality, and well placement (Singh et al. 2010).

### 1.3 Study Objectives and Significance of Study

What are the impacts of the turbulence criteria and effective annular diameter definitions on drilling hydraulics and efficient hole cleaning in directional wells? What are the conclusions from the combined effects of these factors and fluid rheology on directional wells? Can nano-fluids improve rig hydraulics and hole cleaning? In order to answer these questions, the following objectives are proposed.

1. To examine the impact of different equivalent annular diameter definitions on the pressure drop and hole cleaning in directional wells.
2. To examine the possible influence of nano-fluids on drilling hydraulics and hole cleaning.
3. To investigate the impact of fluid rheology on pressure drop and hole cleaning in directional wells.
4. To develop a user-friendly software to facilitate quick evaluation of the optimization process

**Significance of the Study**

A very important aspect of drilling is well control. The equivalent mud weight of the drilling fluid must remain within the mud weight window to avoid kick and lost circulation problems. The presence of drilled cuttings in the annulus increases the Equivalent Circulating Density, and reduces the rate of penetration; thereby increasing cost. Consequently, in drilling, best practices must be followed while drilling to ensure safe, economic, functional, and environmentally responsible delivery of the well. These practices must ensure proper hole cleaning, bit optimization, and formation integrity. This work proposes to examine the impact of the available rheological models and equivalent annular diameter definitions on these important factors, using conventional and nano-based drilling fluids.

**1.4 Research Methodology**

This study examines the impact of conventional rheological models and equivalent annular diameter definitions; and investigates the possible influence of nanofluids on hydraulics optimization under dynamic conditions. Hydraulics optimization, in this context, is the determination of operating conditions that satisfy three requirements: efficient hole cleaning, optimized pressure drop at the bit, and formation integrity by ensuring that the ECD remains within the allowable safe margin. The criteria used for bit optimization are the Hydraulic Horsepower (HHP) and Jet Impact Force (JIF). Rabia,and Baker Hughes INTEQ 1999 report that the HHP requires about 65% of pump pressure to be dissipated across the bit; while the JIF imposes a demand of about 48% of the pump pressure at the bit. Pressure drop and velocity calculations based on the investigated rheological models and equivalent annular diameter definitions are used to determine the pressure drop across the bit. The equations derived by Rudi Rubiandini are used to model the cuttings transport. The equation, for each combination of rheology and equivalent annular diameter, is applied to solve the same problem for a direct comparison.

The effective density (Pak et al. 1998) and effective viscosity (Das, 2012) of the nanofluids are calculated from the respective properties of the base fluids (drilling fluids). Aluminium oxide nanoparticle with a volumetric concentration of 30% is used in for the sensitivity analysis. Consequently, a user-friendly program is developed, using the Matlab® programming language, to facilitate repeated analyses for any combination of rheology, equivalent annular diameter, and bit optimization criteria. The program requires input on well bore profile and geometry; gives the user a choice of rheological models, equivalent annular diameter, and bit optimization criteria; and advises the user based on the results obtained.

### 1.5 Organization of the Thesis

This thesis is organized into five chapters including this Chapter One. Chapter Two covers the theories, principles, and equations that are used in calculating the pressures along the wellbore and across the bit. Chapter Three presents the methodology, and the Computer Program. Chapter Four discusses the results and observations from data analyses. Chapter Five presents the summary, conclusions drawn from the sensitivity analyses, and proposes recommendations for future work.

# CHAPTER 2

# THEORETICAL BACKGROUND

## 2.1 Rotary Rig Hydraulics

Rotary drilling is the act of using torque-powered bits to make holes. The drilling rig consists of six (6) primary sub-systems: the power system; the hoisting system; the rotary system; the circulating system; well control system; and well monitoring system. The circulating system transmits the drilling fluid from the surface through the system and back up the annulus to the surface. Hydraulics optimization is implemented and achievable through careful planning, design, selection, installation, and operation of all the rig components especially the circulating system. Hydraulics is greatly affected by drilling fluid rheology.

## 2.2 Rheology of drilling fluids

Several rheological models are available in the literature to describe the deformation and flow of drilling fluids. These include:

1. Newtonian Model
2. Bingham Plastic Model
3. Power Law Model (Ostwald de Waele's model)
4. Yield Power Law Model (Herchel Bulkley model)
5. API RP 13D Model
6. Other models.

The simplest rheological model is the Newtonian model, which is simply a linear relationship between the shear stress and the shear strain. The Newtonian model has well established fluid flow and pressure loss equations due to its simplicity. Therefore, the related equations developed for Newtonian fluids are the basis for all the other models, except for those that were determined empirically. The definition of an apparent or effective viscosity comparable with the true viscosity of the Newtonian model facilitates the derivations. This apparent viscosity serves as the basis for determining the Reynold's number and defining the fluid flow regime.

### 2.2.1 Newtonian Model

The Newtonian model is a simple linear relationship between shear stress and shear strain rate.

The governing equation is:

$$\tau = \mu\dot{\gamma} \qquad 2.1$$

*where,*

$\tau = shear\ stress(lb_f / 100 ft^2),$

$\mu = viscosity,\ poise\left(centipoise(cp)\right)$

$\dot{\gamma} = shear\ strain\ rate\left(sec^{-1}\right)$

### 2.2.2 Bingham Plastic Model

For decades, the petroleum industry used the Bingham Plastic Model to describe the rheology of drilling fluids. The model incorporates a yield stress necessary to initiate flow.

The governing equation is:

$$\tau = \tau_0 + \mu_p\dot{\gamma} \qquad 2.2$$

*where,*

$\tau = shear\ stress\left(lb_f / 100 ft^2\right)$

$\tau_0 = yield\ \ point,\left(lb_f / 100 ft^2\right)$

$$\mu_p = plastic\ viscosity, cp(centipoise)$$
$$\dot{\gamma} = shear\ strain\ rate\left(sec^{-1}\right)$$

### 2.2.3 Power Law Model (Ostwald de Waele's model)

Ostwald de Waele's Power law model presented a new approach to the study of deformation and flow of drilling fluids. He defined a consistency factor and a flow behaviour index.

The governing equation is:

$$\tau = K\dot{\gamma}^n \qquad 2.3$$

$$where,$$
$$\tau = shear\ stress\left(lb_f / 100\, ft^2\right)$$
$$K = fluid\ consistency\ factor\left(lb - sec^n / 100\, ft^2\right)$$
$$\dot{\gamma} = shear\ strain\ rate\left(sec^{-1}\right)$$
$$n = flow\ behaviour\ index(dimensionless)$$

### 2.2.4 Yield Power Law Model (Herchel-Bulkley Model)

Yield Power law is a modified power law model that incorporates a yield stress that must be overcome in order to initiate flow. It is a generic model that combines the Newtonian, Bingham Plastic and Power Law models. The governing equation is:

$$\tau = \tau_{0HB} + K_{HB}\dot{\gamma}^n \qquad 2.4$$

$where,$

$\tau = shear\ stress\left(lb_f / 100 ft^2\right)$

$\tau_{0HB} = herschel - bulkley\ yield\ \ point, \left(lb_f / 100 ft^2\right)$

$K_{HB} = herschel - bulkley\ Fluid\ Consistency\ Factor\left(lb - sec^n / 100 ft^2\right)$

$\dot{\gamma} = shear\ strain\ rate\left(sec^{-1}\right)$

$n = flow\, behaviour\ index(dimensionless)$

### 2.2.5 API RP 13D Model

The power law model is the basis for the development of this model. The adoption of the model eliminates the need for complex calculations involved in obtaining the rheological parameters of the Herschel-Bulkley model (API RP 13D 2012).

### 2.2.6 Other Models

The Casson model and Robertson-Stiff model, amongst others, have been used to describe the rheology of drilling fluids. The Casson model works better in low shear rate region, especially for drillings fluids laden with cuttings**.** Robertson-Stiff model is similar to the Herschel-Bulkley model. It is also a three-parameter model that reduces to the Newtonian, Bingham Plastic, and Power law models in special cases(INTEQ 1999).

## 2.3 Directional Drilling (Peculiarities and Difficulties)

"Directional drilling is the science and art of deviating a well bore along a planned course to a subsurface target whose location is a given lateral distance and direction from the vertical"(Osisanya 2012). It refers to the controlled process of making horizontal and angled wells along a predetermined path to a predetermined target. Directional drilling has found several applications as necessitated by terrain peculiarities and technical reasons in the petroleum industry. These reasons include sidetracking existing wells; due to restricted or inaccessible surface locations; shoreline drilling; multilateral wells; relief wells; salt dome drilling; fault controlling; to avoid gas and water coning problems; and to maximize production by intersecting several fractures.

### 2.4 Rheological Models Considered

The rheological models considered in this thesis are

- The Newtonian Model
- The Bingham Plastic Model
- The Power Law Model, and
- The API RP 13D Model

### 2.5 Pressure Loss Equations

The pressure loss in a conduit is a function of the fluid properties, flow characteristics, and geometry of the conduit. The pressure gradient in an horizontal conduit is given as:

$$\frac{dp}{dl} = \frac{f \rho \overline{V}^2 d_e}{25.8} \qquad 2.5$$

The friction factor, *f*, in Eq. 2.5 depends on the flow regime of the circulating fluid. The flow regime could be laminar, transitional, or turbulent depending on the value of the Reynold's number. The Reynold's number is the ratio of the inertia forces acting on the fluid to the viscous resistance provided by it. Hence, the Reynold's number is highly dependent on the viscosity of the fluid, whereas the inertia forces depend on largely on the geometry of flow.

Consequently, the rheological model applied has a huge impact on the value of the Reynold's number, and thus the threshold for transition from one flow regime to the other.

The Reynolds number is expressed as

$$\mathrm{Re} = \frac{928\rho\overline{\mathrm{V}}d_e}{\mu_{eff}} \qquad 2.6$$

Another criterion usually used to determine the flow regime is the critical velocity. The critical velocity is simply the velocity at which the fluid changes from the laminar flow regime. This approach simply neglects the existence of the transitional flow regime, thereby creating an abrupt change from laminar to turbulent flow regime. The availability of several definitions for the effective annular diameter further compounds the determination of the flow regime. For simplicity, the industry adopts the hydraulic radius concept stated in Eq. 3.8. The hydraulics radius concept states that the equivalent diameter is the hypothetical circular diameter that is hydraulically equivalent to the actual annular system. The concept is valid as long as the ratio of the external diameter of the pipe to the internal diameter of the casing or open hole is greater than 0.3; a ratio that is almost always exceeded in rotary drilling operations (Bourgoyne et al. 1991).

The following sections discuss the determination of the Reynold's number and flow regime for the various rheological models.

### 2.5.1 Newtonian Fluids

For the Newtonian fluid, the generally accepted threshold for transitional flow is at Re = 2100, and threshold for turbulent flow is at Re = 4000. The fluid is in the transitional flow regime for values of Re between 2100 and 4000.

$$\mu_{eff} = \theta_{300} \qquad 2.7$$

$$f_{lam} = \frac{16}{\mathrm{R_e}}, \mathit{for\ laminar\ flow} \qquad 2.8$$

$$f_{turb} = \frac{0.0791}{\mathrm{Re}^{0.25}}, \mathit{for}\ \text{turbulent}\ \mathit{flow} \qquad 2.9$$

### 2.5.2 Bingham Plastic model

For the Bingham Plastic, the rheological parameters are determined from

$$\mu_p = \theta_{600} - \theta_{300} \qquad 2.10$$

$$\tau_y = \theta_{300} - \mu_p \qquad 2.11$$

The effective viscosity is

$$\mu_{eff} = \frac{\mu_p + 6.66\tau_y d_e}{\overline{V}} \qquad 2.12$$

Where the value of $\overline{V}$ and $d_e$ are as defined in section 3.1.2.2 for pipe and annular flow. The Reynolds number is then calculated from equation 3.1.3. All other equations for the Newtonian Model are valid for the Bingham Plastic Model as long as the appropriate effective viscosity is used.

### 2.5.3 Power Law model

For the Power law model, the rheological parameters are calculated from

$$n = 3.32 \log\left(\frac{\theta_{600}}{\theta_{300}}\right) \qquad 2.13$$

$$K = 510\frac{(\theta_{300})}{511^n} \qquad 2.14$$

$$\mu_{eff} = 100K\left(\frac{144\overline{V}}{d_e}\right)^{n-1} \qquad 2.15$$

The Reynold's number for pipe and annulus are calculated from

$$\mathrm{Re}_{pipe} = \frac{928\rho\overline{V}d_e}{\mu_{eff}\left(\frac{3n+1}{4n}\right)^n}, and \qquad 2.16$$

$$\mathrm{Re}_{annulus} = \frac{928\rho \overline{V} d_e}{\mu_{eff}\left(\frac{2n+1}{3n}\right)^n} \qquad 2.17$$

The flow regime is then determined by comparing the with the threshold values.

$\mathrm{Re}_c = 3470 - 1370n$, threshold for transitional flow 2.18

$\mathrm{Re}_{turb} = 4270 - 1370n$, threshold for turbulent flow 2.19

For flow in pipe

$$f_{lam_{pipe}} = \frac{16}{\mathrm{Re}_{pipe}} \qquad 2.20$$

$$f_{trans_{pipe}} = \frac{16\,\mathrm{Re}_{pipe}}{\mathrm{Re}_c^{\,2}}, \text{and} \qquad 2.21$$

$$f_{turb_{pipe}} = \frac{a}{\left(\mathrm{Re}_{pipe}\right)^b} \qquad 2.22$$

And for flow in the annulus

$$f_{lam_{annulus}} = \frac{24}{\mathrm{Re}_{annulus}} \qquad 2.23$$

$$f_{trans_{annulus}} = \frac{16\,\mathrm{Re}_{annulus}}{\mathrm{Re}_c^{\,2}}, \text{and} \qquad 2.24$$

$$f_{turb_{annulus}} = \frac{a}{\left(\mathrm{Re}_{annulus}\right)^b} \qquad 2.25$$

### 2.5.4 API RP 13D

The API RP 13D is similar to the Power Law model. However, the flow behaviour index, and consistency factor depends on whether the flow is in the drill string or through the annulus.

$$n_p = 3.32\log\left(\frac{\theta_{600}}{\theta_{300}}\right) \quad 2.26$$

$$K_p = 5.11\frac{(\theta_{300})}{511^{n_p}} \quad 2.27$$

$$n_{ann} = 0.5\log\left(\frac{\theta_{300}}{\theta_3}\right) \quad 2.28$$

$$K_{ann} = 5.11\frac{(\theta_{300})}{511^{n_{ann}}} \quad 2.29$$

For pipe flow

$\mathrm{Re}c_{pipe} = 3470 - 1370n_p$, threshold for transitional flow 2.30

$\mathrm{Re}_{turb_{pipe}} = 4270 - 1370n_p$, threshold for turbulent flow 2.31

And for annular flow,

$Re = 3470 - 1370n_{ann}$, threshold for transitional flow 2.32

$Re = 4270 - 1370n_{ann}$, threshold for turbulent flow 2.33

All other equations of the Power Law Model are also applicable to the API RP 13D Model

# CHAPTER 3

# METHODOLOGY AND COMPUTER PROGRAM

## 3.1 Hydraulics

The following assumptions are used to develop the relevant equations (Bourgoyne et al. 1991):

1. The drillstring is placed concentrically in the well bore
2. There is no rotation of the drillstring.
3. The sections of the open hole are circular in shape and of known diameter
4. Drilling fluid is incompressible
5. Flow is isothermal
6. Pipe is smooth.

