QUALITATIVE MODELLING AND SIMULATION OF FREE RUNNING SEMICONDUCTOR LASER

A THESIS SUBMITTED TO THEORETICAL PHYSICS STREAM

AFRICAN UNIVERSITY OF SCIENCE AND TECHNOLOGY, ABUJA

IN

PARTIAL FULFULMENT OF THE REQUIREMENTS FOR THE AWARD OF

M.SC THEORETICAL PHYSICS

BY

MOSES, ISAIAH ABU

UNDER THE SUPERVISION OF

PROF. OLEG YORDANOV

INSTITUE OF ELECTRONICS, BULGARIAN ACADEMY OF SCIENCES

NOVEMBER, 2011

ACKNOWLEDGEMENT

I give thanks to the Lord Almighty for the gift of life, strength, opportunity and wisdom to go through this level of education; “To the only wise God our Saviour, be glory and majesty, dominion and power, both now and ever. Amen” (Jude 25).

I appreciate my supervisor, Prof. Oleg Yordanov who gave me his time despite his busy schedule; thanks so much Prof. Thanks to the AUST Physics Stream Faculty for the tutourship they gave throughout my study. My friends in the Stream were a great motivation to me.

My parents, Mr and Mrs Moses Tsado are a treasure to my life. Thanks for bearing me before the Lord daily; your dutiful labour is not in vain.

Mr & Mrs P. G. Tsado play the role of parent to me, though my brother and Sister they are. Appreciations to my other siblings; Ya sule and his wife, Go’o Muria and her husband, Pastor Mathew and his wife, Go’o Ceci and her husband, Adams and Ezekeil. I am privileged to be blessed with such a family.

Bro Christ and Mrs Suemo offered me immeasurable spiritual oversight and would not withhold their care and guidance from me; thanks so much.

My very special friend, Sis. Susan Kolo must receive the report of the progress of the work at every point. So, I had to work at least for her sake; thanks for been there for me.

ABSTRACT

The thesis presents the modelling and simulation of free running semiconductor laser. The rate equations which were derived on the basis of the fact that there must be a balance between carriers that undergo transition and the photons generated and annihilated were simulated to examine what laser looks like when ran without positive optical feedback nor any optical injection. The result shows the photons get amplified due to stimulated emission but no oscillation of the output is obtained. Such a set up is therefore seen as an optical amplifier.

TABLE OF CONTENT

CHAPTER ONE
Background

1.1 Introduction

The word laser is an acronym for the most significant feature of laser action: Light Amplification by Stimulated Emission of Radiation[3]. The basic component of laser is the material with ability to amplify radiation. Such materials are referred to as active medium. The amplification process results from the phenomenon known as the stimulated emission (this will be explained in chapter two) which was discovered by Albert Einstein in 1916. The first laser was constructed by T. H. Maiman in 1960 and different types have been invented ever since. Common among the different types of laser are the solid state lasers, gas lasers, dye lasers, chemical lasers and semiconductor lasers.

Semiconductor lasers were first demonstrated in 1962 by two US groups led by Rober N. Hall at the General Electrical Research Centre and by Marshall Nathan at the IBM T. J. Watson Research Center[1]. It has applications in a number of areas. It serves as sources as well as amplifiers and multiplexers for optical communications. Semiconductor lasers also serve as pump sources for solid state lasers and sources of beams for spectroscopy. Infrared and red laser diodes are commonly used in CD players, CD-ROMs and DVD technology. Diodes lasers areused in instruments such as laser scanners, pointers, barcode readers and printers.

The attributes of semiconductor lasers which have made them useful in several areas include their relatively small in size and low cost. They are also efficient and require low power current source and have been shown to operate at wide range of wavelengths; from the near ultraviolet to the far infrared.

The operation of semiconductor lasers is similar to that of other lasers with semiconductor material serving as the active medium. The p-n junction is forward biased to produce the pumping process necessary for the stimulated emission or the pumping can be done using another laser, a process known as the optical pumping. The more detail physics of the semiconductor laser is presented in chapter two.

1.2 Aims of the Thesis

The semiconductor lasers have found a wide range of applications as highlighted above. As such, they underwent an intensive research and development since the time of their first operational regime was achieved in 1962. This thesis aims at simulating the free running semiconductor laser using the Matlab as the computational tool. This is done using the rate equations which are represented two systems of coupled ordinary differential equations for carrier density and photons. This gives the continuous wave and the dynamic behaviour of the laser from which the output power of the laser can also be obtained.

1.3 Overview of the Thesis

Chapter two reviews the basic processes involved in the laser action. The processes discussed include spontaneous emission, stimulated emission, optical absorption, population inversion, the threshold condition for lasing and pumping. Homojunction and double-heterojunction lasers are also discussed. The chapter concludes with a brief overview of related previous work.

Chapter three presents a detailed derivation of the rate equations followed by a brief description of the Matlab code used in the simulation.

In chapter four, the results obtained from the simulation were presented and discussed. Conclusion is also given in chapter four.

CHAPTER TWO

Physics of Semiconductor Lasers

"A laser is a device that emits light (electromagnetic radiation) through a process of optical amplification based on the stimulated emission of photons. The term "laser" originated as an acronym for Light Amplification by Stimulated Emission of Radiation."[1]

The technical terms involves in the laser action are discussed hereafter. The operation principles of semiconductor laser are also outlined.[2]

2.1 Spontaneous Emission

Consider two levels 1 and 2 of energy E_1 and E_2 respectively of an atom (Fig.2.1). Suppose $E_1<E_2$ and that the atom is initially at level 2. The atom tends to decay to level 1. If the decay is accompanied by an emission of photon whose energy $h\upsilon_0$ equals the energy difference

$E_2 - E_1$, that is,

$$h\upsilon_0 = E_2 - E_1 \tag{2.1}$$

the process is called spontaneous (or radiative) emission. In Eq. (2.1), h is the Plank's constant and υ_0 is the frequency of the emitted wave.The decay can also be non-radiative in which case the energy difference may be released as either kinetic or internal energy of the surrounding atoms.

Let N_i be the number of atoms per unit volume in a given energy level *i*, at a time *t*. the quantity N_i is called the population of the level.

For spontaneous (radiative) or nonradiatve emission, the rate of decay of the upper state population must be proportional to the population, that is;

$$\frac{dN_2}{dt} = -AN_2 \,. \qquad (2.2)$$

The minus sign accounts for the negative time derivative, *A* is the Einstein's coefficient (so called because it was first obtained by Einstein from thermodynamic considerations), a positive constant also called the rate of spontaneous emission. The physical dimension of A is apparently sec^{-1} – inverse time. It is convenient to introduce $t_{sp} = 1/A$ and $t_{nr} = 1/A$, the spontaneous and nonradiative decay lifetime, respectively. It should be noted that t_{sp} depends only on the particular transition considered, while t_{nr} depends on the characteristics of the surrounding medium as well.

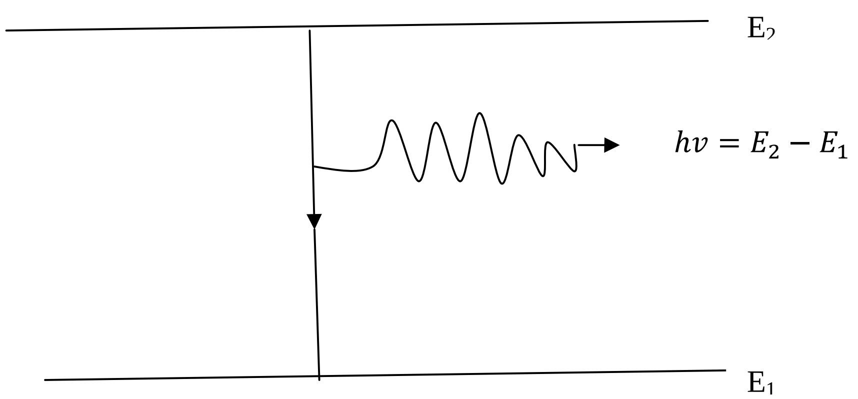


Fig.1.1: Schematic illustration of the Spontaneous emission

2.2 Stimulated Emission

Consider two levels 1 and 2 with the atom initially at level 2. If an electromagnetic wave (em) of frequency $\upsilon = \upsilon_0$ (i.e., equal to that of the spontaneously emitted wave) is incident on the material, there is a likely-hood that this wave will force the atom to undergo the transition $2\ to\ 1$. When this happen the energy difference $E_2 - E_1$ is released in the form of an em wave that adds to the incident wave. This process is called a stimulated emission.

For stimulated emission process, we have the rate at which transition $2 \rightarrow 1$ takes place as

$$\frac{dN_2}{dt} = -W_{21}N_2, \tag{2.3}$$

where W_{21} is the rate of stimulated emission. W_{21} depends on the particular transition as well as on the intensity of the incident em wave.

