Fundamentals of Laser Dynamics

A Thesis
Submitted to the African University of Science and Technology
Abuja-Nigeria
in partial fulfilment of the requirements for

MASTER DEGREE IN THEORETICAL PHYSICS

By

SANNI Dahiru Muhammad

Supervisor:

Prof Oleg Yordanov
Bulgaria Academy of Science
Sofia

April 2013

Fundamentals of Laser Dynamics

A THESIS APPROVED

BY

SUPERVISOR ______________________________

MEMBER ________________________________

MEMBER ________________________________

Abstract

The thesis presents the modeling and simulation of three types of lasers namely; Semicondoctor laser, Solid state laser and CO_2 laser. The rate equations were derived and simulated to examine the dynamic behaviour of the three types of lasers under investigation.
The result shows that the Semicondoctor laser has the longest latency period, highest intensity spikes and takes a longer time to come to relaxation oscillation (RO) while the CO_2 laser has the shortest latency time, the lowest intensity spikes and takes a shorter time to come to relaxation oscillation (RO). The solid state laser lies between the semiconducor laser and the CO_2 laser.
It was also observed from the results that as the pump power A increases the latency time decreases, the intensity increases and it takes a shorter time for the laser to come to relaxation oscillation.

Dedication

This project is dedicated to almighty Allah (SWT) our sustenance, to my late father Mallam Muhammad Sanni may his gentle soul rest in peace,
my mother Mrs. Rabi Muhammad, my Siblings and my late friend UGBEDE Peter.

Acknowlodgement

All praises and adoration are due to Allah the lord of the world who has given me the opportunity in all kinds to acquire this Master Degree. May His peace and blessings be upon our noble prophet Muhammad (S.A.W) and his righteous companions. My appreciation goes to my Mother, siblings and relatives for their love and support; Most especially over the past eighteen months. My sincere appreciation goes to my dynamic supervisor Prof. Oleg Yordanov for his relentless support throughout the course of this thesis. A big thanks to Dr. Omololu Akin-Ojo coordinate of Theoretical Physics Stream for his brotherly contributions and support.

I would like to acknowledge the Nelson Mandela Institution(NMI), The African University of Science and Technology (AUST) and Prof Wole Soboyejo for giving me the the privilege of interacting with numerous distinguished scholars who have helped to shape my perception about science, knowledge and life in general. The efforts of all the staff of AUST have also helped immensely towards making my studies successful.

I must specifically thank Adebayo Fashina, Yoro Diouf, Ismaila Nurudeen Dele (Baba Dee), Idris Malik, Mustpha Aguye, Sharafadeen Adedeji, Musa Yusuf, Hassan Ahmodu, Simon Eboh, Caleb Ede, Owoseni, Usman Bello.

Contents

1 Introduction **1**
1.1 Lasers . . . 1
1.1.1 Concept of stability . . . 2
1.2 Aims of the Thesis . . . 2
1.3 Overview of the Thesis . . . 3

2 Laser operation **4**
2.1 How Laser Emits Light . . . 4
2.2 Active Material . . . 4
2.2.1 Gain . . . 4
2.3 Spontaneous Emission . . . 5
2.4 Absorption . . . 5
2.5 Population Inversion . . . 7
2.6 Non Radiactive Deexcitation . . . 7
2.7 Pumping . . . 7
2.8 Stimulated Emission . . . 8
2.9 Lasing Threshold . . . 9

3 Models of Laser Dynamics **10**
3.1 Simple one-dimensional models . . . 10
3.2 The two-dimensional dynamics . . . 14
3.3 Three-Dimensional Model . . . 19
3.3.1 Physical considerations . . . 19
3.3.2 Model of ruby laser . . . 20
3.3.3 CO_2 laser . . . 23

4 Simulation Results **28**
4.1 Simulation 28
4.2 Switch-on Transient and phase portrait 29
4.3 Comparison of the linear approximation vs. exact numerical solution 41

5 Conclusion and Recommendations **45**
5.1 Conclusion 45
5.2 Recommendation 46

List of Figures

2.1 Spontaneous emission of a photon of energy $h\nu$ 6
2.2 Absorption 6
2.3 Stimulated Emission 8

3.1 Phase diagram for 1D laser model 13
3.2 Bifurcation diagram 16
3.3 Location of the stationary points 17
3.4 19
3.5 Energy levels of the ruby laser 20
3.6 Energy levels of the CO_2 laser 23

4.1 Switch-on transient of semiconductor laser for pump parameter A=2 29
4.2 Switch-on transient of solid state laser for pump parameter A=2 . 30
4.3 Switch-on transient of CO_2 laser for pump parameter A=2 31
4.4 Switch-on transient of semiconductor laser for pump parameter A=5 32
4.5 Switch-on transient of solid state laser for pump parameter A=5 . 33
4.6 Switch-on transient of CO_2 laser for pump parameter A=5 34
4.7 Phase porrait of semiconductor laser for pump parameter A=2 . . 35
4.8 Phase porrait of solid state laser for pump parameter A=2 36
4.9 Phase porrait of CO_2 laser for pump parameter A=2 37
4.10 Phase porrait of semiconductor laser for pump parameter A=5 . . 38
4.11 Phase porrait of solid state laser for pump parameter A=5 39
4.12 Phase porrait of CO_2 laser for pump parameter A=5 40
4.13 41
4.14 42

4.15 43
4.16 43
4.17 44

Chapter 1

Introduction

1.1 Lasers

The origin of laser can be traced back to the Einstein's concept of stimulated emission. The presence of a photon, with appropriate frequency, can stimulate an excited atom to emit a photon, with identical phase, frequency and propagation direction than the incident one [2]. Three components are fundamental in any laser: a medium proving gain/amplification, a pump generating population inversion, and a cavity confining the optical field. The first population inversion was attained in ammonia molecules passing through an electrostatic focuser by Townes and Shawlow [3] in 1958. The constructed device, originally called MASER, emitted light in the microwave range. The first successful laser, operating in the visible spectrum, was constructed by Maiman [4] and consisted of a ruby crystal surrounded by a helicoidal flash tube. This advert was followed, at the ends of the same year, by experimental demostration of working He-Ne gas laser.

Lasers have important applications in communications signal processing and medicine, including optical interconnects, RF links, CD ROM, gyroscopes, surgery, printers and photocopying (to mention but a few). Compared with other optical sources, lasers have a high bandwidth and higher spectral purity, they function as bright cohenrent sources. These properties allow laser emission to be tightly focused, with minimum divergence. Solid state, semiconductor and gas lasers are just a few of the many different types of lasers available in the market. Each of these different types of lasers are important for different applications based upon desired result and cost.

The word laser is an acronym composed of the initial letters of “light amplification by stimulated emission of radiation”. The laser principle emerged from the MASER principle. The word maser is again an acronym standing for “microwave amplification by stimulated emission of radiation”. Light is amplified through the properties of active medium, optical gain, population inversion. The aim of this thesis is to study the dynamical behavior of laser emission and the laser systems. Understanding the dynamical nature of laser will lead to accurate predictions of the emitted irradiance of the laser, which is necessary when studing more complex system with feedback and coupling.

1.1.1 Concept of stability

Stability is an important concept when studying systems over a given time interval. A system is considered stable when a condition converges towards a single point within a set range. On the other hand, a system becomes unstable when conditions diverge from a fixed point and depart from this range. Further, when the system diverge and splits, creating a more complicated system. The locations of these splits are called bifurcation points. These special points can be related to the chaotic behavior of two synchronized lasers systems. This phenomenon of bifurcation points from synchronization may be modeled by Quaratic Maps. In order to find predictions in the system a concentration must be formed between the initial conditions and stability of the system. Further details can be found in a number of books and papers devoted on nonliner science and Chaos theory, see for example [1] and [11].

1.2 Aims of the Thesis

Lasers have a wide range of applications as highlighted above. As such, they underwent an intensive research and development since the time of their first operational regime was achieved.

This thesis aims at studying in detail the rate equations for various types of lasers,

including their phase portrates. The classical approach was used to derive the rate equations for different lasers. In their simplest version, they apply to an idealized active system consisting of only two energy levels coupled to a reservoir. The rate equations will help to explain the stability of a laser (regular or irregular, damped or undamped) intensity spikes commonly seen with the solid states lasers.
The significant of this thesis is that it will help to predict the behavior of lasers.

1.3 Overview of the Thesis

Chapter two reviews the basic processes involved in laser operation. The processes discussed include active materials, spontaneous emission, stimulated emission, optical absorption, population inversion, threshold condition for lasing, optical feedback and pumping.
Chapter three presents a detailed derivation of the rates equations for different lasers followed by brief description of the numerical solution and linear approximation.
In chapter four, the results obtained from the numerical solution were presented and discussed. Followed by the conclusion and recommendation.

Chapter 2

Laser operation

2.1 How Laser Emits Light

The basic components of the laser are current source, an active material and the resonator. Each of these components controls the stimulated emission of the laser and need to be understood in order to perform well-founded computational analysis.

2.2 Active Material

The materials that can be used as the active medium of a laser are so varied that a listing is hardly impossible. Gases, liquid and solids of every sort have been made to lase. The origin of laser photons, is most often in a transition between discrete upper and lower energy states in the medium, regardless of its state of matter. He-Ne, ruby, CO2 and dye lasers are familiar examples, but different materials are frequently used: the excimer laser has an unbound lower state, the semiconductor diode laser depends on the transition between electron bands rather than discrete states and understanding the free-electron laser does not require quantum state. All these materials provide optical gain in the cavity

2.2.1 Gain

Gain is a quantity that is determined by the length of the optical cavity and the number of reflected passes through the active material. For each pass through the optical cavity a loss occurs due to the mirrors that is proportional to the gain. When pumping is applied to the active material, the gain increases for each pass through the optical cavity. Population inversion occurs when the gain reaches a value higher than the loss from reflections [5].

2.3 Spontaneous Emission

Spontaneous emission is the process by which a light source in an excited state undergoes a transition to a state with a lower energy, with the emission of photon. This process occurs spontaneously without any external influence [2],figure 2.1.

Spontaneous emission of light is a fundamental process that plays an essential role in lasers. If the number of light sources in the excited state is given by N, the rate at which N decays is

$$\frac{dN}{dt} = -A_{21}N \tag{2.1}$$

where A_{21} is the rate of spontaneous emission. The rate of spontaneous emission depends on two factors: an atomic part, which describes the internal structure of the light source and a field part which describes the electromagnetic modes of the the environment. The atomic part describes the strength of transition moment.

2.4 Absorption

Absorption of electromagnetic radiation is the way in which the energy of a photon is taken up by matter, typically the electron of an atom. Thus, the electromagnetic energy is transformed to other forms of energy for the example, heat.
In the absorption of photon of energy, $h\nu = E_2 - E_1$, the atom jumps up from level 1 to the higher level 2. The process is induced by an incident photon as shown in figure 2.2.

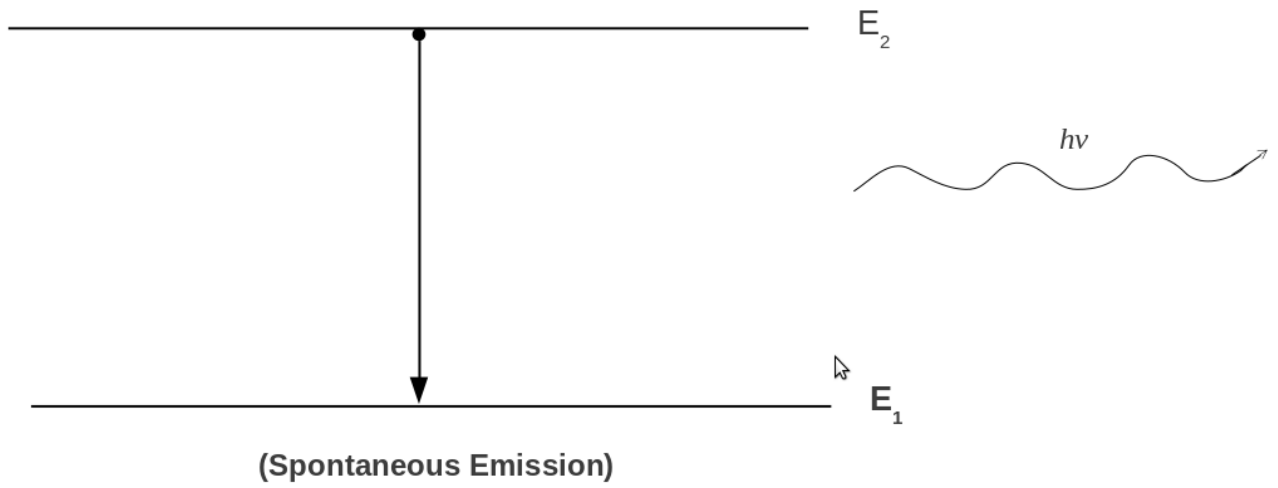


Figure 2.1: Spontaneous emission of a photon of energy $h\nu$

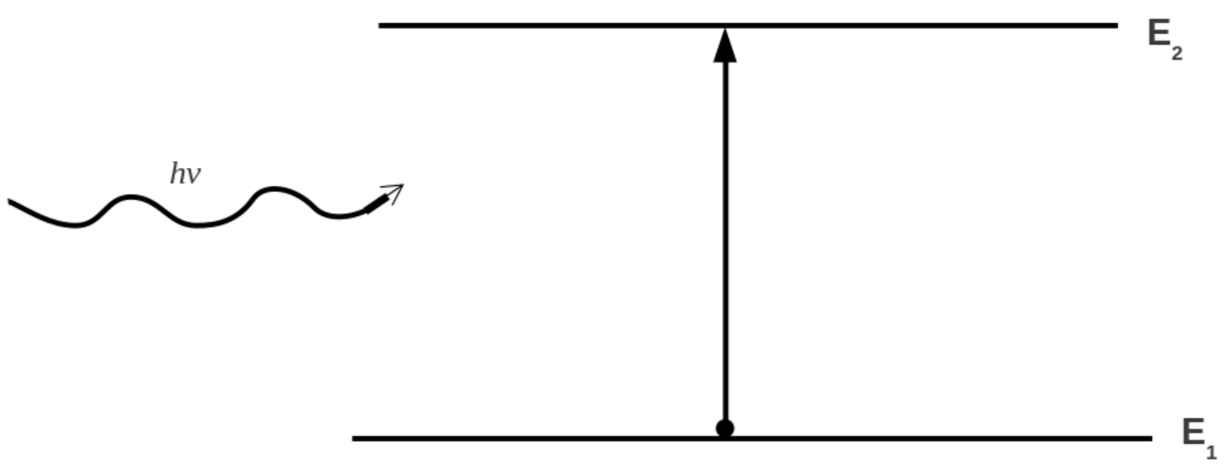


Figure 2.2: Absorption

2.5 Population Inversion

In order for the laser to emit light, population inversion must be created in the system. Population inversion occurs when the initial carrier density is less than the final carrier density, and when the initial energy is less than the final energy. This process occurs from the consquences of pumping in the optical cavity. An optical cavity is necessary to produce a stimulated emission through pumping. This cavity is a region composed of two approximatelly parallel mirrors separated by a define distance.

