ANALYSIS OF REAL OPTION APPROACHES FOR OILFIELD VALUATIONS

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By

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A THESIS APPROVED BY THE PETROLEUM ENGINEERING DEPARTMENT

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ABSTRACT

More than three decades have passed since Stewart Myers coined the term "real options" and introduced this new paradigm into the investment science. In the following years a number of publications in both the finance and decision analysis literature hailed the benefits and power of Real Options for corporate investment decisions. In 2001 Copeland stated that "In ten years, real options will replace NPV (Net Present Value) as the central paradigm for investment decisions." Now, eighteen years hence, the dominating valuation paradigm in the exploration and production industry still seems to be the classical NPV approach. Why?

In the time period following the initial applications of real option valuation to oil & gas investments, a number of approaches were proposed for calculating the value of an uncertain investment. Unfortunately, the assumptions underlying these approaches and the conditions that are appropriate for their application are often not spelled out. Where they are spelled out or can be inferred, they differ from approach to approach and are often contradictory.

In many instances, oil companies struggle with decisions pertaining to petroleum investment. The difficulty partially stems from the uncertainties in many of the inherent variables. Furthermore, conventional investment methods often fail to properly identify available opportunities.

As commonly acknowledged, traditional valuation methods such as Discounted Cash Flow (DCF) and Net Present Value (NPV) analyses are unable to properly portray investment opportunities. Due to large uncertainties and hence risk in Petroleum Exploration and Production (E & P), investors are gradually turning to a more dynamic approach to investment decisions.

Real Options Valuation involves a methodology for evaluating the value of an opportunity, leading to a strategic decision in an uncertain environment. Based on academic research in the finance and business management, Real Options Valuation may be extended from option-pricing tools of the finance sector to that of evaluating E & P projects. In other words, although Real Options thinking has been widely accepted and used in some cases, the wider use of the Real Options approach is still a "hot" debate in the Petroleum Industry.

A permissible definition of "Real Options" may lead to inconsistencies among Real Options approaches. As such, Real Options may be defined as a company having a right, not an obligation, to invest in a future opportunity. The opportunity may involve technical aspects or may be purely commercial in nature. In all cases, a quantitative approach is required. In the work by Borison (2003) and Bratvold et al (2005), the authors have listed five Real Option methodologies: the Classic approach, the Subjective approach, the Market Disclaimer approach (MAD), The Smith approach, and the Luenberger approach. A comparative analysis of these Real Option approaches is presented in this thesis.

In comparing the above-mentioned Real Options approaches, it is apparent that two types of Uncertainties may be considered: technical and market. In the study presented, an offshore petroleum lease project was discussed using the real options approaches.

This research integrates the finance option theory with petroleum engineering projects, as well as project management. As such, it is shown that the petroleum industry could benefit from using the Real Options Valuation in their investment strategy, thus improving business performance.

Keywords: real options, real option approaches, filed development, net present value, decision analysis, marketed asset disclaimer, integrated real option valuation approach.

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Last and most importantly, I would like to take this chance to my family for their understanding and support during my study. Without them, this thesis would not have been possible.

DEDICATION

I dedicate this thesis study to ***My Beloved Mum*** and lovely siblings.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background

The importance of project valuation has been recognized long ago in petroleum science literatures. This is because valuation constitutes an important step during the project development stage. Sound project valuation can inform the firm to make sound investment decisions which ultimately create value. In the literature, there are a wide variety of valuation approaches that attempt to determine the value of an investment. From the traditional Discounted Cash Flow (DCF) techniques to the modern Real Option Analysis (ROA), continuing efforts have been made to develop better methodologies and to support decision making. Practices in many industries also reflect this kind of diversity and progress.

Prior to the 1960s, Payback and Accounting Return on investment were the two primary investment evaluating methods used by large firms (Bierman & Smidt, 2007). In 1951, two books "Capital Budgeting" (Dean, 1951) and "The Theory of Investment of the Firm" (Lutz & Lutz, 1951) were published that opened the door to new managerial techniques for evaluating investments using DCF methods (Bierman & Smidt, 2007). Discounted Cash Flow (DCF) based approaches, such as Net Present Value (NPV), are quite simple and easy to understand. Typically, they predict a stream of cash flows (both cash inflow and cash outflow) over the expected life of a project, discount them at a rate that reflects both the time value of money and the riskiness of those cash flows, and then calculate a net present value. The investment decision rule is then based on simple logic: given two mutually exclusive projects, the one with the larger NPV should be preferred. Also, any project with appositive NPV should be viewed as a good investment. Despite the popularity and simplicity of DCF approaches, they are only acceptable if the expected future prevails. However, they are inherently flawed when analyzing projects with high levels of uncertainty. In these cases, the realized future cash flows may be vastly different from prior expectations. Hence, instead of simply coming up with the value of the project, decision makers must expect that the project will have to serve any one of a range of possibilities, and manage uncertainties proactively (Wang, 2005). In fact, by considering the effect of uncertainty on the valuation process, a growing body of recent research shows that some of the most important aspects of most capital investments are in fact the timing of the investment and the managerial flexibility involved. Thus, it is imperative that a richer framework be established, one that enables decision makers to better understand the effect of uncertainty and to address the issues of managerial flexibility and investment timing more directly.

Indeed, Real Option Analysis has received considerable attention in the petroleum economics literature in recent years. The concept of a real option was developed, at least partly, as a response to the inadequacy of the traditional DCF approaches for the valuation of projects under uncertainty. By using the methods provided by financial option pricing theory, ROA allows analysts to account for traditionally non-easily quantifiable elements such as managerial flexibility and strategic interventions during the development of an investment (Ramirez, 2002). Relative to a passive managerial strategy, in a world in which unexpected change is a rule, an investment strategy that incorporates managerial flexibility in decision-making will respond most efficiently to various possible futures. In many circumstances, if option-like features such as deferment or staged investment are embedded in an investment opportunity, then quantifying the value of these features is significant in the valuation of the investment. Hence, ROA constitutes an advanced way of recognizing how projects are structured and managed, and incorporating this into the valuation method.

Compared to traditional methods, the Real Options pricing model (Real Options Valuation) should be more appealing. The attraction of Real Options Valuation lies not in making the estimated project value more accurate, but in taking the strategy (flexibility) value of the project into account. When decision makers face significant uncertainty, strategic and flexible management can dramatically reduce risks, albeit not totally eliminate them. This flexibility, the result of uncertainty, acted upon quickly and decisively, is able to add significant value to a project. Real option thinking, thus, assigns a value for uncertainty. The more uncertainty, the more flexibility the project has, and thus, the greater the possible upside in project. In other words, flexibility has monetary worth. The Real Options Approach (ROA) thus concentrates on evaluating the monetary worth of flexibility.

Most Real Options approaches have three stages: the identification of uncertainties, the identification of Real Options, and the valuation of flexibilities. The identification of uncertainties involves the recognition of key sources of uncertainty. In principle, there are many sources of uncertainty in petroleum investments, but the two key uncertainties are technical uncertainty and market uncertainty. The second stage of Real Options approach is to understand the types of flexibility or options. If the flexibility comes by solving technical problems, technical data is applied. In contrast, if the flexibility is related to a market situation, market data would be employed. The third stage is then evaluating, via logic and mathematics, the monetary worth of the established flexibility.

1.2 Objectives of the Study

The main objective of this study is to review the various Real Options approaches.

The specific objectives are:

- To enhance the current thinking about real option evaluation techniques for oilfield valuations.
- To develop intuition about the important concepts behind real options approaches and their application condition.
- To examine and compare the conceptual underpinnings of real option approaches.
- To clarify some of the confusion about the choice of the valuation approaches and improve strategic decision making.
- To make recommendations on how these approaches would be of great benefit for oilfield development process.

1.3 Statement of Problem

In the previous works done by many researchers, a number of real option approaches were proposed for calculating the value of an uncertain investment. Unfortunately, the assumptions underlying these various approaches and the conditions that are appropriate for their application are often not spelled out. Where they are spelled out or can be inferred, they differ widely from approach to approach and are even contradictory. Furthermore, the difficulties in implementing these approaches are rarely discussed and the pros and cons of alternative approaches are not explained. This thesis will emphasize on all the aspects of various Real Options approaches by applying “theoretical precision” in the definition of “Real Options” and comparing their valuation frameworks and their models since it was not taken into consideration by the many researchers who discussed the Real Options analysis in oilfield development.

1.4 Organization of the Thesis

The thesis is structured in 5 parts. Chapter 1 unveils the study and gives the background, statement of problems, purpose and objectives of the thesis, and the organization of the thesis. Chapter 2 reviews the literature of previous work done by other researchers on the subject matter. Chapter 3 illustrates five Real Options approaches. Chapter 4 applies the selected real option approaches to value an offshore petroleum lease project. Chapter 5 concludes and gives recommendation followed by references and appendices.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Overview of Investments in the Petroleum Industry

Petroleum exploration and development investment is a high–risk business, involving capital–intensive physical assets, as well as long cycle times. There are numerous, complex geological and market uncertainties that have led E & P investments to be at the more risky end of the investment spectrum.

2.2 Uncertainties in the E & P Industry

Uncertainty denotes a lack of surety. In the uncertain world, investment decisions largely depend on their assessed value. Project valuation, however, is an estimate of the future value derived from numerous uncertain variables. An inappropriate estimate of petroleum project valuation could cause project managers to make a poor decision, leading, in turn, to potential large financial losses for petroleum companies, or to the failure to capture possible upside value. Hence, it is essential for project managers to understand uncertainties in the petroleum industry. Proper understanding has the potential to bring project valuation closer to real market value.

In addition to such possible downsides, uncertainty can bring unexpected opportunities where greater uncertainty gives rise to greater opportunity. Positive opportunities could create benefits for investors. In investment terms, uncertainty allows investors to not only reduce risks but also have the chance of create more upside opportunities based on the work of Rose (2011), some of the more common uncertainties in the E & P industry are discussed below:

2.2.1 Market Uncertainty

Market uncertainty in the petroleum industry mainly results from price volatility. Historically, oil and gas prices have exhibited significant volatility due uncertainty in political situations, and the balance between supply and demand. Viewed from any instant in time, market uncertainty increases with time.

2.2.2 Technical Uncertainty

Technical uncertainties may be exploration and reserve related – involving the hydrocarbon pool size, shape, distribution, pressure, and recovery characteristics etc – as well as filed development uncertainties (involving aquifers, field production rate, and field decline rate). The first category is related primarily to uncertainty in physical characteristics, which are present but not well defined, while the latter relates to man-made implementation which may be well defined a priori but has uncertainty as far as implementation. Although extensive scientific and geotechnical work is indeed essential for successful modern petroleum exploration, it must also be recognized that nearly all the parameters required to assign an expected monetary value to an exploration prospect can only result in an estimate, made under substantial uncertainty (Rose, 2001). The large magnitude of geological uncertainty can initially be reduced by paying for further data or information. Technical uncertainty tends to decrease with time, and as such technical uncertainty may be primarily associated with cost uncertainty.

2.2.3 Cost Uncertainty

Costs associated with any petroleum investment consist of two parts, capital expenditures (Capex) and operational expenditures (Opex), Capex uncertainty arises from uncertainties in future prices of exploration and production equipment or associated project implementation services, such as production facilities and pipelines, and their installation or construction. These costs are estimated with future pricings, and hence, by definition, are uncertain. Opex may include field as well as variable costs. In many cases, fixed costs, such a management costs and maintenance costs, are less flexible, and can be predicted with relative certainty. But variable costs, such as material costs, storage costs, storage costs vary throughout the project life.

2.2.4 Timing Uncertainty

Scheduling and timing uncertainty are inherent in all projects and activities, the result of many influencing factors. All are contingent on the various uncertainties previously discussed. Timing uncertainty impacts project valuation in two ways. First, there is the actual time within the valuation cycle that events are predicted to occur, for example, time to first production, whilst, the second is related to the time value of money. Megill (1988) explained that when a corporation invests in petroleum exploration ventures, they are anticipating the receipt of a series of future annual cash flow revenues. As Ross (2001) has pointed out, assessing the value of such future cash flows requires understanding of the time value of money, especially the concepts of the further value of money, compounding, present value as well as discounting. The project lifetime is directly related to the time value of money.

2.3 Risks in the E & P Industry

2.3.1 High Risks in the E & P Industry

Risk is defined as the potential threat of loss in a petroleum investment (Bratvold et al 2002). The threat of loss is usually measured as monetary investment i.e. capital. The capital loss results, therefore, directly from numerous uncertainties. Like uncertainties, risk cannot be eliminated, but can be reduced and then managed.

Obviously, risks associated with petroleum investments are very high. The outcome of a project is subject to the valuation process, the implementation and change; hence a “bad” outcome can arise from poor estimates as well as “bad” timing about when to deal with uncertainties. Even a “good” decision can bring possible losses for the company because of chance. Bratvold et al (2002) fully elaborated on how a “good” decision can lead to an “outcome in decision – making.” Poor estimates derive from an under or over estimate of the uncertainties previously discussed. It is difficult for decision makers to make profitable decision and precisely predict project returns under uncertainty. Hence, uncertainty and decision quality do not have a linear relationship (Bratvold et al 2002).

On the other hand, decisions would often be made without having all of the information. Lack of information causes a major threat to investment loss. Firstly, lack information for price trends can result in a project with increased price downside risks. Historically, it has been difficult for managers to forecast price trends. Secondly, as mentioned previously, the estimation of hydrocarbon prospect size involves many estimate parameters, each with its own uncertainty, not mention various options in evaluation tools. Exploration professionals generally used probability in estimating the chance, as well as potential size, or for a potential discovery define the probability of discovering a certain volume at different confidence levels, typically “proved, or P90”, “proved plus probable, or P50”, and “proved plus probable plus possible, or P10”. The use of such probabilities may bring significant estimation errors into project valuation and hence potential monetary loss in exploration prospect investment.

2.3.2 Two Types of Risks

There are two types of risks in the oil and gas industry: market and private (Bratvold, 2002). Market risk is the potential loss that arises from oil (or gas) market uncertainty, where price volatility is a main risk factor. Worldwide oil production directly impacts the oil price. All oil and gas companies re exposed to price volatility. Thus, price volatility is critical for economic returns, and it is crucial for project valuation. Furthermore, potential loss due to market uncertainty also arises from supply and demand situations associated with world economic growth.

Private risk, on the other hand, is associated to potential loss that is related to the unique nature of each project. Different projects have different private risks. In the petroleum industry, private potential loss results from uncertainty of the interplay of geological uncertainty, engineering technology uncertainty, as well as corporate business strategies.

2.4 Types of Projects and Options in E & P Projects

2.4.1 Types of Oil and Gas Projects

A typical petroleum project may be divided into three "phases": the exploration phase, the development and production (or exploitation) phase, and concurrently with the latter, the marketing phase (Picoult, 2002).

The exploration phase typically involves seismic surveys and drilling of prospects etc companies conduct these activities to collect geological information of hydrocarbon prospects. High capital costs are inherent in the exploration phase. The drilling of individual offshore prospects costs millions of US dollars for deep-water projects. With investment of such large capital amounts, companies face a high threat of capital loss, the direct result of large geological uncertainty inherent in this phase. Due to poor exploration results, such as a series of "dry holes", companies frequently abandon further exploration of a region.

If exploration results, however, are favorable, a company may proceed to the development phase. This phase involves feasibility studies, appraisal and development drilling, and facilities design and construction. In the development phase, the company installs the necessary field production facilities, such as separators, lifting devices and metering equipment for production of hydrocarbons. The aim is to optimize the overall project, result in the best financial returns. Marketing involves downstream activities, related to refining, selling, and transporting of hydrocarbons. Such activities are very susceptible to price volatility.

Table 2.1 Summarizes typical project aspects, their uncertainties and risks.

2.4.2 Option in E & P Project

As mentioned above, uncertainties lead to possible downsides or risk, but the former also create opportunities related to flexibilities in project investment. **Table 2.1** below summarizes typical aspects, their uncertainties and risks. As ***Table 2.1*** show, projects in the petroleum business may have plenty of flexibilities. First, flexibilities derive from technical uncertainties. Technical flexibility is an opportunity that managers create to solve technical problems. Technical flexibility can add to the project value. Technical uncertainty decreases with time. For an oil or gas lease in the exploration phase, the technical uncertainty is dominant. And thus, there is much technical flexibility; these flexibilities are aimed at increasing the value associated with obtaining new information. The flexibilities are created as the company seeks to reveal more perfect reservoir informant by taking advantage of timing and uncertainty. Once, the value of information increases, the project associated with the project will also increase.

Second, when the company proceeds to the marketing phase, market uncertainty is significant. Market uncertainty increases with time. Oil prices will determine the project value, which in turn reflects the value of the underlying asset to the company. The value of oil reserves needs to be converted from a physical asset to a monetary asset. During the conversion, flexibilities derive largely form market uncertainties, and thus the oil price fluctuation. The company can make use of the oil price volatility to obtain the convenience yield and the option value, and hence, obtains more profits.

Table 2.1: Uncertainty and Risk Categories of Petroleum Industry Projects

Phase or Business	**Example activities**	**Example Uncertainties**	**Example Risks**
Exploration	Seismic surveys drilling (wells)	Trap shape hydrocarbon migration integrity of seals reservoir characterization faults and fractures	Seismic costs drilling costs logging costs chance of success
Development	Facility design & implementation recovery plan well plan R & D well testing well treatment	Size of reservoirs rock volume formation thickness porosity permeability fluid properties producible area recovery factor production rate production decline rate aquifer size& strength reservoir communication	Facility costs production costs drilling costs facilities cost
Marketing	Distribution project refining	Oil/gas price volatility supply and demand political issues	Investment costs distribution costs

2.5 Basic Concepts about Real Options

2.5.1 Definition of Real Option

The real options concept as a branch of financial options was coined by Stewart Myers (1977). He used concepts from financial option valuation to evaluate real assets. A real option is a right but not an obligation for investors to make an investment action. Real options are company strategies aiming at maximizing shareholder value. The options can be viewed as strategies and flexibilities to manage an underlying asset such as oil reserves or oil contracts that an oil company owns.

In Luenberger (1998) – real options are “a series of operation options dealing with real assets”. Numerous operational options provide investors with managerial flexibilities and investment strategies. Investors exploit position opportunities in uncertainties to increment positive cash flows by implementing managerial flexibilities and investment strategies.

In Dixit and Pindyck (1994) – Investment opportunities are analogous to call options on a common stock. It gives the right (which we need not be exercised) to make investment expenditures (at the exercise price of the option) and receive the project (a share of stock) the value of which fluctuates stochastically. The model of irreversible investment demonstrates a close analogy between a company’s option to invest and a financial call option.