For plane wave, we can write

$$W_{12} = \sigma_{21}F, \tag{2.4}$$

where F is the photon flux of the wave and σ_{21} is the stimulated emission cross section, which has physical dimension of an area and depends on the characteristics of the given transition.

The basic difference between the spontaneous and stimulated emitted em wave is in their phases. In the case of the spontaneously emitted wave, there is no relation between the phase of the wave emitted by one atom and that by another.

Also the wave is emitted in any direction. In the case of the stimulated emission, since the process is forced by the incident em wave, the emission of any atom adds in phase to that of the incoming wave and propagates in the same direction.

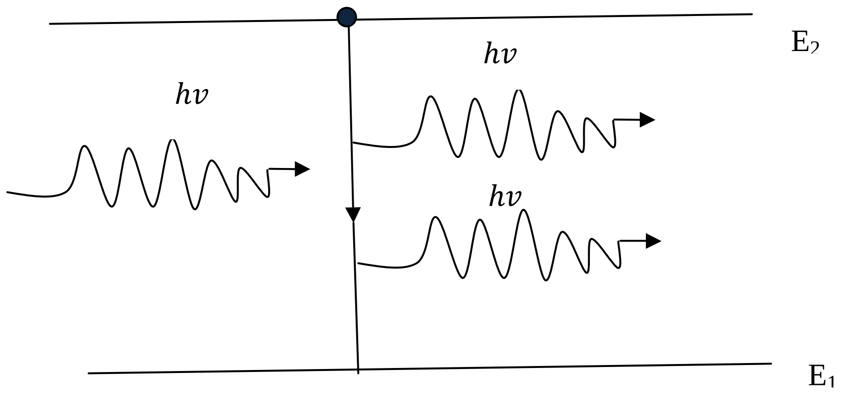


Fig.1.2: Schematic illustration of the Stimulated emission

2.3 Absorption

Consider the two levels 1 and 2 with the atom initially at level 1. If this is the ground level, the atom remains in this level unless some external stimulus is applied. Assume that an em wave of frequency $\upsilon = \upsilon_0$ is incident on the material. In this case there is a finite probability that the atom will be excited to level 2 absorbing a photon of the energy equal to the level difference $E_2 - E_1$. This process is called absorption.

For stimulated absorption rate, we can have the rate of transition 1 to 2 due to absorption

$$\frac{dN_1}{dt} = -W_{12}, \tag{2.5}$$

with

$$W_{12} = \sigma_{12} F, \tag{2.6}$$

where σ_{12} is the absorption cross section (dimension of area) depending only on the particular transition.

As revealed by Einstein at the beginning of the twentieth century, if level 1 is g_1-fold degenerate and level 2 is g_2-fold degenerate, then

$$g_2 W_{21} = g_1 W_{12}, \tag{2.7}$$

which implies

$$g_2 \sigma_{21} = g_1 \sigma_{12} \,. \tag{2.8}$$

From equation (2.7), we can see that if the two levels are non-degenerate or have the same degeneracy, then

$$W_{21} = W_{12} \text{and} \sigma_{21} = \sigma_{12} \tag{2.9}$$

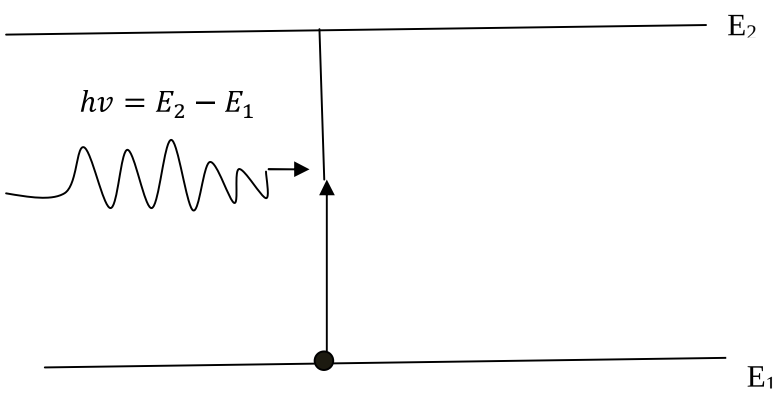


Fig.1.3: Schematic illustration of absorption

2.4 Population Inversion

Fig.1.4 below shows a plane wave with the photon flux F travelling in, say, z-direction in a material. Stimulated absorption and emission phenomena in the material result in an elemental change dF of the flux along the wave direction. The change in number between the outgoing and incoming photons as it passes through the material per unit time is SdF, where S is the cross sectional area of the beam. From the equations (2.3) and (2.5) above we have,

$$SdF = (W_{21}N_2 - W_{12}N_1)(Sdz) \qquad (2.10)$$

That is, the difference between the stimulated emitted and absorbed photons per unit time, where Sdz is the volume of the shaded region. Substituting equations (2.4), (2.6) and (2.8) into (2.10), to obtain

$$dF = \sigma_{21}F[N_2 - \frac{g_2N_1}{g_1}] \qquad (2.11)$$

The spontaneous decays are not considered in the equation above. This is because the photons created by radiative decay are emitted in any direction and so have negligible contribution to the incoming photon flux while the nonradiative decay does not even add new photons. Boltzmann statistics described the populations at thermal equilibrium as

$$\frac{N_2^e}{N_1^e} = \frac{g2}{g1} \exp\,[-(E_2 - E_1)/kT] \qquad (2.12)$$

where N_1^e and N_2^e are the populations at the thermal equilibrium of the two levels, k is the Boltzmann's constant and T is the absolute temperature of the material.

From equation (2.11), the material behaves as an amplifier if $N_2 > g_2N_1/g_1$ and as an absorber if $N_2 < g_2N_1/g_1$. Since the exponential factor in (2.12) is always less than 1, the material therefore, in thermal equilibrium behaves as an absorber at frequency υ_0. If the levels are in nonequilibrium condition it acts as an amplifier. When this happen we say there is a population inversion in the material and the material is said to be an active medium.

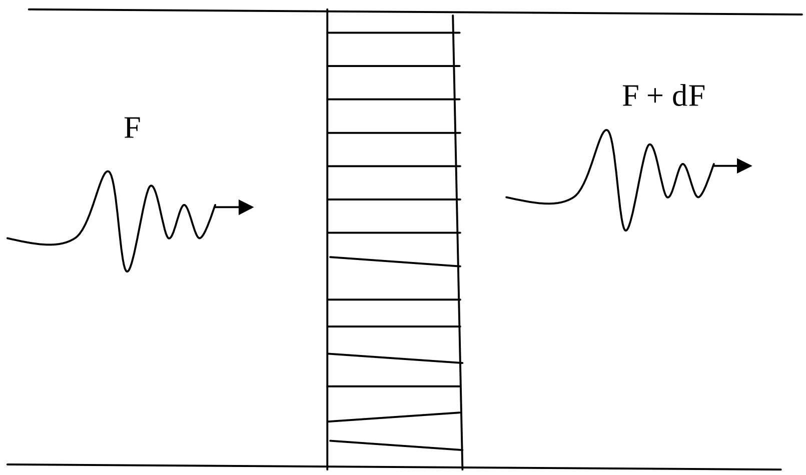


Figure 2.4: An em wave travelling through a material

2.5 Maser and Laser Amplifier

From the foregoing discussion, if υ_0 falls in the microwave region this type of amplifier is called a maser amplifier, with maser being an acronym for microwave amplification by stimulated emission of radiation. If υ_0 falls in the optical region this type of amplifier is called a laser amplifier, with laser being an acronym for light amplification by stimulated emission of radiation.

In the microwave region, an oscillator is made from an amplifier by placing the active material in a resonant cavity with the resonant frequency of υ_0. This is to produce a suitable feedback. The feedback is produced in optical region by placing the active material between two highly reflecting mirrors such as plane parallel mirrors. The plane em wave is amplified on each passage as it travels back and forth perpendicular to the mirrors. An output is obtained by making one of the two mirrors to be partially transparent.

2.6 Threshold Condition for Lasing

The threshold condition for the laser is that the gain of the active material compensates the losses in the laser (e.g., losses due to output coupling). The gain per pass in the active material, that is, the difference between output and input photon flux is given by integrating equation (2.11). This gives

$$F = \exp\{\sigma[N_2 - (g_2 N_1/g_1)]l\} \quad (2.13)$$

where l is the length of the active material and we use $\sigma_{21} = \sigma$. Given that F is the photon flux in the cavity leaving mirror 1 and travelling toward mirror 2 at a given time, then the photon flux F' reflecting from mirror 1 after one round of trip is

$$F' = Fexp\{\sigma[N_2 - (g_2N_1/g_1)]l\} \times (1 - L_i)R_2 \times \exp\{\sigma[N_2 - (g_2N_1/g_1)]l\} \times (1 - L_i)R_1 \quad (2.14)$$

Where R_1 and R_2 are the reflectivities of the two mirrors (Fig.2.4), respectively and L_i is the internal loss per pass in the laser cavity.