2.6 Non Radiactive Deexcitation

In this process the atom jumps down from level 2 to the lower level 1, but no photon is emitted so the energy $E_2 - E_1$ must appear in some other form [e.g increased vibrational or rotational energy in the case of a molecule, or rearrangement (shakeup) of other electron in the atom].

2.7 Pumping

Pumping in laser is the act of energy transfer from an external source into the gain medium of a laser. The energy absorbed in the medium, producing excited state in its atoms. When the number of particles in one excited state exceeds the number of particles in the ground state or a lower-energy-level state, population inversion is achieved. In this condition, the mechanism of *stimulated emission* can take place and the medium can act as a laser or an optical amplifier. The pump power must be higher than the lasing threshold of the laser.

The pump energy is usually provided in the form of light or electric current, but more exotic sources have been used , such as chemical or nuclear reaction.

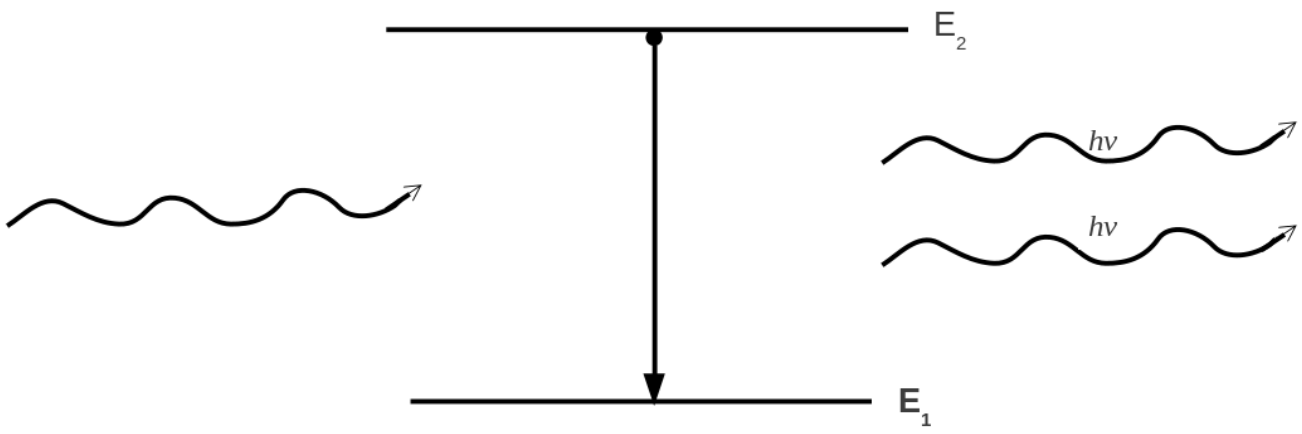


Figure 2.3: Stimulated Emission

2.8 Stimulated Emission

Stimulated emission is the process by which an atom interacting with an electromagnetic wave of a certain frequency may drop to a lower energy level, tranferring its energy to that field. A photon created in this manner has the same phase, frequency, polarization and direction of travel as the photon of the incident wave. This is in contrast to spontaneous emission which occurs without regard to the ambient electromagnetic field.
The atom jumps down from energy level 2 to the lower level 1, and emitted photon of energy. $hv = E_2 - E_1$ is an exact replicant of a photon already present. The process is induced or stimulated by the incident photon [2]. On the other hand, the absorption is taking plase as a competing process; however, when population inversion is present the rate of stimulated emission exceeds that of absorption, and a net optical amplification can be achieved.

2.9 Lasing Threshold

The lasing threshold is the lowest excitation level at which a laser's output is dominated by stimulated rather than by spontaneous emission. Below the threshold, the laser's output power rises slowly with increasing excitation. Above the threshold, the slope of power vs. excitation is orders of magnitude greater. The linewidth of the laser's emission also becomes orders of magnitude smaller above the threshold than it is below. Above the threshold, the laser is said to be lasing.

All these processes occur in the gain medium of a laser. Lasers are often classified according to the nature of the pumping process which is the source of energy for the output laser beam. In electric discharge laser for instance, the pumping occurs as a result of collisions of electons in a gaseous discharge with the atoms of the gain medium.

Chapter 3

Models of Laser Dynamics

In this chapter, we consider several models of laser dynamics with increasing complexity and accuracy. The models are expressed in terns of systems of differential equations and are commonly referred to as *rate equations.* The approach we undertake is gradual: starting from a very simple, involving a single equation model, we incorporate mechanisms and properties neglected at the earlier modeling. The new mechanism in general increases the number of the equations and hence the dimension of the phase space. Taking the dynamical systems terminology we call the models one-dimensional, two-dimensional, and so on.

3.1 Simple one-dimensional models

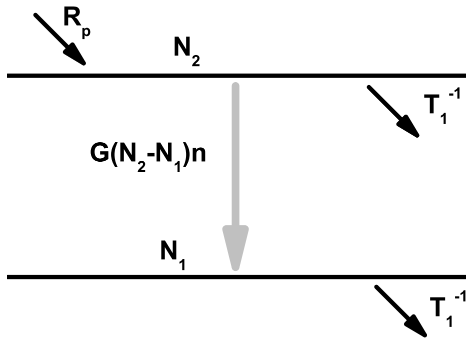
In this section, we begin by defining the simplest model of laser dynamics, which involves a single differential equation. Admittedly, the model is not much realistic; however, it is instructive to work with such a model in terms of both physical considerations and technical analysis.

The model

Consider a system with two energy levels with populations N_1 and N_2. Let:

- ▶ R_p denotes the pumping rate
- ▶ T_1^{-1} be the decay rate of both populations

- n be the population of photons per unit of volume
- $N = N_2 - N_1$ be the population difference between the number of excited atoms and the number of atoms in the lower energy level per unit of volume



The rate of change of photons in the lasing media is given by:

$$\frac{dn}{dT} = GnN - \frac{n}{T_c}, \tag{3.1}$$

where G is called "gain" or "amplification" of the laser with physical dimension $[G] = m^3/s$, and $\gamma = T_c^{-1}$ is the rate at which photons "leak" out. The term GnN describes the increase of photons due to the stimulated emission. Let now,

- N_0 be the population difference due to some pumping mechanism, which excites atoms from the lower level to the upper.

Then,

$$N = N_0 - Bn, \tag{3.2}$$

where B is the coefficient of stimulated emission. Stimulated emission brings an exited atom to the lower energy level and hence lowers N. Substituting in eq. (3.1), we eliminate N making the balance between the creation and loss of photons quadratic function,

$$\frac{dn}{dT} = k_1 n - k_2 n^2 \equiv f(n). \tag{3.3}$$

The coefficients have been aggregated into: $k_1 = (GN_p - \gamma)$ and $k_2 = GB$. In a real experiment, the active laser media has already been chosen; hence both G and T_1^{-1} are fixed. The pumping rate and therefore N_0 is the only quantity in hand, which makes N_0 the control parameter of the system.

Laser system, which is described reasonably well by a single equation of the form (3.3) is referred to as "*class A*" laser [6].

Local analysis of class A lasers

Equation (3.3) has two fixed points $n^\star$ (such that the derivative of $n(T)$ vanish, and hence once reached, $n(T) = n^\star$, remains constant.) Setting $f(n) = 0$, we obtain: $n_0^\star = 0$ and $n_1^\star = k_1/k_2 = \dfrac{GN_0 - \gamma}{GB}$. If N_p, the population difference produced by the pumping mechanism, is viewed as the control parameter, we see that for $N_p < \gamma/G$, $n_1^\star < 0$. Clearly a negative number of photons is not physically relevant; also, $n^\star = 0$ is not an interesting case. For $N_0 > \gamma/G$ two relevant fixed points, $N_{0p} = \gamma/G$ is called "threshold" value of N_0. To study the stability of the fixed points, we introduce $n = n - n^\star$ and linearize the equation.

$$\frac{dn}{dT} = f(n^\star + n) = f(n^\star) + \left.\frac{df}{dn}\right|_{n^\star} n + \cdots \approx (k_1 - 2k_2 n^\star)\, n. \tag{3.4}$$

For $n^\star = n_0^\star = 0$, the solution of the linearized equation is $n(T) = n(T) = \exp(k_1 T)$, which for pumping above the threshold increases with T; (since, $k_1 > 0$). That is to say, the fixed point $n_0^\star$ is unstable or repealing. On the other hand, the solution about the second fixed point is $n(T) = n_1^\star + \exp(-k_1 T)$, i.e. the point is stable or attracting. These properties of the stationary points are illustrated in Fig. 3.1, which is called phase diagram of the model. On the basis of this (linear) analysis we expect that given enough time the laser will settle to a stationary regime with population of photons given by

$$n(T) = n_1^\star = \frac{GN_p - \gamma}{GB}. \tag{3.5}$$

Fig (3.1) represent phase diagram for values of the control parameter $N_0 > \gamma/G$. The plot depicts the two fixed points of eqn. 3.3, the unstable $n_0^\star$ and the stable $n_1^\star$.

In fact, eq. (3.3), which is equivalent to the Verhulst limited logistic growth equation, admits an exact solution. Indeed, changing the variable to $u = k_1/(k_2 n)$, we obtain for $u(T)$ the following (linear) equation

$$\dot{u} = k_1 (1 - u) \tag{3.6}$$

Integrating, we obtain $(u(T) - 1) = Ce^{-k_1 T}$. Setting $T = 0$, it follows that the constant of integration is $C = (u(0) - 1)$ and going back to the original variable

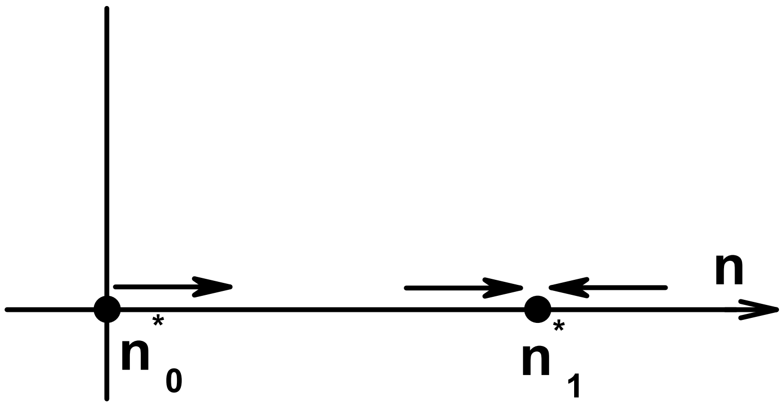


Figure 3.1: Phase diagram for 1D laser model

$n(T)$ we arrive at

$$n(T) = \frac{k_1/k_2}{1 - \left(1 - \frac{k_1}{k_2 n(0)}\right) e^{-k_1 T}}. \tag{3.7}$$

In the above expression, $n(0)$ is the initial value of the photon density. For large T, the solution (3.7),

$$\lim_{T\to\infty} n(T) = \frac{k_1}{k_2} = n_1^\star, \tag{3.8}$$

which is in agreement with the prediction of the linear local analysis. Moreover, both the local analysis and the nonlinear solution show that the laser approaches the stationary operation, for which

$$n = \frac{GN_s - T_1^{-1}}{GB}, \tag{3.9}$$

in a monotonic (aperiodic) manner.

