STATISTICAL DATA-DRIVEN MODELS FOR FORECASTING PRODUCTION PERFORMANCE WITH UNCERTAINTY ANALYSIS

A

THESIS

Presented to the Department of Petroleum Engineering,
The African University of Science and Technology
[Abuja]

In Partial Fulfilment of the Requirements
For the Degree of

MASTER OF SCIENCE

By

LADIPO OLALEKAN AZEEZ

Abuja, Nigeria

December, 2014

STATISTICAL DATA-DRIVEN MODELS FOR FORECASTING PRODUCTION PERFORMANCE WITH UNCERTAINTY ANALYSIS

By

Ladipo Olalekan Azeez

A THESIS APPROVED BY THE PETROLEUM ENGINEERING DEPARTMENT

RECOMMENDED: ………………………………………….

Supervisor, Professor David Ogbe

…………………………………………

Professor Wumi Iledare

…………………………………………

Dr. Alpheus Igbokoyi

………………………………………………………………….

Head, Department of Petroleum Engineering

APPROVED BY: …………………………………………..

Chief Academic Officer

....

Date

DEDICATION

TO THE ALMIGHTY GOD:

The Beneficent, The Gracious, The Merciful.

…for His unending 'divine favour' over my life.

ACKNOWLEDGEMENT

First and foremost, I acknowledge God for His grace in the course of this work, and His exceptional provisions in this journey.

I extend my appreciation to my parents, for their provisions, support and encouragement towards ensuring that an exemplary and worthy life is lived by me. I greatly appreciate a veteran mother, my mum, Mrs. M. A. Ladipo, for her immeasurable contributions to every success I have witnessed so far. You are indeed ‘a heart’ to me. I thank you for your rare strive in keeping the family together.

I will not forget my entire family; I appreciate you all for your support in one way or the other throughout the period. My profound gratitude goes to the African Capacity Building Foundation (ACBF), EEPNL / NNPC, and Paragon Holdings Limited; I thank you for being part of the actualization of this dream.

I greatly appreciate my supervisor, Prof. David Ogbe, for his ‘regular check and support’ in making this work a reality. To all my tutors; Professors W. Iledare, G. Chukwu, M. Onuoha, G. Falade, D. Tiab, as well as Dr. M. Saka, Dr. A. Igbokoyi, Dr. A. Mukhtar, Dr. O. Akin-Ojo, Dr. Djitte and Dr. Degla, I thank you sirs, for your mentoring and evergreen presence. I owe you much!!! A special ‘thank you’ to Mr. Akeem Arinkoola.

I will forever be grateful to the management, staff and students of African University of Science and Technology (AUST) and the Nelson Mandela Institution (NMI). I sincerely appreciate the entire PE PhD-students of AUST; Mr. Onuh, Mrs. Aladeitan, Miss Nwachukwu, Miss Aborishade, Mr. Duru etc.

As usual, I will not end without appreciating my amiable sisters; I appreciate your support and encouragement.

I finally extend my appreciation to all my friends and loved ones, at home and in school, most especially my colleagues and pals at AUST; Maryam, Lizzy, Dammy, Henry, Dickson, Bismark, Ibrahim, Sanni, Tony, Isma’il, Emmanuel, Gabriel, Kizito, Prosper, Aloysius, Chris, Edwin etc. Thanks to you all, you made the period worth it for me. We shall all reach the ‘promised land’.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Data-driven analytical models are important tools employed in the petroleum industry to forecast production rates and reserves of petroleum assets. Studies have shown that existing rate forecasting models project future performance trends by averaging the observed production history data with little or no preferential consideration of the influence of the trends of most recent historical production data. It is also understood that existing empirical data-driven rate forecast models do not account for the uncertainties involved in such future predictions.

In this work, new statistical data-driven models for production performance forecasting are developed. These models forecast future production performance trends using statistical-exponential smoothing of the historically-observed data, attributing more weights to the most recent historically-observed performance trends. Both linear-exponential and double-exponential smoothing algorithms were considered in the study. The uncertainty analysis of these predictions is evaluated using the history-fit errors derived from the observed data and fits to the history. The models' accuracy depends on the segment of the observed historical data selected for the initial data fitting and the trends of most recent historical production data—the tail end of the observed data. The application of the proposed statistical data-driven models was demonstrated on several data sets. The results obtained from the proposed data-driven models compare favorably with the base forecasts derived from existing models such as the Arps' Decline Curve Analysis techniques. The major advantage of the proposed models over existing models is that predicted forecasts include a range of possible performance trends honoring the observed production history with the associated probabilities and confidence intervals.

The major contribution of this work is that coherent statistical data-driven models have been developed for forecasting production rates and reserves of petroleum assets with uncertainty analysis.

1.0 INTRODUCTION

1.1 Why Production Performance Forecast?

The petroleum industry is both capital-intensive and profit-driven, whose needs for conciseness cannot be over-emphasized. These create the need for a regular and specific evaluation of the production deliverability of the ultimate assets of the business, which basically are actively-producing reservoirs and production wells. The projected performance of such asset helps a long way in aiding both technical, developmental, management and economic plans and decisions of the organization. This also provides an avenue towards estimating the ultimate recovery or reserves of such asset till a predetermined economic performance limit, under an on-going production or recovery technique,i.e., primary, secondary or improved.

Furthermore, the optimal hydrocarbon production performance of any petroleum asset cannot be totally effectively sustained throughout the production life of such asset be it a field, reservoir or production well, even under substantial production maintenance schemes. Production declines set in with time due to quite a number of reasons thus, necessitating the need to predict its future deliverability.

Some of the reasons are related to the reservoir conditions; some are volumetric due to variables such as porosity, water saturation, drainage radius, etc. Others are performance related such as permeability and viscosity. Other conditions are mechanical such as well drawdown.

There are declines caused by pressure depletion due to no-flow boundary-dominated (pseudo-steady state) production regimes. With all other reservoir variables including well Bottom-Hole Pressure being held constant, the producing rate will decline as production reduces the pressure of the reservoir through depletion. This serves as the major cause of most production declines.

There are also declines caused by flow restrictions. When the flow capability of a well or field decreases with time, production declines can also occur. This mostly occurs during completion processes. Scaling (Skin) is a common situation.

There is decline caused by increasing producing back pressure. This is a complex type of decline, resulting from the pumping artificial lift method.

Decline caused by transient flow is shown by wells experiencing decreased production as their transient flow period progresses. This drop in production occurs because the effective drainage radius is increasing and while the pressure drawdown remains constant. Since the production has a longer distance to flow through the formation, the rate decreases.

There are also declines caused by water displacement. This is common with reservoirs under water-flood, gas injection, or experiencing a strong aquifer drive. If voidage is maintained so that production in reservoir barrels is balanced by injection, the average reservoir pressure remains somewhat constant. The decline occurs because the displacement of oil by water causes the water-oil-ratio (WOR) to increase. The increase occurs because of fractional flow, sweep efficiency and the displacement nature of these injection processes. Stated simply, the oil rate is being replaced with water rate.

The possibility of any of these or a combination of some is undoubtedly high, and always considered during developmental planning. This champions the need for regular updates and forecasts of the associated asset's production performance.

1.2 Why Forecasting with Uncertainty Analysis?

According to Wikipedia, uncertainty analysis investigates the uncertainty of variables that are used in decision-making problems in which "observations" and "models" represent the knowledge base. In other words, uncertainty analysis aims to make a technical contribution to decision-making through the quantification of uncertainties in the relevant variables. It deals with assessing the uncertainty in a measurement.

It is an experiment designed to determine an effect, demonstrate a law, or estimate the numerical value of a physical variable that will be affected by "errors" due to instrumentation, methodology, presence of confounding effects and so on. Uncertainty estimates are needed to assess the confidence in the results.

Due to the nature of various analytical and empirical models used in making production performance predictions, forecasts are usually based on past performance observations. Significant errors are inculcated with these approaches, either due to the history measurements or the employed methodologies themselves. Therefore, the need for such errors to be accounted for in terms of probability of occurrence of any future performance prediction becomes inevitable.

Also, many deterministic production performance forecasts from these existing models have turned out to be unrealized when such forecasted periods become actually observed, and these have negatively impacted investments and returns in the industry. Over-estimation of future performance is usually avoided and not tolerated in the operations of the upstream petroleum industry. In addition, petroleum assets' performances are preferred to be reported with some degrees of certainty of occurrence. This economically assists managers in taking calculated risks during various decision-making processes. This has also promoted the need for forecasting with uncertainty analysis.

1.3 Need for Statistical Data-driven Models for Forecasting

One of the most important tasks of a petroleum asset management team is the prediction of an asset's production performance based on what it has displayed over some significant period of time in history. Choosing the methodology is critical for accurate forecasts that are, in turn, vital for sound managerial planning.

Extrapolation of production history has long been considered an accurate and defendable method of carrying out this. Data-driven models have played significant roles in various technical and economic activities of the petroleum industry, with the applications of some of them being independent of any reservoir characteristics. They have subsequently shaped the less need for mechanistic and properties-dependent models, whose input parameters are frequently difficult to be accurately estimated due to the complexity and heterogeneity in the nature of petroleum reservoirs.

For example, Decline Curve Analysis-DCA is a method used for the prediction of future hydrocarbon production by analyzing past production performance behavior. Early petroleum engineers realized that by studying the trends of past production from wells and fields, one could forecast their future productions. The early work was empirical, and used mainly for proration and taxation.

These empirical data-driven models give equal weights to both old and recent history performance trends in predicting future performances, and this does not portray the actual performance behaviors of petroleum assets as they usually display more preference for the most recent performance trends. Also, they generate deterministic forecasts, which are not too reliable due to their associated over-estimation or under-estimation of future performance. Statistical models on the other hand attribute more weights to the most recent trends towards predicting probabilistic future production performance; thus, necessitating its application to the subject matter in this work.

2.0 LITERATURE REVIEW

2.1 Introduction

Production forecast is an important aspect of the petroleum industry, which involves future projection of the production performance of an asset, either with economic and/or technical objectives. Various methods have been deployed in carrying out this all-encompassing task, ranging from mechanistic to analytical models. Mechanistic models involve methods that fully take into consideration the properties of such asset (well/reservoir/field) and its surrounding systems. These include the generalized Darcy's flow equation, Material Balance equation, and Fractional flow formula amongst others. The Decline Curve Analysis (DCA) and other empirical methods fall within the analytical models, which are mostly data-driven. The analytical models are data-driven in nature.

Often, data-driven approaches are fundamentally based on analyzing the already-displayed historical production performance trends or behaviors of an asset, bench-marking such trends with the most-prominent behavior, and using the bench-marked historical trend to make future production performance forecast or prediction of such asset, with an assumption that such historical trend is maintained over the future forecast periods. Results have shown that the historical production performance tends to take into consideration averaged-existing production performance properties of the reservoir system as being engaged in the mechanistic models, thus providing dependable future forecasts as well.

The evaluation of the production history data from oil and gas wells can be very beneficial in meeting the production forecasting and ultimately reserves estimation objectives of a petroleum asset's assessment. Many data-driven methods have been developed to address these lofty objectives, such as the automatic history matching with numerical simulation, the decline curve analysis (DCA), as well as the type curve analysis of production decline curves.

2.2 Literature Review

The earliest breakthrough in the production forecasting objectives is the Decline Curve Analysis (DCA), which is credited to the works of **W.W. Cutler, 1924** and **J.J. Arps, 1945.Arps**, **1945,** published several papers on the empirical decline curve equations. He categorized the decline equations into exponential, hyperbolic and harmonic declines, with the hyperbolic exponent serving as the characterizing difference amongst them. The exponential happens to be the most popular and widely used decline model by engineers due to its simplicity and easiness compared to the hyperbolic decline, and also very conservative in its deployment in production forecasting. Really, hyperbolic declines are much more common than exponential, while harmonic declines are quite rare. The works of **Cutler, 1924; Arps, 1945; Brons, 1963; SPE PE Handbook, 1987**, have all shown the excellent applications of these decline equations to production forecasting in primarily-depleted oil and gas reservoirs, and also assets under secondary displacement. Despite the applicability of the DCA method, it was observed that significant number of petroleum assets often showed more than one performance trend in their production history (usually combining the exponential and hyperbolic decline behaviors). This was the origin of the limitation of solely applying any of the Arps' decline models in production forecasting.

In early 1980s, the type curve analysis was introduced into the industry to cater for this limitation. **Fetkovich, 1973,** designed Type Curve models from different studied wells and fields based on Arps' empirical decline equations, flow equations and known reservoir fluid properties; amongst which a match is obtained for the production decline curve derived from any production history data of analysis. This match thereafter serves as the basis upon which the production forecast of such asset is projected.

Long and Davis,1988; Robertson, 1988, further added different approaches to the works of **Arps, 1945.** They tried to look at how the challenge of combining two Arps' decline models could be addressed. In their pursuits, **Long, and Davis, 1988,** developed a log rate-versus-time overlay to cope with this challenge. They also introduced the use of Type Curve Matching method, where the curve from a well production history is matched with already-developed curves from different wells based on different reservoir-well properties for correlation. They also developed new empirical decline model to cater for variations in the decline trends of the history production performance of hydrocarbon assets. Their model incorporated change in decline trend from Arps' hyperbolic decline to an exponential decline after a predetermined decline rate. This was premised on the fact that apparent 'best' Arps' hyperbolic exponent decreases continually with time as practically observed in various studies of production performance of petroleum production history data. The model is limited by the unavailability of 'analogy' studies in the case of new hydrocarbon resources (green fields), from which a transition could be assumedly established. **Robertson, 1988,** also developed a production rate equation that is hyperbolic initially but asymptotically exponential with time. He introduced a dimensionless constant; its value ranges from 0 to 1 and is related to the abandonment pressure, and the rock and fluid properties.

Valko, 2009, took a different approach in modifying the Arps' decline analysis for unconventional resources. He applied the Arps' decline analysis with the view of correcting for different decline trends within the same reservoir, which is peculiar with shale resources. He validated his work for wells with both transient and stabilized flows in the Barnett Shale. Forecast was concluded to be unreliable for history production data less than eighteen months.

Valko *et al*, 2010, later proposed theStretched Exponential Production Decline (SEPD) model, which is a decline model that sums the simultaneous exponential declines in different 'cells' within a reservoir.

Duong, 2010, further extended the use of the Arps' decline models to tight oil and gas reservoirs and the unconventional assets. He developed an empirical decline model for forecasting, which catered for the long-term linear flow regime (fracture plus matrix) associated with these types of hydrocarbon reservoirs.

Duong, 2011, later introduced a modified empirical decline model to account for fracture interference in the use of the pre-existing Duong model. This modified version factored a 'switch decline rate' into the Duong model, after which forecast switches to the empirical Arps' decline analysis. This switch tends to account for the temporal fracture-dominated flow regime in unconventional assets and tight petroleum reservoirs, and usually suitable for short-term production history data.

Ladipo *et al*, 2012, premised on the work of **Khaled, 2006,** provided an empirical solution to the use of combined hyperbolic and exponential decline models in forecasting production and reserves in oil reservoirs. They developed empirical hyperbolic-exponential decline models incorporating the transition time from hyperbolic to exponential decline behaviors. The transition time was further explained with the behavior of the decreasing hyperbolic decline rate with time. A decline rate-versus-time plot was empirically analyzed using the conventional Arps' decline models, from which the practically-effective empirical transition time from hyperbolic to exponential decline was benchmarked, thus averaging the over-estimation and under-estimation of production performance attributable to the sole use of the hyperbolic and exponential decline models, respectively.

All these published works on using the decline curve analysis and type curve matching procedures in meeting the production forecasting and performance evaluation's technical and/or economic objectives have left out the concept of evaluating the uncertainty analysis associated with their predictions and projections despite the significant presence of errors in their respective historical performance trend fits, upon which their future performance projections have been based. They have also not being totally able to show the time-varying impact of external supports and drives on the production performance, which is usually obvious in the history reservoir pressure, water-cut and/or gas-cut data. Nonetheless, majority of the developed models have given good performance predictions over the years.

The Automatic History Matching with numerical simulator approach has contributed immensely in this regards. Asides its foundational responsibility of carrying out performance forecast and reserves evaluation of the reservoir, its ability to present the engineer with different projected performances based on already pre-set developmental scenarios has a great impact in developing economically-viable and technically-realistic integrated reservoir development and management plans. The ability to history-match production and pressure data also helps to generate different combinations of reservoir rock-fluid properties that could provide the same production history as observed. This proffers the basis upon which uncertainty analysis could be carried out on the future production performance forecasts due to the stochastic nature of the inputted rock-fluid properties. In addition, the history matching approach tends to take into consideration the impact of various external supports and drives, as well as the non-constant performance of the production and operating systems in evaluating the history production performance and making future projections. Several numerical simulators have been developed over the years for these tasks, and diverse modifications are routinely researched and published to enhance the performance of these simulators, and diversify their functions.

With these strategic advantages of forecasting with numerical simulators, the process of history matching is a herculean task, and the possibility of generating different combinations of available ranged input parameters that could produce good matches of observed production history and pressure data tends to limit the accuracy of such base forecasts, and subsequently the uncertainty analysis carried out on any of the combinations. It also discourages finalizing technical and economic decisions on such projections, since the only statistically certain thing about forecasts is that they are wrong! Coupled with the cost of obtaining these simulators and software, their use also requires some level of expertise and experience in ensuring that results from them are better technically-analyzed and adjusted if need be. Another major setback in the use of these various highlighted approaches to production predictions and forecasts is the presence of 'noise' in the production history data.

Soleng, 2011, developed a genetic algorithm towards forecasting oil reservoir production with uncertainty estimation. This was further aimed at reducing the variations in production performance forecasts when various existing numerical simulators are used in matching the history production data based on the ranges of existing certain and uncertain geological and geophysical input parameters. Due to the distributions and uncertainties associated with the various input reservoir parameters responsible for the production performance of an oil and gas asset, several possible combinations of such parameters could actually provide a match for the observed production history, but with erroneous forecasts. With the aid of other history performances like pressure and water-cut, the developed genetic algorithm could limit the number of parameters' combinations to significantly compatible parts thus, providing a better estimation of the prediction uncertainty. The method was tested on a synthetic oil field prepared as part of the PUNQ (Production Forecasting with Uncertainty Quantification) project sponsored by the European Community (**Floris *et al*, 1999**). The Modular Oil Reservoir Evaluation (More) simulator (courtesy Smedvig Technologies) was used for the history-matching, and a significant improvement was observed when the forecast results were compared with the existing conventional numerical approach. In conclusion, a statistically-corrected distribution for the geological realizations that actually matched the history well was suggested to be of outmost importance in the analysis.

Irrespective of all these, none of the above-established approaches considers correction for errors generated in its history match or fit towards adjusting its base forecast or prediction. Since the history-fit is the basis upon which forecast is made, the errors in such history-match significantly affect the accuracy of the associated forecast or predictions, and subsequently any uncertainty analysis carried out. Despite its ability to undergo some reliable evaluation and forecasting tasks, studies have however shown the sensitivity and nature of problems arising from the use of the automatic history matching procedures when ‘noisy’ data set is involved. Often, such seem-to-be noisy components of the data set could be vital in the prediction of future production response from the asset. The difficulty with very noisy data is the determination of a reasonably consistent and accurate unique solution, irrespective of the method’s ability to perform intuitive pattern recognition.

Crafton, 1997, developed the Reciprocal Productivity Index (RPI) method towards addressing situations whereby a unique single trend could not be fitted for the production history. This is usually noticed in ‘noisy’ production history data, where historical trend is changing. He employed the use of the conventional well test analysis into his methodology, but with modifications for use in varying pressure and production rate data. His methodology could smoothen and match the observed well’s production history data without the need for a full numerical simulator. It is also graphical, although could be preconditioned by automatic methods. Besides providing the basis for production forecasting and reserves estimation, RPI also gives estimates of reservoir properties such as the permeability, fracture length and drainage area of such well.

Obah *et al*, 2012, also considered the significance of uncertainty analysis on production performance forecasts. They channeled their work towards the impact of a range of sub-surface uncertainties on oil rim recovery, which was captured using the Plackett-Burman Experimental Design (ED) technique. These included the uncertainties in various input parameters used in the development of static or geological models of Niger Delta oil rim reservoirs. A generic dynamic simulation study was carried out to generate oil production profiles, with the use of the least square regression method in developing approximate or base models for oil production forecasting. Production forecast models for the oil rim reservoirs were obtained with the Monte-Carlo simulation approach, which enabled the generation of a probabilistic range of forecasts. Despite integrating the concept of uncertainty analysis into the production forecast, the work assumed a best-fit for the historical fit upon which the stochastic production forecast was made. This was underlain by the fact that the error associated with the linearly-regressed history data was not considered in generating each of its associated possible forecasts. This would have a significant impact on the predicted probabilistic forecasts since the base forecast would have automatically incurred a possible error distribution approximately within the domain of its associated historical errors.

Holdaway, 2012, highlighted how the concept of statistics could be employed for better analysis of production history data towards fitting a better trend based on the empirical Arps' equations. His paper introduced the methodologies to forecast oil and gas production by exploring implementations of procedures in the SAS software. He discussed the SAS reservoir management software, as well as its embedded advantages in matured field development planning for situations when there is a hand-full of history production and pressure data. The software employs the concept of autocorrelation in analyzing the error generated in the history-fit for randomness. The software was originally developed not to displace the use of the Oilfield Manager (OFM) in production forecasting, but to rather assist the Reservoir Management Department (RMD) of Saudi ARAMCO National Oil Company analyze tens of thousands of its producing wells across multiple oil and gas fields that cover hundreds of heterogeneous sandstone and carbonate reservoirs on the Arabian Peninsula. The SAS software for Oilfield Production Forecasting was believed to offer a customized data management and advanced time series analytical approach that addressed the inherent issues of a deterministic workflow by adopting a probabilistic perspective. SAS was described to offer a DCA workflow that rapidly flagged problematic wells with robust data and reliable statistical accuracy indicators. SAS was designed to allow engineers perform dynamic production analysis with more advanced forecasting techniques, which is effective in business planning to generate the production forecast for the new wells and no drill case for the existing fields' wells. To protect the market value of the software, little was discussed about its underlying principles, but their applications to the above-stated management operations were introduced. SAS' method of generating a probabilistic production forecast was not properly explained.

The introduction of software like SAS, and the various published works on probabilistic production forecasting have enormously indicated the relevance of statistics in further analyzing the concept of empirical decline fits that the Arps' decline equations and its numerous modifications have brought to the area of production forecasting in the oil and gas industry. This has been described as a boost towards improving on the accuracy of the possibility of occurrence of the various stochastic production forecasting approaches.

Statistical approach to forecasting is really not recent to the industry.

Statistics has always played a major role in the bench-marking of historical trend on production history data, upon which future performance could be made. These have ranged from the linear regression (LR), auto regression (AR), to moving average (MA), and presently the concept of exponential smoothing (ES). Similar to the Arps' decline equations, as others tend to give equal weight on both old and recent history production performance in fitting a trend on the history data, the exponential smoothing offers more weight on the recent history performance than the older performance. This actually portrays the behavior of hydrocarbon assets in the petroleum industry. This also brings about a reduced error being generated in the history fit thus, improving the accuracy of subsequent forecasts made on such history trend. The smoothing techniques (moving average and exponential) have shown significant improvements on both univariate and multivariate time series analysis.

Kikani, 2013, showed the need for some knowledge of the nature of data as a requirement for an effective smoothing of time-series data. Data smoothing tends to account for the noise in the data, extract real trends and patterns (if any) from the data such that the smoothed data can be modeled and used for prediction purposes. He differentiated between the use of a standard moving average technique in accounting for mere fluctuations that are random in nature, and that of an exponential smoothing algorithm if one trusts the latest data more than the past data (peculiar to the petroleum production data). He also itemized the use of a double-exponential smoothing technique if there is a trend in the history data, and a triple-exponential smoothing technique, also known as the Holt-Winters method (**Chatfield *et al*, 1990)**, if random fluctuation, trend as well as seasonality exist in the data.

Exponential Smoothing

This assigns decreasing weights as the observations get older. The weights that vary from 0 to 1 define the damping level of the averaging technique. Below are the individual smoothed values of a dataset consisting of elements y_i:

$$y_1$$

$$y_2 \quad \rightarrow \quad S_2 = y_1$$

$$y_3 \quad \rightarrow \quad S_3 = \alpha y_2 + (1-\alpha)S_2$$

$$y_4 \quad \rightarrow \quad S_4 = \alpha y_3 + (1-\alpha)S_3$$

$$\dots ..$$

where $S_i$represents the smoothed terms of the series y_i and α is the weighting factor. The term S_2 can be determined a number of ways. One of the ways is to assume the value of y_1. Other techniques could include averaging the first few terms. The general term is given by

$$S_t = \alpha y_{t-1} + (1-\alpha)S_{t-1}\,, \quad 0 < \alpha \leq 1, t \geq 3 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \text{(2.1)}$$

where α ≈ 0 implies slow damping, and α ≈ 1 implies fast damping. It can be shown that the weighting reduces in a geometrical progression as seen from the following equation and evidenced by the decreasing powers of (1-α)

$$S_t = \alpha \sum_{i=1}^{t-2} (1-\alpha)^{t-1} y_{t-1} + (1-\alpha)^{t-2} S_2 \ \ for\ t \geq 2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

The forecast of the new point then becomes

$$S_{t+1} = \alpha y_t + (1-\alpha) S_t \ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ \ (2.3)$$

The exponential smoothing is not good if there is an evident trend in the data.

Double-Exponential Smoothing

This accounts for a trend in the data. The equations are very similar to that of Exponential Smoothing above with a shift parameter β added to the above equations. The general smoothed term is given by

$$S_t = \alpha y_t + (1-\alpha)(S_{t-1} + \beta_{t-1}) \ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

The trend correction is also performed on a moving basis and is given by

$$\beta_t = \gamma(S_t - S_{t-1}) + (1-\gamma)\beta_{t-1} \ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.5)$$

Initial values for β can be set in a number of different ways

$$\beta_1 = y_2 - y_1 \ \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

or

$$\beta_1 = \frac{[(y_2 - y_1) + (y_3 - y_2) + (y_4 - y_3)]}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

where α and γ can be obtained by means of nonlinear optimization techniques such as Marquardt algorithms.

Triple-Exponential Smoothing

This will smoothen the random fluctuations and trend in the data, as well as account for seasonal fluctuations or oscillatory behavior.

Overall smoothing:

$$S_t = \alpha \frac{y_t}{I_{t-L}} + (1 - \alpha)(S_{t-1} + \beta_{t-1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

Trend smoothing:

$$\beta_t = \gamma(S_t - S_{t-1}) + (1 - \gamma)\beta_{t-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$

Seasonal smoothing:

$$I_t = \delta \frac{y_t}{\delta_t} + (1 + \delta)I_{t-L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2.10)$$

The forecast equation becomes

$$S_{t+m} = (S_t + m\beta_t)I_{t-L+m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.11)$$

To initialize this smoothing equation, seasonal data for 1-2 seasons or oscillations is necessary to calibrate the constants. A complete season data in the above has L points or periods.

The triple-exponential smoothing is seldom used in production time-series data analysis, but prominent in petroleum price and supply time-series data analysis.

Foreman, 2014, engaged the use of spreadsheets in analyzing time-series data based on the concept of exponential smoothing, without going through the rigor of the procedures highlighted by **Kikani, 2013**. He discussed the use of spreadsheet in the optimization of the set of history errors generated during the fitting and smoothing of history-trend, and highlighted how the errors could possibly be factored into future predictions towards putting intervals around such forecasts. His work was limited to a linear trend-corrected exponential smoothing technique in an equi-spaced time series history sales data, meanwhile, most production declines usually fit into either the popular and conservative exponential decline, a hyperbolic decline or a combination of both. He categorized the three exponential smoothing techniques into Simple Exponential Smoothing, for random fluctuating data with no trend, Holt's Trend-Corrected Exponential Smoothing, for time series data with a statistically-significant trend, and the Multiplicative Holt-Winters Exponential Smoothing, which incorporates the concept of seasonality or periodicity into a trend-corrected exponential smoothing procedure as discussed earlier.

This thesis work is thereof set on the platform of all these above-cited works in the area of applying statistical analysis in the use of exponential smoothing techniques for production forecasting with uncertainty analysis.

2.3 Study Objectives

This study is set to use developed data-driven empirical exponential smoothing technique in carrying out deterministic production performance forecast (or prediction) of a petroleum production well or field using the observed production history data set, and further generating an uncertainty analysis or stochastic performance prediction range around such forecast based on predetermined certainty or confidence level.

Statistical comparisons are intended to be made between forecast from the methodology employed in this study and that of conventional data-driven approaches.

2.4 Scope of Work

To achieve the objectives of this study, the following scope of work is presented.

This work intends to modify the existing linear-trend exponential smoothing technique for use in a non equi-spaced time series data set as in the case of the petroleum production data, and also develop a new exponential smoothing technique using an exponential-trend, as it is being very conservative and commonly-used in the industry. The two exponential-smoothing techniques as well as the Arps' exponential decline approach would be used in history-matching the production history data of interest, from which production performance forecast or prediction would be made. Monte-Carlo simulation would be used in providing an uncertainty analysis on such forecast by a statistical evaluation of the optimized history-fit errors. The production performance forecast would be done on an Excel Spreadsheet (or other similar programs). The presence of a production history-trend would be firstly established as a major criterion for an exponential-smoothing procedure, followed by the fitting and smoothing of the history data towards developing the history-performance model upon which forecast would be projected. Outlying history data points would be statistically filtered off to improve the accuracy of the fitted production history-performance model. Any hidden seasonality or periodicity in the observed production performance history would be identified using the concept of autocorrelation of history-fit residuals or errors.

3.0 METHODOLOGY

3.1 Introduction

As indicated under the preceding sections of this work, the importance of production performance forecast of a hydrocarbon reservoir towards making technical, developmental and economic decisions in the petroleum industry cannot be over-emphasized. Quite a lot of works have been done with respect to this aspect of the industry. As many of the various oilfields developed globally become matured with multiple years of historical production data, there is the need to understand the significant and vital information about the historical performance and depletion behavior of such wells and their host fields in order to gain some insights intotheir future production performance. As these data begin to become more and more available, the industry starts looking at how such data could be converted into valuable information to aid future developments, reservoir management and further assessments of the fields.

It has been known that the production trends displayed by a well in its history can be used to forecast the future behavior of such well if technical and operational factors are believed to be left unchanged. In establishing this trend of the production history, virtually all existing empirical production forecasting models in the industry attribute equal weights to the historical data irrespective of their time of observation, and these are then used in approximating the expected future responses from such production wells. Several unrealistically erroneous production forecasts have been made via this approach, as the procedurealways happens not to be the practical case in the production behavior of petroleum assets, and the most recent historical trend or behavior showing more influence on the near-future expected performance.

Majority of existing data-driven forecasting models do not usually give the best of fits for the production history, and are not also incorporated with trend-smoothing techniques to improve on the accuracy of their methodologies.

Also, quite a number of existing approaches to production performance forecast only attempt to predict the future, without quantifying the uncertainties around such predictions. Obviously, as the future performance is being forecasted, quantifying such forecast's uncertainty by creating prediction intervals around it is invaluable to decision-making in the industry. It is important to note that the few existing approaches to production performance forecasting developed with uncertainty analysis functions do not consider the errors generated by their historical fits, which definitely have significant impact on the accuracy of their base predictions, upon which their prediction intervals are generated.

Statistical analysis tools have shown tremendous improvements on data science either in the area of historical-data analysis, forecasting, or quantifying predictions' certainty. This provides the leverage foran effective and efficient analysis of the historical behavior of any data set, and prediction of future behavior of such without even incorporating the nature of the system's in-situ parameters generating the data.

This section modifies an existing statistical methodology, as well as develops a new one for analyzing historical petroleum production data to be used in making production forecasts from such analysis, and also quantifying the uncertainty around the forecasts in form of prediction intervals.

3.2 Modification and Development of Methodologies

Statistical analysis is a fundamental aspect of data management, and it has been widely applied to several analytical areas of the petroleum industry ranging from exploration, petro physics, economics, modeling as well as integrated asset management. Statistics has also been introduced into making forecasts with statistical analysis of uncertainties associated with such predictions.

This work combines several of theseapplicable statistical tools in developing a methodology for carrying out forecasts on petroleum production performance based on available production history from oil and gas wells, and it quantifies the uncertainty involved with the forecasts. Theresults could then serve as a benchmark for assessing the production performance of the entire petroleum field, from which management decisions could be based.

