

# **The Design of an On-Chip Silicon Photonic Diode**

**M.Sc. Thesis by**

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## **Certification**

AFRICAN UNIVERSITY OF SCIENCE AND TECHNOLOGY, ABUJA-NIGERIA  
DEPARTMENT OF THEORETICAL PHYSICS

This is to certify that the work described in this thesis entitled **The Design of an On-Chip Silicon  
Photonic Diode.**

By

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Was carried out in the Theoretical Physics Stream under the supervision of:

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Dr. Omololu Akin-Ojo

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(All ideas and concepts taken from other sources, such as books, journal articles are fully referenced herein.)

## Abstract

This work presents numerical calculation of electromagnetic waves in unidirectional on-chip silicon optical diodes. An original optical diode, designed by Wang *et. al.* [*Opt. Express.* 19, 26948-26955 (2011)], is based on breaking of spatial inversion symmetry and the directional band gap difference of two 2D photonic crystals comprising a hetero-junction structure. The dimensions of our structure are however different from those of Wang *et. al.* The electromagnetic waves in the diode obey the Maxwell's equations which were solved with appropriate boundary conditions using the finite difference time domain as implemented in the MEEP software. MEEP is an acronym for M.I.T. Electromagnetic Equation Propagation. Our solutions show the existence of a distinct unidirectional isolation effect in the designed hetero-junction slab. The band structures for the lowest frequency mode of the transverse electric fields in the bulk of each of the 2D crystals comprising the hetero-junctions were also determined by solving the wave equation in frequency space. The MPB (M.I.T. Photonic Bands) software was used. The band structures reveal directional band gaps which are responsible for the optical isolation property of the composite hetero-junction.

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# Chapter 1

## 1.1 Introduction

Just like an electrical diode which allows current to flow in one direction only, a photonic diode allows light to propagate in one direction but blocks counter propagating photons. Why photonic diode? We are living in the information age where there is incessant clamor for faster information processing, more efficient telecommunication, miniaturization of electrical appliances for easy mobility, and availability of quantum computer for scientific purposes. These “basic” needs highlighted above can only be met in the realm of photonics. That is, there is a need to migrate from the use of electrical circuits, which formed the backbone of our information age, in our designs to opto-electrical circuit, the future of our quantum computers. In other words, imagine a world where electrons, which are the active carrier of signal in this age, are replaced with photons. Certainly there would be accelerated growth in information processing, telecommunication, etc. For the new generation of circuits to be functional, there is a need to begin the design of the corresponding electrical circuit components for photonic circuits. One of these components is a diode; in the new case, it is the photonic diode. The successful implementation of this component would serve as a platform for the design of upcoming integrated circuits. Over the years, various schemes have been proposed to construct compact and highly efficient all-optical diodes. Some of the schemes that have been proposed cannot be implemented on a silicon chip because either they require magneto-optical materials or strong magnetic field. We are all aware of the consequence of using magnetic field in systems like computers. Definitely it would affect the storage “organs”. In addition some of these



schemes depend on non-linear media, thus they require high electric field, meaning they are not compatible with complementary metal-oxide-semiconductor (CMOS).

Over the years, all-optical diodes have captured much attention for their potential application in integrated photonic circuits, all optical signal processing, and telecommunications in the future. Various schemes have been proposed to construct compact and highly efficient all-optical diodes [2-9]. Scalora *et. al.* suggested using one-dimensional nonlinear photonic crystals with a spatial graduation in the linear refractive index to realize unidirectional propagation of signals in 1994 [1]. Feise *et al.* demonstrated the bi-stable diode action in an asymmetric multilayer structure consisting of left-handed materials [3]. Philip *et al.* studied numerically and verified experimentally the passive all-optical diode behavior utilizing asymmetric nonlinear absorption [4]. Configurations of a photonic crystal (PC) waveguide with embedded nonlinear PC defects and with asymmetric defect pair are found to display nonreciprocal effects as well [5–8]. However, the achieved transmittance contrasts of all-optical diodes in the schemes mentioned above are all lower than 100. In order to improve the transmission contrast, very recently, Xue *et. al.* suggested using one-dimensional PC-metal heterostructures to achieve highly efficient all-optical diode action [10], and Hu *et. al.* proposed a strategy for obtaining ultrahigh-contrast all-optical diodes based on tunable surface Plasmon polaritons [11]. However, the transmission contrast of the former one is only about 124, still not high enough. For the latter one, although the transmission contrast ratio can be as high as 2166, the forward transmittance is only 0.06, which is too low to be used in the practical applications. Recently, Cai *et. al.* studied the transmission property of two directly coupled nonlinear defects and proposed that it can function as an all-optical bi-stable switch or diode with high transmission contrast if we deliberately and properly misalign the resonant frequencies of the two defects [12]. The application of this kind of configuration in optical switching has already been demonstrated [13]. Cai *et. al.* designed photonic crystal

structures composed of three or four nonlinear defects, and showed that all-optical diodes can be realized for this configuration and the transmission contrast can be further increased by selecting the resonant frequencies of the constitutional defects properly [14]. Li *et. al.* reported a method for making unidirectional on-chip optical diode based on directional band gap difference of two 2D photonic crystals, and confirmed the existence of the isolation effect [16].

Obtaining on-chip optical signal is a primary problem in the integrated photonics. The clamor to overcome this problem is increasingly becoming urgent, specifically with the emergence of silicon nano-scale photonics.

Until now, there have been no techniques that account for complete on-chip signal isolation using materials or processes that are compatible with silicon CMOS processes. Optical isolation can also be achieved using dynamic photonic structures. Wang *et. al.* proposed all-optical diode based on a moving photonic crystal generated in a three level electromagnetically induced medium [18]. Zongfu and Shanhui showed that nonreciprocal isolation with linear and broadband can be achieved by spatial-temporal refractive index modulation [19]. This work demonstrates that on-chip isolation can be achieved with static photonic structures in standard linear and non-magnetic material systems that are widely used for integrated optoelectronics applications. We used the photonic structure designed by Chen *et. al.* in the paper “On-chip optical diode based on Silicon photonic crystal” [16]. However the dimensions of our structure are different. Our work entails numerical simulation; calculation of transmittance and plotting of band structure of the proposed linear and non-magnetic photonics structures that support nonreciprocal optical isolation. The proposed structure is:

On-chip optical isolators based on the directional band gap difference of two dimensional square-lattice photonic crystals comprising a hetero-junction structure and the breaking of the spatial

inversion symmetry. In this structure, effort would be made to simulate the field patterns, calculate the forward and backward transitivity of the isolator and obtain the band structure.

## **1.2 OVERVIEW OF THE THESIS**

1) Chapter two reviews basic concepts of non-reciprocal optical devices used throughout the thesis: band gap, signal contrast, the Maxwell's equations as they apply to dielectric crystal materials, and an introduction to the MEEP software used in the solution of Maxwell's equations. That is, it contains the basic equations governing electromagnetic wave interaction with dielectric functional materials.

2) Chapter three demonstrates the solution of Maxwell's equations as it applies to lossless dielectric structures via a discretization approach and by the use of a (plane wave) basis set.

3) In Chapter four, the results obtained from simulations and calculations are presented and discussed.

A conclusion is also given in this chapter.

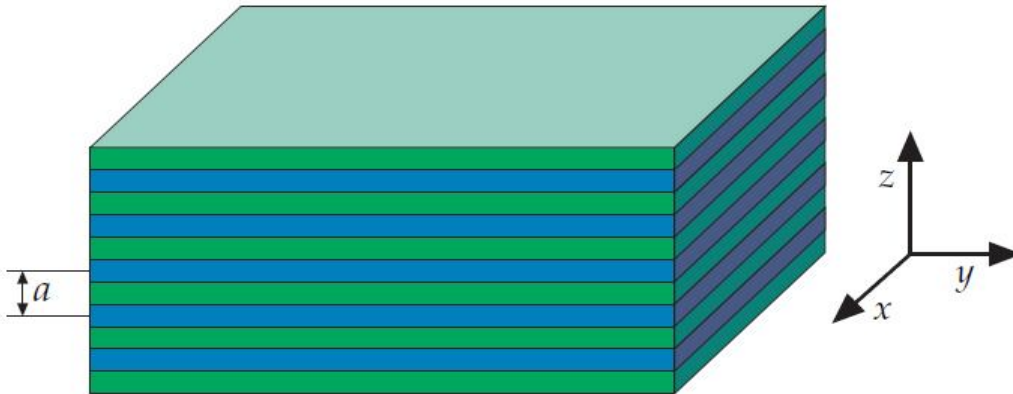
## Chapter 2

This chapter will briefly introduce some of the most important concepts used throughout this thesis. It will serve as a quick reference to readers that are not familiar with these concepts.

### 2.1 Bandgap

When an electromagnetic wave propagates through a dielectric structure (i.e. a photonic crystal), the crystal affects propagation by allowing waves of certain wavelengths to pass while blocking others. A range of blocked wavelengths is called a photonic band gap. For a given frequency range, if a photonic crystal prohibits the propagation of electromagnetic waves of any polarization travelling in any direction from any source, then the crystal is said to have a complete photonic band gap. The optical properties of a dielectric structure may be familiar but by expressing the results in terms of band structures and band gaps, we can observe some phenomena such as omnidirectional reflectivity, optical diode effect, narrow-band filter, etc. We can demonstrate the physical origin of photonic band gap in a periodic dielectric structure using a multilayer film. This multilayer film could be alternating layers of GaAs/GaAlAs or GaAs/Air, etc. The analysis for the band structure of this system can be generalized to two or three dimensional systems.

