IMPLICATION OF VOID PREDICTION IN THE DETERMINATION OF PRESSURE GRADIENT IN VERTICAL PIPES

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By

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ABSTRACT

This research work mainly investigates the implication of void prediction in the estimation of total pressure gradient in vertical pipes for multiphase flow systems. Experimental data was collected for a multiphase flow system with silicone oil and air as the liquid and gas phases. The void fraction prediction was carried out using Microsoft Excel. Ten correlations were used for void estimation in chronological order to include statistical analysis of correlation performance. Nicklin et al. (1962) drift flux correlation gives the best void fraction for bubble flow. The prediction from this correlation shows a fairly constant average absolute error of about 20.98% for low gas rate flow (bubble). Greskovich and Cooper (1975) give the best prediction for void fraction in slug flow regime with about 4.84% average absolute error in void fraction prediction. Hassan and Kabir (1989), show progressively higher accuracy and stability in the direction of increasing gas rate with an average absolute error of 6.99% in the churn flow regime. Hence a good correlation for transitional flow region.

Pressure gradient prediction was carried out using two separate approaches: the Homogeneous model and the Duns and Ros model (1963).The statistical parameters used in this study are percentage absolute average error, average absolute and relative error. The parameters calculated were compared to determine the performance of the different correlations evaluated. The realization of this work was used to develop a quality assurance flow scheme for vertical sections.

Keywords: *Void fraction, pressure gradient, bubble flow, slug flow, churn flow, flow assurance scheme, homogeneous models.*

DEDICATION

This study is dedicated to God Almighty for his faithfulness, love, and grace towards me. And to my beloved wife Abiodun Jatto, my daughter Moyosoreoluwa Jatto, my parents and siblings for their support and encouragement. May God bless you all (Amen).

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Finally to my great friends, Daniel Ocran and Adebayo M and the entire class of AUST-PET-2015-16. May God continue to keep and protect you all. (Amen).

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NOMENCLATURE

A	Flow Area
ρ	Density (kg/m^3)
μ	Viscosity (kg/ms)
U, v	Velocity (m/s)
U_{GM}	Drift flux velocity (m/s)
C_o	Distribution Parameter
C_A	Armand coefficient
D	Pipe diameter (m)
ε	Void fraction
E_k	Dimensionless kinetic energy
ϵ	Pipe roughness (m)
Re	Reynolds number
f	Friction factor
$\frac{f}{f_n}$	Normalized friction factor
g	Gravitational acceleration (m/s^2)
G	Mass flux (kg/s.m^2)
σ	Surface tension (N/m)
Fr	Froude number
L	Length of pipe (m)
N_{LV}	Liquid velocity number

N_{GV}	Gas velocity number
N_d	Diameter number
N_L	Liquid velocity number
N_{ew}	Weber number
Q	Volumetric flow rate (m^3/s)
V	Volume (m^3)
H	Liquid holdup
θ	Pipe inclination
λ	No-slip holdup
S	Slip-velocity number
$\frac{dP}{dZ}$, ΔP	Pressure gradient (Pa/m)

SUBSCRIPTS AND SUPERSCRIPTS

G	Gas phase
g	Gravitational
acc	Accelerational
dn	Downflow
up	Upflow
F	Frictional
L	Liquid phase
SG	Gas superficial
SL	Liquid superficial
M	Mixture
n	No-slip
$GV_{Tr/M}$	Transition/mist boundary
$GV_{S/Tr}$	Slug/transition boundary
$GV_{B/S}$	Bubble/slug boundary
T	Total

CHAPTER ONE

INTRODUCTION

1.1 Background

Any fluid flow with more than one phase or flow species is termed 'multiphase flow'. Most real life flow streams are multiphase. Common examples are hydrocarbon movement either from the reservoir to the wellbore or in transportation lines, blood flow streams in living organisms, nuclear fluids in nuclear reactors, etc. Multiphase systems may be two-phase, three-phase or more in no particular combination of the states of matter (i.e. liquid-liquid such as in oil droplets in water, solid-liquid such as in suspensions or gas-liquid-water found in common hydrocarbon traps).

Multiphase flow is characterized by the simultaneous flow of the components of the flow stream. Therefore the parameter to be accounted for in any multiphase system design includes, the volumetric flow rate (total and phase) [m^3/s] , the mass flow rate [kg/s], the mass flux [kg/m^2], phase fraction, distribution term, flow velocities [m/s], slip values, drift factor and variations of fluid properties as a result of changes in flow stream (flow patterns).

The summation of the volumetric fraction of all the species in any multiphase stream is unity and each phase moves with a superficial velocity as a result of interference by the other phase(s). The mixture velocity is obtained as the algebraic sum of the superficial velocities of all the species.

A common subject of interest by investigators in the field of multiphase streams are the flow regimes: prediction, identification, and marching, liquid holdup (or void fraction), convective heat transfers (due to mixing effects), pressure drop prediction and estimation, waxing and hydrate formation. One of the most challenging factors in a multiphase investigation or monitoring is the high tendency for flow stream modifications (i.e. changes in flow regimes), this is because each of the flow patterns has its unique impact on the flow parameters. The flow pattern is also very sensitivity to flow line orientation. Another important factor that affects the

flow regime is the fluid characteristics of the two phases. Most works in literature are reported for air-water flow map, kerosene-air flow map, air-glycerin, and air-oil flow map.

1.1.1 Bubble flow

This type of flow pattern is characterized by a small free-gas phase with the pipe almost completely filled with the liquid phase. Hence a liquid dominated flow. The gas phase is randomly distributed as small bubbles with varying diameters. The individual gas bubble moves with unique velocities as a function of its diameter[8]. In a riser, the liquid moves up the pipe at a fairly uniform velocity and, except for its density, the gas phase has little effect on the pressure-gradient.

The pipe inclination is another important factor that influences flow pattern. When the pipe is tilted at angle $\theta > 0^o$ to the horizontal, the discrete gas bubbles dispersed in the liquid continuum flow through the upper part of the cross-section for the low gas rate. But as the gas rate is increased, the gas bubbles occupy an increasing flow area in the pipe making the small bubbles to or tend to coalesce to form larger gas bubbles[5].

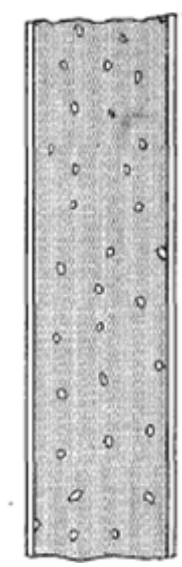


Figure 1.1- **Bubble flow scheme[8]**

1.1.2 Slug flow

The liquid phase is still the continuous phase of the flow, but the gas phase is now more pronounced than in bubble flow[13]. This is as a result of the coalesced gas bubbles leading to the

formation of stable bubbles of approximately equal size and shape which may be approximately the pipe diameter. The stable bubbles are separated by liquid slugs and each gas bubble is surrounded by a thin film of liquid which moves with a low velocity concurrently or counter to the bulk flow. The stable bubbles now move with a velocity greater than that of the liquid and in the direction of the bulk flow, while the liquid moves with a smaller velocity. These varying liquid velocities result in varying liquid hold up (or void fraction) and leading to varying frictional pressure drops along the flow line[3]. Because of the sensitivity of frictional pressure drop to the interaction of both the stable bubble and liquid slug, the total pressure gradient is therefore dependent on both gas and liquid phases[7].