The methodology encompasses the following steps, summarized with a flow chart in Figure 3.1:

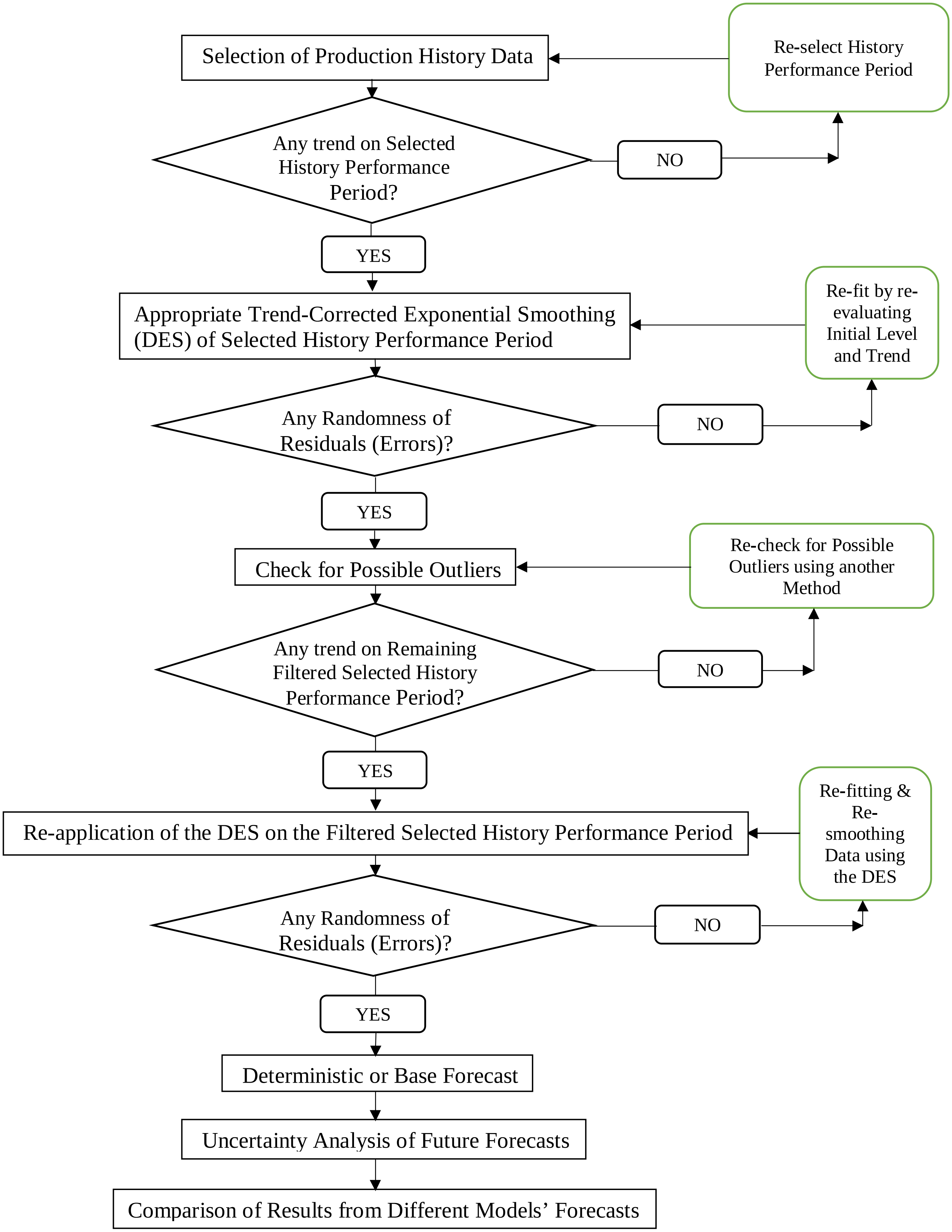

Fig. 3.1: FLOW CHART FOR THE STEPWISE METHODOLOGY PROCEDURE

3.2.1 Production History Data Analysis

This is the pre-analysis stage at which asub set of the well's production history data set that is recent and has shown a significant and dominant period of production pattern (decline) is selected. This stage is very crucial in any forecasting operation as its analysis serves as the basis upon which any forecast is made. The time duration of the selected data set in this region depends on the consistency and reliability of its associated pattern. There is no reservation in its selection, as the engineer just only needs to ensure that it is a region where the well and/or reservoir has been subjected to a uniform production or withdrawal technique(s) in ensuring that fluctuations in the nature of the data is not as a result of any external influence.

3.2.2 Trend-Check on Selected Production Decline Period

This important step in the methodology is designed to confirm the presence of a statistically-significant trend in the selected fluctuating history data set. The presence of a dominant production trend could actually be viewed without any statistical check, but it is safer to confirm its nature with at least a statistical tool. This involves primarily checking that the trend is of a non-zero slope, and secondarily using a 't-test' to confirm the statistical significance of such a non-zero slope. A 't-test' works on the principle of 'null hypothesis'. In other words, it checks for the statistical significance of the primarily-calculated non-zero slope of the data set's trend by assuming that there exists another slope within a possible statistical distribution of slopes that could fit the trend of such data set of interest. It then estimates the level of significance (confidence level) of such a possible slope by calculating if its probability value (percentile or p-value) is less than

5%. To compute such p-value, a ‘t-test’ requires the trend-fitted non-zero slope, its standard error and degree of freedom.

3.2.3 Double-Exponential Smoothing (DES) of the Trend-History

As it was pointed out under the literature review of this work, the exponential smoothing approach of fitting history data and subsequently making forecasts has been a more realistic statistical tool for analyzing petroleum production data than either the automatic regression or the moving average. Apart from attributing more weights to the most recent data which is peculiar to production behavior, it also ensures limited errors in its history-fit. This it doesby making use of additional smoothing parameter(s) to correct for a fluctuating trend, besides the incorporation of ‘seasonality factor(s)’ in case of the presence of any periodicity.

Besides the need for a forecast-strategy using a most recent and dominant history-trend in making future predictions, the double-exponential smoothing (DES) is also used in this work because of its trend-correctness approach which makes the use of a simple exponential smoothing impossible, and the virtual impossibility of the presence of seasonality or periodicity in a petroleum production data set due to its technical and engineering nature, which also makes the application of a triple-exponential smoothing unnecessary.

The available double-exponential smoothing technique uses a linear trend-corrected exponential smoothing approach to fit the history and subsequently make forecast. The application of this in petroleum production forecasting is limited as petroleum production declines usually exhibit exponential trend behaviors. This has been underscored by the widespread industrial use of the exponential decline curve analysis (DCA), usually at early stages when secondary displacements are not yet engaged or the reservoir is designed to be completely produced under a full depletion mechanism. Even in situations when water-flooding is involved, managers and economists prefer the use of the exponential decline approach because of its level of conservativeness in forecasting.

Also, this linear trend-corrected exponential-smoothing technique has been designed for time series data set. Situations occur in the oilfield production-data measurements that make data for some periods unavailable (either as a result of errors in meter readings or shut-down) or initially available but filtered-off as outliers, this necessitates the need for some adjustments in the applications of the existing double-exponential smoothing governing equations.

It is in line with this that this work further modifies the existing linear trend-corrected exponential smoothing model for a non equi-spaced univariate time series data set. The modified linear trend-corrected exponential smoothing technique is thereafter used to develop the exponential trend-corrected exponential-smoothing model.

The selection of the appropriate exponential-smoothing technique for any forecast could be based on which between the modified linear trend-corrected and exponential trend-corrected smoothing techniques gives some least statistical parameters that ensure a better fit. This actually to a large extent could depend on the type of history data set being analyzed.

3.2.3.1 Modified Linear Double-Exponential Smoothing

The existing linear trend-corrected exponential-smoothing procedure for equi-spaced time seriesdata set has been properly analyzed under the literature review (Kikani[18] and Foreman[20]). To modify it for applications in non equi-spaced univariate time series data set, the various smoothing equations governing the linear double-exponential smoothing technique are adjusted, and thus, highlighted below.

For an observed production history data set (y_1, y_2, y_3, …, y_n) with a linear trend, β, the history-observed production value at time t

$$y_t = S_t + random\ error\ around\ S_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.1)}$$

where S_t is the forecast value for time t, and is expressed as

$$S_t = S_{e,t-1} + \left(t - (t-1)\right) * \beta_{e,t-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.2)}$$

$S_{e,\ t-1}$ and $\beta_{e,\ t-1}$ are the error-adjusted forecast and error-adjusted trend values at t-1.

Also,

Error-adjusted forecast value at time, t

$$S_{e,t} = S_t + \alpha * (y_t - S_t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.3)}$$

where α is the forecast-smoothing parameter, and (y_t – S_t) is the random error around each period forecast as indicated in equation (3.1).

Forecast error at every period of time, t, could be defined as

$$e_t = y_t - S_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.4)}$$

Error-adjusted trend value at time, t

$$\beta_{e,t} = \beta_{e,t-1} + \gamma * \alpha * (y_t - S_t) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.5)$$

where γ is the trend-smoothing parameter.

In form of the error term in equation (3.4), equations (3.3) and (3.5) respectively become

$$S_{e,t} = S_t + \alpha * e_t \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.6)$$

and

$$\beta_{e,t} = \beta_{e,t-1} + \gamma * \alpha * e_t \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.7)$$

3.2.3.2 Exponential Trend-Corrected Exponential Smoothing

As stated earlier, exponential decline trend has been a more prominentproduction decline trend in the industry than the linear trend used in developing the conventional double-exponential smoothing procedure thus, an exponential trend-corrected exponential smoothing would be a more realistic fittingtechnique with less error in some production history data than some. It is in lieu of this that this work goes further to upgrade the above non equi-spaced univariate time series linear trend-corrected exponential smoothing model into developing an exponential trend-corrected exponential smoothing model.

From the general Arps' exponential decline equation, production rate at a time, t

$$q_t = q_o e^{D\Delta t} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.8)$$

Taking the natural logarithm of both sides of equation (3.8) gives

$$\ln q_t = \ln q_o + D\Delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.9)$$

where q_t and q_o are the production rates at time, t, and a reference time, t_o respectively. D is the constant exponential rate (which is negative for decline, and positive for a build-up), and Δt ($\Delta t = t - t_o$) is the time or period lapse or difference between time, t, and the reference time, t_o.

Equation (3.9) can also be written as

$$\ln q_t = \ln q_o + (t - t_o) * D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.10)$$

Expressing equation (3.10) in a form of the history-observed production value,

$$\ln y_t = \ln y_o + (t - t_o) * D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.11)$$

In terms of a reference time period that equals a preceding time period, equation (3.11) can be expressed as

$$\ln y_t = \ln y_{t-1} + \big(t - (t-1)\big) * D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.12)$$

Comparing equation (3.2) and equation (3.12) in expressing S_t, which is the forecast value for time t,

$$\ln S_t = \ln S_{e,t-1} + \big(t - (t-1)\big) * \beta_{e,t-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.13)$$

Therefore,

$$S_t = e^{\ln(S_{e,t-1}) + (t-(t-1)) * \beta_{e,t-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.14)$$

where $S_{e,t-1}$ and $\beta_{e,t-1}$ are the error-adjusted forecast and error-adjusted trend values at t-1, and it can be concludedfor an unbiased or a less-fluctuating fit that,

$$D \approx \beta_{e,t-1} \approx \beta_{e,t} = constant \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.15)}$$

Equation (3.15) would be practically confirmed from a field application of this model in the next chapter.

Equation (3.15) implies that equations (3.13) and (3.14) can be expressed as

$$\ln S_t = \ln S_{e,t-1} + \big(t - (t-1)\big) * D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.16)}$$

$$S_t = e^{\ln(S_{e,t-1}) + (t-(t-1))*D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.17)}$$

Error-adjusted forecast value at time, t

$$\ln S_{e,t} = \ln S_t + \alpha_{exp} * \left(\ln\left(\frac{y_t}{S_t}\right)\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.18)}$$

where α_{exp} is the exponential forecast-smoothing parameter.

Forecast error at every period of time, t, remains

$$e_t = y_t - S_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.1)}$$

From equation (3.5),

Error-adjusted trend value at time, t

$$\beta_{e,t} = \beta_{e,t-1} + \gamma_{exp} * \alpha_{exp} * ln\left(\frac{y_t}{S_t}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.19)}$$

where γ_{exp} is the exponential trend-smoothing parameter.

Substituting equation (3.15) for $\beta_{e,t}$ and $\beta_{e,t-1}$ in equation (3.19) for an unbiased or a less-fluctuating fit,

$$\gamma_{exp} * \alpha_{exp} * ln\left(\frac{y_t}{S_t}\right) = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3.20)$$

Since

$$ln\left(\frac{y_t}{S_t}\right) \neq 0 \text{ ,}$$

Therefore,

$$\alpha_{exp} \; or \; \gamma_{exp} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.21)$$

Equation (3.21) would be confirmed from a practical application of the model, when the exponential smoothing parameters are optimized for minimum errors.

Either of the two models above could be used for the double-exponential smoothing procedure depending on the type of data being analyzed. To confirm which is better, both could be employed, and the one with the less generated history standard error is preferable most especially if such forecast is to be further subjected to an uncertainty analysis.

Also, it should be noted that irrespective of adopting an exponential trend-corrected exponential smoothing for history-trend fitting, it does not automatically mean that such approach would give a better base forecast than the modified linear trend-corrected exponential smoothing. A major advantage of this approach is the shrinking of the prediction intervals during uncertainty analysis of the forecasts, which is an appreciative tool for economical evaluation of a well/reservoir performance forecast. This crucial point would be validated in the Results and Discussion chapter of this work.

The application of the appropriate DES (either modified linear trend-corrected or exponential trend-corrected) on the production history data for generating a history-fit still follows the same procedure as highlighted in the literature review (Kikani[18] and Foreman[20]). Initial values of the forecast and trend($S_0$and β_0) could be estimated as stated under the literature review (Kikani[18] and Foreman[20]), or from a linear or exponential fit on the early data points of the production history for a linear trend-corrected or an exponential trend-corrected DES, respectively. The early portion of the production history data set used in estimating the initial values of forecast and trend ($S_0$and β_0) must not be too far from the onset of the history data set, in ensuring that the expected more weight is given to the first few early observed production history values. This also helps in reducing the fit-errors for the early-time region. The smoothing parameters could initially be assumed, but later optimized with any of the non-linear optimization software to minimize the generated forecast errors after fitting and smoothing. For example, the ‘Solver’ add-in in Excel Spreadsheet can be used to calculate the parameters.

Following the estimation of the initial parameters (forecast and trend), the exponential smoothing model could then be employed over the entire production history data using the various equations derived above (depending on which double-exponential smoothing technique is chosen) and the initially-assumed values of the smoothing parameters (α and γ).

As the history fitting and smoothing is being done, each history period's forecast error (e_t) is calculated. At the end of history fitting, the sum of squared error (SSE) should be estimated using the formula:

$$SSE = \sum_{t=1}^{n} e_t^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.22)}$$

The standard error, σ_e could then be estimated as:

$$\sigma_e = \sqrt{\frac{SSE}{DOF}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.23)}$$

where:

DOF is the Degree of Freedom, which is equal to the difference between the number of analyzed history periods or data values and the number of coefficients involved in the history-fit (same as the number of smoothing parameters in the smoothing model).

The standard error is very important in optimizing the smoothing parameters (like in the case of using Excel Spreadsheet).

After exponentially smoothing the history data set and generating the associated errors set, the smoothing parameters could then be optimized to minimize the history errors, and subsequently the standard error.

3.2.4 Autocorrelation of Generated Minimized History Residuals (Errors)

This procedure is designed to confirm that a possibly best-fit has been matched for the observed historical data set, using the concept of randomness of generated errors. The procedure could also help to confirm if there exists any seasonality pattern hidden in the data set being analyzed. The seasonality presence or absence confirms if the DES is suitable for the history-smoothing, history-fitting and subsequently forecasting, or a better model is required.

Autocorrelation is a statistical tool which is basically used to either detect non-randomness in a data set, or identify an appropriate time series model if the data are not random. The latter involves identifying if there exists any seasonality or periodicity in the behavior of the data set, which could assist in selecting a more appropriate time series model (for example the Triple-exponential Smoothing model) that could better analyze, smoothen and fit the historical data for a more reliable forecasting. Autocorrelation itself is a correlation coefficient. Instead of correlating two different variables, the correlation is between two values of the same variable at times t_i and t_{i+k}.

For this methodology, the variable is the generated minimized history error, and the behavior of such errors set when lined up with itself shifted (lagged) by a production period or more (to check if it moves in sync), indicates if the model is doing a good work.

Given minimized errors $e_1,e_2,\ldots,e_N$ at period/time $t_1,t_2,\ldots,t_N$, the lag k autocorrelation function is defined as:

$$r_k = \frac{\sum_{i=1}^{N-k}(e_i - \bar{e})\,(e_{i+k} - \bar{e})}{\sum_{i=1}^{N}(e_i - \bar{e})^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.24)$$

where $\bar{e}$is the average or mean of the entire history errors set.

When autocorrelation is used to detect non-randomness, it is usually only the first (Lag 1) autocorrelation that is of interest. For this case, the error autocorrelation coefficient for Lag-1 (r_1) must be close to zero. When it is used to identify any seasonality or periodicity, the autocorrelations are usually plotted for many lags. For a non-seasonal data set, the absolute values of the error autocorrelation coefficients for as many lags as calculated must fall below a threshold value, which is defined as:

$$r_{threshold} = \frac{2}{\sqrt{Number\ of\ Data\ Points}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.25)}$$

Meanwhile, as stated earlier, petroleum production data are not expected to show any observable seasonality or periodicity in their behavior or trend.

Randomness assumption must be valid before any statistical parameters are used in analyzing a data set, and this is better ensured by checking if the errors generated by the exponential smoothing model are indeed random, and do not exhibit any hidden pattern or seasonal behavior.

If the Lag-1 absolute value is below the threshold (e.g., random errors) and the absence of seasonality is confirmed through subsequent lags, it shows that the engaged model is capturing the structure of the production history data set. Thus, the methodology moves ahead to check for outliers.

3.2.5 Check for Possible Outliers

Once errors are random, presence of any outliers amongst the data values must be checked. Outliers could go as far as erroneously increasing the standard error, which is very important when it comes to making uncertainty analysis of forecasts. Due to the nature of how production data are obtained, there are high chances of noisy and off-trend data to be reported, which might not actuallyhave originated from the well/reservoir response. They could be due to metering or mistakes on the path of the reporting personnel. To maintain the integrity of the history trend, there is need to check for outliers and be removed. Usually, engineers do eye-fit trends, and expunge out any off-trend data values to smoothen their production fit. This works most times, but data values conveying important information about the historical production decline behavior that could actually shape the behavior of the forecast are occasionally removed. This necessitates the need for a statistical approach in checking and removing outliers from a production data set while still maintaining the integrity of the original data values within a statistically-tolerant range.

To identify the outliers,this methodology uses the minimized errors generated from the fitting and smoothing procedures in grouping the history data set. The methodology incorporates the use of two tested methods to check and remove outliers from production data set. These are:

1. Deviation Method / Thompson's τ-Technique
2. Tukey Fences Method

The Deviation method is a popular statistical way of identifying outliers. In engaging it for this methodology, production history data values with minimized errors outside the range of twice the standard deviation from the mean of the errors set are considered as outliers.

The Tukey Fences method is usually suitable for one-dimensional data, where the definition of an outlier is an extreme value of the normal range. It involves creation of a statistical distribution for the production history data errors, and estimating the 25th and 75th percentiles from such a distribution. The two percentile values are then used in calculating the Inter-quartile Range (IQR), which is the difference between the 25th and 75th percentiles. Two fences (inner and outer fences) could thereof be defined from the IQR, 25th and 75th percentiles.

For the Inner Fences, the Lower Inner Fence (LIF) and the Upper Inner Fence (UIF) are calculated as:

$$LIF = 25th\ Percentile - 1.5 * IQR \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.26)$$

$$UIF = 75th\ Percentile + 1.5 * IQR \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.27)$$

For the Outer Fences, the Lower Outer Fence (LOF) and the Upper Outer Fence (UOF) are calculated as:

$$LOF = 25th\ Percentile - 3 * IQR \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.28)$$

$$UOF = 75th\ Percentile + 3 * IQR \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.29)$$

The Inner Fences of the Tukey Fences Method give a slimmer range of acceptable data points than the Deviation Method, while the Outer Fences provide room for a wider range of acceptable data points than the Deviation Method.

3.2.6 Trend-Check on Filtered Selected Production History Data

Once the originally-selected production history data set has been filtered for any identified outliers, the remaining data set is re-subjected to another trend-check as initially done for the original production history data in the trend-checking stage above. This is to re-confirm that the remaining data set after the removal of outliers still maintains a non-zero slope that is statistically significant. This test is a major requirement in the application of any of the double-exponential smoothing techniques, and it further re-affirms that the outlier-checking stage has not over-filtered key data points that are significant in portraying the historical production trend. It ensures that the integrity of the information conveyed by the originally-selected production history data set is still maintained even after filtering-off the outliers (if any).

3.2.7 Double-Exponential Smoothing of the Filtered Selected Historical Production Trends

As procedurally carried out in the preceding DES stage of this methodology, the appropriate DES technique is re-applied on the remaining historical production trends. The same procedures for estimating the initial values of the forecast and trend, initial assumption of the smoothing parameters, fitting and smoothing, history-errors generation, calculation of the SSE and the standard error, and optimization of the smoothing parameters to achieve the minimal standard error, are all sequentially followed again towards establishing the possibly ‘best or perfect history-fit’ for the remaining history data set.

3.2.8 Autocorrelation of Residuals

The minimized set of production history data errors is further auto-correlated as before, to check and confirm randomness as well as presence of a seasonality pattern. If no seasonality is present and errors are random, fit is considered good. Then, deterministic or base forecast can be carried out.

3.2.9 Deterministic Prediction or Forecast

Once all the preceding procedural steps in the proposed methodology have been carried out and the results are consideredacceptable, in terms of minimized standard error, random errors and no seasonality presence (for the case of a double-exponential smoothing technique), the deterministic forecasts can then be established.

For a linear trend-corrected exponential smoothing, the future periods are forecasted using:

$$S_{t\_f} = S_{e,last} + \beta_{e,last} * \left(t_f - t_{last}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.30)$$

While for an exponential trend-corrected exponential smoothing, forecasts of future periods are calculated from:

$$S_{t\_f} = e^{\ln(S_{e,last})+(t_f-t_{last})*\beta_{e,last}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.31)$$

where:

S_{t_f} is the forecast value for period t in the future (with reference to the history periods),

$S_{e,last}$ is the last error-adjusted forecast value in history,

$\beta_{e,last}$ is the last error-adjusted trend in history,

t_f is the future time of forecast, and

t_{last} is the last history period.

3.2.10 Uncertainty Analysis of Future Forecasts or Predictions

This step involves putting some prediction intervals (lower and upper bounds) around the established deterministic future forecasts in the immediate stage described above, with some confidence level of certainty.

This stage involves the use of Monte Carlo simulation to generate as many possible forecast values for each forecast period as possible, from which the lower and upper percentiles of the respectively desired confidence or certainty level could be estimated to benchmark the prediction intervals.

It is noted that the generated minimizedproduction history data errors are usedin generating the possible forecasts for each forecast period. It is statistically most likely that for any history-fit, there exist some errors associated with it, and in as much as these errors occurred in the fitting and smoothing of history, it specifically points out that such history-fit when projected into the future would inherit at least a distribution of those errors in its forecast values. Therefore, this minimizedhistory errors set could then be used to generate a statistical distribution of errors from which different possible errors are extracted and used to correct the deterministic forecast of any of the future periods.

The presence of a distribution of errors that are possibly incurable on any future forecast, simply means that a distribution of possible forecast values could be generated for each forecast period. It is this distribution of possible forecast values per forecast period that could be analyzed for different percentiles, from which a prediction interval could be established for each desirable confidence or certainty level.

For a linear trend-corrected exponential smoothing,

To calculate a possible forecast, y_{t_f}, for any possible error (e_{t_f}) at a future period **right after** the last history period, t_{last}includes the following:

The base forecast value for such future period,

$$S_{t\_f} = S_{e,last} + (t_f - t_{last}) * \beta_{e,last} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.30)$$

The possible forecast value,

$$y_{t\_f} = S_{t\_f} + e_{t\_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.32)$$

Error-adjusted forecast value at time, t_f

$$S_{e,t\_f} = S_{t\_f} + \alpha * e_{t\_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.33)$$

Error-adjusted trend at time, t_f

$$\beta_{e,t\_f} = \beta_{e,t_f-1} + \gamma * \alpha * e_{t\_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.34)$$

For an exponential trend-corrected exponential smoothing,

To calculate a possible forecast, y_{t_f}, for any possible error (e_{t_f}) at a future period **right after** the last history period, t_{last}includes the following:

The base forecast value for such future period,

$$S_{t\_f} = e^{\ln(S_{e,last})+(t_f-t_{last})*\beta_{e,last}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.31)$$

The possible forecast value is the same as equation (3.32) above for the linear trend-corrected exponential smoothing.

Error-adjusted forecast value at time, t_f

$$S_{e,t\_f} = S_{t\_f} + \alpha_{exp} * e_{t\_f} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.35)$$

Error-adjusted trend at time, t_f

$$\beta_{e,t\_f} = \beta_{e,t_f-1} + \gamma_{exp} * \alpha_{exp} * ln\left(\frac{y_{t\_f}}{S_{t\_f}}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.36)$$

Recalling for an unbiased less-fluctuating history-fit,

$$\alpha_{exp}\ or\ \gamma_{exp} = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.21)$$

Therefore,

$$\beta_{e,t\_f} = \beta_{e,t_f-1} = constant\ (D) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (3.37)$$

In both techniques, where a current future period's forecast error is negative with a negative preceding future period's forecast trend, the calculated error-adjusted forecast trend for such current period will be positive (a rise in possible production performance). This does not represent the actual production performance decline from petroleum reservoirs due to a finite resource volume. To correct this and fit a practically-feasible decline for the future performance, this methodology adopts that the future periods' forecast trend for both techniques is kept constant from the last error-adjusted trend in history. This tends to build the uncertainty analysis around the deterministic base forecast. This limitation is prominent with the linear DES, as the exponential DES often corrects this by usually fitting history with a zero trend-smoothing parameter.

The deterministic future forecast and uncertainty analysis stages would be better carried-out using an Excel Spreadsheet as described in the Results and Discussion chapter.

3.2.11 Comparison of Forecast Models

The validation of any prediction model is to test how well it predicts when compared with actual data values. To do this, either remove the last few given observations or find the next few actual observations.Different prediction models can be statistically compared by analyzing the deviations of their individual predictions from what have actually been observed in history within the forecast periods. This may be done using the Root Mean Squared Error of Prediction (RMSEP) concept. The formula for calculating this statistic is given as:

$$RMSEP = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}(y_t - \hat{y}_t)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3.38)}$$

where: y_t is the model's prediction, $\hat{y}_t$ is the actual value observed at the predicted period, and τ is the number of predictions being used in the calculation of RMSEP.

4.0 RESULTS AND DISCUSSION

4.1 Introduction

As described in the earlier sections of this work, stochastic predictions of a hydrocarbon reservoirperformance have always been preferred when making technical and economic developmental decisions on petroleum assets. This tends to give predictions of a range of possible production performance with a measured degree of certainty of occurrence, rather than the deterministic method. The possibility ofstochastic behavior in the future production performance of a reservoir has always been attributed to the heterogeneous nature of the reservoir. To account for reservoir heterogeneities requires considering the varying distributions of several reservoir rock and fluid properties which serve as input parameters for predicting the production performance of wells producing from the host reservoirs. These parameters used in the development of either the geological or dynamic model of the reservoir include high levels of uncertainty in their measured values, and therefore necessitate the need for the quantification of the uncertainties in the predicted production performance generated from combinations of their values.

Out of the previously highlighted major data-driven approaches for making production performance forecasts, the history matching technique (which involves the use of the various reservoir simulation software) has been widely applied in carrying out production performance forecast with uncertainty analysis. There has been few published works describing the conversion of the well-established, deterministic production performance forecast procedures of the decline-curve and type-curve analyses into stochastic predictions. This is a major objective of this work.

4.2 Data Analysis, Results and Discussion

This section looks at the application and validation of the modified linear trend-corrected exponential smoothing technique and the newly-developed exponential trend-corrected exponential smoothing technique discussed under the methodology section of this work. The production performance forecasts with uncertainty analysis is presented using a set of oil production history data obtained from a Niger-Delta oilfield. In carrying out this work, a selected portion of the production history data is analyzed and subjected to the methodology of this study, probabilistic production performance predictions are made on the selected portion of history, and the generated stochastic predictions are compared with what were actually observed thereafter in history, starting from the last analyzed period of history.

Both the linear trend-corrected and the exponential trend-corrected exponential smoothing techniques areemployed in making the production forecasts. This is to show the merits and demerits involved in the use of each of the exponential smoothing techniques. The analysis of the production history data also involved evaluating the effectiveness and application of the two proposed methodologies of filtering-out data outliers. The chapter concludes by comparing the forecasts made by the developed methodological approaches and the results predicted using other existing empirical approaches.

4.2.1 Production History Data of Interest

The production history data from the Niger Delta candidate well used for the validation of the methodology is represented in Figure 4.1. This comprises of a production history period of over sixteen years.

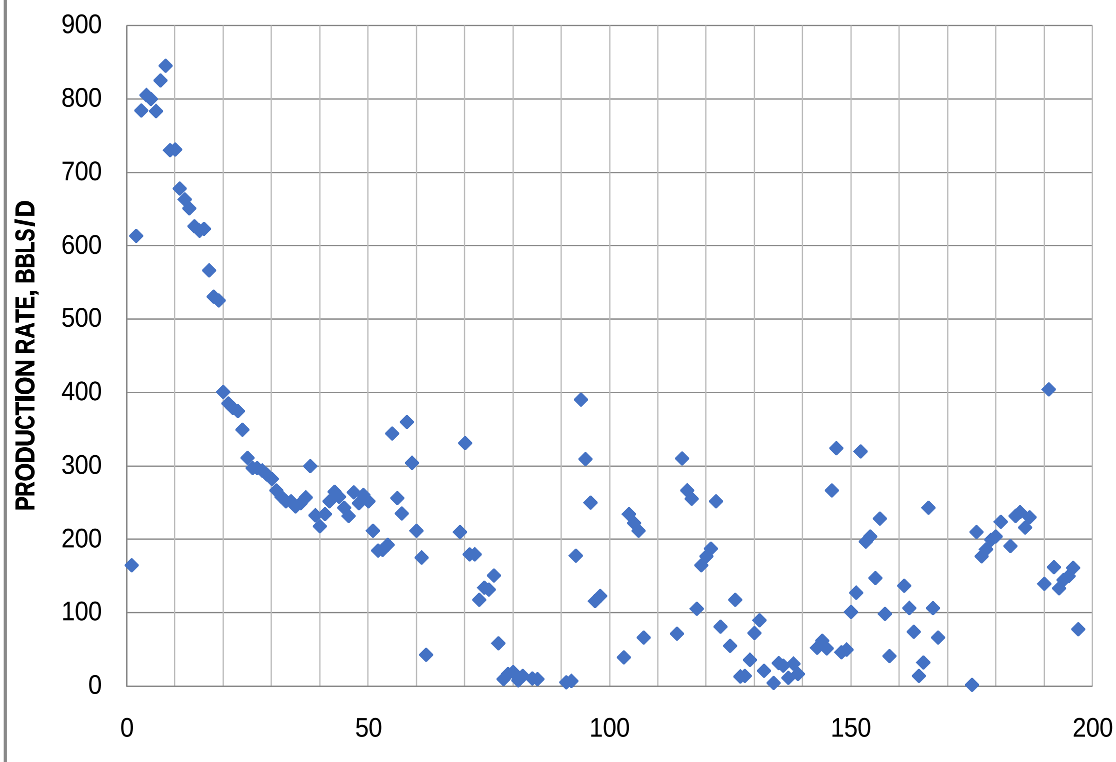

Fig. 4.1: HISTORICAL RATE -TIME PLOT

Due to the declining nature of the well production performance, after an initial depletion of the reservoir for about five years of production, the well was subsequently subjected to several work-overs and production enhancement operations. This is obvious with the display of random patterns in the well's production performance after the initial five years of production. For the sake of this work and to properly analyze the declining nature of the well's production, the initial four years of production of the well is studied. This is represented in Figure 4.2.

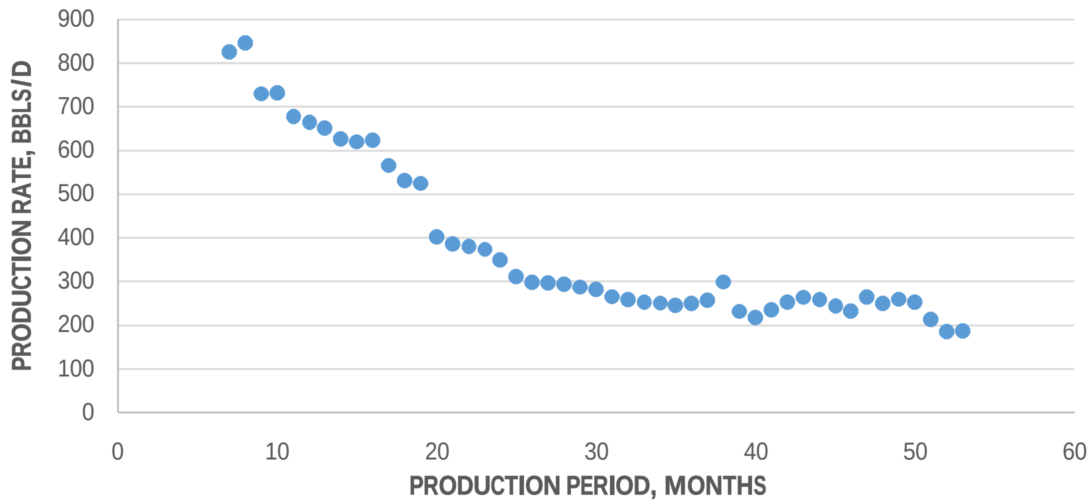

Fig. 4.2: TOTAL OBSERVED DECLINING PRODUCTION HISTORY

To properly confirm the consistency of the methodology used as stated above, an earlier sub-section of the initial declining behavior was used to generate a stochastic production forecast for the remaining section of the initial production decline period. The resulting forecast or prediction was then compared with what was actually observed in history. The initial declining sub-section comprised of the production history from the 7^{th} to the 42^{nd} production month. This is presented in Figure 4.3. This historic period represented a region in history that had shown a significantly-dominant production decline.