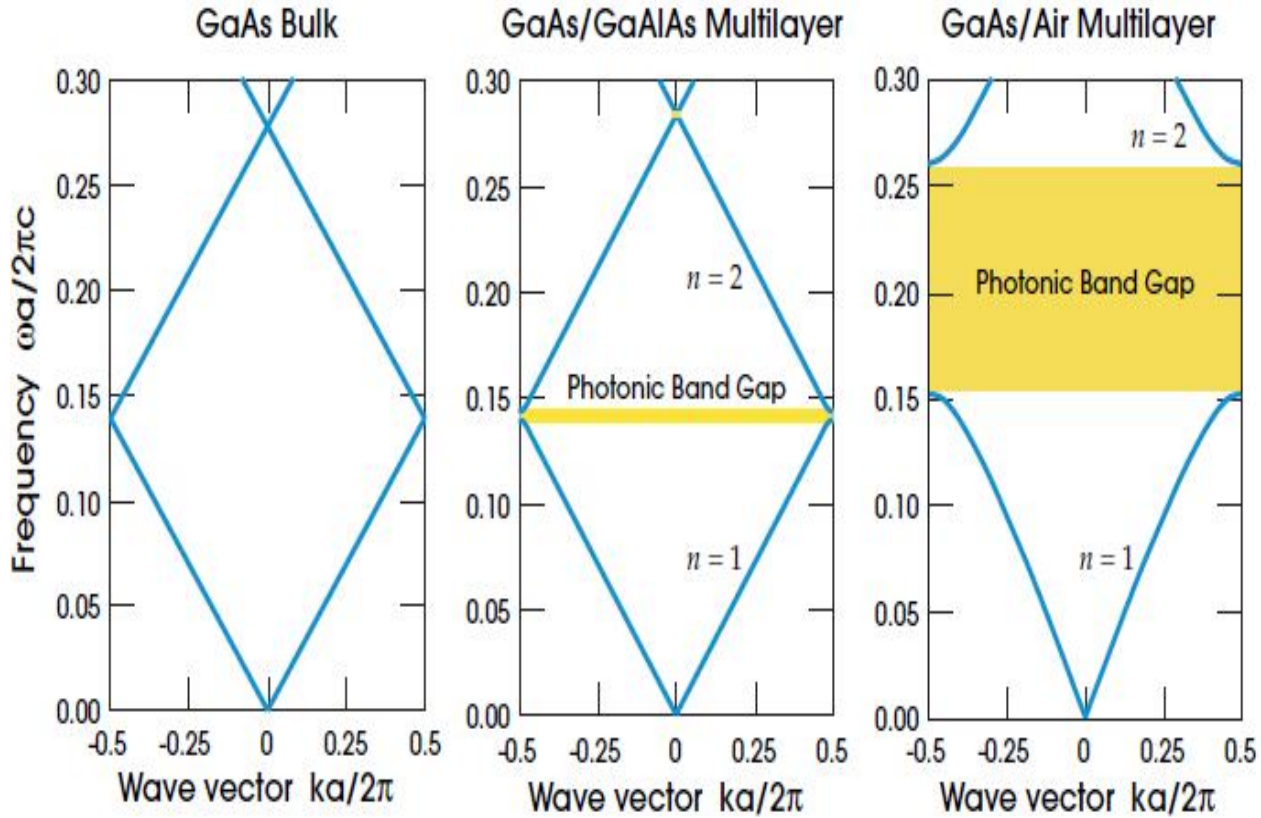
## THE MULTILAYER FILM WITH ON-AXIS PROPAGATION



**Fig1: One dimensional photonic crystal with dielectric function  $\epsilon(z)$ . (source: Joannopoulos *et al.* 2008)**

A multilayer film (as shown in Fig1 above) consists of alternating layers of materials with different dielectric constants. It is homogenous along the  $xy$  plane, and periodic in  $z$ -direction. For instance, at normal angle of incidence, if waves propagate in the  $z$ -direction of these layers, then  $\mathbf{k} = (0, 0, k_z)$  and  $k_{\parallel} = 0$ . Where  $k_z$ ,  $k_{\parallel}$  and  $n$  are the wave vector in the  $z$ -direction, wave vector in the plane, wave number, and are used to classify modes. By plotting frequency  $\omega_n$  against wave vector  $k$  for three different multilayer films, we get the band structure in Fig. 2.

In Fig. 2, the plot on the left is for a system with a homogenous dielectric medium. The center plot is for a structure with alternating dielectric constants of 13 and 12; the right hand plot is for a system with dielectric constants 13 and 1.



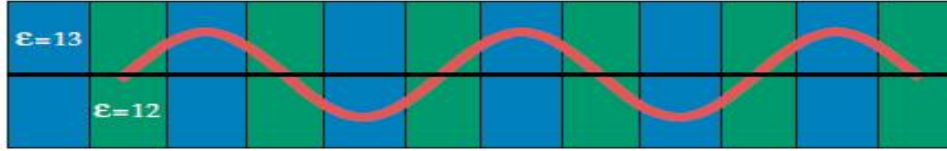
**Figure 2: Photonic band structure for on axis propagation as computed for three different multilayer films. (Source: Joannopoulos *et al.* 2008)**

For a given system with two different dielectric constants, the ratio of the higher to the lower dielectric constant is called the dielectric contrast. The center plot with dielectric contrast of approximate one looks like the homogenous case (left plot) with one major difference: there is a gap in frequency between the upper and lower branches of the lines. The right-hand plot shows that the gap widens considerably with a higher dielectric contrast.

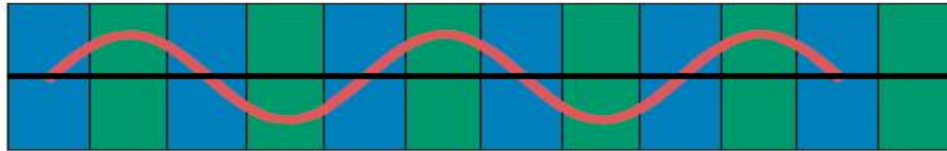
What is the physical origin of photonic bandgaps? Through the electric field mode profiles around the gap, the physical origin of photonic band gap can be understood. For  $k=\pi/a$ , using the center plot in Fig. 2, which can be considered as a perturbation of the homogenous system, the modes have a wavelength of  $2a$ , twice the lattice constant. To center a mode of this type, there are two ways to do it:

the node can be aligned in each low- $\epsilon$  layer, as indicated in fig 3(a), or in each high- $\epsilon$  layer, as in fig 3(b).

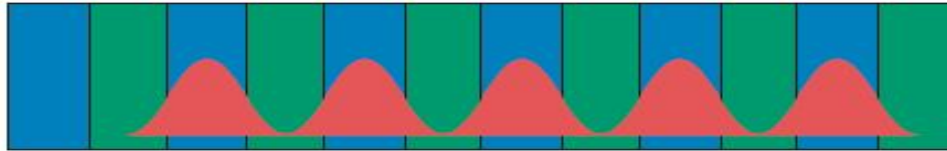
(a)  $E$ -field for mode at top of band 1



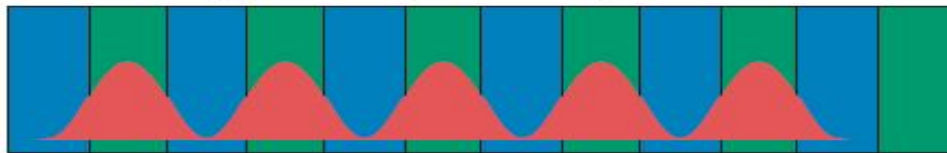
(b)  $E$ -field for mode at bottom of band 2



(c) Local energy density in  $E$ -field, top of band 1



(d) Local energy density in  $E$ -field, bottom of band 2



**Figure3: The modes associated with the lowest band gap of the band structure plotted in the center panel of figure 2, at  $K=\pi/a$ . (a) electric field of band 1;(b) electric field of band2; (c) electric-field energy density  $\epsilon |\mathbf{E}|^2 / 8\pi$  of band 1; (d) electric-field energy density of band 2. In the depiction of the multilayer film, blue indicates the region of higher dielectric constant ( $\epsilon=13$ ). (source: Joannopoulos *et al.* 2008)**

According to Joannopoulos *et al.* [15], “low-frequency modes concentrate their energy in the high- $\epsilon$  regions, and the high-frequency modes have a larger fraction of their energy (although not necessarily a majority) in the low- $\epsilon$  regions”. This explains why there is a frequency difference between the two cases. Just below the gap, the mode has more of its energy concentrated in the  $\epsilon=13$  regions as indicated in Fig .3c, and gives it a lower frequency than the next band, which has most of its energy is

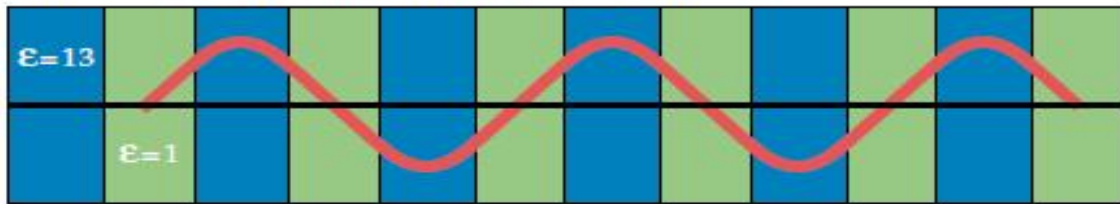
in  $\epsilon=12$  regions as shown in Fig .3 (d). For configuration with a higher contrast, it can be found that field energy for both bands is primarily concentrated in the high- $\epsilon$  layers, the first band being more concentrated than the second. The fields are graphically illustrated in Fig.4, corresponding to the right-hand plot of Fig. 2. The gap results from the difference in field energy location. In conclusion, in one dimension, a gap usually occurs between every set of bands, at either the Brillouin zone's edge or its center [15]. This is shown for the band structure of a multilayer film in Fig. 5. As a point of emphasis, bands gaps always appear in a one dimensional photonic crystal for any dielectric contrast. The smaller the dielectric contrast, the smaller the gaps.

## 2.2 The Size of the Band Gap

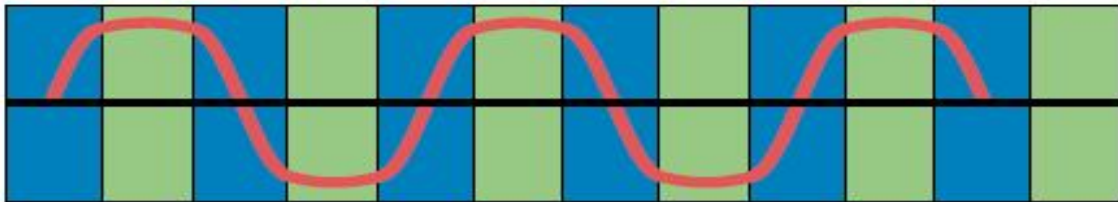
The size of a photonic band gap can be characterized by its frequency width  $\Delta\omega$ , but this is not a good practice because Maxwell Equations are scale invariant. That is, if the crystal were increased by a factor "s", the corresponding band gap would have a width  $\Delta\omega/s$ . A better useful characterization is the "gap-midgap ratio", which is independent of the length of the crystal. Letting  $\omega_m$  be the frequency at the middle of the gap, the gap-midgap ratio is defined as  $\Delta\omega/\omega_m$  and expressed as a percentage (e.g., a "30% gap" refers to a gap-midgap ratio of 0.3). This quantity remains the same, even if the system is scaled up or down. Therefore, it should be noted here that, the size of a gap refers to, generally, the midgap ratio [15]. The basic concepts of periodic systems so far emphasized can be equally applied to more complicated two or three-dimensional structures.