Real options are options over real assets while financial options are often options over stocks. Both concepts are similar, but investors deal with different assets. In financial options, investors handle financial assets. In real options, investors handle real assets. A financial option contract give investors the right, but not the obligation, to buy or sell a security at a future stated price, Analogously, with real options, investors (or holders of real assets) have the right, but not the obligation, to exploit positive payoffs from the strategic management in future.

Real options thinking centers attention on two key points: uncertainty and time. Firstly, uncertainty is not necessarily an enemy. Great uncertainty is not always equal to great loss. Great uncertainty can generate positive opportunities. Investors may capture “unexpected” monetary values from the positive opportunities. Secondly, time is on the investors’ side. In other words, investors can take advantage of time to resolve uncertainty and avoid downside risks. Time plays an important role when solving investment uncertainties with real options. “Wait and see”, or “a trial investment” is often used as a strategy with real options. Time is not only associated with chances that generate positive payoffs for the company, it is also associated with the time value of money. For example, a petroleum engineering project (such as a reservoir management project or a drilling project) needs time to operate. Time allows investors to change their minds to avoid potential project losses. In fact, real option thinking represents a conservative approach for investment in a long run.

2.5.2 Real Options Categories

As with financial options, there are several categories of real options. According to Jafarizadeh (2009), the categories of real options are based on the following types of strategies:

i. The deferral call option
ii. The abandonment option
iii. The expansion option
iv. The compound option

1. ***The deferral call option:*** here the investor has the right to delay capital investment to a deferral date. It is a call option. If the deferral date is stipulated in a contract, this option is analogous to a European Call Option (ECO). The investors only can start a project at a contracted date. If the deferral date is not stipulated in a contract, it is analogous to an American Call Option (ACO). The investors can start the project at any time.

2. ***The abandonment option:*** here investors have the right to give up a project by selling an underlying asset (or a project) at a given price. It is viewed as an American Put Option (APO) when the price of the project is not favorable for investors (when the price of an underlying asset decreases), investors can abandon the project at a profitable price. For example, investors may begin with a small trial investment, if the investment results are unsatisfactory, the investors could abandon the project.

3. ***The expansion option:*** If an initial investment goes well, investors have the right to expand the project scale by investing additional costs.

4. ***A compound option:*** different types of options are being exercised in one project at its different phases. It is a combination of options.

So flexibility is the kernel idea of real option, due to uncertainty, investors make investments more strategically, and create more opportunities. Investors take advantage of opportunities, and they obtain greater potential benefits form uncertainties. In this way, risks (losses) of investments can be partly reduced.

Real options valuation is a method to appraise an underlying property or a project. One focuses on the value of flexibilities under uncertainty, and the asset present value and its further potential value. The key issue in real options valuation is to assess the value of this flexibility. To do this, investors need to analyze sources of uncertainties. What types of uncertainties bring about risks? How does one model these uncertainties?

Actually, l real options valuation is a tool for the project valuation specially to assign value to flexibility. It is difficult to value these flexibilities under uncertainties in other ways.

2.6 Project Valuation and Valuation Errors

Project valuation is a process to appraise the market value of an investment project. With project valuation, decision makers estimate the monetary worth of the investment project. It is a basis by which decision makers make optimal decisions in an uncertain world. However, decision makers may not make decisions based only on the results of the project valuation. The reason is partly due to valuation errors in it. As most researchers and professionals agree, a project valuation always has some valuation errors.

“A valuation error occurs when there is a difference between the market valuation of a contract and the value assigned by a valuation system” (Picoult, 2002). The errors of a valuation system arise from valuation formulas and assumptions of the formulas, the transformation of data, inputs of observed market data, as well as related parameters. The lost project value (positive and negative) in the valuation arises from errors in input of the market data and related parameters, and errors in formulas and assumptions of the formulas. A given project might be valued differently with different formulas, even if valuation systems have the same inputs. The errors in inputs of the market data and related parameters are unique, and the errors in formulas and their assumptions are systematic.

The errors of a project valuation will never be eliminated, but can be decreased (see figure 2.1). Theoretically, errors are associated with any part of a project valuation system. Uncertain inputs in formulas and uncertain formulas in a valuation system could result in an uncertain value for a project. That is why decision makers are reluctant to take the project valuation as the only criterion n decision making.

In the finance and corporation management literature, there are two commonly used project valuation methods: the traditional discounted cash flow method (the traditional DCF method) and real options valuation. The errors in the traditional DCF method arise from its formulas and assumptions. As researchers and practitioners agree, the traditional DCF method is very simple and practitioners readily apply it. The “simple” here comes from two aspects: explicit mathematical formulas, and a direct transformation of data. The simple transformation of data would possibly ignore the values of an investment project.

In contract, real options valuation is a valuation method that focuses on the value of flexibility. The mathematical sophistication of the real options pricing model is often a barrier for practitioners to apply it. Its "sophistication" contains abroad assumptions, such as the assumption of a complete market, and the assumption that markets are arbitrage free. Its "sophistication" also contains the probabilities, such as risk neutral probabilities. Moreover, its "sophistication" also contains complex formulas for the valuation process. However, the abroad assumptions, complex formulas and the valuation process, which are used for valuing the flexibility, exactly compensate for weaknesses of the traditional DCF method.

Thereby, real options valuation is intended to revise the valuation errors of the traditional discounted cash flow method. See Figure 2.1

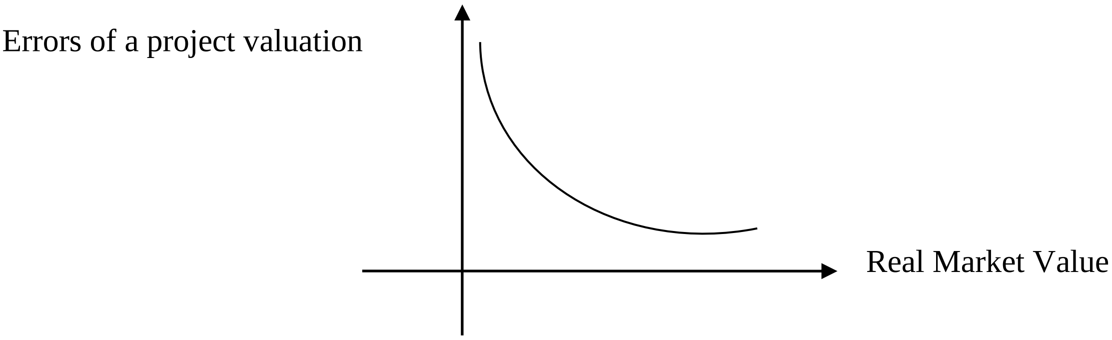


Figure 2.1: Project Valuation Errors

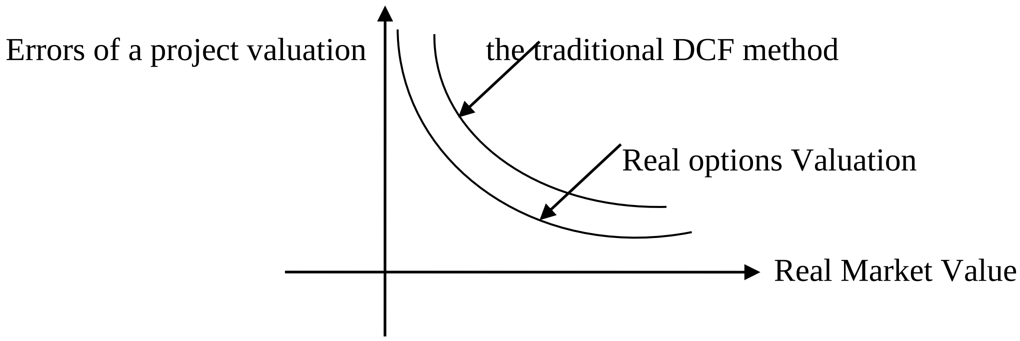


Figure 2.2: Real Option Valuation vs. Project Valuation Errors

2.7 Review of Traditional Project Valuation Methods

The increasing interest in real options valuation research is triggered by long term unsatisfactory investment valuation based on the traditional discounted cash flow method, it is widely accepted that the valuation error of the traditional discounted cash flow method can be large, especially when it is used for contingent investment decisions (Laughton, et al 2000, and Copeland et al 2001).

2.7.1 Traditional DCF Method

The traditional discounted cash flow method essentially assumes that a series of future cash flows of a project are certain, and then discounts them according to the project risks, adding them together, and using formula 2.1. In the process, project managers need to estimate a discount rate that accounts for the project risks. The total value of net discounted cash flows is Present Value (PV). The Net Present Value (NPV) is then this Present Value minus the initial investment and is assumed to be the project value. By calculating NPV or its derivations, such as the Discounted Payback (DP), the Internal Rate of Return (IRR), the Growth Rate of Return (GRR), project managers could predict the minimum capital requirement and payoffs of the investments, and the time value of capital (Iledare, 2019). In this approach, the condition NPV > 0 indicates that decision makers should invest. Conversely, if NPV < 0, then mangers should abandon the investment.

$$PV = \sum_{i=1}^{T} \frac{CF_t}{(1+r)^t} \qquad 2.1$$

Where:

PV	=	*the present value*
CF_t	=	*forecast cash flows after corporate taxes at time t*
r	=	*the discount rate*
T	=	*the project life*

2.7.2 Limitations – A Valuation System Error Rather than a Model Error

The traditional DCF criterion is able to estimate the inherent value of an investment project. It represents the values regarding the time value of money and the minimum capital requirement. When project cash flows are not uncertain, the traditional DCF method provides a project value that is equal to the market value. In another words, the traditional DCF methods is suitable for valuing cash flows in an explicit manner.

Despite this, the traditional DCF method is used by many practitioners and has long been used to value oil exploration and development projects. But they have found this approach fails to account for the flexibility of future management.

The main reasons given are following:

i. ***The method assumes cash flows are certain***

This assumption in the formulas is too restrictive to reflect the changeable movement of cash flows in real investments. In reality, project cash flows are usually uncertain. The level of uncertainty of cash flows depends largely on investment strategies and managerial flexibilities. When managers respond to unfolding unexpected events, the traditional DCF method does not allow for the changed cash flows. Thus, it does not consider how many project cash flows are impacted by strategies decision that manager's make. Valuation errors of the traditional DCF method occur when uncertain events unexpectedly occur. Thus, the traditional DCF method has a system valuation error rather than a model error.

As Dixit and Pindyck (1994) also argued, many investment realities do not meet the assumptions of the DCF method. When unexpected chances occur, the DCF technique cannot adjust quickly, responding to the new situation, and evaluate the alternatives. The traditional DCF method fails because it neglects to account for the value of strategies and managerial flexibilities.

It is also recognized by Campbell (2001). He argued that, there are three critical pitfalls. The first pitfall is the source of cash flows. All project cash flows come from a single investment case. The investment decision is irreversible. In reality, project cash flows could derive from many sources. An investment decision can be cancelled, changed, as well as reshaped. These decisions bring about changes to cash flows. Project cash flows can be always changed, reshaped and optimized.

ii. ***The discount rate***

Campbell mentioned another critical pitfall result from the discount rate. The risks are liable to occur from any aspect of a project. A single discount rate cannot represent all the risks in different periods of an investment.

As Claeys et al (1999) and Newendorp et al (2000) also point out, it simply assumes that a risk adjust discount rate could not reflect future opportunity costs. The opportunity of the capital growth is intangible. Can a risk adjusted discount rate account for the intangible opportunity costs? When future opportunity values are implicit, the valuation of the opportunity is complex. Simply depending on a sign discount rate to value all flexibilities (opportunities), one would not obtain the real market value of an investment.

iii. ***The utility of capital***

Different projects have different amounts of invest capital, they involve different risks, but they might have the same NPV. The traditional DCF method does not consider the utility of capital. Newendorp and Schuyler (2000) also elaborated this aspect. For example, the present value of project A is $3,500 and $500, then the net present value of project A is $1,500 and the project B is $1,500. Their NPVs are same, but investors will more readily choose the project B, since it costs less. The traditional DCF method does not tell investors that the utility of capital in the project B is less.

In summary, the traditional DCF method accounts for the time value of investment capitals. But it ignores management strategies and flexibilities that are taken in response to the uncertain market.

Due to the weaknesses of the traditional DCF method, improvements over this traditional valuation method is needed. Real options valuation is a new approach to project valuation, and thus, could attract attention from investment managers.

2.8 Previous Studies of Real Options Valuation

Previous research of real options valuation originated from the valuation of natural resources investments, partly because they have higher risks and are more uncertain than other investments, and partly because oil companies use flexibilities for investment managements.

Real options valuation is a valuation method, the research on real options began in the 1980s. Researchers applied financial option theory to value assets in real investments. Financial option pricing models such as Black – Scholes Option Pricing Model (Black and Scholes, 1973) and Binominal Tree Option Pricing Model (Cox, Ross and Rubinsten, 1979) are widely being used to value managerial flexibility of real assets.

Brennan and Schwartz (1985) applied financial options to model the copper mine investment decision problem. According with the fluctuation of the copper price, the miner has three choices: open the mine, defer mining, and abandon mining. These are all managerial options. They developed a one factor model for this investment decision evaluation.

McDonald and Siegel (1986) also used financial option valuation to appraise a project. They introduced a stochastic geometric Brownian motion to determine the present values and investment costs. They supposed that the life of project is unlimited, and that project cash flows could be zero at times during the life of the investment.

Paddock, Siegel & Smith (1988) wrote a paper using the option theory to study the value of an offshore lease and the development investment timing. The Paddock, Siegel and Smith's model is one of the most popular models for petroleum real options applications. They used the typical Black–Scholes option pricing model to evaluate real assets, they believed that there is “the market equivalent value” for the real underlying assets. They connected the stock price with the real underlying asset (We will discuss this in the next chapter).

Copeland and Antikarov (2001) published a book “Real Option” and systematically illustrated the real options concept and a valuation assumption called – “Market Disclaimer Assets” (MAD). They introduced the binominal tree and the risk neutral probabilities. The crucial assumption of MAD is: “the present value of cash flows of the project without flexibility (i.e. the traditional PV)” is the best unbiased estimates of the market value of a project were it is traded asset. When the flexibility comes in, the real options value is the market value including the flexibility value.

These are advocates for real option valuation. However, there also exist critical voices about real options valuation.

Ross and Lohrenz (1996) used Black–Scholes option pricing model (Black–Scholes Model) to value oil and gas assets. They compared the values from Black–Scholes option pricing model with the one from the traditional DCF valuation. They proved the values from Black–Scholes option pricing model are greater than PV from the traditional DCF method.

Burns, Lewis, and Sick (1992) argued, though the traditional DCF method “tends to undervalue real investments”, they supported “the assertion (from advocates of real options) are correct from assets with values subject only to the effects of the oil and gas price fluctuations, but are not necessary correct and may be horrendously in error for real oil and gas asset values”. In other words, they agreed real options valuation that are based on Black–Scholes Model and can account for management flexibilities form the price uncertainty, but it does not correctly account for the project specific uncertainty. Thus, their conclusion is that: there is “no panacea” to apply real options pricing model in every case.

Investors are less happy now to accept real options valuation as a valid method, because real options valuation does not necessarily make the important distinction for decision making between what is possible and what is do - able. Even if opportunity value looks attractive, they add values in project valuation but not in real investments.

The past literature presents pros and cons for real options valuation. As most recent researchers agree, and as Dayer (2002) points out, and as we have discussed above, the *"real options approach is intended to supplement, not replace, capital budgeting analyses based on standard DCF methodologies."*

2.9 Real Option Valuation and Decision – Making

Decision making is the cognitive process of selecting a series of actions from among multiple alternatives. Real options valuation is a quantitative analysis of an investment project fro decision makers to judge investment decisions. Thus, real options valuation and decision making are inter–related. Both are needed for decision makers. Gigerenzer et al (1999) and Kahneman et al (1982) point out: "understanding human decision – making processes has been a central enterprise for the cognitive sciences, as well as the focus of applied research across disciplines like psychology, economics, business, marketing, and the health sciences".

The mathematical sophistication of real options valuation does not simplify the process and criteria for decision – making. For instance the real options technique needs to use high quality probability assumptions. The real options approach does not make decisions easier since making decisions are inherently difficult and complex.

However, an optimal investment decision needs to understand impacts of valuation tools and decision processes. Real options valuation does open up investors' minds in a new way. Real options valuation provides investors and managers with an insight on understanding how to deal with uncertain events.

As a consequence, improving valuation tools with real options valuation is cone way that leads to improve decision making. Despite this, one still needs to prove that with real options valuation there is a decrease in errors in the project valuation.

Therefore, to make an optimal decision, decision makers still need to rely on a combination of real options valuation and improved traditional techniques, as well as other related disciplines.

2.10 Real Options Theory

Real options theory is the analysis for values of a series of strategies and flexibilities in investments. It is related to the finance theory, decision – making, economics, corporation management as well as other related disciplines. Real options theory originates from the financial option theory.

To value flexibilities of an investment project using real options valuation, we necessarily review financial options and their valuations.

2.10.1 Financial Options

2.10.1.1 An Introduction to Financial Option

In finance, an option is a contract where an investor has a right (but not the obligation) to exercise a contract (the option) on or before a future date (the exercise date).

In option markets, trading financial options can have advantages over trading the stocks themselves. Investors trade options to speculate on prices movements of a stock. Generally, it is cheaper to trade options than the stocks themselves. It takes less capital than directly investing into underlying stocks. From his aspect, there are a number of opportunities and flexibilities in the investments.

Basically, there are two kinds of options: American options and European options. An American option permits the owners to exercise at any time before or at tits expiration. The owner of a European option can exercise only at its expiration.

There are two popular numerical options pricing models: binomial tree model and Black – Scholes option pricing model.

2.10.1.2 Option Monetary Value

If the option does not have positive net payoffs, it is viewed as “out of money”. If the option yields positive payoffs, it is viewed as “in the money”. When the option is “out of money” (generating negative payoffs) at its expired date, the holders will simply “abandon the option”. It will expire worthless. As a consequence, based on above definitions for the option, the option itself would never have value less than zero (figure 2.3).

This aspect is very important for real options valuation. According to the financial option theory, when one evaluates the flexibility using option – pricing models, one always obtains a positive value of the flexibility. However, flexibility creates opportunities, but it is not equal to generate positive payoffs (see figure 2.4). Only positive opportunities can generate positive payoffs. When one is not able to control uncertainties in investments, even if there are a plenty of flexibilities, investors could not obtain profits.

In other words, when one evaluates the project using real options valuation, one should not create unrealistic strategies. Otherwise, it is same as increasing income in the valuation but not in the real investment.