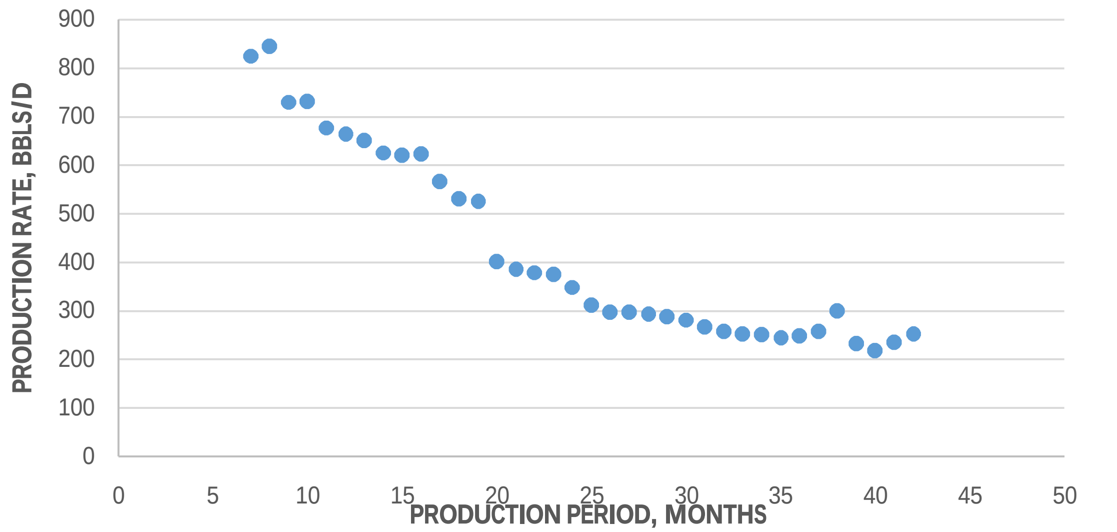

Fig. 4.3: SELECTED SECTION OF THE TOTAL OBSERVED DECLINING PRODUCTION HISTORY

4.2.2 Trend-Check on Selected Production Decline Period

In confirming the existence of a required possible trend along the selected production history, the data set was subjected to a trend-check to estimate the statistically-significant non-zero slope. As stated under the methodology section of this work, the slope, its standard error and the associated degree of freedom were calculated using the INDEX and LINEST commands in Excel Spreadsheet as approximately -17.32 bbls/d-month, 1.14 bbls/d-month and 34 respectively. The statistical significance of the slope was then checked by a t-test, done using the TDIST command in Excel. This estimated a p-value of 9.00E-17. The slope was actually non-zero as required, while the negative nature of its value showed a declining trend. An estimated p-value less than 0.05 (5%) further confirmed the statistical significance of the slope.

4.2.3 Modified Linear Double-Exponential Smoothing Technique

4.2.3.1 Linear Trend-Corrected Exponential Smoothing of the Trendy-History

The linear trend-corrected smoothing technique was applied at this stage to the trendy production history data. This represented a preliminary stage of fitting and smoothing the history data towards identifying the outliers present among the production history data values. A fitted linear regression on the whole trendy-history data set could have been used in doing this, but in ensuring that important production data values were not filtered-off as outliers by such fitted linear regression, it was more accurate and safer to use the right smoothing and fitting technique that would bring as many data points close to the history-fit as possible.

As shown in Figure 4.4, a linear regression was fitted on the earliest 6-month production data. This was to establish the initial forecast value (S_0) and the initial trend value (β_0). This is shown in Fig. 4.4 as 1101.1 bbls/d for the initial forecast and -37.458 bbls/d-month as the initial trend.

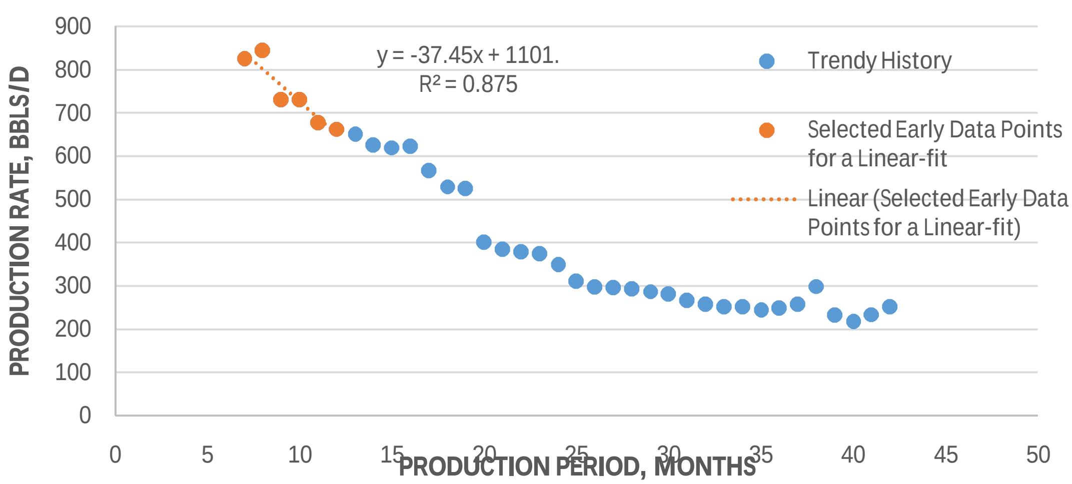

Fig. 4.4: ESTIMATION OF THE INITIAL FORECAST LEVEL AND TREND VALUES FROM TRENDY-HISTORY

Subjecting the selected production history data to a linear trend-corrected smoothing technique using equations (3.1) through to (3.7) established under the methodology, estimating the corresponding errors or residuals, and non-linearly optimizing the associated standard error using the Solver add-in in Excel, generated a forecast-smoothing parameter, $\alpha \approx 0.6695$, trend-smoothing parameter, $\gamma \approx 0.1957$, and an optimized standard error, $\sigma_e \approx 32.45$ bbls/d. The standard error was calculated using equations (3.22) and (3.23).

4.2.3.2 Autocorrelation of Generated History Residuals or Errors

Autocorrelation of the optimized history errors showed that the errors were random, and the established history-fitting and smoothing model was representing the history data well. This is shown in the correlogram of Figure 4.5 below with a Lag-1 autocorrelation coefficient of -0.0598, compared with a corresponding threshold of -0.3333. These were estimated using equations (3.24) and (3.25) respectively.

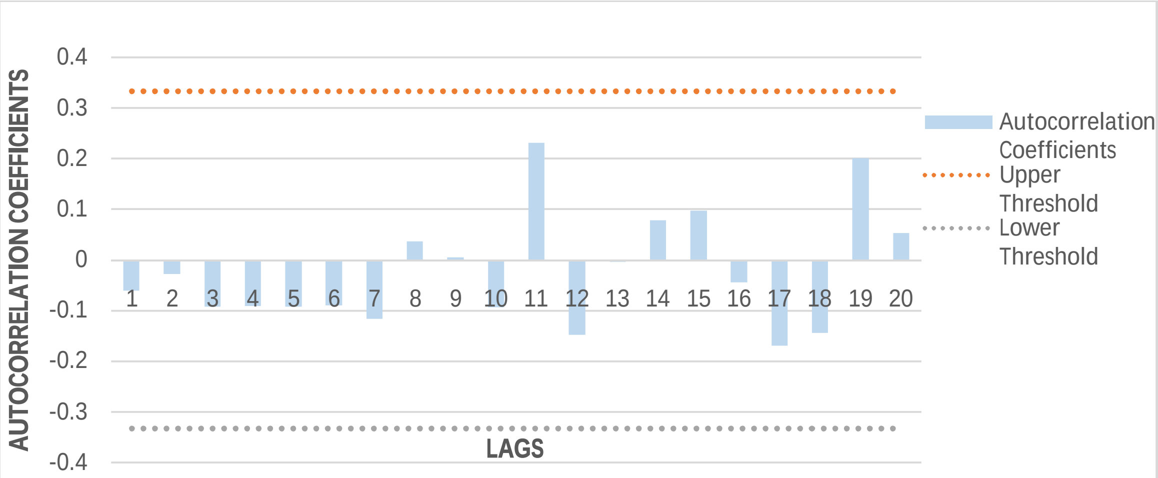

Fig. 4.5: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS

The randomness of these history errors would permit any statistical analysis to be carried out on the data set towards identifying and filtering-out any existing outliers. The correlogram also shows that none of the twenty-period lagged autocorrelation coefficients was outside the threshold range thus, there was no seasonality or periodicity pattern hidden in the history production performance of the well.

4.2.3.3 Checking for Possible Outliers

Outliers were checked for using both the recommended Deviation and Tukey Fences (Thompson's τ) Methods, by applying each independently to the optimized history residuals or errors set. This involved the use of equations (3.26) through (3.29).

The Deviation method showed a mean error and standard deviation values of 7.5319 bbls/d and 30.6267 bbls/d respectively, thereby establishing an upper and lower error bounds of 68.7854 bbls/d and -53.7215 bbls/d respectively.

On the other side, the Tukey Fences method calculated a 25th, 75th and Inter-quartile Range (IQR) of -6.2086 bbls/d, 28.2190 bbls/d and 34.4275 bbls/d respectively. This established an inner fence of lower and upper bounds of -57.8499 bbls/d and 79.8603 bbls/d respectively, as well as an outer fence of lower and upper bounds of -109.4910 bbls/d and 131.5016 bbls/d respectively. Both methods identified the 9th and 20th periods' production data values as outliers within the production history data set, with optimized errors of -64.4061 bbls/d and -94.6540 bbls/d respectively.

4.2.3.4 Trend-Check on Filtered Selected History Data

Subsequent to the removal of the identified outliers from the original history data, the remaining selected data set was re-subjected to another trend-check for the purpose of maintaining the production behavior or pattern of the history data set. The remaining selected history data is represented in Figure 4.6.

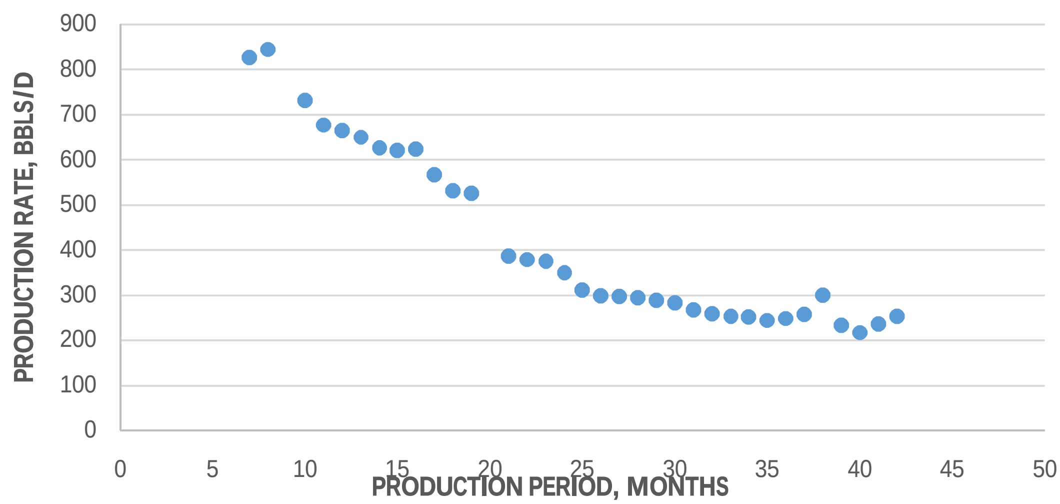

Fig. 4.6: REMAINING PRODUCTION HISTORY DATA AFTER REMOVING OUTLIERS

The slope, its standard error, the degree of freedom and the p-value were estimated same way it was done for the initial trend-check, and values of -17.2933 bbls/d-month, 1.1707 bbls/d-month, 32 and 7.64E-16 were obtained for the parameters, respectively. The new slope value indicated that the removal of the outliers had not affected the original trend, while the change in the degree of freedom value also showed the removal of two outliers. Since the removal of outliers still kept the slope to be non-zero and statistically significant, we could therefore go ahead with the smoothing of the remaining history data set.

4.2.3.5 Linear Double-Exponential Smoothing (DES) of the Filtered Selected Production History Data

The linear trend-corrected smoothing technique was re-applied to the remaining filtered production history data set as done initially when identifying outliers. At this stage, the main objective of the technique was to carry out the smoothing and fitting of data, from which the production forecasts were made. As done earlier, a new linear regression was fitted on the earliest six-month production data to determine the initial forecast and trend values to kick-start the smoothing and fitting procedures. This generated values of 1088.9 bbls/d and -35.108 bbls/d-month, respectively for the initial forecast level (S_0) and trend (β_0). It was observed that these were quite different from the results initially obtained at the earlier smoothing and fitting stage for outliers' identification. This was attributable to the removal of the 9th-period production data as an outlier. Figure 4.7 shows the fit and its regression values.

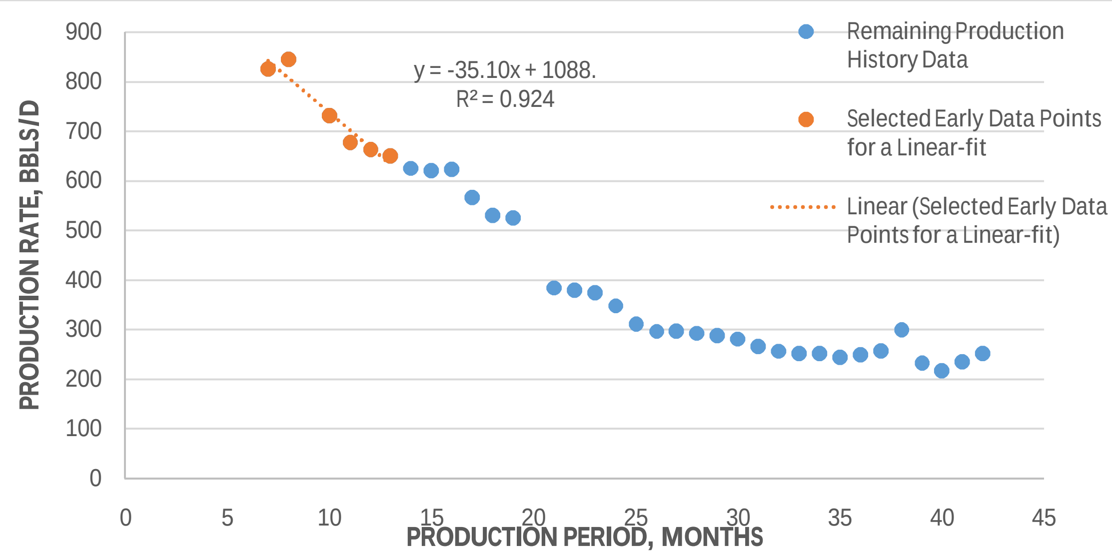

Fig. 4.7: ESTIMATION OF THE INITIAL FORECAST LEVEL AND TREND VALUES FROM REMAINING PRODUCTION HISTORY DATA

It should be noted that the earliest production data points were selected in making the linear-fit to determine the initial forecast and trend values. This was to ensure appropriately larger weight was given to the earlier data points whose values were paramount in determining the initial values of forecast and trend. If the late data points had been used, they would have generated quite more errors on the initial history-fit values, which would have significantly affected the optimized standard error.

After smoothing and fitting the history, as well as estimating the respective history errors, the optimized smoothing parameters were $\alpha \approx 0.7229$ and $\gamma \approx 0.1598$. The standard error (σ_e) was approximately 30.71 bbls/d. Figure 4.8 shows both the production history-observed data and the production history-match for comparison.

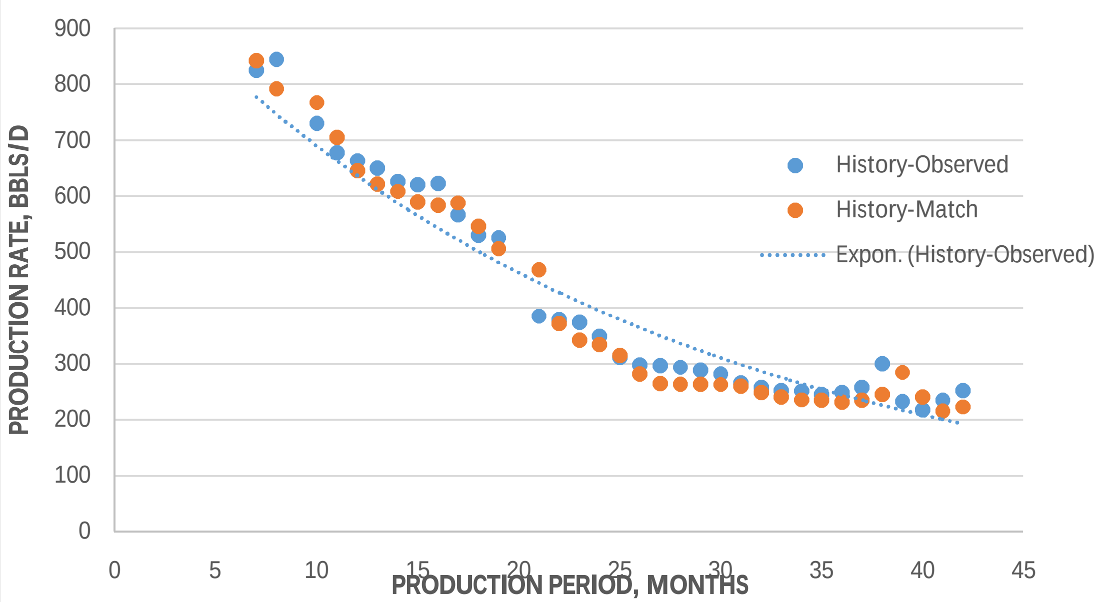

Fig. 4.8: PLOTS OF THE HISTORY-OBSERVED, LINEAR DES HISTORY-MATCH, AND EXPONENTIAL ARPS' FIT (HISTORY-OBSERVED)

The results shown in Figure 4.8 also indicate the application of conventional Arps' exponential decline fit to the production history data. The advantage of the smoothing technique over the popular Arps' exponential decline model can be easily recognized, as the former captured more data points than the latter.

The statistical difference between them is shown by calculating the sum of squared errors (SSE) for both models using equation (3.22). The linear trend-corrected exponential smoothing gave a SSE of 30,173 $bbls^2/d^2$ while the Arps' exponential decline model generated a SSE of 70,280 $bbls^2/d^2$. This clearly showed that the linear trend-corrected exponential smoothing technique was a better fit. It should be noted that the SSE does not guarantee that a smoothing model is doing a good job until its generated errors pass the autocorrelation check.

4.2.3.6 Autocorrelation of the Residuals

To confirm that the fitted history-model captured the history-trend, the set of generated history errors was further subjected to an autocorrelation procedure to determine its randomness, and to ensure no seasonality pattern was left unaccounted for. The calculated Lag-1 autocorrelation coefficient was -0.032, quite close to zero and below its corresponding threshold of -0.343. This confirmed the randomness of the errors. All the 20-period error lags were also within the threshold range of +0.343 to -0.343. This confirmed that there was still no seasonality or periodicity pattern hidden within the filtered production history data, and that no better smoothing model or technique (triple-exponential smoothing) was required to cater for any seasonal behavior. The correlogram in Figure 4.9 summarizes the autocorrelation results.

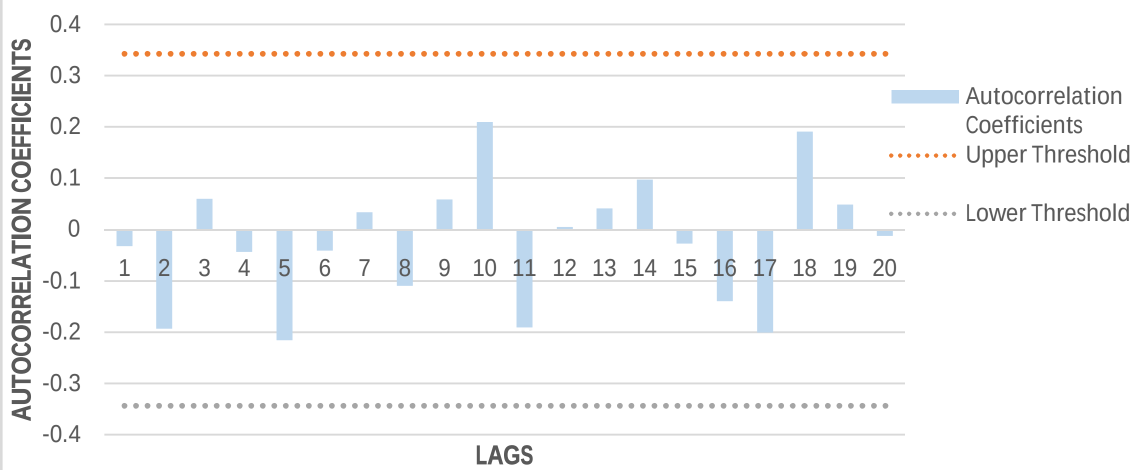

Fig. 4.9: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS

4.2.3.7 Deterministic or Base Forecast

Since the model had been confirmed to be a good representation of the production history data and their associated varying trend, then the models can be used to make the base forecasts. Equation (3.30) was used in estimating the base forecasts from the last history period (42^{nd} period) till the 54^{th} production period (which actually had been observed in history). In doing this, the last history-fit error-adjusted forecast ($S_{e,\,42}$) and trend ($\beta_{e,\,42}$) were required in equation (3.30). The values of the last history-fit error-adjusted forecast and the last history-fit error-adjusted trend were 243.9580 bbls/d and -2.9129 bbls/d-month, respectively. These were used in equation (3.30) to generate the base forecasts till the 54^{th}-production period (1 year-forecast). Also, the base forecasts till the 54^{th} period using the Arps' exponential decline model were generated for comparison. All these results are graphically represented in Figures 4.10(A) and 4.10(B).

In addition to the plots on Figure 4.10(A), Figure 4.10(B) also includes the shifted version of the Arps' exponential decline forecasts. This was done to show how the Arps' decline forecasts could also be interpreted in some production decline curve analysis (DCA) software like the Oilfield Manager (OFM), where the history decline pattern is fitted to one of the last and most recent production history performance data, assuming that the future production performance would usually follow from such most recent production history decline performance.

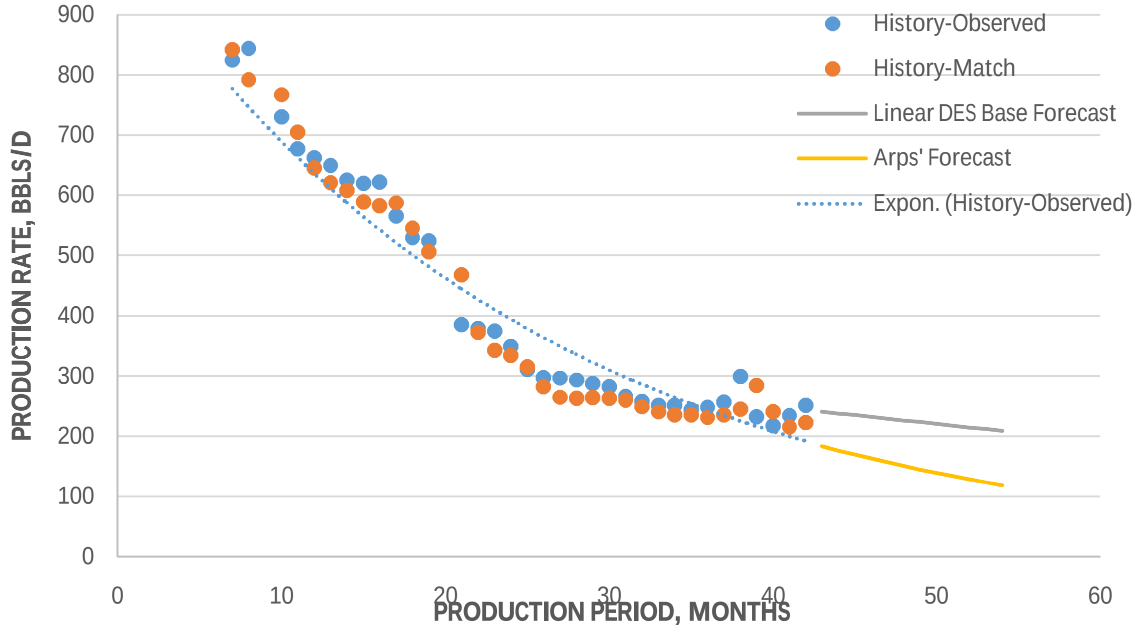

Fig. 4.10(A): PLOTS OF THE HISTORY-OBSERVED, HISTORY-FITS AND BASE FORECASTS FOR BOTH THE LINEAR DES AND ARPS' EXPONENTIAL DECLINE

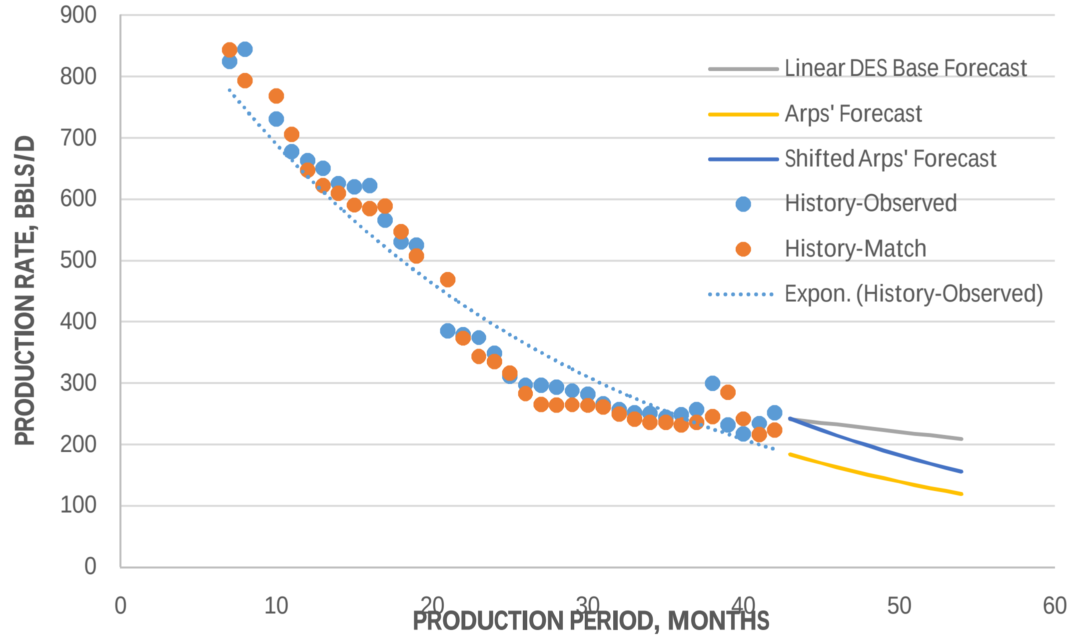

Fig. 4.10(B): PLOTS OF THE HISTORY-OBSERVED, HISTORY-FITS AND LINEAR DES BASE FORECASTS, ARPS' EXPONENTIAL DECLINE AND SHIFTED ARPS' EXPONENTIAL DECLINE

4.2.3.8 Impacts of the Selected Early Data Points Used in Estimating the Initial Forecast and Trend Values on the Forecast

As noted above, it is statistically better to select the first few early production data points in fitting the linear regression for estimating the initial forecast (S_0) and trend (β_0) values. This is to ensure appropriate weight is allotted to these values in calculating their preceding estimates (forecast and trend). Besides, this also ensures that the generated optimized standard error is minimized, as it has a great influence on the simulation of possible production forecasts upon which an uncertainty analysis could be established.

To show how this selection could affect the base forecast, and subsequently its associated possible forecasts, an analysis was made by selecting more early production data points. The analysis was divided into two scenarios (Scenarios 1 and 2). Scenario 1 involved selecting half of the entire production history data, while Scenario 2 was done with the selection of the entire production history data to fit the linear regression used in estimating the initial forecast and trend values.

As indicated in Figure 4.11, Scenario 1 gave an initial forecast value (S_0) of 1035.3 bbls/d and an initial trend value (β_0) of -28.793 bbls/d-month, after a linear fit was made on half of the entire production history data. Optimizing the standard error (σ_e) using the non-linear option of the Solver add-in in Excel, produced a forecast-smoothing parameter, α of 0.7381, a trend-smoothing parameter, γ of 0.1430, and a standard error, σ_e of 30.99 bbls/d (quite close to that analyzed above when the first 6-month early production data points were used).

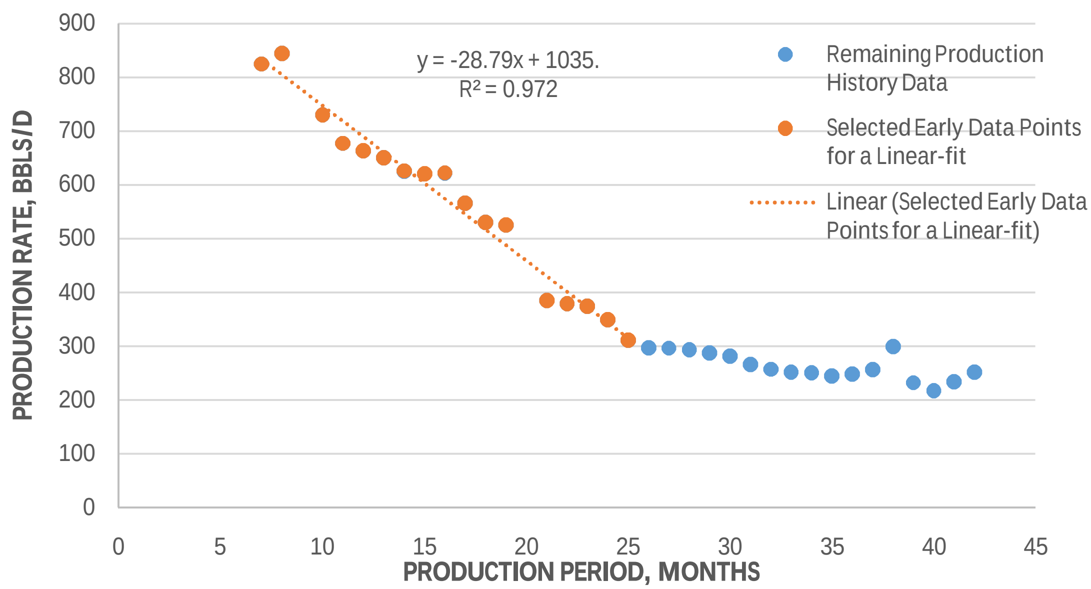

Fig. 4.11: ESTIMATION OF THE INITIAL FORECAST AND TREND VALUES BY LINEARLY FITTING HALF OF THE REMAINING PRODUCTION HISTORY DATA (SCENARIO 1)

The autocorrelation of the generated minimized history errors set for Scenario I is shown on the correlogram in Figure 4.12. This clearly indicates that the errors were random, and no seasonality existed within the observed production history data.

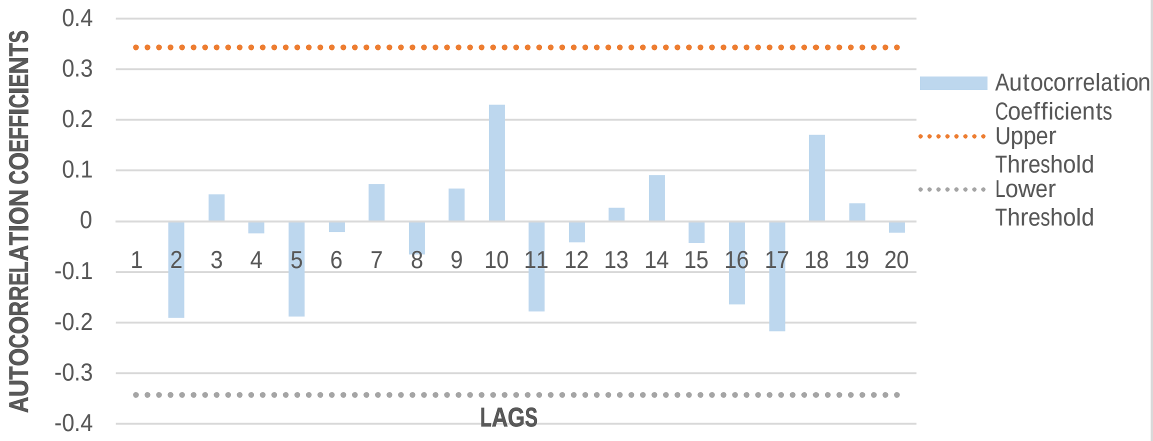

Fig. 4.12: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS

The linear DES technique also generated a last history-fit error-adjusted forecast level and trend values of 244.2440 bbls/d and -3.6542 bbls/d-month respectively, upon which the base forecasts till the 54th period were made. The history-observed, history-matched and base forecasts are highlighted in Figure 4.13.