At threshold we must have $F = F'$ and soEquation (2.14) implies that the population inversion $N = N_2 - g_2N_1/g_1$ must reach a critical inversion N_c for the threshold to be reached. N_c is given by:

$$N_c = -\frac{[ln\, R_1 \ln R_2 + 2ln(1 - L_i)]}{2\sigma l} \quad (2.15)$$

Simplifying equation (2.15), we define

$$\gamma_1 = -lnR_1 = -ln(1 - T_1) \quad (2.16)$$

$$\gamma_2 = -lnR_2 = -ln(1 - T_2) \quad (2.17)$$

$$\gamma_i = -[ln(1 - L_i)] \quad (2.18)$$

where T_1 and T_2 are mirror transmissions where we have assumed that the mirror absorption is negligible. Therefore we can write

$$N_c = \frac{\gamma}{\sigma l} \quad (2.19)$$

Where

$$\gamma = \gamma_i + \frac{(\gamma_1 + \gamma_2)}{2} \quad (2.20)$$

The quantity γ_i, defined by eq.(2.18), can be called the logarithmic internal loss of the cavity. In fact when $L_i<<1$, as usually occurs, one has γ_i approximately equals L_i. Similarly, since both T_1 and T_2 represent a loss for the cavity, γ_1 and γ_2, defined above can be called the logarithmic losses of the two cavity mirrors. Thus the quantity γ defined by eq. (2.20) can be called the single-pass loss of the cavity.

Once the critical inversion is reached, oscillation builds up from spontaneous emission. Photons spontaneously emitted along the cavity axis in fact initiate the amplification process.[2]

2.7 Pumping

An external stimulus is required to produce stimulated emission in lasers. In four-level lasers, atoms have to be excited with em wave with frequency υ_0 from the ground level to level 3. The process of doing this is known as pumping.

It must be noted that it is not possible to produce population inversion with just two-level scheme. This is because at thermal equilibrium, $N_1>g_1N_2/g_2$ and absorption is predominant over the stimulated emission. The idea is, more transitions 1 to 2 than the transition 2 to 1 is produced by the incident wave.

If however, the level are say at least three 1(ground) 2 and 3, and the atoms are excited by some process to level 3, from where they decay fast to level 2 (for example by nonradiative decay), a population inversion can be achieved between levels 1 and 2.

Semiconductor lasers are four-level lasers (ground 0, 1, 2 and 3) in which atoms are excited from ground level to the highest level 3. If the atom then decay from level 3 to 2 rapidly (e.g. by rapid nonradiative decay), a population inversion can be obtained between levels 2 and 1. Atoms are transferred to level 1 through stimulated emission soon after the commencement of oscillation in the four-level laser. It is also necessary that the transition from 1 to ground level be very rapid for a continuous wave operation.

The sketches below give the three and four- level schemes. In general population inversion is easier in four-level than in three-level laser.

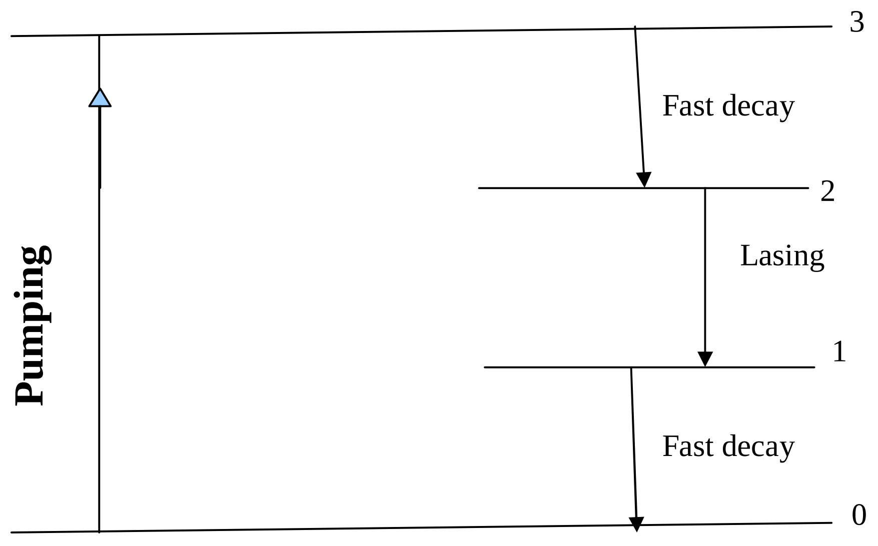


Fig. 2.5(a) Four- level laser scheme

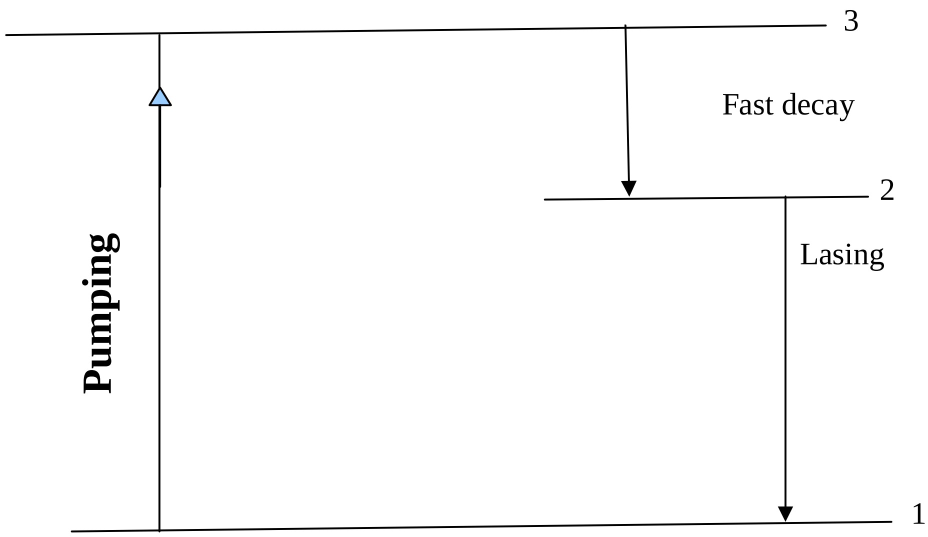


Fig. 2.5(b) Three- level laser scheme

2.8 Operation Principles of Semiconductor Lasers

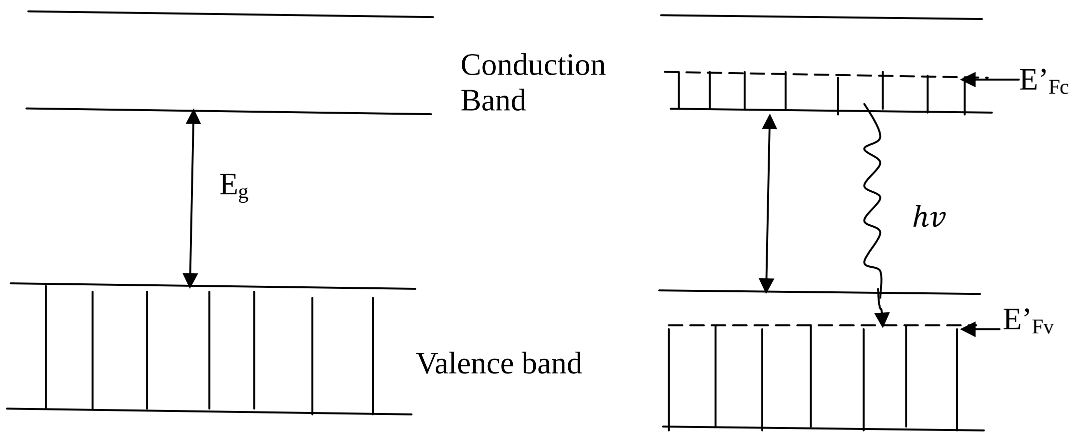


Fig. 2.6 Operation principle of semiconductor laser

A semiconductor with a conduction band and valence band separated by the energy gap E_g is shown schematically in Figure 2.6. At temperature 0K the valence band is completely filled with electrons while the conduction band is completely empty. If the electrons in the valence band are raised by pumping into the conduction band, they will drop to the minimum unoccupied levels of the band after a very short time (~1 ps). Also the electrons on the top of the valence band will drop the lowest unoccupied levels of the valence band. This result in what is referred to as the quasi-Fermi levels. The quasi-Fermi level for the conduction band E'_{Fc} is the highest energy level occupied by electrons in the conduction band and it is higher than what we had before pumping process. The quasi-Fermi level for the valence band, E'_{Fv}, is the highest energy level occupied by electrons in the valence band and it is lower than what we had before pumping process.