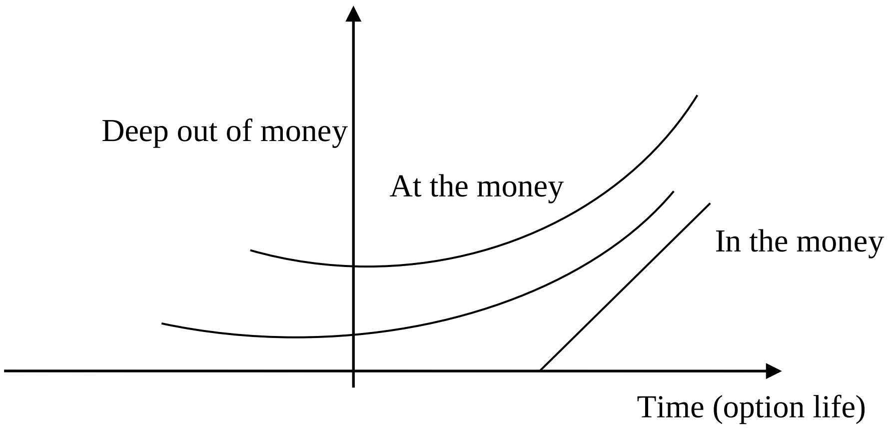


Figure 2.3: Option Monetary Values

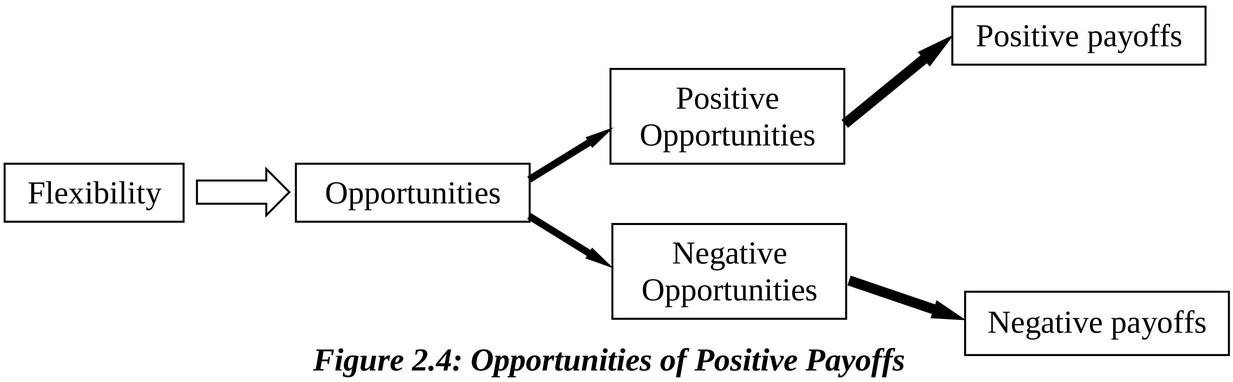


Figure 2.4: Opportunities of Positive Payoffs

2.10.2 Options Pricing Models

Common financial option pricing model are the Black–Schools Option Pricing Model (BSOPM) and the binomial tree option model (BTOM), in which the former can be based on the latter. Binomial tree option model is a discrete model for the option valuation. Black–Scholes option pricing model uses a continuous model. Given small time steps in the BTOM, the option value form BSOPM is close to the result from this binomial tree model.

2.10.2.1 The Binomial Tree Option Model

The binomial tree options model (BTOM) provides a numerical method for option valuation. The binomial tree option model was first proposed by Cox, Ross and Rubinstein (1979). The model is a discrete time model to trace the evolution of variable option values, using a binomial lattice. In the model, the prices vary over time. Essentially, the binomial options model is an application of the risk neutral valuation over the life of the option. The valuation process is iterative, starting at the present price, deduce the variable prices form node to node till the final nodes, and then working backwards deducing option costs. The option value is the value at the initial node of the second binomial tree. The binomial tree model process is the following:

Calculating the magnitude of the movement

a. Time steps (Δt)

In the binomial lattice, the whole option life is divided into given time steps. The given time steps begin at the valuation date, and end at the option expiration date. Each node in the lattice presents a possible price of the option at the given time.

b. Up and down factors (u and d)

It is assumed that the stock price movement is allowed to move in two directions: up and down. An up or down factor is calculated by equation 2.4 and 2.5 respectively. The up and down factor, and time steps control price changes, given so as the current price, the price in the next period will either be S_{up} or S_{down}, using equation 2.2 and 2.3. Thus the price tree is produced from the start date to the option expired date.

$$S_{up} = S_o \times up\ factor \tag{2.2}$$

$$S_{down} = S_o \times down\ factor \tag{2.3}$$

The up and down factors are calculated using the stock price volatility σ and the time step t.

$$u = up\ factor = e^{\sigma}\sqrt{\Delta t} \tag{2.4}$$

$$d = down\ factor = e^{-\sigma}\sqrt{\Delta t} = \frac{1}{u} \tag{2.5}$$

Generating price trees:

Price Tree:

$S_{1up} = S \times u$ → $S_{2up} = S_{1up} \times u$, $S_{updown} = S \times u \times d$

S

$S_{1down} = S \times d$ → $S_{updown} = S \times u \times d$, $S_{2down} = S_{1down} \times d$

Calculating of the option value as each mode

Option Value Tree:

$V_{1up} = [pV_2 + (1-p)V_{updown}]/(1+r)$ — $V_{2up} = max[S_{2up} - C_o),0]$, $V_{updown} = max[S_{updown} - C_o),0]$

V_0

$V_{1down} = [pV_{2down} + (1-p)V_{updown}]/(1+r)$ — $V_{updown} = max[S_{updown} - C_o),0]$, $V_{2down} = max[S_{2down} - C_o),0]$

Where Co is the exercise cost of the (call) option

Through the backward calculation, the value V_o at the first node is the option value. From the formulas above, the value of the option consists of two components: the intrinsic value and the time value, the first component being the value we obtain if we could exercise the option immediately, and the time value of the option is always positive and declines with time, reaching zero at the expiration date. The up or down factors could also be functions of time and the stock price volatility.

2.10.2.2 The Black–Scholes European Call Options Model

Black–Scholes European call option model (Black – Scholes model) is built upon following assumptions:

* Asset prices adjust to prevent arbitrage
* Stock prices change continuously
* Stock returns flow a lognormal distribution
* The interest rate and the volatility of the stock remain constant over the life of the option;
* The model holds for European call options on stocks with no dividends

The Black–Scholes European Call Option Pricing Model:

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \quad 2.6$$

Where:

C_t = *the price of an European call option at time t*

S_t -= *the stock price at the time t*

$N(\ )$ = *the cumulative normal density function*

$$d_1 = \frac{ln\left[\frac{S_t}{X}\right] + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \quad 2.7$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad 2.8$$

Where:

X = *the exercise price*

T = *the expiration date of the option*

t = *the current time*

r = *the risk free rate of interest*

σ^2 = *the variance of the lognormal distribution of the stock return process*

Based on equation 2.6, 2.7, 2.8, one should note that the Black–Scholes option model do not depend on the discount rate that affect investors risk preference, stock prices, time, the risk free interest rate, and the volatility, do not represent investors attitude for risk tolerances. All these variables are independent on risk preferences.

With real options, parameters and methods to value the flexibility can be borrowed from financial option. The value of real options depends on six variables (Copeland and Antikarov, 2001).

* The value of the risky assets
* The exercise price
* The expiration time of option
* The standard deviation of the return rate of the underlying risky asset
* The risk free rate of interest
* The dividends

2.10.3 Differentiating Risk Definitions between finance and Decision Analysis

The meanings of the term “risk”that is used in finance and in decision analysis are different. The different definitions of “risk” have consequences for different quantitative analysis.

Picoult (2002) presents an explicit definition on risk. He pointed out: “risk” can be defined as the magnitude of a potential loss, or “risk” can be defined as the standard deviation of the potential revenue (or income) of a trading or investment portfolio over some period of times. In decision analysis, “risk” always is understood as the potential downside monetary loss, where the mean of the project PV is less than zero.

When we discuss “risk” in real options valuation, as usual, we focus on the risk as the potential lost, and the uncertainty as the standard deviation of project returns. These definitions apply though out the thesis.

2.10.4 Differences between Real Options and Financial Options

The real options concept replicates the financial option theory to manage risks and uncertainties for physically tangible underlying assets. The real options concept is analogous to financial options concepts, but investors deal with different assets in different markets. Actually, real option is a process of risk management from the perspective of the finance.

Firstly, we will distinguish the markets between finance option and real investments. In finance, the efficient market means that a commodity price is decided by the time and its intrinsic value. The intrinsic value is marginal production costs of a commodity. In other words, the market price reflects marginal production costs and time values of the commodity.

The market information is instantaneously available for anybody. There are no risks less arbitrage chances in the market; every investor is able to access the commodity that they want to buy at any time. Market liquidity is smooth. The efficient market theory is based on macroeconomics "rational expectation theory" (Muth 1961). The future price change is a function of the price volatility and times. And thus, there is no analysis for human behavior.

However, referring to real investments, investors are not always rational, human behavior effects a decision, its investment process and of course, its investment results. In real options, investors deal with tangible real assets. Human behavior plays an important role in the real market and in real options and impact on the value of the real assets.

Real assets have their own characteristics. There is a comparison between real options and financial options in the table below:

Table 2.2: Comparison between real options assets and financial assets

Items	**Financial Options**	**Real Options**
Market	Complete market	Incomplete market
Expiration time	Months	Years or months
Arbitrage ability	Limited	High
Carrying option costs	Limited	High
Depreciation of assets	None	Have
Initial value of option	Unchanged (limited)	Changed according to market volatility and depreciation
Capital liquidity	High	Low
Physical constraints and technology impact	None	Plenty
Sources of uncertainties	Price uncertainty	Price uncertainty technology uncertainty
The value effected by human behavior	Market aspect	Market and technology aspects
Flexibility in management	Low	High

From this comparison, we would conclude that:
Real options assets are more complex than financial assets.

Real options deal with two problems: the investment problem and the financial problem (Smith, 1998). Some investment projects do not meet all assumptions of financial option pricing models.

As such, in some case, a project could not be viewed as a European call option, because they can be exercised at any time. Furthermore, real assets are often not in the complete and efficient market. If one directly replicated the financial option pricing model to value opportunities, it would lead to valuation errors for real option.

2.10.5 Flexibility in Real Assets

With respect to an option on real assets, the option here pertains to the flexibility (or strategy) in the investment. Despite that, a real option is a right (but not an obligation) for investors to take investment action. They have different definitions of the flexibility with real options. The options can category into two types: flexibilities in the technology management and flexibility with respect to financial markets. That is, there are options in management and options in pricing and trading.

The flexibility in real options is defined in a ready capability to adapt to new, different, or changing requirements in uncertain markets and in changing the technological management. Based on main types of uncertainties: the market uncertainty and the technical uncertainty, the flexibility consists of two aspects: strategies used for solving market uncertainties, and strategies used for solving technical uncertainties.Figure 2.5 lists the origination and valuation of the types of flexibility.

Strategies for market uncertainty

The strategies for market uncertainty cope with market uncertainty. The market uncertainties are the price uncertainty and the market liquidity uncertainty. As such, they are "trading project strategies" (Smith et al, 1998).

Strategies for technical uncertainties

The strategies for technical uncertainties are related with the technology management. Investors use technical strategies, when they cope with technical uncertainties. As such, they could be a well treatment for a production well, or a horizontal drilling strategy for the oil exploration. They are strategies used within the company.

By reviewing a number of real options papers, we led to conclude that there is a belief amongst finance researchers that "the value of a non-traded project is the price the project would have when it is traded in the market" (Mason and Merton, 1985). The fundamental assumptions of real options arise from financial options. The flexibilities of the project are strategies by selling or buying projects after the initial capital has been invested. In other words, the flexibilities could be derived from trading the project. Thus, the flexibility value is mean to be the value of the trading project. These strategies are essentially related with the price volatility. And therefore, based on numerous published papers, most researches focus on the price uncertainty (the market uncertainty). We refer to N. Dickens, Ross and Lohrenz (summer 1996), Brennan, M. J. and Schwartz, E. S. (1985).

Another side comes from management researchers. They believe flexibilities in a project are strategies used for technology management. Managers should actively and quickly response to technically uncertain events. Managers implement strategies in order to optimize technical flexibility values. Strategic management creates opportunities that have positive payoffs. Managers exploit these opportunities to increase the project profits. Thus, the flexibility value is the value of technical strategies. Even if the invested project is not traded in the market, there still exist the flexibility values in technical strategies. As such, Luenberger (1998) discussed the option to expand and how to value it. The project in his example is on a nontrade project. The value of flexibility derives from management strategies. Also, Gallil, Armstrong, and Dias, (2004) and Dias (2002) discuss the valuation of technical strategies as real options. As most researchers agree, there are two parts of the values in investments. One part of the value is the present value, arising from the direct investment. Another part of the value is the managerial flexibility or the opportunity value, arising from flexible investment strategies. Based on the real options literature, the former is evaluated by the traditional DCF method, and the latter is evaluated by real options valuation.

Therefore, based on these two meanings for strategies, numerous real options valuation approaches arise in the real options literature.

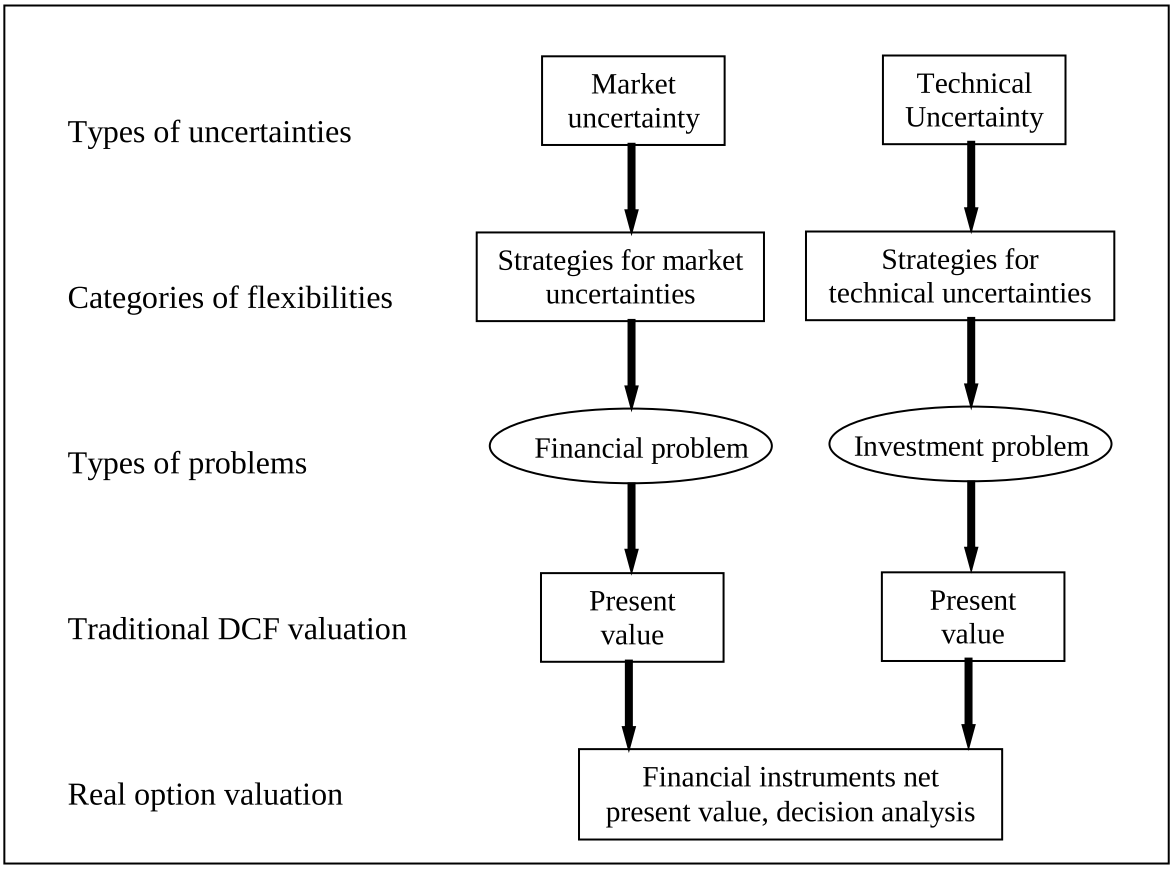


Figure 2.5: Flexibilities and Uncertainties in Real Options Valuation

2.11 Modelling Uncertainty in Stochastic Processes

Real options thinking (Begg et al 2002) guides people to exploit more uncertainties, they believe that more uncertainties bring investors more opportunities and flexibilities in the investment. If opportunities were correctly dealt with, the more value there would be. The critical issue in real options valuation is how to understand uncertainties, and then how to value flexibilities.

Two mathematical models have been used to describe the price uncertainty in petroleum project valuations: Geometric Brownian Motion (GBM) and Mean Reverting Oil Price Model (MROPM). The two processes assume that the oil price varies continuously over time in a random or pardy random way. The underlying asset value is stochastic. Therefore, investment decisions are made under uncertainty.

Geometric Brownian motion also can be used to model the cost uncertainty and production uncertainty.

2.11.1 Geometric Brownian Motion

A geometric Brownian Motion (GBM) (Brown, 1828) is a continuous (time) stochastic process. Its mathematical model is appropriate for some financial phenomena, such as the stock price or an option price movement, because absolute numerical changes of the stock price vary stochastically. The equation is:

$$dx = \alpha x dt + \sigma x dz \qquad 2.9$$

Where:

dz is the increment of a wiener process

dt is a time interval

α is the drift parameter. In a price forecast, the drift usually is the risk free rate.

σ is the volatility of the stock price

Based on the equation 2.9 above, as Dixit and Pindyck elaborated (1994), GBM contains three key assumptions:

Markov process

This means the future price depends only on its current value not its past value. The future price path is determinate only by the current price.

Wiener process

The wiener process describes the changes always exist in the process. Each change in the log price at any time involves an independent increment *dz*.

Normal probability distribution

The changes in the log price at any time t are distributed normally. The variance of this distribution could be a function of time.

2.11.2 A Popular Oil pricing Model – The Mean Reverting Model

Mean reverting model is fundamentally based on Brownian motion. The general equation of a mean reverting model (Ornstein – Unhlenbeck process) is:

$$dx = \eta(\bar{x} - x)\, dt + \sigma dz \qquad 2.10$$

A “normal level” – $\bar{x}$ controls the drift in the mean reverting model (Dixit and Pindyck, 1994) $\bar{x}$ is assumed to be a long run equilibrium level. Also, it can be viewed as a long run marginal production cost.