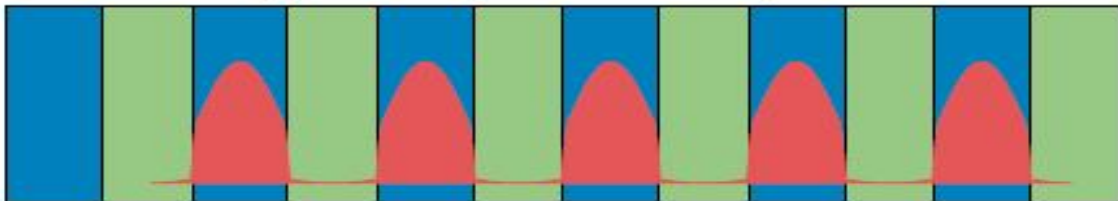
(a)  $E$ -field for mode at top of band 1



(b)  $E$ -field for mode at bottom of band 2



(c) Local energy density in  $E$ -field, top of band 1



(d) Local energy density in  $E$ -field, bottom of band 2

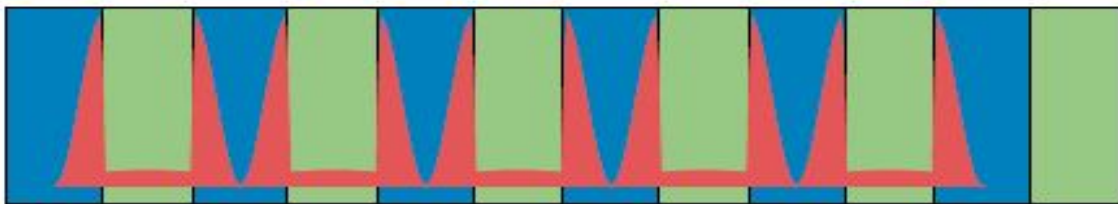


Figure 5: The modes associated with the lowest band gap that is shown in the band structure of the right-hand plot of fig 2, at  $k=\pi/a$ . This case shares similarity with that of fig 3, but the dielectric contrast is larger. The blue and green regions correspond to dielectric constant of 13 and 1, respectively. (source: Joannopoulos *et al.* 2008)

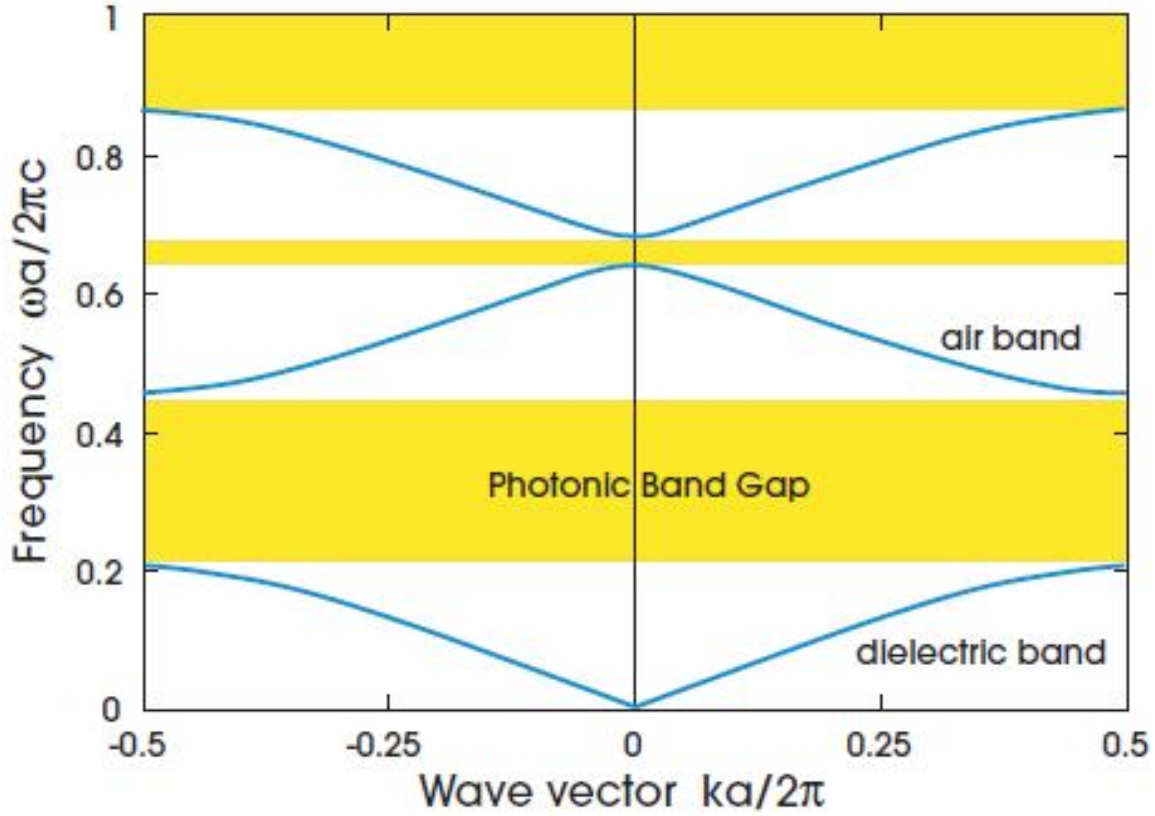


Figure 5: The photonic band structure of a multilayer film with lattice constant “a” and alternating layers of different widths. The width of the  $\epsilon=13$  layer is  $0.2a$ , and the width of the  $\epsilon=1$  layer is  $0.8a$ . (Source: Joannopoulos *et al.* 2008).

### 2.3 Signal Contrast (S)

$$S = \frac{T_f - T_b}{T_f + T_b} \quad (2.1)$$

$T_F$  and  $T_B$  denote the forward and backward transmission coefficient, respectively.

A diode with  $S=1.0$  is a perfect isolator. Hence, the closer the signal contrast of a photonic diode is to one, the more efficient it is as a unidirectional optical device.

## 2.4 Maxwell's equations

Maxwell's equations describe the interactions of electric ( $\mathbf{E}$ ) and magnetic ( $\mathbf{H}$ ) fields with one another and with matter and sources. In particular, the equations for the evolution of the fields are [21]:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \nabla \cdot \mathbf{B} = \mathbf{0} \qquad (2.2)$$

$$\frac{\partial \mathbf{D}}{\partial t} = -\nabla \times \mathbf{H} - \mathbf{J} \qquad \nabla \cdot \mathbf{D} = \rho \qquad (2.3)$$

Where  $\mathbf{D}$  is the displacement field,  $\mathbf{J}$  is the current density (of electric charge),  $\mathbf{B}$  is the magnetic flux density (also called the magnetic induction), and  $\mathbf{H}$  is the magnetic field. For linear isotropic media, the following constitutive relations connect  $\mathbf{D}$  with  $\mathbf{E}$  and  $\mathbf{B}$  with  $\mathbf{H}$ :

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{D} = \varepsilon \mathbf{E} \qquad (2.4)$$

where  $\varepsilon$  is the dielectric constant and  $\mu$  is the magnetic permeability of the medium.

More generally,  $\varepsilon$  depends not only on position but also on frequency (material dispersion) and on the field  $\mathbf{E}$  itself (nonlinearity), and may include loss or gain.

## 2.5 MEEP

MEEP (an acronym for M.I.T. Electromagnetic Equation Propagation) is a program for solving electromagnetic related problems. It uses the finite-difference time-domain (FDTD) scheme [20] for solving the Maxwell's equations with the associated boundary conditions. FDTD is a well known method in which space is discretized and the fields are evolved in time using discrete time steps—as the grid and the time steps are made smaller and smaller, this becomes a closer and closer approximation for the true continuous equations and one can simulate many practical problems

essentially exactly. MEEP is free and open source, developed by scientists at the Massachusetts Institute of Technology (MIT). [17]

## 2.6 MPB

As we will see in chapter three, under certain conditions, Maxwell's equations can be re-cast into an eigenvalue equation which can be solved for periodic structures by expanding the field as linear combinations of plane waves. The free and open software MPB (MIT Photonic Bands) was used in this thesis to obtain the eigenvalues of this equation. A plot of the eigenvalues against the corresponding wave vectors  $\mathbf{k}$  of the plane waves is called the band structures. For some structures, there are certain frequency ranges for which there are no eigenvalues for any  $\mathbf{k}$ . Such a range is called the bandgap as explained in section 2.1 above.

## 2.7 Units

In this thesis we shall work in dimensionless units. This is possible because Maxwell's equations are scale invariant (i.e the equation look the same if  $\vec{r} \rightarrow \gamma\vec{r}$  and  $t \rightarrow t\gamma$ ). Furthermore, most of the quantities we are interested in (e.g, the transmission coefficient which is the ratio of the transmitted to the incident intensities, or the contrast ratio) are ratios anyway, and the units end up "canceling out" so that they are dimensionless. In particular, we shall choose a characteristic length scale "a" and set c, the speed of light, to unity. Lengths will be quoted in units of "a" and, since, frequency  $f=c/\lambda=c/a$  ( $a/\lambda$ ), frequencies will be quoted as ( $a/\lambda$ ) in units of ( $c/a$ ) where  $\lambda$  is the wavelength of waves involved.

## 2.8 Boundary conditions

In this work open boundary conditions are simulated, in which the boundaries absorb all incident waves on them but reflected none. This is implemented with perfectly matched layers (PML) in the

MEEP software. PML ,is technically,not a boundary condition-rather, it is a special absorbing material placed adjacent to the boundaries. PML is not real material, and designed to have zero reflections at its surface. It has zero reflections in the theoretical continuous system but, in the actual discretized system it has some reflections. As such, the PML is given some finite thickness in which the absorption gradually “turns on”[17].

### Chapter 3

This chapter introduces Finite Difference Time- Domain method for solving Maxwell's equations in isotropic media of lossless materials, because MEEP computes electromagnetic related problems using this technique (FDTD). The basic Maxwell's equations are:

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0 \dots\dots\dots(3.01)$$

$$-\frac{\partial \vec{D}}{\partial t} + \vec{\nabla} \times \vec{H} = \vec{J} \dots\dots\dots(3.02)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \dots\dots\dots(3.03)$$

$$\vec{\nabla} \cdot \vec{D} = \rho \dots\dots\dots(3.04)$$

For a linear, isotropic and non-dispersive medium, the following relations hold:

$$\vec{B} = \mu \vec{H} \dots\dots\dots(3.05a)$$

$$\vec{D} = \epsilon \vec{E} \dots\dots\dots(3.05b)$$

Explicitly , equations (3.01) and (3.02) in 3D are :

$$-\frac{\partial B_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \dots\dots\dots(3.06)$$

$$-\frac{\partial B_y}{\partial t} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \dots\dots\dots(3.07)$$

$$\frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \dots\dots\dots(3.08)$$

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_x \dots\dots\dots(3.09)$$

$$\frac{\partial D_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - J_y \dots\dots\dots(3.10)$$

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z \dots\dots\dots(3.11)$$

### 3.1 Maxwell's Equations in Two Dimensions

If the field is uniform along z-axis, then partial derivatives of the fields' components with respect to z equal to zero. If in addition, there are no sources ( $\rho=0, \vec{J} = 0$ ) and  $\epsilon$  and  $\mu$  are constants, then equations (3.05) to (3.11) yield two sets of equations.

$$\text{(from 3.08)} \quad \mu \frac{\partial H_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \dots\dots\dots(3.12)$$

$$\text{(from 3.09)} \quad \epsilon \frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} \dots\dots\dots(3.13)$$

$$\text{(from 3.10)} \quad \epsilon \frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \dots\dots\dots(3.14)$$

which involve only  $H_z, E_x$  and  $E_y$  and

$$\text{(from 3.06)} \quad -\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} \dots\dots\dots(3.15)$$

$$\text{(from 3.07)} \quad -\mu \frac{\partial H_y}{\partial t} = -\frac{\partial E_z}{\partial x} \dots\dots\dots(3.16)$$

$$\text{(from 3.11)} \quad \varepsilon \frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \dots\dots\dots(3.17)$$

which involve only  $E_z$  ,  $H_x$ , and  $H_y$  .