Light is emitted when an electron in the conduction band falls to the valence band to recombine with a hole. This is the process by which light is emitted in light-emitting diodes (LED). If the condition below holds:

$$E_g \leq h\nu \leq E'_{Fc} - E'_{Fv} \tag{2.21}$$

then the photon will be amplified with the gain exceeding absorption losses. The values of E'_{Fc} and E'_{Fv}depend on the density of electrons raised to the conduction band, that is, the satisfaction of equation (2.21) depends on the intensity of the pumping. The electron density N must exceed some critical value given by the condition:

$$E'_{Fc}(N) - E'_{Fv}(N) = E_g \tag{2.22}$$

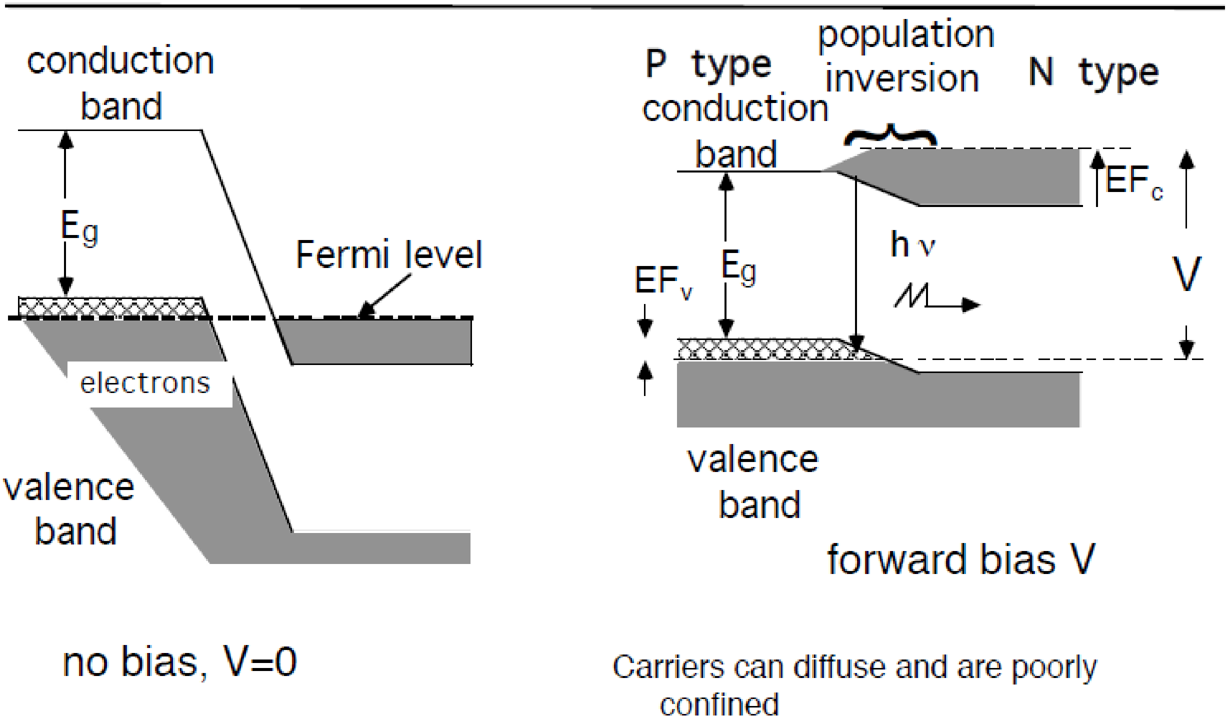
Where the injected carrier density satisfying eq.(2.22) is known as the carrier density at transparency, N_{tr}. For lasing to occur the transparency density N_{tr} must be exceeded by some value of the injected carrier density high enough to result in a net gain.–The gain will make up for the cavity losses when the active medium is placed in a suitable cavity. Thus the injected carriers must reach some threshold value N_{th}.

Forward biasing the semiconductor diode has been found to be the most convenient pumping mechanism for the semiconductor lasers. Other ways of pumping include using beam of another laser or an auxiliary electron beam. Laser action in a semiconductor was first observed in 1962 by using a p-n junction diode. Four groups made the observation with three of them using GaAs.[2]

We call homojunction lasers the lasers in which the same material is used for both the p and n type semiconductor. These were developed at the early stage of semiconductor laser research. The homojunction lasers were superseded by the double-heterostructure (DH) laser in which we have active medium sandwiched with a different material between p and n sides of the junction.

2.9 Homojunction Semiconductor Lasers

In the homojunction laser, both the p-type and n-type regions are of the same material. The donor and acceptor concentrations are so large($about\ 10^{18}\ atoms/cm^3$)that the Fermi level, E_{Fc}, of the conduction band in the n-type falls in the valence band of the p-type and the Fermi level of the valence band in the p-type, E_{Fv} falls in the conduction band of the n-type.



Figure, 2.7[20]: (a) band structure of p-n junction (b) Forward biased p-n junction

If no junction voltage is applied the two Fermi levels across the barrier aligns, see Fig. 2.7a. If the junction is forward biased with a $voltage\ V$, the Fermi levels are separated by the energy gap $\Delta E = eV$, Fig.2.7b. Electrons from the n-type region are injected into the conduction band while the holes from the p-type region are injected into the valence band. The transparency and laser threshold can therefore be reached with appropriate value of current density.

The homojunction laser operates at a very high current density at room temperature $J_{th} \cong 10^5\, A/cm^2$. This is due to the very low potential barrier an electron in the conduction band encounters on reaching the p region of the junction. The electrons therefore penetrate in to the p-type and combine with holes. The penetration depth is $d = \sqrt{D\tau}$ according to the diffusion theory, where D is the diffusion coefficient and τ the electron lifetime, i.e. the time needed to recombine with a hole. A typical values for these parameters are $D = 10\,\frac{cm^2}{s}\ and\ \tau \cong 3\ ns$ (for GaAs), which gives $d \approx 1\ \mu m$. Hence, the width of active region is determined by the penetration depth being thicker compared with the depletion layer $(\cong 0.1\mu m)$. Due to the high threshold frequency it is not possible to operate the laser as cw at room temperature without suffering destruction in a very short time. Thus, homojunction laser can operate a cw only at cryogenic temperature[10-13], typically at a liquid nitrogen temperature, T = 77 K.

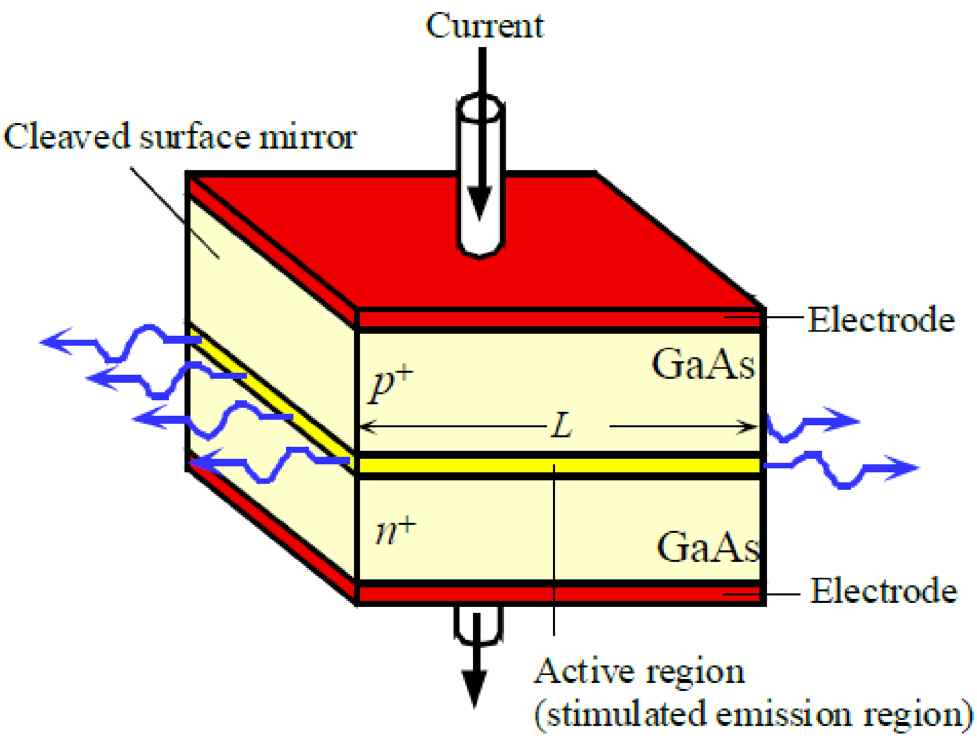


Figure 2.8[21]: Homojunction Semiconductor Laser

2.10 Double-Heterostructure Lasers

A schematic diagram of Double-Heterostructure laser (or DH) is shown below. In DH, we have the active medium between two different materials. It is operated at about two order of magnitude of current density less than that of the homojunction devices and hence the cw operation is possible at room temperature. This is achieved due to the features of photon confinement, carrier confinement and reduced absorption by the material.

The heterostructure widths can be chosen to produce either a bulk, or a quantum-well gain medium.

In the sketch below, GaAs acts as the active layer between $Al_xGa_{1-x}As$ as the p-type and n-type material.