3.2 The two-dimensional dynamics

The model

The one-dimensional model considered in the preceding section neglects the dynamics of the energy levels populations. As a drawback,the model cannot predict transient and oscillating phenomena, which are frequently observed in the experiments. In this section, we incorporate the population rate of change, considering again for simplicity a two-level system with populations N_1 and N_2, respectively.

$$\frac{dn}{dT} = G\left(N_2 - N_1\right)n - \frac{n}{T_c}, \tag{3.10a}$$

$$\frac{dN_2}{dT} = R_p - \frac{N_2}{T_1} - G\left(N_2 - N_1\right)n, \tag{3.10b}$$

$$\frac{dN_1}{dT} = -\frac{N_1}{T_1} + G\left(N_2 - N_1\right)n. \tag{3.10c}$$

If we introduce the population difference, $N = N_2 - N_1$,

$$\frac{dn}{dT} = GNn - \frac{n}{T_c} \tag{3.11a}$$

$$\frac{dN}{dT} = -\frac{1}{T_1}\left(N - N_0\right) - 2GNn, \tag{3.11b}$$

where, $N_0 = T_1 R_p$ and the following dimensionless quantities have been introduced: $I = 2GT_1 n$, $D = GT_c N$, $t = T/T_c$, $A = GT_c N_s$, and $\gamma = T_c/T_1$.

The dimensionaless version of system (3.11) reads

$$\frac{dI}{dt} = I(D-1) \tag{3.12a}$$

$$\frac{dD}{dt} = \gamma\left[A - D(1+I)\right], \tag{3.12b}$$

in which the free parameters are reduced to just two, A and γ. Typical values for them are $A \sim 1 \div 10$ and $\gamma \sim 10^{-3} << 1$. $\frac{dD}{dt} \approx 0$ $D = \frac{A}{(1+I)}$. Since the parameter γ is small, $\frac{dD}{dt} \approx 0$, and hence the dimensionless population difference is constant. In addition, if we consider the case, in which the second equation is in a fixed point, we can substitute $D = \frac{A}{(I+1)}$ in the first equation, obtaining

$$\frac{dI}{dt} = I\left(\frac{A}{1+I} - 1\right).$$

If further $I < 1$, we can expand $\frac{1}{(I+1)} \approx 1 - I$ we arrive at an equation similar to the equation defining the 1D model,

$$\frac{dI}{dt} = (A-1)I - AI^2,$$

however, now for the dimensionless photon density (dimensionless intensity of the laser emission).

Local Analysis of the 2d model.

As in the case of the 1D model we begin by finding the fixed (stationary) points I_s and D_s of the system (3.12). From the first equation, we have $I(D-1) = 0$ and therefore two cases:

Case OFF: $I_s = 0$, and thus $D_s = A$.

Case ON: $D_s = 1$ and then the second equation gives $I_s = A - 1$, which exist physically for $A > 1$.

What is the location of the fixed points depending on the values of the control parameter A? For $A < 1$, the regime OFF as physically realistic option exist only. (Physically, the population N_2 is so low that a photon created in a spontanious transition ($N_2 \mapsto N_1$) has much higher probability to excite an electron ($N_1 \mapsto N_2$) compared to the probability to create another photon through stimulated emission. The outcome is a state of equilibrium with population diffrence $D_s = A$, or $N = N_0$ in the corresponding dimension variable.)

$R_p = \frac{1}{GT_1T_c}$.

$u(t) = I(t) - I_s$, $v(t) = D(t) - D_s$

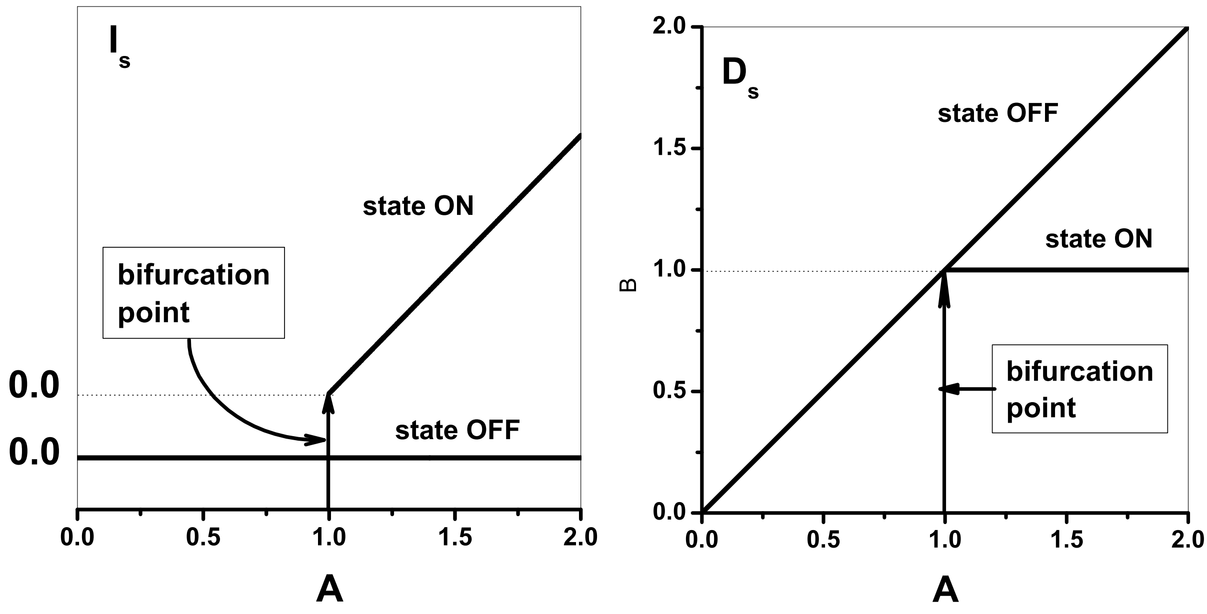


Figure 3.2: **Bifurcation diagram. Left panel:** The dependess of the stationary dimensionless intensity I_s on control parameter A (dimensionless pumping). For the sake of clarity, the state ON is shifted vertically. The dotted line is an indication, that for $A < 1$, the state ON does not exists. The arrow marks the bifurcation point $A = 1$, the treshhold value, at which the state ON is "born". **Right panel:** The same for the dimenssionless stationary population difference D_s. See Fig. 3.2

$$\frac{du}{dt} = (D_s - 1)u + I_s v, \tag{3.13a}$$

$$\frac{dv}{dt} = \gamma \left[-D_s u - (1 + I_s)v \right]. \tag{3.13b}$$

$$\sigma^2 + [\gamma(1 + I_s) - D_s + 1]\sigma + \gamma(1 + I_s - D_s) = 0. \tag{3.14}$$

$$\sigma^2 - (\gamma - A + 1)\sigma + \gamma(1 - A) = 0 \tag{3.15}$$

Which has solutions: $\sigma_1 = A - 1$, $\sigma_2 = -\gamma$, hence for $A > 1$, $\sigma_2 > 0$ showing that the OFF regime is unstable.

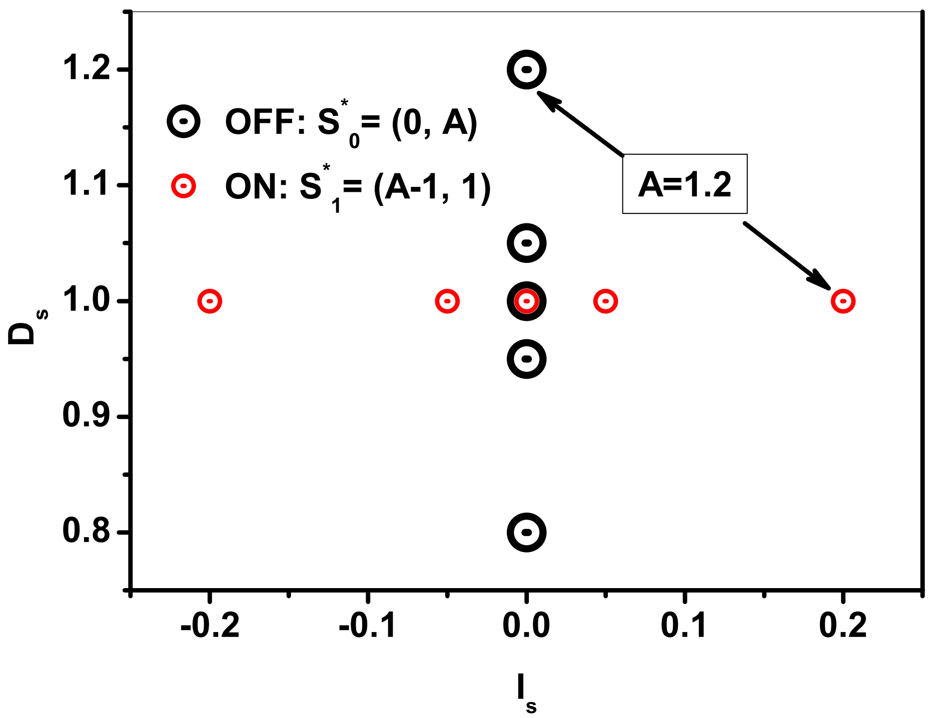


Figure 3.3: Location of the stationary points of Eq. (3.12) OFF and ON (see, the legends) in the phase space for five different values of the control parameter A: 0.8, 0.95, 1.0, 1.05, and 1.2

$$\sigma^2 + (\gamma A)\sigma + \gamma(A-1) = 0. \tag{3.16}$$

$$\sigma_1\sigma_2 = \gamma(A-1) > 0$$

$$\sigma_1 + \sigma_2 = -\gamma A < 0$$

and therefore both $Re\sigma_1 < 0$, $Re\sigma_2 < 0$.

Solving equation (3.16), we

$$\sigma_{1,2} = -\frac{1}{2}\gamma A \pm i\sqrt{(A-1)\gamma - \gamma^2 A^2/4} \tag{3.17}$$

$$\sigma_{1,2} \approx -\frac{1}{2}\gamma A \pm i\sqrt{(A-1)\gamma} + O(\gamma^{\frac{3}{2}}) \tag{3.18}$$

$$u(t) = c\exp(\sigma_1 t) + c^\star \exp(\sigma_2 t), \tag{3.19}$$

where the star in the second term denote complex conjugation.

$c = C\exp(i\phi)$

$$u(t) = C\exp\left(-\frac{1}{2}\gamma A t\right)\cos\left[\sqrt{\gamma(A-1)}t + \phi\right], \tag{3.20}$$

$\widetilde{\omega}_R = \sqrt{\gamma(A-1)}$ and $\widetilde{\Gamma} = \gamma A/2$, $\sqrt{\gamma}$ and γ.

$$u(T) = C\exp\left(-\frac{1}{2}\Gamma T\right)\cos(\omega_R T + \phi), \tag{3.21}$$

$$\Gamma = G T_c R_p, \tag{3.22}$$

$$\omega_R = [G R_p - (T_1 T_c)^{-1}]^{\frac{1}{2}}. \tag{3.23}$$

G, R_p, T_c^{-1}.

Dynamical Equations for the electric field in the laser's cavity

To this end, the first of eq. (3.10a)

$$\frac{dE}{dT} = \frac{GN}{2}E - \frac{E}{2T_c}. \tag{3.24}$$

Equation (3.24) is obtained $n = |E|^2$.

$$\frac{dE^\star}{dT} = \frac{GN}{2}E^\star - \frac{E^\star}{2T_c}.$$

$E^\star \frac{dE}{dT} + E\frac{dE^\star}{dT} = \frac{d}{dT}|E|^2 = \frac{dn}{dT}$

The second of the equations (3.11) modifies accourding to

$$\frac{dN}{dT} = -\frac{1}{T_1}(N - N_0) - 2GN|E|^2. \tag{3.25}$$

$\mathcal{E} = \sqrt{2GT_1}E$

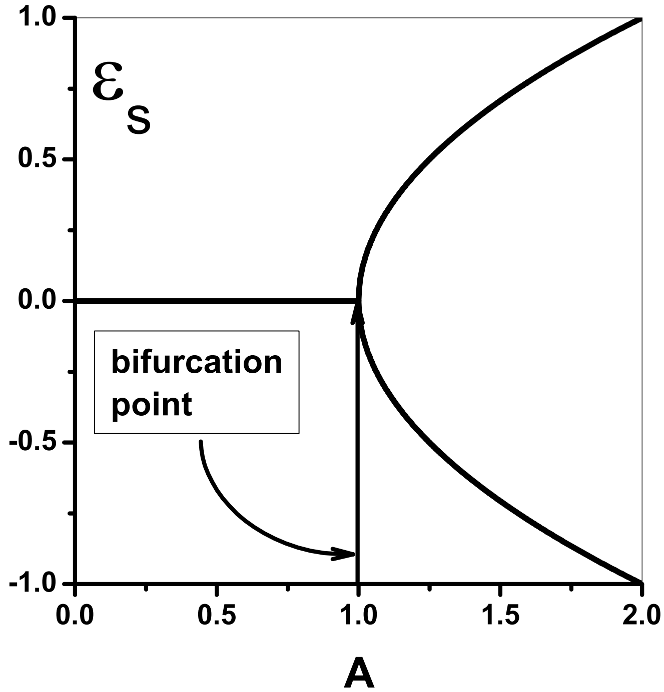


Figure 3.4:

$$\frac{d\mathcal{E}}{dt} = \frac{1}{2}\mathcal{E}(D-1) \tag{3.26a}$$

$$\frac{dD}{dt} = \gamma\left[A - D(1+\mathcal{E}^2)\right]. \tag{3.26b}$$

$\mathcal{E}_s = 0$, $D_s = A$, $D_s = 1$, $\mathcal{E}_s = \pm\sqrt{(A-1)}$, $I_s = \mathcal{E}_s^2$

3.3 Three-Dimensional Model

3.3.1 Physical considerations

In this section we extend the standard 2d rate equations (S2dRE) to higher dimensions. Physically, higher dimensional models correspond to lasers the operation of which involves three or more energy levels. Clearly, the model is dictated by the nature of the active medium (lasing material). For the majority of lasing media,

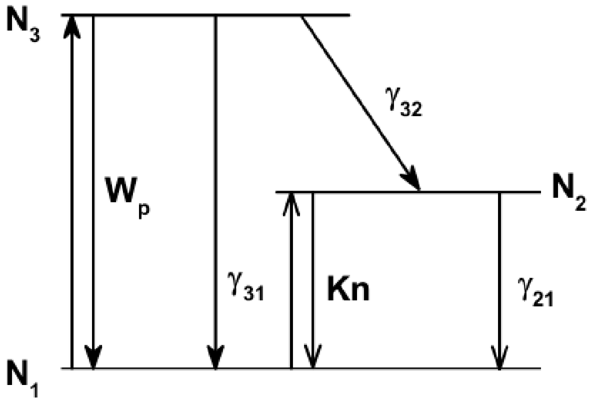


Figure 3.5: Energy levels of the ruby laser with their relaxation rates γ_{ij} indicated. The pumping is carried out from level (1), population N_1, to level (3), population N_3, with rate W_p. Level (3) is short-lived and hence the electrons exited to (3) drop almost instantaneously to levels (1) and (2). The stimulated emission occurs between the long lived levers (2) $\mapsto$ (1) with rate Kn, where n denotes the number density of the photons.

the S2dRE model does not provide adequate description. On the other hand, to access aspects like power conversation efficiency, response time, etc, it is often good enough to consider three- and four-dimensional models. The benefit of this fact is that the laser dynamics depends and therefore the laser operation can be controled by just few few physical parameters.