As stated in chapter 1, the main factors of a hydraulics program are the fluid properties (rheology and density), well bore geometry (inclination, hole or casing diameter, drill pipe and drill collar diameters and lengths), and formation characteristics (pore pressure, fracture gradient, and maximum allowable ECD). The subsequent sections present the various rheological models equation as well as pressure drop equations used in this work.

## 3.2 Pressure, and Velocity Calculations Along The Well Bore

In rotary drilling operations, the circulating system transmits the drilling fluid from the surface through the well bore and back to the surface. The mud pumps provide the energy that transmits the drilling fluid from the mud pit through the suction line to the standpipe, down the drillstring, out through the drill bits, up through the annulus, to the mud treatment devices through the return line, and back to the mud pit. The mud treatment equipment includes the shaleshaker, desander, desilter, and degasser which are used to remove solids, gases and other impurities from the drilling fluid. Thus, from the principle of conservation of energy, the sum of the pressures along the well geometry (assuming no influx or efflux of formation fluid) must be equal to the pump pressure. From the perspective of bit hydraulics optimization, all other pressures are said to be parasitic.

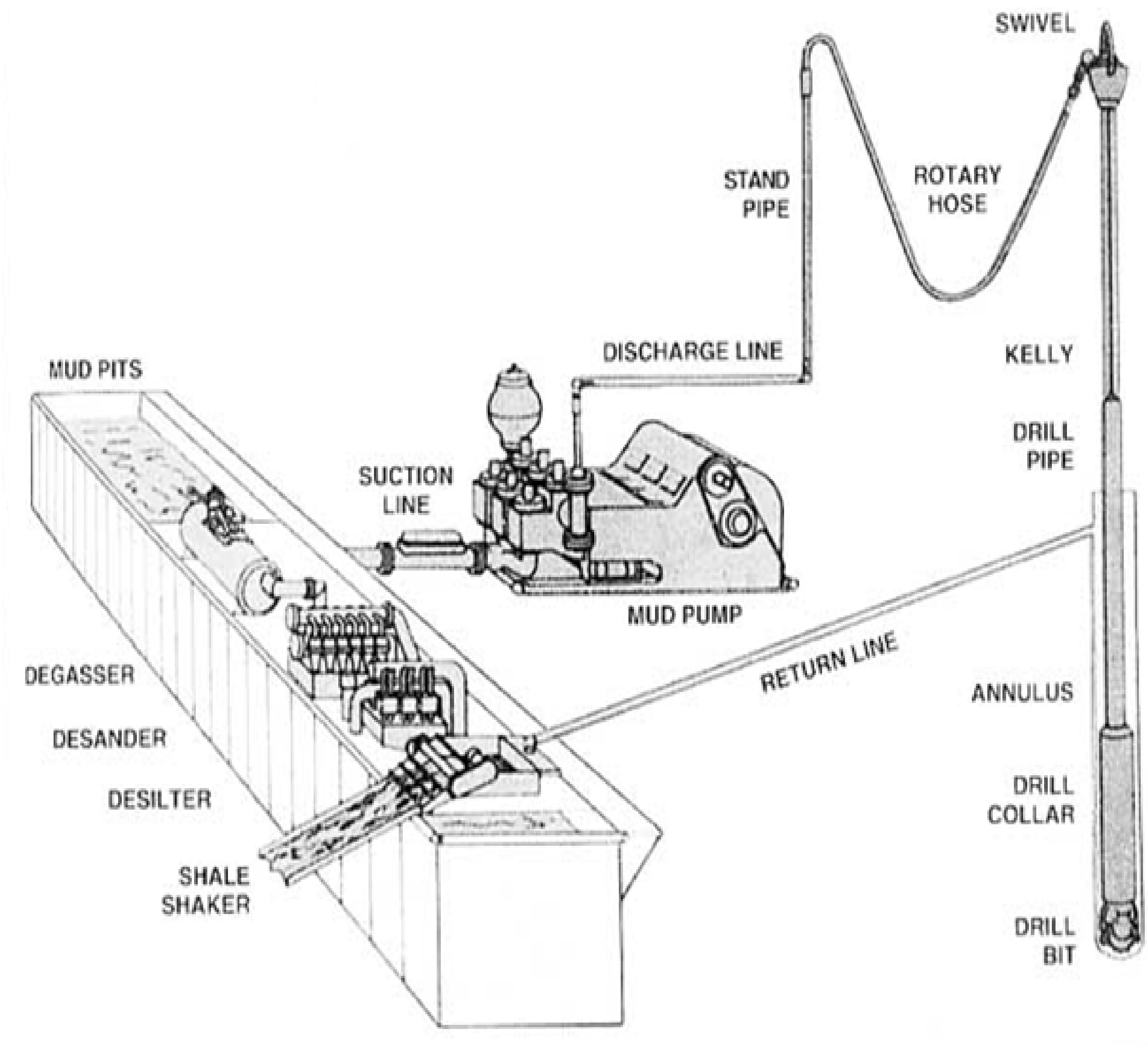


Fig. 3.1: The Circulating System of a Rotary Drilling Rig

Thus:

$$P_{pump} = \Delta P_b + \Delta P_{sc} + \Delta P_{dp} + \Delta P_{dc} + \Delta P_{dpa} + \Delta P_{dca} \qquad 3.1$$

$$\text{Parasitic Pressure losses} = \Delta P_{sc} + \Delta P_{dp} + \Delta P_{dc} + \Delta P_{dpa} + \Delta P_{dca} \qquad 3.2$$

*Therefore,*

$$P_{pump} = \Delta P_b + \text{Parasitic Pressure losses} \qquad 3.3$$

*Where ,*

$P_{pump}$ = *Available System pressure* = *Standpipe pressure* $(psi)$

$\Delta P_{sc}$ = *Pressure loss in surface connections* $(psi)$

$\Delta P_{dp}$ = *Pressure loss in drill pipe* $(psi)$

$\Delta P_{dc}$ = *Pressure loss in drill collar* $(psi)$

$\Delta P_b$ = *Pressure loss across the drill bit* $(psi)$

$\Delta P_{annular}$ = *Annular pressure loss around drill pipe and drill collar* $(psi)$

$= \Delta P_{dpa} + \Delta P_{dca}$

**3.2.1 Pressure Loss Through Surface Connections**

The pressure loss through the surface connections depends on the type of connection, the fluid density, and the flow rate. It is calculated from(API RP 13D 2012)

$$\Delta P_{sc} = C_{sc}\rho\left(\frac{q}{100}\right)^{1.86} \qquad 3.4$$

where,

$C_{sc} = a$ constant depending on the type of connection

$\rho = mudweight$(ppg), and

q = volumetric flow rate(gpm)

The surface connections are broadly classified into four, depending on the length and internal diameter of the standpipe, hose, swivel, and kelly. The table below shows the classification.

Table 3.1: Surface connection Pressure loss constant (API RP 13D 2012)

| Type | Standpipe | | Hose | | Swivel | | Kelly | | $C_{sc}$ |
|---|---|---|---|---|---|---|---|---|---|
| | Length (ft) | I.D (in) | Length (ft) | I.D (in) | Length (ft) | I.D (in) | Length (ft) | I.D (in) | |
| 1 | 40 | 3.0 | 45 | 2.0 | 4 | 2.0 | 40 | 2.25 | 1.00 |
| 2 | 40 | 3.5 | 55 | 2.5 | 5 | 2.5 | 40 | 3.25 | 0.36 |
| 3 | 45 | 4.0 | 55 | 3.0 | 5 | 2.5 | 40 | 3.25 | 0.22 |
| 4 | 45 | 4.0 | 55 | 3.0 | 6 | 3.0 | 40 | 4.00 | 0.15 |

### 3.2.2 Pressure Drop in and Around the Drillstring

Figure 3.2 shows the schematic of an angled well inclined at θ degrees from the vertical. The pressure drop, for fully developed flow, in the drillstring and through the drillstring annulus of an angled well can be expressed (in field units) as:

$$\frac{dp}{dl} = \frac{\rho g \cos\theta}{620} \pm \frac{f \rho \overline{V}^2 d_e}{25.8} \qquad 3.5$$

The dynamic pressure is the pressure due to fluid circulation. The total pressure is the sum of the two pressures for flow down the wellbore, and difference for flow up the annulus i.e. when flow is against gravity.

The effective diameter for flow in the drillstring is the internal diameter of the drillstring, and the average velocity is given as:

$$\overline{V} = \frac{q}{2.448 d_i^2} \qquad 3.6$$

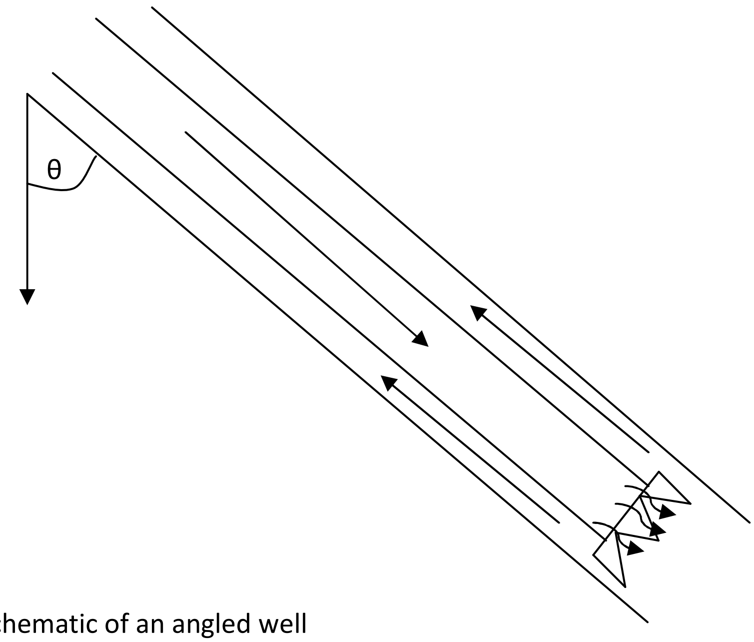


Fig. 3.2 Schematic of an angled well

*where*,

$\frac{dp}{dl} = pressure$ gradient along well bore(psi/ft)

$l = measured$ depth along the well path(ft)

$g = acceleration$ due to gravity $= 32.2\, ft/s^2$

$f = friction\ factor$(dimensionless)

$\rho = fluid$ density(ppg)

$\overline{V} = average$ velocity in pipe or annulus(ft/s)

$d_e = effective$ diameter(in)

For annular flow, several effective diameter definitions are available in the literature. The effective diameters considered are:

1. Hydraulic radius concept

$$d_e = d_0 - d_i \qquad 3.7$$

2. Slot representation

$$d_e = 0.816(d_o - d_i) \tag{3.8}$$

3. Lamb's equation

$$d_e = \sqrt{d_0^{\,2} + d_i^{\,2} - \frac{d_0^{\,2} - d_i^{\,2}}{ln\left(\frac{d_o}{d_i}\right)}} \tag{3.9}$$

4. Crittendon correlation

$$d_e = \frac{\sqrt[4]{d_0^{\,4} - d_i^{\,4} - \frac{(d_0^{\,2} - d_i^{\,2})^2}{ln\left(\frac{d_o}{d_i}\right)}} + \sqrt{d_0^{\,2} - d_i^{\,2}}}{2} \tag{3.10}$$

*where,*

$d_o$ = *internal diameter of open-hole or casing* (*inches*)

$d_i$ = *outside diameter of drill pipe or drill collar* (*inches*)

$d_e$ = *effective* diameter(in)

Two definitions of average velocity are applicable for annular flow.

1. True average velocity, this is used for all the equivalent diameter definitions except the Crittendon criteria. The true average velocity is

$$\bar{V} = \frac{q}{2.448\left(d_0^{\,2} - d_i^{\,2}\right)} \tag{3.11}$$

2. The fictitious average velocity applies to the Crittendon criteria. It is expressed as :

$$\bar{V} = \frac{q}{2.448 d_e^{\,2}} \tag{3.12}$$

### 3.3 Bit Optimization Criteria

Bit optimization refers to the process of ensuring adequate pressure drop across the bit to achieve good bit and hole cleaning. The pressure drop across the bit is optimized by minimizing the parasitic pressure losses along the well bore.

The two main criteria adopted for bit optimization are the Hydraulic Horsepower (HHP), and the Jet Impact Force (JIF) criteria. Both methods involve the calculation of an exponent based on a power law relationship between the parasitic pressure losses and the volumetric flow rate.

$$P_{parasitic} = mq^n \qquad 3.13$$

A log-log plot of $P_{parasitic}$ and q, gives the constants m and n as intercept and slope respectively. The constants m and n are evaluated on the field by using pressure data at two flow rates. A log-log plot of $P_{parasitic}$ against q will give a straight line with slope n and intercept m.

The value obtained is then used to define the optimization criteria. The following sections explain the procedure.

### 3.3.1 Jet Velocity Criterion

The jet velocity criterion demands that the velocity of the fluid exiting the nozzles of the bit should be high enough to clean the surface of the bit and carry the cuttings away as soon as the cuttings are generated. This is to ensure that the rate of penetration is not affected by the cuttings generated; that the bit is in constant touch with new formation and the cuttings are not being regrounded by the bit.

### 3.3.2 Hydraulic Horsepower (HHP) Criterion

The Hydraulic Horsepower criterion seeks to maximize the horsepower at the bit.

$$\Delta P_{bit} = \left(\frac{n}{n+1}\right) P_{pump} \qquad 3.14$$

$For$ example, if $n = 1.8$

$$\Delta P_{bit} = \left(\frac{1.8}{2.8}\right) \mathrm{P_{pump}}$$

$$\Rightarrow \Delta P_{bit} \approx 64\% \text{ of } \mathrm{P_{pump}}, and$$

Therefore, based on the Hydraulic Horsepower criterion, 64% of the pump pressure should be exerted across the bit.