Photon confinement is achieved as the refractive index of the active layer, n = 3.6 is larger than that of the p-side and n-side cladding-layers. This confines the photon to the active region.

Carrier confinement is achieved in the sense that the band gap of the active layer is significantly smaller than that of the cladding layers. This forms energy barriers at the two junction planes and thereby confines the injected carriers within the active layer.

Reduced absorption is also achieved by carefully arranging the differences in the band gaps between the active and the cladding layers. The smaller value of the active layer band gap and consequently that of the frequency of the laser beam made the absorption by the cladding layers to be significantly smaller.

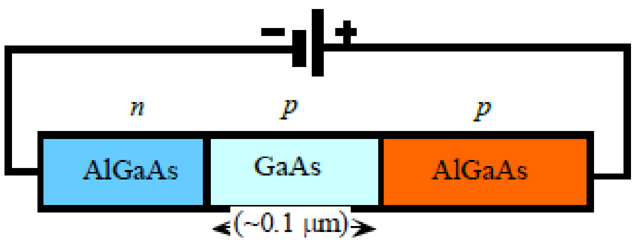


Figure 2.9[21]: Double-heterostructure Semiconductor Laser

2.11 Rate Equations and Semiconductor Laser

Since its first operation in 1962[4-7], further developments and improvements have been made to semiconductor lasers. On the theoretical side, the rate equations have been particularly important reference point for the study of dynamics of semiconductor lasers. Hereby, I present a review of some of the key issues related to these equations.

In 1974, the light output and the electron density within the active layer of a semiconductor laser were calculated from the steady state solution of a rate equation approach by R. Salathe et al[8]. They found a simple analytical approximation formula which was found to be in good agreement with the numerical results from the rate equations. A. N. Pisarchik and F. R. Ruiz-Oliveras[9] studied the dynamic of semiconductor laser with two external cavities. In their paper, the complex dynamics of the semiconductor laser with one and two external cavities were examined numerically. It was shown that the dynamical regimes of the laser could be controlled with the addition of a second external cavity.

The ICUE report presented by Toby Schaer et al[10] presented a dynamics simulation model for semiconductor laser. The work presented a method of using MATLAB and simulink to simulate thee behaviour of distributed feedback quantum-well semiconductor laser diodes using the rate equations. The technique used is suitable for investigating both analogue and digital modulation performance of frequency and pulse response.

A host of other researchers have also used the rate equations for the study of the semiconductor laser[11-14]. Rate equations have therefore been used extensively to study the dynamics of the semiconductor lasers. This thesis presents the MATLAB simulation of the rate equations with the aim of examining the qualitative basic transient behaviour of semiconductor laser with neither external optical injection nor optical feedback.

CHAPTER 3

Modelling and Simulation

3.1 The Rate Equations

In this chapter I derive the rate equations for the four-level laser. The rate equations are derived on the basis of the assumption that there is a balance between the atoms undergoing transition and the photons emitted or absorbed. Examples of four-level lasers are He-Ne laser, Argon laser and the semiconductor laser, the latter being the main focus of my work. Other lasers are classified as either three-level e.g. gas lasers (examples are Co_2, Cu vapour, N_2, Ar^+, He-Ne, He-Cd and HF) or quasi-three-level lasers (examples are rare-earth ions in crystal or glass hosts eg. Yb,Ho, Tm:Ho, Tm, Er, and Nd).

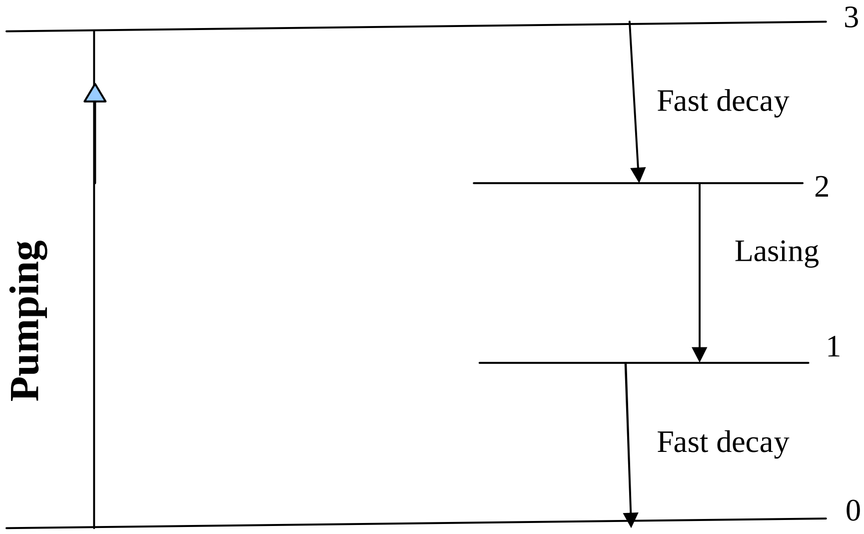


Figure. 3.1 Four- level laser scheme

An idealized scheme is shown in Figure 3.1. In this idealisation, it is assumed that there is only one pump level/band 3 (from the ground level 0). The analysis, however still holds even if there is more than 1 pump level provided the transition from this level (say 4) to level 2 is still very rapid. And the decay from level 3 to 2 as well as 1 to 0 is very fast. Within this approximation, we can assume that $N_1 \cong N_3 \cong 0$ for the populations of level 1 and 3. The considerations are therefore limited to the population N_2 of level 2 and N_g of the ground level.

It is also assumed that the laser is oscillating on only one cavity mode.

3.2 Deriving The Rate Equations.

In this case we assume that the laser is oscillating on a single mode and pumping and mode energy densities are uniform within the laser material. In this simple case the equations are referred to as space-independent rate equations. The treatment that follows only applies to a unidirectional ring resonator, with uniform transverse beam profile, where pumping is uniformly distributed in the active medium. It is a simplified case which helps in understanding many basic properties of laser behaviour.

The rate of change of carrier density is given by

$$\frac{dN}{dt} = R_{in} - R_{out}, \tag{3.1}$$

where $R_{in} = R_p = \frac{I}{eV}$, with V denoting the volume of the active region.

R_{in} is referred to as the rate of carrier generation and R_{out} the rate of carrier dissipation. The rate of carrier dissipation is accounted for by the spontaneous emission, non-radiative recombination, leakage from active region and stimulated emission. The dissipation due to the non-radiative recombination and leakage from the active region is ignored in favour of the other dissipations. So we have

$$R_{out} = R_{stim} + \frac{N}{\tau}$$

Where R_{stim} is the rate of stimulate emission and N/τ account for the spontaneous emission with τ being the lifetime of the upper-laser level. Thus equation (3.1a) becomes

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau} - R_{stim} \qquad (3.1a)$$

The population *N* in Equation (3.1b) is coupled to the photon density via R_{stim}, the rate of stimulated recombination.

The photon density generation rate is given by

$$R_{ph} = \frac{V}{V_{ph}} R_{stim} = \Gamma R_{stim},$$

where V_{ph} is the volume occupied by the photons. It is larger than the active carrier volume in guided structure and the ratio is known as the optical confinement, *Γ*.

The rate of change of the photon density with time is therefore given by

$$\frac{d\phi}{dt} = \Gamma R_{stim} + \Gamma \beta_{sp} R_{sp} - \frac{\phi}{\tau_c} \qquad (3.1b)$$

Where $\frac{\phi}{\tau_c}$ is the loss of photons from the ends of the cavity via the photons lifetime. The latter accounts for the removal of photons due to cavity losses.

Further, the term $\Gamma\beta_{sp}R_{sp}$ accounts for the spontaneously emitted photons.

The spontaneous emission is random and does not contribute to laser output. The fraction emitted into the lateral mode of the laser, that is, in the same angular direction and spectral bandwidth of the mode is that which does. That explains the introduction of β_{sp}, the spontaneous emission coupling factor.

In both (3.1a) and (3.1b), R_{stim} is given by BϕN where B is defined as the stimulated emission coefficient per photon per mode. Note that in equation (3.1b), the rate of photon generation due to the spontaneous emission is ignored compared to that obtained by the stimulated emission. The two rate equations can therefore be written as

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau} - B\phi N \qquad (3.1c)$$

$$\frac{d\phi}{dt} = B\phi N - \frac{\phi}{\tau_c} \qquad (3.1d)$$

These equations give the cw (continues wave) and dynamic behaviour of semiconductor laser.

Stimulated Emission Coefficient per Photon per Mode, B

Simulation of the laser behaviour requires the explicit expressions for the stimulated emission coefficient B as well as for the lifetime τ_c and the volume of the mode in the active medium V_a which are obtained next.

Let *I* be the intensity of the beam at a given cavity position at time t = 0.

Let *L* denotes the length of the resonator in which an active medium of length *l* and refractive index *n* is inserted.