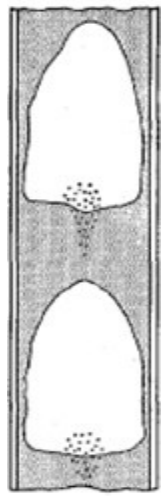


Figure 1.2- **Slug flow scheme**[8]

1.1.3 Transition flow

This type of flow describes a change from liquid dominated flow to gas dominated flow. The liquid slug separating the stable bubbles from liquid slug progressively disappears and the amount of entrained liquid become significant. Although the effects of the liquid phase are significant, the gas phase is more predominant.

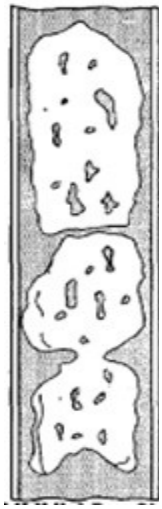


Figure 1.3- **Annular-slug transition scheme**[8]

1.1.4 Annular-mist flow

This type of flow is gas phase dominated with entrained liquid forming a thin film wetting phase on the walls of the pipe. The gas phase velocity controls the two-phase flow[5]. Regardless of pipe orientation, pipe size, and flow direction, increasing the gas rate at a constant liquid rate will eventually lead to annular-mist flow[3]. The minimum gas rate at a fixed liquid rate, at which annular flow starts to form, is called the transition gas rate for that constant liquid rate[17].

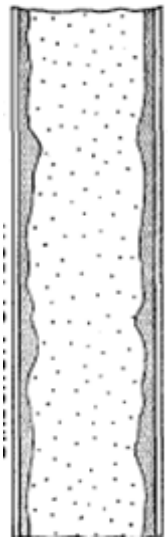


Figure 1.4 - **Annular-mist flow scheme**

1.2 Problem Statement

Pressure drop fluctuation is a common feature of most multiphase fluid flow systems regardless of the type of flow orientation either in vertical pipes (risers or downcomers), horizontal pipelines or in inclined configurations. The size of these pressure fluctuations could be of the same order of magnitude as the pressure drop itself[4].

The prediction of pressure drop in multiphase flow streams is therefore of particular interest for the Oil and Gas Industry since most fluid flow in this area are multiphase hence the knowledge of the pressure drop is needed for the design of surface facilities in offshore fields, and for better instrumentation decision making.

Studies indicate a combined influence of void fraction (liquid holdup), pipe size, flow orientation, fluid characteristics as the likely contributing factors to pressure drop. This served as the basis for this research work, to study the implication of void prediction (liquid holdup) to pressure gradient in vertical pipes[12].

1.3 Objectives

1. To evaluate the accuracy of several multiphase void fraction correlations: Homogenous correlations, Drift flux correlations and General correlations with air-silicon experimental data for 39 test points.
2. To study the impact of using void fractions prediction from different empirical correlations on overall pressure gradient in vertical pipes.
3. To develop a scheme for better quality assurance in multiphase flow systems with a high degree of flow pattern variations in risers.

1.4 The Work Outline

This research work comprises of five chapters. Chapter one introduces the concept of multiphase flow, the different types of flow regimes and the issue of pressure gradient. Chapter two is the literature review of the studies. Here a concise review of past works on void fraction comparison was done to include major contributions and developments spanning the whole of the last

century. The chapter also contains pressure drop models and selection criteria. Chapter three is the methodology including presentation of empirical data, mathematical computation of void fractions and pressure gradient estimation. Chapter four shows results presentation, analysis, and interpretation. Chapter five is conclusions and recommendation.

1.5 Methodology

1. Experimental data collection and loaded onto Microsoft Excel workspace.
2. Selected correlations, programmes and void fraction predictions.
3. Correlation performance evaluation.
4. Identification of flow regimes.
5. Correlation performance evaluation based on flow regimes identified.
6. Pressure gradient prediction using "slip consideration, no flow pattern" model,
7. Pressure gradient prediction using "slip consideration, flow pattern consideration" Duns and Ros (1963) Model.
8. Results analysis: Error estimation, absolute average error, relative error and scatter plots
9. Development of flow schemes.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

It is a fact that most industrial flow streams are multiphase from oil and gas to nuclear, chemical, biotechnology, medical industry and so forth. Different investigators in different industries have made significant contributions to the prediction of liquid holdup (void fraction), correlation formulation, flow map designs, and pressure drop evaluation.

2.2 The Concept of Void Fraction

In any multiphase flow characterization, a very important parameter to be considered is the void fraction (liquid holdup). There are a few different understandings on void fraction which may include: local fraction, cross-sectional void fraction, and volumetric void fraction[13]. These definitions are based on the technique used to measure the void fraction: the flow space and geometric consideration, and fluid distribution across a pipe section. These definitions also reflect the method of measurement which may include: the electrochemical method, optical method, radioactivity absorption and scattering method, impedance and the use of mathematical correlation[19].

The cross-sectional void fraction is defined as $\varepsilon = A_g / (A_g + A_l)$ where A_g represents the pipe (channel) cross-sectional area occupied by the gas phase while A_l represents the pipe (channel) cross-sectional area occupied by the liquid phase. In most of the experimental studies of two-phase flow, the void fraction is measured using the quick closing valves method and hence volumetric void fraction is defined as, $\varepsilon = V_g / (V_g + V_l)$, where V_g and V_l are the volume of channel occupied by gas and liquid phases respectively.

2.2.1 Void fraction correlations

Sokolov et al. (1969) relates the downward void fraction to the upward void fraction in his proposed correlation. The correlation has a simple form shown below;

$$\varepsilon_{dn} = 2\beta - \alpha_{up} \qquad 2.1$$

Yijun and Rezkallah (1993) found a general agreement when they compared Sokolov et al. (1969) correlation against their own data except at very low values of void fraction of $\varepsilon < 0.2$. Yihun and Rezkallah (1993), give a graphical representation of this comparison. They found that in a region of void fraction the Sokolov et al. (1969) correlation can be replaced by the Armand (1946) correlation as;

$$\varepsilon_{dn} = \frac{2\varepsilon_{up}}{C_A} - \varepsilon_{up} = 1.4\varepsilon_{up} \qquad 2.2$$

Where C_A is the Armand coefficient with a value of 0.83 in this region of low void fraction.