### 3.3.3 Jet Impact Force Criteria

$$\Delta P_{bit} = \left(\frac{n}{n+2}\right) \mathrm{P_{pump}} \qquad 3.15$$

$For$ example, if $n = 1.8$

$$\Delta P_{bit} = \left(\frac{1.8}{3.8}\right) \mathrm{P_{pump}}$$

$$\Rightarrow \Delta P_{bit} \approx 47\% \text{ of } \mathrm{P_{pump}}, and$$

Therefore, 47% of the pump pressure should be exerted across the bit.

## 3.4 Cuttings Transport Modelling

The equations developed by Rubiandini 1999 are adopted for cuttings modelling.

Assumptions related to the modelling of nanofluids application in directional drilling include:

1. The equations developed for the Bingham Plastic model is also applicable to other non-newtonian fluids with adjustment to the apparent viscosity.
2. The equations developed for the non-newtonian fluids are adoptable for nanofluids.
3. The nanofluids do not settle within the time considered.

All the assumptions stated for hydraulics have inherent flaws as indeed stated by the authors. None of the assumptions is valid.

However, the assumptions on cuttings transport are reasonable, since Rubiandini equations were derived from the dimensionless plots of the empirical correlations of Larsen, Ford, and Moore.

Nanoparticles typically take several months to settle. Rubiandini's model is presented in Appendix A.

### 3.5 Nanofluid Application: Development of Equations

The equations of fluid flow are applicable to nanofluids by appropriate substitution of the effective density and viscosity of the nanofluids.

#### 3.5.1 Effective Density

Pak et al. 1998 presented the following equation for calculating the effective density of a nanofluid.

$$\rho_{nf} = \phi\rho_p + (1-\phi)\rho_{bf} \qquad 3.16$$

$$\begin{aligned}
&where, \\
&\phi = volumetric\ concentration\ of\ the\ nanoparticles \\
&\rho_{nf} = density\ of\ nanofluid \\
&\rho_p = density\ of\ nanoparticles \\
&\rho_{bf} = density\ of\ base\ fluid
\end{aligned}$$

#### 3.5.2 Effective Viscosity

Several correlations exist in the literature to compute the effective viscosity of a nanofluid. The following correlations are considered:

1. Einstein's Correlation

$$\mu_{nf} = \mu_{bf}\left(1+2.5\phi\right) \qquad 3.17$$

2. Batchelor's Correlation

$$\mu_{nf} = \mu_{bf}\left(1+2.5\phi+6.25\phi^2\right) \qquad 3.18$$

3. Brinkmann's Correlation

$$\mu_{nf} = \frac{\mu_{bf}}{(1-\phi)^{2.5}} \qquad 3.19$$

$$\begin{aligned} &where, \\ &\mu_{nf} = viscosity\ of\ nanofluid \\ &\mu_{bf} = viscosity\ of\ basefluid \\ &\phi = volumetric\ concentration \end{aligned}$$

These equations are valid for any set of consistent units, since the volumetric concentration is dimensionless.

### 3.5.3 Reynolds Number

The general expression for the Reynold's number in field units is

$$R_{e_{nf}} = \frac{928 \rho_{nf} \bar{V} d_e}{\mu_{nf}} \qquad 3.20$$

Consequently, the equations for calculating the Reynolds number for nanofluids are based on several correlations, stated as follows:

Einstein's correlation,

$$R_{e_{nf}} = \frac{928\left[\phi\rho_p + (1\text{-}\phi)\rho_{bf}\right]\bar{V}d_e}{\mu_{bf}(1+2.5\phi)} \qquad 3.21$$

Batchelor's correlation,

$$R_{e_{nf}} = \frac{928\left[\phi\rho_p + (1\text{-}\phi)\rho_{bf}\right]\bar{V}d_e}{\mu_{bf}\left(1+2.5\phi+6.25\phi^2\right)} \qquad 3.22$$

And, Brinkmann's correlation,

$$R_{e_{nf}} = \frac{928(1-\phi)^{2.5}\left[\phi\rho_p + (1-\phi)\rho_{bf}\right]\bar{V}d_e}{\mu_{bf}} \qquad 3.23$$

Equations 3.21 to 3.23 are substituted in place of the Reynold's number for fluid flow and cuttings transport.

**3.6 Use of the Computer Program**

The program was written in the MATLAB® programming language. It consists of 12 mfiles for the various rheological models, equivalent annular diameter, and other necessary computations. The program prompts the user for inputs that describe the fluid rheology, wellbore geometry, and type of surface connection. It allows the user to make a choice of fluid rheology, equivalent annular diameter, and bit optimization criteria. It then advises the user based on the results obtained from the analysis. The advice indicates the part of the well planning design that needs to be reviewed to achieve bit optimization, efficient hole cleaning, and avoid formation fracture.

**Required User inputs**

- Rheological inputs required.
    - The $\theta_3$, $\theta_{300}$, and $\theta_{600}$ fann viscometer readings.
    - The available pump pressure
    - The available flow rate
    - The density of the drilling fluid
- Wellbore geometry inputs
    - The number of sections in the geometry
    - The diameter of the open hole or casing in each section
    - The lengths, and diameters of the drill pipe and/or drill collar in each section
    - The angle of inclination of each section taken clockwise from the vertical

- Inputs for cuttings transport modelling and ECD management
    - Average diameter , and density of the cuttings
    - The rate of penetration
    - The rotational speed(RPM) of the drill string
    - The maximum permissible ECD of the formation

**Available Choices**

- Surface connection type as defined in Table 3.1
    - Type 1
    - Type 2
    - Type 3
    - Type 4
    - Neglect losses through Surface connections
- Fluid rheological models
    - Newtonian model
    - Bingham Plastic model
    - Power Law model
    - API RP 13D model
- Equivalent annular geometry
    - Hydraulic radius concept
    - Slot approximation
    - Lamb's approach
    - Crittendon correlation
- Bit optimization

- Hydraulics Horsepower
- Jet Impact force

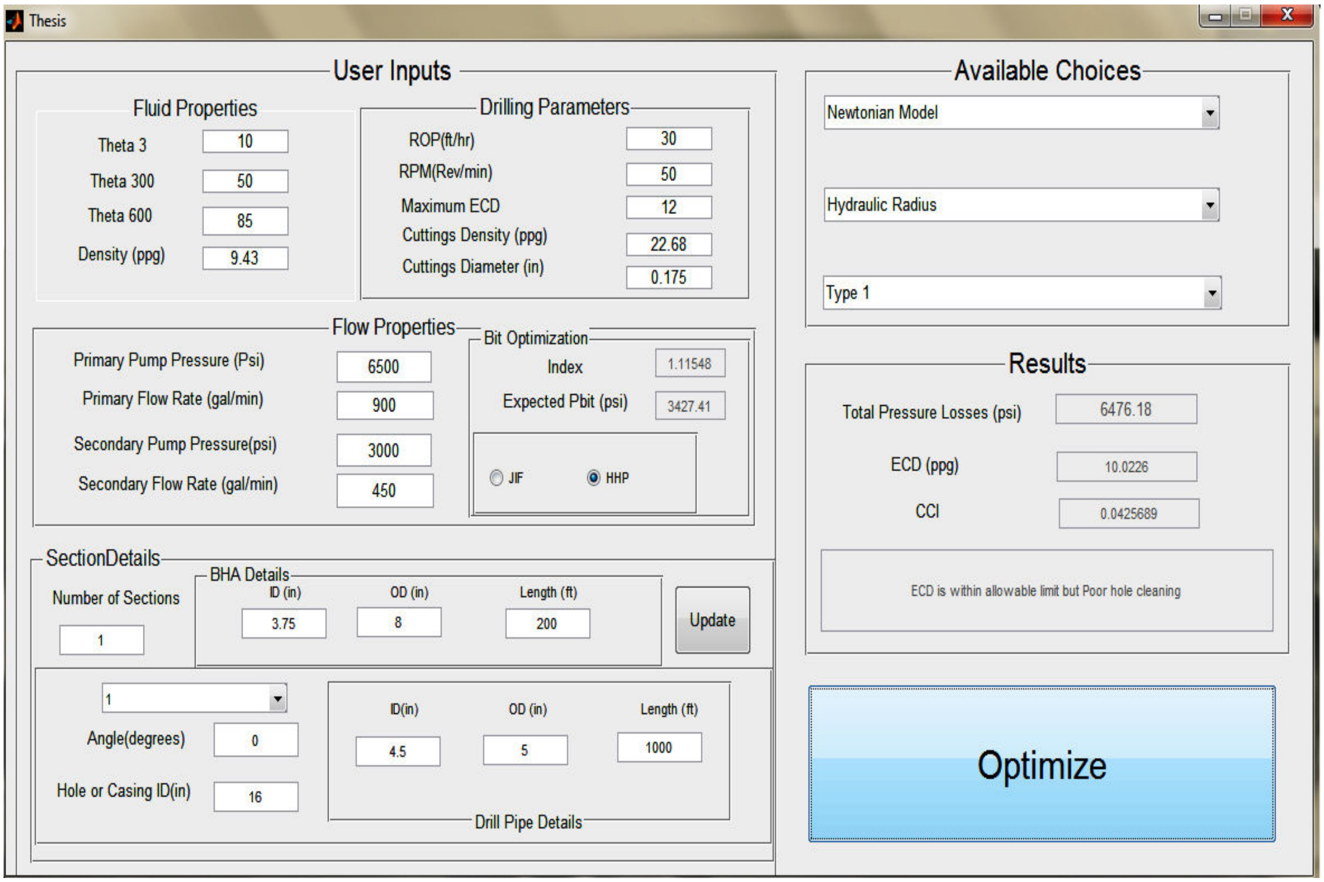


Fig. 3.3: The Graphical User Interface of the Program

In summary, this chapter presented the methodology and equations used to determine the pressure along the wellbore, the bit optimization criteria, and the equivalent annular diameter definitions. The applicable equations for the determination of the density, viscosity, and Reynolds number for the nanofluids were also presented. Finally, the features and use of the computer program were presented. Fig.3.3 shows the results displayed by the program, further details on use of the program are given in appendix C.

# CHAPTER 4

# DATA ANALYSIS, RESULTS, DISCUSSIONS, AND SENSITIVITY ANALYSIS

This chapter is divided into 2 sections:

1. To perform sensitivity analysis of the effect of inclination, rheological models, and equivalent annular diameter definitions on annular pressure and equivalent circulating density
2. To examine the impact of nanoparticles on annular pressure and equivalent circulating density (ECD). The influence of nanofluids is examined by the addition of aluminium oxide nanoparticles to a base fluid.

For these analyses, a 5 in OD drill pipe, 12.25 in hole, and 150 gal/min flow rate are used. The rheological models and equivalent annular diameter equations are applied to the same problem to have a single basis for comparison.

## 4.1 Sensitivity of Annular Pressure to Inclination

Fig. 4.1 to 4.3 shows the profiles of pressure gradient against inclination for the different rheological models and equivalent annular diameter definitions. In general, the annular pressure gradient decreases with increasing angle of inclination from the vertical; it has the least value at $90^0$, which corresponds to a horizontal wellbore. The values of the annular pressure gradients estimated by the Slot Approximation and Lamb's Method coincide, and are the most conservative for all the rheological models considered. For the Bingham Plastic Model, the Hydraulic Radius concept showed an intermediate behaviour, while the Crittendon correlation gave the highest value. The API RP 13D Model showed similar trend as the Bingham Plastic

Model. The pressure gradient for all the equivalent annular diameter definitions coincides for the Power Law Model.

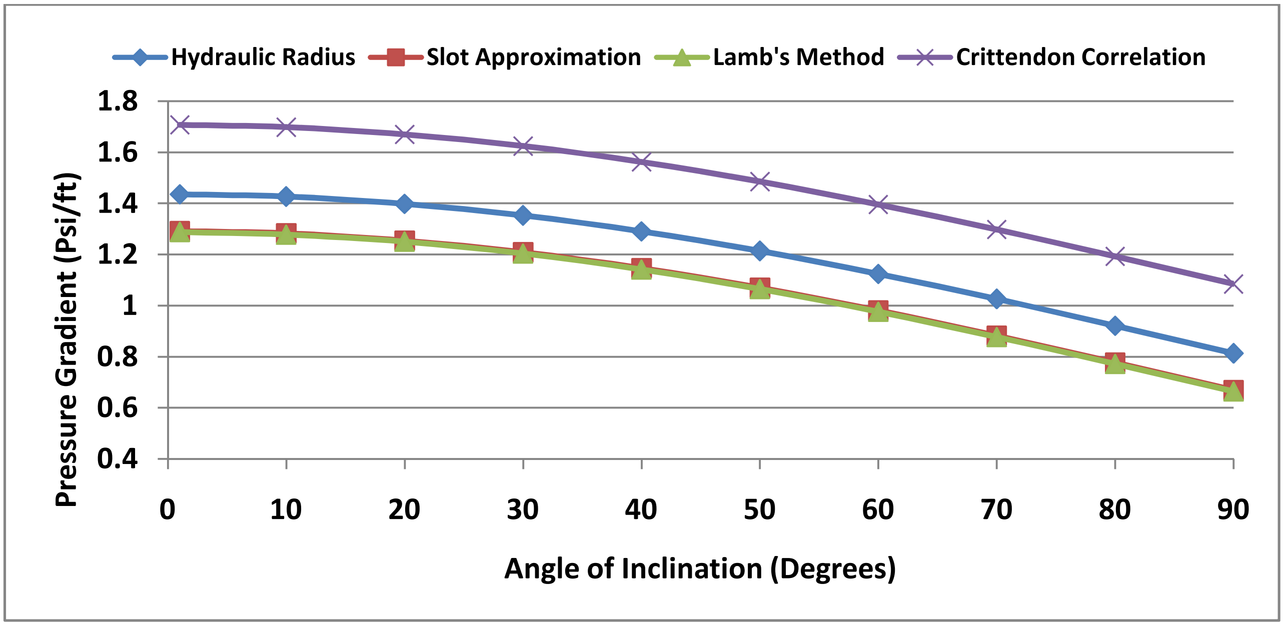


Fig. 4.1: Effect of Angle of Inclination on Annular Pressure Gradient for Bingham Plastic Fluid