Observe that the first set of equations are completely decoupled from the second so that each set of equations can be solved separately. The solutions of the first set of equations with  $E_z=0$  (i.e. with zero electric field along the direction of propagation, z) are called transverse electric (TE) waves.

Similarly, transverse magnetic (TE) waves are the solutions of the second set of equations and with  $H_z=0$  (i.e. zero magnetic field along the direction of propagation).

Let us focus on the TE waves, since our structure in 2D has a bandgap only for the TE modes. We can solve equations (3.12) to (3.14) numerically following the discretization method by Yee [20]. In this scheme we discretized space and time and let  $F^n(i,j)$  denote any function of space and time  $F(i\Delta x, j\Delta y, n\Delta t)$  where  $i, j, n$  are integers,  $\Delta x, \Delta y, \Delta t$  are the grid spacings in the x,y and t ‘direction’ and  $x=i\Delta x$  ,  $y= j\Delta y$  and  $t= n\Delta t$  are the x and y coordinates and time, respectively, at which the function F is evaluated. Yee’s approach, which is a popular finite difference time domain (FDTD) scheme evaluates the partial derivatives at midpoints on the grid as follows .

$$\left. \frac{\partial H_z}{\partial t} \right|_{i+1/2, j+1/2}^{t=n\Delta t} \approx \frac{H_z^{n+1/2}(i+1/2, J+1/2) - H_z^{n-1/2}(i+1/2, j+1/2)}{\Delta t} \dots\dots\dots(3.18)$$

(i.e. by the central-difference approach)

$$\left. \frac{\partial E_x}{\partial y} \right|_{i+1/2, j+1/2}^{t=n\Delta t} \approx \frac{E_x^n(i+\frac{1}{2}, j+1) - E_x^n(i+\frac{1}{2}, j)}{\Delta y} \dots\dots\dots(3.19a)$$



$$\left. \frac{\partial E_x}{\partial t} \right|_{i+1/2, j}^{n+1/2} \approx \frac{E_x^{n+1}(i+\frac{1}{2}, j) - E_x^n(i+\frac{1}{2}, j)}{\Delta t} \dots\dots\dots(3.19b)$$

Similar expressions can be used to approximate the other derivatives in equations (3.12) to (3.14) to obtain the following equations after re-arranging:

From (3.12):

$$H_z^{n+1/2}(i+\frac{1}{2}, j+\frac{1}{2}) \approx H_z^{n-1/2}(i+\frac{1}{2}, j+\frac{1}{2}) - \frac{1}{z} \frac{\Delta \tau}{\Delta x} [E_y^n(i+1, j+\frac{1}{2}) - E_y^n(i, j+\frac{1}{2})] + \frac{1}{z} \frac{\Delta \tau}{\Delta y} [E_x^n(i+\frac{1}{2}, j+1) - E_x^n(i+\frac{1}{2}, j)] \dots\dots\dots(3.20)$$

From (3.13):

$$E_x^{n+1}(i+\frac{1}{2}, j) \approx E_x^n(i+\frac{1}{2}, j) + z \frac{\Delta \tau}{\Delta y} [H_z^{n+1/2}(i+\frac{1}{2}, j+\frac{1}{2}) - H_z^{n+1/2}(i+\frac{1}{2}, j-\frac{1}{2})] \dots\dots\dots(3.21)$$

From (3.14) :

$$E_y^{n+1}(i, j+\frac{1}{2}) \approx E_y^n(i, j+\frac{1}{2}) - z \frac{\Delta \tau}{\Delta x} [H_z^{n+1/2}(i+\frac{1}{2}, j+\frac{1}{2}) - H_z^{n+1/2}(i-\frac{1}{2}, j+\frac{1}{2})] \dots\dots\dots(3.22)$$

Where  $\Delta \tau = c\Delta t = \frac{\Delta t}{\sqrt{\mu \epsilon}}$  and  $z = \sqrt{\frac{\mu}{\epsilon}}$

The initial values  $E_x^0(i+\frac{1}{2}, j)$  and  $E_y^0(i, j+\frac{1}{2})$  are obtained from the incident wave while  $H_z^{-\frac{1}{2}}(i+\frac{1}{2}, j+\frac{1}{2})$  can be approximated from equation (3.12) by using the ‘backward difference’:

$$H_z^{-1/2}(i+\frac{1}{2}, j+\frac{1}{2}) \approx H_z^0(i+\frac{1}{2}, j+\frac{1}{2}) + \frac{1}{2z} \frac{\Delta \tau}{\Delta x} [E_y^0(i+1, j+\frac{1}{2}) - E_y^0(i, j+\frac{1}{2})] - \frac{1}{2z} \frac{\Delta \tau}{\Delta y} [E_x^0(i+\frac{1}{2}, j+1) - E_x^0(i+\frac{1}{2}, j)] \dots\dots\dots(3.23)$$

Subsequent values of  $E_x^{n+1}$ ,  $E_y^{n+1}$  and  $H_z^{n+1/2}$  can be obtained from equations (3.17) to (3.19).

With constant values of  $\epsilon$  and  $\mu$ , the FDTD scheme by Yee is stable provided [20]

$$\frac{1}{\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}}} > c\Delta t = \frac{\Delta t}{\sqrt{\mu \epsilon}}$$

Thus, for the chosen  $\Delta x$  and  $\Delta y$ , this criterion puts a restriction on  $\Delta t$ .

### 3.2 Calculation of the Transmission Coefficient

FDTD provide the efficient way of computing a broad-spectrum response by Fourier-transforming the signal to short pulse. The transmitted power is given by Poynting vector  $P(w)$ (in the normal direction)[19]

$$P(w) = \text{Re } \hat{n} \cdot \int E_w(x)^* \times H_w(x) d^2x \dots\dots\dots(3.24)$$

where  $\hat{n}$  is the unit normal vector. This is the transmitted power of the structure defined over a given plane. Here, the Fourier transform of  $E_w(x)$  and  $H_w(x)$  are accumulated for every point in the flux

plane via summation over the discrete time steps  $n$ : 
$$\tilde{f}(w) = \frac{1}{\sqrt{2\pi}} \sum_n e^{iwn\Delta t} f(n\Delta t)\Delta t \approx \frac{1}{\sqrt{2\pi}} \int e^{iwt} f(t)dt$$

[19].

MEEP does all these steps automatically. What the user does is to specify where, the frequency, and the region over which to compute the flux. For details on how to use MEEP to calculate the flux, check the appendix of this thesis. Although there are other calculation that can be done with MEEP,we limit the scope to the calculation of the transmitted power.

### 3.3 Calculation of the Photonic Band Structure

Dividing equation (3.01) by  $\epsilon$  and taking the curl gives:

$$-\frac{\partial(\vec{\nabla} \times \vec{E})}{\partial t} + \vec{\nabla} \times \left( \frac{\vec{\nabla}}{\epsilon} \times \vec{H} \right) = 0 \quad (\text{since there are no sources or sinks, } \vec{J}=0, \rho=0)$$

Substituting for  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$  from (3.01) yields

$$\frac{\partial^2 \vec{H}}{\partial t^2} + \vec{\nabla} \times \left( \frac{\vec{\nabla}}{\epsilon} \times \vec{H} \right) = 0 \dots\dots\dots(3.25)$$

where we have supposed that  $\mu$  is time/frequency independent. Assuming a sinusoidal time dependence of the magnetic field  $\vec{H}(\vec{r}, t) = \vec{H}(\vec{r})e^{-i\omega t}$  in equation (3.25), we obtain :

$$\vec{\nabla} \times \left( \frac{1}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \right) = \omega^2 \mu \vec{H}(\vec{r}) . \text{ Multiplying this by } \epsilon_o, \text{ gives } \vec{\nabla} \times \left( \frac{\epsilon_o}{\epsilon} \vec{\nabla} \times \vec{H}(\vec{r}) \right) = \omega^2 \epsilon_o \mu \vec{H}(\vec{r}) .$$

Materials like silicon suitable for complementary metal oxide silicon (CMOS) technology have  $\mu \approx \mu_o$  so that the equation above gives

$$\vec{\nabla} \times \left( \frac{1}{\epsilon_r(\vec{r})} \vec{\nabla} \times \vec{H}(\vec{r}) \right) = \left( \frac{\omega}{c} \right)^2 \vec{H}(\vec{r}) \dots\dots\dots(3.26)$$

where  $\epsilon_r(r) = \frac{\epsilon(r)}{\epsilon_o}$  is the relative permittivity of the medium and  $c = \frac{1}{\sqrt{\mu_o \epsilon_o}}$  is speed of light in the vacuum. Observed that equation (3.26) is an eigenvalue equation (just like the time-independent Schrodinger equation in Quantum mechanics). This can be seen clearly if define the operator

$\hat{\Theta} \equiv \nabla \times \left( \frac{\nabla}{\epsilon(r)} \times \right)$  so that equation (3.26) becomes:  $\hat{\Theta} \vec{H}(r) = \left( \frac{w}{c} \right)^2 \vec{H}(r)$  . This is the equation to be

solved to obtain the frequencies  $w(\mathbf{k})$  at different wave vectors  $\mathbf{k}$ . This is the band structure. Once  $\vec{H}(r)$  and ‘w’ are found,  $\vec{E}(r, t) = \vec{E}(r) e^{-iwt}$  can also be determined by plugging this expression into (3.02) to yield:

$$i w \epsilon \vec{E}(r) + \nabla \times \vec{H} = 0 \text{ i.e. } \vec{E}(r) = \frac{i}{w \epsilon} \nabla \times \vec{H} \dots\dots\dots(3.27)$$

Note that the solution of (3.26) and (3.27) must also satisfy the remaining Maxwell’s equations :

$\vec{\nabla} \cdot \vec{D} = \rho$  and  $\vec{\nabla} \cdot \vec{B} = 0$  which, with our current assumptions, reduce to  $\vec{\nabla} \cdot \vec{E} = 0 = \vec{\nabla} \cdot \vec{H}$  . This is called the transversality requirement since it reduces to  $\vec{H} \cdot \vec{k} = \vec{E} \cdot \vec{k} = 0$  in the case of the plane waves

$\vec{H} = \vec{H}_o e^{i\vec{k} \cdot \vec{r}}$  and  $\vec{E} = \vec{E}_o e^{i\vec{k} \cdot \vec{r}}$  where  $\vec{E}_o$  and  $\vec{H}_o$  are constant vectors.