Yihun and Rezkallah (1993) carried out an experimental investigation of two-phase flow in 95 mm ID pipe using an air-water fluid combination. They concluded that the void fraction should be a function of the physical characteristics of the multiphase fluids, velocities of each phase and the pipe diameter. They presented correlations where they proposed that the void fraction correlation for a downward flow could be represented in terms of the void fraction in an upward flow.

$$for\ \Re_l < 17400\ \varepsilon_{dn} = 0.076 + 0.074\,\Re_l^{0.05}\,\varepsilon_{up} \qquad 2.3$$

For $\Re_l > 17400\ \varepsilon_{dn} = 0.025 + 0.058\,\Re_l^{0.05}\,\varepsilon$ 2.4

The $\Re_l$ is the superficial Reynolds number denoted

The upward flow void fraction reported by Yijun and Rezkallah (1993) is written as:

$$\frac{\varepsilon_{up}}{1-\varepsilon_{up}} = \frac{x}{1-zx} \qquad 2.5$$

And $$z = \Re_l^n \left[\Re_g Fr_g^2\right]^{-m} \qquad 2.6$$

z is a weighting factor which is a function of the flow pattern and indicates the increase in the void fraction with an increase in the gas flow rate. The values of the powers obtained were n = 0.95 and m = 0.332. Yijun and Rezkallah (1993) verified the upward void fraction correlation against the data of other investigators but the downward void fraction correlation was compared only with the correlation of Sokolov et.al. (1969) and for their own data. The percentage error between the observed and predicted value of void fraction was found to be within ± 25 .

Beggs (1972) investigated the behavior of an air-water two-phase flow in a 25mm and 38mm ID pipe at various pipe orientations spanning 0 to ± 90 ^{o} and recommended a flow pattern dependent correlation to predict liquid holdup (void fraction) at all pipe inclinations. The correlation is given below:

$$\varepsilon_\theta = \varepsilon_o \left[1 + C\left(\sin(1.8\theta) + \frac{1}{3}\sin^3(1.8\theta)\right)\right] \qquad 2.7$$

Where the void fraction ε_o for different flow patterns can be obtained as follows:

Segregated flow pattern $$\varepsilon_o = \frac{0.98\,\lambda^{0.4846}}{Fr^{0.868}} \qquad 2.8$$

Intermittent flow pattern $$\varepsilon_o = \frac{0.845\,\lambda^{0.5351}}{Fr^{0.0173}} \qquad 2.9$$

Distributed flow pattern $$\varepsilon_o = \frac{1.065\,\lambda^{0.5821}}{Fr^{0.609}} \qquad 2.10$$

$$C = (1-\lambda)\ln\left[\frac{4.7\,N_{LU}^{0.1244}}{\lambda^{0.3692}\,Fr^{0.5056}}\right] \qquad 2.11$$

These correlations are based on the flow patterns in horizontal flow which can be determined by the following criteria as reported by Beggs (1972).

If Fr < L_1 flow pattern is segregated

If Fr > L_1 and > L_2 the flow pattern is distributed

If L_1 < Fr < L_2 the flow pattern is intermittent

The L_1 and L_2 are found out using:

$$L_1 = \exp\left(-4.62 - 3.757\,X - 0.481\,X^2 - 0.207\,X^3\right) \qquad 2.12$$

$$L_2 = \exp\left(1.061 - 4.602\,X - 1.609\,X^2 - 0.179\,X^3 + 0.000635\,X^5\right) \qquad 2.13$$

And $$x = \ln(\lambda) \qquad 2.14$$

Where $\varepsilon_o, \lambda \wedge Fr, N_{LV}$ is the void fraction in horizontal flow, input liquid content, Froude number, and liquid velocity number respectively. Beggs (1972) did not compare his correlation with other experimental data or other existing correlations. Hence the validity of this correlation is not known.

Mukherjee (1979) investigated and analyzed the two-phase flow behavior for air-kerosene and air-oil two-phase flow in 38 mm ID pipe at orientation and suggested a flow pattern dependent

on empirical correlation for downhill flow. Mukherjee (1979), used different regression coefficients in the derivation of the correlation, which predicts void fraction in stratified downhill and other flow regimes. The form of the correlation is given below:

$$1-\varepsilon=\exp\left(C_1+C_2 sin\theta+C_2\sin^2\theta+C_4 N_L\right)\frac{N_{GV}^{c_5}}{N_{LV}^{c_6}} \qquad 2.15$$

Mukherjee (1979), reported the following dimensionless numbers N_L, N_{GV}, N_{LV}, meaning the liquid phase viscosity number, the gas phase velocity number and the liquid phase velocity number respectively.

The BIOMED non-linear regression program was used by Mukherjee (1979) to develop the correlation and the results compared to the measured data. The void fraction predicted by this correlation was found to match the measured value within ± 30 band. Table 1.1 shows the regression coefficients used in the correlation.

Table 1.1 **Regression coefficients used in Mukherjee (1979) void fraction correlation**[5]

	Values of regression coefficients					
Flow Patterns	C_1	C_2	C_3	C_4	C_5	C_6
Stratified	-1.33028	4.808139	4.171584	56.26227	0.079551	0.504887
Others	-0.51664	0.789805	0.551627	15.51921	0.371771	0.393852

The validity of this correlation is not known for two reasons: firstly, Mukherjee (1979), did not include data points with large errors in the final derivation; secondly, he did not compare the correlation with other experimental data or with existing correlations. Hence the accuracy and spread of the usability of the correlation cannot be defined with all certainty.

2.3 Drift Flux Model

The drift flux model is arguably the most popular and versatile model for predicting void fraction (or liquid hold up) in gas-liquid two-phase flow[13]. Zuber and Findlay (1964) were the first to introduce the concept of drift flux in gas-liquid two-phase systems as a general technique for predicting void fraction (or liquid hold up). The method was later developed by Wallis and many other researchers. Wallis (1969), defines the drift flux model essentially as the existence of relative motion between the two phases rather than the notion of motion of an individual phase. This new understanding provides a better way of explaining the influence of the interaction of gravity, upthrust (buoyancy) and pressure on the two phases and ultimately the flow pattern. The sharp contrast between the drift flux model and the homogenous flow model is the fact that the former accounts for the slip between the two phases in contrast to the homogenous model which assumes that the phases flow at the same velocity. The general form of the drift flux model is given below:

$$\varepsilon = \frac{U_{SG}}{C_o U_M \pm U_{GM}} \qquad 2.16$$

U_M and U_{SG} are the mixture velocity and the gas phase average superficial velocity respectively using weighting average definition as follows:

$$\left(U_M\right) = \left(Q_G + Q_L\right)/A \qquad 2.17$$

$$\left(U_{SG}\right) = Q_G/A \qquad 2.18$$

Where, Q_G , Q_L and A, are the gas phase flow rate, liquid phase flow rate and A is the flow area. The very important ingredient of the drift flux model as seen in equation 2.16 above is the distribution parameter C_o and the drift velocity U_{GM}. The distribution parameter accounts for the distribution of discrete phase with respect to the mixture in the pipe cross-section (for example, the distribution parameter accounts for the dispersals of tiny gas bubbles in a liquid continuum in

a bubble flow or distribution of entrained liquid in annular-mist flow etc.). The drift velocity accounts for the relative velocity of the gas phase with respect to the mixture velocity. Since velocity is associated with direction (i.e. vector quantity), it implies that the drift velocity may be assigned a positive or negative sign since the gas phase may travel in or opposite to the direction of the mean flow.