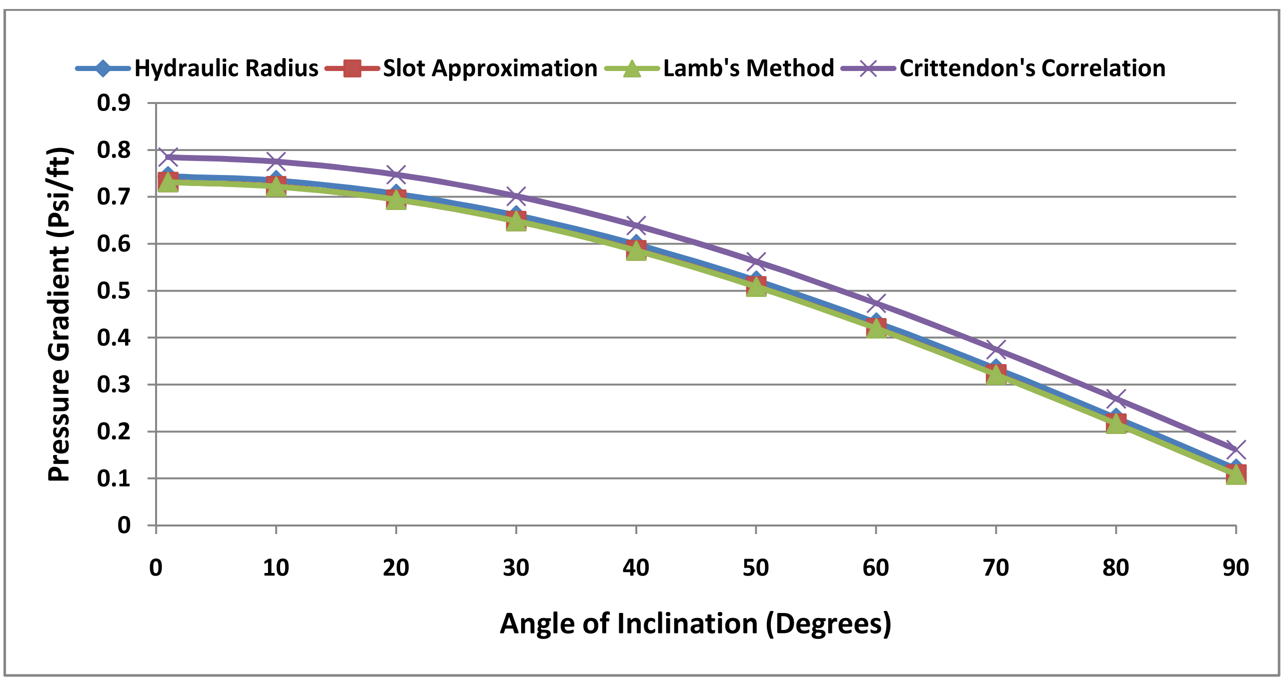


Fig. 4.2: Effect of Angle of Inclination on Annular Pressure Gradient for API RP 13D Fluid

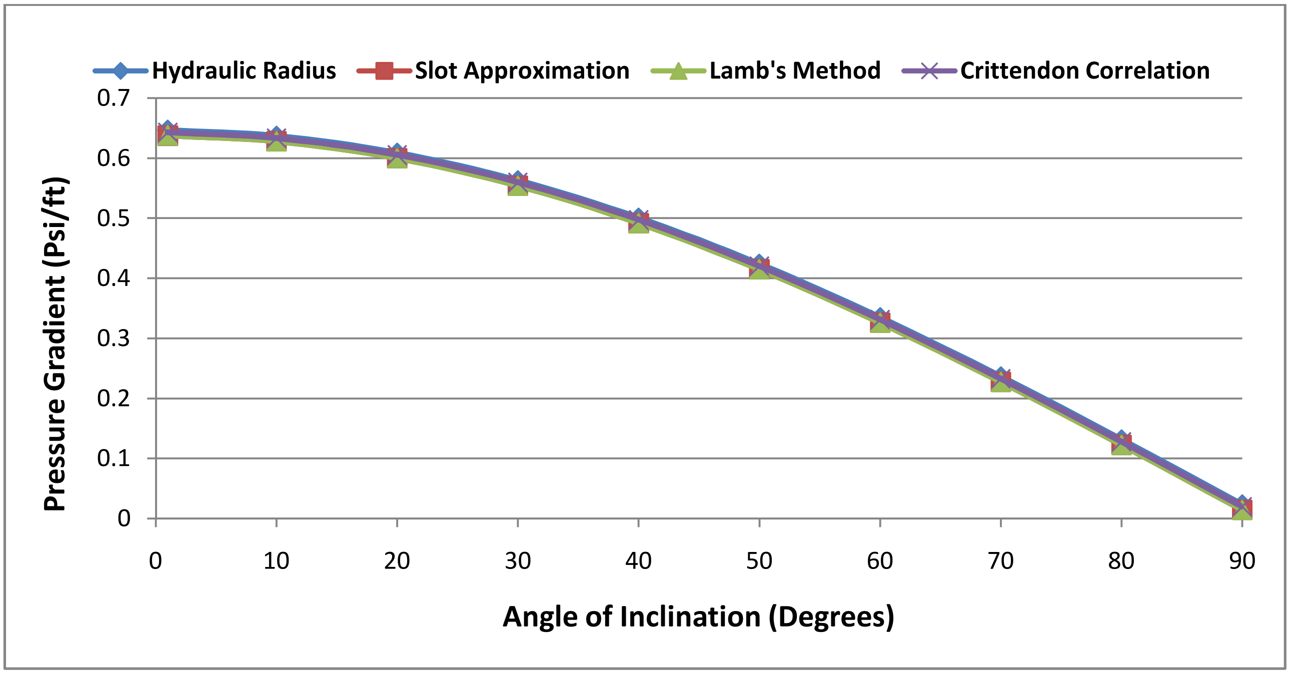


Fig. 4.3: Effect of Angle of Inclination on Annular Pressure Gradient for Power Law Fluid

**4.2 Impact of Rheological Model and Equivalent Annular Diameter**

The influence of rheology and equivalent annular diameter is examined by considering the case study of a Well A drilled in the Niger-Delta region to a depth 3750ft (MD). The well consists of three sections, a vertical section drilled to 500ft (TVD), an inclined section with an average inclination angle of $46^0$ drilled to 1333ft (TD), and a 2000ft, horizontal section. Tables 4.1 and 4.2 give the details of the case study.

Fig. 4.5 to 4.8 shown in Appendix B depicts the results obtained from the analysis of Well A for separate rheological models. Fig. 4.9 shows the combined effect for all the rheological models. It is observed that, in general, the Crittendon correlation gave the highest value for the annular pressure. The Bingham Plastic Model showed remarkable sensitivity to the equivalent annular diameter definition with the Crittendon correlation being the highest, the Hydraulic Radius definition and Lamb's Method gave similar values for the annular pressure, while Slot Approximation gave the lowest value. The Power Law Model showed the same trend as the Bingham Plastic Model with the Slot Approximation and Lamb's Method giving similar values. The API RP 13D Model showed no significant variation in the annular pressure for all equivalent annular diameter definitions.

Table 4.1: Fluid and Flow Properties of Case Study

| Fluid and Flow Properties | | |
|---|---|---|
| Fluid Properties | | |
| Density | 12 | ppg |
| Viscometer Readings | | |
| θ3 | 3 | |
| θ6 | 5 | |
| θ300 | 25 | |
| θ600 | 45 | |
| Flow Properties | | |
| Pump Pressure | 6000 | Psi |
| Flow Rate | 550 | gal/min |

Table 4.2: Wellbore Geometry and Profile of Case Study

| Hole Section Geometry | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Section (From Bottom) | Angle | Diameter (Hole or Casing) | Drill Collar | | | Drill pipe | | |
| | (degrees) | (in) | ID (in) | OD (in) | Length (ft) | ID (in) | OD (in) | Length (ft) |
| 1 | 90 | 12.25 | 2.813 | 6 | 577 | 4.276 | 5 | 1,923 |
| 2 | 46 | 12.25 | 0 | 0 | 0 | 4.276 | 5 | 1,266 |
| 3 | 0 | 17.5 | 0 | 0 | 0 | 4.276 | 5 | 436 |

Fig. 4.7 shows the comparison among the rheological models and equivalent annular diameter definitions on the annular pressure. The Power Law Model gave the most conservative value within each equivalent annular diameter definition.

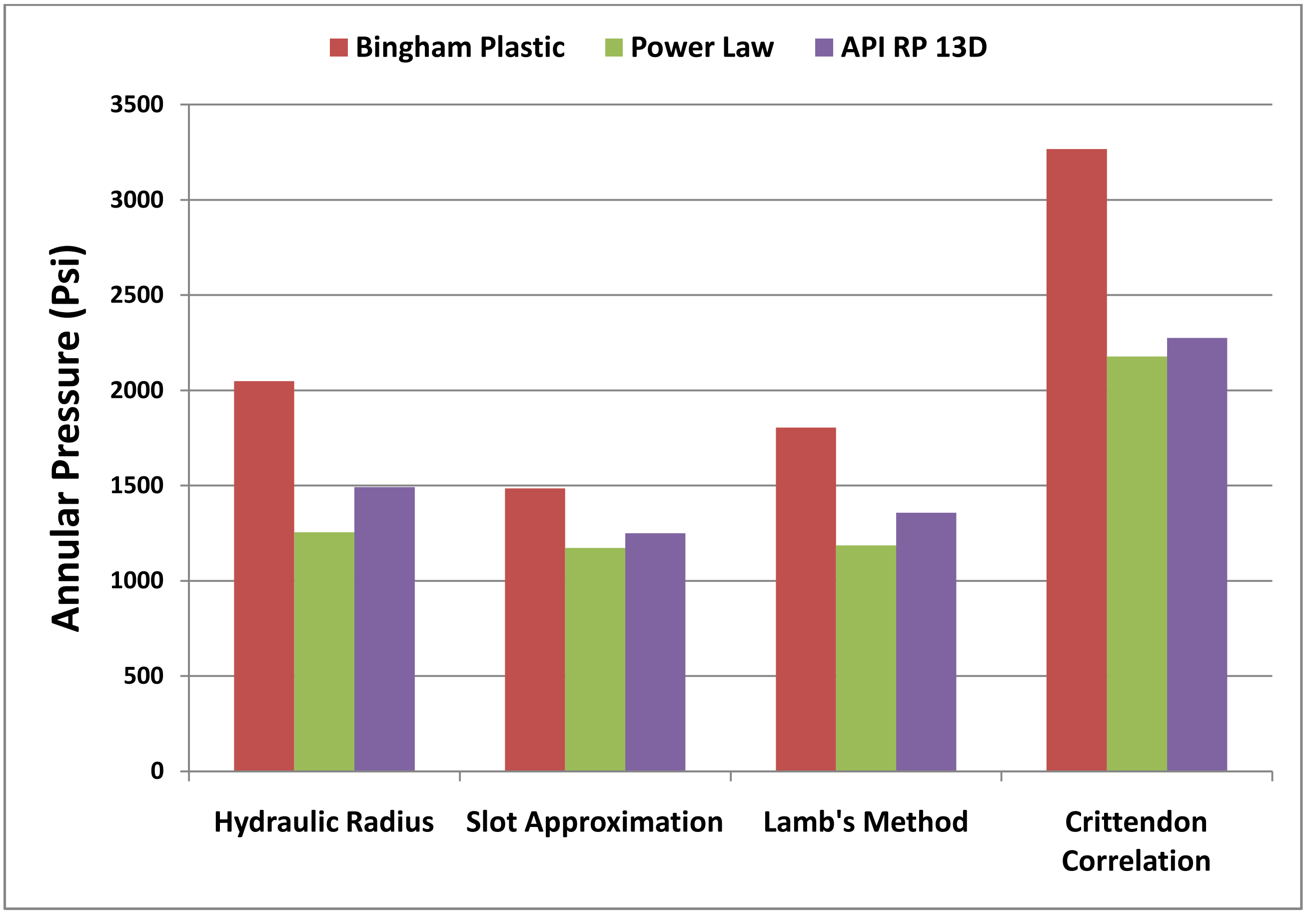


Fig. 4.7: Effect of Rheological Model and Equivalent Annular Diameter on Annular Pressure

Fig. 4.8 shows the effect of fluid rheology and equivalent annular diameter on the equivalent circulating density. It is observed that the Crittendon Correlation has the highest values for the equivalent circulating density for all the rheological models while the Bingham Plastic Model also has the highest values for all the equivalent diameter definitions.

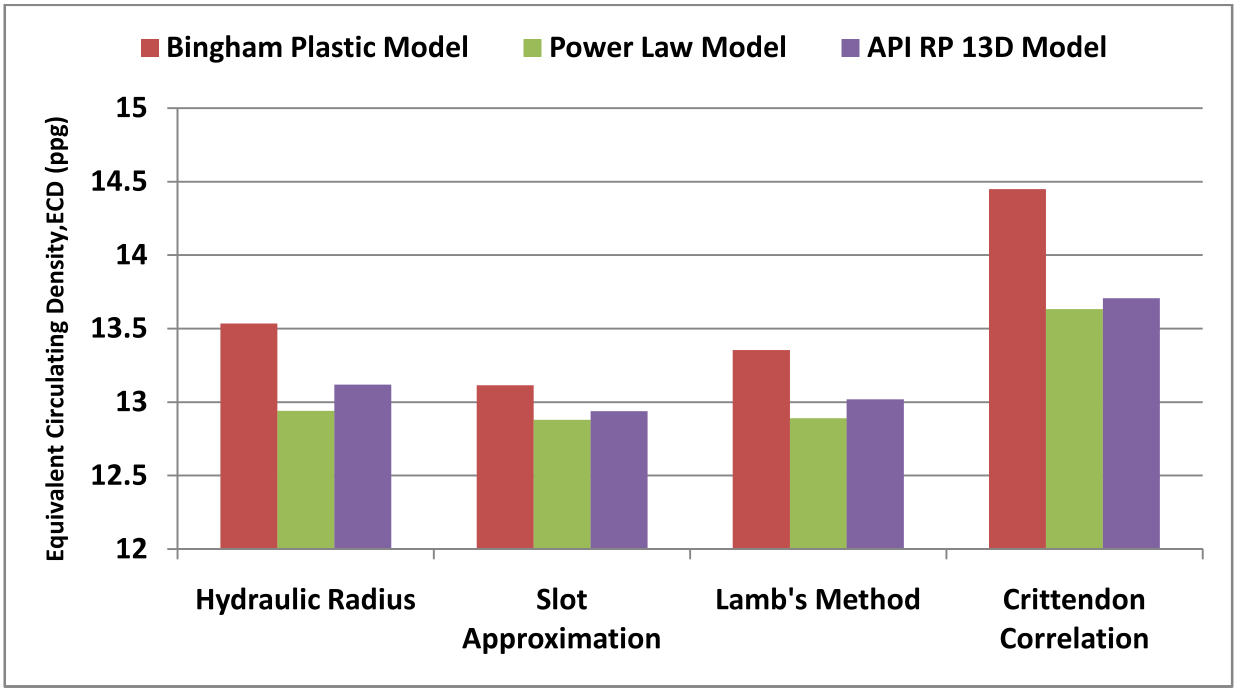


Fig. 4.8: Effect of Rheological Model and Equivalent Annular Diameter on ECD

### 4.3 Impact of Nanofluids on Hydraulics Optimization

Fig. 4.9 and Fig. 4.10 present typical plots of the density (using Aluminum oxide nanoparticles with density of 33.1 ppg) and viscosity of nanofluids as a function of the volumetric concentration of the nanoparticles respectively. The density of nanofluids has a linear relationship with the volumetric concentration, having an intercept equal to the density of the original (base) fluid. The viscosity of the nanofluid also has an intercept equal to the viscosity of the base fluid. The viscosity increases with concentration of nanofluid; the manner of increase is governed by the equation applied. The variance in the effective viscosities determined from the predefined correlations increases as the concentration of the nanoparticle increased.