Rather than adopting a general approach to the problem, we take up specific examples of laser systems. These are the ruby laser – active material Cr^{3+}: Al_2O_3 – and the CO_2 laser. They are also important in the practical applications.

3.3.2 Model of ruby laser

The dynamics of the ruby laser could directly be described initially within the framework of a three-dimensional model. The pumping scheme involving three-levels was originally suggested in [7] and further elucidated in [8]; see Fig. 3.5 and its description.

$$\frac{dN_1}{dT} = \gamma_{21}N_2 - W_p(N_1 - N_3) + kn(N_2 - N_1) + \gamma_{31}N_3 \tag{3.27a}$$

$$\frac{dN_2}{dT} = \gamma_{32}N_3 - \gamma_{21}N_2 - kn(N_2 - N_1) \tag{3.27b}$$

$$\frac{dN_3}{dT} = W_p(N_1 - N_3) - \gamma_{32}N_3 - \gamma_{31}N_3 \tag{3.27c}$$

$$N_1 + N_2 + N_3 = N_T, \tag{3.28}$$

where N_T is a constant. To see how this property comes out, we add eqs. (3.27) together, obtaining readily $\frac{dN}{dT}(N_1 + N_2 + N_3) = 0$. For the ruby crystal, the the lifetime of the upper laser level, that is level (2), is exceptionally long; according to some measurements, $\gamma_{21}^{-1} = 3$ ms and according to others $\gamma_{21}^{-1} = 4.3$ ms. On the other hand, the relaxation rates from level 3 to level 2 or from level 3 to level 1 are fast compared to γ_{21}^{-1}. (γ_{32}^{-1} and γ_{31}^{-1} are of the order of 10^{-4} ms)
Furthermore, we note the inequalities

$$\gamma_{32}^{-1} << W_p^{-1}, \gamma_{31}^{-1} \tag{3.29}$$

As soon as one atom is excited from 1 to level 3, it will almost instantaneously be de-excited to level 2 and N_3 will remain small. Mathematically,we assume N_3 is small compared to N_1 and that $\gamma 31 N_3$ and N_3' are both small compared to W_pN_1 eq (3.1) and (3.2) simplifies as

$$\frac{dN_1}{dT} = \gamma_{21}N_2 - W_pN_1 + kn(N_2 - N_1) \tag{3.30a}$$

$$\frac{dN_2}{dT} = \gamma_{32}(N_3 - N_3) - \gamma_{21}N_2 - kn(N_2 - N_1) \tag{3.30b}$$

$$0 = W_pN_1 - \gamma_{32}N_3 \tag{3.30c}$$

$$N_T = N_1 + N_2 \tag{3.30d}$$

solving equation (3.4c) we have

$$N_3 = \frac{W_pN_1}{\gamma_{32}} < 1 \tag{3.31}$$

using equation (3.5) and (3.4b) we further simplify as

$$\frac{dN_2}{dT} = W_p N_1 - \gamma_{21} N_2 - kn(N_2 - N_1) \tag{3.32}$$

Introducing the inversion of population$N = N_2 - N_2$ and using equations (3.4a),(3.5)and (3.6) we determine equation for N

$$\frac{dN}{dT} = -\gamma_{21}(N + N_T) - W_p(N - N_T) - 2knN \tag{3.33}$$

The right hand side of equation (3.7) displays the three main processes appearing in laser action. The first term models the relaxation to equilibrium in the absence of pumping: N relaxes towards -N_T since the population accumulates in level 1 under the influence of single relaxation process. The second term describes the pumping process which creates the inversion of population (if $W_p > \gamma_{21}$): in case of very strong pumping if ($W_p >> \gamma_{21}$), and in the absence of laser emission ($n = 0$), the population accumulates in level 2 ($N = N_T$). The last term indicates the nonlinear coupling between population and intensity as the result of stimulated emission. Equation (3.7) for N is coupled to an equation for the laser number of photons given by

$$\frac{dn}{dT} = n(-\gamma_c + 2kN) \tag{3.34}$$

equation (3.8) is identical to equation (2.11a); with $k = G$ and $\gamma_c = T_c^{-1}$ introducing the new variables

$$t \equiv \gamma_c T, I \equiv \frac{2kn}{\gamma_{21} + W_p} and D \equiv \frac{kN}{\gamma_c} \tag{3.35}$$

$$\frac{dI}{dT} = I(-1 + D) \tag{3.36a}$$

$$\frac{dD}{dT} = \gamma \left[A - D(1 + I)\right] \tag{3.36b}$$

where $\gamma = \dfrac{\gamma_{21} + W_p}{\gamma_c}$ and $A = \dfrac{(W_p - \gamma_{21})kN_T}{(\gamma_{21} + W_p)\gamma_c}$
equations (3.10) are identical to equations (2.12)

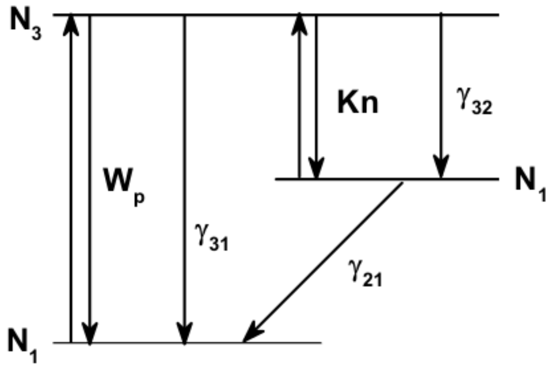


Figure 3.6: Energy levels of the CO_2 laser with their relaxation rates γ_{ij} indicated. The pumping is carried out from level (1), population N_1, to level (3), population N_3, with rate W_p. The stimulated emission occurs between the long-lived level (3) and level (2) with rate Kn, where n denotes the number density of the photons. Level (2) is short-lived and hence the electrons drop almost instantaneously to level (1).

3.3.3 CO_2 laser

Assuming independence of the three basic processe (pumping, relaxation, stimulated emission), the population equation for N_1, N_2, and N_3 are now given by

$$\frac{dN_1}{dT} = -W_p(N_1 - N_3) + \gamma_{21}N_2 + \gamma_{31}N_3 \tag{3.37a}$$

$$\frac{dN_2}{dT} = \gamma_{32}N_3 - \gamma_{21}N_2 + kn(N_3 - N_2) \tag{3.37b}$$

$$\frac{dN_3}{dT} = W_p(N_1 - N3) - \gamma_{32}N_3 - \gamma_{31}N_3 - kn(N_3 - N_2) \tag{3.37c}$$

As for the ruby laser we assume coherent pumping, i.e the pumping mechanism induces back and forth transition between levels 1 and 3. For

$$\frac{dN_1}{dT} + \frac{dN_2}{dT} + \frac{dN_3}{dT} = 0$$

We would take advatage of the relatively small values ofγ_{32} and W_p compared to either γ_{21} or γ_{31} The large value of γ_{21} and γ_{31} means that N_2 and N_3 are small compared to N_1 because they rapidly relax to their equilibrium values. From equation (3.2) $N_1 = N_T$. With $N_1 = N_T$ and neglecting all $\gamma_{32}N_3$ terms equations (3.11) simplify as

$$\frac{dN_1}{dT} = -W_pN_T + \gamma_{21}N_2 + \gamma_{31}N_3 \tag{3.38a}$$
$$\frac{dN_2}{dT} = -\gamma_{21}N_2 + kn(N_3 - N_2) \tag{3.38b}$$
$$\frac{dN_3}{dT} = W_pN_T - \gamma_{31}N_3 - kn(N_3 - N_2) \tag{3.38c}$$

We introduced the inversion of population

$$N = N_3 - N_2 \tag{3.39}$$

and express N_2 interms of N_3 and N as N_2=$N_3 - N$, from equation (3.12)

$$\frac{dN_1}{dT} = -W_pN_T + \gamma_{21}(N_3 - N) + \gamma_{31}N_3 \tag{3.40a}$$
$$\frac{dN_1}{dT} = W_pN_T - \gamma_{31}N_3 - 2knN + \gamma_{21}(N_3 - N) \tag{3.40b}$$

since the total population is
$N_1 + N_2 + N_3 = N_1 + 2N_3 - N = N_T$

$$N_3 = \frac{N_T - N_1 + N}{2} \tag{3.41}$$

substitute equation (3.15) into (3.14)

$$\frac{dN_1}{dT} = \gamma_1 N + \gamma_2(N_T - N_1) - W_pN_T \tag{3.42a}$$
$$\frac{dN}{dT} = -\gamma_1(N_T - N_1) - 2knN - \gamma_2 N + W_pN_T \tag{3.42b}$$

Where $\gamma_1 = \dfrac{\gamma_{31} - \gamma_{21}}{2}$ and $\gamma_2 = \dfrac{\gamma_{21} + \gamma_{31}}{2}$

We now introduce the following dimensionaless variables $t = \gamma_c T$, $I = \dfrac{2kn}{\gamma_2}$ and $u = \dfrac{kN}{\gamma_c}$

$$W = \frac{k}{\gamma_c\gamma_2}(-\gamma_1(N_T - N_1) + W_pN_T) \tag{3.43}$$

From equations (3.8) and (3.16) we obtain

$$\frac{dI}{dT} = I(-1 + u) \tag{3.44a}$$

$$\frac{du}{dT} = \mathcal{E}(W - u(1 + I)) \tag{3.44b}$$

$$\frac{dW}{dT} = \mathcal{E}(A + bu - W) \tag{3.44c}$$

$\mathcal{E}\equiv\frac{\gamma_2}{\gamma_c}$, $b\equiv(\frac{\gamma_1}{\gamma_2})^2$ and $A\equiv\frac{kW_pN_T}{\gamma_c\gamma_2}(1 - \frac{\gamma_1}{\gamma_2})$

From equation (3.18)the population inversion U which is coupled to a reservoir population W. Both U and W are slow variables because the right hand sides of the equations for U and W are proportional to $\mathcal{E}$ which is a small parameter. A is the control parameter and there are only two fixed parameters, b and E. Using the values of the parameters given by Lefrac et al. We find $b = 0.85$ and $\mathcal{E} = 0.1375$. From equation (3.18), we find the following steady state solution.

$$I = 0, W = U = \frac{A}{1 - b} \tag{3.45a}$$

$$I = A + b - 1 \geq 0, U = 1, W = A + b \tag{3.45b}$$

corresponding to OFF and ON states, respectively. The value of b is close to 1 because $\gamma_{31} << \gamma_{21}$. However we cannot set b equal to 1 because equation (3.19a) is singular at b=1.

From equation (3.19b) we find that the lasing threshold is $A = A_{th} = 1-b$ suggesting a drastic reduction of the lasing threshold from a two to a three level system. This is however not the case because the definition of A is quite different in the two and three level problems. Practically A is not calculated from the physical constants but it is normalized using the threshold pump as a reference since it can be determine experimentally. If this is done, the OFF and ON steady states are $(I, U)\equiv(0, A/A_{th})$ and $(I, U)\equiv(A - A_{th}, 1)$, respectively.