2.3.1 Drift flux correlations

Many investigators have derived values for distribution parameter and drift flux coefficients. Some examples are listed below:

Hughmark (1965) considered slug flow in a horizontal pipe and developed the correlation:

$$\varepsilon = \frac{U_{SG}}{1.2\left(U_{SL}+U_{SG}\right)} \qquad 2.19$$

Gregory and Scott (1969) in line with the work of Nicklin et al. (1962) derived the correlation:

$$\varepsilon = \frac{U_{SG}}{1.19\,U_M} \qquad 2.20$$

Clark and Flemmer (1985) analyzed the original drift flux model by Zuber and Findlay (1964) for both upflow and downflow scenarios. They carried out experiments using an air-water two-phase system in a 1.0 ID pipe. Regression analysis was used to analyze the data using an equation proposed by Wallis (1969). They found that the bubbles rise to a velocity of 0.25 m/s approx. which was in good agreement to the equation proposed by Haramthy (1960). And a distribution parameter of 1.165 for bubbly flow downflow and 1.07 for upflow. This finding was in contrast to the proposition by Zuber and Findlay (1964) that the distribution parameter would remain the same for both upflow and downflow. Clark and Flemmer (1985) suggested the same drift flux velocity to that proposed by Nicklin et al (1962) equation given below to relate the distribution parameter and void fraction for slug flow regime:

$$C_o = 1.5221\left(1-3.67\,\varepsilon\right) \qquad 2.21$$

This linear equation between distribution parameter and void fraction may not be valid for other pipe geometries since the relationship between these two dimensionless parameters is pipe diameter dependent. This equation is therefore open to further evaluations based on two reasons: firstly, the correlation was not compared with any other data set and secondly, the percentage accuracy of spread of the void fraction was not stated.

Arosio and Stogia (1976) used a curve fitting technique to study two-phase flow phenomenon in large diameter pipes. They analyzed their experimental data for 44 mm and 90 mm ID pipes and data reported by De Rauz (1976). Based on the drift flux model, they found that the distribution parameter C_o should assume a value of 1.03 approx. with a drift velocity of 0.24 m/s.

Rouhani and Axelsson (1970), used the drift flux analysis of Zuber and Findlay (1965) to develop two correlations for different regimes for test data of a two-phase boiling stream that covers a wide range of heat fluxes, pressure, mass rates and subcooling. The correlation is given a:

$$\varepsilon = \frac{\frac{x}{\rho_G}}{\left[C_o\left(\frac{x}{\rho_G}+\frac{1-x}{\rho_L}\right)+\frac{U_{GM}}{G}\right]} \qquad 2.22$$

Where

$$U_{GM} = \left[\frac{1.18}{\sqrt{\rho_L}}\right]\left(g\sigma\left(\rho_L-\rho_G\right)\right)^{0.25} \qquad 2.23$$

Rouhani I:

$$C_o = 1+0.2\left(1-x\right) \qquad 2.24$$

Rouhani II:

$$C_o = 1+0.2\left(1-x\right)\left(gD\right)^{0.25}\left(\frac{\rho_G}{G}\right)^{0.5} \qquad 2.25$$

Bonnecaze et al. (1971), gave a variation of Nicklin et al. (1962) correlation as:

$$\varepsilon = \frac{U_{SG}}{1.2\,U_M+0.35\left[1-\frac{\rho_G}{\rho_L}\right]\sqrt{gD}} \qquad 2.26$$

Kokal and Stanislav (1989), investigated an air-oil two-phase system and developed correlations for horizontal and near horizontal ($\pm 9^o$¿ pipe orientation using the drift flux model for all the regime.

$$\varepsilon = \frac{U_{SG}}{1.2\,U_M + 0.35\left[\frac{gD(\rho_L - \rho_G)}{\rho_L}\right]^{1/2}}$$

2.27

Coddington and Macian (2002), analyzed void fraction correlation of Jowitt (1981) and developed parameter for drift flux expression as:

$$C_o = 1 + 0.79\exp\left[-0.061\sqrt{\frac{\rho_L}{\rho_G}}\right] \qquad 2.28$$

$$U_{GM} = 0.034\left[\sqrt{\frac{\rho_L}{\rho_G}} - 1\right] \qquad 2.29$$

Coddington and Macian (2002) also derived the factors of the Bestion (1985) correlation presented as: $C_o = 1$ and $U_{GM} = 0.188\left[\frac{gD(\rho_L - \rho_C)}{\rho_G}\right]^{0.5}$ 2.30

Toshiba (1989) developed values for these constants as:

$$C_o = 1 \quad \text{and} \quad U_{GM} = 0.45 \qquad 2.31$$

2.4 General Correlations

General correlations are basically empirical in nature. Flanigan (1958) derived a general type of correlation from a number of curves. This type of correlation assumes that pipe orientation has no effect on void fraction (or liquid hold up) but depends only on the gas phase superficial velocity as indicated by equation 2.32

$$\varepsilon = \frac{1}{1 + 3.0637\, U_{SG}^{-1.006}} \qquad 2.32$$

Wallis (1969), correlated an expression that best fit the data of Lockhart – Martinelli which depends only on the Lockhart – Martinelli variable.

$$\varepsilon = \left[1 + X_{tt}^{0.8}\right]^{-0.38} \qquad 2.33$$

Neal and Bankoff (1975), developed their correlation report by varying the mass flow rates and the relative volume which was determined from experimental data collected.

$$\varepsilon = 1.25\left(\frac{U_{SG}}{U_M}\right)^{1.88}\left(\frac{U_{SL}^2}{gD}\right)^{0.2} \qquad 2.34$$

2.5 Direct Measurement Method

Direct measurement is the most commonly used method for determining void fraction (or liquid hold up). Direct measurement is an experimental technique where the two-phase flow is trapped between two quick closing valves mounted along the test section after which the trapped volume of liquid is drained. The required void fraction (or liquid hold up) is obtained as a ratio of the measured amount to that of the total volume of the test section between the valves and is predetermined by design.

Simple equipment designs, construction, and low cost are the most noticeable merits of this technique. Also, this method can be used to calibrate and/or validate void fraction (or liquid hold up) obtained from other methods. This method requires a bypass line that helps to prevent damage to the flow system. The major demerit of this technique is that it is time-consuming and would not capture transient properties of the flow. This method is also not practicable for high pressure and temperature applications[20].

The work of Hewitt (1978) where he discussed some methods on top the ones presented above with an appreciable number of references is recommended for someone thinking of building up or putting together a system for measuring void fraction or liquid hold up.

2.6 Impedance Method

This technique has been used more commonly to measure void fraction[7]. It incorporates the appealing nature of being a nonintrusive continuous measurement, which is less expensive and simple in construction. The basic principle of the method is based on the changes in the electrical impedance (or resistance) between electrodes separated by dielectric (insulator) material which in this case is the two-phase flow stream. Since the two phases: gas and liquid have different conductivities, the impedance would, therefore, be triggered as a result of vibration in a void fraction.