Just like the wave function  $\psi_k(\vec{r})$  of electrons in a periodic structure satisfy Bloch’s theorem  $\psi_k(\vec{r}) = e^{i\vec{k} \cdot \vec{r}} u_k(\vec{r})$  for some wave vector  $\mathbf{k}$  and periodic function  $u_k(\vec{r})$  which has the periodicity of the structure, we also have  $\vec{H} = e^{i\vec{k} \cdot \vec{r}} \vec{u}_k(\vec{r})$  for the spatial part of the electromagnetic wave. Using this form in Eq. (3.26) yields:

$$[(i\vec{k} + \nabla) \times \frac{1}{\epsilon_r(r)} (i\vec{k} + \nabla) \times] \vec{u}_k(\vec{r}) = \left[ \frac{w(\mathbf{k})}{c} \right]^2 \vec{u}_k(\vec{r}) \dots\dots\dots(3.28)$$

This is the equation to be solved and  $\vec{u}_k(\vec{r})$  must satisfy transversality condition:

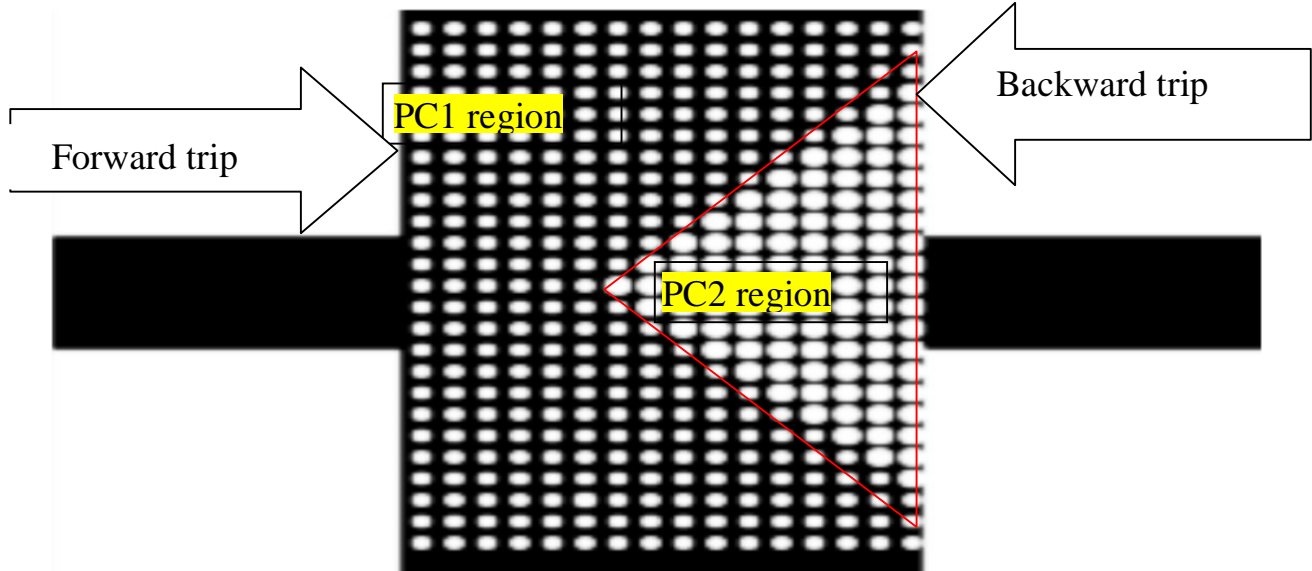
$$0 = \vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \vec{H} = \nabla \cdot (e^{i\vec{k} \cdot \vec{r}} \vec{u}_k(\vec{r})) \text{ i.e. } (i\vec{k} + \nabla) \cdot \vec{u}_k = 0 .$$

The general way (3.28) is solved is by expanding in plane waves:  $\vec{u}_k(\vec{r}) = \sum_{\vec{G}} \vec{c}_{\vec{G}}(\vec{k}) e^{i\vec{G} \cdot \vec{r}}$  where the coefficients  $c_{\vec{G}}(\vec{k})$  need to be determined

and  $\mathbf{G}$  is the so-called reciprocal lattice vectors – the same as those in solid-state physics. Once the expression is plugged into (3.28) and both sides of the equation are Fourier transformed, one arrives at a set of matrix equations for the set of  $\{c_{\vec{G}}(\vec{k})\}$  which can be solved to obtain the eigenvalues  $\left[\frac{w(\vec{k})}{c}\right]^2$  and the corresponding eigenvectors  $\vec{u}_{\vec{k}}(\vec{r})$ . (See the appendix of reference [15] for more details on the method of solution used in MPB program which was used in this work. MPB is an acronym for MIT Photonic Bands) .

## Chapter 4

### 4.1 Results and Discussions



**Fig .6: The dielectric structure of the optical isolator**

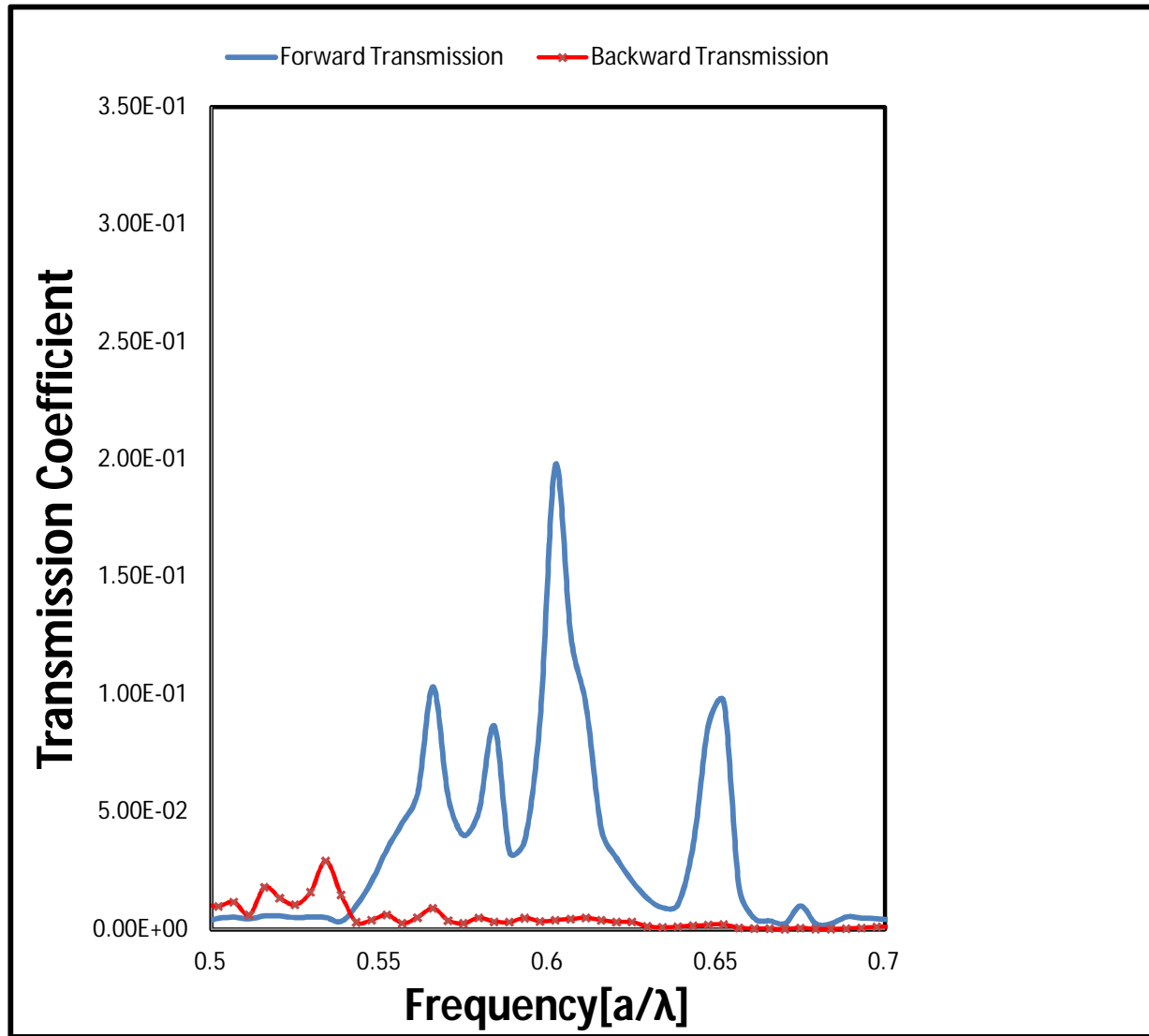
This work reports a method for making one-way on-chip silicon optical isolators based on the directional band gap difference of two 2D square-lattice photonic crystals comprising a hetero-junction structure and the break of the spatial inversion symmetry. The original design is from photonic structure designed by Wang *et. al* [16]. Simulations show the existence of a distinct unidirectional isolation effect in the designed hetero-junction slab. This device would exhibit the same fundamental role as the electrical diode in photonic circuits and pave the way for linear or non-magnetic silicon on-chip optical logical devices, and facilitate the photonic network integration.

Figure 6 depicts the case study of dielectric function of the photonic diode structure , which consists of two PC slab regions (PC1 and PC2) with the same lattice constant  $a_l = 0.375a$  but different air hole sizes ( $r_1$  and  $r_2$ , respectively) comprising a hetero-junction structure. These two Photonic crystal (PC)

regions are situated on a silicon slab (dark area in Figure .6).While PC2 region forms a triangular pattern on the slab, both the two regions have a square-lattice configuration of air holes (white holes in Figure6 ), with the hetero-interface separating them . The hetero-interface consists of two parts separating PC1 and PC2. One part of the hetero-interface is along positive  $\Gamma$ -M direction ( $45^0$ ), and the other is along negative  $\Gamma$ -M direction ( $-45^0$ ). The two hole radii were set as  $r_1=0.100a$  and  $r_2=0.16a$ , where  $a=1\mu\text{m}$  is MEEP lattice parameter. The whole set-up comprises of two  $4a$ -long and  $2a$ -wide similar arms as waveguides. The two arms are integrated, as shown in Fig .6, to the central connector (comprises of the two PCs domains) which is  $6a$ -long and  $10a$ -wide. To have a lossless system, the whole area is enclosed by a perfectly matched layer (pml).