Linear Stability Analysis

We wis to find how a small perturbation of the either (3.19a) or (3.19b) will grow or decay. let insert $I = I_s + i$, $U = U_s + u$ and $W = W_s = w$ into equation (3.18) where (I_s, U_s, W_s) denotes the zero intensity steady state of equation (3.19b). Simplifying the resulting equations and neglecting all nonlinear contributions in $i, uandw$. We obtain the following linear equation for i,u and w.

$$\frac{d}{dt}\begin{pmatrix} i \\ u \\ w \end{pmatrix} = \begin{pmatrix} U_s - 1 & I_s & 0 \\ -\mathcal{E}U_s & -\mathcal{E}(1 + I_s) & \mathcal{E} \\ 0 & \mathcal{E}b & -\mathcal{E} - \sigma \end{pmatrix}\begin{pmatrix} i \\ u \\ w \end{pmatrix} \tag{3.46}$$

The general solution is a linear combination of the exponential solution of the form

$$i = c_1 exp(\sigma t), u = c_2 exp(\sigma t).andw = c_3 exp(\sigma t) \tag{3.47}$$

Where σ is the growth rate and the c_j are constants. substituting (3.21) into (3.20), we have a homogeneous system of equations for c_1, c_2, c_3. A trivial solution is possible only if

$$\begin{bmatrix} U_s - 1 - \sigma & I_s & 0 \\ -\mathcal{E}U_s & -\mathcal{E}(1 + I_s) - \sigma & \mathcal{E} \\ 0 & \mathcal{E}b & -\mathcal{E} - \sigma \end{bmatrix} = 0 \tag{3.48}$$

For the zero intensity steady state (3.19a) and (3.22) leads to the following characteristic equation

$$\left(\frac{A}{1-b} - 1 - \sigma\right)(\sigma^2 + 2\mathcal{E}\sigma + \mathcal{E}^2(1 - b)) = 0 \tag{3.49}$$

$\sigma_1 = \dfrac{A}{1-b} - 1$. σ_2 and σ_3 satisfy $\sigma^2 + 2\mathcal{E}\sigma + \mathcal{E}^2(1 - b) = 0$. From the sign of coefficient and since $b < 1$ the real part is always negative. The stability is determine by σ_1 only. σ_1 changes sign at $A = A_{th} = 1 - b$ and the solution is stable (unstable) if $A < A_{th}$ $(ifA > A_{th})$

For the non-zero intensity steady state (3.19b) and (3.22) leads to the following characteristic equation for σ

$$\sigma^3 + c_1\sigma^2 + c_3\sigma + c_3 = 0 \tag{3.50}$$

$c_1 \equiv 2\mathcal{E} + \mathcal{E}I_s$, $c_2 \equiv \mathcal{E}I_s + \mathcal{E}^2 A$, $c_3 \equiv \mathcal{E}^2 I_s$
The necessary and sufficient conditions for σ to have a negative real part are known as the Routh-Hurwitz conditions, these requires the following inequalities on the coefficient c_j
c_j (j=1,2,3)
$c_1 > 0$, $c_3 > 0$ and $c_1c_2 - c_3 > 0$.
They are easily verified since $\mathcal{E}$ and I_s are both negative. The non-zero intensity solutionis always stable.

Chapter 4

Simulation Results

4.1 Simulation

In this chapter the two-dimensional model of the rate equations derived in the previous chapter were simulated using Matlab and Runge-Kutta.

Matlab was used to simulate equations (3.12a) and (3.12b). The simulation was done for various values of the pump parameters A and for three different types of lasers namely:
Semicondoctor Laser
CO_2 Laser
Solid State Laser.
The results of the simulation was used to discribed the dynamical behaviour of the lasers.

Laser	$T_c(s)$	$T_1(s)$	γ
CO_2	10^{-8}	4×10^{-6}	2.5×10^{-3}
Solid State ($Nd^{+3}:YAG$)	10^{-6}	2.5×10^{-4}	4×10^{-3}
Semiconductor($AsGa$)	10^{-12}	10^{-9}	10^{-3}

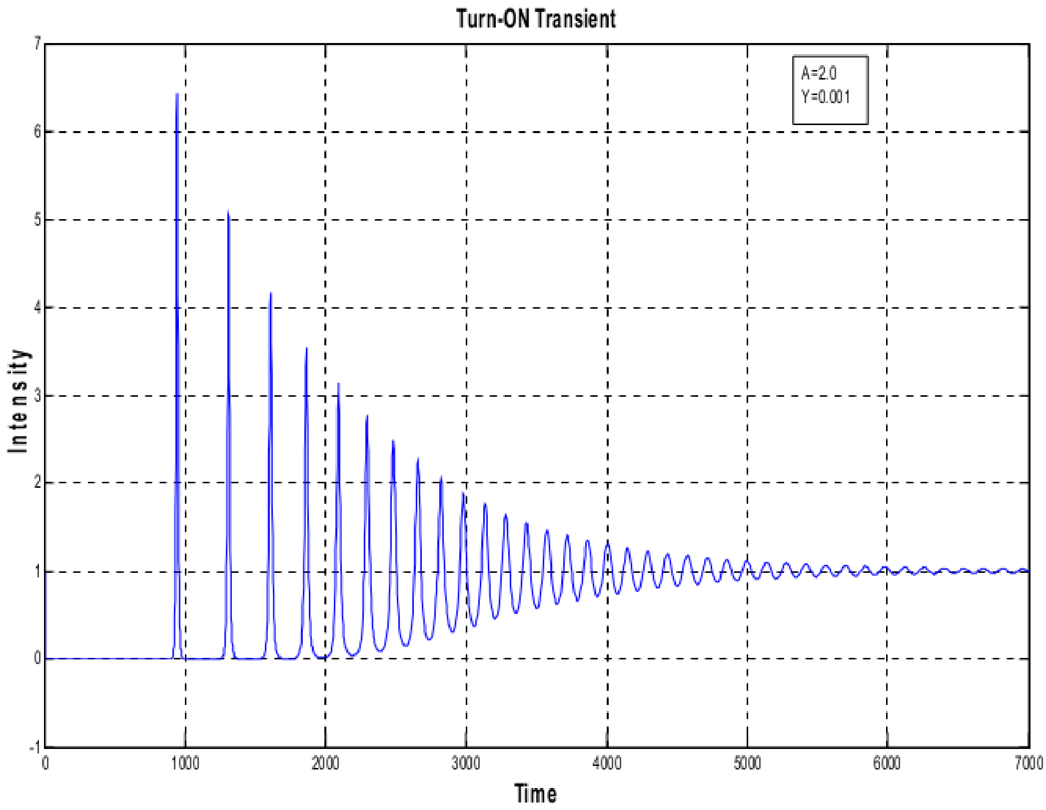


Figure 4.1: Switch-on transient of semiconductor laser for pump parameter A=2

4.2 Switch-on Transient and phase portrait

The intensity was plotted against time fig. 4.1 to fig. 4.6. The phase portrait is also plotted for population density and intensity fig. 4.7 to fig. 4.12.