Investigators have developed a number of devices based on either the conductance or capacitance or both. Examples are the works by Song et al. (1998), Elkow and Rezkalla (1996), Mukherjee (1979), Roschart et al. (1975) and Gregory and Mattar (1973). The high sensitivity of impedance of materials on temperature, the need for high excitation frequency for large conductivity fluid, calibrations and instrumentation and effects of impurities are the common setbacks of this technique[20]

2.7 Radioactive Absorption and Scattering Method

The radioactive absorption and scattering method relies on the properties of a two-phase fluid to attenuate or scatter a given radioactive beam. Researchers have used streams of beta rays, gamma rays, and x-rays with gamma ray being the most commonly used due to its electromagnetic field neutrality and high frequency (or energy). This technique gives the instantaneous (local) void fraction. Average void fraction (representative value) can be obtained by integrating the instantaneous void fraction over time. A major advantage of this method is that it does not disrupt flow lines like in the direct measurement method and the incorporation of a traversing mechanism that scans the entire length of the text section while the complexity of equipment construction and design, inaccuracy of measurement and high cost are the setbacks associated with this method[7].

2.8 Review of Previous Works

2.8.1 Duckler et al. (1964)

Duckler et al. (1964), was the first to carry out void fraction correlation comparison. The investigation consists of 706 refined void fraction data of Hoogendoom (1959) obtained from tests run in 1, 2, 3, 5 and 5.5 inch ID horizontal pipes with liquid phase viscosities of 3cp 20cp. The correlations used for void fraction prediction are those developed by Lockhart and Martinelli (1949), Hoogendoon (1959) and Hughmark (1962). The statistical tools used includes, arithmetic mean deviation, standard deviation, and a new variable was developed to account for the fractional deviation which includes 68% of the population to measure the spread of data. Of the three correlations used, Duckler et al. (1964) found Hughmark (1962), to perform better than the other two.

2.8.2 Marcano (1973)

Marcano (1973) carried out comparison using five void fraction correlations: Beggs (1972), Duckler et al. (1969), Eaton et al. (1967), Guzhov et al. (1967), Hughmmark (1962) and Lockhart and Martinelli (1949) using data of Eaton (1966), with 238 data points of natural gas-water two-phase system and Beggs (1972), with 58 data points of air-water two-phase system. It was found that Eaton et al. (1967) and Beggs (1972) correlations performed well because the data used for the comparison is the data from which these correlations were developed. Also, the prediction from correlations of Duckler et al. (1969) and Lockhart and Martinelli (1949) were found to be acceptable while the results from Hughmark (1962) and Guzhov et al. 91967) correlations were unsatisfactory.

Marcano dropped out the unreliable low liquid hold up from the data and carried out further comparison. It was observed that the predictions of the correlations were improved. The performance of the correlations for specific void fractions ranges were also analyzed and Eaton et al. (1967), Guzhov et al (1967) and Beggs (1972) correlations were found to do better for void fraction less than 0.65 while on the higher range of void fraction, only the Eaton et al. (1967) correlation predicted acceptable results. For very high void fractions above 0.9, all the correlations gave unsatisfactory results, but the no-slip model with Duckler et al. (1969)

correlation gave best estimates. One possible explanation for this observation could be as a result of the unreliability of the measurement of data.

2.8.3 Palmer (1975)

Palmer (1975) used a 51 mm ID pipe with flow line in three uphill scenarios of 4.2°, 7.1° and 7.5° from the horizontal and three downhill scenarios 4.3°, 3.8° and 6.3° from the horizontal. Palmer (1975) carried out the comparison using correlations of Flanigan (1958), Guzhov et al. (1967) and Beggs (1972). The statistical tools of percentage error, average percentage error, and standard deviation were estimated and comparison was made. It was found that Beggs (1972) correlation gave good predictions of the void fraction for uphill flow while the Flanigan (1958) correlation gave the least accuracy which is expected since it did not consider downhill flow.

2.8.4 Mandhane et al (1975)

Mandhane et al. (1975), used void fraction (liquid hold up) data from the University of Calgary multiphase pipe flow data bank. A two-step procedure was used for the computation of the void fraction. Firstly, the flow pattern was predicted secondly, a correlation was developed for that flow pattern and the void fraction (liquid hold up) computed. The flow map of Mandhane et al. (1974) was used, and void fraction prediction from the correlations of Hoogendoorn (1959), Levy (1960), Scott (1962), Hughmark (1962), Hughmark (1965), Guzhov et al. (1967), Eaton et al. (1967), Beggs (1972), and Agrawal et al. (1973).

Statistical tools of root-mean-square error, mean absolute error, simple mean error, mean-percentage absolute error and the mean-percentage error were used. An arbitrary designation of void fraction ranges to the flow regime was made to see the predictive capability of the correlations within each range of void fraction in the flow regime. Using the flow pattern map of Mandhane et al. (1974), the following recommendations were made: The Hughmark (1962) correlation is recommended for the bubble, elongated slug flow regime, while Agrawal et al. (1973) gave acceptable prediction for stratified flow regime, Lockhart and Martinelli (1949) correlation was recommended for annular and annular-mist regime, and Beggs (1972) correlation for the dispersed-bubble flow pattern. The summary of this work is that no single correlation was recommended for the entire data points although the error limits for all parameters were stated.

A further comparison of the performance of these correlations using flow maps of Baker (1954), Hoogendoorn (1959), Govier and Aziz (1972) and Govier and Aziz (1972), Again the Hughmark (1962) correlation predicted bubble, elongated bubble regimes in the four flow maps but slug flow in Baker (1954). Agraval et al. (1973) predicted stratified in both Mandhane et al. (1974) and Baker (1954) maps. Duckler et al. (1964) correlation predicts that stratified regime in maps of Hoogendoorn (1959) and Govier and Aziz (1972). This is easily explainable as it has an advantage over the others due to the fact that the correlation was developed from these two data sets. Lockhart and Martinelli (1949), predicted the annular and annular-mist flow regimes in all the flow maps. Beggs (1972) correlation predicted the dispersed-bubble regime in the Mandhane et al. (1975) map, while Hughmark (1962) predicted that of Hoogendoorn (1959) and Govier and Aziz (1972). Hoogendoorn (1959), predicted the data in Baker (1954) map quite satisfactorily. In all, again no one correlation gave satisfactory results in the annular, annular-mist or in all the flow regimes.

2.8.5 Papathanassiou (1983)

By considering the wall shear stresses between the two components of a two-phase system that exists and changes as a result of the range within which the two-phase flow operating conditions and specified fluid characteristics vary, leading to a range of void fraction and slip ratio a spectrum graph can be drawn which may be used to compare the correlations such as those of Hughmark (1962), Bankoff (1960), Hoogendoorn (1959), and Lockhart and Martinelli (1949). They all gave similar trend in their prediction with exception only to Bankoff (1960) seen to underpredict the void fraction mainly due to the fact that it was developed for vertical flow. At a void fraction of 0.4 approx., the Lockhart and Martinelli (1949) correlation gave physically unrealistic values. Bankoff (1960), correlation at a high void fraction of about 0.7, also gave predictions that are unrealistic. The explanation to the disagreement between predictions from these correlations for lower end and upper end values of void fraction ranges is due to the fact that few experimental results exist at these conditions when the correlations were developed.