At every transmission exercise, a Gaussian light source is fixed inside symmetrically to the width of the waveguide. The transverse electric field  $E_y$  (TE-like) light wave propagating in the forward (from left to right) or backward (from right to left) direction was simulated using MEEP in 2D . The dielectric constants of 11.56 and 1.00, corresponding to that of silicon and air respectively at 1,550 nm are used for the dielectric structure . The slab thickness was taken to be insignificant, because the simulation was done in 2D. It should be emphasized that all the geometric objects parameters are constant and normalized by the Meep default lattice parameter  $a=1\mu\text{m}$ ; the frequency is normalized by  $(a/\lambda)$ . Figure 7 shows the plot of the transmission spectra in the forward (blue line) and backward (red line) direction. From the plot, it can be seen that there is an isolation band (ranging from  $(0.548(a/\lambda)$  to  $0.657(a/\lambda))$  peaks at different intervals: i.e. the forward transmission forms a peak at  $0.566(a/\lambda)$  with a signal contrast of 0.840, at  $0.584(a/\lambda)$  with a signal contrast of 0.927, at  $0.602(a/\lambda)$  with a signal contrast of 0.961, and at  $0.652(a/\lambda)$  with a signal contrast of 0.958 .

The maximum  $S$  of this diode equals 0.961 which determines the various relevant parameters necessary for optimum and efficient operation of the diode. At this operation regime this structure shows an extreme unidirectional transport phenomenon.

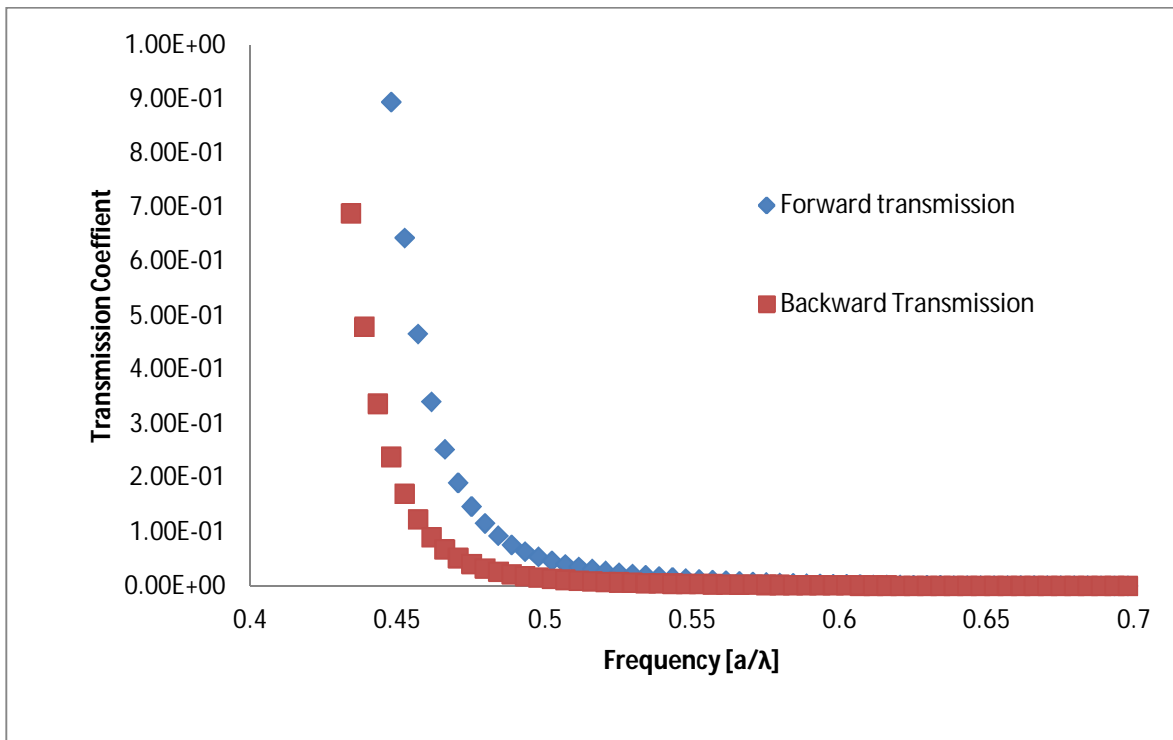


**Figure 7:**the transmission spectra in the forward direction(blue color) and backward direction(red color)

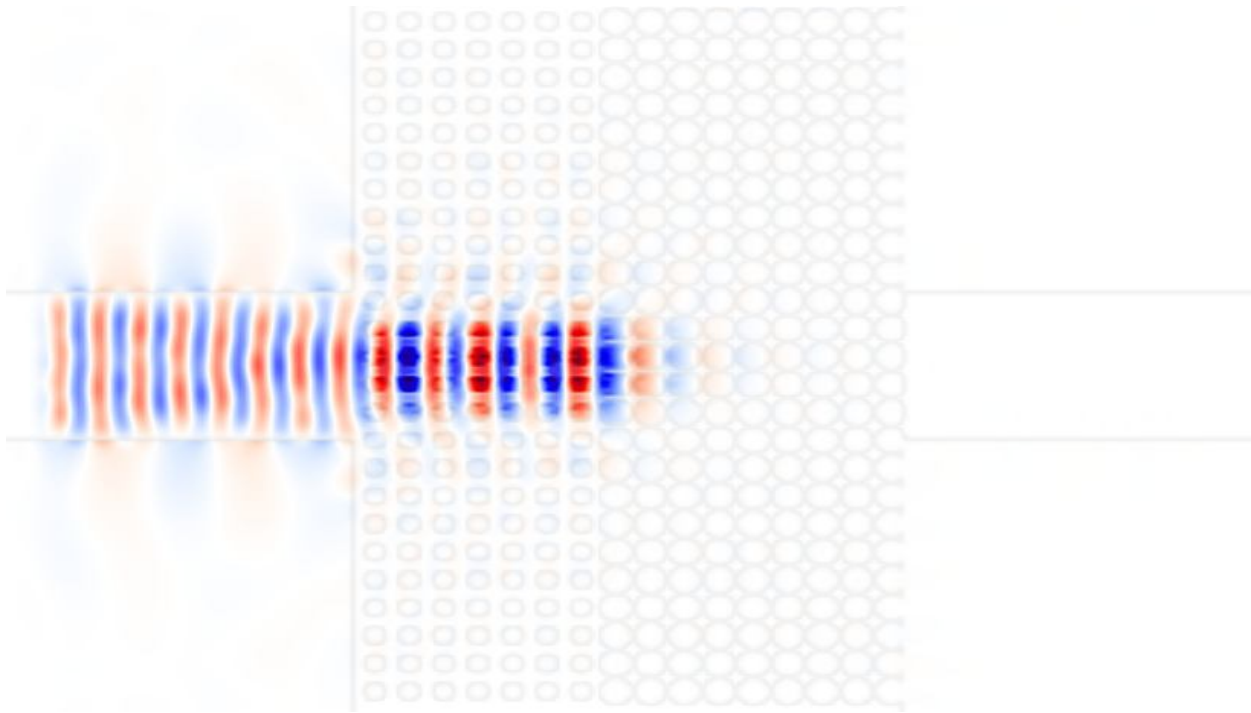
When the interface of the hetero-junction between  $PC_1$  and  $PC_2$  is not tilted, the simulation result shows reciprocal response in optical transport through the isolator. This indicates that the unitary



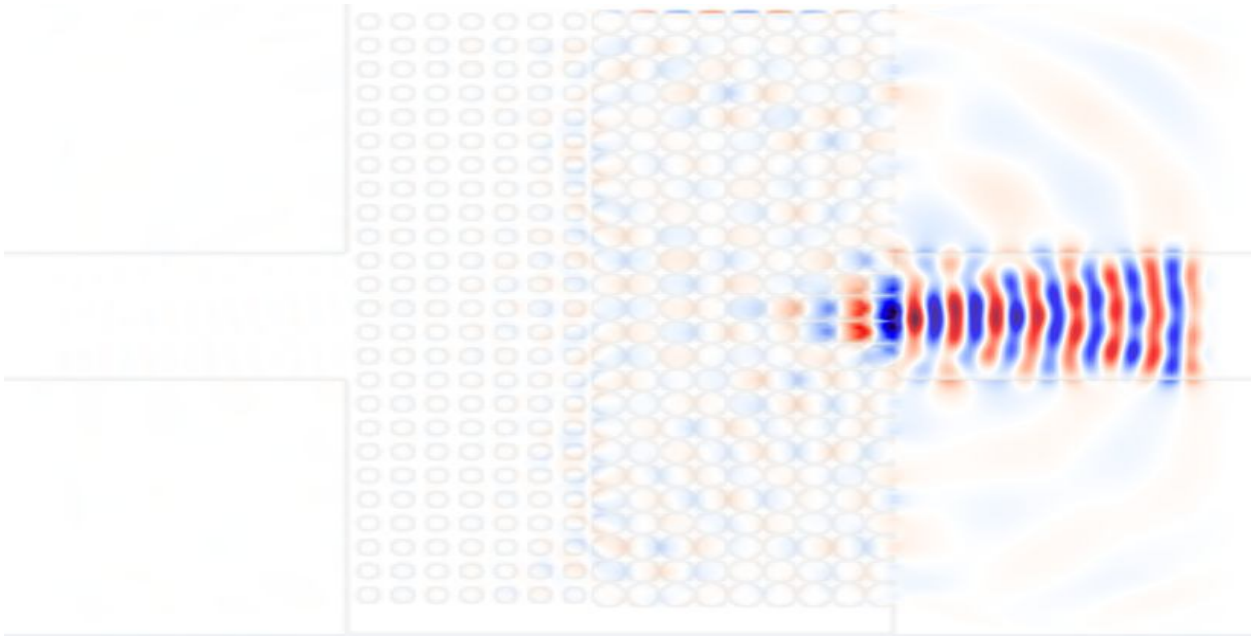
propagation of photons in the structure occurs only if the hetero-junction is tilted. Fig .8 shows the transmission spectra in forward and backward direction of the structure when the hetero-junction is not tilted. Note the frequency's axis in Fig .8 has been extended to  $0.4[a/\lambda]$  to show the contrast in spectra during forward (Blue color) and backward transmission (Red color).