The intensity I', after one cavity round-trip, is[2]

$$I' = Iexp(\sigma N_2 l) \times (1 - L_i)R_2 \times \exp(\sigma N_2 l) \times (1 - L_i)R_1.$$

This gives:

$$I' = IR_1R_2(1 - L_i)^2 \exp(2\sigma N_2 l) \quad (3.2)$$

Where $R_1$and R_2 are the power reflectivities of the two mirrors are, L_i is the single-pass internal loss of the cavity, $(1 - L_i)^2$ is the round-trip cavity transmission, $(2\sigma N_2 l)$ is the round-trip gain of the active medium.

Using $R_1 = 1 - a_1 - T_1$ and$R_2 = 1 - a_2 - T_2$ where T_1 and T_2 are the power transmissions of the two mirrors and a_1 and a_2 are the corresponding fractional mirror losses.

The change of intensity,$= I' - I$ for a cavity round-trip is then:

$$\Delta I = [(1 - a_1 - T_1)(1 - a_2 - T_2)(1 - L_i)^2 \exp(2\sigma N_2 l) - 1]I \quad (3.3)$$

It is assumed that the mirror losses are equal, that is, $a_1 = a_2 = a$ and so small that we can set $(1 - a - T_1) \cong (1 - a)(1 - T_1)and(1 - a - T_2) \cong (1 - a)(1 - T_2)$. Equation (3.3) therefore becomes

$$\Delta I = [(1 - T_1)(1 - T_2)(1 - a)^2(1 - L_i)^2 \exp(2\sigma N_2 l) - 1]I \quad (3.4)$$

For future convenience, the logarithmic losses per pass are introduced:

$$\gamma_1 = -ln(1 - T_1) \quad (3.5a)$$

$$\gamma_2 = -ln(1 - T_2) \quad (3.5b)$$

$$\gamma_i = -[ln(1 - a) + ln(1 - L_i)] \quad (3.5c)$$

Where γ_1 and γ_2 are the logarithmic losses per pass due to the mirror transmission and γ_i is the logarithmic internal loss per pass. (γ_1 and γ_2 are called mirror losses for short and γ_i the internal loss).The total logarithmic loss per pass is defined as:

$$\gamma = \gamma_i + \frac{\gamma_1 + \gamma_2}{2}. \quad (3.6)$$

I now substitute equation (3.6) into (3.4) taking into account (3.5a), (3.5b) and (3.5c) to obtain:

$$\Delta I = [\exp(-2\gamma) \exp(2\sigma N_2 l) - 1]I \quad (3.7)$$

This gives

$$\Delta I = [\exp [2(\sigma N_2 l - \gamma)] - 1]I \quad (3.8)$$

Using the additional assumption:

$$(\sigma N_2 l - \gamma) \ll 1, \quad (3.9)$$

and expanding the exponential function in equation (3.8) retaining the linear term only results in:

$$\Delta I = 2(\sigma N_2 l - \gamma)I \quad (3.10)$$

The time taken for the light to make one cavity round-trip is

$$\Delta t = {}^{2L_e}/_{c}, \tag{3.11}$$

Where L_e is the optical length of the resonator given by

$$L_e = L + (n-1)l \tag{3.12}$$

Dividing eq.(3.10) by (3.11) and using the approximation,

$$\frac{\Delta I}{\Delta t} \cong \frac{dI}{dt}$$

gives

$$\frac{dI}{dt} = \left(\frac{\sigma lc}{L_e}N_2 - \frac{\gamma c}{L_e}\right)I. \tag{3.13}$$

~~But~~ However, since the number of the photons in the cavity is proportional to the intensity, if we compare eq.(3.13) with (3.1b), we observe that

$$B = \frac{\sigma lc}{V_a L_e} \tag{3.14}$$

and

$$\tau_c = \frac{L_e}{\gamma c} \tag{3.15}$$

Putting

$$V = \frac{L_e}{l}V_a\ , \tag{3.16}$$

eq.(3.14) becomes

$$B = \frac{\sigma c}{V}\ . \tag{3.17}$$

The quantity V is called the mode volume within the laser cavity. Finally the rate equations used in the next subsections to infer the laser dynamics are written as:

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{\tau} - \frac{\sigma c \phi N l}{L_e V_a} \tag{3.18}$$

$$\frac{d\phi}{dt} = \frac{\sigma c \phi N l}{L_e V_a} - \frac{\phi}{\tau_c} \tag{3.19}$$

3.3 Matlab Simulation of The Rate Equations

Matlab was used to simulate equations (3.1c) and (3.1d) using the codes below:

```
% THE FUNCTION DEFINING THE DIFFERENTIAL EQUATIONS FOR
%       POPULATION AND THE PHOTON


function dNdt = eqN(t,N)
global gamma c n l L I e Va sigma taou          % the simulation parameters


 Le = L + (n-1)*l; V = Le*Va/c; Rp = I/(e*V);
 B = sigma/V; taouc = Le/gamma*c;


dNdt = zeros(2,1);                    % this creates an array of two rows and one
                                      % column for the solution
                                    % of the differential equations
 % THE DIFFERENTIAL EQUATIONS
```

```
dNdt(1) = Rp – B*N(2)*N(1) – N(1)/taou;          % the differential equation for
                                                  % population
dNdt(2) = (B*Va*N(1) – 1/taouc)*N(2);            % the differential equation
                                                        %for the photon
% THE CODE FOR SOLVING AND PLOTING THE DIFFERENTIAL
%EQUATIONS ABOVE


% Declaring the parameters
global gamma c n l L I e Va sigma taou
gamma = 0.443; c = 3*10^8; n = 3.6; l = 8*10^-3; L = 6*10^-3; I = 3*10^-6;
e = 1.6*10^-19; Va = 22*10^-10; sigma = 3.6*10^-14; taou = 10^-10;
% THE INITIAL CONDITIONS AS WELL AS THE FINAL TIME OF
%SIMULATION
finaltime = 5*10^-18;
N10 = 0;         % The initial condition for carrier density is zero
N20 = 1;         % The initial condition for the photon is 1
% SOLVING THE DIFFERENTIAL EQUATION
[t,N] = ode45('odeq', [0 finaltime], [N10 N20]);
% PLOTING THE SOLUTION
plot(t,N(:,1), t, N(:,2)/Va, 'red'), axis([0 3*10^-18 -2*10^14 3*10^14])
xlabel('t'), ylabel('carrier and photon density'), title('Laser dynamics')
```

Symbol	**Parameter**	**Value**
e	**Electronic charge**	**1.6e-19C**
γ	**Single pass loss of the cavity**	**0.443**
c	**Speed of light**	**3e8m**
N		**3.6**
L	**Length of resonator**	**6e-3m**
I	**Pumping current**	3e-6A
σ	**Absorption cross section**	2.6e-14m^2
τ	**Effective lifetime of the upper-laser level**	10e-10s
V_a	**Volume of the mode in the active medium**	22e-10m^3
l	**Length of active medium**	3e-6m
L_e	**Optical length of the resonator**	

CHAPTER 4

The Results Obtained and Discussions

4.1 The Plots of the Dynamics

The code in chapter 3 was used to generate the Fig. (4.1) below:

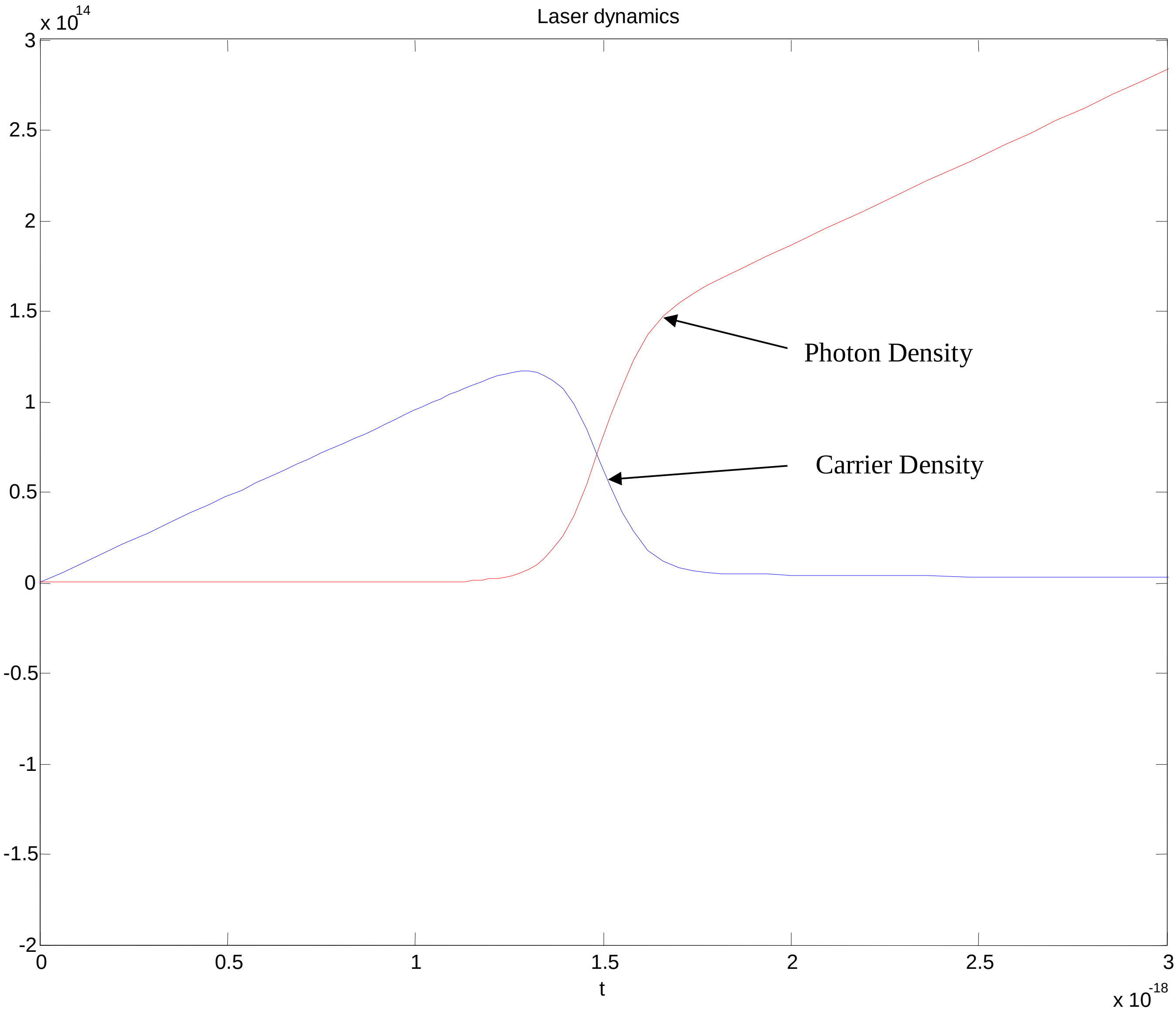


Figure 4.1: Photon and Carrier Density

4.2 The Carrier density

To obtain the carrier density plot below, the same code in chapter 3 was used with only the plot replaced by the plot code;
plot(t,N(:,1)), xlabel('t'), ylabel('carrier density')

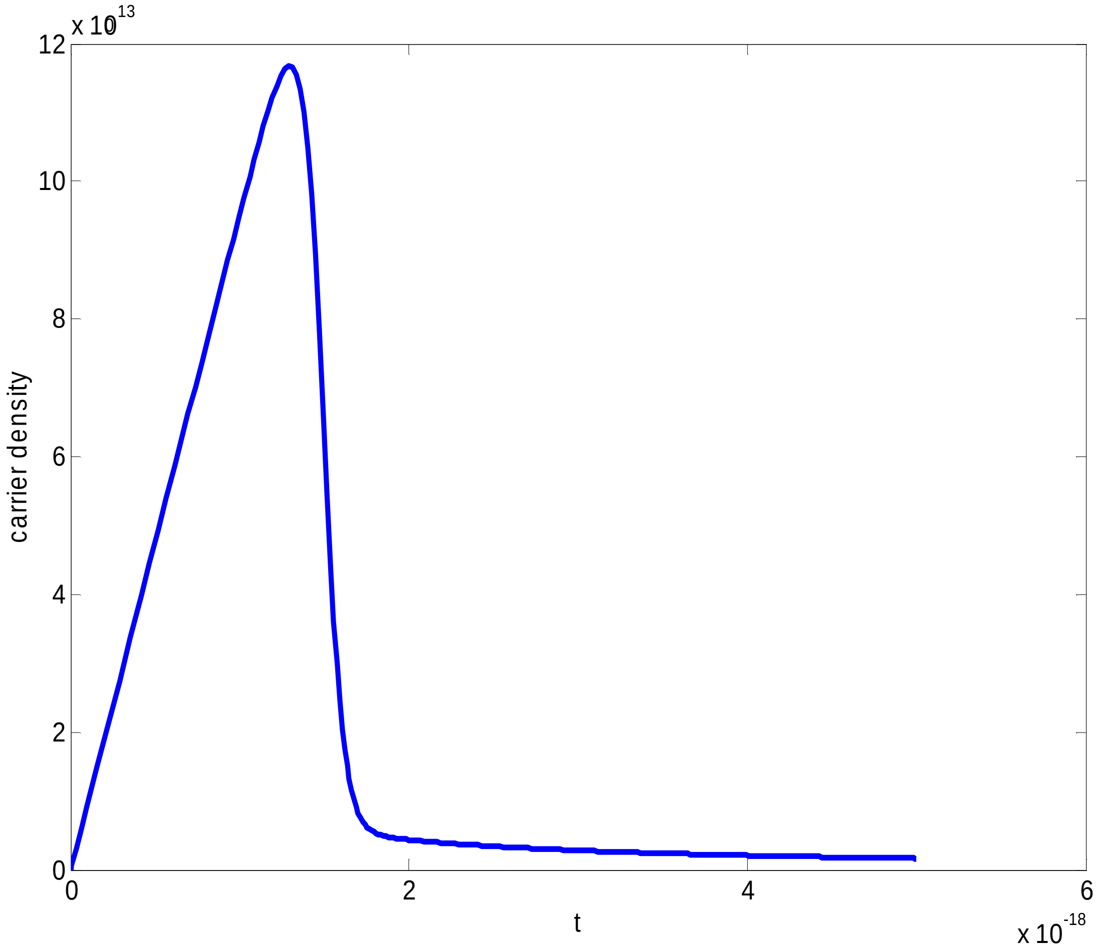


Fig. (4.2): The Plot of Carrier Density with time

4.3 The Photon Density

To obtain the plot for the photon below, thesame code in chapter 3 was used with only the plot code replaced by the code:

```
plot(t,N(:,2),'red'), xlabel('t'), ylabel('photon'):
```