2.8.6 Spedding et al. (1990)

In this work, Spedding et al. (1990) considered 60 correlations using the data of Spedding and Nguyen (1976) for upward flow with a pipe orientation of 2.75^{o} from the horizontal to make the comparison of the performance of the selected void fraction correlations. The acceptable limits

of ± 15 and a $\pm$ 30% was used to define the level of spread. The following conclusions were made: for upward inclination, Nicklin et al. 91962) correlation gave an acceptable prediction for bubble and slug flows for all inclination angles. Lockhart and Martinelli (1949), Premoli et al (1970) and Bonnecaze et al. (1971), correlations were found to perform fairly well in the slug flow regime while the joint work of Spedding and Chen (1984) led to the development of a correlation that was able to predict annular flow regime and associated wave patterns. In conclusion, all the correlations considered were found adequate for the droplet flow pattern.

2.8.7 Abdulmajeed (1996)

Abdulmajeed (1996) tries to simplify the mechanistic model of Taitel and Duckler (1976) by developing a new correlation. To achieve this, he collected 88 air-kerosene void fraction data for a 51 mm ID horizontal pipe and compared 12 correlations of; Abdulmajeed (1996), Minami and Brill (1987, Mukherjee and Brill (1983), Chen and Spedding (1981,1983), Brill et al. (1981), Gregory et al. (1978), Beggs (1972), Guzhov et al. (1967), Eaton et al. (1967), Hughmark (1962), Hughmark and Pressburgh (1961), and Armad(1946). The tools used for comparison were average percentage error, absolute average percentage error, and the standard deviation. The new correlation developed by Abdulmajeed (1996) predicted void fraction in the stratified, slug and annular flow regimes which cover a wider range of flow regime than the original form of the Taitel and Duckler (1976) which was originally designed for stratified flow.

2.8.8 Spedding (1997)

Spedding (1997) used air-water data of him and his coworker for five years (1993, 1991, 1989, 1979 and 1976) to carry out an extensive work for over 100 void fraction correlations. 26 mm to 95.3 mm ID range of diameter was used. Predictions that fall within ± 15 of the data with spread of individual points within ± 30 were considered satisfactory. The performance capability of the correlations was carried out for the 18 flow regimes identified by the study. The pipe orientation used for this work was ± 90 ^{o}. Spedding (1997) found that no one single correlation could give a satisfactory prediction in all the flow regimes and pipe orientation. Therefore, different void fraction correlations were recommended for different flow regimes and different pipe inclinations and flow directions.

2.9 Pressure Drop in Multiphase Flow Systems

A very important requirement for the development and optimum exploitation of any oil field is the prediction of the multiphase pressure gradient in vertical production strings flowing with a mix of gas, oil, and water. An important engineering consideration for the design of surface facilities such as separators, gas-lift installation, flow lines, heat exchangers stock tanks and the likes is the accurate prediction of the pressure drop to be encountered from the wellbore flowing string. Therefore, to design a general multiphase flow model for flow assurance, there is a need for reliable pressure drop data. Many investigators have developed a variety of models and empirical correlations for evaluation of pressure drop (or gradient) associated with multiphase flow streams.

A considerable number of experimental and theoretical investigations has been carried out over the years to understand the dynamics of multiphase flow in vertical wells leading to the development of some correlations. The performance of empirical correlation is significantly affected by the following variables; fluid property, the scale of the experiment, and flow conditions. Therefore the validity of any correlation outside the original input variables is not advisable as it usually results in a large error. It is, therefore, a best practice that data and conditions best described by a given correlation should originate from the author of the correlation. Because multiphase flow in pipes has a variety of field conditions, the selection of correlation most appropriate has to be the one that gives a more conservative result to obtain the pipeline bore.

2.9.1 Pressure drop models in vertical pipe

Four distinct flow regimes have been identified for multiphase flow in vertical pipes to include bubble, slug, transition, and mist flow regimes. Clearly, there is a different model for estimating pressure drop in multiphase systems depending on the understanding of the flow system.

The models used for predicting pressure drop in multiphase systems are categorized into three[8:]

1. **Category "a" No-slip, no flow pattern consideration**

   This model is based on the assumption that the gas phase and liquid phase travel at the same velocity (no-slip). The model does not distinguish between different flow regimes

hence mixture density and viscosities are calculated from the input gas/liquid ratio. The only correlation required in this model is the two-phase friction factor.

2. **Category "b" Slip considered, no flow pattern considered**
   This model assumes that the gas phase and liquid phase travel with different velocities hence a need for relation for both liquid holdup and void fraction. To obtain these values, a method must be provided to predict the instantaneous gas/liquid ratio or the portion of the pipe occupied by gas phase or liquid phase at any location and time. This model does not consider variation in flow pattern hence the same correlations used for the liquid holdup and friction factor are used for all flow regimes.

3. **Category "c" Slip considered, flow pattern considered**
   This model assumes that the two phases travel with different velocities and also that the flow pattern is not the same throughout the flow life of any multiphase stream. There is, therefore, the need for correlations to predict liquid holdup, (void fraction), friction factor and the flow regime. In this method, flow regimes are determined first followed by the selection of appropriate correlations for estimating liquid holdup (void fraction), and friction factor.

Table 2.1 – **Categorization of Pressure drop models in vertical pipes**[1]

Vertical Flow Models	Category
Poettmann and Carpenter	a
Baxendell and Thomas	a
Fancher and Brown	a
Hagedorn and Brown	b
Duns and Ros	c
Orkiszewski	c

Aziz, Govier and Fogarazi	c
Chierief and CiucciandSclocehi	c
Beggs and Brills	c

Beggs and Brill (1973) model

Beggs and Brill (1973) developed a model for predicting pressure drop in pipes, inclined between -90º to 90º. The multiphase system is air-water in pipes with a diameter range of 1 to 1.5 inch. The model considered the following flow regimes of segregated, intermittent and distributed horizontal flow. They calculated the liquid holdup for a horizontal pipeline and the results corrected for pipes of different inclinations. The flow pattern map developed by Beggs and Brill (1973) is presented as a function of the Froude number and the no-slip liquid holdup. The total pressure drop is the summation of the frictional pressure loss, gravitational pressure drop, and acceleration pressure drop.

Beggs and Brill (1973) model, developed correlations from 584 measured tests. They proposed the following pressure-gradient equation for pipes at any inclination:

$$\frac{dP}{dL}=\frac{\frac{f\rho_m V_m^2}{2d}+\rho_n g sin\theta}{1-E_k} \qquad 2.35$$

$$\rho_n=\rho_n H_{L(\theta)}+\rho_n[1-H_{L(\theta)}] \qquad 2.36$$

The equations for the modified flow pattern transition boundaries are:

$$L_1 = 316 \ \lambda_L^{0.302}$$

2.37

$$L_2 = 0.00925 \quad \lambda_L^{-2.468}$$

2.38

$$L_3 = 0.10\,\lambda_L^{-1.452}$$

2.39

And

$$L_4 = 0.5\,\lambda_L^{-6.738} \qquad 2.40$$

Segregated flow regime identification:

$\lambda_L < 0.01$ and $N_{Fr} < L_1$ or

$\lambda_L \geq 0.01$ and $N_{Fr} < L_2$

Transition flow regime identification:

$\lambda_L \geq 0.01$ and $L_2 \leq N_{Fr} \leq L_3$

Intermittent flow regime identification:

$0.01 \leq \lambda_L < 0.4$ and $L_3 < N_{Fr} \leq L_1$ or

$\lambda_L \geq 0.4 \wedge L_3 < N_{Fr} \leq L_4$

Distributed flow regime identification:

$\lambda_L < 0.4$ and $N_{Fr} \geq L_1$ or

$\lambda_L \geq 0.4$ and $N_{Fr} < L_4$

$$N_{Fr} \; ¿ \frac{V_m^2}{gd} \qquad 2.41$$

$$H_{L(\theta)} = \frac{a\lambda_L^b}{N_{Fr}^c} \qquad 2.42$$

Table 2.2 – **Beggs and Brill (1973) Empirical coefficients for horizontal liquid holdup**[16]

Flow pattern	A	B	C
Segregated	0.980	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributive	1.065	0.5824	0.0609

$$H_{L(\theta)} = H_{L(\theta)}\Psi \qquad 2.43$$

$$\Psi = 1.0 + C\,\text{¿}\,\sin(1.8\theta) - 0.333\sin^3(1.8\theta)\,] \qquad 2.44$$

$$C = \text{¿}\,(1.0 - \lambda_L)\,e\,\lambda_L^f N_{LV}^g N_{Fr}^h\,\text{¿¿}\,) \qquad 2.45$$

Table 2.3 – **Beggs and Brill (1973) Empirical coefficients for C** [16]

Flow pattern	e	F	g	h
Segregated uphill	0.011	-3.768	3.539	-1.614
Intermittent uphill	2.96	0.305	-0.4473	0.0978
Distributive	C = 0	Ψ = 1		
All pattern downhill	4.7	-0.3692	0.1244	-0.5056

The liquid holdup must be interpolated between the segregated and intermittent liquid holdup values when the flow pattern falls in the transition region, as:

$$H_{L(\theta)} = A H_{L(\theta)Seg} + (1-A) H_{L(\theta)\int ¿} \qquad 2.46$$

Where,

$$A = \frac{L_3 - N_{Fr}}{L_3 - L_2} \qquad 2.47$$

$$N_{\Re} = \frac{\rho_n V_m d}{\mu_n} \qquad 2.48$$

Beggs and Brill (1973) experimental data was used to correct the ratio of the two-phase friction factor to the normalizing friction factor, resulting in the exponential relation:

$$\frac{f}{f_n} = e^s \qquad 2.49$$

Where

$$s = \frac{\ln y}{-0.0523 + 3.182 \ln y - 0.8725 \ln y} \qquad 2.50$$

And

$$y = \frac{\lambda_L}{H^2_{L(\vartheta)}}$$

Beggs and Brill (1973) model – summary

Beggs and Brill (1973) developed a model for predicting pressure drop in pipes, inclined between -90° to 90°. The multiphase system is air-water in pipes with a diameter range of 1 to 1.5 inch. The model considered the following flow regimes of segregated, intermittent and

distributed for horizontal flow. They calculated the liquid holdup for horizontal pipeline and the results corrected for pipes of different inclinations. The summary below gives the performance of the model:

- **Tubing size:** The pressure losses are accurately estimated for the range in which the experimental investigation was conducted (i.e., tubing sizes between 1 to 1.5 in. Over prediction in the pressure loss occurs for any further increase in tubing size.
- **Oil Gravity:** A reasonably good performance is obtained over a broad spectrum of oil gravities.
- **Gas-Liquid Ratio (GLR):** In general, with an increasing GLR, an over-predicted pressure drop is obtained. For GLR greater than 5000, the error margin becomes very large.
- **Water-Cut:** The pressure drop prediction generally gives a very good accuracy for water-cut up to about 10%.

Payne et al.

Payne et al. found out that there was limited data, the Beggs and Brill (1973) model over-predicted liquid holdup both in the uphill and downhill flow. Payne et al. developed correction factors to improve liquid holdup values.

$$If\ \theta > 0, H_{L(\theta)} = 0.924\ \ H_{L(\theta)}{}^{*} \qquad 2.52$$

$$If\ \theta < 0, H_{L(\theta)} = 0.685\ \ H_{L(\theta)}{}^{*}$$

2.53

However, the resulting liquid holding for $\theta > 0^{o}$ should not be less than λ_L. The original Beggs and Brill (1973), method has been found to over-predict pressure drops in producing wells. Consequently, if the Paynes et.al. liquid holdup correction factor is applied for wells improved results should be obtained.

Payne et al. found that the Beggs and Brill (1973) model under-predicted friction factors. This under-prediction of the Beggs and Brill method was because it was originally developed based on data obtained in a smooth pipe. Payne et al. recommended that the normalizing friction factor, f_n, be obtained from the Moody diagram or from Reynolds number estimation for an appropriate

value of roughness. This change improved the pressure drop predictions for the Beggs and Brill method for rough pipes.

Hagedorn and Brown (1965) Model

The Hagedorn and Brown (1965) model was developed for pressure losses in multiphase wellbore based on data obtained from a 1500-ft-deep producing vertical experimental well. Air was the gas phase, and four different liquids were used with varying flow rates; water and crude oils with viscosities of about 10, 30 and 110cp. Tubing with 1.0, 1.25 and 1.5 inch nominal diameters was used.

Hagedorn and Brown (1965) data represent some of the most extensive large-scale tests ever reported. However, it is important to recognize that they did not measure liquid holdup. The model developed a pressure-gradient equation that permitted the calculation of pseudo liquid holdup values for each test to match measured pressure gradients after assuming a friction factor correlation. Therefore, the values used to develop the liquid holdup correlation were not true measures of the portion of pipe occupied by the liquid phase.

Hagedorn and Brown (1965), developed this pressure-gradient equation for vertical multiphase flow. The model utilizes the same dimensionless numbers of liquid viscosity number, liquid velocity number, gas velocity number and the pipe diameter number developed by Duns and Ros (1963) model which were also used by Eaton (1967) correlation.

Total pressure gradient:

$$\frac{dP}{dL}=\frac{f\,\rho_n^2 V_m^2}{2\rho_s d}+\rho_s g+\frac{\rho_s \Delta\left(V_m^2\right)}{2\,dZ} \qquad 2.54$$