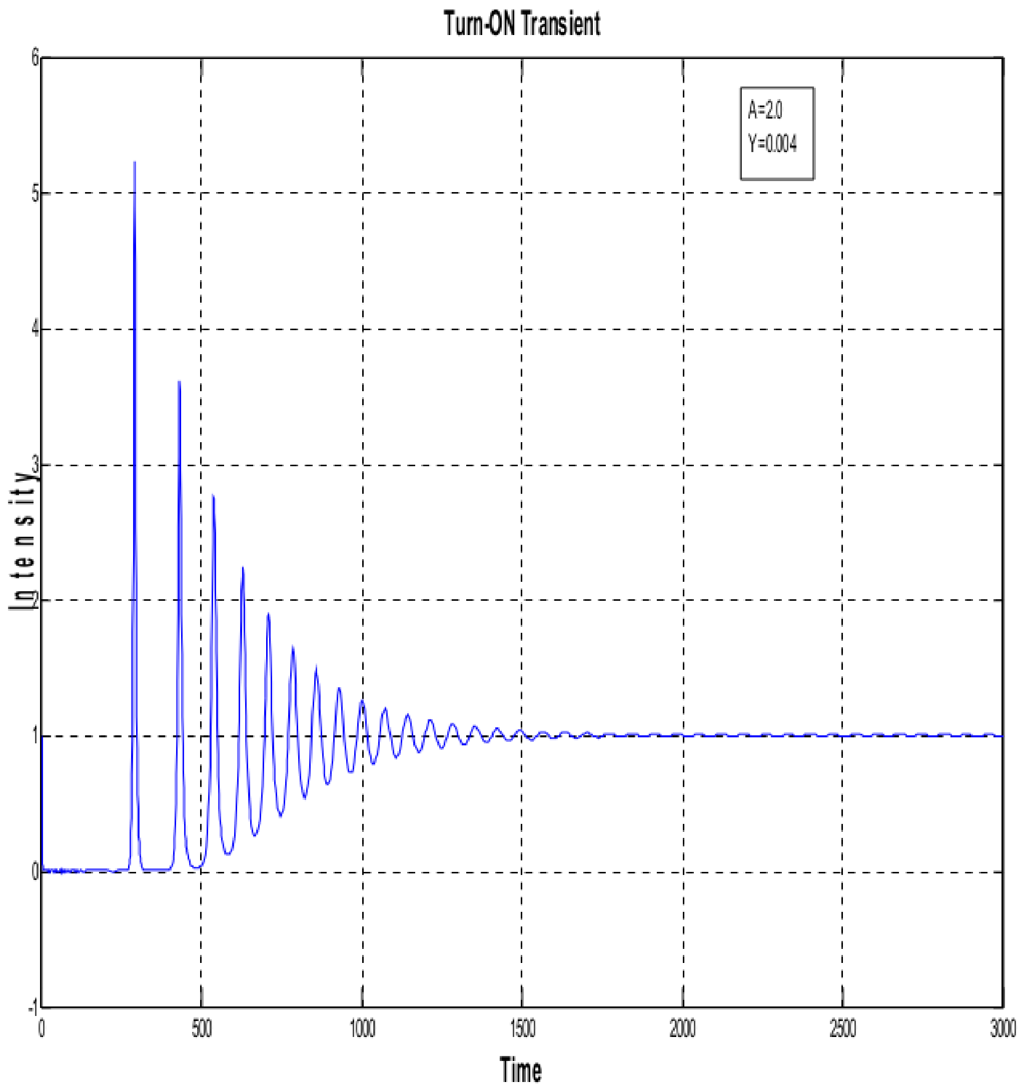


Figure 4.2: Switch-on transient of solid state laser for pump parameter A=2

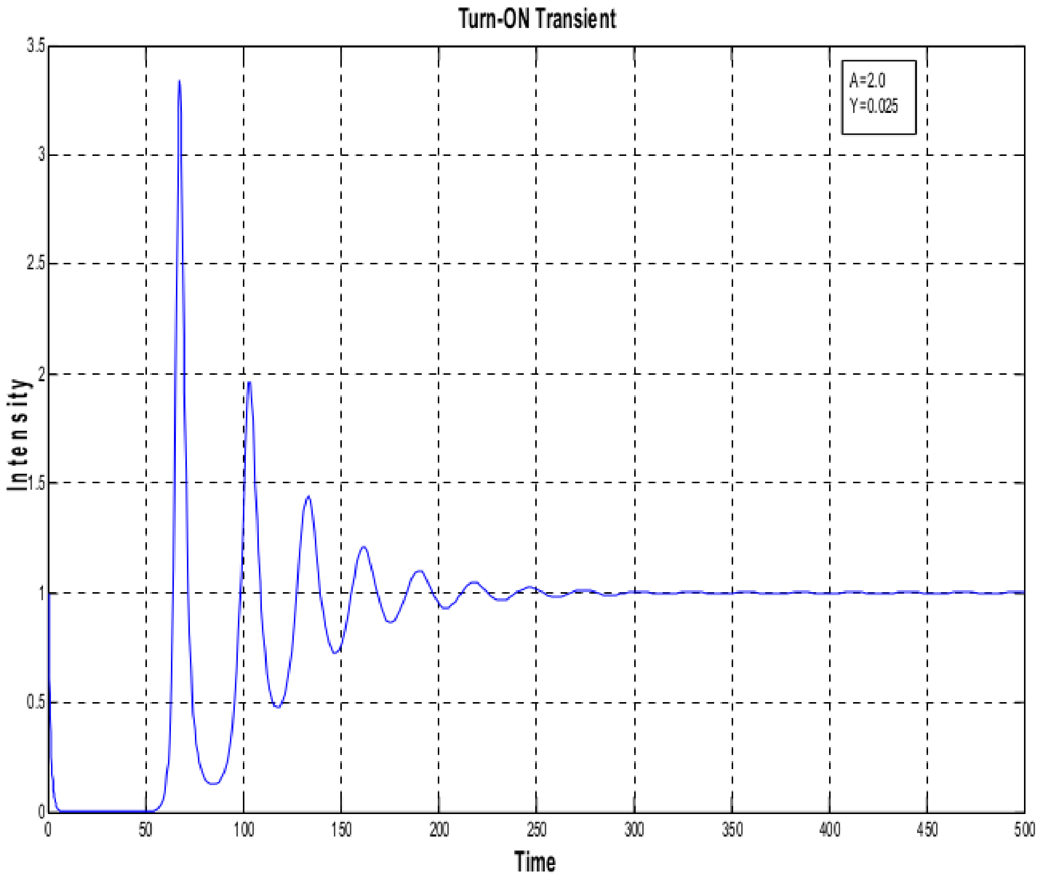


Figure 4.3: Switch-on transient of CO_2 laser for pump parameter A=2

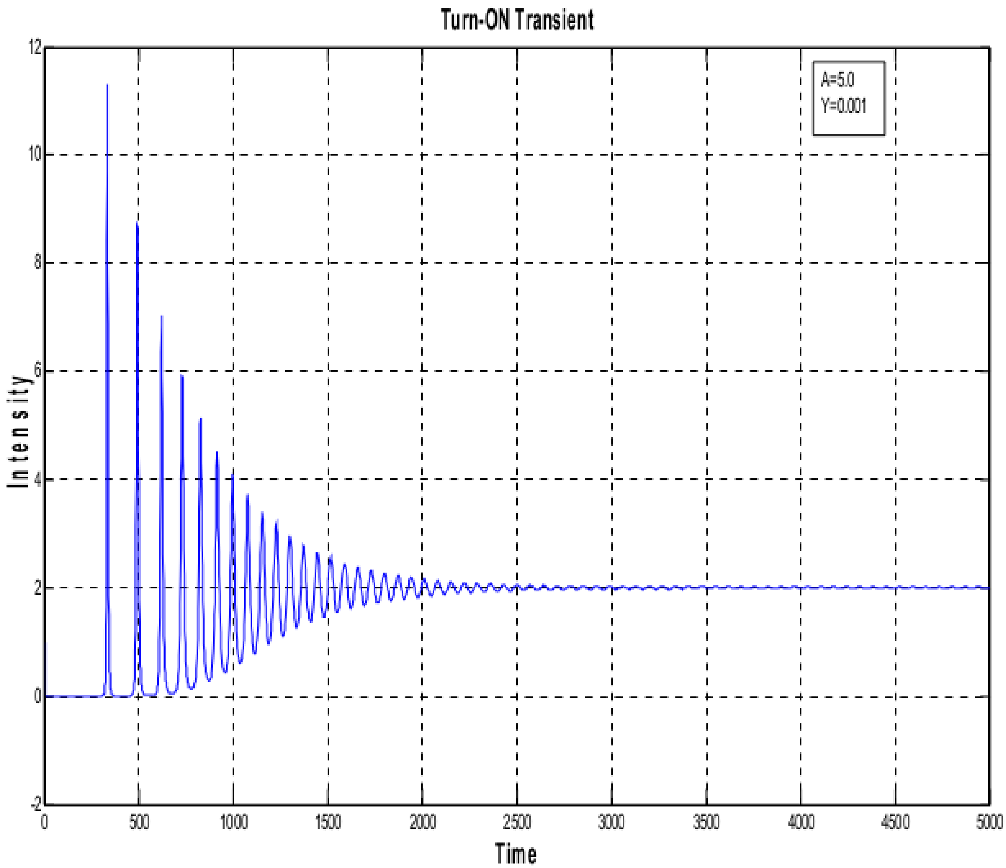


Figure 4.4: Switch-on transient of semiconductor laser for pump parameter A=5

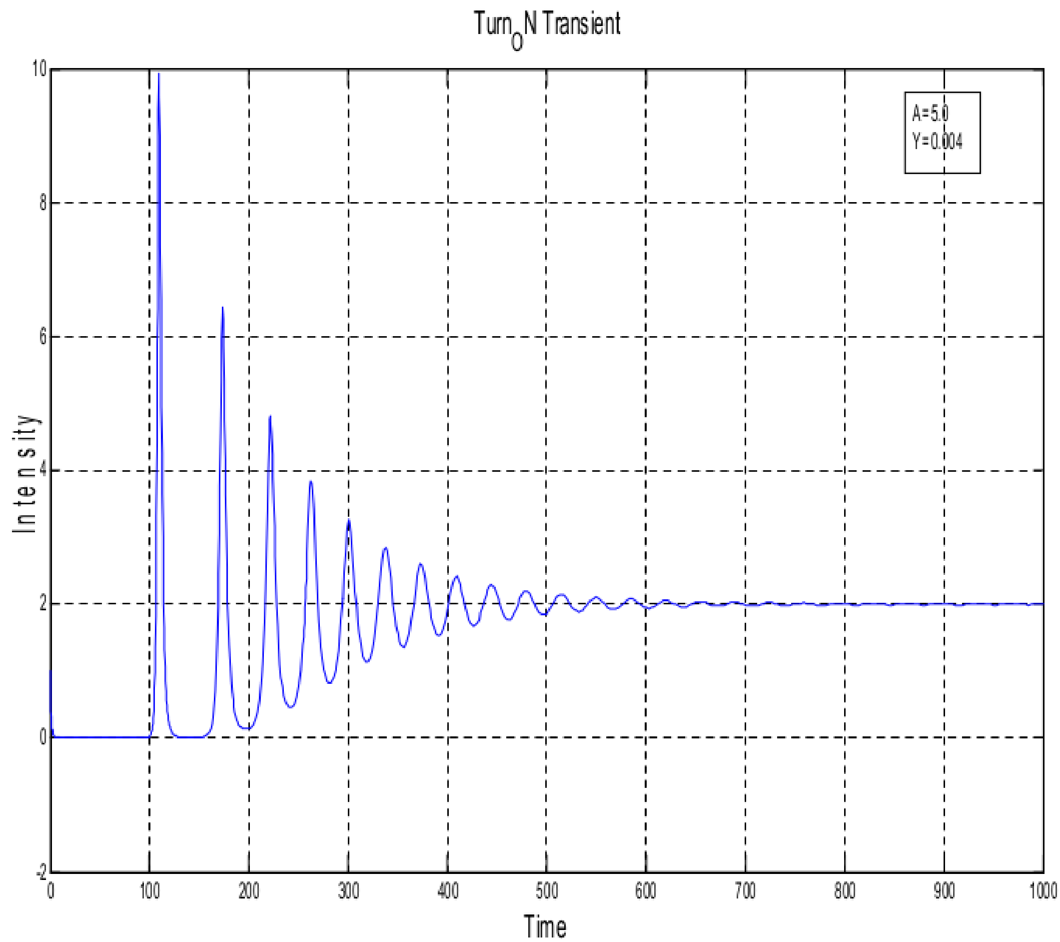


Figure 4.5: Switch-on transient of solid state laser for pump parameter A=5

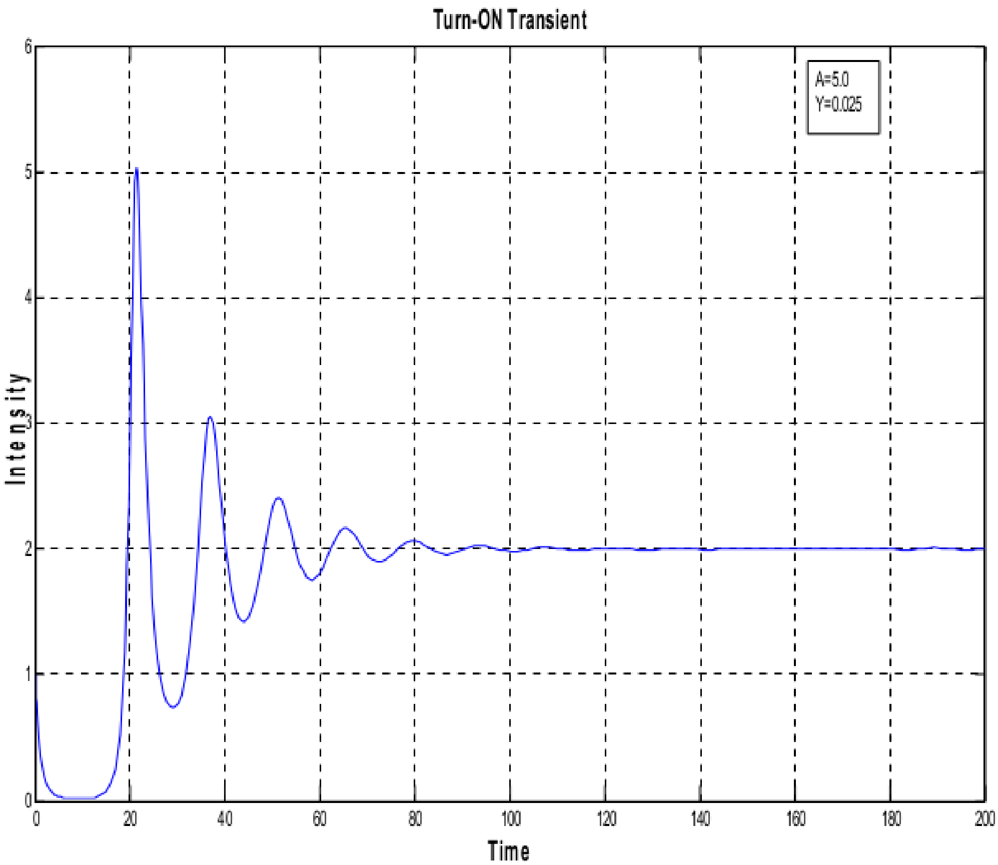


Figure 4.6: Switch-on transient of CO_2 laser for pump parameter A=5

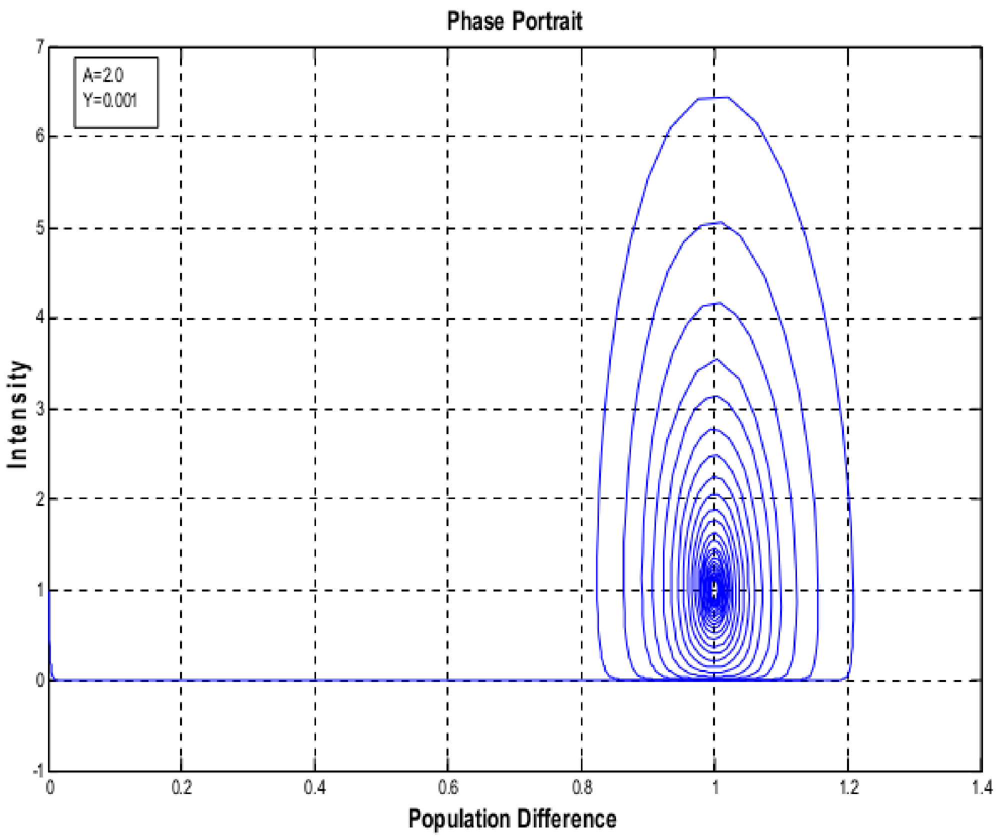


Figure 4.7: Phase porrait of semiconductor laser for pump parameter A=2

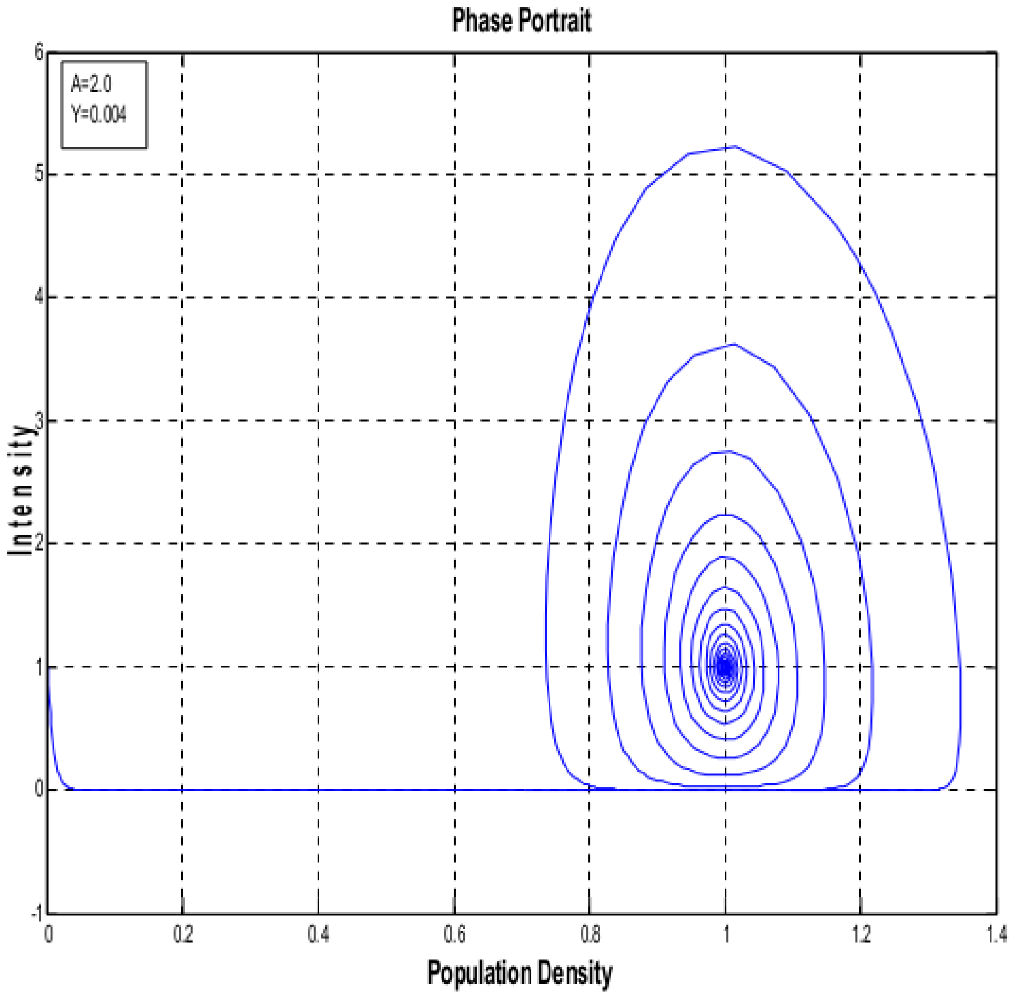


Figure 4.8: Phase porrait of solid state laser for pump parameter A=2

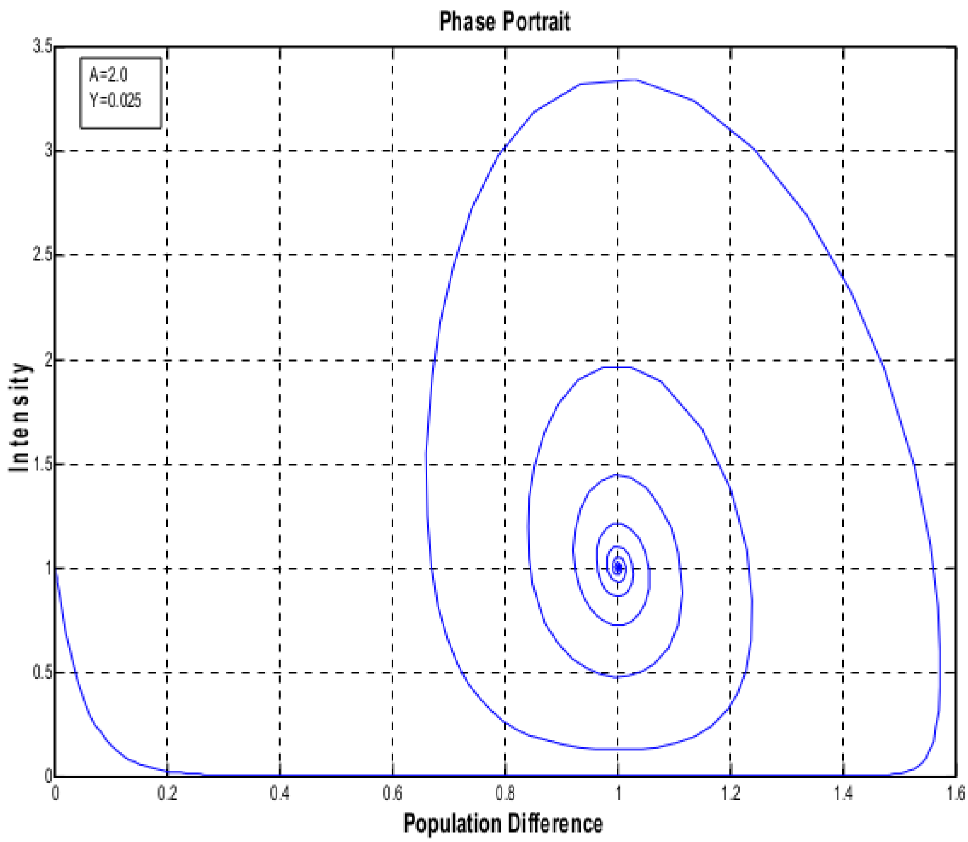