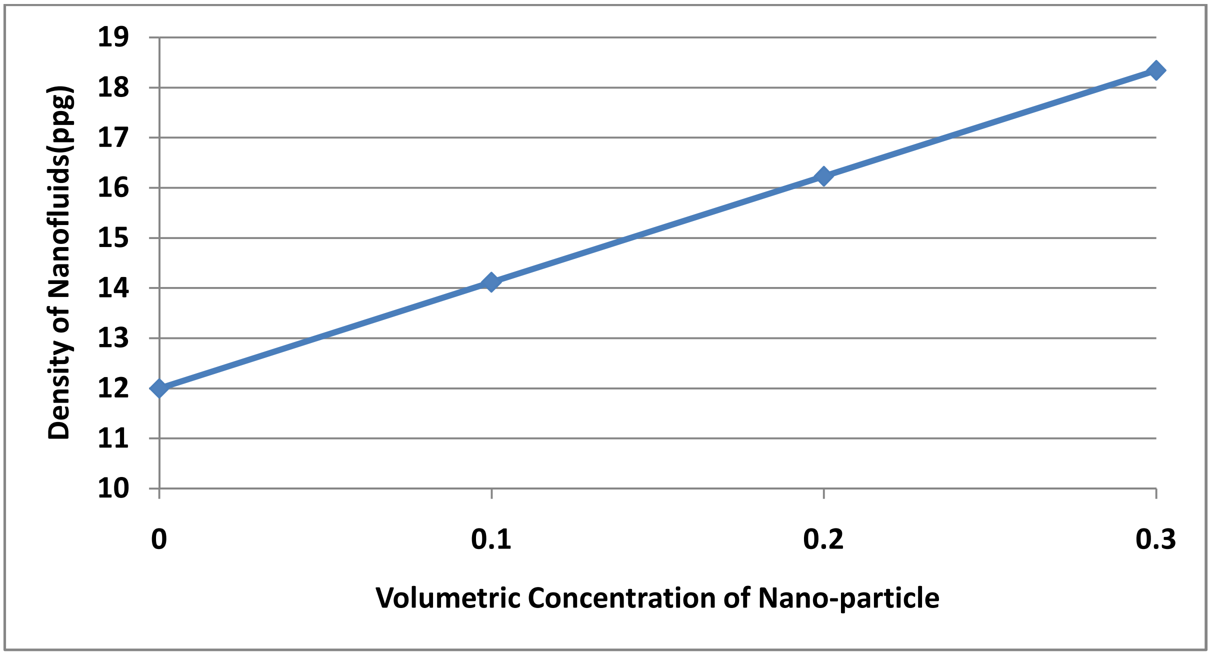


Fig. 4.9: Density of nanofluid as a function of the Volumetric Concentration

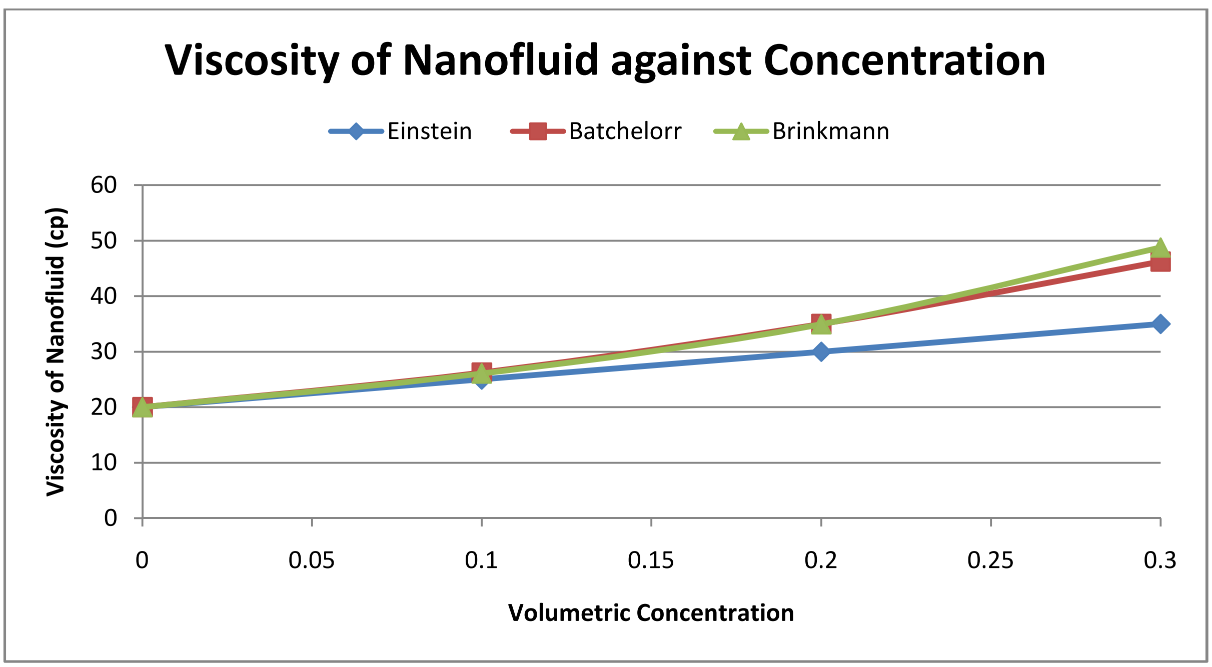


Fig. 4.10: Viscosity of nanofluid as a function of the volumetric concentration

The combined-effect of nano-based fluid and equivalent annular diameter definition on hydraulics optimization is investigated by analyzing the case study of Well A presented in the previous section. A base fluid of density 12 ppg and viscosity of 20 cp is used. A volumetric concentration of 30% is used for the analysis. Fig. 4.11 shows the effect of the nanofluid on the annular pressure for all equivalent annular diameter definitions. It is observed that Einstein's correlation for effective viscosity of nanofluids gave the most conservative values of annular pressure for all equivalent annular diameter definitions while Brinkmann's correlation gave the highest values. The Crittendon correlation for annular diameter gave the highest annular pressure values in comparison to other equivalent annular diameter definitions for all the viscosity correlations.

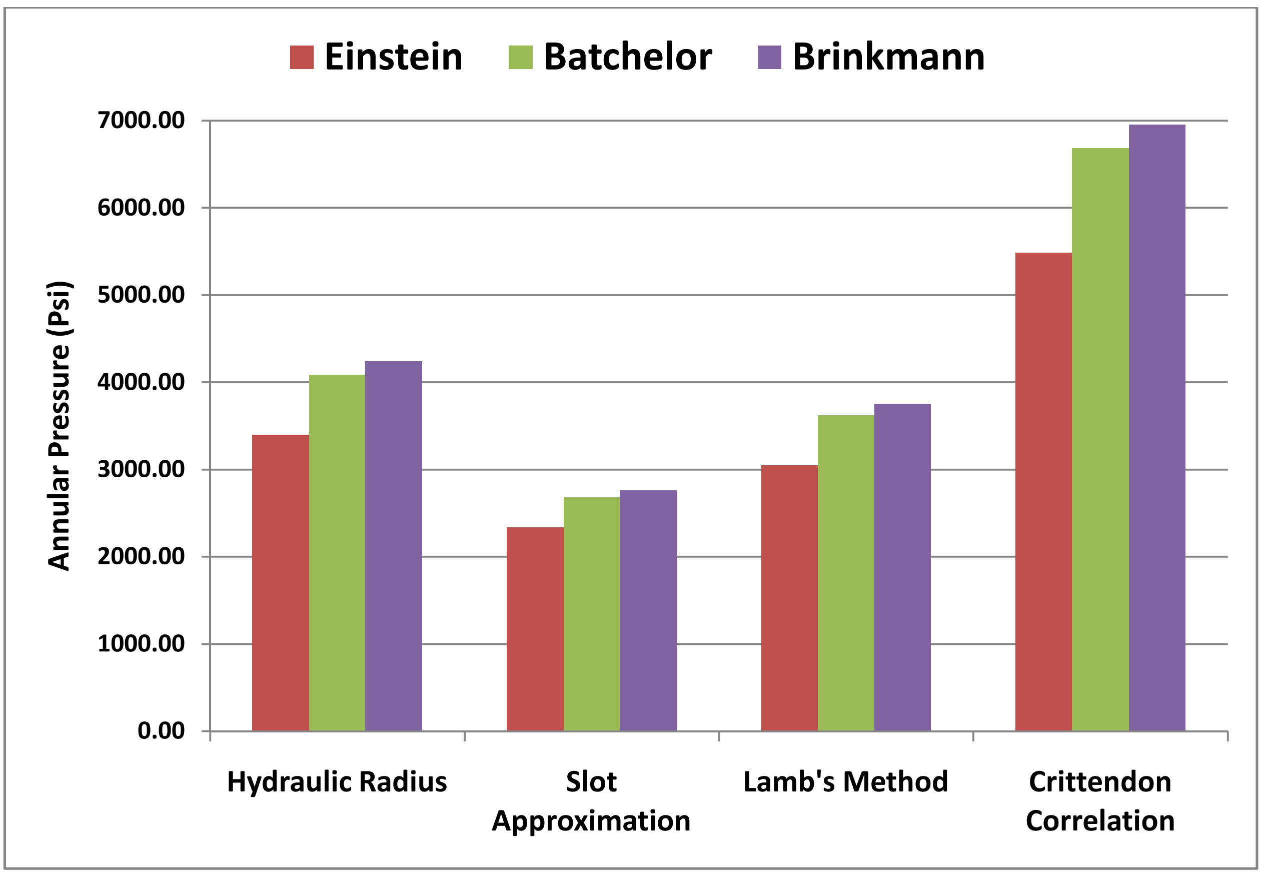


Fig. 4.11: Effect of Viscosity Correlation and Equivalent Annular Diameter on Annular Pressure

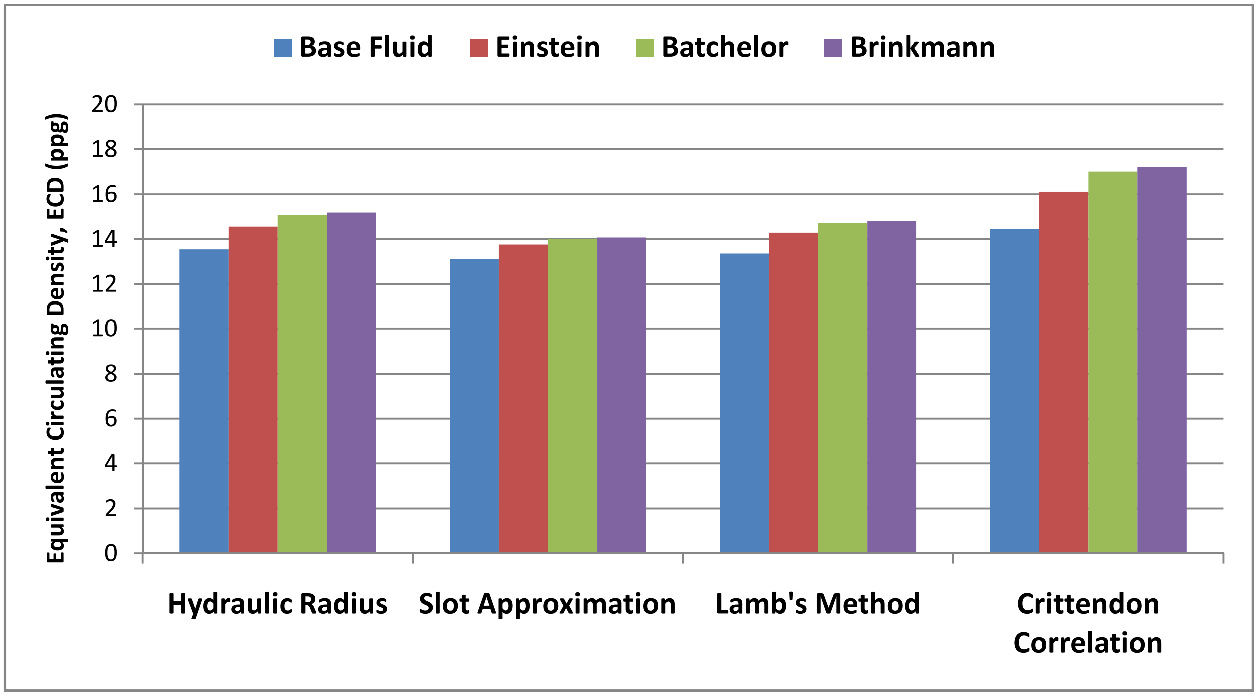


Fig. 4.12: Comparison of the Effects of Base Fluid and Nanofluid on ECD

The Crittendon correlation gave the highest value of equivalent circulating density for all viscosity correlations. Fig. 4.12 shows the comparative effects of the original fluid and the nanofluid on the equivalent circulating density. There is a significant increase in the ECD using the nanofluid when compared to the original fluid.

### 4.4 Discussion of Results

The Bingham Plastic model gives the highest values. The use of offset Pressure-While-Drilling (PWD) data could help to indicate the fluid model that is most appropriate for a given drilling location. The Crittendon correlation always gives the highest value, this is perhaps due to the use of the apparent annular velocity instead of the actual annular velocity; there is little variation in the values obtained from the other equivalent annular diameter definitions especially between the Slot Approximation and Lamb's Method. Einstein's correlation is the most conservative viscosity correlation for nanofluids, while Batchelor and Brinkmann's correlations show similar values for low concentration nanofluids. Regardless of the viscosity correlation, and annular diameter definition used, the nanofluid increases the annular pressure, and ECD of the well bore, which could lead to formation fracture and other associated problems, especially in areas with a narrow mud weight window. Another issue related to nanofluid application is the pumpability of the drilling mud, the higher the volumetric concentration, the higher the density and viscosity of the nanofluid. These increase in density and viscosity could, however, be limited by using low density, and low viscosity base fluids.