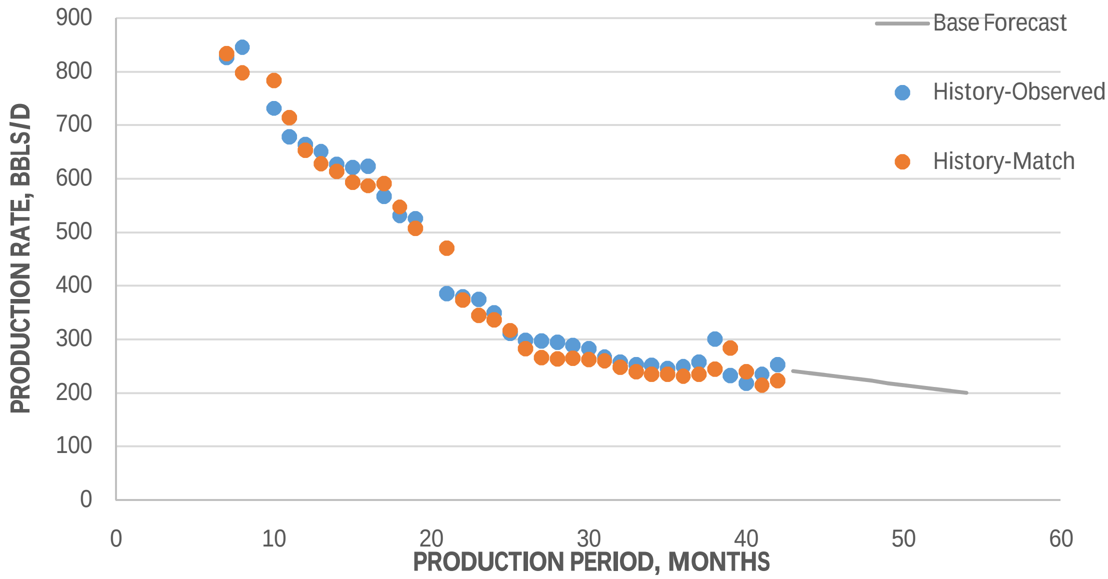

Fig. 4.13: PLOTS OF THE HISTORY-OBSERVED, HISTORY-MATCH AND BASE FORECASTS (SCENARIO 1)

Apart from a higher standard error generated by this approach compared to that generated when less early production history data were used to estimate the initial forecast and trend values, which would definitely generate a wider possible errors distribution upon which uncertainty analysis would be based, it would also be observed from Figure 4.13 that the base forecasts produced by this scenario were declining at a faster rate than the ones estimated when less data were used. This was as a result of a high last history-fit error-adjusted trend value given by this scenario.

Scenario 2 on the other hand generated an initial forecast level and trend values of 850.1 bbls/d and -17.293 bbls/d-month respectively. This is shown in Figure 4.14. It also produced corresponding values of $\alpha = 1$, $\gamma = 0$, and standard error, $\sigma_e \approx 36.95$ bbls/d after optimization.

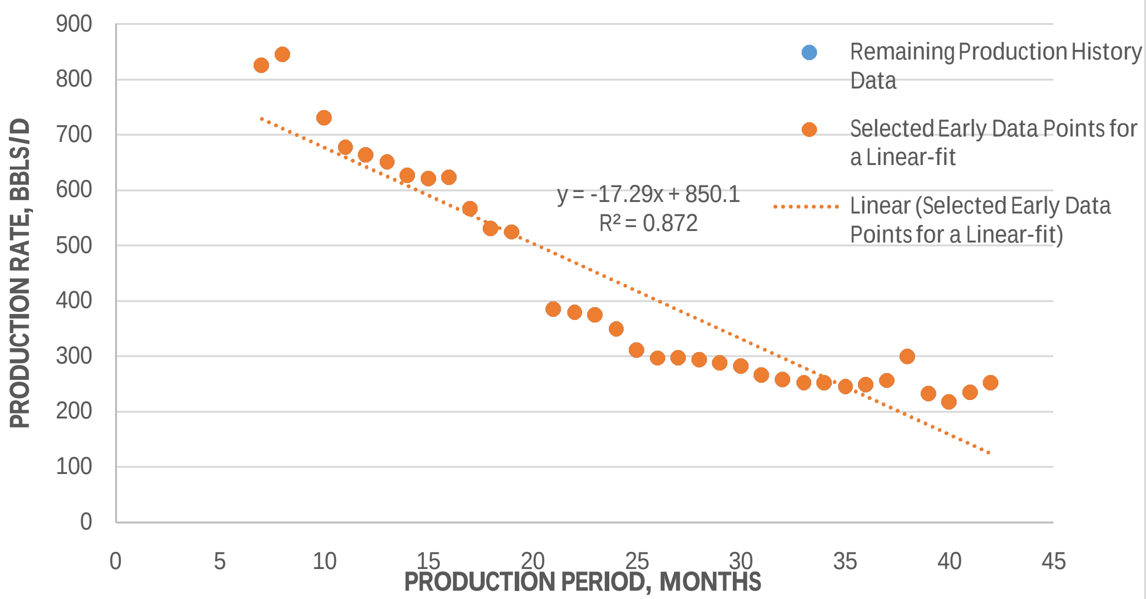

Fig. 4.14: ESTIMATION OF THE INITIAL FORECAST AND TREND VALUES BY LINEARLY FITTING ALL OF THE REMAINING PRODUCTION HISTORY DATA (SCENARIO 2)

The standard error value was way higher than that of Scenario 1.It can be concluded that the more the early data points used to estimate the initial forecast and trend values for a distinct perfect fit, the more the standard error generated, and the wider the possible distribution of errors used for the uncertainty analysis. Also, the forecast-smoothing parameter, α, generated by this scenario showed that the whole of the history error generated by each current forecast was used in smoothing for the next forecast (that is generating the error-adjusted forecast for the next period). It also pointed out that the trend was not error-adjusted throughout the smoothing and fitting of the history data. This was underlain by a trend-smoothing parameter of 0. The generated history errors set produced an autocorrelation that indicated the errors were random, and no seasonality was present in the history data set. This is represented by the correlogram in Figure 4.15.

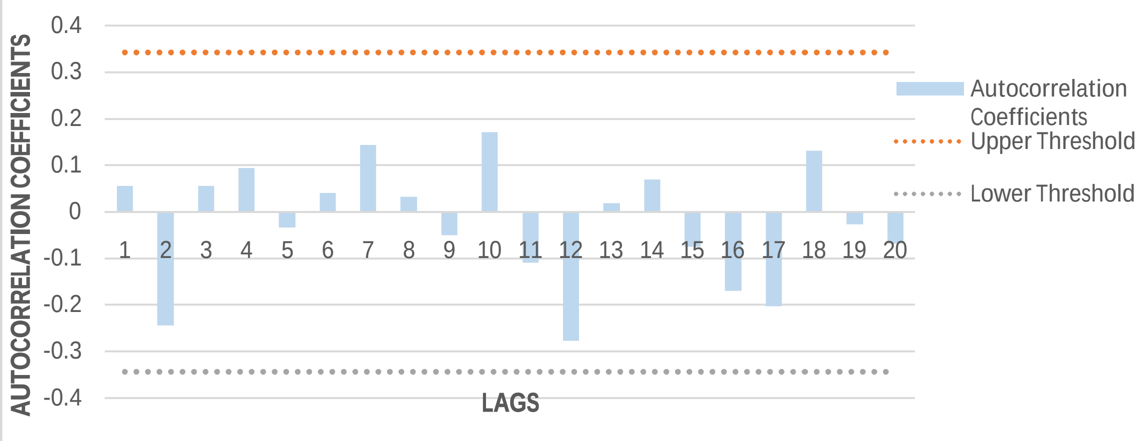

Fig. 4.15: CORRELOGRAM SHOWING AUTOCORRELATION RESULTS (SCENARIO 2)

Scenario 2 also led to a last history-fit error-adjusted forecast level value of 251.940 bbls/d, and a last history-fit error-adjusted trend value of -17.293 bbls/d-month (same as the initial trend value, due to a zero trend-smoothing parameter). Both values were used to generate the base forecasts till the 54th production period, using the same equation

(3.30) as applied previously. Figure 4.16 (A) shows the history-observed, history-match and base forecasts for this scenario. It is shown with Figure 4.16 (B) that the base forecasts obtained from Scenario 2 were decliningat a faster rate than that obtained from Scenario 1. The impact of using many early data points to estimate the initial forecast and trend values could also be concluded as producing forecasts whose base trend (decline) might not replicate the most recent history-observed trend. This is linked to the equal weighting of all history-observed trends as a result of fitting many history-observed data to estimate the initial values, thereby generating a more unbiased rather than a perfect history-fit upon which subsequent forecasts are based.

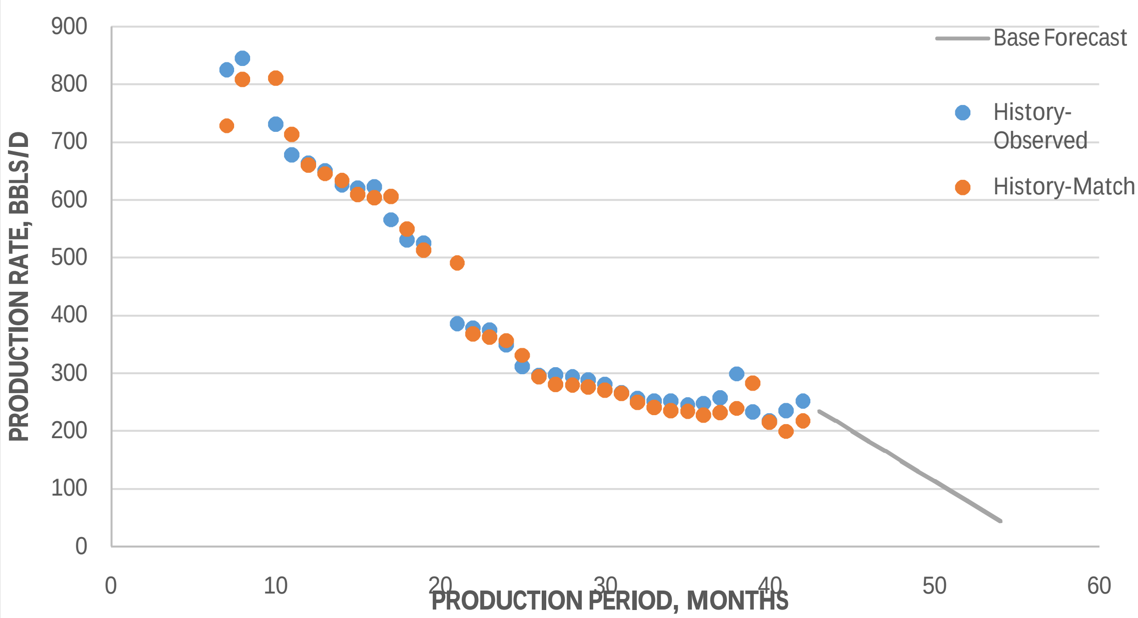

Fig. 4.16 (A): PLOTS OF THE HISTORY-OBSERVED, HISTORY-MATCH AND BASE FORECASTS (SCENARIO 2)

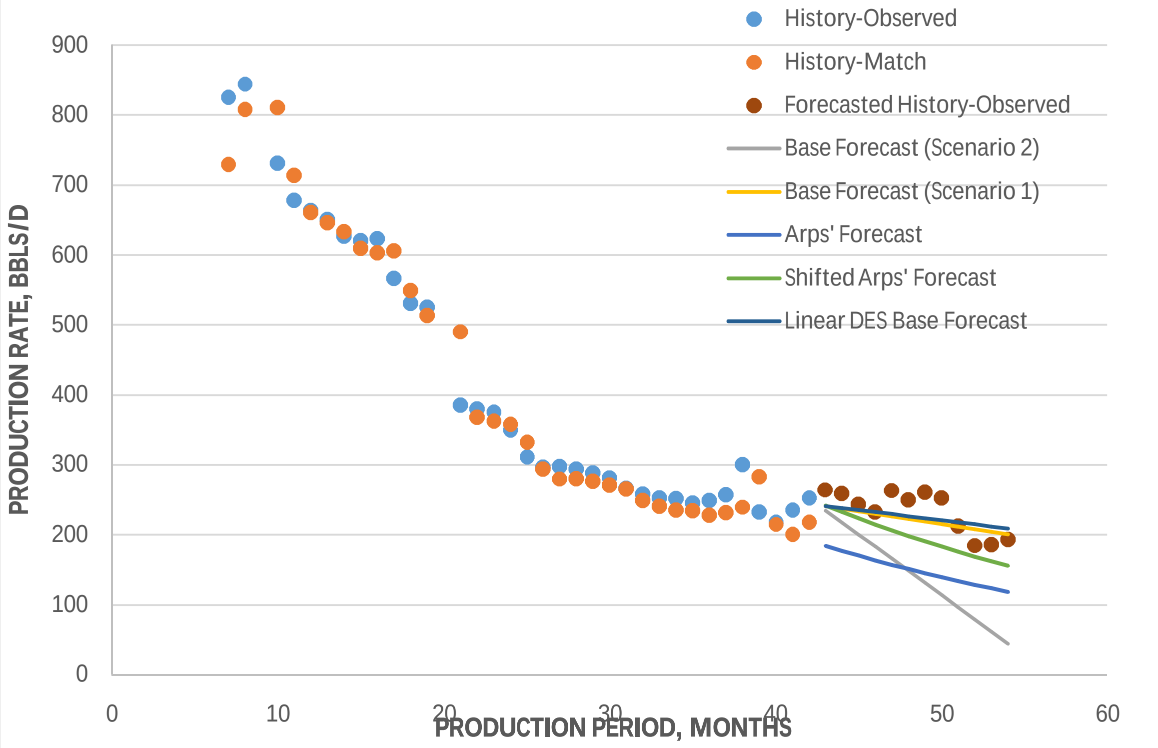

Fig. 4.16 (B): PLOTS OF THE HISTORY-OBSERVED, HISTORY-MATCH, BASE FORECASTS, FORECASTED HISTORY-OBSERVED, SCENARIOS 1 & 2 FORECASTS, AND ARPS' FORECAST

4.2.4 Exponential Double-Exponential Smoothing Technique

Before an uncertainty analysis was carried out on the previously generated forecast model from the modified linear trend-corrected exponential smoothing technique, an exponential trend-corrected exponential smoothing procedure developed in this work, was also applied on the production history data to compare the accuracy of both models.The objective was to analyze how the exponential trend-corrected procedure would reduce the optimized standard error, therefore reducing the span of the prediction intervals obtainable on any uncertainty analysis carried out on its production forecast model as compared to that obtainable through the modified linear trend-corrected exponential smoothing technique. It was also used to correct the linearly declining base forecasts to a more industry-adopted exponential decline.

For the exponential trend-corrected exponential smoothing technique, the same production history data used for the linear trend-corrected technique as shown in Figure 4.3 was used.

To smoothen the production history data towards establishing a fit which could be used to statistically identify any outliers, the exponential trend-corrected smoothing technique was applied. First, use an exponential fit on the first 8-month early production history data points to estimate the initial forecast level value (S_0) and the initial forecast trend value (β_0). These were reported as 1121.5 bbls/d and -0.043 per month, respectively. These are shown in Figure 4.17.

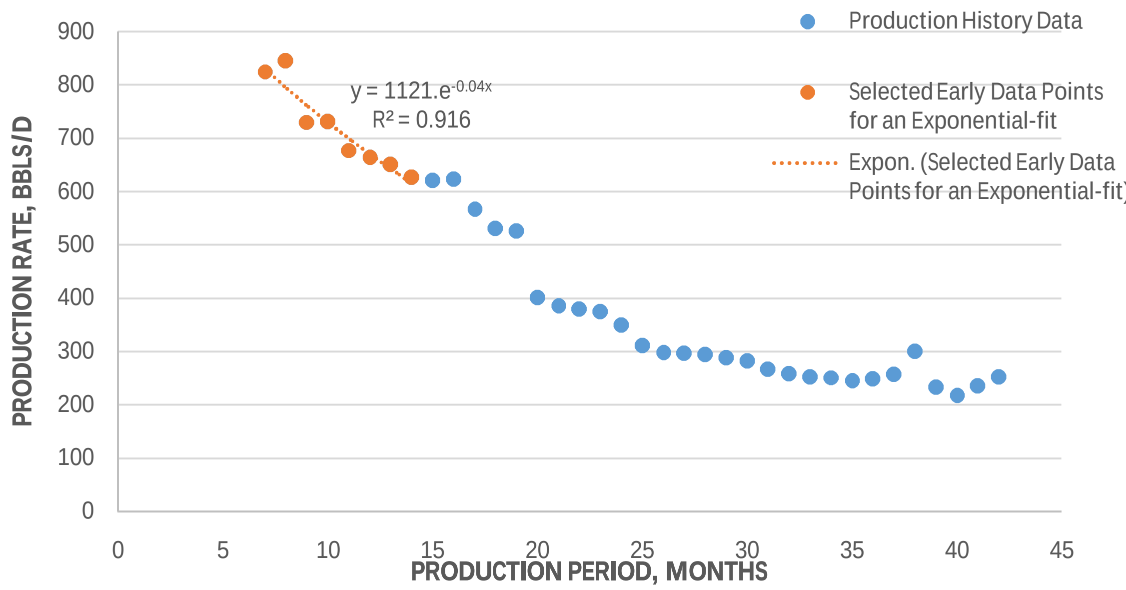

Fig. 4.17: ESTIMATION OF THE INITIAL FORECAST AND TREND VALUES FROM THE TRENDY-HISTORY DATA USING AN EXPONENTIAL-FIT

To begin the exponential trend-corrected smoothing and fitting, initial values of 0.5 was assumed for both the forecast and trend-smoothing parameters (α and γ respectively). Equations (3.13), (3.14), (3.1), (3.18) and (3.19) were used in calculating the corresponding forecast, forecast error, error-adjusted forecast and error-adjusted trend values for the whole history data points. The sum of squared error (SSE) and its associated standard error (σ_e) were also estimated with equations (3.22) and (3.23) respectively. The standard error was then optimized with the non-linear Excel Solver add-in, using the forecast and trend-smoothing parameters. At the end of optimization, optimized forecast and trend-smoothing parameters values of 0.6493 and 0 were obtained. The product of these two parameters (which was zero) tended to validate equations (3.15), (3.20) and (3.21), as stated earlier in the methodology section of this work. A minimized standard error (σ) value of 29.65 bbls/d was also obtained. It must be noted that this is peculiar to a less-fluctuating fit, where an unbiased fit tends to portray the quality of a best or perfect fit for the historical production data. For a scenario where the fit is highly fluctuating, and a best or perfect fit is significantly different from an unbiased fit for the same historical production data, the trend-smoothing parameter that optimizes the standard error may be non-zero.

The generated history forecast residuals or errors were then subjected to an autocorrelation check, to confirm their randomness and absence of any seasonality. Figure 4.18 shows the correlogram highlighting the autocorrelation coefficient values at all the lagged periods, which were all within the threshold range.

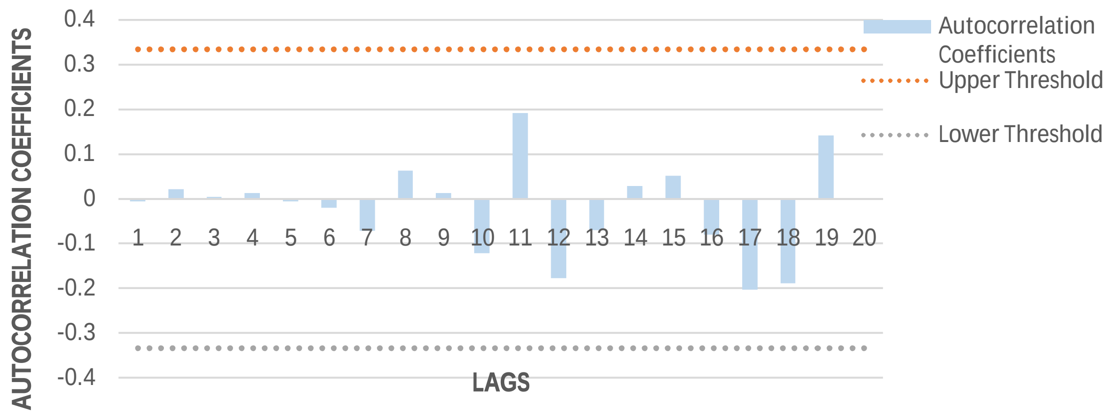

Fig. 4.18: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS FOR DES ANALYSIS

In identifying the outliers, the production history data set was subjected to both the Deviation and Tukey Fences methods. A mean error, standard deviation, upper and lower bound values of 2.1118 bbls/d, 28.7343 bbls/d, 59.5805 bbls/d and -55.3569 bbls/d were obtained respectively from the Deviation Method. With these bounds, the Deviation Method pointed out the 9th, 20th and 38th periods' data points as outliers with error values of -60.85 bbls/d, -98.21 bbls/d and 62.00 bbls/d respectively. The Tukey Fences Method generated the 25th and 75th percentile error values of -10.4468 bbls/d and 13.5577 bbls/d respectively, with an Inter-quartile Range of 24.0045 bbls/d. These gave an inner fence of lower and upper values of -46.4536 bbls/d and 49.5645 bbls/d respectively, as well as an outer fence of lower and upper values of -82.4604 bbls/d and 85.5713 bbls/d, respectively. The 8th, 9th, 20th and 38th periods' data points were pointed out as outliers by the Tukey Fences Method, with error values of 52.78 bbls/d, -60.85 bbls/d, -98.21 bbls/d and 62.00 bbls/d, respectively.

Both of the remaining filtered production history data sets (after removing the outliers) from the Deviation and Tukey Fences methods were individually analyzed with the exponential trend-corrected exponential smoothing technique to check for the one with the lower standard error, upon which the base forecasts and the corresponding prediction intervals were made.

Figure 4.19 is a plot of the remaining production history data after set of outliers was removed using the Deviation method. On the plot is the exponential fit using the first 7-month early production data points to estimate the initial forecast level (S_0) and trend (β_0) values, which were 1151.1 bbls/d and -0.045 per month, respectively.

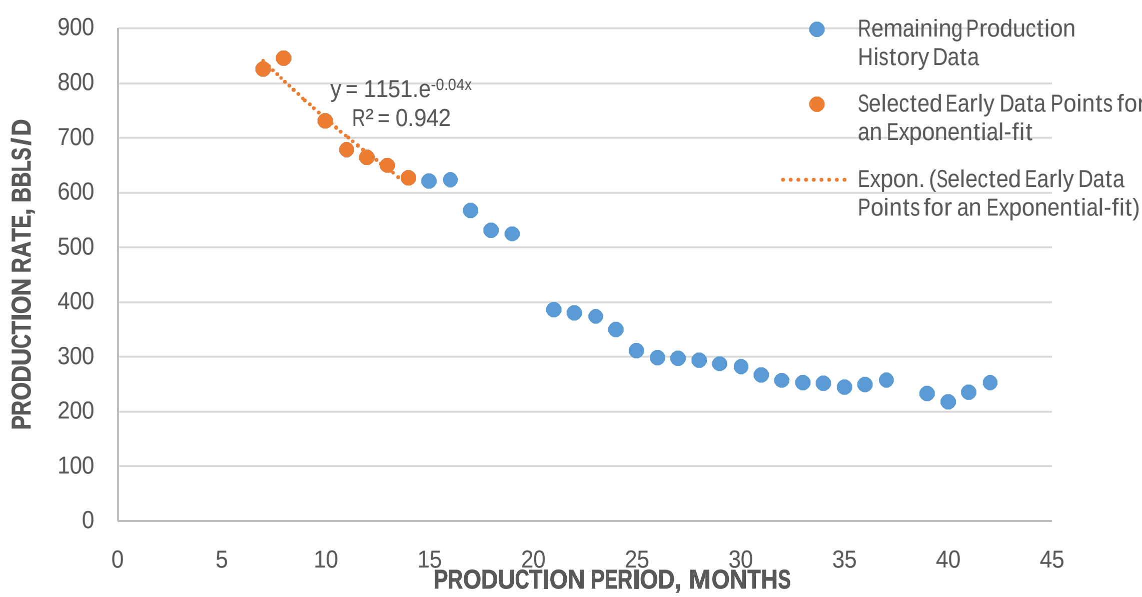

Fig. 4.19: ESTIMATION OF THE INITIAL FORECAST AND TREND VALUES FROM REMAINING PRODUCTION HISTORY DATA USING AN EXPONENTIAL-FIT (DEVIATION METHOD)

Applying the exponential trend-corrected exponential smoothing technique to the remaining production history data as done previously and optimizing the history errors, a forecast-smoothing parameter of value 0.7727, and a trend-smoothing parameter of value 0 were obtained. An optimized standard error of 25.65 bbls/d was also obtained. Figure

4.20 shows the smoothing and fitting of the observed production history data by the exponential trend-corrected smoothing technique.

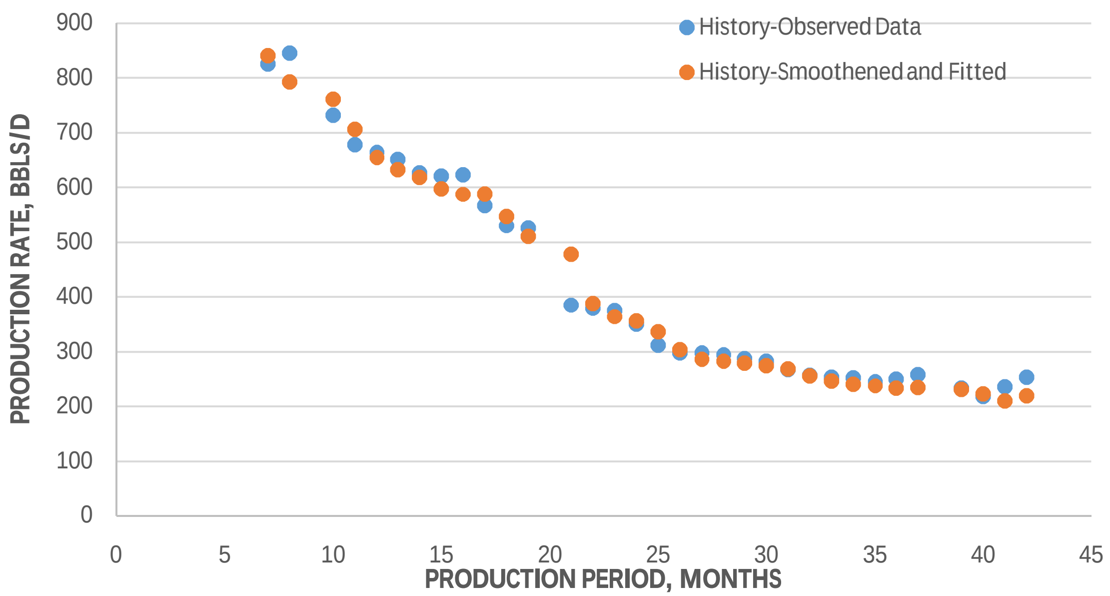

Fig. 4.20: SMOOTHING AND FITTING OF THE HISTORY-OBSERVED BY THE EXPONENTIAL DES TECHNIQUE

An autocorrelation of theset of the history residuals was also done to show that the fitted model was capturing the production history properly; by checking for randomness of error and absence of any hidden periodicity within the production history data. The correlogram produced by this check is represented with Figure 4.21, with all the calculated autocorrelation coefficients falling within the established threshold range.

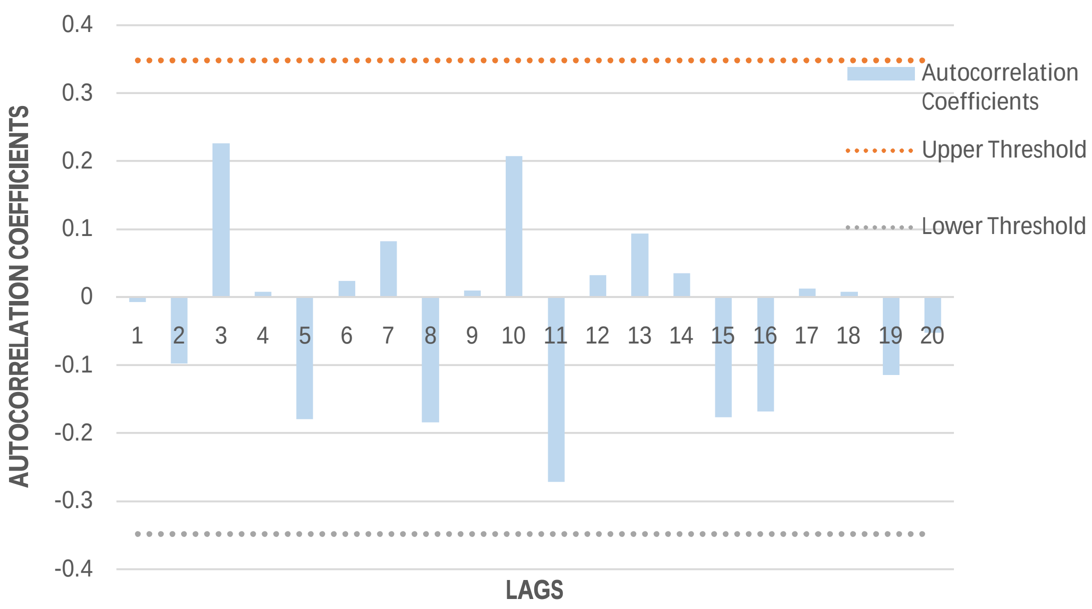

Fig. 4.21: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS (DEVIATION METHOD)

Note that the remaining production history data set after removing the outliers using the Tukey Fences method were subjected to the exponential trend-corrected smoothing procedure, and initial forecast level and trend values obtained were 1079.5 bbls/d and -0.04 per month, respectively. The results of the fitting of an exponential regression on the first 6-month early production history data points are shown in Figure 4.22.

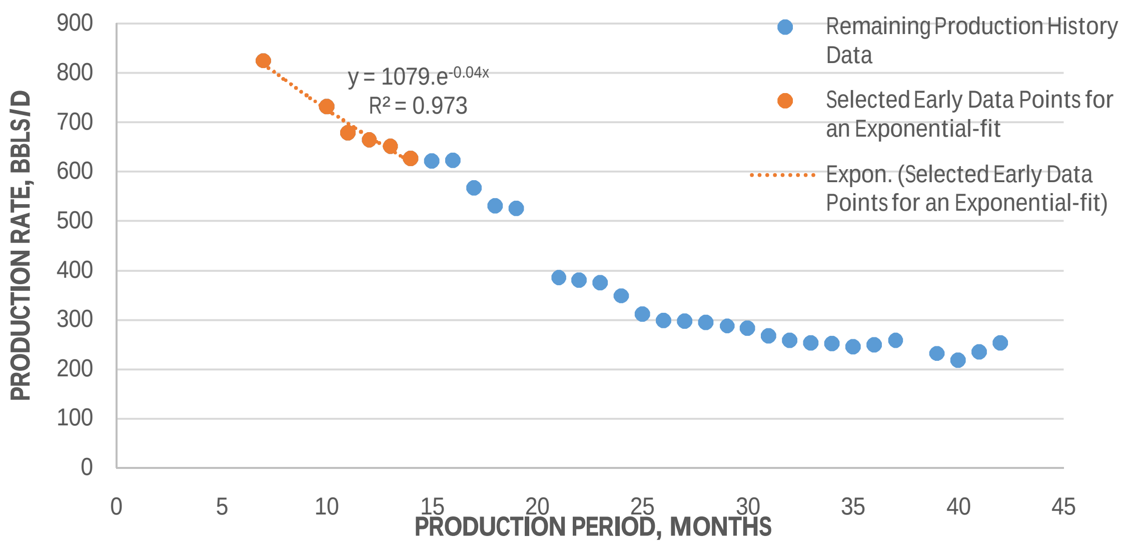

Fig. 4.22: ESTIMATION OF THE INITIAL FORECAST AND TREND VALUES FROM REMAINING PRODUCTION HISTORY DATA USING AN EXPONENTIAL-FIT (TUKEY FENCES METHOD)

The smoothing technique also generated a forecast-smoothing parameter, $\alpha \approx 0.8843$, a trend-smoothing parameter, $\gamma \approx 0$, and a standard error, $\sigma_e \approx 23.48$ bbls/d, after a non-linear optimization.