**Figure 8: Transmission spectra of the structure when the hetero-junction is not tilted**

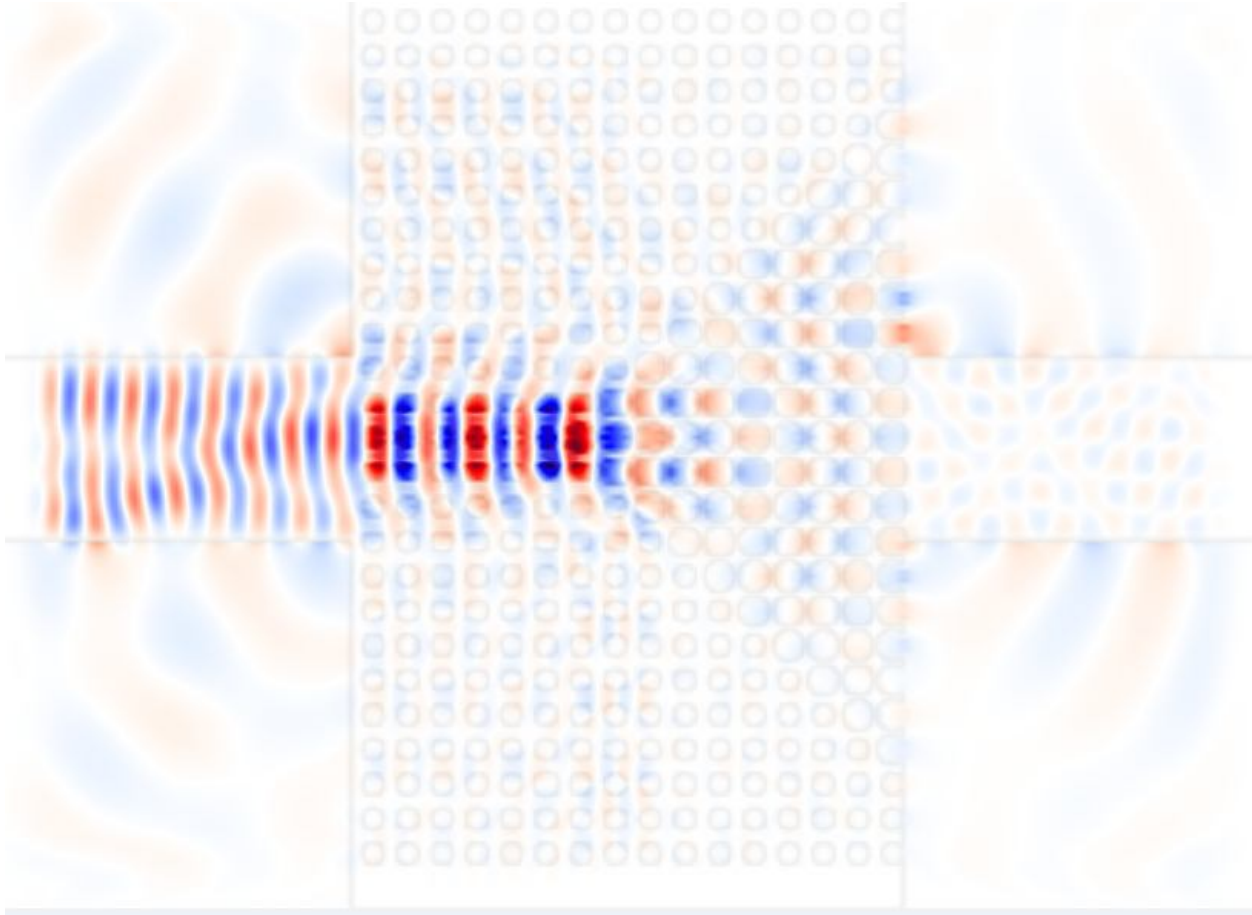


**Fig .9: The electric field  $E_y$  (TE-mode) patterns at forward transmission**

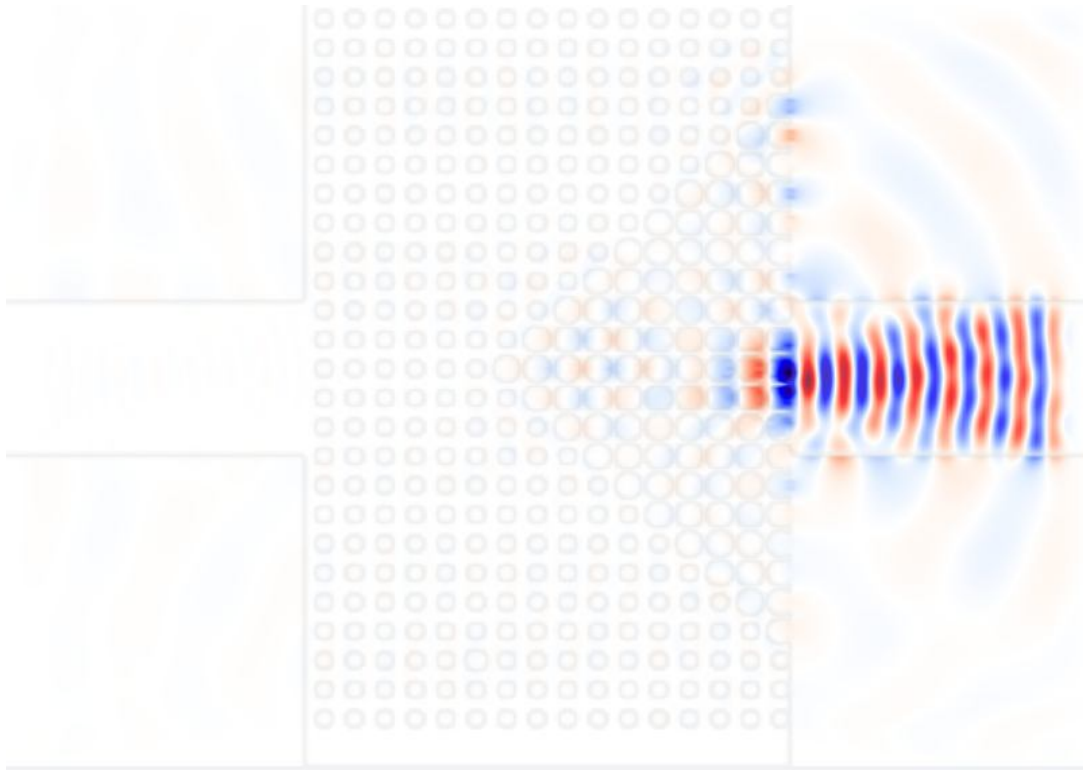


**Fig .10: The electric field  $E_y$  (TE-mode ) patterns at forward transmission**

Fig .9 and Fig. 10 show electricfields (TE mode) distribution in the isolator during forward and backward transmission respectively , and that the unitary propagation of photons in the structure can only occurs if the hetero-junction is tilted.



**Figure 11: the electric field ( $E_y$ ) pattern in forward direction**

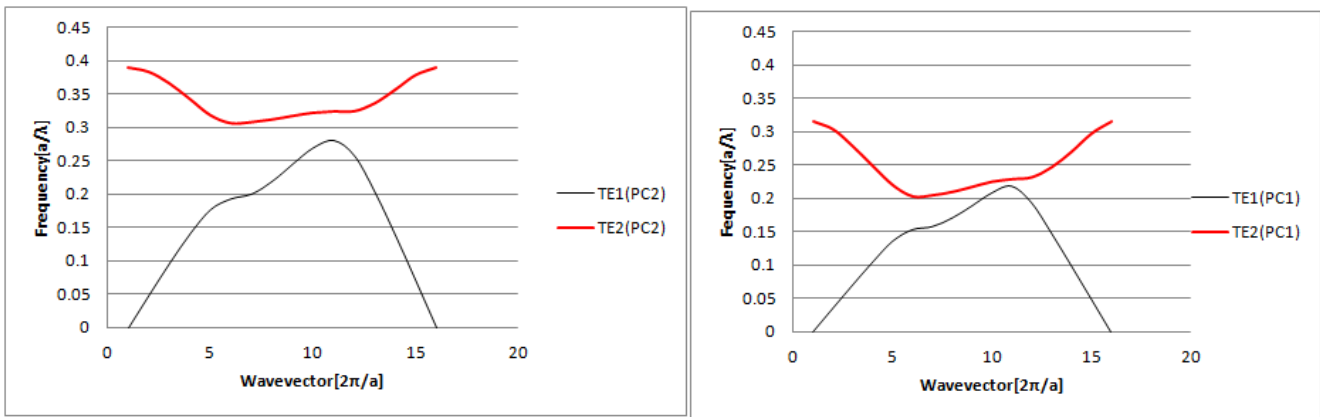


**Figure 12: the electric field ( $E_y$ ) pattern in backward direction**

To have a field visualization of the characteristic of optical isolation, Fig. 11 and Fig .12 show the TE-like electromagnetic field distributions of the forward and backward transmission of light at  $0.65 (a/\lambda)$  are simulated.

In Fig. 11, light travels through PC1 and on reaching the hetero-junction interface, part of it gets reflected downward, and the other propagate along the junction and diffuses into PC2, which eventually outputs from PC2 into the waveguide. But in Fig. 12, light cannot propagate straight but separates into the positive and negative 45°-direction paths. Most of light is lost and there is little light reaching the output waveguide. Thus the difference in losses of light energies that account for the difference between the forward and backward transmission [16].

The two operational modes of the diode structure are the even and odd modes defined with respect to the size of the holes in the two regions.



**Fig .13: modal dispersion curve for PC1(r=0.267) PC2(r=0.427)**

Fig. 13 show the dispersion curves for PC1 and PC2 at different Gamma points i.e. (Gamma,  $\Gamma -X$ ,  $\Gamma -M$ )

1) Forward: here the light travels across PC1 and on getting to the hetero-junction along the X - direction, it cannot move further in the X direction in PC2 because of the X- directional gap. But the

hetero-junction is along the  $\Gamma$ -M direction, so light turns to the hetero-junction and diffracts at any  $\Gamma$ -M direction into PC2.

2) Backward: here light moves directly into PC2 , it turns to the two  $\Gamma$ -M direction paths and cannot move in PC1 in the  $\Gamma$ -X direction . As such it eventually leak out so that it doesn't output.

Due to the arbitrariness of choosing the lattice constant  $a$ , the isolation frequency can be adjusted to a desired value. This could be necessary for the design of realistic photonic devices.

The working principle of the optical diode presented above is based on the difference between the directional band gaps of the two PC domains making up the structure. For more efficient performance of the diode, it depends on how to maximize the peak of the forward transmission and minimize the backward transmission.

## 4.2 Conclusion

- The result shows that the photonic diode exhibits promising performance of optical non-reciprocal response with forward transmission coefficient higher than backwards' in the infrared region.
- The design is compact, linear, non-magnetic and can be implemented on CMOS with size in Micro-range.
- Though Wang *el. tal* design has a maximum signal contrast of 0.92 at  $0.283(a/\lambda)$  , our oscillator has a signal contrast of 0.961 at  $0.602(a/\lambda)$ .
- The design will stimulate the exploration of optical integrated circuit.
- The design leads to a loss in intensity of the incident light both for the forward and the backward directions. We observe that even for the forward direction, the intensity of the transmitted light is only 20% of the incident light.

## 4.3 Recommendation

The forward transmission can be enhanced by adjusting various relevant parameters of the diode.

That is, by changing the air hole size of PC1 and PC2, one can alter the directional band gap. Or one can change the relative size of the input and output waveguides.

To reduce the backward transmission, the directional band gap of PC2 should be enhanced to attenuate the backward signal. Or eliminate the reflection of light from outside of the slab by incorporating the absorbing metal dots near the structure.