In summary, this chapter presented the results obtained from the analysis of a Well A using conventional and nano-based fluid, by considering several combinations of rheological models, equivalent annular diameter definitions, and effective nanofluid viscosity correlations.

# CHAPTER 5

# SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

## 5.1 Summary

Hydraulics optimization is an attempt to maximize the pressure drop across the bit in order to ensure adequate rate of penetration, and effective hole cleaning. The hydraulics system is centered around well geometry and fluid properties, and thus drilling fluids and hydraulics have common purposes. The total pressure loss along the wellbore is the sum of the pressure drops in the drillstring and in the drillstring annulus. The pressure drops are functions of the flow regime, which depends on the Reynold's number. This study proposed to provide a means for quick evaluation of the well plan, examine the impact of available fluid rheological models and equivalent annular diameter definitions on hydraulics optimization, and to examine the possible impact of nanofluids on hydraulics optimization using different effective viscosity correlations.

## 5.2 Conclusions

The following conclusions were drawn from the analysis:

1. The Hydraulic Radius concept, Slot Approximation, and Lamb's Approach give almost the same pressure gradients for various rheological models.
2. The Crittendon correlation overestimates the values of the annular pressure gradient and ECD for various rheological models. This phenomenon is compounded when the fluid is not in the laminar flow regime.
3. The pressure gradient decreases with increasing inclination from the vertical for all rheology and equivalent annular diameter definitions.

4. The increase in density and viscosity of the nanofluid compared to the base fluid leads to the need for a higher capacity pump to flow the system. However, using a low density and low viscosity base fluid, with lower density and concentration of nanoparticles would reduce the effect of increase in the density and viscosity of the mixture.
5. A user-friendly computer program in MATLAB was developed to facilitate the computation of the pressure gradients. This definitely eliminates error in hand calculations

## 5.3 Recommendations

The API and IADC give detailed guidelines of procedures for hole cleaning and other drilling and completions operations. The computer program is a means of quick evaluation. The use thereof, does not eliminate the need for sound engineering practices and principles, to ensure safe, economic, and environmentally responsible delivery of the well. Consequently, the following recommendations for further studies are made.

1. This work ignores the effect of nanoparticles on fluid rheology. Future studies should incorporate the rheology of nanofluids.
2. The computer program should be developed further in order to improve its robustness. It uses the Rubiandini's cuttings transport model. The API, however, recommends the use of a cuttings chart.

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# APPENDICES

## Appendix A: Additional Plots

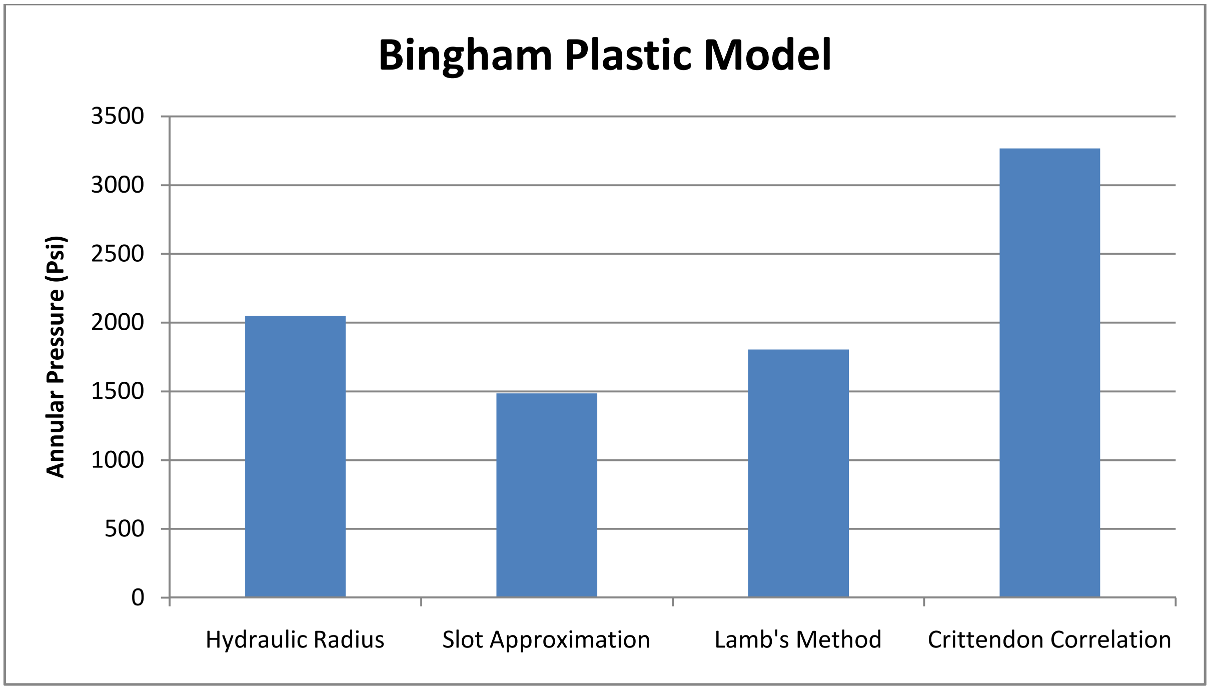



Fig. 4.4: Effect of Equivalent Annular Diameter on Annular Pressure for Bingham Plastic Fluid

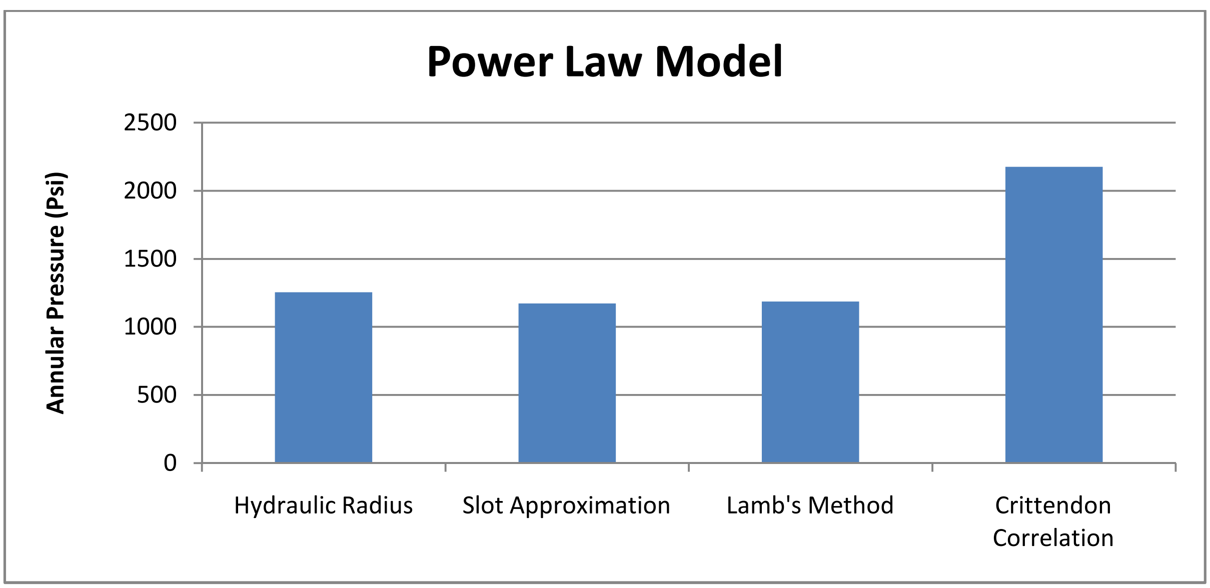



Fig. 4.5: Effect of Equivalent Annular Diameter on Annular Pressure for a Power Law Fluid

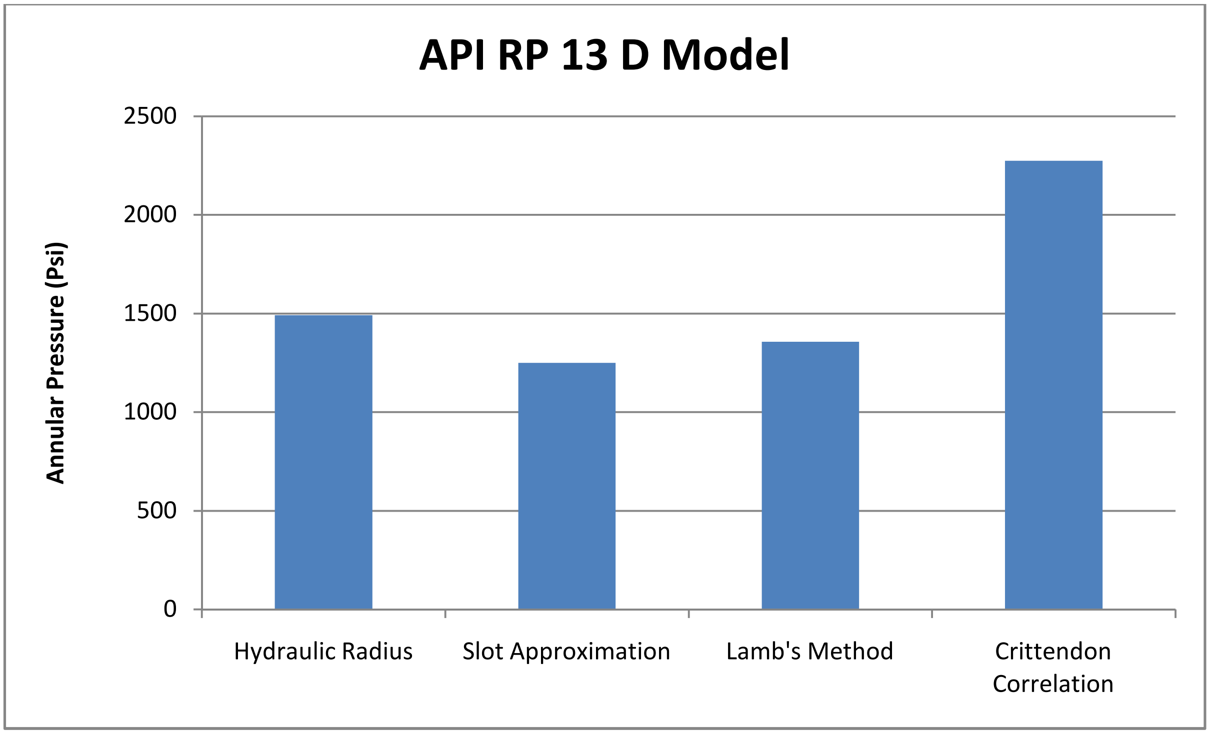



Fig. 4.6: Effect of Equivalent Annular Diameter on Annular Pressure for API RP 13D Model