The remaining production history data also passed the randomness and seasonality checks after applying an autocorrelation check on the history errors. The associated correlogram is shown in Figure 4.23.

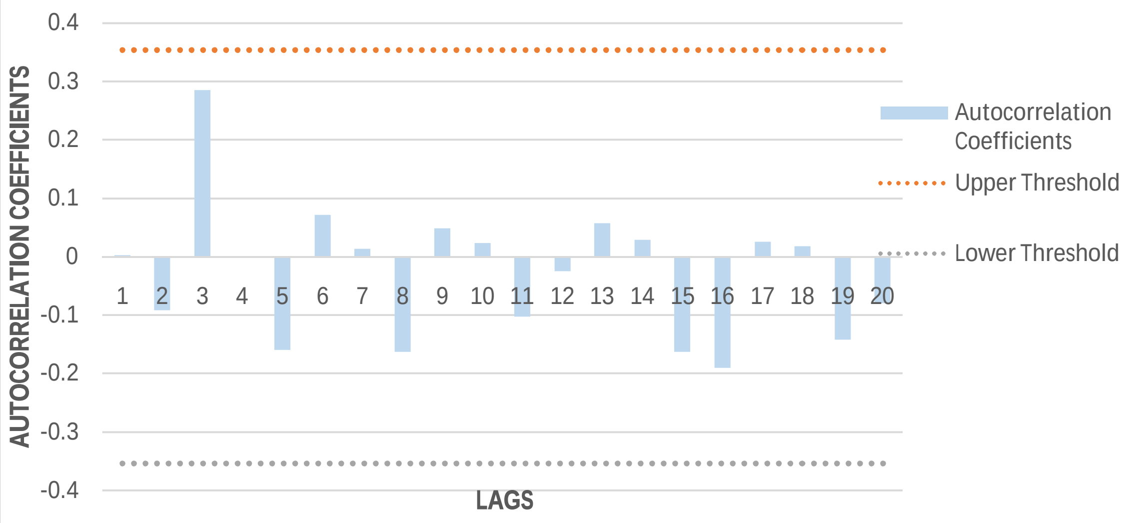

Fig. 4.23: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS (TUKEY FENCES METHOD)

It is observed that the optimized standard error generated by fitting and smoothing the production history data set when Tukey Fences method was used to remove the outliers was smaller than that generated when the Deviation method was applied. This behavior is subjective and depends on the data set being analyzed. The tendency of the Tukey Fences method to yield smaller standard error necessitated the adoption of its remaining production history data to run the production forecasts.

The case with the lower standard error generated a last history-fit error-adjusted forecast level value ($S_{e,\ last}$) of 248.350 bbls/d and a last history-fit error-adjusted trend value of -0.040 per month. These were used in equation (3.31) to generate the base forecast values till the 54th production period. Figure 4.24(A) shows the history-observed, exponential smoothing history-match, base forecasts, Arps' exponential history-match and the Arps' exponential base forecasts plots for comparison.

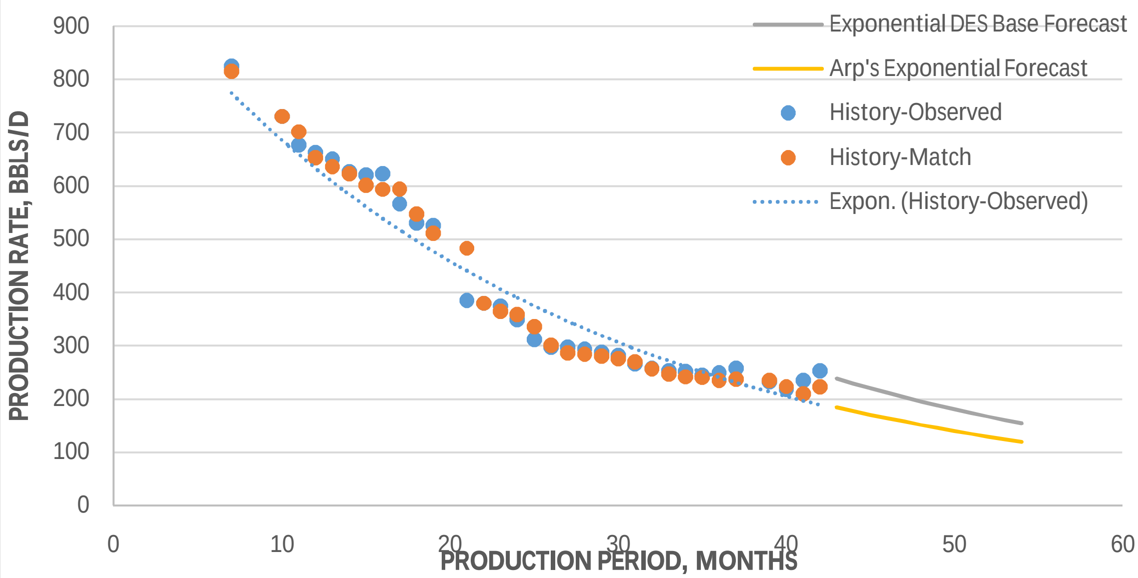

Fig. 4.24(A): PLOTS OF THE HISTORY-OBSERVED, HISTORY-FITS AND BASE FORECASTS FOR BOTH THE EXPONENTIAL DES TECHNIQUE AND ARPS' EXPONENTIAL DECLINE

Figure 4.24(B) further includes the shifted version of the Arps' exponential decline forecasts. This was to show how the Arps' decline forecasts could also be applied in some production decline curve analysis (DCA) software like the Oilfield Manager (OFM). The results look pretty similar to the exponential DES base forecasts within the stipulated forecastperiods.

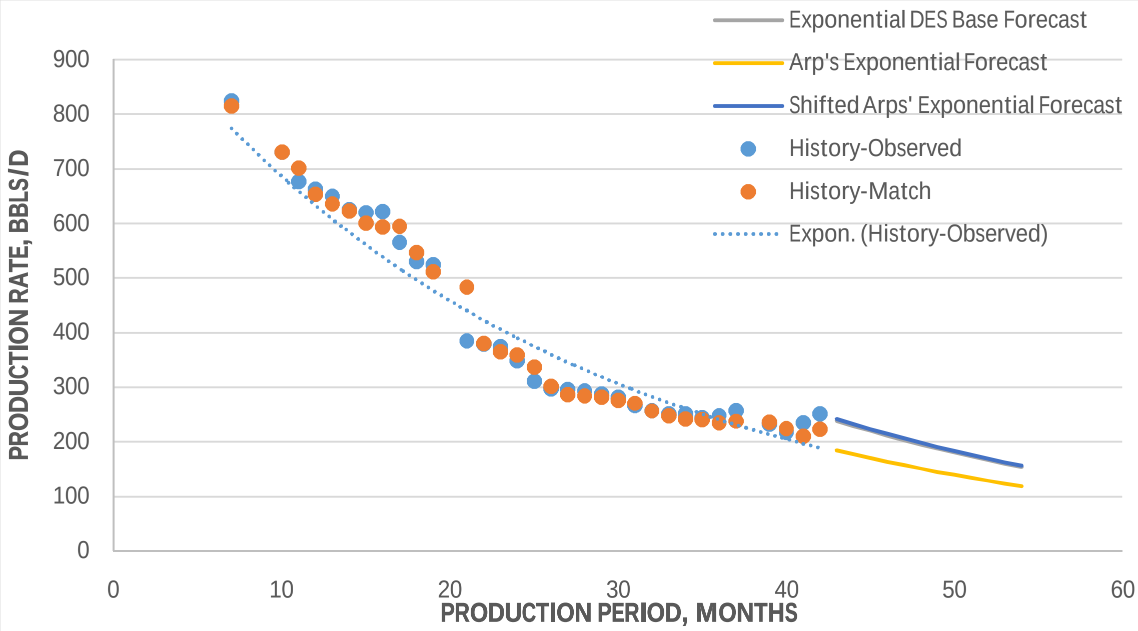

Fig. 4.24(B): PLOTS OF THE HISTORY-OBSERVED, HISTORY-FITS AND BASE FORECASTS FOR THE EXPONENTIAL DES TECHNIQUE, ARPS' EXPONENTIAL DECLINE AND SHIFTED ARPS' EXPONENTIAL DECLINE

The more-declining base forecasts obtained using the exponential trend-corrected smoothing technique can be compared with that of the modified linear trend-corrected smoothing technique. This provided a forecast trend similar to that of the shifted version of the Arps' exponential decline behavior. Note that the conventional non-shifted Arps' exponential decline under-estimated the base forecasts. The lower optimized sum of squared error and standard error obtained from using this DES technique compared with the ones obtained from both the linear trend-corrected smoothing technique and the Arps' exponential decline approach can also be observed. This observation is very important during any uncertainty analysis on the base forecasts, as would be seen later.

4.2.5 Combined Linear and Exponential Double Exponential Smoothing Techniques

It is pointed out that this work also examined how the standard error could be minimized. The approach taken involved the use of the linear trend-corrected exponential smoothing technique in fitting and smoothing the original production history data towards identifying the outliers, and subsequently subjecting the remaining filtered data set to an exponential trend-corrected exponential smoothing technique, from which an optimized standard error could be calculated, with the autocorrelation of the history errors passing the randomness and seasonality checks. This approach generated an optimized standard error of 28.1887 bbls/d, which was higher than either of the preceding techniques. This result could actually go in either way, depending on the data set being analyzed.

In terms of the optimized standard errors, this DES approach further helped to confirm that the selected exponential trend-corrected smoothing technique was still the most effective in meeting an objective of minimal history-fit errors, since all their smoothened fits actually passed the autocorrelation stage.

The plot of the exponential regression used to estimate the initial forecast level and trend values after outliers had been removed, is represented with Figure 4.25, while the history-observed and history-match plots using the exponential trend-corrected smoothing technique are shown on Figure 4.26. Figure 4.27 shows the correlogram obtained from the autocorrelation of the history errors.

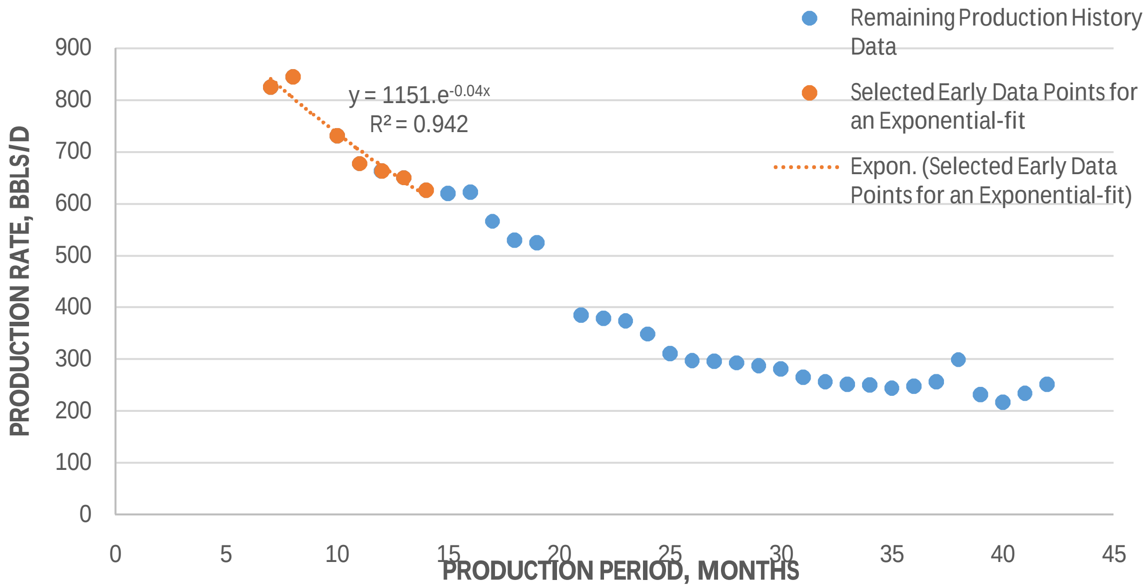

Fig. 4.25: ESTIMATION OF THE INITIAL FORECAST LEVEL AND TREND VALUES FROM REMAINING LINEARLY-FILTERED PRODUCTION HISTORY DATA USING AN EXPONENTIAL-FIT

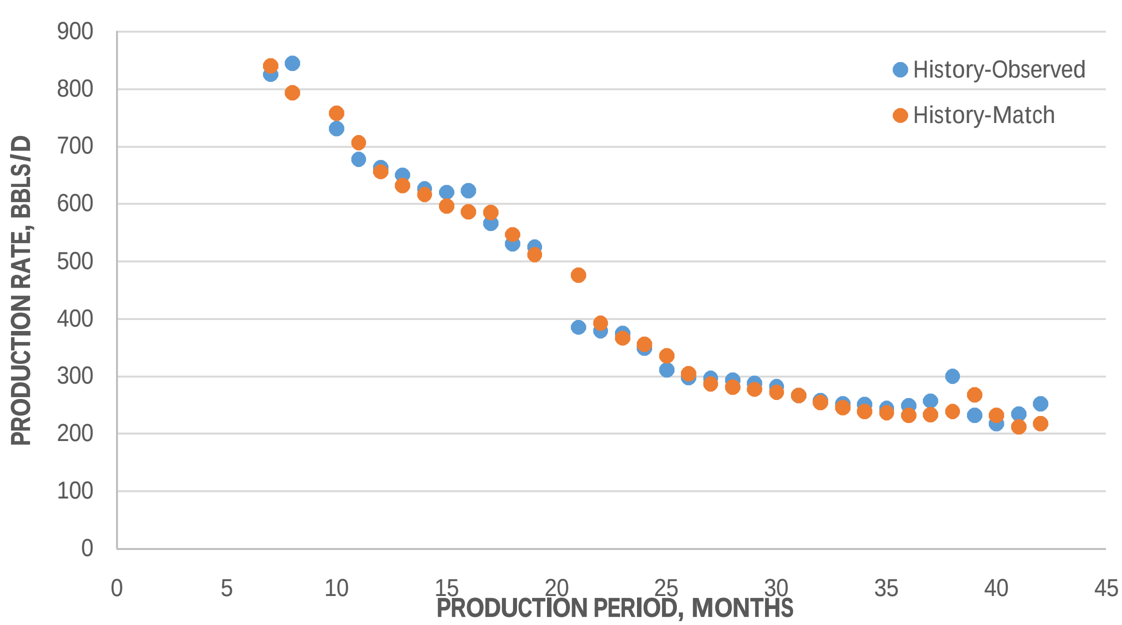

Fig. 4.26: PLOTS OF THE HISTORY-OBSERVED AND HISTORY-MATCH USING THE EXPONENTIAL TREND-CORRECTED SMOOTHING TECHNIQUE

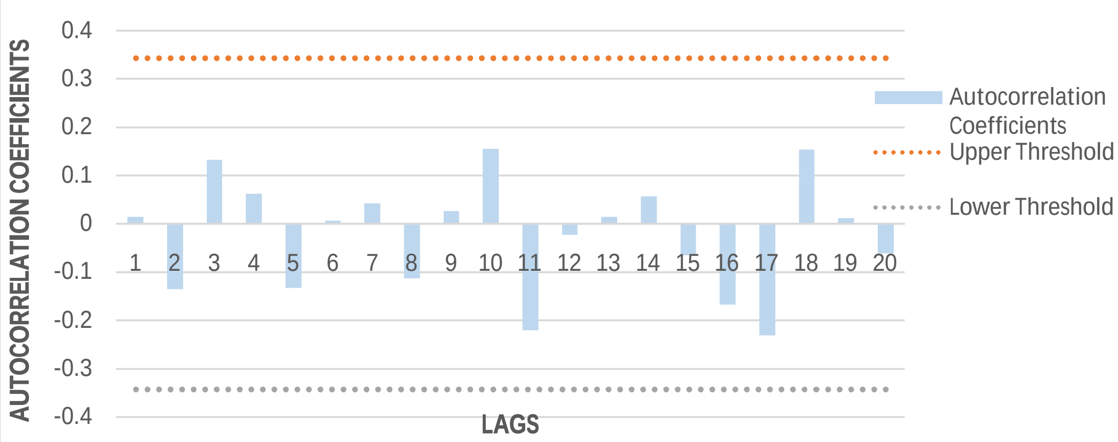

Fig. 4.27: CORRELOGRAM SHOWING THE AUTOCORRELATION RESULTS

4.2.6 Uncertainty Analysis on Base Forecasts

The uncertainty analysis was done on both the base forecasts from the linear and exponential trend-corrected exponential smoothing techniques. To run the uncertainty analysis involves simulating various possible forecasts for each of the forecast periods, so as to establish a set of possible distribution of the forecasts. A user specified confidence level can then be used to establish a range of values for each period’s possible forecasts usingthe distribution that satisfies the certainty interval.

In this work, a distribution of possible forecasts for a production period was generated using the optimized history errors. The optimized history errors were used to develop a distribution of possible errors, from where errors were randomly sampled to adjust the forecast level for each forecast period. In this procedure several possible forecasts were simulated for each future production period, and these were subsequently subjected to an uncertainty analysis whereby a prediction interval was estimated with a user specified degree of certainty of its occurrence.

To generate a distribution of possible errors, the NORM.INV function in Excel Spreadsheet was used with the optimized standard error, and an assumed mean error of zero (assuming the history-fit or history-match would overall produce an unbiased fit) for each of the exponential smoothing techniques. This was a Monte Carlo simulation, and it would randomly pop up a possible error from the distribution of errors, which could then be used to adjust the forecast level, and subsequently generate a possible forecast using equations (3.30) through (3.37) developed under the methodology. As the Spreadsheet was being refreshed, a new set of possible errors would be popped up for the forecast periods. The inverse normal (Gaussian) distribution is selected because of its ability to give a better distribution of a variable that could show a large possible value (sudden spike) off its normal trend.

Excel Macro was used to record and execute the commands used in generating the various possible forecasts.This was done to ensure that a wide portion of the possible forecasts' distribution was captured for each forecast period. The upper and lower percentiles of the various simulated possible forecasts for each period were then calculated using the PERCENTILE function in Excel. This percentile range would usually depend on the desired confidence interval or certainty level.

Figures 4.28, 4.29 and 4.30 show some possible forecast plots obtained when the procedure was used on the modified linear trend-corrected smoothing technique.

NoteFigures 4.31, 4.32 and 4.33 show the results obtained from applying the exponential trend-corrected smoothing technique.

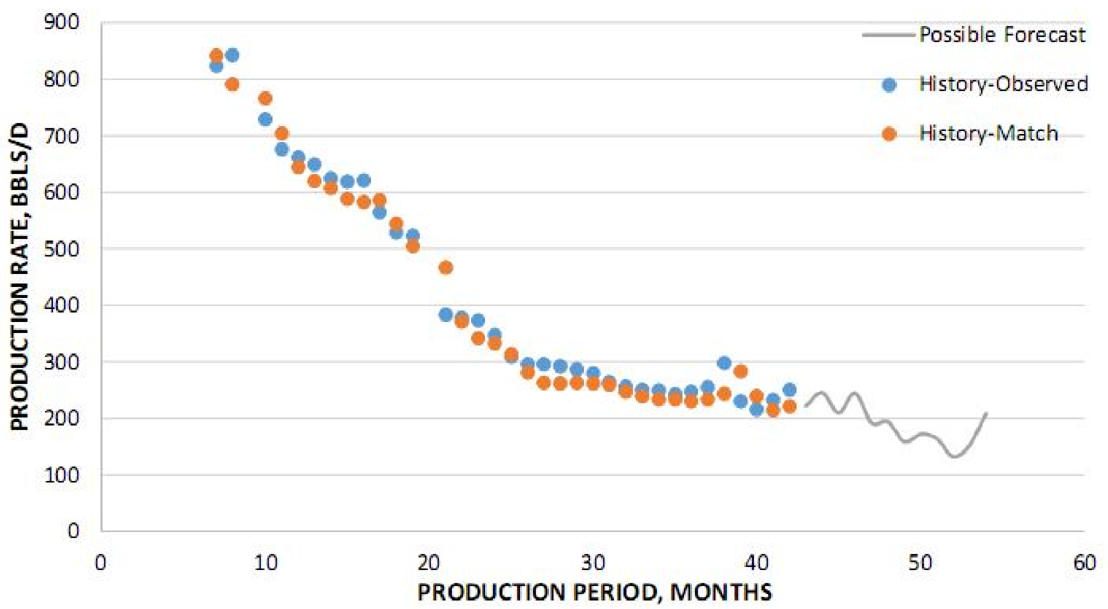

Fig. 4.28: PLOT OF A POSSIBLE FORECAST FROM THE MODIFIED LINEAR TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

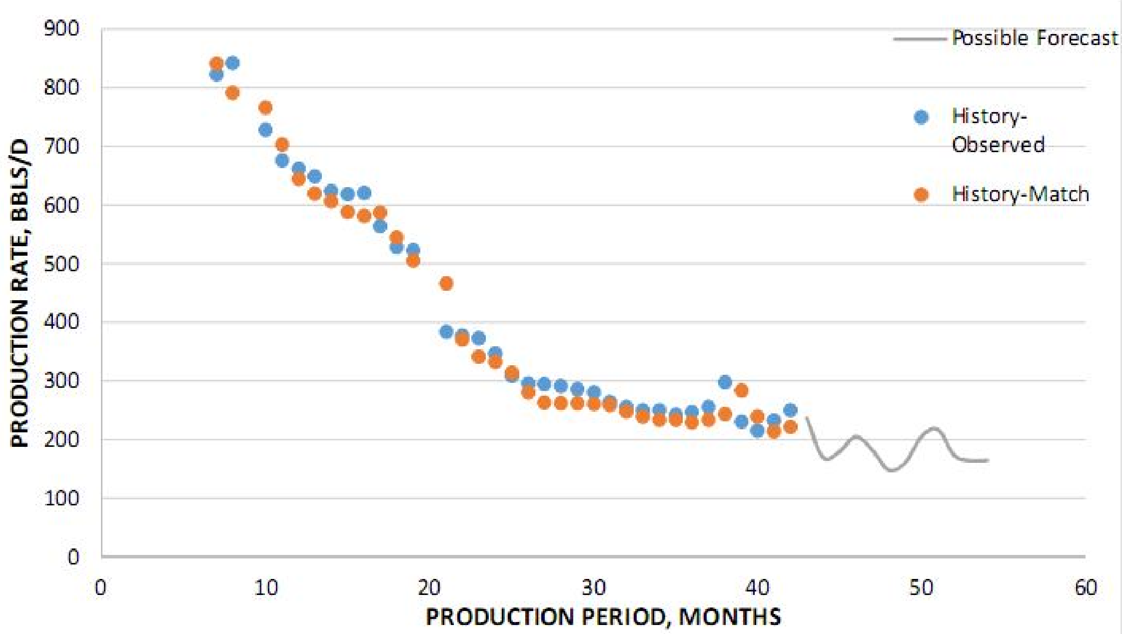

Fig. 4.29: PLOT OF A POSSIBLE FORECAST FROM THE MODIFIED LINEAR TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

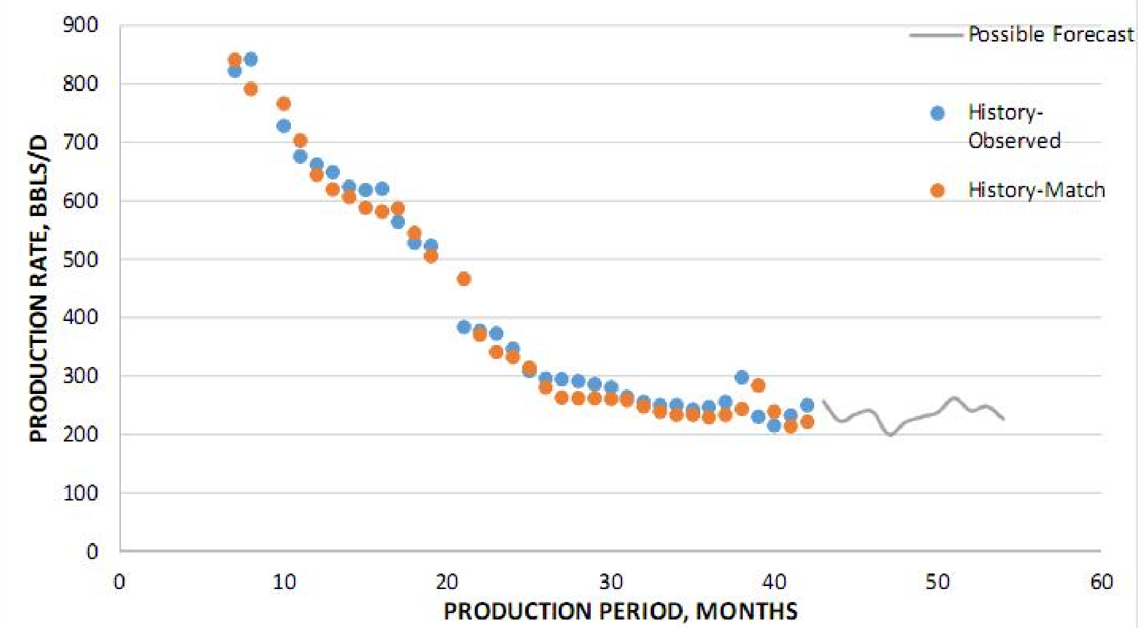

Fig. 4.30: PLOT OF A POSSIBLE FORECAST FROM THE MODIFIED LINEAR TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

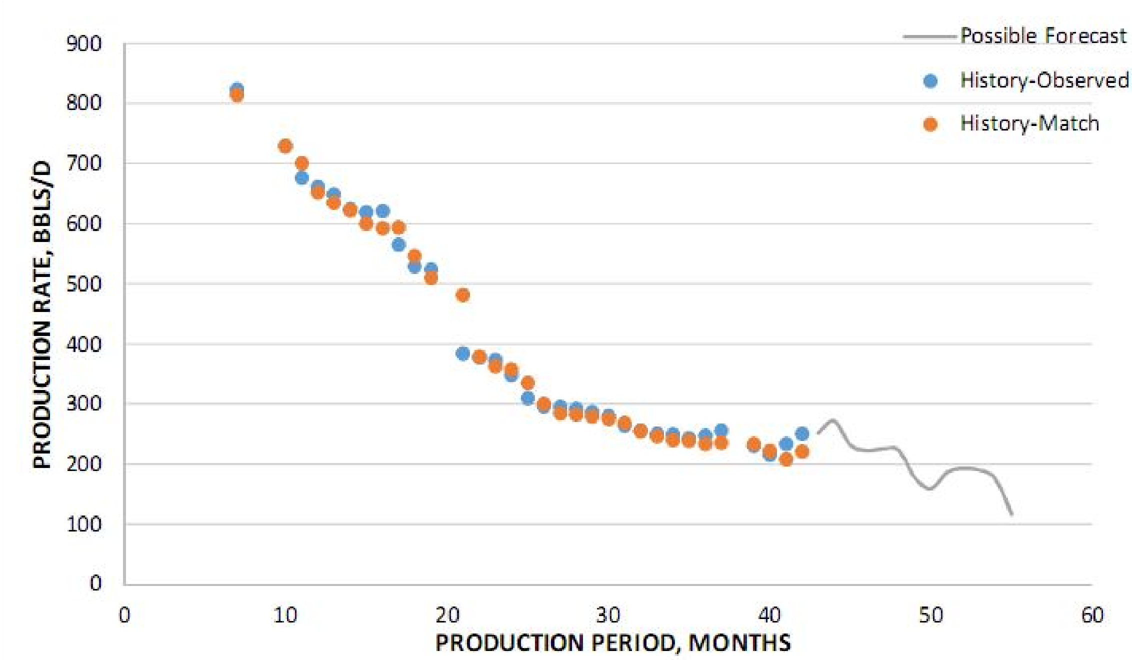

Fig. 4.31: PLOT OF A POSSIBLE FORECAST FROM THE EXPONENTIAL TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

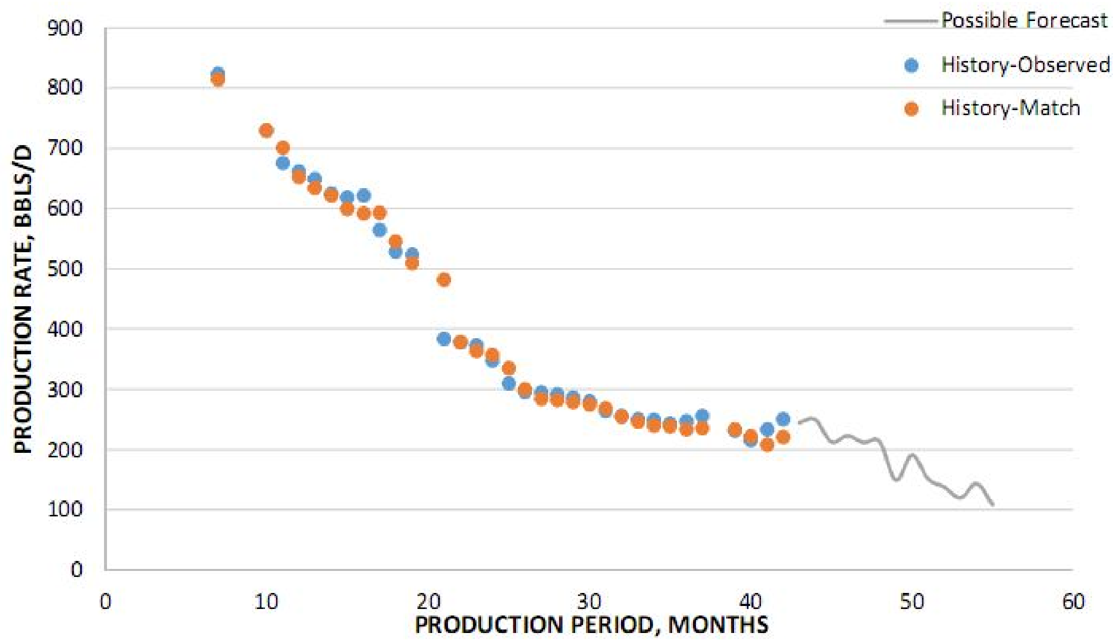

Fig. 4.32: PLOT OF A POSSIBLE FORECAST FROM THE EXPONENTIAL TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

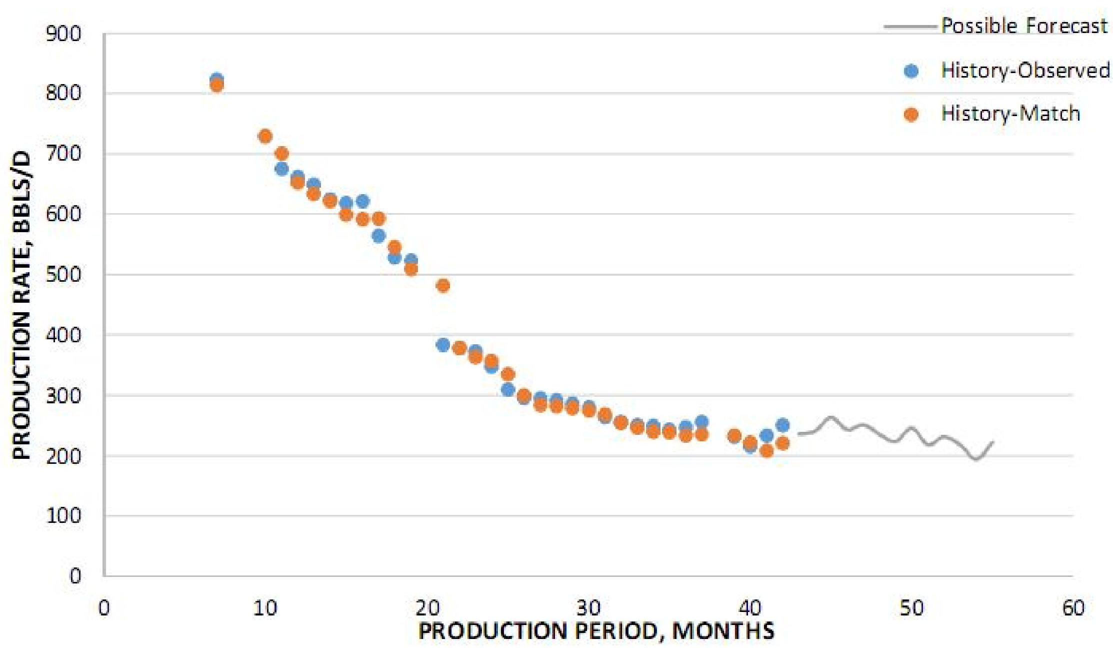

Fig. 4.33: PLOT OF A POSSIBLE FORECAST FROM THE EXPONENTIAL TREND-CORRECTED EXPONENTIAL SMOOTHING APPROACH

To carry out the uncertainty analysis used in establishing a prediction interval, over twenty-five thousand possible forecasts were simulated for the case of the linear trend-corrected exponential smoothing, while over twenty-four thousand possible forecasts were simulated for the exponential trend-corrected exponential smoothing scenario. The modified linear DES uncertainty plot is shown in Figure 4.34 with the forecast results of the observed history falling within 68% confidence level of the range of distribution of possible forecasts for each forecast period. The exponential DES uncertainty plot is represented with Figure 4.35 with the results of the forecast from the observed history falling within 90% confidence interval for the forecast period.