Figure 4.9: Phase porrait of CO_2 laser for pump parameter A=2

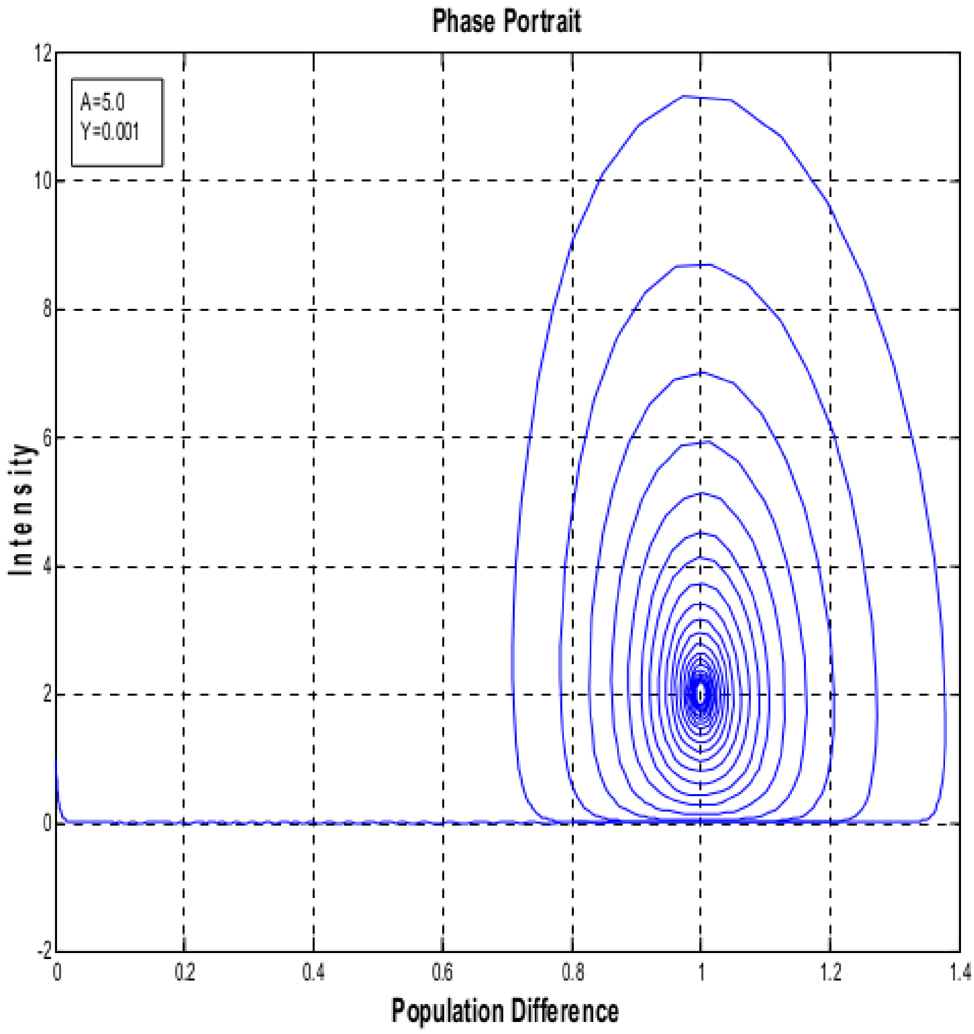


Figure 4.10: Phase porrait of semiconductor laser for pump parameter A=5

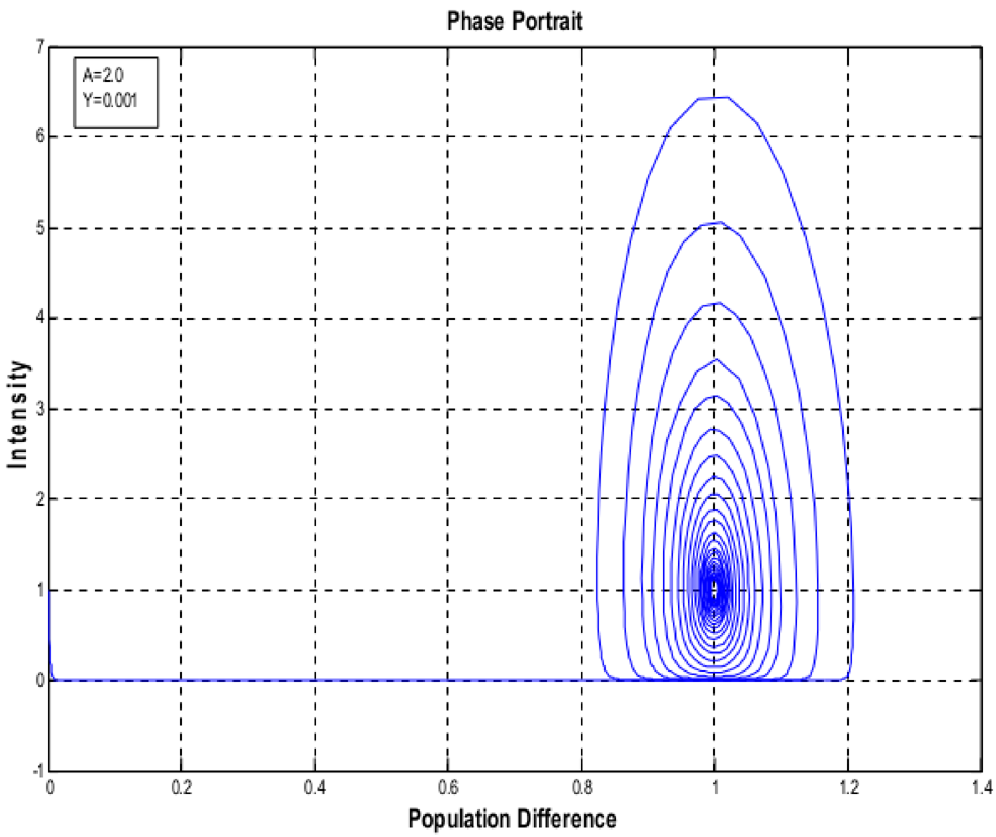


Figure 4.11: Phase porrait of solid state laser for pump parameter A=5

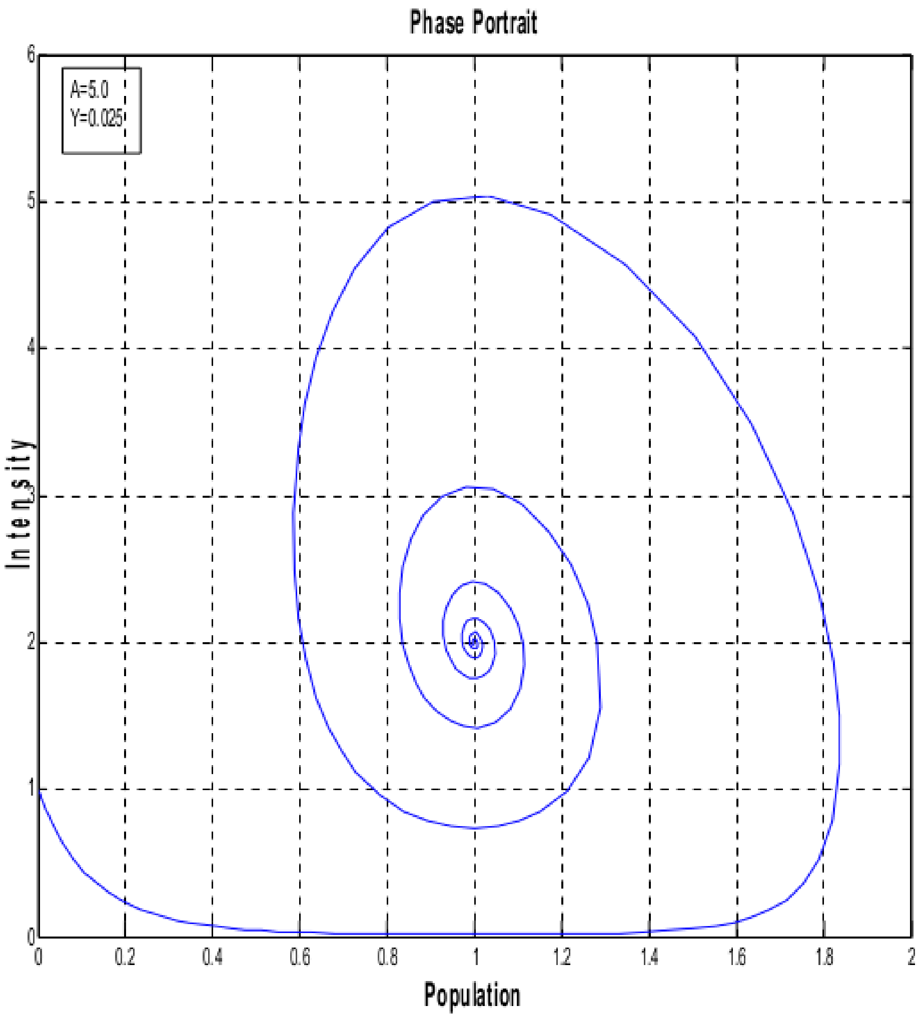


Figure 4.12: Phase porrait of CO_2 laser for pump parameter A=5

4.3 Comparison of the linear approximation vs. exact numerical solution

In this section we obtain numerical sulutions of the 2d nonlinear laser model of equations (3.26). This is carried out using a common variable-degree Runge-Kutta procedure with well controlled precision. The solution for the value of the parameter $A = 2$ for both the dimensionless field $\mathcal{E}$ and population density difference D is shown in Fig. (4.14) by the solid lines. The phase portrait of equations (3.26) is shown in fig. (4.15).