The model represents:

* Despite that the price sensibly oscillates in short term, the price of a commodity follows its marginal production costs in long run.
* The drift can be positive and negative in the process. When the market price goes far from the equilibrium level (becomes “unreasonably” high or low), the equilibrium level will draw back the price. In other words, the long run marginal production costs constrain the commodity price trend. Like price leverage, the marginal production cost constrains price changes.
* Based on the microeconomics theory, the model reflects the “rational expectations theory”. That is, the price of the commodity is decided by time and its intrinsic value. The intrinsic value means the marginal production cost.

As widely agreed by academics and practitioners, the mean reverting process is centred on the long – term marginal production costs. It presents the nature of physically tangible commodities, such as the oil and gas.

Referring to the oil price, although it is volatile due to political issues, geological structures, exploration and production technologies, and labour costs, for example, the long–term oil price is tied with marginal production (exploration and development) costs of Organization for Petroleum Exporting Countries (OPEC).

2.12 Using Monte Carlo Simulation to Determine the Project Volatility

Monte Carlo simulation is used to “simulate repeatedly the random processes by governing random variables (production rates or prices) from statistical or mathematical models” (Dowd, 1998). It is named after the small country famous for its casinos. Each result is a possible value from the statistical or mathematical model. The model repeatedly generates numerous results. The results from Monte Carlo simulation depend on the model that one uses to describe the variables. The mean and variance of the distribution could indicate the model characteristics Copeland et al (2001) used Monte Carlo simulation to estimate the volatility of the project returns. When the risk factor is non linear with project revenues, Monte Carlo simulation can be used to work out a distribution for project revenues, but the computation is intensive because one needs many repetitions.

2.13 Convenience Yield

The existence of the convenience yield is important for the investment management. Financial theory assumes the market is efficient. A price protection (or a price leverage) arises from the efficient market. The asset prices always reflect demands and supplies of the market, and always balance the two. The diversity of investor opinions results in human behavior that can lead to the price volatile, because investors do not always make reasonable decisions. Thus, the convenience yield actually is arbitrage benefits from the price volatility. Investors could exploit the convenience value by "surfing" the volatility of commodity prices.

2.13.1 Definition of Convenience Yield

In finance, the convenience yield is the dividend on a stock, and it is a yield per unit of a stock price. But for a real commodity, the definition is slightly modified. The convenience yield is the flow of benefits that the marginal stored unit provides (Dixit and Pindyck, 1994), or in other words a liquidity premium arising from the storage of a commodity (Gibson, 1990). It is a measure of the value of remaining physical control of the commodity.

A firm often holds inventories to be able to smooth production. It is convenient for the firm to access productions. By this method, a firm would avoid future shortfalls. During the high demand market, a firm might be willing to hold the commodity in order to get opportunities to arbitrage. During the high supply market, the firm might be willing to relinquish the commodity to avoid its physical depreciation.

The convenience yield means that the company has benefits when the firm stores a commodity. "Inconvenience" always carries costs. The convenience yield represents the relation between the carrying commodity costs and carrying commodity benefits. For example, the gold has a zero convenience yield, because its price changes are very small, and generally complaint with changes of the risk frees interest rate. So keeping gold is same as keeping money in the bank.

2.13.2 Convenience Yield and Oil Price

Convenience yield is closely related to the oil price. Based on the definition of the convenience yield, it arises from the price changes. If the oil price volatility is zero, the convenience yield is constraint and equal to the discounted rate. The convenience yield is an indicator of the oil price risk.

Although the convenience yield is associated closely with the market risk (public risk) of the project, it makes less sense for private risks.

2.13.3 Convenience Yield and Project Cash Flows

The convenience yield could significantly impact on the firm cash flows. The firm buys or sells commodities (inventories) to obtain or reduce cash flows. Bu this way, the firm also can optimize cash flow portfolios. Form the components of cash flows, the convenience yield could impact on operating cost, annual revenues, and taxes of the firm. The firm buyer could acquire the tax refunds by arbitraging the convenience value. Different investors can generate different cash flows from same investment using arbitrage strategies, in essence, this is arbitrage behaviour.

2.13.4 Convenience Yield and Storage Costs

Inconveniently accessing to inventories could cause extra costs, when the firm needs those inventories urgently. The company would pay more, such as long distant transportation expenses, if there were shortfalls of the commodity in the demanding market. These costs could be avoided by holding suitable inventories for emergency needs. In petroleum investments, investors hold reserves as inventories for the development and obtain the oil convenience value. Also, investors could hold reserves for trading project purposes.

However, it is important to distinguish between gas and oil. In terms of gas, it could have the convenience liquidity premium in theory due to changes of gas price. In reality, its storage is so costly that few investors exploit the real options value, rather than carry thegas as the commodity. Then storage costs can be considered as keeping option costs, such as costs to delay for development gas field.

2.13.5 Convenience Yield in Real Options Valuation

In real options, the price volatility pushes inventors to delay for developing oil fields. On the contrary, the convenience yield pushes investors to exercise option early. When the weighted average cost of capital (WACC) is larger than the growth rate of the oil price, the convenience yield is positive. The time value of capital that is occupied by invested reserves is higher than the option value of the reserves, investors keep options with high costs. So, the project should be developed early.

Assuming the convenience yield is very high at nearly 100%, PV of the project is close to the real options value of it. It represents the situation where there is not too much flexibility in the project. The project should be invested in right now.

2.13.6 A Model of Convenience Yield

Based on the concept of the convenience yield, the basic model is defined by:

$F_{t+1}+1 \times (1+\delta_t) = F_t \times (1+r_f)$ *2.11*

Where:

F_t *= spot price at time t*

F_{t+1} *= future prices at time t +1*

r_f *= risk free interest rate*

δt *= the convenience yield at time t*

According to the formula 2.11, the future value of the commodity with the convenience yield is equivalent to the inherent value of this commodity plus the time value of money. The inherent value of the commodity today is equivalent to the inherent value of the commodity plus its time value tomorrow.

But considering carrying commodity costs and the time value of cash flows occupied by the commodity, Sick (1997) proposed a modified model for the oil convenience value for the petroleum industry. This is:

$$\delta_t = k_t - 1n\left[\frac{E_0[\acute{S}_{t+1}]}{E_0[\acute{S}_t]}\right] \qquad 2.12$$

Where:

k_t *is the risk adjusted discount rate*

$E_0(\acute{S}_{t+1})$ *= the expected future spot price at t+1*

$E_0(\acute{S}_t)$ *= the expected spot price at t*

Based on the formula 2.12 above, when the future oil prices decrease, $1n\left[\frac{E_0[\acute{S}_{t+1}]}{E_0[\acute{S}_t]}\right] < 0$ and one receives the high convenience value, and a high liquidity premium at present. Investors will not control more physical inventories because the commodity price could be accessed with lowers costs in future. When the future oil price increases, $1n\left[\frac{E_0[\acute{S}_{t+1}]}{E_0[\acute{S}_t]}\right] > 0$ investors will get the low convenience value, and will control more inventories because of its low liquidity value at present. The equation shows the price volatility is sensitive to the convenience yield.

In equation 2(12), $1n\left[\frac{E_0[\acute{S}_{t+1}]}{E_0[\acute{S}_t]}\right]$ is the growth rate of the expected spot price. The oil convenience value is the discounted value minus the expected growth value of the oil price (that is, WACC minus Growth Rate of the oil price).

Sick considered three aspects into his model, which are the costs of capital, the time value of capital, and investment opportunity costs.

Embedded with real options, if WACC is larger than the growth rate of the oil price, t is high, the time value of capital that are occupied by investment reserves is higher than the growth rate of the oil price. Investors should develop the project immediately. The option should be exercised early. If WACC is less than the inherent price growth rate, investors keep options with low costs. Investors should delay in exercising the option. If WACC is equal to the growth rate of the oil price, the carrying option costs is equal to the growth rate of the oil price, there is no benefits to control the physical petroleum assets. Whether one develops the project latter or develops the project now, the investors would receive the same returns.

2.14 Risk Neutral Probabilities

The term of "risk neutral" in economics is used to describe an investor who only cares about the expected outcome of an investment. He never pays less to avoid risks (the potential loss) or positively take more risks. The investor always sits on the fence for risks. In the risk neutral valuation, the expected return rate of holding underlying assets is the risk free interest rate. The value of an underlying asset is discounted by the risk free interest rate.

The risk neutral probability is a measurement for risks.

The equations are:

$u = upfactor = e^{\sigma\sqrt{t}}$

$d = downfactor = e^{-\sigma\sqrt{t}} = \frac{1}{u}$

$$\frac{(1+r_f)-d}{u-d} + \frac{u-(1+r_f)}{u-d} = 1 \qquad 2.13$$

$$up\_probability = \frac{(1+r_f)-d}{u-d} \qquad 2.14$$

$$down\_probability = 1 - up\_probability \qquad 2.15$$

Where:

σ *= the price return volatility*

Δt *= time step*

r_f *= risk free rate*

Based on equation 2.13, 2.14, 2.15 one could infer that the risk neutral probability is only related with the time and the price volatility. This proves what "the efficient market theory" holds. The price of a financial underlying asset is a function of time and the price volatility. All (attainable) financial assets have the same expected rate of return, regardless of the price volatility.

However, real assets do not have same expected rate of return, since assets have their own characterized private risks.

2.15 Value of Information

The Value of Information (VoI) is a quantitative measure of the value of knowing the outcome of uncertainty variables prior to making decision. It includes values of "perfect information" and "imperfect information", "perfect information" describes the fact that is fully revealed. It is known that information plays an important role in the process of investment decision making. The value of information can be viewed as a "learning option" (Dias, 2002) in real options, since obtaining information involves monetary costs, so information has monetary values. When high uncertainties arise perfect information has high value for the investment, and increases the project value.

Value of information is related closely with technical flexibility. This is because the whole process of implementing technical flexibility is essentially a process of revealing information.

CHAPTER THREE

3.0 Real Options Approaches

3.1 Introduction

Since 1980s, real options have been valued by a variety of approaches. Currently, most oil companies still use the traditional DCF method to evaluate investment projects rather than using real options valuation. The main reason for this is that there are disputes about the correct approach to the valuation of real options.

Five real option approaches will be elaborated in the thesis. As named by Borison (2003), they are the Classic Approach (Paddock, Siegel and Smith, 1988), the subjective approach (Luehrman, 1998), the Market Asset Disclaimer (MAD) approach (Copeland et al, 2001), the Smith approach (Smith and McCardle, 1998) and the Luenberger approach (Luenberger, 1998).

All of these focus on valuing the flexibility. They are built upon their own assumptions and mechanisms respectively. The assumptions used in different approaches are aimed to model uncertainty and assign value to flexibility.

The goal of this research is to compare the different real options approaches by applying them to realistic petroleum lease projects example, to help petroleum managers comprehend different real option valuation approaches, and to apply real options valuation in the E & P projects. Also we identify characteristics of the real options approaches, indicate their applicability and then draw some constructive conclusions.

3.2 Real Options Approaches

Borison published a paper "Real Options Analysis: where are the emperor's clothes?" He discussed five real options approaches in this paper. They are listed as following:

* Classic Approach
* Subjective approach
* MAD Approach
* Revised Classic Approach (Amram and Kulatilaka, 2000)
* Integrated Approach (Smith Approach)

In addition to these approaches, there is another real options approach – the Luenberger approach. It will be discussed in the section 3.6

3.2.1 Reasons for various Real Options Approaches

Current researchers agree that the real options concept is a valuation innovation for investment management and finance. However, various real options approaches being used are inconsistent. The literature presents different definition for real options. Real options are "a series of operational options dealing with real assets" (Luenberger, 1998). The "operational options" are composed of two parts: the financing flexibility (trading projects) and the technical flexibility. Financial people would like to think real options are a series of financing operational flexibilities in the market. Technical people would prefer to regard real options as a series of technical operational flexibilities. The options should be evaluated differently because of the different characteristics of the flexibility. Also Lund (2002) in "Real Options in Offshore Oil Field development Projects" says that most real options papers simplify the types of flexibility, and focus only on the market uncertain variable. As a result, "it is hard to discern the benefit of flexibility in a realistic oil field development project from contributions reported in the literature".

And as a result, it is difficult for practitioners to choose a sound implementation. Borison mentions, the difficulty of implementing real options approaches is rarely being discussed in the literature. Moreover, when there are errors in project valuation, wrong investment decisions can be made. These wrong decisions could be costly for shareholders.

Based on their assumptions, applicability, and mechanisms, Borison contrasted and criticized those real options approaches. Also, he studied an example from oil investment projects to interpret these approaches. Through the comparison, Borison pointed out pros and cons for each approach, and suggested how one should apply each real options approach.

3.3 The Classic Approach

3.3.1 Assumptions (no arbitrage market data)

The classic approach is based on paddock et al (1988). The assumptions are analogous to those used with financial options:

- It supposes the investment market is complete and efficient.
- There are no arbitrage opportunities in the complete market, so no profitable project exists without paying costs for it.
- There is a replicating portfolio between a financial asset and a real investment. The financial asset and the underlying asset are similar; the underlying asset can thus be traded and priced by financial instruments.

Given a no arbitrage condition, similar assets must necessarily have similar prices. The price of the underlying asset is a stochastic process, and the value follows a geometric Brownian motion.

Paddock et al demonstrated the market equilibrium for the underlying real asset. By integrating typical black schools option pricing model to value a delay option of developed petroleum reserves, they derived a value for the real options. They hold that the option valuation methodology can avoid the need to use a risk adjusted discount rate.

They supported project evaluators need to understand the real asset value (petroleum reserve value) is in equilibrium to the market stock value. They modeled petroleum reserve values with option pricing techniques, valuing the development option and the option to explore.

3.3.2 Mechanism of the Classic Approach

3.3.2.1 Petroleum reserves Market Equilibrium

There is equilibrium between the value of petroleum reserves and the market value. The stock price of the oil company is essentially equivalent to the net asset value of the company. For the oil company, the main assets are petroleum reserves. The value of company assets is equivalent to the value of developed oil reserves. Thus, the stock price multiplied by the number of stock shares of the company can be equal to the market value of development oil reserve. That is the market equilibrium value of developed petroleum reserves.

When the stock price of the oil company goes up, the value of oil reserves goes up accordingly. The present value of holding a proven oil reserve can compensate the owner for the opportunity costs of investing in the reserve.

3.3.2.2 Important Inputs of the Classic Approach

Table 3.1:Inputs of the classic approach

Current stock price	Value of developed reserve discounted for development lag
Variance of the stock price	Variance of rate of change of the value of a developed reserve (variance of the oil and gas price)
Exercise price	Per unit development price
Time to expiration	Relinquishment requirement
Riskless rate of interest	Riskless rate interest
Dividend	Net production revenue less depletion

3.3.2.3 The Model to Value Real Option

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \tag{3.1}$$

$$d_1 = \frac{ln\left[\frac{S_t}{X}\right] + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C_t = Price of real (deal) option at time t

S_t = Value of reserves discounted by delaying lag

N( ) = The cumulative normal density function

X = Exercise price

T = Expiration date of the option

t = Present time

r = Risk–free rate of interest

σ^2 = Variance on the return of gas prices

3.3.3 Conclusion from the Classic Approach

C_t is the real options value for the developed oil reserves. It is a pure financial solution. The real options value is in the value of financial flexibilities. The approach focuses on the price uncertainty. The classic approach does not have a good solution for valuing technical flexibilities. The value of technical flexibilities is ignored.

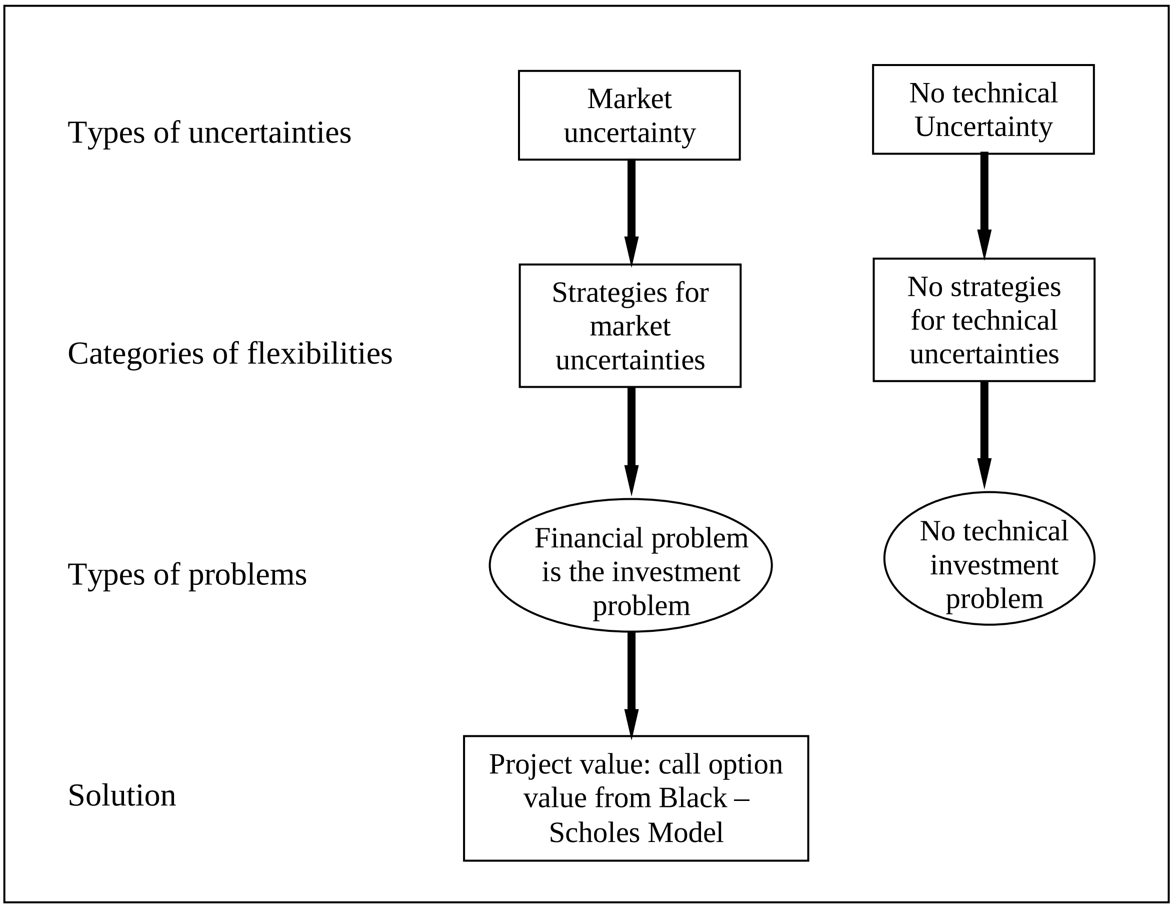


Figure 3.1: Modelling Uncertainties in the Classic Approach

3.4 The Subjective Approach

3.4.1 Assumptions (no arbitrage, subjective data)

The subject approach is based on Luehrman, T. A. (1997/1998a/1998b). He agreed that the investment valuation is still based on the capital budget analysis. A corporate investment opportunity is like a call option because the corporate has the right, but not obligation, to acquire payoffs form the opportunity. Thus, the investment value composes of two problems: a capital investment problem and an investment opportunity problem.