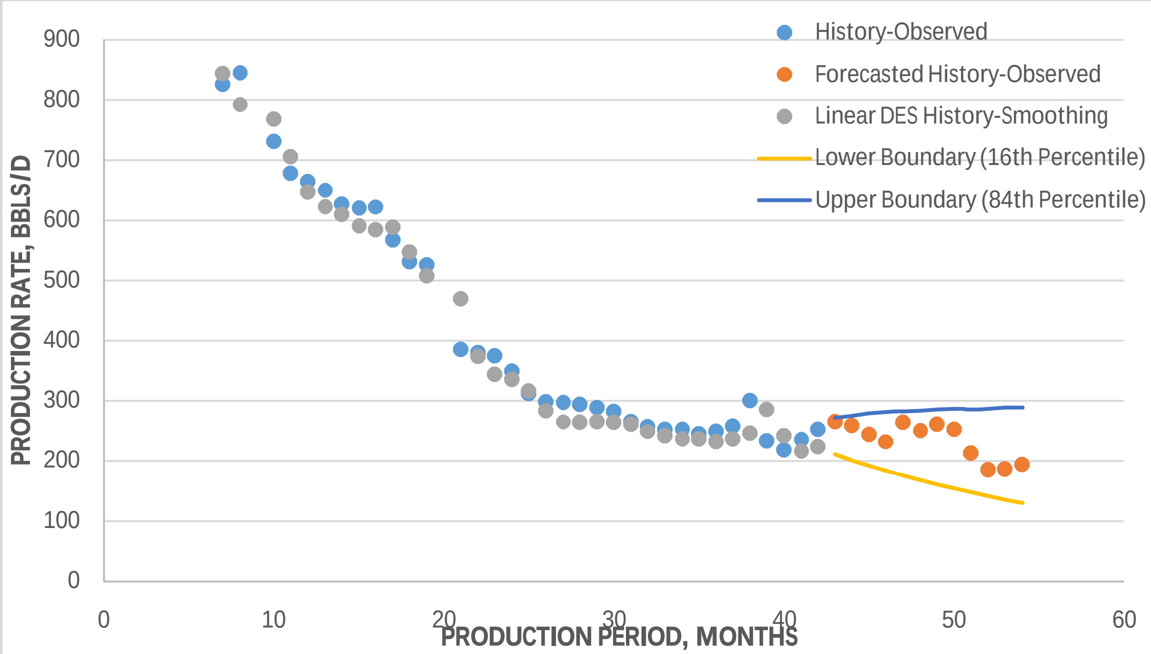

Fig. 4.34 (A): MODIFIED LINEAR DES UNCERTAINTY PLOT
(68% CONFIDENCE INTERVAL)

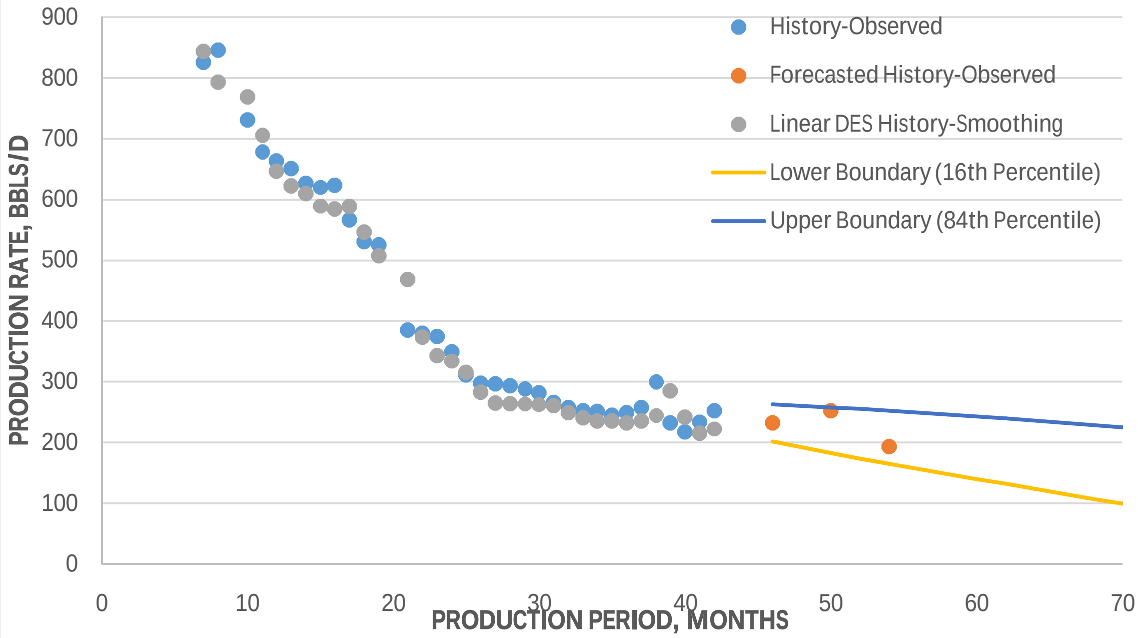

Fig. 4.34 (B): MODIFIED LINEAR DES UNCERTAINTY PLOT (68% CONFIDENCE INTERVAL)

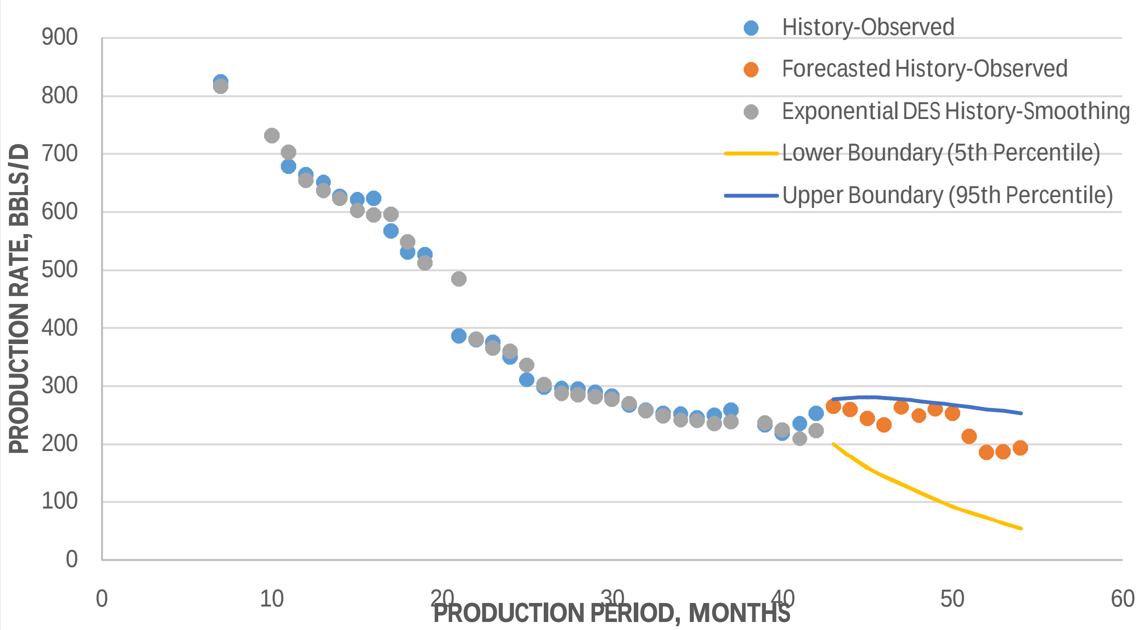

Fig. 4.35 (A): EXPONENTIAL DES UNCERTAINTY PLOT (90% CONFIDENCE INTERVAL)

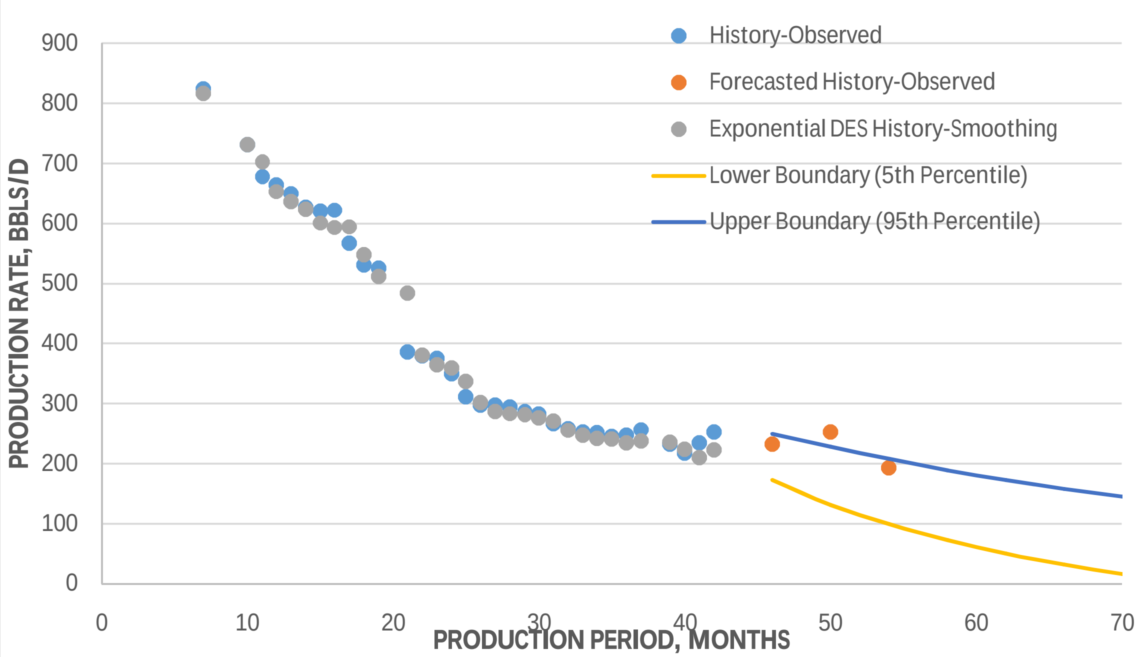

Fig. 4.35 (B): EXPONENTIAL DES UNCERTAINTY PLOT (90% CONFIDENCE INTERVAL)

It was further deduced that for an equal certainty interval, the linear DES generated a wider range of prediction interval when compared with the exponential DES. This is a demerit when such forecast is to be reported most especially for economics purpose, as it predicts a wider range of uncertainty. Note thatthe generation of a reduced optimized standard error is also another advantage that the developed exponential DES technique has over the linear DES technique.

Figure 4.36 shows the prediction interval for the linear DES technique at 90% confidence interval, which canbe compared with the resultsobtained from the exponential DES in Figure 4.35 at the same confidence interval.

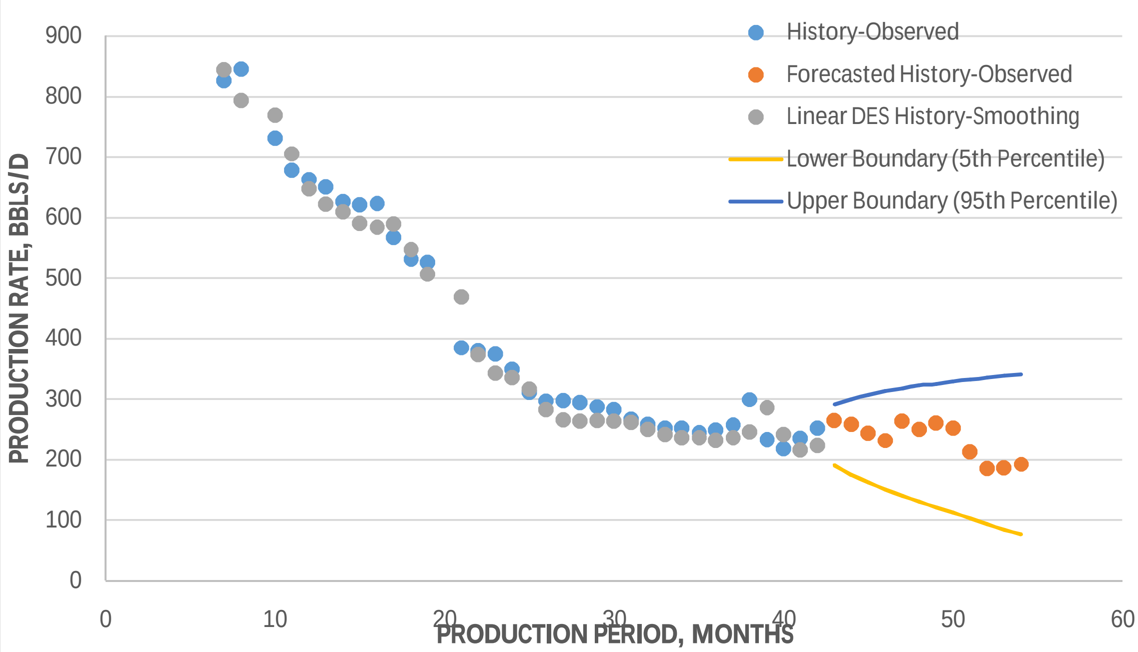

Fig. 4.36 (A): MODIFIED LINEAR DES UNCERTAINTY PLOT
(90% CONFIDENCE INTERVAL)

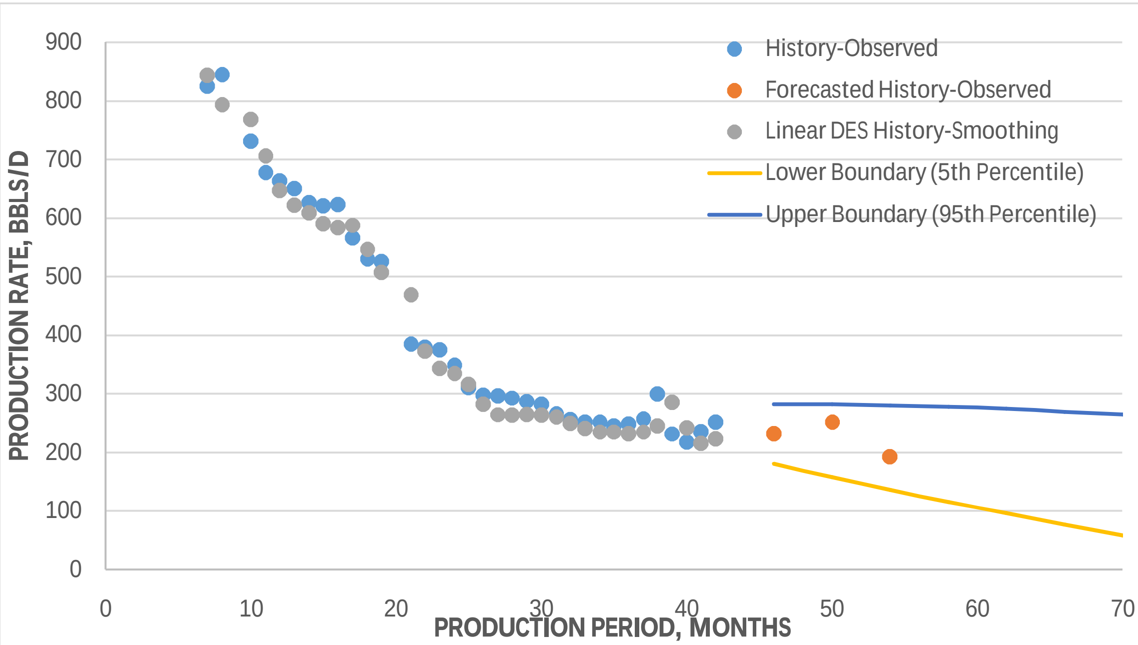

Fig. 4.36 (B): MODIFIED LINEAR DES UNCERTAINTY PLOT
(90% CONFIDENCE INTERVAL)

Also, Figures 4.37 and 4.38 show the corresponding P10, P50 and P90 production performance forecasts for both the linear and exponential DES. Using the P50 forecast as benchmark for reporting, it can be observed that only the exponential trend-corrected exponential smoothing gave the P50 forecast that met what was actually observed subsequently in history.

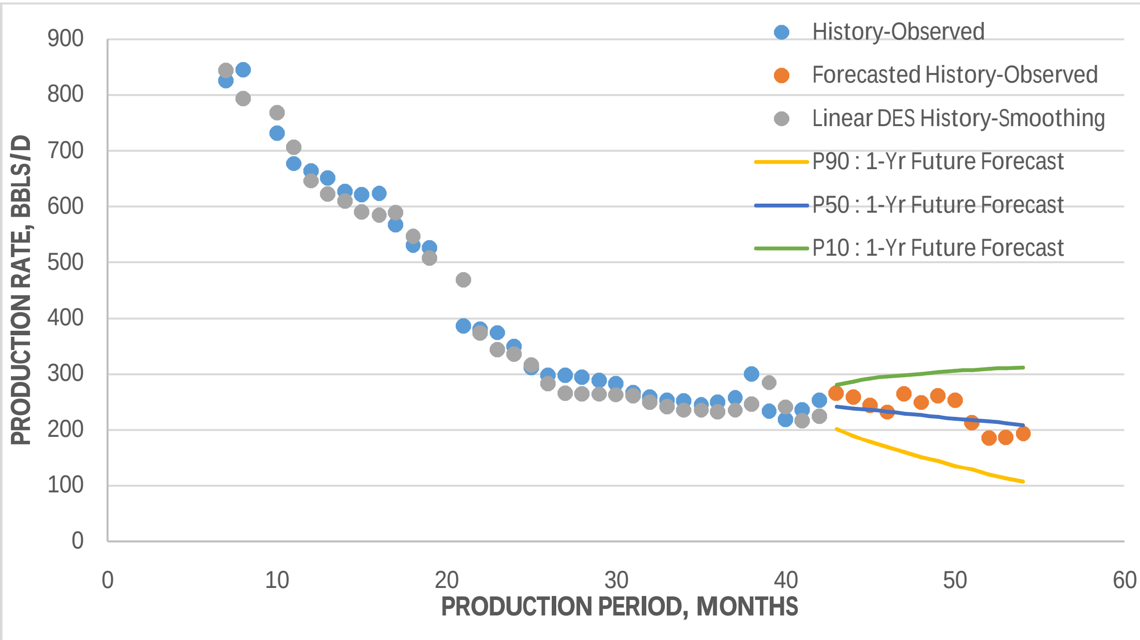

Fig. 4.37 (A): MODIFIED LINEAR DES FORECAST PLOTS OF P10, P50 AND P90

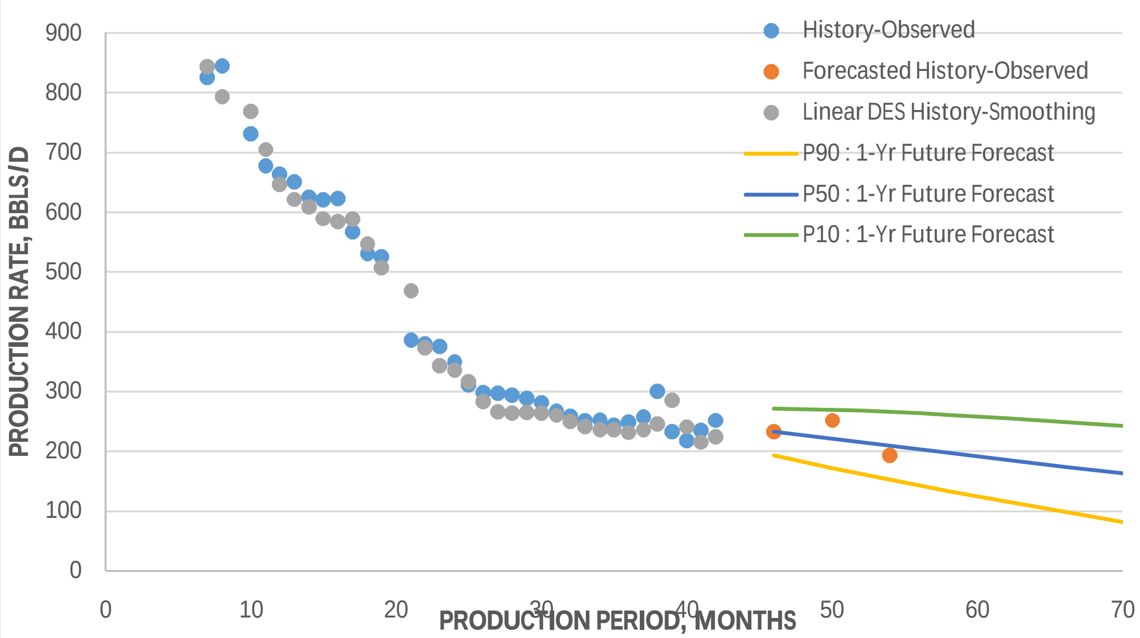

Fig. 4.37 (B): MODIFIED LINEAR DES FORECAST PLOTS OF P10, P50 AND P90

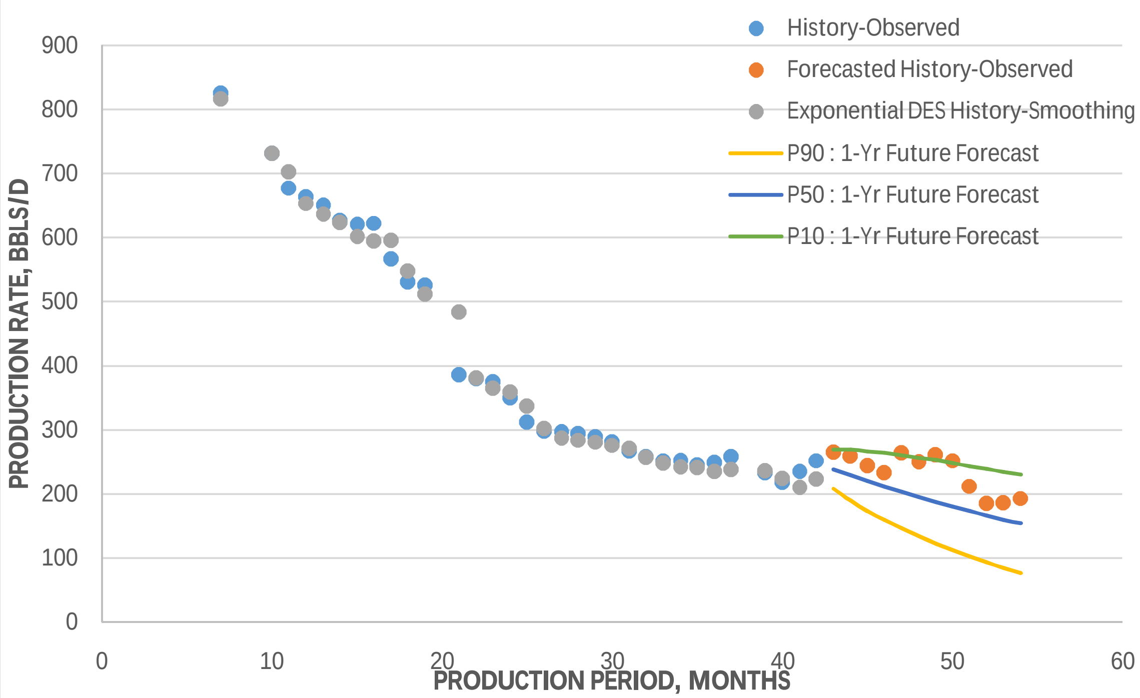

Fig. 4.38 (A): EXPONENTIAL DES FORECAST PLOTS OF P10, P50 AND P90

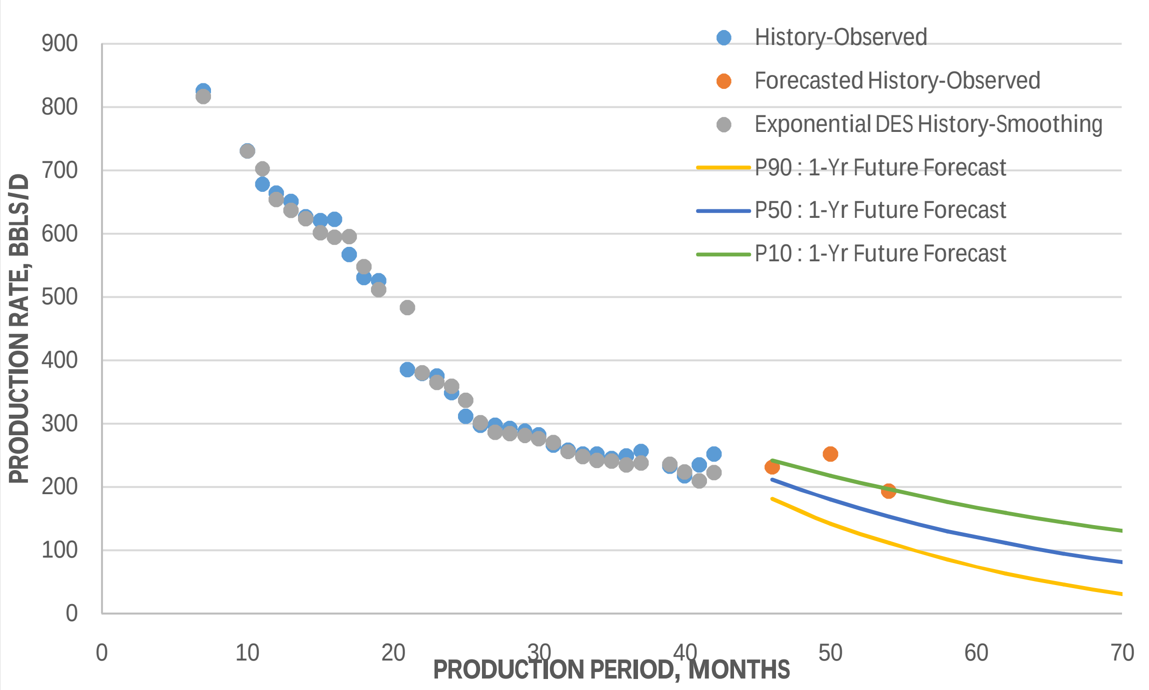

Fig. 4.38 (B): EXPONENTIAL DES FORECAST PLOTS OF P10, P50 AND P90

It should be noted that the inclining (rising) performance forecasts from the modified linear DES technique is a limitation of the technique when used for short forecast durations or with a history-fit producing high value of minimized standard error. This can be linked to its consideration of the history-observed performance rises in its future possible forecasts with limited periods or durations of spreading such effects. Meanwhile, the exponential DES displays better analysis of the rises in history-observed performance thus, showing better performance decline in its predictions as expected. Both techniques are actually prone to this, but the effect can be managed by either extending the forecast durations or significantly increasing the number of simulations especially for the modified linear DES where it is prominent. Figures 34 (B), 35 (B), 36 (B), 37 (B) and 38 (B) show improvements through increment in the forecast durations or periods, with about thirty thousand simulations of possible forecasts for each.

The results of an uncertainty analysis done on the conventional Arps' exponential decline base forecasts using the history-errors distribution approach are presented in Figure 4.39. This was achieved after simulating over sixteen-thousand possible forecasts for each prediction period. The Arps' exponential decline prediction range was only able to capture a few of the actual history-observed production performances within the forecasted one-year period, even at a 95% confidence interval. Theresults also indicated the advantage of the exponential smoothing techniques over the conventional Arps' exponential decline procedure, not only in making production forecast as shown earlier, but also in making such forecast with an evaluation of the degree of certainty of its occurrence.

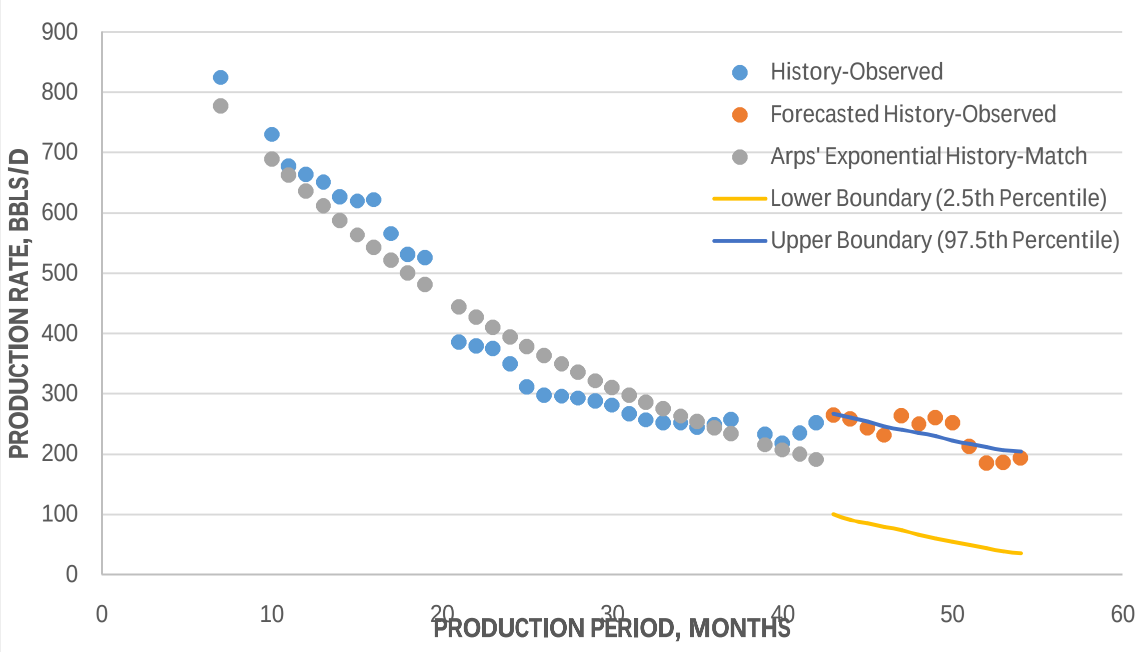

Fig. 4.39: ARPS' EXPONENTIAL DECLINE FORECAST UNCERTAINTY PLOT (95% CONFIDENCE INTERVAL)

4.2.7 Comparison of Production Forecasts from the Different Models (Techniques)

Comparing the base forecasts generated by each of the exponential smoothing techniques with what was actually observed in history within the one-year forecast period for the set of production history data used in this work, it was deduced that the linear DES generated a closer deterministic or base forecast to the actual history-observed performance than the exponential DES. This was further confirmed statistically by using the Root Mean Squared Error of Prediction (RMSEP) approach. Using equation (3.38), the linear DES gave a RMSEP of approximately 24.00 while the exponential DES produced a RMSEP of approximately 44.19, similar to the 41.71 generated by the shifted version of the Arps' exponential decline. The conventional non-shifted Arps' exponential decline forecast gave the highest RMSEP of approximately 86.18, indicatingthe largest deviation of all the prediction models. This may be a subjective conclusion, as RMSEP values obtained largely depend on the production history data set being used for the forecast. Also, since the purpose of this work is to run stochastic predictions rather than the base or only deterministic forecasts, this observation was not consideredduring the comparison of the two exponential smoothing techniques. Figure 4.40 compares the base or deterministic forecasts of the three approaches with the actual history-observed production data.

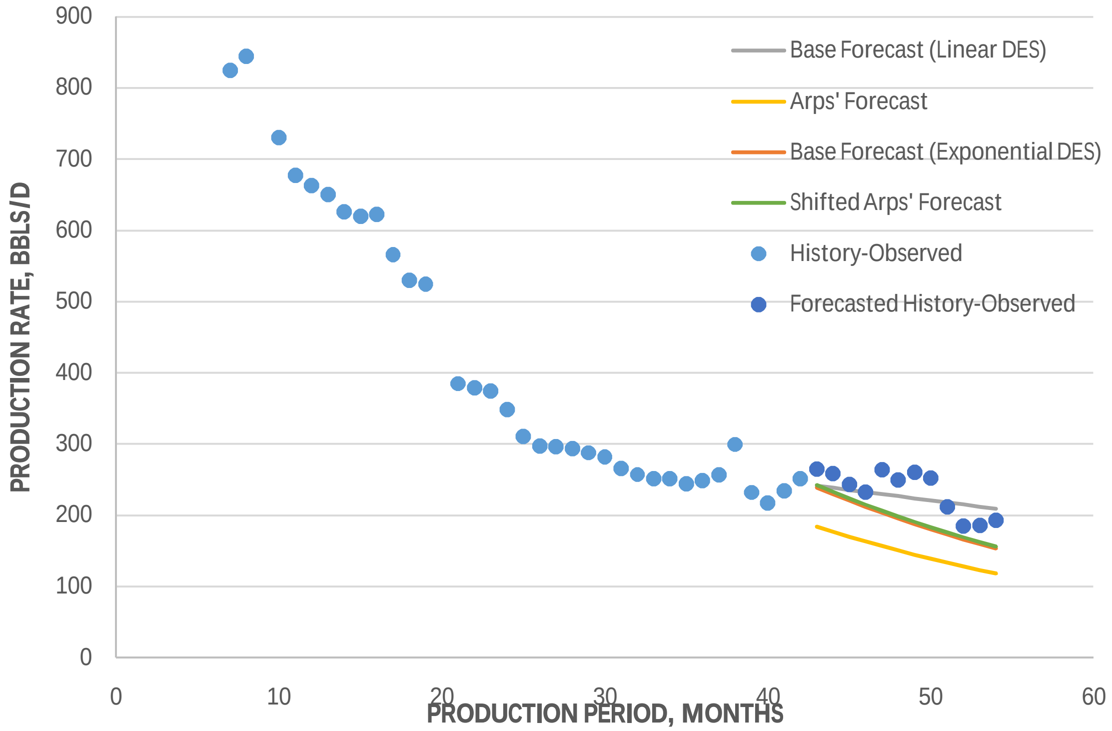

Fig. 4.40: COMPARISON OF THE DIFFERENT TECHNIQUES' BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED

In summary, this chapter has been able to show the importance of carrying out an uncertainty analysis on production performance predictions, with the application of the techniques discussed. The nature of the selected history performance goes a long way in determining the accuracy of its subsequent projections. There must be a statistically-significant non-zero trend in the history-observed performance before the DES can be applied. The history-fit errors must also be confirmed random before any base forecasts are made. The outlier-removing methods do not usually generate significant differences in their forecasts thus, either of them can be employed in as much as minimum standard error is set as the objective.