**Appendix B: Some MATLAB® codes**

Rubiandini Cuttings Transport Model

```
%% Program CuttingsTransport_Rubidiani.m
%%% This Program evaluates the transport of cuttings from the wellbore %%%%
%%%%%% as a function of the wellbore geometry from the vertical to %%%%%%%%
%%%%%%%%%% the horizontal using the Rudi Rubiandini's Model
%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SPE 143675
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 function Rubiandini =
CuttingsTransport_Rubiandini(sections,rheology,cuttings,fluid_choice,edc)
% Some initializations
Rho = rheology.Rho;
theta300 = rheology.theta300;
theta3 = rheology.theta3;
theta600 = rheology.theta600;
 ROP = cuttings.ROP;
 RPM = cuttings.RPM;
 Rho_s = cuttings.Rho_s;
 d_cut = cuttings.d_cut;
   for i=1:length(sections)
       dpi = sections(i).dpi;
       Lc = sections(i).Lc;
       theta = sections(i).theta;
       Lp = sections(i).Lp;
       dci = sections(i).dci;
       dhole_casing = sections(i).dhole_casing;
       dep = edc.dep(i); dec = edc.dec(i) ;
   eps = 0.001;
   if Lp ~= 0 && Lc == 0
   V_cut_pipe = ROP/(36*(1-(dpi/dhole_casing)^2)*(0.64 + 18.16/ROP));
   Vs = 0.1; Vsv = 0.5;
   while abs(Vsv-Vs)> eps
       V_min_pipe = V_cut_pipe + Vs;
        if fluid_choice == 1 % Newtonian fluid
           mhu_apparent = theta300;
           Rep = 928*Rho*d_cut*Vs/mhu_apparent;
          if Rep < 3
             f = 40/Rep;
             Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
```

```
  elseif 3 < Rep < 300
     f = 22/(sqrt(Rep));
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  else
     f = 1.54;
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  end
  Vs = 0.5*(Vsv+Vs);
elseif fluid_choice==2 % Powerlaw fluid
   n = 3.32* log10((theta600/theta300));
   K = 5.11*theta600/(1022^n);
  mhu_apparent_pipe = 100*K*((144*V_min_pipe/dep)^(n-1));
  Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
  if Rep < 3
     f = 40/Rep;
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  elseif 3 < Rep < 300
     f = 22/(sqrt(Rep));
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  else
     f = 1.54;
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  end
  Vs = 0.5*(Vsv+Vs);
elseif fluid_choice==3 % API RP 13D 2010
   n_ann = 0.5* log10((theta300/theta3));
   K_ann = 5.11*theta300/(511^n_ann);
  mhu_apparent_pipe = 100*K_ann*((144*V_min_pipe/dep)^(n_ann-1));
  Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
  if Rep < 3
     f = 40/Rep;
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  elseif 3 < Rep < 300
     f = 22/(sqrt(Rep));
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  else
     f = 1.54;
     Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
  end
  Vs = 0.5*(Vsv+Vs);
```

```
    elseif fluid_choice == 4 % Bingham Plastic fluid
      PV = theta600-theta300;
      YP = 2*theta300-theta300;
      mhu_apparent_pipe = PV + (5*YP*dep)/V_min_pipe;

     Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
     if Rep < 3
        f = 40/Rep;
        Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
     elseif 3 < Rep < 300
        f = 22/(sqrt(Rep));
        Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
     else
        f = 1.54;
        Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
     end
     Vs = 0.5*(Vsv+Vs);
    end
end
elseif Lc~=0 && Lp == 0
   V_cut_collar = ROP/(36*(1-(dci/dhole_casing)^2)*(0.64 + 18.16/ROP));
   Vsc = 0.1; Vsvc = 0.5;
   while abs(Vsvc-Vsc)> eps
      V_min_collar = V_cut_collar + Vsc;
       if fluid_choice == 1 % Newtonian fluid
         mhu_apparent = theta300;
         Rec = 928*Rho*d_cut*Vsc/mhu_apparent;
        if Rec < 3
           fc = 40/Rec;
           Vsv = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        elseif 3 < Rec < 300
           fc = 22/(sqrt(Rec));
           Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        else
           fc = 1.54;
           Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vsc = 0.5*(Vsvc+Vsc);
       elseif fluid_choice==2 % Powerlaw fluid
         n = 3.32* log10((theta600/theta300));
```

```
 K = 5.11*theta600/(1022^n);
mhu_apparent_collar = 100*K*((144*V_min_collar/dec)^(n-1));
Rec = 928*Rho*d_cut*Vs/mhu_apparent_collar;
if Rec < 3
   fc = 40/Rec;
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
elseif 3 < Rec < 300
   fc = 22/(sqrt(Rec));
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
else
   fc = 1.54;
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
end
Vsc = 0.5*(Vsvc+Vsc);
  elseif fluid_choice==3 % API RP 13D 2010
 n_ann = 0.5* log10((theta300/theta3));
 K_ann = 5.11*theta300/(511^n_ann);
mhu_apparent_collar = 100*K_ann*((144*V_min_collar/dec)^(n_ann-1));
Rec = 928*Rho*d_cut*Vs/mhu_apparent_collar;
if Rec < 3
   fc = 40/Rec;
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
elseif 3 < Rec < 300
   fc = 22/(sqrt(Rec));
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
else
   fc = 1.54;
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
end
Vsc = 0.5*(Vsvc+Vsc);
elseif fluid_choice ==4 % Bingham Plastic fluid
PV = theta600-theta300;
YP = 2*theta300-theta300;
mhu_apparent_collar = PV + (5*YP*dec)/V_min;
Rec = 928*Rho*d_cut*Vs/mhu_apparent_collar;
if Rec < 3
   fc = 40/Rec;
   Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
elseif 3 < Rec < 300
   fc = 22/(sqrt(Rec));
```

```
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          else
            fc = 1.54;
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          end
          Vsc = 0.5*(Vsvc+Vsc);
        end
      end
    elseif Lp~=0 && Lc~=0

      V_cut_pipe = ROP/(36*(1-(dpi/dhole_casing)^2)*(0.64 + 18.16/ROP));
      Vs = 0.1; Vsv = 0.5;
      while abs(Vsv-Vs)> eps
        V_min_pipe = V_cut_pipe + Vs;
         if fluid_choice == 1 % Newtonian fluid
           mhu_apparent = theta300;
           Rep = 928*Rho*d_cut*Vs/mhu_apparent;
          if Rep < 3
            f = 40/Rep;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
          elseif 3 < Rep < 300
            f = 22/(sqrt(Rep));
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
          else
            f = 1.54;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
          end
          Vs = 0.5*(Vsv+Vs);
         elseif fluid_choice==2 % Powerlaw fluid
           n = 3.32* log10((theta600/theta300));
           K = 5.11*theta600/(1022^n);
          mhu_apparent_pipe = 100*K*((144*V_min_pipe/dep)^(n-1));
          Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
          if Rep < 3
            f = 40/Rep;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
          elseif 3 < Rep < 300
            f = 22/(sqrt(Rep));
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
          else
```

```
            f = 1.54;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vs = 0.5*(Vsv+Vs);
    elseif fluid_choice==3 % API RP 13D 2010
         n_ann = 0.5* log10((theta300/theta3));
         K_ann = 5.11*theta300/(511^n_ann);
        mhu_apparent_pipe =100*K_ann*((144*V_min_pipe/dep)^(n_ann1));
        Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
        if Rep < 3
            f = 40/Rep;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        elseif 3 < Rep < 300
            f = 22/(sqrt(Rep));
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        else
            f = 1.54;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vs = 0.5*(Vsv+Vs);
    elseif fluid_choice==4 % Bingham Plastic fluid
         PV = theta600-theta300;
         YP = 2*theta300-theta300;
         mhu_apparent_pipe = PV + (5*YP*dep)/V_min_pipe;
        Rep = 928*Rho*d_cut*Vs/mhu_apparent_pipe;
        if Rep < 3
            f = 40/Rep;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        elseif 3 < Rep < 300
            f = 22/(sqrt(Rep));
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        else
            f = 1.54;
            Vsv = f* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vs = 0.5*(Vsv+Vs);
    end
end
V_cut_collar = ROP/(36*(1-(dci/dhole_casing)^2)*(0.64 + 18.16/ROP));
Vsc = 0.1; Vsvc = 0.5;
```

```
while abs(Vsvc-Vsc)> eps
    V_min_collar = V_cut_collar + Vsc;
     if fluid_choice == 1 % Newtonian fluid
         mhu_apparent = theta300;
         Rec = 928*Rho*d_cut*Vsc/mhu_apparent;
        if Rec < 3
            fc = 40/Rec;
            Vsv = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        elseif 3 < Rec < 300
            fc = 22/(sqrt(Rec));
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        else
            fc = 1.54;
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vsc = 0.5*(Vsvc+Vsc);
     elseif fluid_choice==2 % Powerlaw fluid
         n = 3.32* log10((theta600/theta300));
         K = 5.11*theta600/(1022^n);
        mhu_apparent_collar = 100*K*((144*V_min_collar/dec)^(n-1));
        Rec = 928*Rho*d_cut*Vsc/mhu_apparent_collar;
        if Rec < 3
            fc = 40/Rec;
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        elseif 3 < Rec < 300
            fc = 22/(sqrt(Rec));
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        else
            fc = 1.54;
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
        end
        Vsc = 0.5*(Vsvc+Vsc);
     elseif fluid_choice==3 % API RP 13D 2010
         n_ann = 0.5* log10((theta300/theta3));
         K_ann = 5.11*theta300/(511^n_ann);
   mhu_apparent_collar =100*K_ann*((144*V_min_collar/dec)^(n_ann-1));
        Rec = 928*Rho*d_cut*Vsc/mhu_apparent_collar;
        if Rec < 3
            fc = 40/Rec;
            Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
```

```
          elseif 3 < Rec < 300
             fc = 22/(sqrt(Rec));
             Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          else
             fc = 1.54;
             Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          end
          Vsc = 0.5*(Vsvc+Vsc);
        elseif fluid_choice ==4 % Bingham Plastic fluid
         PV = theta600-theta300;
          YP = 2*theta300-theta300;
          mhu_apparent_collar = PV + (5*YP*dec)/V_min_collar;
          Rec = 928*Rho*d_cut*Vsc/mhu_apparent_collar;
          if Rec < 3
             fc = 40/Rec;
             Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          elseif 3 < Rec < 300
             fc = 22/(sqrt(Rec));
             Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          else
             fc = 1.54;
             Vsvc = fc* sqrt(d_cut*(Rho_s - Rho)/Rho);
          end
          Vsc = 0.5*(Vsvc+Vsc);
       end
    end
end
if theta <= 45
   V_slip_pipe = Vsv*(1+ theta*(600-RPM)*(3+ Rho))/202500;
   V_min_pipe = V_cut_pipe + V_slip_pipe;
   V_slip_collar = Vsvc*(1+ theta*(600-RPM)*(3+ Rho))/202500;
   V_min_collar = V_cut_collar + V_slip_collar;
   V_min = V_min_pipe + V_min_collar;
elseif theta >= 45
   V_slip_pipe = Vsv*(600-RPM)*(3+ Rho)/3000;
   V_min_pipe = V_cut_pipe + V_slip_pipe;
   V_slip_collar = Vsvc*(600-RPM)*(3+ Rho)/3000;
   V_min_collar = V_cut_collar + V_slip_collar;
else
   disp('Make a right selection for angle of inclination')
```

```
    end
      V_min(i,1) = V_min_pipe;V_min(i,2) = V_min_collar;
    end
    Rubiandini.V_min(i,:)=V_min(i,:);
end
```

Equivalent Annular Diameter Definitions

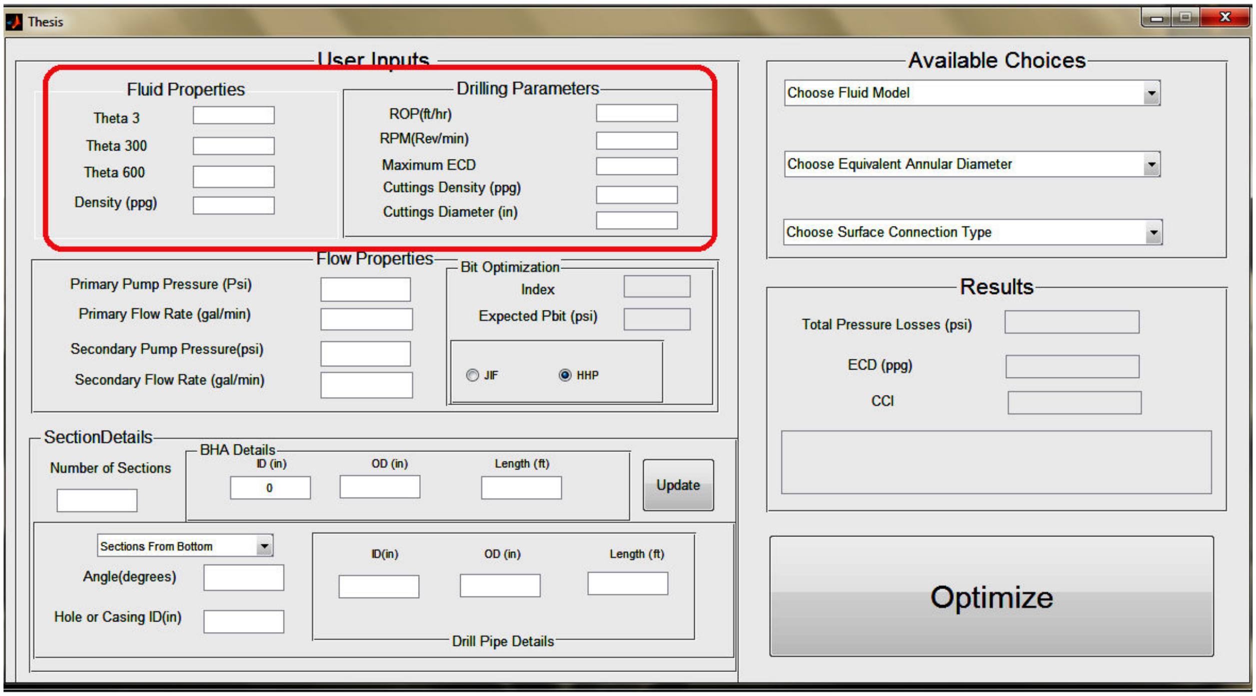
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%
% get_edc.m - March,2013                              %
% Author: Babawale, Olatunde Paul                      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%
%--------------------------------------------------------------------
%% Program get_edc.m
% Purpose: This program evaluates the equivalent annular diameter and
% annular velocity based on the user's choice

function edc = get_edc(sections, q,DiameterChoice)
for i=1:length(sections)
    dpi = sections(i).dpi;
    dpo = sections(i).dpo;
    Lp = sections(i).Lp;
    dci = sections(i).dci;
    dco = sections(i).dco;
    dhole_casing = sections(i).dhole_casing;
    Lc = sections(i).Lc;
    if DiameterChoice==1
    disp('HYDRAULIC RADIUS APPROXIMATION')
    dep = dhole_casing - dpo;
    dec = dhole_casing - dco;
      if Lp~=0 && Lc~=0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
        V_dp = (0.408*q)/(dpi^2);
        V_dc = (0.408*q)/(dci^2);
      elseif Lp ~=0 && Lc == 0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = 0;
        V_dp = (0.408*q)/(dpi^2);
        V_dc = 0;
      elseif Lp ==0 && Lc~=0
        Va_dp =0;
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
        V_dp = 0;
        V_dc = (0.408*q)/(dci^2);
```