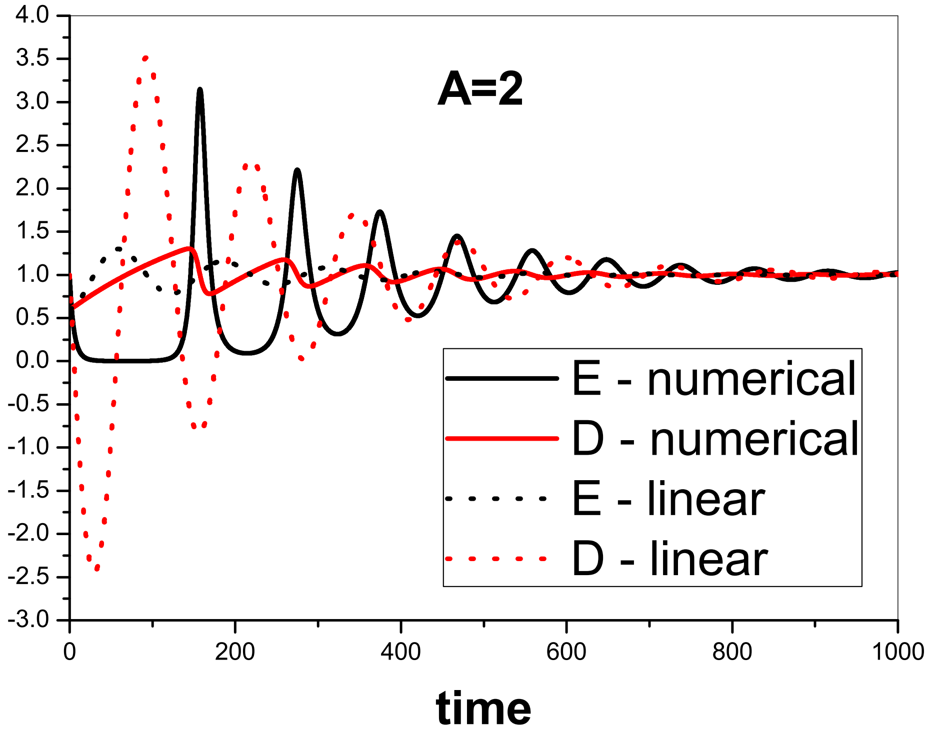
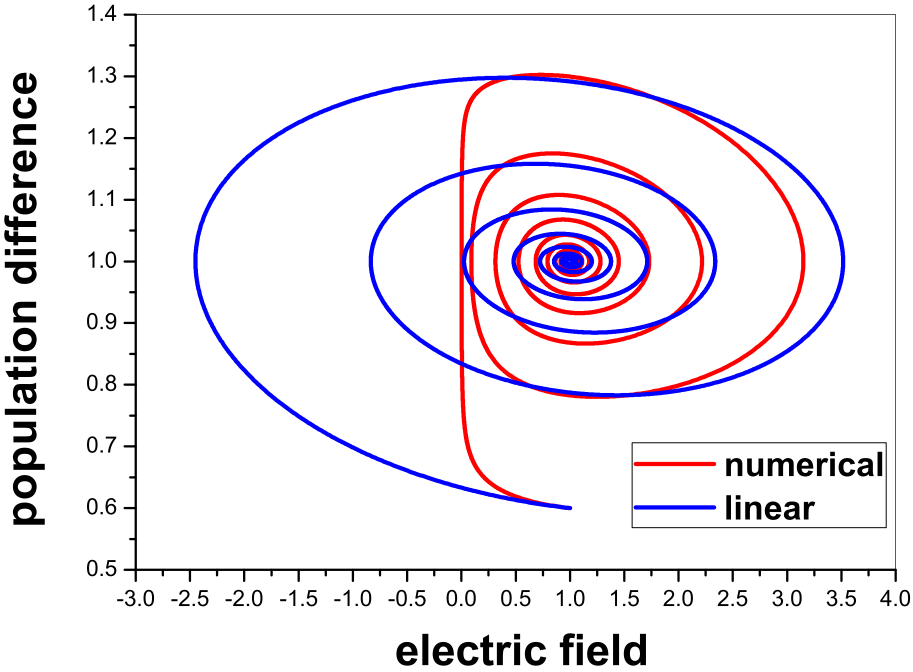


Figure 4.13:



1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
population difference
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
electric field
numerical
linear

Figure 4.14:

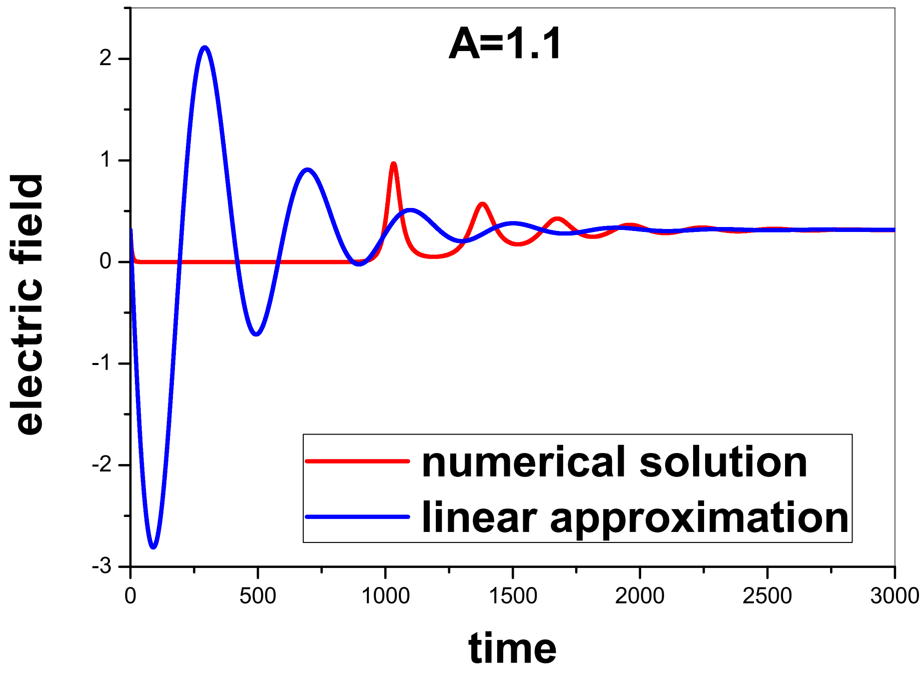


Figure 4.15:

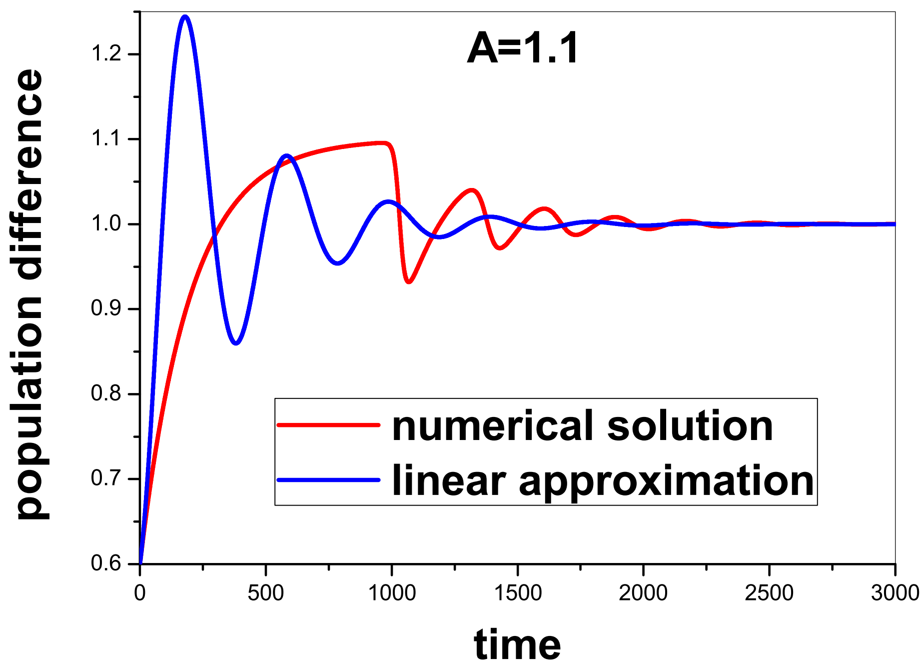


Figure 4.16:

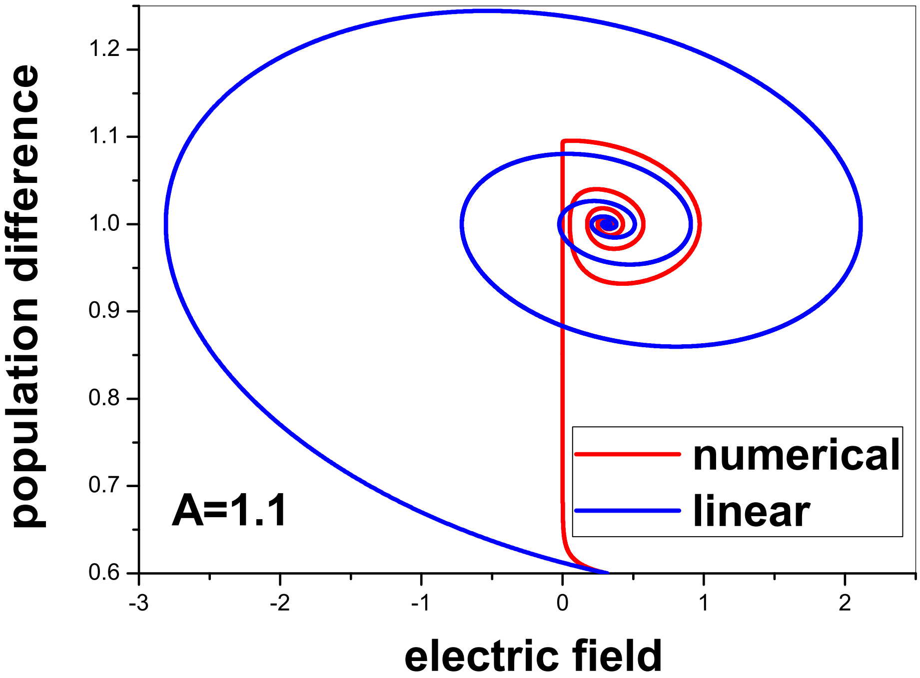

population difference
1.2
1.1
1.0
0.9
0.8
0.7
0.6
-3
-2
-1
0
1
2
electric field
A=1.1
numerical
linear

Figure 4.17:

Chapter 5

Conclusion and Recommendations

5.1 Conclusion

Fig (4.1) to (4.6) show a typical turn on time for the most common lasers used today (solid state, CO_2, and semiconductor lasers). We switch the pump from an above threshold value and observe the time evolution of the intensity. Three distinct regimes were observed.
1. A time interval where the laser power remain very low. This region is called the " latency," "lethargy," or "turn-on" region. The delay of the laser transition is called turn-on time or delay.
2.A strongly pulsating intensity regime during which the laser emitts a series of sharp spikes separated by periods of very low emission.
3.A region of damped oscillation as the laser approaches its steady state through exponentially damped sinusoidal oscillations.

For pump parameter A=2, from fig. (4.1) it was observed that the semiconductor laser has the highest latency period, the highest intensity spikes and it takes a longer time for the laser to come to steady state.
CO_2 laser has the lowest latency period
lowest intensity spikes and
it takes a shorter time for the laser to come to steady state.
From the graph it was also observed that as the pump parameter A increases the latency time reduces, the intensity increases and it takes a shorter period for the

laser to come to steady state.
The graphs of the turn-on transient and phase portraits are compactable, they clearly explain the trajectory of the dynamical system of lasers.

The 2D version of the rate equations were simulated. It was observed from the results that as the pump power A increases the latency time decreases, the intensity increases and it takes a shorter time for the laser to come to relaxation oscillation. From the results it was also observed that the semiconductor laser has the highest latency period because it takes some time to move the electron from the valence band to the conduction band. The CO_2 laser has the shortest latency time, lowest intensity spikes and it takes a shorter time for the laser to come to relaxation oscillation. The solid state laser lies between the semiconducor and the CO_2 laser.

5.2 Recommendations

The following are suggestions for future reseach:
Quantum and semiclassical approach should be used to derive the rate equations.
The numerical solution should be extended to 3D.
The research should be done for more lasers.

Bibliography

[1] Manus J. Donahue *The Chaos Theory.*

[2] H. Haken, Light *(North-Holland, Amsterdam, 1985)*

[3] A.L Schawlow and C.H Townes, *phys. Rev. 112, 1958*

[4] T.H Maiman, *Nature 187, 493 (1960)*

[5] Bernard, Jaffe. *A compact Science Dictionary. Edited by G. E speck.* London: Ward, Lock and Co, 1954.

[6] Y.I. Khanin, Principles of Laser Dynamics, Elsevier, Amsterdam (1995)

[7] About possible methods for obtaining active molecules for a modlecular oscillator, *Zh. Eksp. Teor. Fiz.* **28/2,** 249–250 (1955), translated in *Sov. Phys. JETP* **1**, 184–185 (1956).

[8] N. Blombergen, Proposal for a new type solid state maser, *Phys. Rev.* **104**, 324–327 (1957).

[9] J.R. Tredicce, F.T. Arecchi, G.L. Lippi, and G.P. Puccioni, Instabilities in lasers with an injected signal, J. Opt. Soc. Am. B 2, 173–183 (1985)

[10] W.E. Boyce, R.C. DiPrima, Elementary Differential Equations, 4th edn, JohnWiley, New York (1986)

[11] G. Nicolis, Introduction to Nonlinear Science, Cambridge University Press, Cambridge (1995)

[12] R. Haberman, Mathematical Models: Mechanical Vibrations, Population Dynamics, and Traffic Flow. An Introduction to Applied Mathematics, Prentice-Hall, Englewood Cliffs, NJ (1977), reprinted SIAM Classic Series 21, Society for Industrial and Applied Mathematics, Philadelphia, PA (1998)