It has also indicated that the standard history-fit error plays a major role in selecting a method for generating stochastic predictions. Both techniques can be applied first to any selected history performance, then the one with a lower standard error can be employed for making the forecasts. From the results above, the developed exponential DES produces a better stochastic forecasts as a result of its often lower standard error, which replicate the declining trend of reservoir production performance over time, than the modified linear DES. More applications of these techniques are put in the appendices for further evaluation of their processes, results and limitations.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Most existing data-driven empirical models for predicting the production performance of petroleum assets are deterministic and tend to generate forecasts that may become economically unrealistic. The need for a more industrially-reliable stochastic forecast has risen over the years. Also, existing models tend to average the entire history-observed petroleum production performance trends of such an asset upon which the future forecasts are made. This has been widely noticed as another major pitfall of these existing models as production performance of petroleum assets usually gives more preference to the most recent history-observed performance trends. It was on this premise that this research work was carried out.

In this work, a new approach was developedfor forecasting the production performance of petroleum assets. It involves the exponential smoothing of the history-observed, statistically-proven production performance trends. This approach gives more weight to the most recent history-observed production performance trends towards making future performance forecasts; by establishing a perfect-fit (reduced history-match standard error) rather than the usual unbiased-fit (reduced history-match mean error) associated with most of the preceding models. Also, making subsequent probabilistic production forecasts were shown to be possible with this approach using a statistical distribution of the history-fit exponential smoothing errors.

Under this approach, an existing univariate time series linear trend-corrected exponential smoothing model was modified for non equi-spaced univariate data, which is peculiar with petroleum production performance. A new exponential trend-corrected exponential smoothing model was also developed. This was to cater for the conventionally conservative exponential decline behavior observed with production performance of petroleum assets with time; and also to adjust the possibly rising stochastic production performance forecasts occasionally associated with the modified linear DES which does not portray the expected declining behavior from petroleum reservoirs. A statistical procedure for production performance forecasting was also developed for each of the above-stated models.

Asides the dependency of the accuracy of either of the models on the nature of the history-observed production performance data being analyzed, the exponential trend-corrected exponential smoothing methodology further shrinks the range of any stochastic future predictions (improvement on the uncertainty) made on such selected history-observed production data when compared with the results obtained from the modified linear trend-corrected exponential smoothing methodology. This it does as a result of its fitting and smoothing procedure often generating less history-match optimized standard error, which is of prime importance when carrying out an uncertainty analysis on any base predictions. More emphasis should be placed on the "optimized standard error" in the final selection of which of the models to be used for the uncertainty analysis, as the outliers' checking methods studied in this work do not usually exhibit significant differences in either of their overall base or stochastic predictions.

The production performance prediction strength of these models is also highly influenced, and different from the conventional linear or exponential regression by the nature of the selected 'initial forecast level' and "initial forecast trend' values. The initial values must be evaluated in such a way that a minimal optimized standard error is achieved (to enable the establishment of a possibly best or perfect-fit), and its associated forecast level-smoothing parameter is non-zero (to ensure the exponential smoothing procedure does not return back to the conventional linear or exponential unbiased-fit).

The modified models in this work are therefore applicable to either gas or oil producing assets, as they are statistical and totally data-driven.

Additionally, it must be noted that irrespective of the validated performance of these models in making production forecasts with uncertainty analysis, they are highly dependent on the most recent history-observed performances within the entire production history data being analyzed, and as such, an average declining history-observed trend could generate an increasing performance base-forecast if an actual rise in performance is dominant towards the end of its associated history-observed performance. This is just to draw attention to the need for selecting appropriate portion of the entire history-observed production performance data before engaging any of the models in this work. Usually, this is aided by a detailed review of various production activities carried out so far in the history of such asset.

5.2 Recommendations

The procedures for the effective application of the methodologies in this work are very robust and cumbersome, despite being simplified with the use of spreadsheet. A priority of any addition work to this research will be to develop a software program that can effectively and efficiently carry out the proposed methodology with much ease and speed.

Upon the validation of the models in this work with various kinds of conventional petroleum resources, it is recommended that further studies should bethe application ofstatistical models to the unconventional resources as the developmentof unconventional resources is presently gradually springing up across the various global petroliferous regions.

Besides the development of the “exponential trend-corrected exponential smoothing” technique in this work, the development of a “hyperbolic trend-corrected exponential smoothing” technique should as well be looked into in subsequent works.

REFERENCES

1. Cutler, W.W., “Estimation of Underground Oil Reserves by Oil-Well Production Curves”, 1924.
2. Arps, J.J., “Analysis of Decline Curves”, Trans., AIME, Vol. 160, 1945, pp. 228-247.
3. Brons, F., “On the Use and Misuse of Production Decline Curves”, Producers Monthly, 1963.
4. Society of Petroleum Engineers, “Petroleum Engineering Handbook”, 1987.
5. Fetkovich, M.J., “Decline Curve Analysis Using Type Curves”, SPE Paper-4629 presented at the SPE 48th Annual Fall Meeting, Las Vegas, Nev., Sept. 30-Oct. 3, 1973.
6. Long, D.R., and Davis, M.J., “A New Approach to the Hyperbolic Curve”, Journal of Petroleum Technology, Vol. 40, 1988, pp. 909-912.
7. Robertson, S., “Generalized Hyperbolic Equation”, Unsolicited, SPE 18731, 1988.
8. Valko, P.P, “Assigning Value to Stimulation in The Barnett Shale-A simultaneous Analysis of 7000 Plus Production Histories and Well Completion Records”, paper SPE 119369, presented at the 2009 SPE Hydraulic Fracturing Technology Conference held in the Woodlands, Texas, USA, 19-21 Jan., 2009.
9. Valko, P.P. and Lee, W.J., “A Better Way to Forecast Production from Unconventional Gas Wells”, paper SPE 134231, presented at the SPE Annual Technical Conference and Exhibition held in Florence, Italy, 19-22 September 2010.

10. Duong, A.N., “An Unconventional Rate Decline Approach for Tight and Fracture-Dominated Gas Wells”, paper SPE 137748, presented at the Canadian Unconventional Resources & International Petroleum Conference held in Calgary, Alberta, Canada, 19–21 October 2010.

11. Duong, A.N., “Rate-Decline Analysis for Fracture-Dominated Shale Reservoir”, SPE 137748, SPE Reservoir Evaluation and Engineering, 14(3):377-387.

12. Khaled, A.F., “Predicting Production Performance Using a Simplified Model”, Production Technology-World Oil, April, 2006.

13. Ladipo, A.O., Makinde, F.A., Orodu, O.D., and Anawe, P.A.L., “Cumulative Production Forecast of an Oil Well Using Simplified ‘Hyperbolic-Exponential’ Decline Models”, Global Journal of Researches in Engineering (J), Vol. 12, Issue 2, Version 1.0, May, 2012.

14. Soleng, H.H., “Oil Reservoir Production Forecasting with Uncertainty Estimation Using Genetic Algorithms”, Norwegian Computing Centre, Publication No. 2928, 2011.

15. Crafton, J.W., “Oil and Gas Well Evaluation Using the Reciprocal Productivity Index Method”, SPE Paper No. 37409, presented at SPE Production Operations Symposium, Oklahoma City, OK., Mar. 9-11, 1997.

16. Obah, B., Aniefiok, L., and Chima, M.E., “Simplified Models for Forecasting Oil Production: Niger Delta Oil Rim Reservoirs Case”, Petroleum Technology Development Journal (ISSN 1595-9104), Vol. 2, No. 2, July, 2012.

17. Holdaway, K.R., “Let Oil and Gas Talk to You: Predicting Production Performance”, Paper 342-2012, SAS Global Forum – Statistics and Data Analysis, SAS Institute Inc., Cary, NC, USA, 2012.

18. Kikani, J., “Reservoir Surveillance”, ISBN 978-1-61399-304-0, Society of Petroleum Engineers, USA, 2013.

19. Chatfield, C., and Yar, M., “Prediction Intervals for the Holt-Winters Forecasting Procedure”, International Journal of Forecasting, Vol. 6, pp 127-137, North Holland, 1990.

20. Foreman, J.W., “Data Smart: Using Data Science to Transform Information into Insight”, ISBN 978-1-118-66146-8, John Wiley & Sons, Inc., Indiana, 2014.

21. Floris, F.J.T., Bush, M.D., Cuypers, M., Roggero, F., and Syversveen, A.R., “Comparison of Production Forecast Uncertainty Quantification Methods: An Integrated Study”, *work in progress*, 1999.

22. Jochen, V.A., and Spivey, J.P., “Probabilistic Reserves Estimation Using Decline Curve Analysis with the Boostrap Method”, SPE 36633, 1996.

23. Jochen, V.A., Spivey, J.P., and S.A. Holditch & Associates, Inc., “Probabilistic Reserves Estimation Using Decline Curve Analysis with the Boostrap Method”, SPE 36633-MS, 1996.

24. Mattar, L., and Anderson, D.M., “A Systematic and Comprehensive Methodology for Advanced Analysis of Production Data”, SPE 84472, 2003.

25. Kewen, L.H., and Roland, N., “A Decline Curve Analysis Model Based on Fluid Flow Mechanics”, SPE 83470, 2003.

26. Chen, Sh., “A Generalized Hyperbolic Decline Equation with Rate-Time Dependent Function”, SPE 80909, Presented at the SPE Production and Operations Symposium held in Oklahoma City, Oklahoma, USA, 22-25 March, 2003.

27. Chen, Sh., “A Generalized Hyperbolic Decline Equation with Rate-Time and Rate-Cumulative Relationships”, SPE 81427, Presented at the SPE 13th Middle East Show & Conference held in Bahrain, 5-8 April, 2003.

28. Foster, G.A., and Wong, D.W., “Reducing Uncertainty in Reservoir Management Using Semi-Analytical Modeling and Long Term Daily Data Acquisition”, SPE 60309, Presented at the 2000 SPE Rocky Mountain Regional/Low Permeability Reservoirs Symposium and Exhibition held in Denver, Colorado, 12-15 March, 2000.

29. Shirman, E.I., “Universal Approach to the Decline Curve Analysis”, Journal of Canadian Petroleum Technology, Vol. 38, No. 13, 1999, pp. 1-4.

30. Huffman, C.H., and Thompson, R.S., “Probability Ranges for Reserves Estimates from Decline Curve Analysis”, SPE 28333, 1994.

31. Towler, B.F., and Bansal, S., “Hyperbolic Decline-Curve Analysis”, Journal of Petroleum Science and Technology, 1993, pp. 257-268.

32. Blasingame, T.A., McCray, T.L., and Lee, W.J., “Decline Curve Analysis for Variable Pressure Drop/Variable Flowrate Systems”, 1991.

33. Purvis, R.A., “Further Analysis of Production-Performance Graphs”, Journal of Canadian Petroleum Technology, April, 1987, pp. 74-79.

34. Spivey, J.P., “A New Algorithm for Hyperbolic Decline Curve Fitting”, SPE Paper 15293, Presented at Petroleum Industry Applications of Microcomputers, Silver Greek, June 18-20, 1986.

35. Nind, T.E.W., “Principles of Oil Well Production”, 2nd Ed., McGraw-Hill Inc., New York, 1981.

36. Rowland, D.A., and Chung, L., “Computer Model Solution Proposed”, Oil and Gas Journal, Jan. 21, 1985, pp. 77-80.

37. Thompson, R.S., Wright, J.D., and Digert, S.A., “The Error in Estimating Reserves Using Decline Curves”, Paper SPE 16925, Presented at the SPE Hydrocarbon Economics and Evaluation Symposium, Texas, March 2-3, 1987.

38. McNulty, R.R., and Knapp, R.M., “Statistical Decline Curve Analysis”, Paper SPE 10279, Presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Oct. 5-7, 1981.

39. Koederitz, C.F., Harvey, A.H., and Honarpour, M., “Introduction to Petroleum Reservoir Analysis”, Gulf Publishing Company, Houston, Texas, 1989.

40. Gentry, R.W., “Decline-Curve Analysis”, SPE Paper- 3356, 1972.

APPENDICES

APPENDIX A: HISTORY-OBSERVED AND DETERMINISTIC FORECASTS PRODUCTION PERFORMANCE DATA AND RESULTS

TABLE A.1: TOTAL OBSERVED DECLINING PRODUCTION HISTORY

Date	Time (Months)	Rate (bbls/day)
1/15/1976	7	825.16
2/14/1976	8	844.83
3/15/1976	9	730.32
4/15/1976	10	730.67
5/15/1976	11	677.42
6/15/1976	12	663.33
7/15/1976	13	650.32
8/15/1976	14	626.13
9/15/1976	15	620.33
10/15/1976	16	622.58
11/15/1976	17	566.33
12/15/1976	18	530.32
1/15/1977	19	525.16
2/14/1977	20	401.07
3/15/1977	21	385.16
4/15/1977	22	379.33
5/15/1977	23	374.52
6/15/1977	24	349
7/15/1977	25	310.97
8/15/1977	26	297.42
9/15/1977	27	296.67
10/15/1977	28	293.55
11/15/1977	29	288
12/15/1977	30	281.94
1/15/1978	31	266.13
2/14/1978	32	257.5
3/15/1978	33	251.94
4/15/1978	34	251.33
5/15/1978	35	244.52
6/15/1978	36	248.67
7/15/1978	37	257.1
8/15/1978	38	299.68
9/15/1978	39	232.33
10/15/1978	40	217.42
11/15/1978	41	234.67
12/15/1978	42	251.94
1/15/1979	43	264.52
2/14/1979	44	258.21
3/15/1979	45	243.23

4/15/1979	46	232
5/15/1979	47	263.55
6/15/1979	48	249.33
7/15/1979	49	260.32
8/15/1979	50	251.94
9/15/1979	51	212
10/15/1979	52	184.84
11/15/1979	53	185.67
12/15/1979	54	192.9

TABLE A.2: SELECTED OBSERVED DECLINING PRODUCTION HISTORY

Date	Time (Months)	Rate (bbls/day)
1/15/1976	7	825.16
2/14/1976	8	844.83
3/15/1976	9	730.32
4/15/1976	10	730.67
5/15/1976	11	677.42
6/15/1976	12	663.33
7/15/1976	13	650.32
8/15/1976	14	626.13
9/15/1976	15	620.33
10/15/1976	16	622.58
11/15/1976	17	566.33
12/15/1976	18	530.32
1/15/1977	19	525.16
2/14/1977	20	401.07
3/15/1977	21	385.16
4/15/1977	22	379.33
5/15/1977	23	374.52
6/15/1977	24	349
7/15/1977	25	310.97
8/15/1977	26	297.42
9/15/1977	27	296.67
10/15/1977	28	293.55
11/15/1977	29	288
12/15/1977	30	281.94
1/15/1978	31	266.13
2/14/1978	32	257.5
3/15/1978	33	251.94
4/15/1978	34	251.33
5/15/1978	35	244.52
6/15/1978	36	248.67
7/15/1978	37	257.1
8/15/1978	38	299.68
9/15/1978	39	232.33
10/15/1978	40	217.42
11/15/1978	41	234.67
12/15/1978	42	251.94

TABLE A.3: DETERMINISTIC OR BASE PRODUCTION PERFORMANCE FORECASTS

Time (Months)	Rate (bbls/day)				
	History-Observed	Arps'	Linear DES	Exponential DES	Shifted Arps'
43	264.52	184.17	241.05	238.61	242.06
44	258.21	176.95	238.13	229.25	232.57
45	243.23	170.01	235.22	220.26	223.45
46	232	163.34	232.31	211.63	214.69
47	263.55	156.94	229.39	203.33	206.27
48	249.33	150.79	226.48	195.36	198.18
49	260.32	144.87	223.57	187.70	190.41
50	251.94	139.19	220.65	180.34	182.95
51	212	133.73	217.74	173.27	175.77
52	184.84	128.49	214.83	166.47	168.88
53	185.67	123.45	211.92	159.94	162.26
54	192.9	118.61	209.00	153.67	155.90

APPENDIX B: VALIDATION OF DEVELOPED MODELS AND COMPARISON USING ANOTHER SET OF HISTORY-OBSERVED PRODUCTION PERFORMANCE DATA

TABLE B.1: TOTAL AVAILABLE HISTORY-OBSERVED PRODUCTION PERFORMANCE DATA SET 2

Periods	Rate	Periods	Rate	Periods	Rate

(Months)	(bbl/d)	(Months)	(bbl/d)	(Months)	(bbl/d)
26	2795.06	157	363.16	284	994.19
27	3148.71	158	325.87	285	811
28	2651.29	159	321.32	286	545.43
29	2559.43	160	339.29	287	439.74
30	1902.61	161	245.47	288	426.63
31	1866.13	162	352.84	289	100.48
32	1762.1	163	356	292	134.55
33	1512.23	164	247.42	294	270.39
34	1327.03	165	301.65	295	441.07
35	1310.45	166	324.6	296	371.94
36	1290.5	167	293.26	297	544.71
37	1242.87	168	328.97	298	770.67
38	1331.29	169	147.55	299	263.65
39	951.14	170	18.19	300	203.7
40	1296.13	171	28	301	234.45
41	1242.7	172	310.35	302	287.54
42	691.61	173	310.5	303	388.39
43	1300.87	174	309.77	304	435.15
44	527.97	175	314.57	305	368.42
45	1266.65	176	379.45	306	464.3
46	1308.3	177	440.74	307	453.01
47	1268.87	178	492.67	321	585.04
48	1265.63	179	535.29	322	1650.99
49	1259.87	180	440.17	323	1910.42
50	1207.1	181	330.32	324	1680.87
51	1204.55	182	368	325	1442.43
52	1666.19	183	334.71	326	1211.53
53	2048.8	184	433.48	327	1806.5
54	2216.16	185	383.27	328	888.23
55	1966.2	186	348.52	329	134.95
56	807.39	187	311.4	330	552.63
58	949.17	190	98.8	331	605.29
59	1605.03	191	526.61	332	2881.69
60	1854	192	550.8	333	3523.48
61	1895.81	193	611.45	334	3972.65
62	1369.29	194	569.55	335	3214.85
63	1368.68	195	395.79	336	3480.21
64	1401.81	196	486.35	337	2085.2
65	1278.97	197	403.43	338	4531.17
66	1270.48	198	405.58	339	4744.71
67	1314.37	199	414.3	340	3803.89
68	1236.81	200	414.1	341	2612.43
69	936.06	201	400.94	342	1783.87
70	1247.23	202	451.3	343	1823.95
71	688.42	203	448.45	344	1849.75
72	786.63	204	374.03	345	2657.93
73	758.87	205	437.68	346	1233.98
74	1013.55	206	531.97	348	2479.97

76	971.45	207	514.43	349	575.54
77	1381.23	208	467.35	350	1837.52
78	1400.81	209	476.1	351	2811.61
79	1136.73	210	387.9	352	1910.23
80	1167.94	211	408	353	2558.82
81	1198.52	212	488.19	354	1939.04
82	1295.1	213	279.16	355	2086.51
83	1342.71	214	259.53	356	1016.02
84	1433.4	215	259.13	357	687.69
85	1407.52	216	227.83	358	2162.87
86	1382.71	217	263.87	359	1878.34
87	1349.86	218	183.87	360	2028.34
88	1459.74	219	171	361	2159.89
89	1424.6	220	134.1	362	2084.06
90	1374.58	221	120.3	363	1043.36
91	1402.4	222	147.13	364	1661.26
92	1472	223	233.1	365	1438.49
93	1271.94	224	300.26	366	2079.15
94	1217	225	298.87	367	1487.46
95	1372.65	226	275.17	368	1470.76
96	1326.73	227	263.29	369	1095.36
97	1356.97	228	328.97	370	922.64
98	1397.74	229	372.87	371	860.83
99	1452.86	230	397.13	372	912.23
100	1428.9	231	406.25	373	614.73
101	1158.2	232	388.9	374	856.14
102	1109.48	233	295.63	375	770.43
103	1195.6	234	274.55	376	648.94
104	1126.55	235	262.03	377	630.42
105	1093.16	236	259.97	378	651.69
106	241.63	237	312.16	379	719.37
107	927.74	238	200.37	380	697.21
108	931.27	239	309.45	381	700.78
109	1060.65	240	510.7	382	688.09
110	1125.16	241	458.39	383	710.21
111	1054.36	242	902.45	384	685.15
112	354.52	243	1038.38	385	684.39
113	927.9	243	958	386	635.32
114	82.26	245	1234.83	387	637.24
115	448.17	246	1117.13	388	665.62
118	483.3	247	1046.53	389	707.4
119	513.32	248	1010.32	390	680.28
120	928.27	249	1117.26	391	618.57
121	1007.23	250	1141.67	392	625.34
122	948.9	251	973.58	393	579.82
123	1149.43	252	1034.07	394	611.43
124	42.97	253	898.87	395	540.58
126	1024.03	254	1067.19	396	536.54
127	1360.33	255	1006.86	397	491.45

128	1123.13	256	1078.16	398	454.11
129	813.77	257	1060.33	399	527.67
130	933.9	258	1100.77	400	482.05
131	1029.42	259	1109.13	401	604.14
132	882.27	260	1018.23	402	677.69
133	1070.55	261	973.26	403	649.43
134	973.81	262	931.97	404	602.28
135	1202.82	263	880.81	405	535.43
136	1339.65	264	1061.57	406	319.94
137	1148.53	265	1124.35	407	521.66
138	1345.1	266	1104.65	408	589.26
139	1092.3	267	642.79	409	628.33
140	1423.9	268	764.03	410	560.01
141	1191.39	269	968.63	411	421.21
142	966.87	270	1032.65	412	254.66
143	891.42	271	900.73	413	591.16
144	370.63	272	1209.16	414	579.71
145	445.26	273	1068.68	415	543.28
146	452.06	274	1077.97	416	576.97
147	476.55	275	1272.1	417	473.99
148	368.19	276	1213.47	418	473.7
150	386.48	277	997.48	419	262.14
151	435.17	278	1010.68	420	546.75
152	484.29	279	1100.18	421	425.97
153	510.23	280	1088.06	422	332.02
154	465.87	281	959.7	423	531.02
155	457.06	282	1005.97	424	465.17
156	370.67	283	1063.87	425	338.91

TABLE B.2: SELECTED HISTORY-OBSERVED PRODUCTION PERFORMANCE DATA SET 2 (1ST ANALYSIS)

Periods (Months)	Rate (bbl/d)	Periods (Months)	Rate (bbl/d)	Periods (Months)	Rate (bbl/d)
26	2795.06	50	1207.1	76	971.45
27	3148.71	51	1204.55	77	1381.23
28	2651.29	52	1666.19	78	1400.81
29	2559.43	53	2048.8	79	1136.73
30	1902.61	54	2216.16	80	1167.94
31	1866.13	55	1966.2	81	1198.52
32	1762.1	56	807.39	82	1295.1
33	1512.23	58	949.17	83	1342.71
34	1327.03	59	1605.03	84	1433.4
35	1310.45	60	1854	85	1407.52
36	1290.5	61	1895.81	86	1382.71
37	1242.87	62	1369.29	87	1349.86
38	1331.29	63	1368.68	88	1459.74
39	951.14	64	1401.81	89	1424.6
40	1296.13	65	1278.97	90	1374.58

41	1242.7	66	1270.48	91	1402.4
42	691.61	67	1314.37	92	1472
43	1300.87	68	1236.81	93	1271.94
44	527.97	69	936.06	94	1217
45	1266.65	70	1247.23	95	1372.65
46	1308.3	71	688.42	96	1326.73
47	1268.87	72	786.63	97	1356.97
48	1265.63	73	758.87	98	1397.74
49	1259.87	74	1013.55	99	1452.86

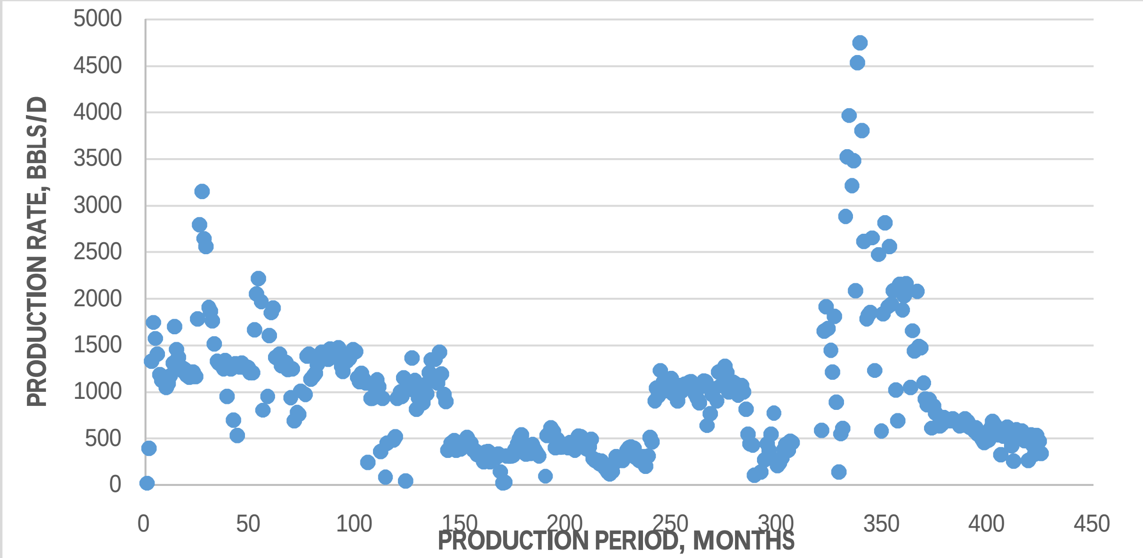

Fig. B.1: TOTAL HISTORY-OBSERVED PRODUCTION PERFORMANCE

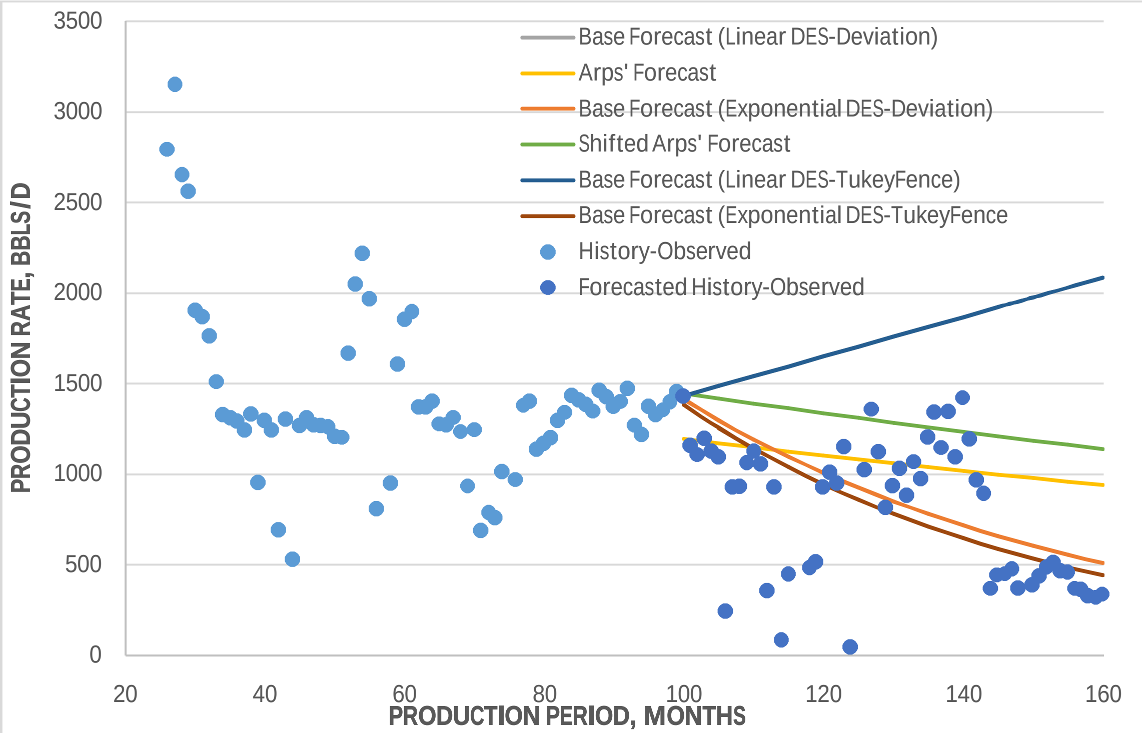

Fig. B.2: COMPARISON OF THE DIFFERENT TECHNIQUES' NEXT 5-YEAR BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED (1ST ANALYSIS)

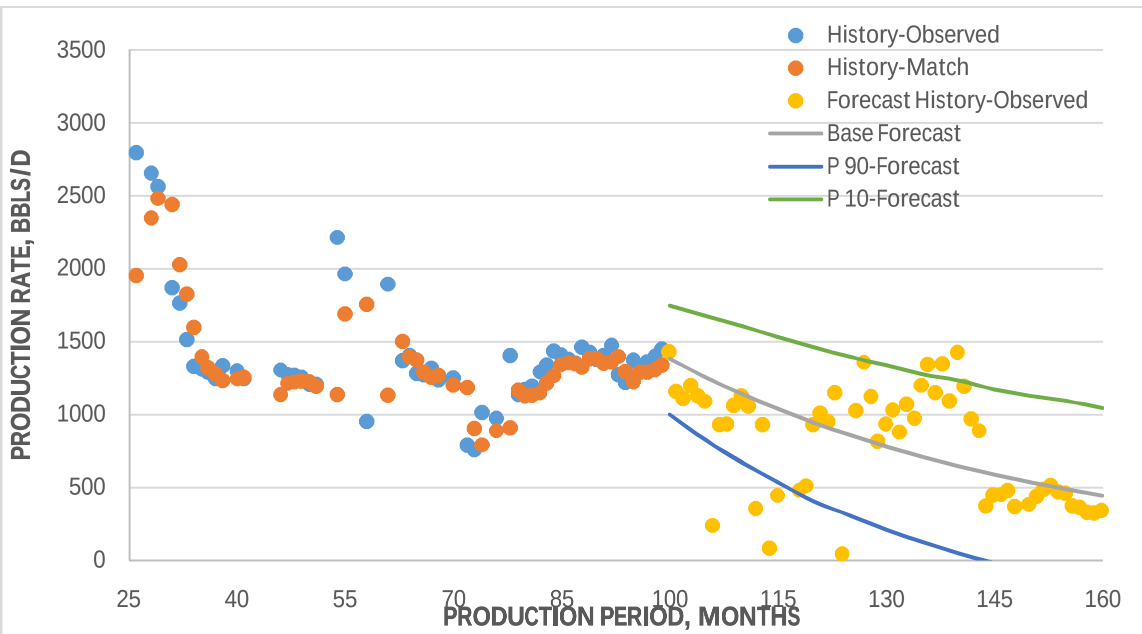

Fig. B.3: UNCERTAINTY ANALYSIS - 80% CONFIDENCE (EXPONENTIAL DES - TUKEY FENCE)

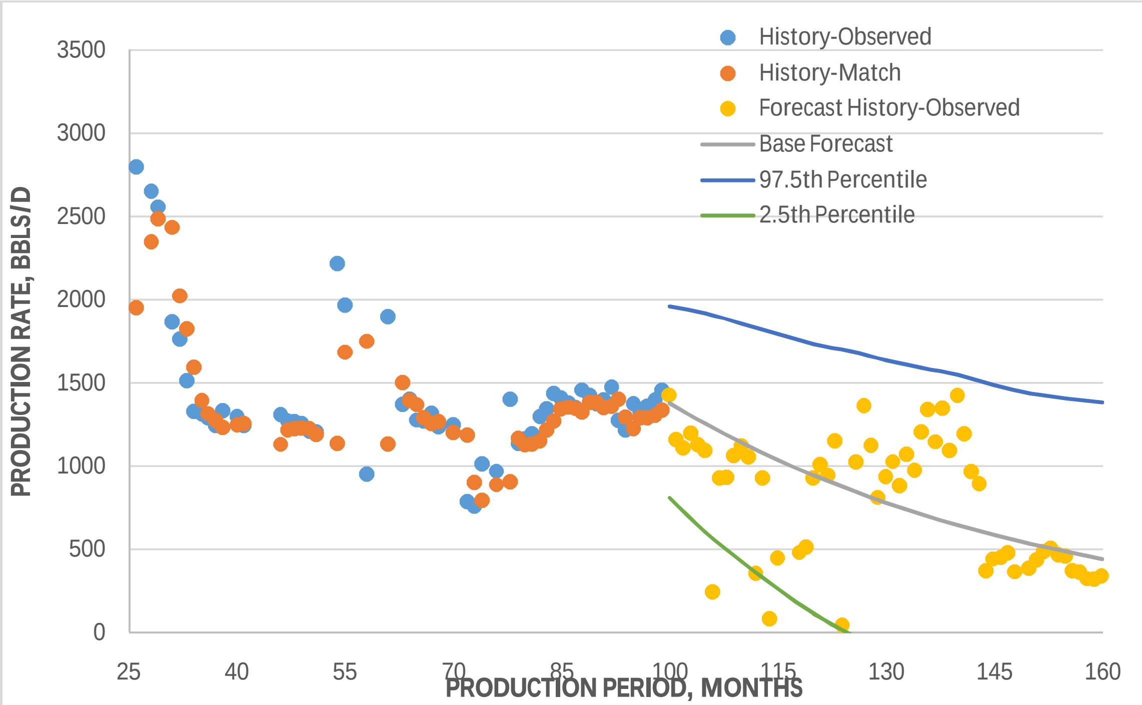

Fig. B.4: UNCERTAINTY ANALYSIS - 95% CONFIDENCE
(EXPONENTIAL DES - TUKEY FENCE)

Figure B.2 indicates a situation when the modified linear DES fails due to the nature of the selected history-observed performance, whereby it predicts an incline or rise in future productions rather the expected decline. When this happens, the exponential DES technique becomes unrivalled for future performance predictions.