The assumptions of subjective approach are as follows:

* The real options value is the opportunity value. It can be viewed as the value of a European call option.
* There is a replicating portfolio between a financial asset and a real investment. The project is tradable.
* The project (the underlying asset) is in a party complete market. There is no arbitrage in the tradable project market.

* If assume that the dynamic value of a project follows geometric Brownian motion process.

3.4.2 Mechanism of the Subjective Approach

3.4.2.1 Project Value

Luehrman claims “project valuators who want to begin using the option pricing technique need not discard their current DCF – based systems”. The net present value is the investment intrinsic value. NPV can be obtained from the traditional DC method. The conventional NPV of the traditional discounted cash flow valuation is a starting point of a project. The real options value is the value of an opportunity in the project. The real options value can be derived from Black–Scholes option pricing model. The total project value is:

$$\text{Project Value} = NPV + \text{Option Value} \qquad 3.2$$

$$NPV = PV - \text{investment costs} \qquad 3.3$$

3.4.2.2 Inputs of the Subjective approach

Table 3.2: Inputs of the Subjective Approach

Financial Option	Symbol	Real Options
Stock price	S	Present value of a project
Exercise price	X	Investment costs
Expiration time	T	Time unit the opportunity elapse
Stock price volatility	σ	Standard deviation of project returns
Risk free interest rate	rf	Time value of money

S = Present value of a project, which is from the traditional DCF method

X = Expenditure required to acquire the project assets

T = Length of time the decision may be deferred

rf = Risk free interest rate

σ = standard deviation of project returns (see chapter 3.3.2.3)

3.4.2.3 Project Return Vitality

Luehrman suggested “the variance is a summary measure of the likelihood of drawing a value far away from the average value”. It is obviously that high variance assets are riskier than low variance assets. Variance is a measure of uncertainty. A time dimension is needed as well; how much prices can change while investors wait depends on how long investors can afford to wait. Time is relevant to option costs. Timing as an opportunity, investors must pay to keep an opportunity. For business projects, business circumstances could change much more if investors wait two years than if an investor waits only two months. Thus, in the option valuation, Luehrman applied the term "variance per period". He suggested using "the variance of project returns percentage gained (or loss) per year" rather than using the variance of project values. The volatility is the square root of variance. He ensured the uncertainty is associated with time and the volatility of project returns.

He proposed that the Monte Carlo simulation technique could be applied to synthesize a probability distribution of project returns, when the project risk is nonlinear with project returns.

3.4.2.4 Evaluation Value of Flexibility

$$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \qquad 3.4$$

$$d_1 = \frac{ln\left[\frac{S_t}{X}\right] + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Where:

C_t *= price of real (delay) option at time t*

S_t *= project present value*

N( ) = the cumulative normal density function

*X = exercise price (*x *= costs of delaying for bidding + costs of investment)*

T = Expiration date of the option

t = Present time

rf = Risk – free interest rate

σ^2 *= Variance of project returns*

3.4.3 Conclusion from the Subjective Approach

Ct is the opportunity value of an investment project. From equation 3.4, the opportunity value depends on time, investment costs, and the volatility of project returns. Luehrman suggested that the volatility σ quantifies all elements of uncertainty including technical and market.

Technical uncertainty

Luehrman assumed that the traditional DCF method and Monte Carlo simulation could model technical uncertainty. The option pricing model could value the technical flexibility. This value of the technical flexibility is associated with investment costs. The investors must allocate costs to have this option. Based on equation 3.3, the value of the technical flexibility increases, the investment costs increase, and the net present value of the project decrease as the volatility increases.

Market uncertainty

Based on equation 3.3 and 3.4, Luehrman also take the market opportunity value account into the volatility of project returns. The volatility of the project returns also represents the market uncertainty. He claims that the market flexibility derives form the price uncertainty. The price uncertainty is associated with investment costs, which is considered in the net present value.

However, Mason and Merton (1985) pointed out "the value of a non-traded project is the price the project would have when it is traded in the market". The financing flexibility arises from trading projects and managing price fluctuations: when arbitrage opportunities arise in the market, the value of trading projects will be ignored in this approach.

The real options value, as the opportunity value, arises from financing flexibilities and technical flexibilities. Therefore, the subjective approach misses the value of the flexibilities for trading project.

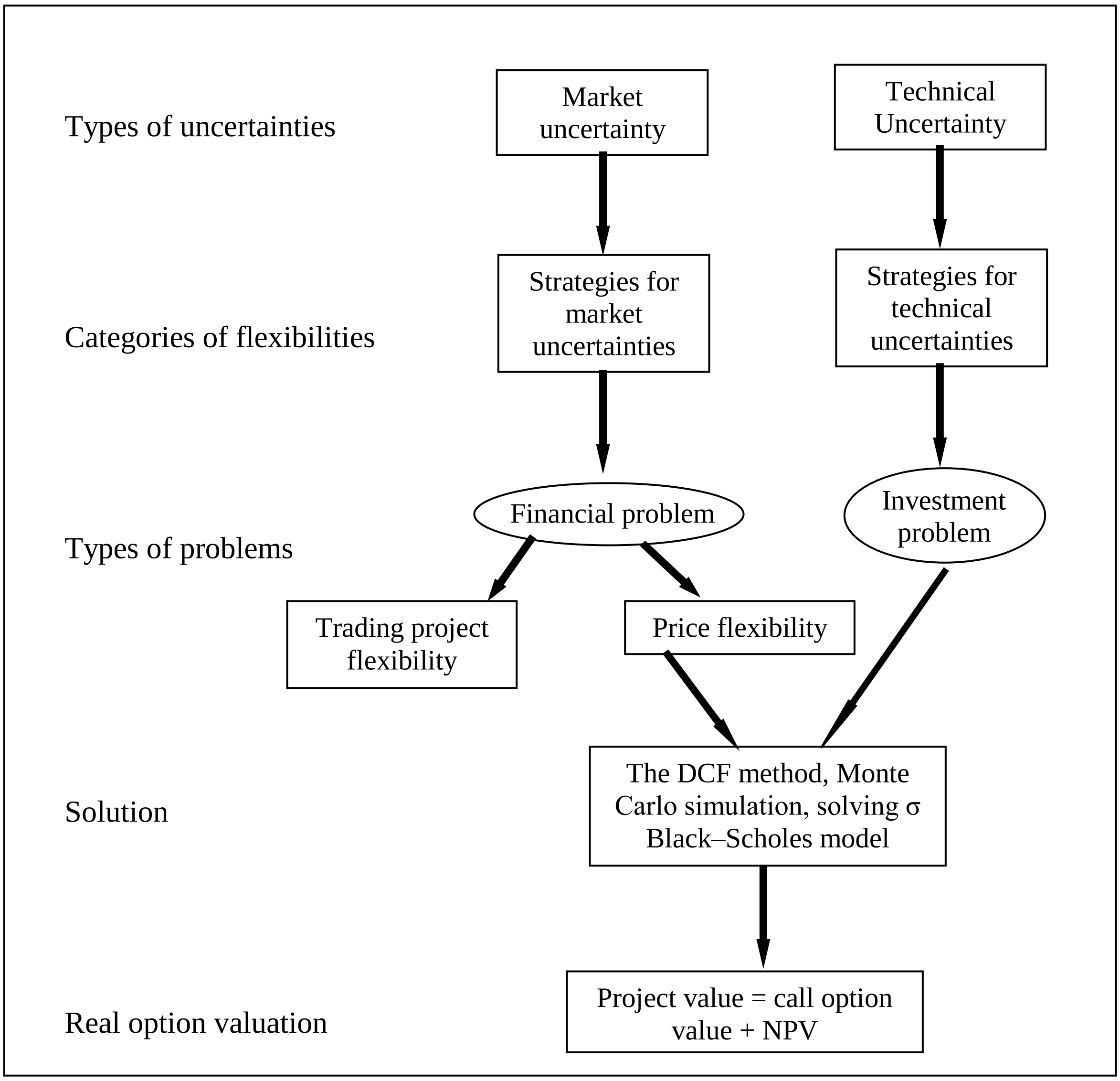


Figure 3.2: Modelling Uncertainties in the Subjective Approach

3.5 Market Asset Disclaimer (MAD)

3.5.1 Assumption

According to Copeland et al (2000), the MAD approach essentially supports "the present value of a traded project is an unbiased estimate of its market value when there are no flexibilities and opportunities in investments". As other approaches, the real options value here arises from technical flexibilities and market opportunities. The magnitude of the value of flexibility depends on the uncertainties. The more uncertainty (or volatility), the higher is the value of the project flexibility.

The assumptions of MAD are as follows:

* The market of real assets is not complete, or it is "partly" and arbitrage free
* There is no replicating portfolio between a financial asset and a real investment

The volatility of oil prices in the financial market is not same as the volatility of a commodity (oil reserve) in the real investment.

* The present value is the inherent value of the company investment.
  That is, the project value = present value + real options value
* The price of the underlying asset moves stochastically. The price follows a geometric Brownian motion.
* Investors' attitude is risk neutral

They applied the risk neutral probability approach. They argued that the traditional DCF method could value the intrinsic value for an investment. NPV could not evaluate opportunity. Uncertainties may affect future cash flows throughout the project whole life. And thus, they integrate the traditional DCF method with the option pricing model to evaluate project opportunities; instead of the Black–Scholes Operation Pricing Model they apply the Binomial Tree Model. Binomial tree model makes fewer assumptions than Black–Scholes Model. The Binomial tree Model permits the underlying asset to be in an arbitrage free and complete market. The model has much wider application than the Black–Scholes Model.

3.5.2 Four Steps in the Market Asset Disclaimer Approach

3.5.2.1 Build up a Present Value Spreadsheet

The commodity price uncertainty and the production uncertainty are first defined in a present value spreadsheet. The present value is discounted by WACC.

$$PV = \sum_{t=0}^{T} \frac{Cf(t)}{(1+WACC)^t} \qquad 3.5$$

Where:

PV = Present Value at time t

Cf(t) = Free Cash Flow at time t

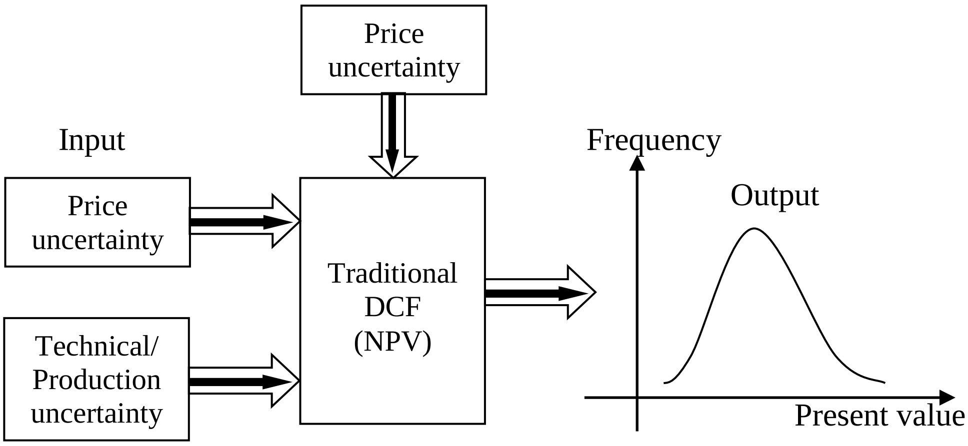
WACC = Weighted Average Capital Costs Rate

T = Project Life

t = Project at time t

3.5.2.2 Define Uncertainty Variables

Because of the non – linear relationship between risks and the project returns, Copeland et al used Monte Carlo simulation to estimate the volatility of project returns.



$$z = ln\frac{PV_t+FCf_t}{E(PV_0)} \quad 3.6$$

$$E(PV_0) = \frac{E(FCf_t)}{(1+WACC)^t} + \frac{E(PV_t)}{(1+WACC)^t} \quad 3.7$$

$$PV_t = \sum_{t=2}^{T}\frac{FCf_t}{(1+WACC)^{(t-1)}} \quad 3.8$$

When the simulations are run, one will obtain:

$$\sigma = StdDev(z) \quad 3.9$$

In the thesis, Copeland's example is replicated for estimating the volatility (the project returns volatility).

3.5.2.3 Binomial Tree

Binomial tree forecasts the evolution of the project value (including option costs)

3.5.2.3.1 Up and Down Factor

The up and down factor are functions of the time increment and the project return volatility. See equations 3.10 and 3.11.

$$u = e^{\sigma\sqrt{dt}} \quad 3.10$$

$$d = \frac{1}{u} \quad 3.11$$

3.5.2.3.2 Rick Neutral Probabilities

$$\frac{(1+r_f)-d}{u-d} + \frac{u-(1+r_f)}{u-d} = 1 \quad 3.12$$

$$p = \frac{(1+r_f)-d}{u-d} \quad 3.13$$

3.5.2.3.3 Evolution of Project value and Net Option Value

1. Evolution of the net project value:

$$\begin{tikzcd}[column sep=large]
 & V_{u1}=PV\text{x}u \arrow[r] \arrow[dr] & V_{u2} = PV \text{ x } u^{2} \\
PV \arrow[ur] \arrow[dr] & & V_{no\ min\ al} = PV \text{ x } u \text{ x } d \\
 & V_{d1}=PV\text{x}d \arrow[ur] \arrow[r] & V_{d2} = PV \text{ x } d^{2}
\end{tikzcd}$$

2. Net Option Value

$$\begin{tikzcd}[column sep=large]
 & V'_{u1} \arrow[r] \arrow[dr] & V'_{u2}= max\ [(V_{u2} - C_{option}),0] \\
V_{option} \arrow[ur] \arrow[dr] & & V'_{ud}= max\ [(V_{ud} - C_{option}),0] \\
 & V'_{d1} \arrow[ur] \arrow[r] & V'_{d2}= max\ [(V_{u2} - C_{option}),0]
\end{tikzcd}$$

$$V'_{u1}= [pV'_{u2} + (1 - p)V'_{ud}]/(1+r_{f}) \qquad 3.11$$

$$V'_{d1}= [pV'_{u2} + (1 - p)V'_{d2}]/(1+r_{f}) \qquad 3.12$$

$$V_{option}= [pV'_{u1} + (1- p)V'_{d1}]/(1+r_{f}) \qquad 3.13$$

$$V_{project} = V_{option} + PV \qquad 3.14$$

Equation 3.13 shows how to computer the project value using risk neutral probabilities. The value of the project with the (embedded) option is given by 3.14

3.5.3 Conclusion from the Market Asset Disclaimer Approach

As shown in figure 13 below the market asset disclaimer approach is similar as the subjective approach. The difference between the two is the former use the binomial tree model while the latter uses the Black–Scholes Model.

Technical uncertainty

The MAD approach assumes that the technical uncertainty can be valued by the traditional DCF method. The value of the technical flexibility is the opportunity value in the technical uncertainty and is modeled by the binomial tree model.

Market uncertainty

The price flexibility is opportunities in the market uncertainty. It is also modeled by a binomial tree mode. As the subjective approach, the value of trading projects is also missed in MAD, since the project volatility used is the price volatility

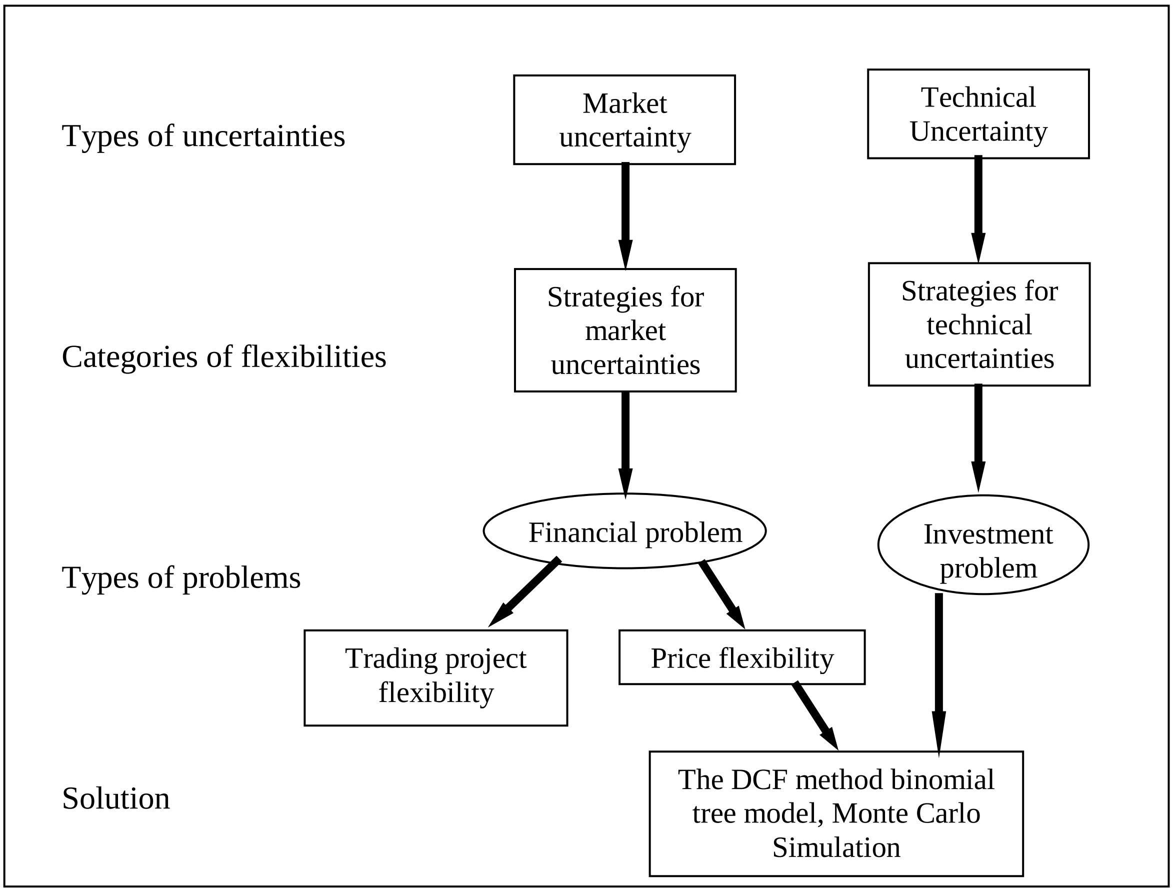


Figure 3.3: Modelling uncertainties in MAD

3.6 The Smith Approach

3.6.1 Assumption (two types of risks)

Smith et al (1995/1998a/1998b) introduced the decision tree analysis into real options valuation. The assumptions of the Smith approach are presented as follows:

* The market is not complete and arbitraged, which is closer to reality
* Throughout their papers, they assume the project could be traded as the financial security. The firm can obtain projects by trading the project without taking risks or expending additional capital, because the market is arbitrage free.
* Stochastic process: the prices and the production rate follow random walk.