```
    end
elseif DiameterChoice==3
disp('GEOMETRIC TERM CRITERION')
dep = sqrt((dhole_casing^2) + (dpo^2)-(((dhole_casing^2) - ...
    (dpo^2))/log(dhole_casing/dpo)));
dec = sqrt((dhole_casing^2) + (dco^2)-(((dhole_casing^2) - ...
    (dco^2))/log(dhole_casing/dco)));
    if Lp~=0 && Lc~=0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
        V_dp = (0.408*q)/(dpi^2);
        V_dc = (0.408*q)/(dci^2);
    elseif Lp ~=0 && Lc == 0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = 0;
        V_dp = (0.408*q)/(dpi^2);
        V_dc = 0;
    elseif Lp ==0 && Lc~=0
        Va_dp =0;
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
        V_dp = 0;
        V_dc = (0.408*q)/(dci^2);
    end
elseif DiameterChoice==2
disp('SLOT REPRESENTATION')
dep = 0.816*((dhole_casing) - (dpo));
dec = 0.816*((dhole_casing) - (dco));
    if Lp~=0 && Lc~=0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
        V_dp = (0.408*q)/(dpi^2);
        V_dc = (0.408*q)/(dci^2);
    elseif Lp ~=0 && Lc == 0
        Va_dp = (0.408*q)/((dhole_casing^2) - (dpo^2));
        Va_dc = 0;
        V_dp = (0.408*q)/(dpi^2);
        V_dc = 0;
    elseif Lp ==0 && Lc~=0
        Va_dp =0;
        Va_dc = (0.408*q)/((dhole_casing^2) - (dco^2));
```

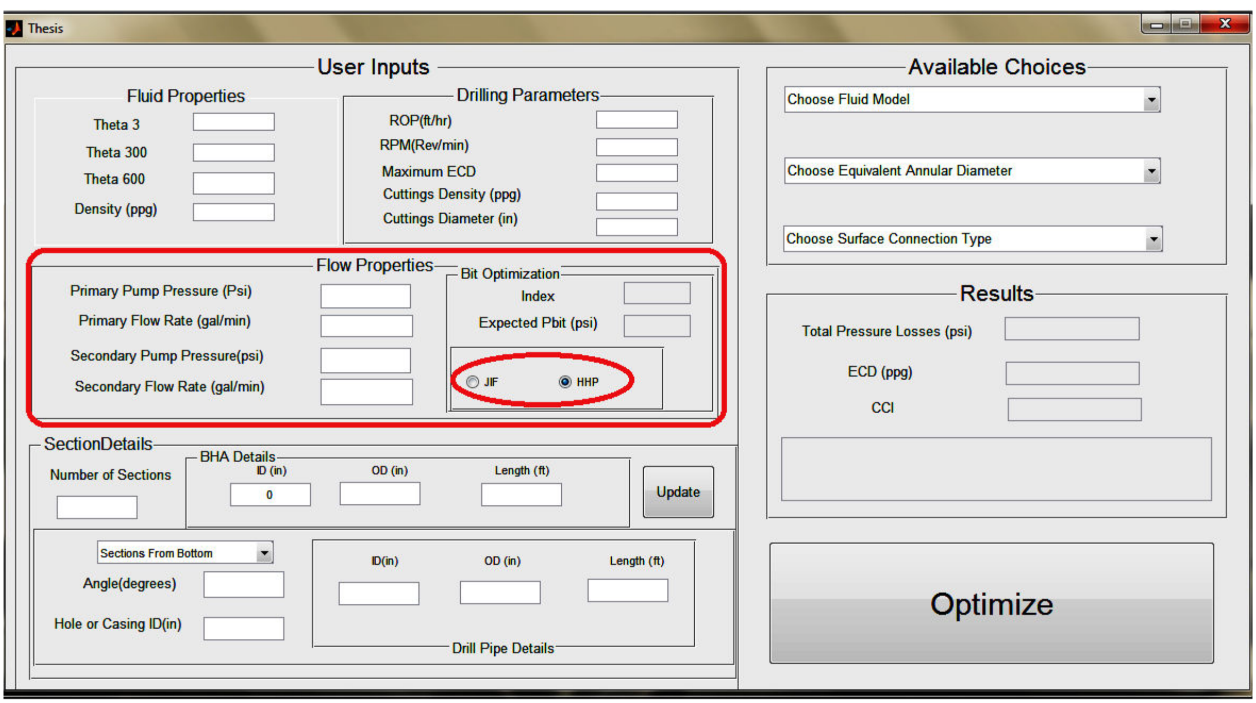
```
            V_dp = 0;
            V_dc = (0.408*q)/(dci^2);
          end
        elseif DiameterChoice==4
        disp('CRITTENDON CORRELATION')
        dep = 0.5*(sqrt((dhole_casing^2) - (dpo^2)) + ((dhole_casing^4) - (dpo^4)-...
          ((((dhole_casing^2) - (dpo^2))^2)/log(dhole_casing/dpo)))^0.25);
        dec = 0.5*(sqrt((dhole_casing^2) - (dco^2)) + ((dhole_casing^4) - (dco^4)-...
          ((((dhole_casing^2) - (dco^2))^2)/(log(dhole_casing/dco))))^0.25);
           if Lp~=0 && Lc~=0
            Va_dp = (0.408*q)/(dep^2);
            Va_dc = (0.408*q)/(dec^2);
            V_dp = (0.408*q)/(dpi^2);
            V_dc = (0.408*q)/(dci^2);
          elseif Lp ~=0 && Lc == 0
            Va_dp = (0.408*q)/(dep^2);
            Va_dc = 0; V_dc = 0;
            V_dp = (0.408*q)/(dpi^2);
          elseif Lp ==0 && Lc~=0
            Va_dp =0; V_dp = 0;
            Va_dc = (0.408*q)/(dec^2);
      V_dc = (0.408*q)/(dci^2);
          end
        end
        edc.dep(i) = dep; edc.dec(i) = dec; edc.V_dp(i) = V_dp;
    edc.V_dc(i) = V_dc; edc.Va_dp(i) = Va_dp; edc.Va_dc(i) = Va_dc;
    end
    end
```

## Appendix C: The Computer Program

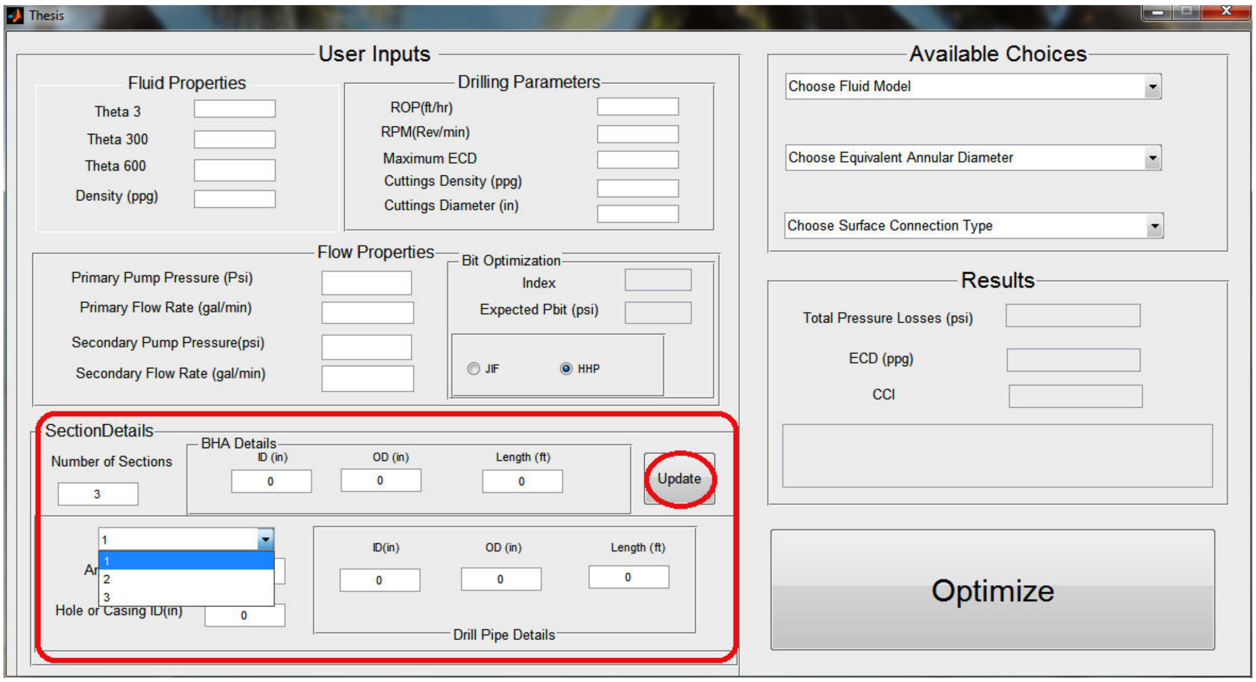
1. Fill in the values of the fluid properties and drilling parameters



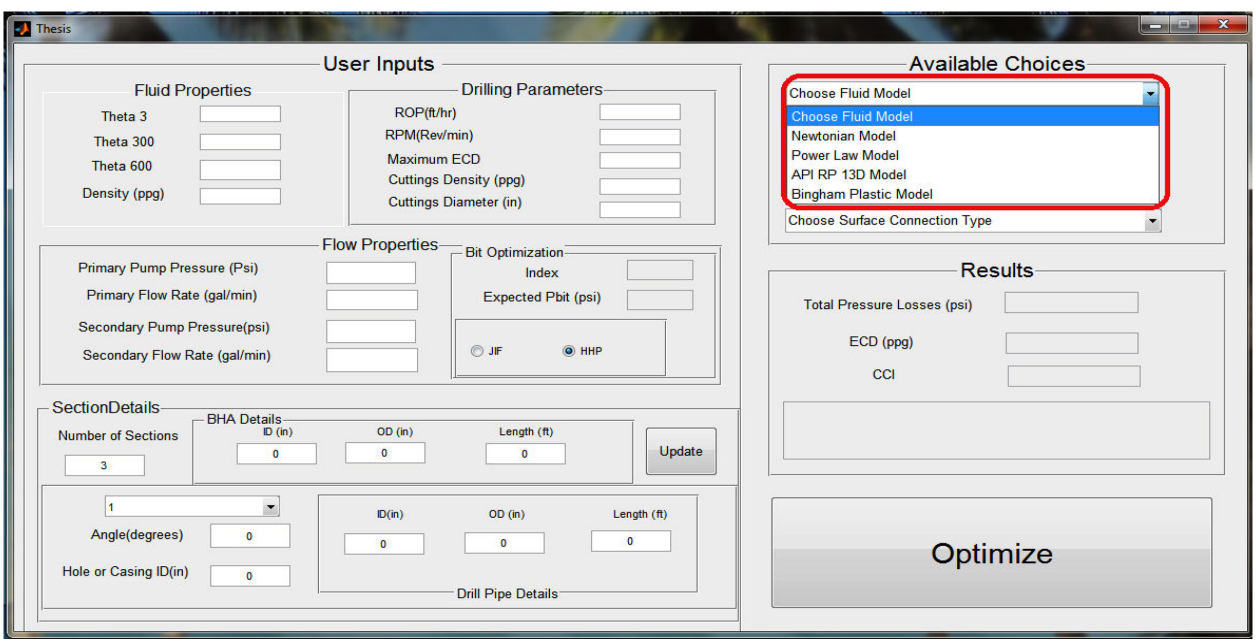

2. Enter values for two Pump and Flow Conditions. Make selection of bit optimization criterion. The optimization index and expected pressure drop across bit are displayed.



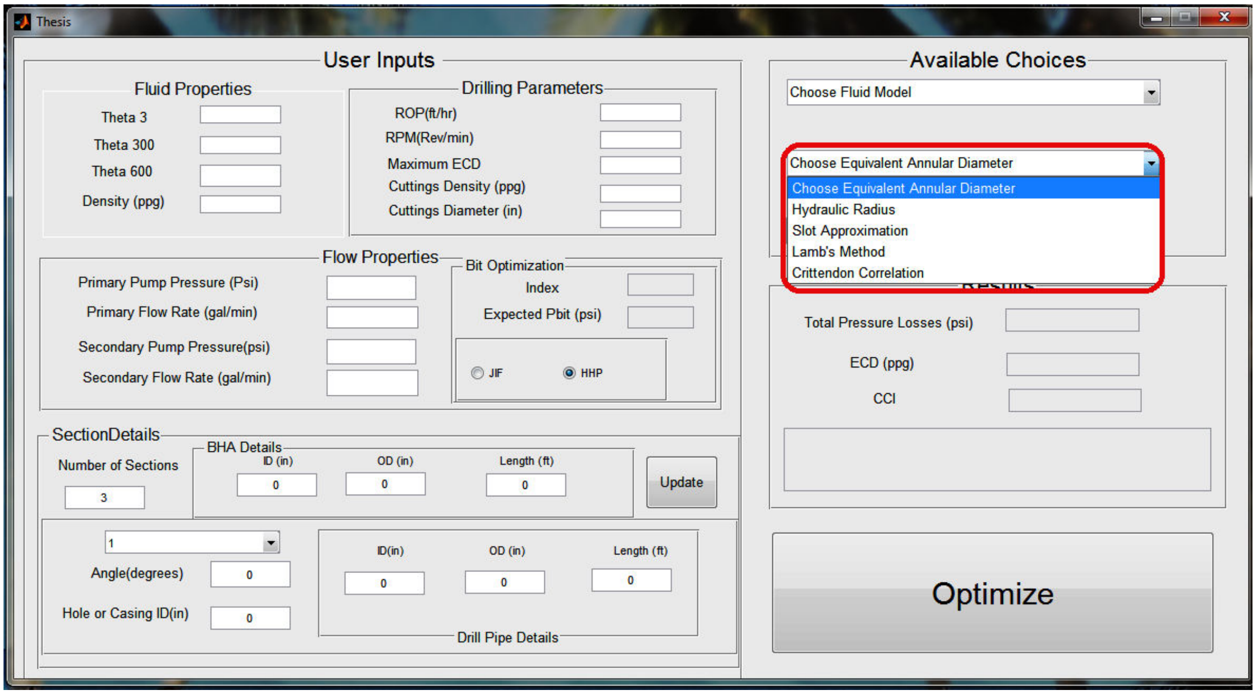
3. Fill in the details of the wellbore profile by following these steps
   - Enter the number of sections
   - Select a section, and fill in the hole and drillstring parameters
   - Click the “Update” button
   - Fill in details for other sections, clicking “Update” at the end of each section entry



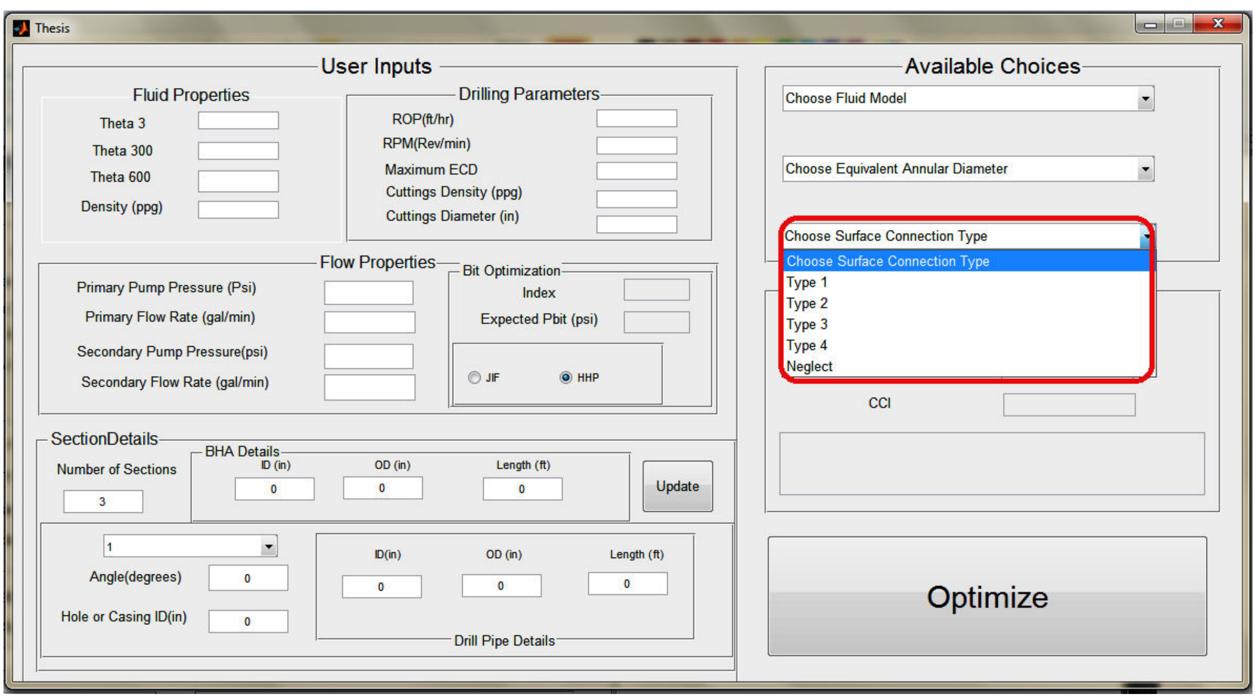

4. Make a choice of rheological model: Newtonian, Power Law, API RP 13D, and Bingham Plastic models.



5. Make a choice of equivalent annular diameter: Hydraulic Radius, Slot Approximation,




6. Make a choice of surface connection type or select neglect to ignore surface connection type.



7. Click the “Optimize” button