TABLE B.3: SELECTED HISTORY-OBSERVED PRODUCTION PERFORMANCE DATA SET 2 (2ND ANALYSIS)

Periods (Months)	Rate (bbl/d)	Periods (Months)	Rate (bbl/d)	Periods (Months)	Rate (bbl/d)
26	2795.06	70	1247.23	114	82.26
27	3148.71	71	688.42	115	448.17
28	2651.29	72	786.63	118	483.3
29	2559.43	73	758.87	119	513.32
30	1902.61	74	1013.55	120	928.27
31	1866.13	76	971.45	121	1007.23
32	1762.1	77	1381.23	122	948.9
33	1512.23	78	1400.81	123	1149.43
34	1327.03	79	1136.73	124	42.97
35	1310.45	80	1167.94	126	1024.03
36	1290.5	81	1198.52	127	1360.33
37	1242.87	82	1295.1	128	1123.13
38	1331.29	83	1342.71	129	813.77
39	951.14	84	1433.4	130	933.9
40	1296.13	85	1407.52	131	1029.42
41	1242.7	86	1382.71	132	882.27
42	691.61	87	1349.86	133	1070.55
43	1300.87	88	1459.74	134	973.81
44	527.97	89	1424.6	135	1202.82
45	1266.65	90	1374.58	136	1339.65
46	1308.3	91	1402.4	137	1148.53
47	1268.87	92	1472	138	1345.1
48	1265.63	93	1271.94	139	1092.3
49	1259.87	94	1217	140	1423.9
50	1207.1	95	1372.65	141	1191.39
51	1204.55	96	1326.73	142	966.87
52	1666.19	97	1356.97	143	891.42
53	2048.8	98	1397.74	144	370.63
54	2216.16	99	1452.86	145	445.26
55	1966.2	100	1428.9	146	452.06
56	807.39	101	1158.2	147	476.55
58	949.17	102	1109.48	148	368.19
59	1605.03	103	1195.6	150	386.48
60	1854	104	1126.55	151	435.17
61	1895.81	105	1093.16	152	484.29
62	1369.29	106	241.63	153	510.23
63	1368.68	107	927.74	154	465.87
64	1401.81	108	931.27	155	457.06
65	1278.97	109	1060.65	156	370.67
66	1270.48	110	1125.16	157	363.16
67	1314.37	111	1054.36	158	325.87
68	1236.81	112	354.52	159	321.32
69	936.06	113	927.9	160	339.29

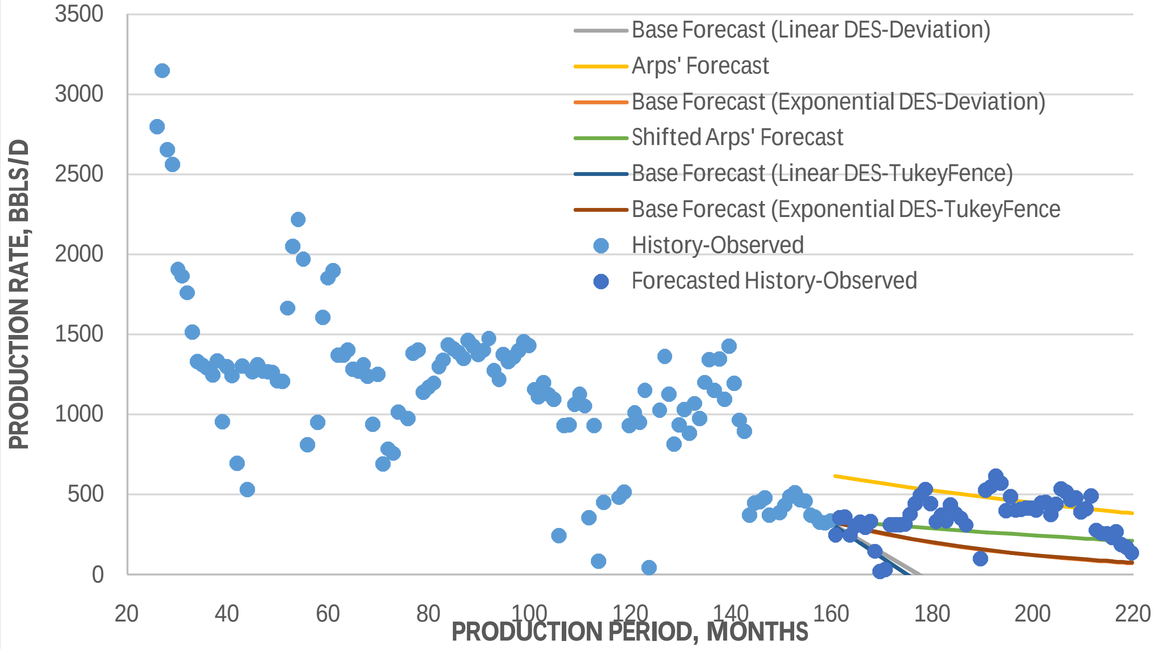

Fig. B.5: COMPARISON OF THE DIFFERENT TECHNIQUES' NEXT 5-YEAR BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED (2ND ANALYSIS)

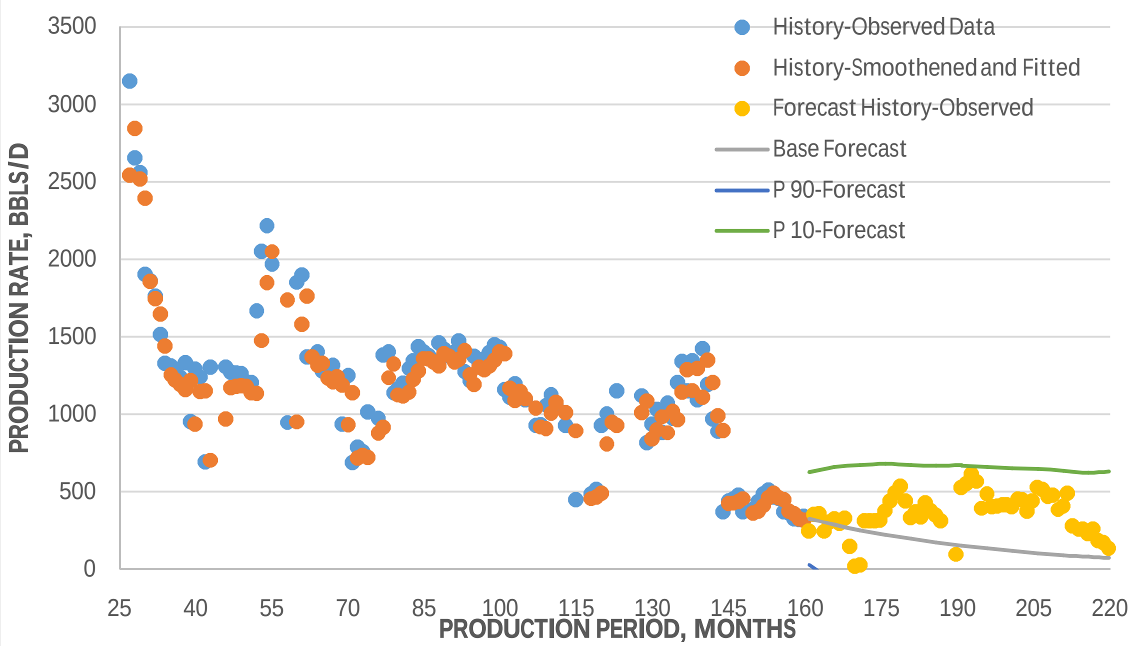

Fig. B.6: UNCERTAINTY ANALYSIS - 80% CONFIDENCE (EXPONENTIAL DES)

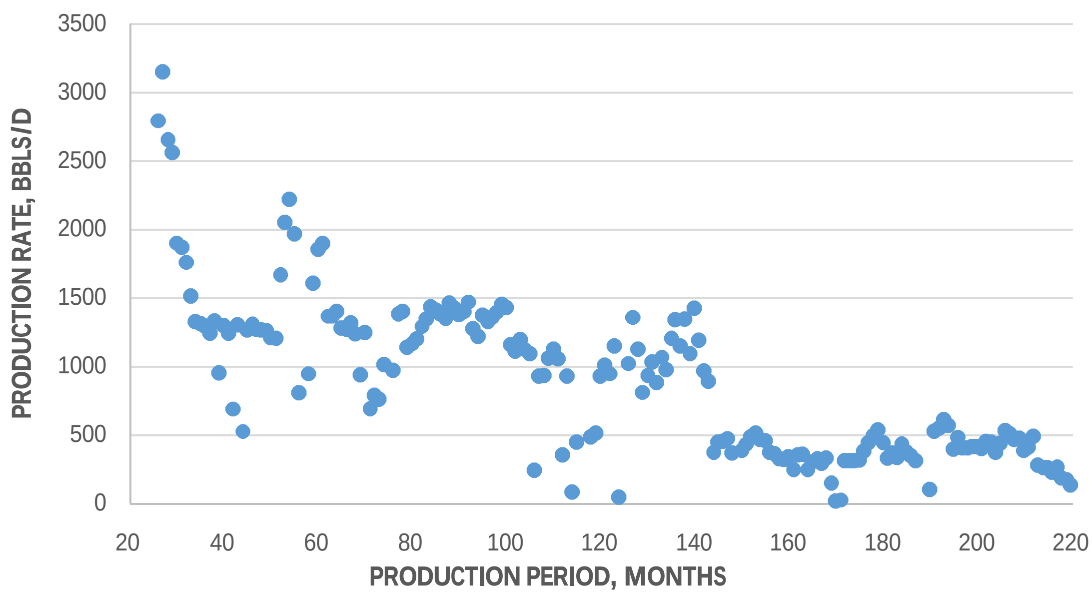

Fig. B.7: TOTAL OBSERVED DECLINING PRODUCTION HISTORY (3RD ANALYSIS)

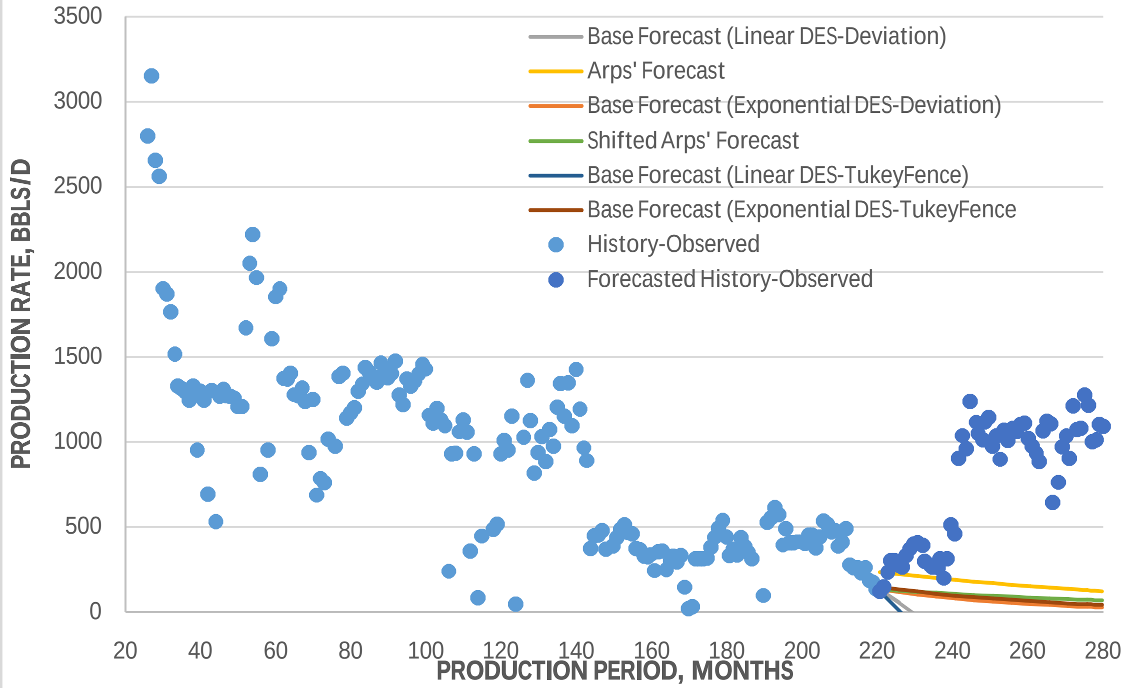

Fig. B.8: COMPARISON OF THE DIFFERENT TECHNIQUES' NEXT 5-YEARS BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED (3RD ANALYSIS)

Figure B.8 shows the effect of production history literature in selecting history periods, which in turns affect the performance forecasts. The accuracy of the models as stated earlier, is affected by the selected data. Availability of information on production activities carried out during the production history of the asset helps in selecting the most appropriate decline regions in order to avoid erroneous forecasts. This is shown in Figure B.9 where impractical (incline) behavior was obtained when uncertainty analysis was run on the base forecast in Figure B.8. Asides the selected history portion, this was due to the numerous inclines in history performance noticed earlier in the production history.

The figures also indicate that some production-enhancement activities were put in place around the forecasted history-observed periods, which led to the sudden subsequent rise in production. Performance forecasts also help the company to evaluate the necessity for a production-boosting program on an asset.

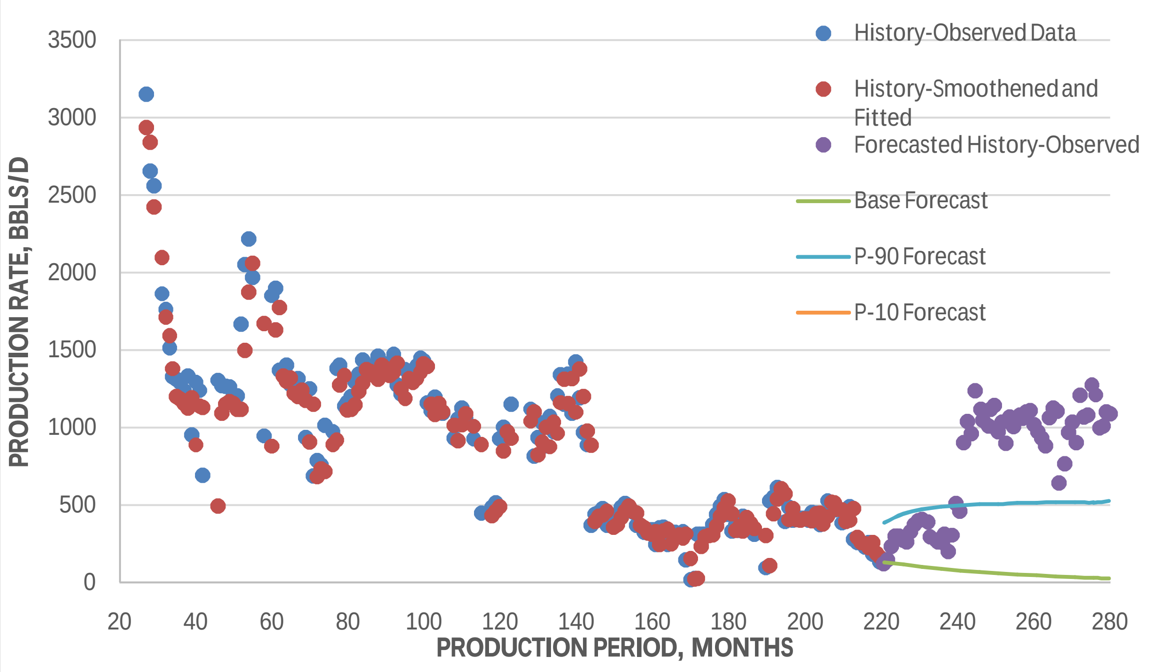

Fig. B.9: UNCERTAINTY ANALYSIS - 80% CONFIDENCE (EXPONENTIAL - DES)

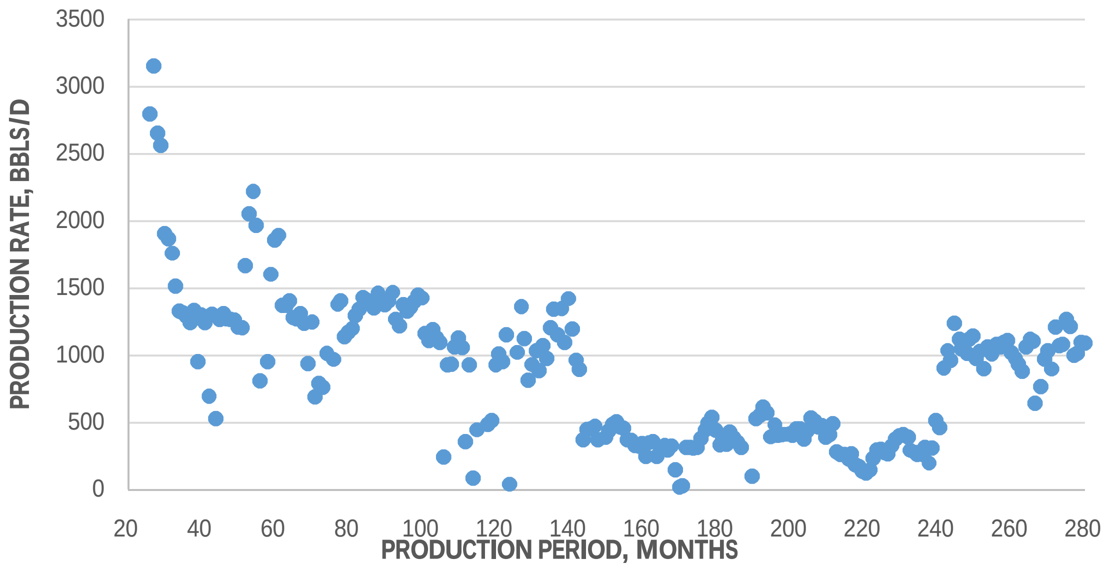

Fig. B.10: TOTAL OBSERVED DECLINING PRODUCTION HISTORY (4TH ANALYSIS)

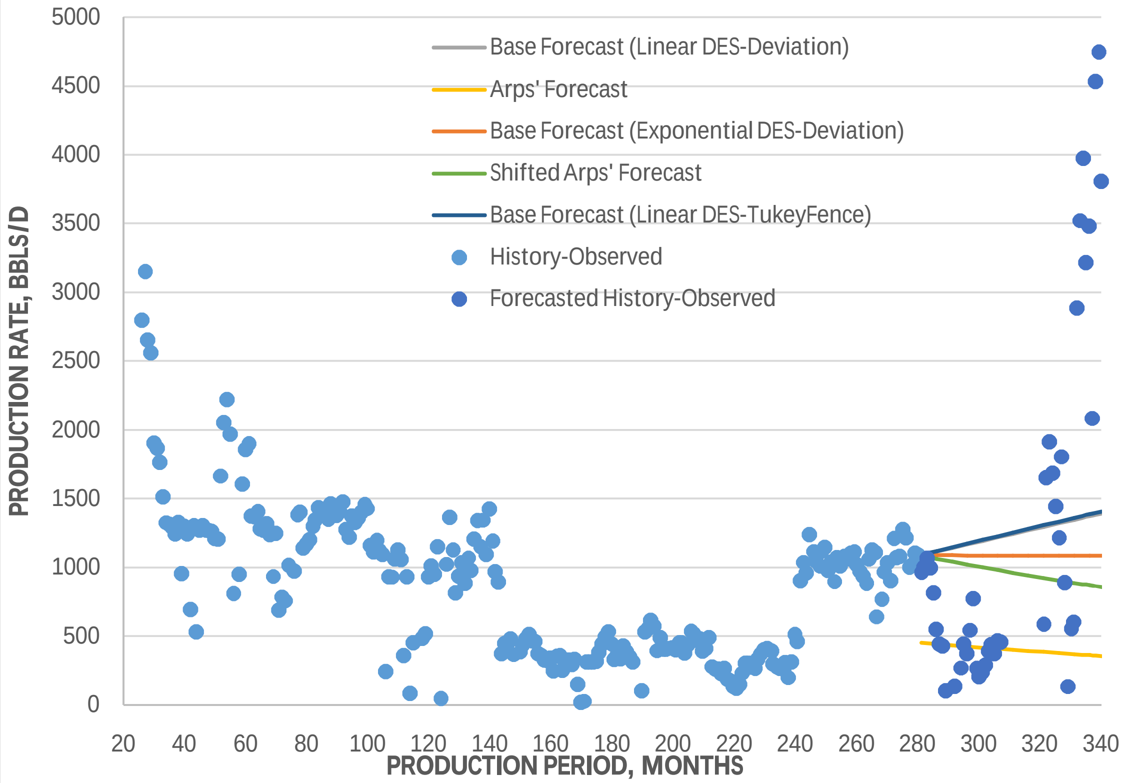

Fig. B.11: COMPARISON OF THE DIFFERENT TECHNIQUES' NEXT 5-YEAR BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED (4TH ANALYSIS)

Figure B.11 also indicates an introduction of some changes to the initial production program, and as such distorted the original production performance trends. The figure further shows the high dependency of the developed models on the most recent history-observed performance trends. This is indicated by the production rise of the models' forecasts, as the most recent history-observed performance showed a rise in production. The impact of an appropriate selection of history-observed production performance region cannot be over-emphasized in the accuracy of any analytical or empirical performance prediction approach.

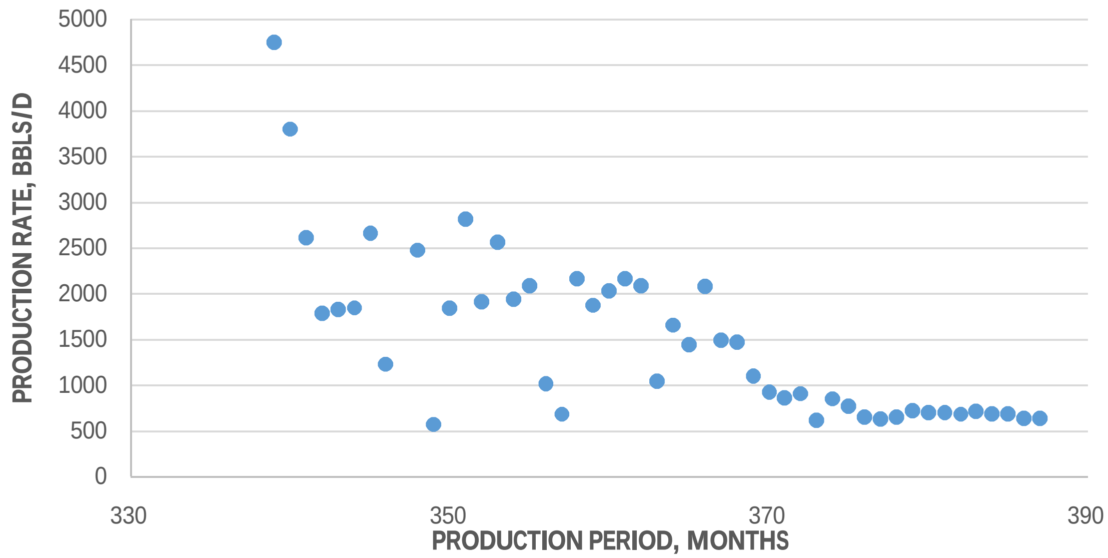

Fig. B.12: TOTAL OBSERVED DECLINING PRODUCTION HISTORY (5TH ANALYSIS)

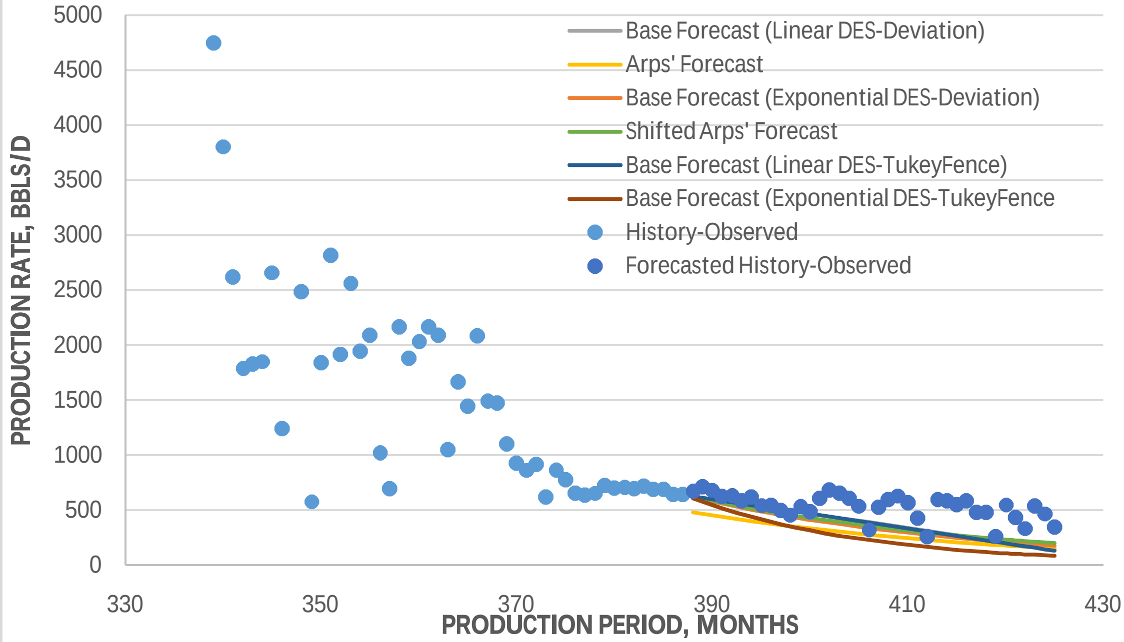

Fig. B.13: COMPARISON OF THE DIFFERENT TECHNIQUES' NEXT 5-YEAR BASE-FORECASTS WITH THE ACTUAL FORECASTED HISTORY-OBSERVED (5TH ANALYSIS)

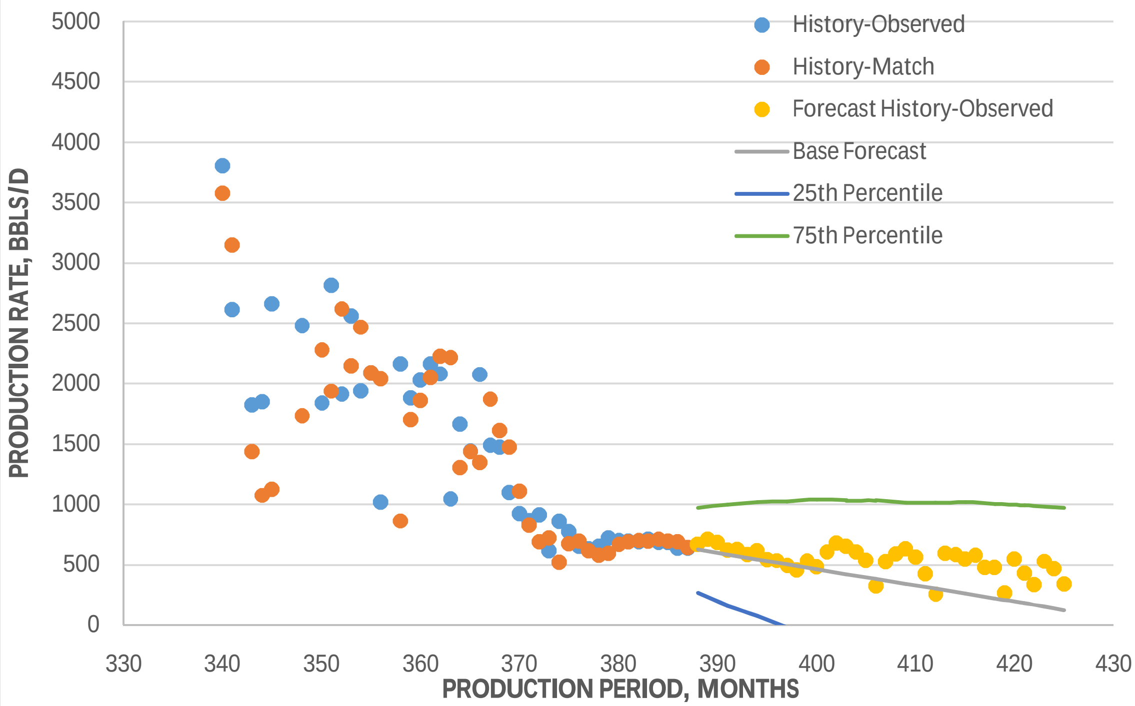

Fig. B.14: UNCERTAINTY ANALYSIS - 50% CONFIDENCE (LINEAR DES)

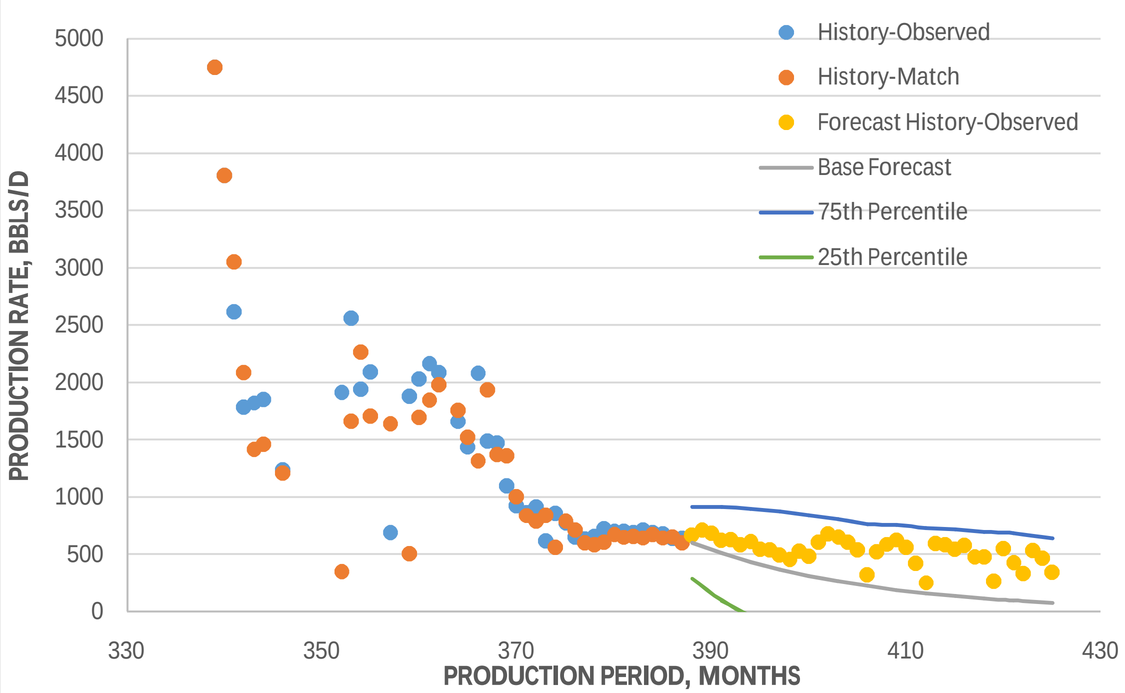

Fig. B.15: UNCERTAINTY ANALYSIS - 50% CONFIDENCE (EXPONENTIAL DES)

Figures B.14 and B.15 practically buttress a critical advantage of the exponential DES over the linear DES for economic analysis. The exponential DES always gives a slimmer range of predictions for each confidence.

Note that all the models show economically acceptable base or deterministic forecasts on Figure B.13, with the exponential DES being the most conservative. Their difference becomes prominent during the stochastic forecast which is a crucial focus or objective of this work.