  There are two kinds of risks in the investment market risk and private risk. On the other hand, the private risk arises from the individual project. It is unsystematic. The private risk is mainly associated with the technical uncertainty.

These two types of risks should be modeled in different ways.

* An investment decision problem can be separated into two subs – problems Smith et al. suggest problem one: the project state. It is an investment problem, in which project

investors' focus on the capital budget decisions. Problem two is a financing problem – the security stage, in which investors focus on the opportunity to buy or sell the project.

In the project state, decision makers face a pure investment problem. Investors invest capital to own the project. There are private risks in this stage. Private risks are associated with uncertainties such as in the commodity quantity (large or small oil reserves) and in the techniques (the success or failure of operated drilling techniques or well treatments). They assume PV (using discounting by the risk free rate) could represent capital costs; and it is a pure investment issue. Project risks in this state mainly come from capital investments.

After inventors obtain the ownership of the project, the investment enters the second stage – the security state, investors now face a financial problem. Uncertainties in this stage come mainly from the market. "If the security market is complete, all project risks can be perfectly hedged by trading the security." Smith suggested project managers can trade or borrow the project, just as one can trade a financial security. Downside risks can be hedged in this way.

3.6.2 Integration of Decision Analysis and Option Price Techniques

Smith and Nau (19950 proposed, "Decision analysis and option price techniques can be profitably integrated". The option pricing method – Binomial tree model or the risk neutral pricing techniques can value public or market uncertainties. Decision tree analysis can model private uncertainties. They also suggested that if the option pricing model and decision analysis methods are correctly applied, two methods must give consistent results.

3.6.3 Three Steps in the Smith Approach

3.6.3.1 The Investment Problem in the Incomplete Market

For private risks, the project is an investment problem in an incomplete market.

* Sketching (constructing) a decision tree for the investment
* Calculating the present value of each end of the node;

$$s(0) = \sum \frac{s(t)}{(1+r_f)^t} \qquad 3.15$$

* Using subjective probabilities and the certainty equivalence

3.6.3.2 A Financing Problem in the Complete Market

For market risks, the project is a financial problem in a complete market

* Using risk neutral probabilities
* Using market information

$$u = e^{\sigma\sqrt{dt}} \qquad 3.16$$

$$d = \frac{1}{u} \qquad 3.17$$

$$\frac{(1+r_f)-d}{u-d} + \frac{u-(1+r_f)}{u-d} = 1 \qquad 3.18$$

$$p = \frac{(1+r_f)-d}{u-d} \qquad 3.19$$

* Replacing the expected present value with the present value

$$s(0) = \sum \frac{\pi(t)}{(1+r_f)^t} s(t) = E_\pi\left[\frac{s(t)}{(1+r_f)^t}\right] \qquad 3.20$$

π is risk neutral probabilities. T is time

3.6.3.3 Choosing Branch with the Maximum Value

$$v' = \max E_\pi\left[\sum_{t=0}^{T} \frac{C_\alpha(t)}{(1+r_f)^t}\right] \qquad 3.21$$

* Upper bounds of the project value in the incomplete market

$$\bar{v} = c(0) + \binom{min}{\beta}\{\beta(0)s(0): [\beta(t-1) - \beta(t)]s(t) \geq c(t)\};\ t > 0$$

$$\bar{v} = \binom{sup}{\pi\epsilon\prod} E_\pi\left[\sum_{t=0}^{T} \frac{c(t)}{(1+r_f)^t}\right] \qquad 3.22$$

* Lower bounds of the project value in the incomplete market

$$\underline{v} = c(0) + \binom{max}{\beta}\{\beta(0)s(0): [\beta(t-1) - \beta(t)]s(t) \leq c(t)\};\ t > 0 \qquad 3.23$$

$$\underline{v} = \binom{inf}{\pi\epsilon\prod} E_\pi\left[\sum_{t=0}^{T} \frac{c(t)}{(1+r_f)^t}\right] \qquad 3.24$$

Where

$s(t)$ = *project price at time t*

w_t = *given state of information at time t*

$x(t,w)$ = *risky cash flow streams at time t in state w*

c_α = *pay offs generated from a strategy* α

βr = *a replicating trading strategy*

3.6.4 Price and Production Models

Project price follow a geometric Brownian motion model

$$ds(t) = \mu_p s(t)dt + \sigma_p s(t)dz_p(t) \qquad 3.25$$

Production output also follows geometric Brownian motion mode;

$$dq(t) = \mu_q[q(t) - L]dt + \sigma_q[q(t) - L]dz_q(t) \qquad 3.26$$

3.6.5 Conclusion from the Smith Approach

* The option pricing model and decision analysis approaches are able to model the opportunity. The decision tree, or dynamic programming model, and the option pricing model complement each other. See Figure 3.4 below.
* Investing in the project is a priority. The initial investment seems to be a “ticket”. Only after investor buy this “ticket”, and they have rights to implement financing flexibilities. In other words, investors need to invest capital to obtain the project, and then they have opportunities to own flexible options in the trading project period. During the investment period, investors compute the project NPV by using the risk free rate.
* During the security period, the project likes a security. Investors could buy or sell the project, just as trading a security in the financial market. There are a variety of trading opportunities in this period.
* Discrete decisions tree

The decision tree in the Smith approach is a “discrete tree”, split by time periods of the cash flows. It is different from the traditional decision tree.

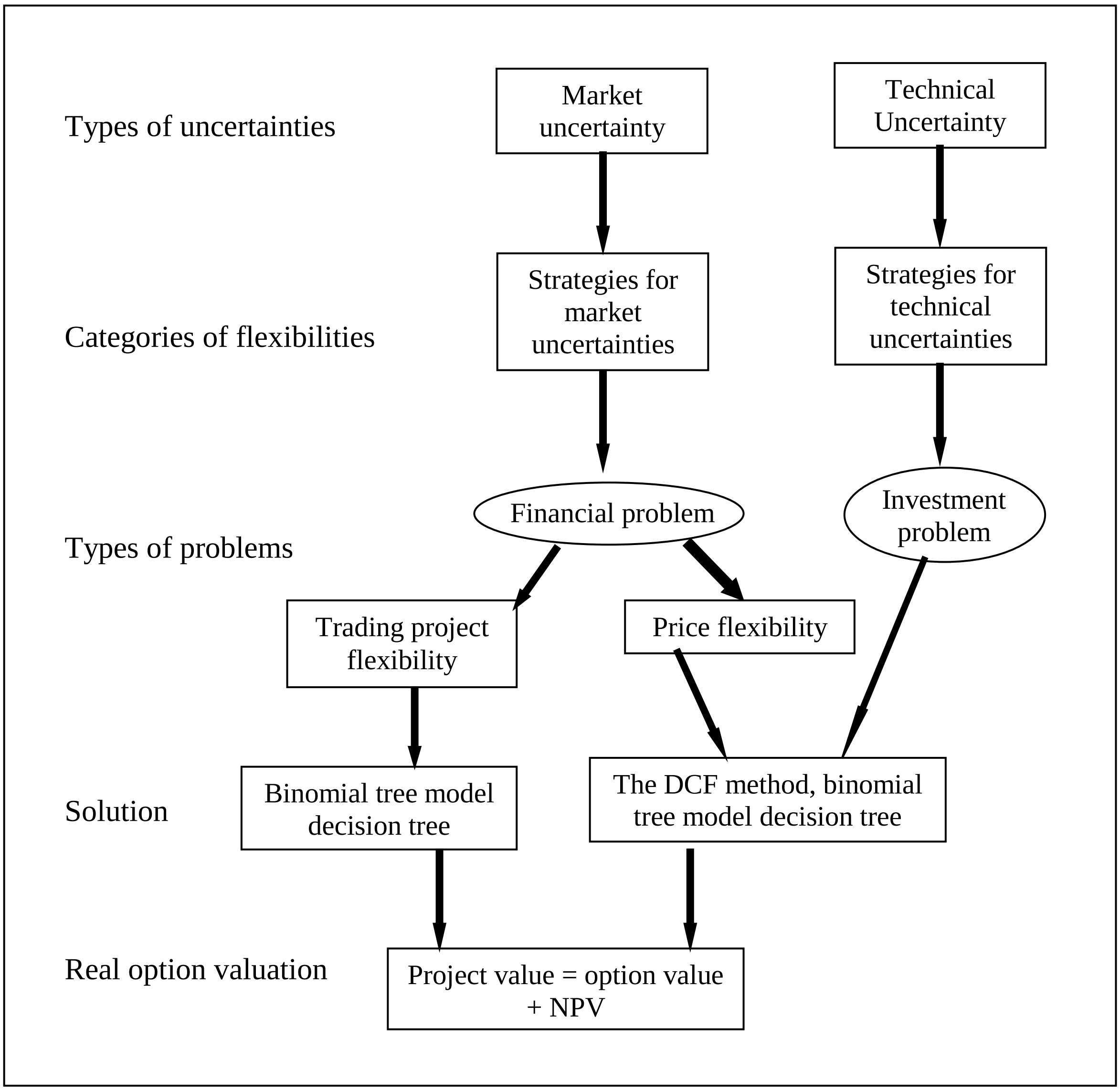


Figure 3.4: Modelling Uncertainties in the Smith Approach

3.7 The Luenberger Approach

3.7.1 Assumptions (dual decision trees)

Luenberger (1998) also developed a similar approach; he assumed investors should purchase the project and then carry it on as an option.

* The market is not compete and arbitrage free
* There are two types of uncertainty market uncertainty and private uncertainty
* The value of the underlying asset follows the GBM stochastic process
* Luenberger assumes the project starts at the "zero level pricing"

3.7.2 The Mechanism of the Luenberger Approach

In this example, he applies subjective probabilities to the increasing and decreasing of oil production with private uncertainty. He implemented the standard binominal lattice approximation to determine the up and down factors, as ell as risk neutral probabilities to handle the market uncertainty.

3.7.2.1 Private uncertainty

For the investment problem, the project is in the incomplete market:

* Calculating the present value of each end of the node

$$s(0) = \sum \frac{s(t,w)}{(1+r_f)^t} \qquad 3.27$$

* Using subjective probabilities and the certainty equivalent

3.7.2.2 Market uncertainty

* Using market information and the risk neutral probability approach

$$u = e^{\sigma\sqrt{dt}} \qquad 3.28$$

$$d = \frac{1}{u} \qquad 3.29$$

$$\frac{(1+r_f)-d}{u-d} + \frac{u-(1+r_f)}{u-d} = 1 \qquad 3.30$$

$$p = \frac{(1+r_f)-d}{u-d} \qquad 3.31$$

* Replacing the expected present value with the present value

$$s(0) = \sum \frac{\pi(w_t)}{(1+r_f)^t} s(t,w) = E_\pi\left[\frac{s(t)}{(1+r_f)^t}\right] \qquad 3.32$$

π is risk neutral probabilities. The solution of the risk neutral probabilities is same as that of the MAD approach.

3.7.2.3 Project Value Model

The project starts at the "zero level pricing".

V = Revenues - cost + 1/R (risk neutral value of next period) 3.33

V = flow x *oil price - (fixed costs + variable cost) + 1/1.4* x *(risk neutral value of next period)* 3.34

3.7.2.4 Time Discrete Decision Tree

Just as in the Smith approach, Luenberger also integrates the decision analysis with option price techniques. This also uses it time discrete decision tree approach. The decision tree is split by time periods of cash flows.

3.7.3 Conclusion from the Luenberger Approach.

The Luenberger approach is similar to the Smith approach. The difference is that the project starts at "the zero price" which is the modified present value.

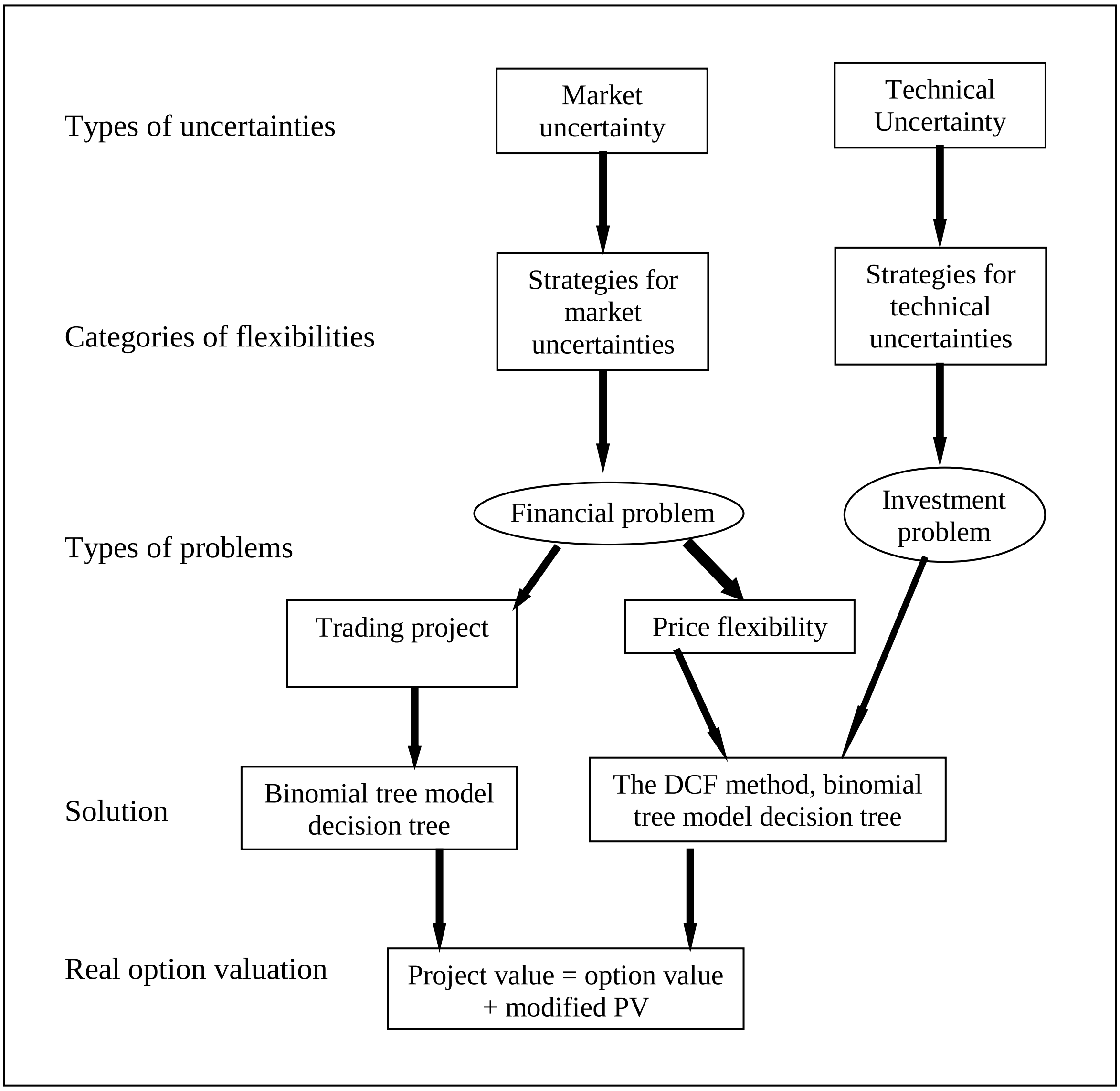


Figure 3.5: Modelling Uncertainties in the Luenberger Approach

CHAPTER FOUR

4.0 REAL OPTION VALUATION – AN OFFSHORE PETROLEUM LEASE EXAMPLE

In this chapter, an offshore petroleum lease project is used to illustrate the correct application conditions for the selected real option approaches. The empirical example is constructed based on the pioneering papers of Paddock *et al.* (1988) and Smit (1997). The development of an offshore petroleum lease consists of sequential investments in exploration, development and extraction (production) of oil. During the stages of development, managers are often able to get a better sense about revenues or costs regarding the investment, and so review and change their decisions based on new information. Traditional DCF techniques are unable to capture all important aspects of the investment and often cannot generate a proper value of the investment. They fall short in reflecting the varying uncertainty of future cash flow and capturing managerial flexibility. As a result, application of the standard DCF to such investment decisions would not maximize the value of a firm. For example, the firm would only choose to continue with the investment if the current expectations for future revenue are such that the further investment remains profitable. As such, an offshore petroleum lease project is often modeled as a sequential compound real option problem. That is, ROA can explicitly incorporate the various sources of managerial flexibility that are attached to the investment opportunity and that allow the firm to commit itself sequentially to further investment decisions.

From a methodological standpoint, a real option approach offers a rich understanding of the dynamic nature of investment and its relationship with the capital market. In terms of real options, analysts seek objective market information to determine project values, real option values and optimal strategies. One of the major theory-practice gaps of the real option approach is, however, that the process of identifying objective market information can be difficult to execute in many real-world situations. This is particularly true when the uncertainties are not well-defined by the capital market, and hence are difficult to characterize, or the uncertain ties are out of the market scope, and hence are unable to be characterized. Thus, there are unresolved methodological gaps that constrain the application of real options to major investment decisions (Mahnovski, 2006) and a Classic Real Option approach can rarely be used as the sole basis for project valuation. The practical solutions offered in this thesis are to relax some severe assumptions regarding perfect replication by allowing for a "self-constructing" technique if the market proxy is difficult to determine precisely; and/or the combination of some supplemental methods (e.g., Decision Analysis) to deal with un-tracked uncertainty.

The development of an oil reserve requires substantial investments which are subject to at least two important sources of uncertainties: market uncertainty (mainly oil prices), and technical uncertainty (the properties of the reserve). Uncertainties and managerial flexibility (options) are critical in order to maximize the value of the investment opportunity. Hence, an offshore petroleum lease project needs to be valued by a dynamic option pricing framework in a partially complete market condition.[13] For market uncertainty, the valuation process relies either upon finding information about the traded asset (e.g., oil), or by assuming that the present value of the cash flows of the project without flexibility is the market price, as if the project were traded (i.e., MAD assumption). For technical uncertainty, the valuation process has to rely on subjective assumptions and estimates. Hence, for an offshore petroleum lease project, the valuation process typically requires different kinds of real option valuation approaches in order to properly capture market uncertainty, geological uncertainty, and different types of flexibility.