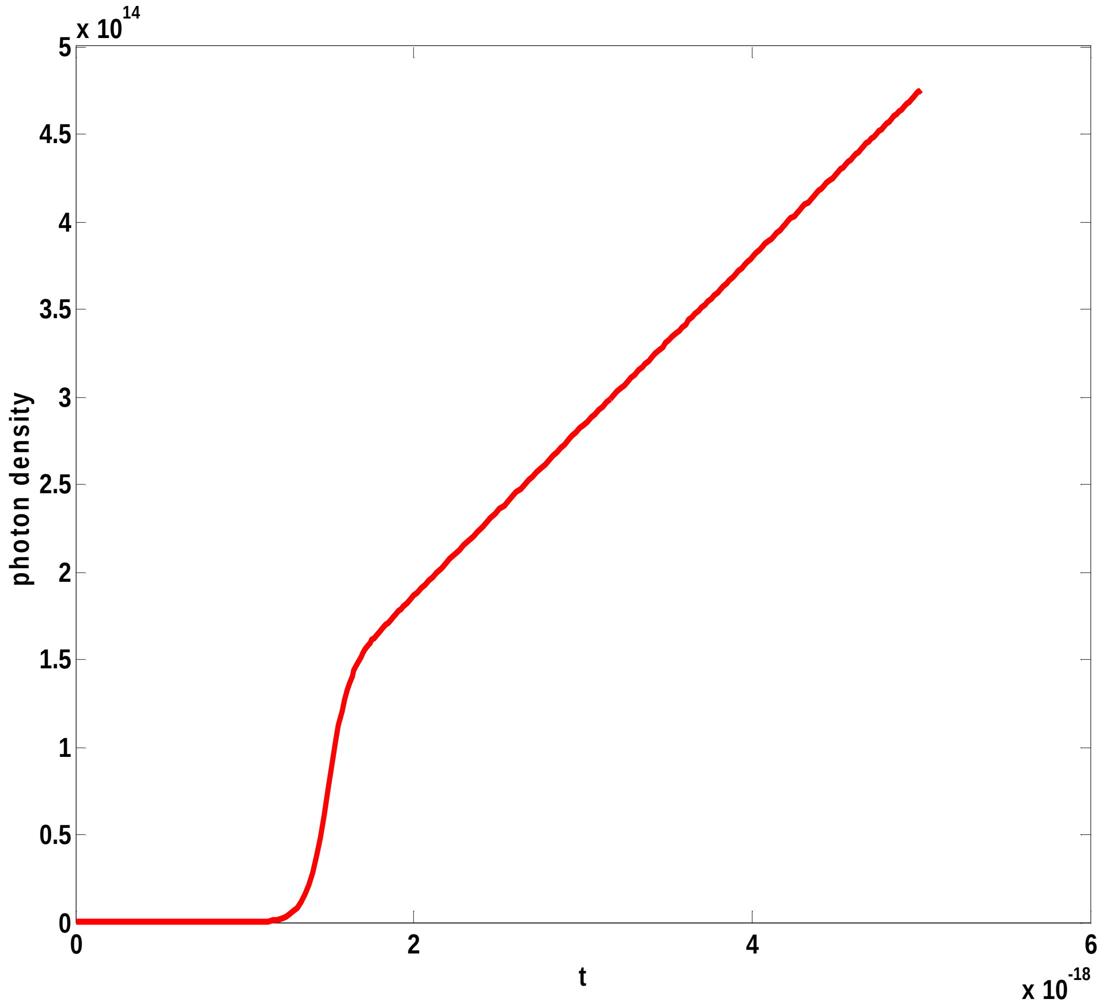


Fig. (4.3): The plot of photon emitted with time

4.4 Discussion of Results

Figure 4.1 gives a clear picture of the transient behaviour of the laser. The initial carrier density was set at zero. As the pumping commenced, the carrier density increased with time. This is what is expected as electrons were excited to the conduction (higher) band of the active medium. It is interesting to note that the photon density remains constant as the carrier density increased and before it reaches its threshold value requires for amplification of stimulated emission. Before the threshold, the major photon interaction with the active medium is absorption and spontaneous emission. At threshold value of carrier density, the stimulated emission dominates and the photon density increases rapidly while the carrier density decreases. The surge of stimulated emission begins to rapidly deplete the carrier density, causing it to decrease. The photon density got amplified with increase in stimulated emission. The carrier density continues to deplete while the photon density continues to increase due to the dominant stimulated emission of photon.

The result obtained shows that the optical feedback is suppressed. This account for the reason why optical oscillation as would be observed with optical feedback is not obtained (see Fig. 5 below). Thus the amplification of photons observed is mainly due to the domination of the stimulated emission of the photon attained to due to the pumping and not optical feedback from the laser cavity and hence the qualitative transient behaviour of “free running semiconductor laser” is observed.

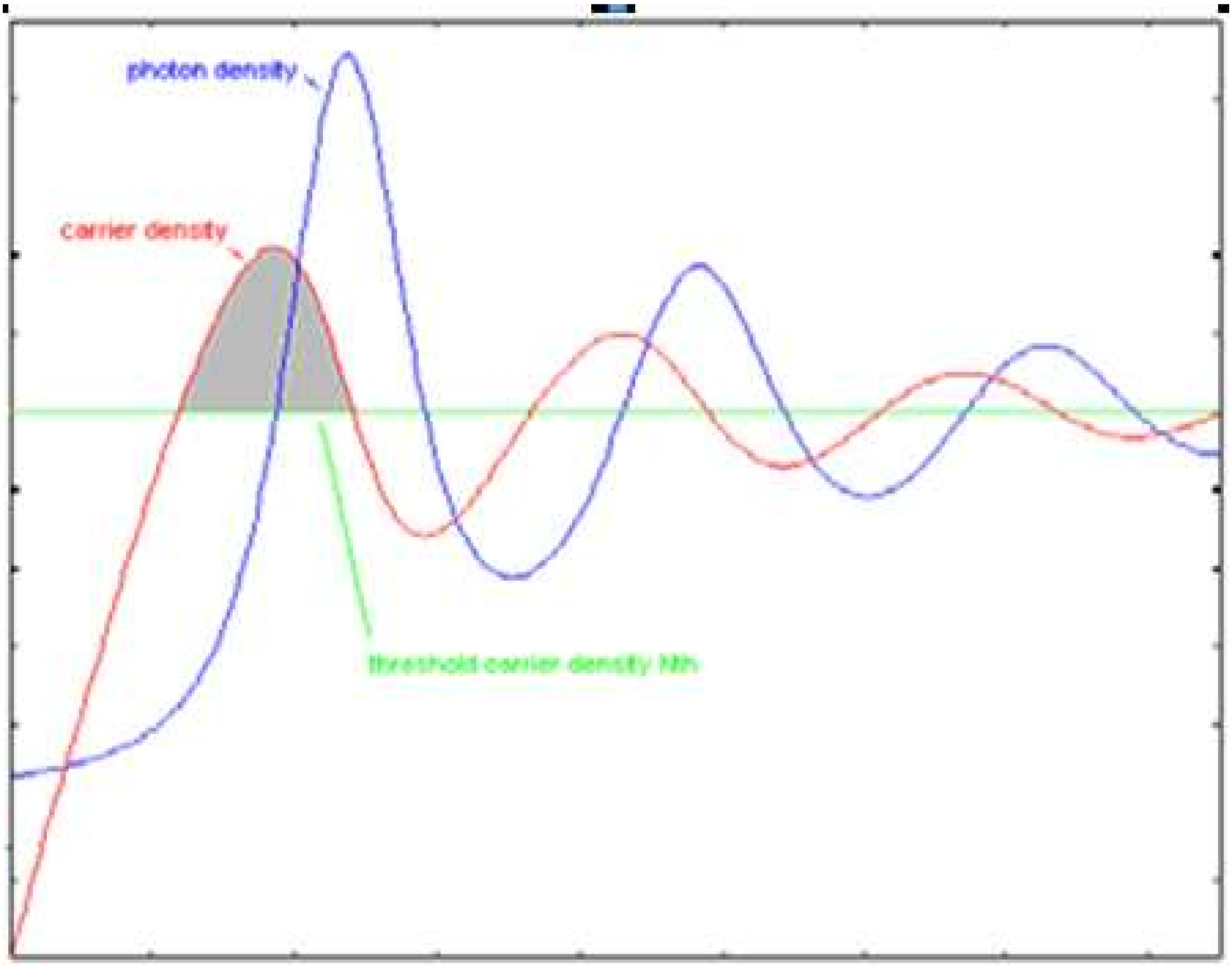


Figure 4.4: Laser dynamic[5]

Conclusion

The rate equations describing the density of carrier undergoing transition and the photon generated and dissipated were simulated. It was observed from the results that if laser is operated with the suppression of positive optical feedback, the photons get amplified due to stimulated emission but no oscillation of the output is obtained. It can therefore be concluded that 'laser' without a positive feedback is rather an amplifier. If the photon falls in the microwave region, it's called a maser amplifier and a laser amplifier if it falls in the optical region.

References

[1] "Laser Diode", http://en.wikipedia.org/wiki/laser_diodes, [Online]

[2] O. Svelto, "Principles of Lasers", Plenum Press, New York, 1998

[3] http://wikipedia.org/wiki/laser_diode_rate-equations, [Online]

[4] R. N. Hall et al, "Coherent light emission from GaAs junctions", phys. Rev. Lett., vol.9, pp.366-367,1962

[5] M. I. Nathan et al, "Stimulated emission of radiation from GaAs p-n junctions", Appl. Phys. Lett., vol.1, pp. 62-63, 1962

[6] N. Holanyak Jr and A. F. Bevacqul, "Coherent (visible) light emission for GasAsP junctions", App. Phys. Lett., vol.1, pp. 82-83, 1962

[7] T. M. Quist et al, "Semiconductor maser of GaAs", Appl. Phys. Lett., vol.1, pp. 91-92, 1962

[8] R. Salathe, C. Voumard, H. Weber, "Rate equation approache for diode lasers", Opto-electronics, vol.6, pp. 451-456, 1974

[9] A. N. Pisarchuk and F. R. Ruiz-Oliveras, "Dynamics of Semiconductor Laser with Two External Cavities", PIERS ONLINE, vol.3, No.4, 2007

[10] T. Schaer et al, " A Dynamic Simulation Model for Semiconductor Laser Diodes", ICUE report

[11] A. Gavrielides et at, “Coexisting periodic aattractors in injection-locked diode lasers”, Quantum Semiclass. Opt.9, pp. 785-796, 1997

[12] S. M. Wieczorek, B. Krauskopf and D. Lenstra, “A unifying view of bifurcations in a semiconductor laser subject to optical injection”, Opt. Comm. 172, pp. 279-295, 1999

[13] T. B. Simpson et al, “Nonlinear dynamics induced by external optical injection in semiconductor lasers”, Qaunt. Semiclass. Opt.9, pp. 765-784, 1997

[14] J. E. Carroll, J. Whiteaway, D. Plumb, “Distributed feedback semiconductor asers”, IEE and SPIE Optical Engineering Press, 1998

[15] D. Sands, “Diode Lasers”, Bristol: Institute of Physics, 2005

[16] Britney Spears Guide to Semiconductor Physics, [Online], http://britneysspears.ac/lasers.htm

[17] P. W. Milonni and J. H. Eberly, “Laser Physics”, John Wiley & Sons, Inc., New Jersey, 2010

[18] J. T. Verdeyen, “Laser Electronics”, 3rd ed. Prentice Hall, New Jersey, 1995

[19] A. E. Siegman, “Lasers”, University of Science Books, Califonia, 1986

[20] P. Brosson, “Semiconductor Lasers and integrated devices”, ESO, 2005

[21] S. D. Kasap, “Optoelectronics”, Prentice Hall, 1999

[22] D. McMahon, “MATLAB Demystified”, McGraw-Hill, New York, 2007