## Appendix

### Appendix A

This appendix gives the Meep code for generating dielectric function, displaying and visualizing the field patterns, and calculating polarized electric field flux for forward or backward transmission of the structure. Note to perform the various actions listed above, some lines of the code must be omitted or modified and then the program can be executed with appropriate commands. For an example, to generate the required data for forward transmission spectra, the following commands should be typed ,in order, at the UNIX command prompt: `meep no-connector?=true file-name.ctl | tee name.out` .(2) `meep file-name.ctl | tee name2.out`. where “file-name” is the name of the control file (ctl). Note as was explained in chapter two, in MEEP, transmission coefficient is defined as the ratio of the transmitted flux to the incident flux. The definition lends credence from the fact that , every unit in MEEP is normalized to unity. Thus ,the first command generates the required result for the incident flux, while the second’s gives that of the transmitted flux. The same procedure can be adopted to get backward transmission spectra ,but the detection and the source point are interchanged.

#### **MEEP CODE FOR TRANSMISSION COEFFICIENT**

```
(define-param Ra 0.100)
(define-param Rb 0.160)
(define-param R -6)
(define-param F 3)
(define-param no-connector? false) ; if true ,have no center
connector
(set! geometry-lattice (make lattice (size 14 10 no-size)))
(set! geometry
  (if no-connector?
    (list
      (make block (center 0 0) (size infinity 2 infinity)
        (material (make dielectric (epsilon 11.56))))))
    (list
      (make block (center -5 0) (size 4 2 infinity))
```



```

(material (make dielectric (epsilon 11.56))))
(make block (center 0 0) (size 6 10 infinity)
(material (make dielectric (epsilon 12))))
(make cylinder (epsilon 1) (center -2.75 -
4.375 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
4.000 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
3.625 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
3.250 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
2.875 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
2.500 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
2.125 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
1.750 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
1.375 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
1.000 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
0.625 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75 -
0.250 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
0.125 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
0.500 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
0.875 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
1.250 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
1.625 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
2.000 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
2.375 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
2.750 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
3.125 ) (radius Ra) (height infinity))
(make cylinder (epsilon 1) (center -2.75
3.500 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center -2.75
3.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.75
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.75
4.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
2.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
1.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
1.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
1.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
0.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375 -
0.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
0.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
0.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
0.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
1.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
1.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
2.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
2.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
2.750 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center -2.375
3.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
3.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
3.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.375
4.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
2.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
1.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
1.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
1.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
0.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000 -
0.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
0.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
0.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
0.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
1.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
1.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -2.000
2.000 ) (radius Ra) (height infinity))

```

```

                (make cylinder (epsilon 1) (center -2.000
2.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
2.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
3.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
3.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -2.000
4.625 ) (radius Ra) (height infinity))

                (make cylinder (epsilon 1) (center -1.625 -
4.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
4.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
3.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
3.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
2.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
2.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
2.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
1.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
1.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
1.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
0.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625 -
0.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625
0.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625
0.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625
0.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center -1.625
1.250 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center -1.625
1.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
2.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
2.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
2.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
3.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
3.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
3.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.625
4.625 ) (radius Ra) (height infinity))

      (make cylinder (epsilon 1) (center -1.250 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
2.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
1.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
1.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
1.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
0.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250 -
0.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250
0.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center -1.250
0.500 ) (radius Ra) (height infinity))

```

```

                                (make cylinder (epsilon 1) (center -1.250
0.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
1.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
1.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
2.000 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
2.375 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
2.750 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
3.125 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
3.500 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
3.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
4.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -1.250
4.625 ) (radius Ra) (height infinity))

                                (make cylinder (epsilon 1) (center -0.875 -
4.375 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
4.000 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
3.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
3.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
2.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
2.500 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
2.125 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
1.750 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
1.375 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
1.000 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
0.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.875 -
0.250 ) (radius Ra) (height infinity))

```

```

    (make cylinder (epsilon 1) (center -0.875
0.125 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
0.500 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
0.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
1.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
1.625 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
2.000 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
2.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
2.750 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
3.125 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
3.500 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
3.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
4.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.875
4.625 ) (radius Ra) (height infinity))

    (make cylinder (epsilon 1) (center -0.500 -
4.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
4.000 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
3.625 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
3.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
2.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
2.500 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
2.125 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
1.750 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
1.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.500 -
1.000 ) (radius Ra) (height infinity))

```

```

                                (make cylinder (epsilon 1) (center -0.500 -
0.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500 -
0.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
0.125 ) (radius Rb) (height infinity));
                                (make cylinder (epsilon 1) (center -0.500
0.500 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
0.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
1.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
1.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
2.000 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
2.375 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
2.750 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
3.125 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
3.500 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
3.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
4.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.500
4.625 ) (radius Ra) (height infinity))

                                (make cylinder (epsilon 1) (center -0.125 -
4.375 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
4.000 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
3.625 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
3.250 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
2.875 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
2.500 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
2.125 ) (radius Ra) (height infinity))
                                (make cylinder (epsilon 1) (center -0.125 -
1.750 ) (radius Ra) (height infinity))

```



```

    (make cylinder (epsilon 1) (center -0.125 -
1.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125 -
1.000 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125 -
0.625 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125 -
0.25) (radius Rb) (height infinity));
    (make cylinder (epsilon 1) (center -0.125
0.125 ) (radius Rb) (height infinity));
    (make cylinder (epsilon 1) (center -0.125
0.500 ) (radius Rb) (height infinity));
    (make cylinder (epsilon 1) (center -0.125
0.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
1.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
1.625 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
2.000 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
2.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
2.750 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
3.125 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
3.500 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
3.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
4.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center -0.125
4.625 ) (radius Ra) (height infinity))

    (make cylinder (epsilon 1) (center 0.25 -
4.375 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center 0.25 -
4.000 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center 0.25 -
3.625 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center 0.25 -
3.250 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center 0.25 -
2.875 ) (radius Ra) (height infinity))
    (make cylinder (epsilon 1) (center 0.25 -
2.500 ) (radius Ra) (height infinity))

```

```

                (make cylinder (epsilon 1) (center 0.25 -
2.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25 -
1.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25 -
1.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25 -
1.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25 -
0.625 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.25 -
0.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
0.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
0.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
0.875 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
1.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
1.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
2.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
2.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
2.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
3.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
3.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.25
4.625 ) (radius Ra) (height infinity))

                (make cylinder (epsilon 1) (center 0.625 -
4.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
4.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
3.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
3.250 ) (radius Ra) (height infinity))

```

```

                (make cylinder (epsilon 1) (center 0.625 -
2.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
2.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
2.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
1.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
1.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
1.000 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
0.625 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625 -
0.25 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
0.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
0.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
0.875 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
1.250) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
1.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
2.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
2.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
2.750 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
3.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
3.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 0.625
4.625 ) (radius Ra) (height infinity))

                (make cylinder (epsilon 1) (center 1.00 -
4.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 1.00 -
4.000 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center 1.00 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
2.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
1.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
1.375 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
1.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
0.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00 -
0.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
0.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
0.500 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
0.875 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
1.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
1.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
2.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
2.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
2.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
3.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
3.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
3.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.00
4.625 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center 1.375 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
2.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
1.75 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
1.375 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
1.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
0.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375 -
0.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
0.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
0.500 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
0.875 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
1.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
1.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
2.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
2.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
2.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
3.125 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
3.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
3.875 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center 1.375
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.375
4.625 ) (radius Ra) (height infinity))

      (make cylinder (epsilon 1) (center 1.75 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
2.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
2.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
1.750 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
1.375 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
1.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
0.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75 -
0.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
0.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
0.500 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
0.875 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
1.250) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
1.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
2.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
2.375 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
2.750 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
3.125 ) (radius Ra) (height infinity))

```

```

      (make cylinder (epsilon 1) (center 1.75
3.500 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
3.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
4.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 1.75
4.625 ) (radius Ra) (height infinity))

      (make cylinder (epsilon 1) (center 2.125 -
4.375 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
4.000 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
3.625 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
3.250 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
2.875 ) (radius Ra) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
2.500 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
2.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
1.750 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
1.375 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
1.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
0.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125 -
0.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
0.125 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
0.500 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
0.875 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
1.250 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
1.625 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
2.000 ) (radius Rb) (height infinity))
      (make cylinder (epsilon 1) (center 2.125
2.375 ) (radius Rb) (height infinity))

```

```

                (make cylinder (epsilon 1) (center 2.125
2.750 ) (radius Rb ) (height infinity))
                (make cylinder (epsilon 1) (center 2.125
3.125 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.125
3.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.125
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.125
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.125
4.625 ) (radius Ra) (height infinity))

                (make cylinder (epsilon 1) (center 2.5 -
4.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
4.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
3.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
3.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
2.875 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
2.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
2.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
1.750 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
1.375 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
1.000 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
0.625 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5 -
0.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
0.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
0.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
0.875 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
1.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
1.625 ) (radius Rb) (height infinity))

```



```

                (make cylinder (epsilon 1) (center 2.5
2.000 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
2.375 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
2.750 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
3.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
3.500 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.5
4.625 ) (radius Ra) (height infinity))

                (make cylinder (epsilon 1) (center 2.875 -
4.375 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
4.000 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
3.625 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
3.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
2.875 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
2.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
2.125) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
1.750 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
1.375 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
1.000 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
0.625 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875 -
0.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
0.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
0.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
0.875 ) (radius Rb) (height infinity))

```

```

                (make cylinder (epsilon 1) (center 2.875
1.250 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
1.625 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
2.000 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
2.375 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
2.750 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
3.125 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
3.500 ) (radius Rb) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
3.875 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
4.250 ) (radius Ra) (height infinity))
                (make cylinder (epsilon 1) (center 2.875
4.625 ) (radius Ra) (height infinity))

                ;(make cylinder (epsilon 1) (center 3.25 -
4.375 ) (radius 0.120) (height infinity))
                ;(make cylinder (epsilon 1) (center 3.25 -
4.000 ) (radius 0.120) (height infinity))

                ;(make cylinder (epsilon 1) (center 3.25
4.250 ) (radius 0.120) (height infinity))
                ;(make cylinder (epsilon 1) (center 3.25
4.625 ) (radius 0.120) (height infinity))

                ;(make cylinder (epsilon 1) (center 3.625 -
4.375 ) (radius 0.120) (height infinity))

                ;(make cylinder (epsilon 1) (center 3.625
4.625 ) (radius 0.120) (height infinity))

                (make block (center 5 0) (size 4 2 infinity)
                (material (make dielectric (epsilon
11.56))))))

(define-param fcen 0.65)
(define-param df 0.45)
(set! sources (list
                (make source

```

```

                (src (make gaussian-src (frequency fcen) (fwidth
df)))
                (component Ey)
                (center R 0) (size 0 2))))

(set! resolution 20)
(set! pml-layers (list (make pml (thickness 1.0))))
(define-param nfreq 100)
(define trans
  (add-flux fcen df nfreq
    (if no-connector?
      (make flux-region
        (center F 0 ) (size 0 10))
      (make flux-region
        (center (* -1 F) 0) (size 0 10))))))

(run-sources+
  (stop-when-fields-decayed 50 Ey
    (if no-connector?
      (vector3 F 0)
      (vector3 (* -1 F) 0))
    1e-3))

(display-fluxes trans)

```

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