4.1 Stages of an Offshore Petroleum Development

Oil production is a multi-stage process involving sequential investment decisions. According to Paddock *et al.* (1988), a typical offshore petroleum lease has three sequential development stages: the exploration, development and extraction (production) of oil. The stage-investment decisions are determined by joint geological and market uncertainty. Option-based valuation approaches are appealing because each stage typically involves different kinds of market conditions, project uncertainties and managerial flexibilities. An analysis of flexibility must therefore account for these factors before making irreversible investments. There are three stages as follows.

Firstly, the exploration stage involves seismic and drilling activities to obtain geological information on the presence of hydrocarbons, the size of the reserves, and the cost of extracting oil. When the firm acquires an unexplored reserve, it has the option to decide whether and when to invest in the exploratory drilling (e.g., the option to start test drilling and the option to invest in appraisal wells) to receive an explored but undeveloped reserve. This is analogous to a financial call option, where the stockholder has the right to pay the exercise price and receive the stock. Since the decision to explore a reserve entails an irreversible investment, the firm should accurately analyze the project's future cash flow expectations, which in turn depend on both geological and market uncertainty.

Secondly, the development stage involves development activities (e.g., construct platforms) to convert an undeveloped reserve into a developed reserve. This stage only occurs if the quantity of hydrocarbons, the magnitude of development costs and the current expectations for future revenue are favorable. As the development stage requires the largest capital expenditures, and they are not easily recovered once investment is undertaken, this is where option value is most important (Smith, 1997). Dixit and Pindyck (1994) argue that the option to delay the development stage is the most valuable option in the oil industry. For example, given that a firm inherits some flexibility in deciding whether and when it is optimal to develop, such a project is always worth more than a project without this flexibility.

Finally, the extraction (production) stage involves using the installed capacity to extract oil over some period of years. At this stage, most geological and technical uncertainties have already been resolved through exploration and development. As such, the project uncertainty is now dominated by exogenous market uncertainty (e.g., oil price uncertainty). As the market uncertainty evolves over the life of the project, the firm may have the operating options to scale (e.g., expand or contract) or abandon the producing field. For instance, the firm may alter the production rate in response to oil price changes. In addition, it may be possible to expand or contract the capacity of the facility. More extreme options may involve shutting down reserves temporarily or even abandoning the producing field early. Figure 4.1 gives an overview of the staged development processes.

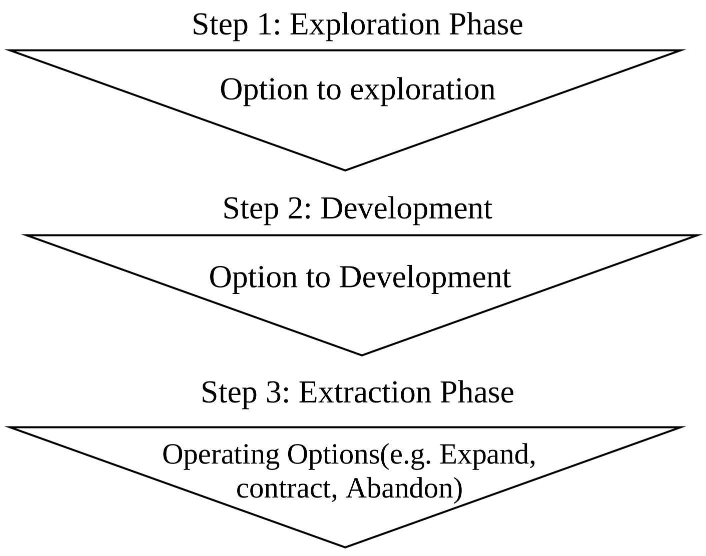


Figure 4.1: Staged Investment Processes for an Offshore Petroleum Lease Project

4.2 Modeling of Uncertainty

Performing a real option valuation requires that we make assumptions about which variables affect the value of the project and its associated real options (Grafström & Lundquist, 2002). In the real option literature, the evolution of these variables over time is typically modeled as some sort of stochastic process. Option-based models may be highly sensitive to the way the behavior of the critical variables is modeled (Triantis, 2005). In our case, three critical variables are considered: the oil price, the variable operating costs and the reservoir volume. They are specified asfollows:

4.2.1 Oil Price

Oil price uncertainty is clearly a market-priced uncertainty, which evolves and changes over time. It has long been modeled using no-arbitrage finance models in both continuous-time and discrete-time settings. The vast majority of real option applications model the commodity price stochastic process as a Geometric Brownian Motion (GBM) (Paddock *et al.* 1988). This assumption implies that future commodity price is lognormal distributed. Another way of modeling the evolution of commodity price is to assume that it follows an Ornstein-Uhlenbeck (OU) mean-reverting process (Laughton & Jacoby 1993). The OU process describes a commodity price that has a tendency to revert back to some long-run average level over time. The commodity price could also be modeled by a jump process, given that the commodity has a tendency to be exposed to price shocks (Grafström & Lundquist, 2002). In addition, Arnold, Crack and Schwartz (2007) show how to model a commodity price using an implied binomial tree that allows for general distributions. More advanced modeling practice assumes that oil price uncertainty is driven by several sources and applies multi-factor models to capture its resolution over time. For example, Gibson and Schwartz (1990) develop a two-factor model where the spot prices follow a GBM stochastic process and the instantaneous convenience yields are mean reverting. Detailed mechanics of these stochastic processes can be found in Dixit and Pindyck (1994), Hull (1999), McDonald (2002), Gibson and Schwartz (1990), and Wilmott (1998).

Another important parameter to model the evolution of oil prices in the GBM setting is the volatility of the oil price. One problem encountered when estimating the volatility is that there are no long-term financial instruments (such as long-term oil futures or oil options) available, which match the time horizons of the project, and allow us calculate the implied volatility. Alternatively, we can use the historical spot oil prices to estimate the volatility. Based on the discussion of Smith (1997), one way to obtain the volatility of the oil price is to calculate the standard deviation from the time series of spot-market oil prices and use this historical standard deviation as an estimate for the volatility of oil prices. The actual volatility may change over time and the historical data may or may not be a good predictor of the future (see Sharma, 1998 and Kroner *et al.*, 1995 about the comparisons of historical volatility, implied volatility and realized volatility for commodity price).

To mimic a GBM stochastic process in a discrete-time model, for each time step Δt the future oil price can move up or down by multiplicative factors u or d respectively. These factors u and d are determined by the estimate of the oil price's volatility (equation 4.1). The risk neutral probability, p, is based on adynamic replication strategy using a traded oil instrument and a risk-free bond (equation4.2).

$$u = e^{\sigma\sqrt{\Delta t}}, \; d = 1/u \qquad 4.1$$

$$p = (e^{(r-\delta)\Delta t} - d)/(u-d) \qquad 4.2$$

4.2.2 Variable operating costs

Due to the high uncertainty about the operating cost structure of the project (e.g., unexpected cost and labor cost etc), we assume that the project has an additional source of market uncertainty: variable operating costs. However, unlike the evolution of future oil prices, the uncertainty about the variable operating costs does not have rich and observable market information with which to measure it. Thus, we project this uncertainty and its evolution over time based on the management's subjective opinion. We assume it follows a GBM stochastic diffusion process with a mean annual rate of increase of 1% and an annual volatility of 3% (see Brandão *et al.* (2005) and Borison, (2005) who make similar assumptions).

4.2.3 Reservoir volume

Reservoir volume uncertainty is unrelated to the overall economy, and is therefore project specific. Reservoir volume uncertainty is high during the initial exploration phase. Once exploration investment is undertaken, uncertainty on the reservoir volume decreases and the market uncertainties such as oil prices become comparatively more important in the later phases.

4.3 Real Option Valuation Procedure

The valuations and decisions relating to whether, when and how the investment should actually proceed depend on a series of development milestones being successfully achieved. At each stage, the uncertainties surrounding the project resolve, evolve and change over time. For example, the initial decision to invest in exploration is reached by analyzing all the future consequences taking into account technical uncertainty and market uncertainty over the life of the project. If the exploration phase proves economically exploitable reserves, the decision to invest in development can be based on the geological information discovered during the exploration phase, the capital expenditures required for development and the uncertainty about future market conditions. It indicates that an offshore petroleum lease project can be modeled as a sequential compound option; each stage provides an option to complete the next stage (Paddock *et al.*, 1988). There are many important previous studies which demonstrate how the theory and methods of real options are applied in an oil project. For example, Paddock *et al*. (1988) evaluates an offshore oil reserves using standard option-pricing technique. Smith and McCardle (1998, 1999) illustrate how to apply an integrated option pricing approach in the context of oil projects and give a discussion of lessons learned in the application.

In order to perform real option valuation and quantitatively analyze the value of the real option embedded in the offshore petroleum lease project, a backward induction process is applied. That is, the analytical convention is to start valuing the producing fields at the extraction (production) stage and the undeveloped reserves at the development stage with each potential size of reserves, and then working backward in time to value the unexplored reserve at the exploration phase. For the purpose of this thesis, we make some assumptions to simplify the valuation process. The structure of the valuation process proposed in this thesis is, however, able to deal with more complex situations. To conduct the valuation, like many real world oilfield applications, the staged investment is separated into two market scenarios. At the extraction (production) and development stages, the technical uncertainty regarding the quantity of the reserve is largely resolved by exploratory drilling. So, it is assume that the project uncertainty is dominated by market uncertainty. This is called the "market uncertainty dominated scenario". At the initial exploration stage, the option to invest in exploration highly exposes us to market uncertainty but also technical uncertainty. This is called the "mix-uncertainty scenario".

4.4 Valuation of Undeveloped Oilfields under a Complete Market Scenario

According to the discussion of the previous section, in order to value an unexplored reserve we start by valuing a set of undeveloped oilfields. Then, we value the option to invest in exploration based on the valuation results and the probability distribution of potential reserves based on the prior geological and geophysical data. In this section, two common real option valuation approaches will be used to value undeveloped oilfields. One implements the Classic

Real Option approach through a dynamic replication strategy, and the other uses a DCF calculation with "add-on" flexibility based on the MAD assumption. The application conditions for these two methods shall be examined and compared under a complete market scenario.

4.4.1 Classic Real Option Approach through Replication

In this section the Classic Real Option approach based on dynamic replication shall be applied to value undeveloped oilfields contingent on oil price. The basic idea behind this approach is to develop the market value of the project and its associated value of flexibilities through the value of traded securities based on the no arbitrage principle. In order to determine the market value of the asset through replication, it has to meet two basic requirements:

1. The capital market is complete with respect to the project uncertainties.
2. A "twin-security" can be constructed and do a reasonable job of replicating.

In the literature, a common replication strategy is to assume that the values of the project and the real options are driven solely by the fluctuation of oil price (Kemna, 1993 and Smith, 1997). Other variables such as development costs, operating costs and the production rate are considered as deterministic. In this way, the uncertainty of the operating cash flow of the project is determined only by the movement of the oil prices. The modeling practice regarding the movement of oil prices can be based on the publicly traded instruments that include spot oil, oil futures, and a host of derivatives.

4.4.1.1 Estimating the Market Value of the Producing Field

In this case, the value of the underlying asset (the value of the producing field) and its resolution over time can be determined based on operating cash flows, which in turn relate to the fluctuation of oil price. It is clear that the oil price dynamics would result in a closely related, dynamic movement of the operating cash flows. For the future oil price in each state, the operating cash flow equals the oil price times the production rate, minus the operating cost and the fixed cost (For simplicity, we ignore some real world costs such as royalties and taxes). The procedure to determine the market value of the producing field can start at the terminal nodes of the cash flow tree and work backward to the beginning of the production phase. At the terminal nodes, the state project value equals the corresponding state operating cash flow. For the early nodes, equation 4.3 is applied to sum the state operating cash flow when stepping backward in time. For example, the project value at time t is simply the present value of the remaining project cash flows. In this way, the value of the producing field is replicated by the tracking portfolios comprised of oil securities.

$PV_t = CF_t + (pPV^+_{t+1} + (1\text{-}p)PV^-_{t+1})/e^{r\Delta t}$ 4.3

Where:

PV = Project Value,

p = risk neutral probability,

CF = Operating Cash Flow and the "+" and "-" superscripts refer to value in up and down states respectively at the next node of the tree.

4.4.1.2 Real Option Valuation

After determining the value of the underlying asset and its resolution over time, we now can solve for a set of real options. A recursive valuation procedure will be used to determine the value of the producing field with the abandonment option through risk neutral valuation. Equation 4.4 is used to determine the value at the terminal nodes. Equation4.5 is used to determine the value at the intermediate nodes when stepping backward in time. $FPV_0$is the current value of the producing field with the abandonment option.

$FPV_{t+1} = MAX\ (AV, PV_{t+1})$ 4.4

$FPV_t = MAX\ (AV, (pFPV^+_{t+1} + (1\text{-}p)FPV^-_{t+1})/e^{r\Delta t} + CF_t)$ 4.5

Where:

FPV = Project Value with flexibility

PV = Project Value

AV = Abandonment Value

p = risk neutral probability

CF = CashFlow

Now we move to the early development phase. To commence the production, the firm has to decide whether and when to make capital investment in infrastructure (e.g., a drilling platform). There is, of course, no obligation to start the development of the oilfield immediately, but rather a right which will only be excised when the expectations for future revenue are most favorable (Kemna, 1993). The time to maturity of the deferment option is not always so clear-cut for most real applications. However, in the light of high uncertainty of future oil price and huge irreversibility outlay, this wait-and-see approach clearly has value. For example, if the value of the deferment option is worth more the costs, management can extend the investment phase and wait for higher oil prices. In other words, if the firm invests today, it kills the opportunity of investing in the future, when the market conditions may be more favorable. To value this deferment option, we use the value of a producing field including the option to abandon as the value of the underlying asset to calculate the early deferment option. At year 1, we have the option to invest or abandon at each state (Equation 4.6). At year 0, we have the option to invest now, abandon, or wait-to-see (Equation 4.7). $NPV_0$is the current value of the oilfield with the compound option.

$$NPV_1 = MAX(FPV_1\text{-}I,0) \qquad 4.6$$

$$NPV_0 = MAX(FPV_0\text{-}I,\ 0,\ (pNPV^{+}{}_{1} + (1\text{-}p)NPV^{-})/e^{r\Delta t}) \qquad 4.7$$

Where: $_1$

FPV = Project Value with the Abandonment Option,

p = Risk Neutral Probability

I=Development Cost

NPV = Project Value with the Compound Option

It should be noticed that the value of this compound option is not simply the value of the abandonment option plus the value of the deferment option. This is because the multiple options embedded in a project might interact; hence value activity may break down (Trigeorgis, 1993). In our case, the presence of a later abandonment option enhances the value of the underling asset for a prior deferment option, while exercising an earlier deferment option may alter the scale of the later option.

4.4.2 MAD Approach through a "Self-construction" Technique

In the previous section, traded oil securities was used to serve as a "twin security" for a producing oilfield. The empirical question arising from the previous section is: does the modeled oil price fluctuation really do a reasonable job of replication for a producing oilfield? The answer should vary from project to project. For example, oilfields operated in developing countries may suffer some additional market uncertainties relating to risks such as political change, uncertain inflation, and continually changeable taxes and royalties. This could potentially result in the stochastic fluctuation of the operating costs over time. In general, the real underlying variable is frequently exposed to multiple sources of uncertainty. In this section, we illustrate the MAD approach to value an undeveloped oil reserve using an expanded version of the previous section. In addition to being uncertain about oil prices, we assume that the project has an additional source of market uncertainty: variable operating costs. While there are well-developed financial markets for managing oil price uncertainty, there is very limited market opportunity for hedging operating cost uncertainty. This adds significant difficulties to the process of identifying a replicating portfolio.

4.4.2.1 The Valuation Procedure of the MAD Approach

Roughly speaking, the MAD real option valuation approach relies on the *marketed asset disclaimer* (MAD) assumption. This assumes that the value of the project without flexibility is the best unbiased estimator of the market value of the project. This value is typically obtained through a traditional DCF calculation and serves as the value of the underlying asset for an option valuation model. Then the value of the underlying asset is assumed to evolve over time based on the characters of the uncertain factors, and is often modeled as some sort of stochastic process in a discrete-time setting. Typically, the MAD approach uses the following steps to identify the value of the real options (Copeland & Antikarov, 2001):

1. Estimate the value and volatility of the underlying asset.
2. Build an event tree to model the underlying assets value.
3. Conduct the Real Option Analysis.

Step One: Estimation of the Value and volatility of the Underlying Asset

Value of Underlying Asset: The value of the underlying asset is the present value of a producing oilfield without flexibility. The present value of a producing oilfield is derived from a standard discounted cash flow model that incorporates annual production level, the expected future oil price, and the ongoing operating costs over the assumed economic life of the project. The firm's weighted average cost of capital (WACC) can be used as the discount rate for the project, which can be assumed. The calculated NPV then serves as the market value of the underlying asset for the option valuation model. Nonetheless it is important to bear in mind that this is a very significant assumption that may alter the precision of the results.

Leakage in Underlying Value: The leakage in the underlying value represents a series of payouts (like dividend for stock options) or competitive losses from the underlying assets. It could be the expected net cash flows accruing from a project, the opportunity cost of delaying, or the loss of market share to competitors. This is an important factor in the case of real options involving the delay, abandonment, expansion, contraction, or extension of a project. However, the accurate modelling of project payouts or competitive losses is very difficult as the timing and amount may be dependent on exogenous influences (Miller & Park, 2002).

Volatility: Another important input parameter required to perform real option valuation is the volatility of the return to the underlying asset. According to Copeland and Antikarov (2001), the underlying asset volatility is the standard deviation of the rate of return on the underlying asset (in GBM setting). This standard deviation of the returns, or volatility of the underlying asset, can be estimated through a Monte Carlo Simulation of the underlying asset returns. To conduct a simulation, key project uncertainties (oil price and variable operating costs) are entered as simulation input variables in the project cash flow *pro forma* worksheet. The statistical properties of these input variables such as mean values; standard deviations and probability distributions need to be assigned before running a simulation.

Step Two: Build an Event Tree to Model the Underlying Asset's Value

The second step is to build an event tree to model the evolution of the value of underlying asset over time. As mentioned in chapter 3, the application of the MAD approach implies that the value of the underlying asset follows a random walk. A Geometric Brownian Motion (GBM) stochastic process can be applied to model the evolution of the present value of the producing oilfield. Quirk and Ruthrauff (2006) show that the value of oilfield reserves frequently follow a lognormal distribution. To keep it simple, it can be assumed that the volatility of the underlying asset and project payout derived in the previous section remain constant over the modeling period. This assumption implies that the values of the risk neutral probabilities are constant throughout the lattice. Figure 4.3 shows a binomial model for the evolution of the value of the underlying asset with a single constant volatility measure (i.e., a recombining tree structure).

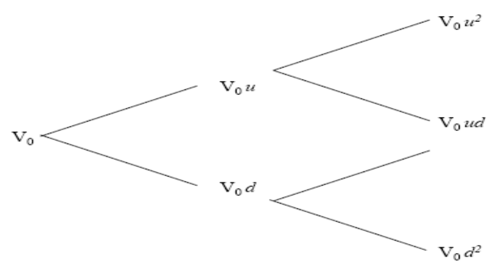


Figure 4.3: The Evolution of the Value of the Underlying Asset

V = The Value of Underlying Asset

u = Upward Multiplicative Factor

d = Downward Multiplicative Factor

Step Three: Conduct the Real Option Analysis

The event tree in the previous step does not have any decisions built into it. The third step in the process of estimating the real option value is putting the decisions that management may make into the nodes of the event tree to turn it into a decision tree (Copeland & Antikarov, 2001). From a valuation standpoint, once the project without options is modeled as a stochastic process, options can be added to the decision tree. This is called an "add-on" approach. To calculate the option value, the risk neutral valuation technique developed by Cox *et al.* (1979) is applied. In this section, like the previous section, we value an abandonment option at the extraction (production) stage and a deferment option at the development stage.

4.4.3 The Comparison of two Approaches in Practical Implementation

Under complete market scenario, in principle, the Classic Real Option approach and the MAD approach are fundamentally the same. As we mentioned in the previous chapter, if the value of a non-traded asset can be perfectly replicated through a corresponding traded asset (i.e., Classic Real Option approach), then the same value also can be deduced in a DCF calculation by employing an appropriate discount rate (MAD approach).

However, in practice, one of the great challenges in implementing the Real Option Analysis is the fact that many market uncertainties are poorly understood and difficult to benchmark from historical data. For many real world applications, whether or not the project value and the real options can be tracked in financial markets remains an operational and subjective judgment. Pendharkar (2010) classifies the market uncertainty faced by a project into two categories: the industry aspect of market uncertainty and the financial market aspect of market uncertainty. The industry aspect of the market uncertainty includes industry characteristics such as operating costs, development costs, demand, supply and competition. The financial market aspect of market uncertainty involves market trading activities such as interest rates, commodity prices and other assets traded in a well-functions market. The financial market aspect of market uncertainty usually has the richness and public availability of historic and near real time data, which allow statistical methods to be used to calculate the structural form of the uncertainty and the degree of volatility. On the other hand, the industry aspect of market uncertainty usually has poor and limited information to measure it, especially for some innovation-related investments. For example, the demand uncertainty for a new drug is hard to track due to the lack of market information. In general, the farther we move away from financial markets, the more difficult and costly it is to track assets and their real options (Amram & Kulatilaka, 1999).

The Classic Real Option approach requires that information about the real investment be projected onto the capital market. The valuation process is straightforward, once we identify the replicating portfolio and the size of the investment relative to the replicating portfolio, and apply standard financial option pricing tools. While it is possible to use market information to track the movement of oil prices, it is doubtful whether there is enough market information available to track an entire oil project. In other words, the value or the volatility of an oilfield is not necessarily replicated by the value or the volatility of the traded oil instruments. In practice, finding a replicating portfolio will be very difficult, if not impossible, for a complicated project. This is because that the information that comes from the capital markets is not always the most appropriate or complete to assess a specific project with specific risk structure. For example, investment projects usually involve various types of market uncertainties with some form of correlation structure. In these situations, the MAD approach is a good alternative if the market value of the investment cannot be replicated in a meaningful way but where market valuation remains the goal. Copeland and Antikarov (2005) point out that the assumptions and conditions necessary to justify the use of the MAD assumption are the same as those that support the use of DCF analysis—and that the MAD approach can be used in any setting where DCF is appropriate. In addition, the MAD approach has great modeling flexibility to deal with muti-source uncertainty which drives the value of the underlying asset. The MAD approach can reduce many sources of uncertainty taking into account their correlations to only one in a consolidated approach, where add an additional input variable does not impact the subsequent computational burden. It also can model different sources of uncertainty and their correlation over time in a separate way (i.e., a separate approach) when it is necessary to do so.

4.5 Valuation of an Unexplored Oilfield – The Luenberger and the Integrated Real Option Approaches

Now we consider valuations and decisions at the earlier *exploration* phase. When the firm considers the development of a tract of land at the exploration stage, the decision problems are even more complicated than previously because the valuation problems are affected not only by exogenous market uncertainties, but also by endogenous project-specific uncertainties. For example, before the exploration phase begins, there are many alternative levels of reserves that could eventually be obtained. The no-arbitrage enforced real option models have difficulty in incorporating various possible specific characteristics of a project into the valuation. The presence of project-specific uncertainty challenges the foundation (e.g., no arbitrage principle) of option theory. This is because capital market contains no information regarding the project-specific uncertainty. We can extend the option pricing approach to distinguish between market uncertainty and project-specific uncertainty (Smith

& Nau, 1995, Smith & McCardle, 1998). We can then value the project through an integrating valuation procedure (e.g., integrating Real Option Analysis and Decision Analysis).

The Integrated Real Option approach works as follows. We first construct a decision tree that uses risk-neutral probability for market uncertainty and physical probability for project-specific uncertainty. Then we propagate the decision tree backwards (i.e., from future nodes back to today) to determine the optimal strategy and its associated values. This procedure takes into account both project-specific uncertainty and the market trading opportunities related to the project. In order to estimate the value of an unexplored oilfield, we use the corresponding values of the undeveloped reserves (including the real options) where each potential reserve size represents the potential value at the end of the exploration phase (i.e., the market state). We then multiply these values by the physical probability of finding the corresponding quantity. To simplify matters, other geological uncertainties such as the quality of the reserve are not taken into account in the valuation. Figure 5.4 shows the corresponding decision tree for the unexplored oilfield.

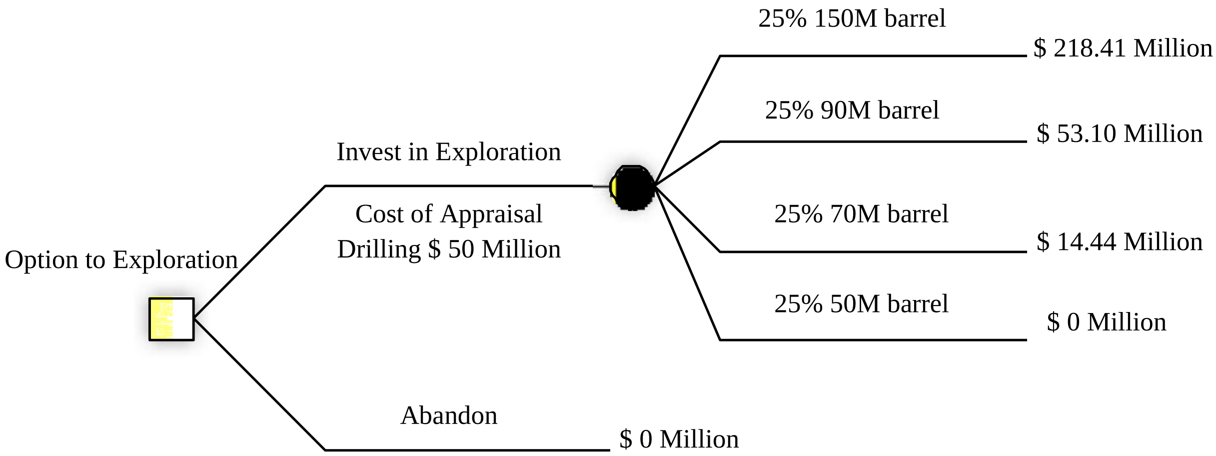


Figure 4.4: Decision Tree for the Unexplored Oilfield

In practice, considering the size of most oil companies, it is reasonable to assume that project-specific technical uncertainty can be effectively diversified. Hence, management is effectively risk neutral towards this uncertainty. Under the Luenberger Real Option approach, we can value this unexplored oilfield by discounting the expected value of the undeveloped reserves at a risk-free rate (assuming one year of appraisal drilling) and subtract the present value of the exploration costs.

On the other hand, if the firm or the investor either cannot diversify the project-specific uncertainty or intentionally retains it because of some comparative advantages or strategic reasons, decision makers' subjective beliefs and preferences for bearing this uncertainty come into play. For example, if the project constitutes a significant part of the firm's portfolio, and thus cannot be effectively diversified, decision makers often behave in a risk-averse fashion towards the project-specific uncertainty. Mattar and Cheah (2006) give two possible reasons for this phenomenon: managerial self-interest and the possibility of financial distress (especially when considering large risky and irreversible investments).

In the case where the oil company cannot diversify the project-specific technical uncertainty, this uncertainty can be valued by calculating a certainty equivalent through an Integrated Real Option Approach. To do so we need a utility function. Let us assumethat the decision maker's time and risk preferences for consumption $X_1$and X_0can be represented by an exponential utility function:

$$U(X_0, X_1) = 1-exp(-(X_0+X_1/k_1)/p), \qquad 4.8$$

Where p denotes the decision maker's risk tolerance and k_1 describes the decision maker's time preference. In this case, $X_0$and $X_1$are the time-0 and time-1 net cash flows.

4.6 Conclusion from the Offshore Petroleum Lease Valuation Example

In this chapter, an offshore petroleum lease example was discussed in order to develop intuition about important concepts behind real option approaches and their application conditions. The development of an offshore oilfield is a task characterized by its highly irreversible costs in exploration and infrastructure, long project life cycle in construction and operation, and high degree of uncertainty. In addition, the presence of option or option-like features means that it is difficult to derive an accurate estimate of value based on a standard DCF calculation. In response to this, many researchers recommend real option approaches to value the oil development investments.

An oil development investment typically experiences various types of uncertainties throughout its life cycle. These uncertainties may involve either technical uncertainties related to the properties of the reserve, or market uncertainties related to the future market conditions. Hence, a decision to invest in the development of an oil reserve requires an in-depth analysis of several uncertainties. The evaluation approach must be chosen appropriately. For an undeveloped oilfield contingent on oil price, Classic Real Option approaches are useful. The tracking portfolio is made by observing traded oil securities such as oil futures and options. For an undeveloped oilfield contingent on both oil price and variable operating costs, the MAD approach is probably more suitable as the process of determining a perfect tracking portfolio is a challenge. Under the complete market scenario, data availability and quality is the key to the application of option valuation approaches. Classic Real Option approaches require collecting sufficient data to justify a replication strategy. In practice, many market uncertainties are, however, poorly understood and there is a lack of sufficient data to measure them. For example, market uncertainties highly related to the financial market (e.g., commodity price) are easier to track than market uncertainties related to industry aspects (e.g., variable operating costs). Hence, if there is no traded asset or portfolio of traded assets that tracks the properties of the project reasonably well, the MAD approach is a good alternative. Nonetheless it is important to bear in mind that an accurate modeling practice even for financial assets with a long time series data available is a complex task. Furthermore, some of the significant sources of uncertainty that affect the value of strategic options are not caused by the general movement of the market, but by project-specific events. The value of project-specific uncertainty depends on a diversification argument and/or some strategic reasons. For example, the firm will not require additional compensation other than risk-free rate if the project-specific uncertainty can be effectively diversified. However, if the firm cannot diversify or intentionally retain the project-specific uncertainty, decision makers' subjective beliefs and preferences for bearing this uncertainty come into play. Hence, taking a different treatment about the project-specific uncertainty does affect the value of the project and strategies. Under a mix-uncertainty scenario, we apply both the Luenberger and Integrated Real Option approaches to value an exploration investment taking into account the uncertainty of the reserve size.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The first conclusion from this study is that the ambiguous definition of "Real Options" is one of reasons that have led to numerous real options approaches. Real options are operational strategies that companies could have. The flexible strategies are composed of technical strategies and market strategies. The ways to appraise technical strategies and market strategies need not be said.

The second conclusion from this study is that among the five real options approaches, all real options approaches are still based on the traditional DCF method, except the classic approach. They still employ the discount rate to value the private risk. Real options valuation is not a replacement for the traditional DCF method. It extends the traditional DCF method. In fact, it is an improvement for the project valuation.

The third conclusion is that there are five techniques applied in real options valuation. These are the traditional DCF method, the Black–Scholes Model, the binominal tree model, decision analysis, and Monte Carlo simulation. We list the techniques applied in the real options approaches below:

Table 5.1: Techniques Applied in Real Options Approaches

Real Options Approach	Traditional DCF	Option pricing model		Decision Analysis	Monte Carlo Simulation
		Black schools model	Binominal tree model		
Classic approach		√			
Subjective approach	√	√			√
Market asset disclaimer	√		√		√
Integrated approach	√		√	√	√
Luenberger approach	√		√	√	

Different methods lead to different project values. Similar methods have similar numerical results, because they use similar techniques. The subjective approach and the MAD approach have similar project values, because both of them replicate option – pricing models: Black–

Scholes model and the binominal tree model. The Smith approach and the Luenberger approach have similarity due to application of the time discrete decision tree analysis.

The fourth conclusion is that the real options approaches have their own way to value flexibility. See Table 5.2 below:

Table 5.2: Comparisons of Real Options Approaches in Valuing Flexibilities

Real option approaches	Authors	The value of flexibility			Real options valuation	
		Technical	Market		Value of technical flexibility	Value of market flexibility
			Trading project	Price		
Classic approach	Paddock et al 1988	Missing	√	√	-	Black-Scholes model
Subjective approach	Luehrman, 1998a	√	Ambiguous	√	Traditional DCF Monte Carlo Simulation	Black-Scholes model
Market asset disclaimer	Copeland et al 2001	√	Ambiguous	√	Traditional DCF Monte Carlo Simulation	Binominal tree model risk neutral probabilities
Smith approach	Smith and McCardle	√	√	√	Traditional DCF decision analysis	Binominal tree model risk neutral probabilities decision analysis
Luenberger approach	Luenberger	√	√	√	Modified traditional DCF decision analysis	Binominal tree model risk neutral probabilities decision analysis

√: to be considered

The fifth conclusion is that most real options approaches are more suitable for valuing the market strategy, because real options valuation is derived from financial option valuation methods. Seldom do the described real options approaches value well the technical strategy. To model technical uncertainty, we need to apply technical information. In addition, the appraisal of the value of technical flexibility is more complicated. As dixit (1994) pointed out, the decision to invest or to wait depends on the parameters that specify the model, most importantly the extent of uncertainty (which determines the downside risk avoided by waiting) and the discount rate (which measures the relative importance of the future versus the present). And hence, valuing technical flexibility requires high quality parameters. There his quality parameters are derived from the analysis of evaluators who understand the essence of technical problems and real options theory.

5.2 Recommendation

When we focus on oil and gas applications, we need to classify the types of flexibility. In principle, these are technical flexibility and market flexibility. Despite that the valuation of a project starts at PV – the inherent value of the project, technical flexibility and market flexibility in the project should be valued in different ways.

If technical flexibility is dominant in the project, we should concentrate on modeling technical uncertainty. The technical information, such as the reservoir permeability, porosity, reservoir pressures, as well as relative reservoir parameters, etc., will be used for the appraisal of the value of technical flexibility. Based on the analysis and conclusion of thesis, Isuggest applying decision tree techniques. Monte Carlo Simulation, and risk neutral probabilities. Since the decision tree analysis and Monte Carlo Simulation can value the technical flexibility, when the risk is non – linear with costs. Meanwhile, the risk neutral probability technique can give one the solution of valuing flexibility, because it is able to represent the relation between uncertainty and time. Therefore, the Smith Approach and the Luenberger Approach are more suitable for valuing the technical flexibility.

On the other hand, if market uncertainty is dominant in the project, to value the market flexibility, we should focus on modeling market uncertainty. It is a financing problem. We need to employ the market information to evaluate the market flexibility. The price volatility, investment timing, oil prices forecast, are necessary. In this case, I suggest using Monte Carlo Simulation, risk neutral probabilities, financial option–pricing instruments, and decision tree technique as well. These techniques are suitable for the appraisal of market flexibility. Consequently, we recommend the subjective approach, MAD, the Smith Approach, and the Luenberger Approach. These approaches are suitable for market flexibility valuation.

However, I could not agree that one method is the best beyond the other approaches, since each approach has its own strengths and weaknesses for the flexibility valuation. The choice and application of a real options approach could be determined on the case–by–case basis.

5.3 Future Research

Further research needs researchers to understand the flexibility variables that are used for modelling technical uncertainty and market uncertainty, and to value real options in the realistic, and “option – doable” way.

Finally, I believe that more research is required focusing on the relationship between industry-specific characteristics and the structure of the real option approaches so as to provide use-friendly frameworks for specific industry users.

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NOMENCLATURE

BBL	Barrel
BOE	Barrels of Oil Equivalent
BOPD	Barrels of Oil per Day
CAPEX	Capital Expenditure
CF	Cash Flows
DA	Decision Analysis
EV	Expected Value
IRR	Internal Rate of Return
NPV	Net Present Value
OIIP	Oil Initially in place
OPEX	Operating Expenditures
RO	Real Options
ROA	Real Options Analysis

APPENDIX

Appendix A: Model Equations

$C_t = S_t N(d_1) - Xe^{-r(T-t)} N(d_2)$

$$d_1 = \frac{ln\left[\frac{S_t}{X}\right] + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Appendix B: Black-Scholes Model

		Conventional Method		
Prob	**Initial Investment (MM)**	**NCF for each of the following five years (MM)**	**NPV (MM)**	**Prob X NPV(MM)**
0.3	$ 20.00	$ 12.00	$ 20.23	$ 6.07
0.4	$ 20.00	$ 8.50	$ 8.49	$ 3.40
0.3	$ 20.00	$ 2.00	$ (13.30)	$ (3.99)
		Expected NPV		$ 5.48

Real Option Method (Black-Scholes Pricing Model)

Prob.	**NCF for each of the 5 following years (MM)**	**PV (MM)**	**Prob. X Pv**		
0.3	$ 12.00	34.97900972	10.4937029	49.34908854	
0.4	$ 8.50	24.77679855	9.91071942	2.752945022	
0.3	$ 2.00	5.829834953	1.74895049	79.93736656	
	Expected NPV		$ 22.15	11.49083984	standard deviation
	Coefficient of variation		0.51869482		
	Volatility		0.48812305		
	Stock Price		$ 22.15		
	Strike Price		$ 20.00		
	risk free rate		6%		
	Volatility		49%		
	Time		1	year	
	d1		0.57647255		
	d2		0.0883495		
	N(d1)		0.71785209		
	N(d2)		0.53520055		
	Option Value		$ 5.82		