Liquid velocity number:

$$N_{LV}=V_{SL}\sqrt[4]{\frac{\rho_L}{g\,\sigma_L}} \qquad 2.55$$

Gas velocity number:

$$N_{GV} = V_{SG} \sqrt[4]{\frac{\rho_L}{g \sigma_L}} \qquad 2.56$$

Pipe diameter number:

$$N_d = d \sqrt{\frac{\rho_L g}{\sigma_L}} \qquad 2.57$$

Liquid viscosity number:

$$N_L = \mu_L \sqrt[4]{\frac{g}{\rho_L \sigma_L^3}} \qquad 2.58$$

If constants are included:

$$N_{LV} = 1.938\, V_{SL} \sqrt[4]{\frac{\rho_L}{\sigma_L}} \qquad 2.59$$

$$N_{GV} = 1.938\, V_{SG} \sqrt[4]{\frac{\rho_L}{\sigma_L}} \qquad 2.60$$

$$N_d = 120.872\, d \sqrt{\frac{\rho_L}{\sigma_L}}$$

2.61

$$N_L = 0.1572\, \mu_L \sqrt[4]{\frac{1}{\rho_L \sigma_L^3}} \qquad 2.62$$

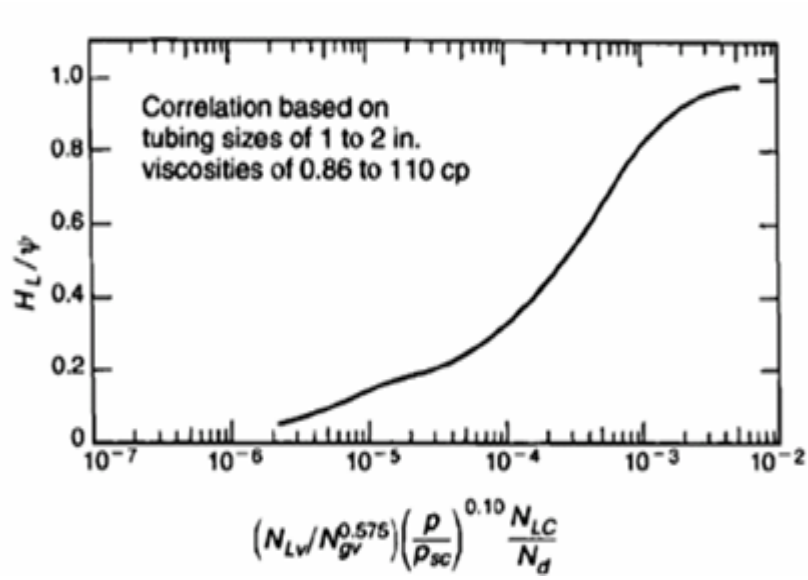


Figure 2.1 – **Hagedorn and Brown (1965) correlation for normalized liquid holdup**[21]

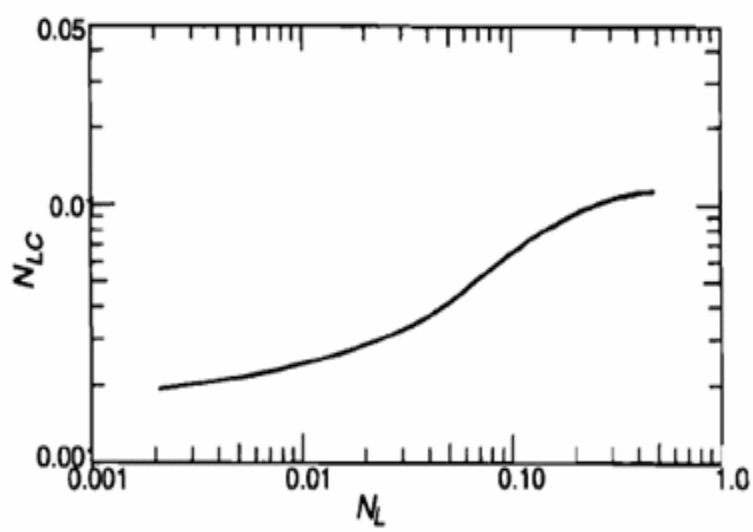


Figure 2.2 – **Hagedorn and Brown (1965) correlation for N_{LC}**[21]

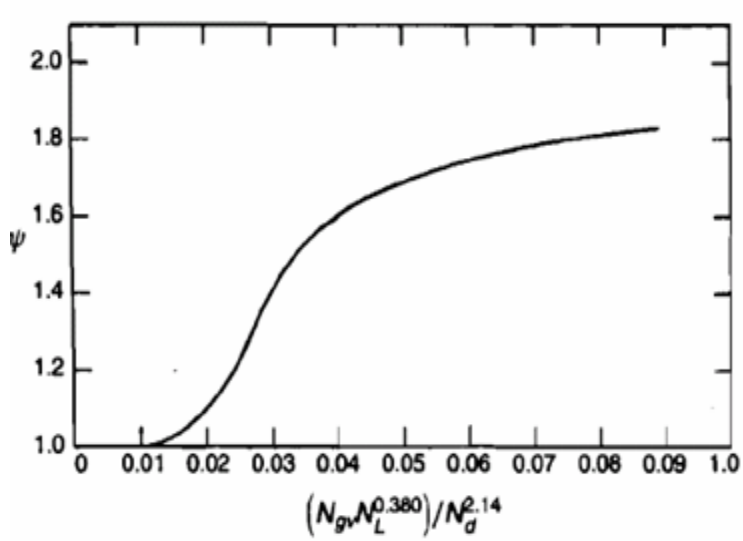


Figure2.3 – **Hagedorn and Brown (1965) correlation for** Ψ [21]

Hagedorn and Brown (1965), obtained friction factor from the Moody diagram because the model assumed that two-phase friction factors could be predicted in the same way as single-phase friction factors. Hence for a given relative roughness, a two-phase Reynolds number is defined as:

$$N_{\Re} = \frac{\rho_n V_m d}{\mu_s} \qquad 2.63$$

Acceleration term: The pressure gradient resulting from acceleration is given by:

$$\left(\frac{dP}{dZ}\right)_{acc} = \frac{\rho_s \Delta\left(V_m^2\right)}{2\,dZ}$$

2.64

Where

$$\Delta\left(V_m^2\right) = V_{m1}^2 - V_{m2}^2 \qquad 2.65$$

V_{m1} and V_{m2} designate downstream and upstream ends of a calculation increment, respectively.

Hagedorn and Brown (1965) model - summary

This correlation was developed using data obtained from a 1500-ft vertical well. Tubing diameters ranging from 1-2 inch were considered in the experimental analysis along with five different fluid types, mainly water and four types of oil and varying flow rate with viscosities ranging between 10cp and 110cp at 80°F.

Hagedorn and Brown (1965) data represent some of the most extensive large-scale tests ever reported. The model developed a pressure-gradient equation that permitted the calculation of pseudo liquid holdup values for each test to match measured pressure gradients after assuming a friction factor correlation. Therefore, the values used to develop the liquid holdup correlation were not true measures of the portion of pipe occupied by the liquid phase. The correlation developed is independent of flow pattern and its performance is briefly outlined below:

- **Tubing Size**: For tubing sizes between 1 and 1.5 inch, the model accurately predicted the pressure drop since it is the range in which the experimental investigation was conducted. But a further increase in tubing size causes the pressure drop to be over-predicted.
- **Oil Gravity:** For heavier oils (13 – 25 ^{o}API) Hagedorn-Brown (1965) model is seen to over-predict pressure loss and under predict the pressure profile for lighter oils (40 – 56 ^{o}API).
- **Gas-Liquid Ration (GLR):** for a gas-liquid ratio (GLR) greater than 5000, the model over-predicted pressure drop.
- **Water-Cut:** The accuracy of the pressure profile prediction is generally good for a wide range of water cuts.
- The Hagedorn and Brown (1965) model is:
    - Simple and straightforward.
    - Inadequate for bubble and mist flow.
    - Good for slug flow regime.
    - Not accurate enough for acceleration pressure drop.
    - Liquid holdup is not accurately correlated and most of the time physically meaningless.

The Duns and Ros (1963) model

The Duns and Ros (1963) carried out an extensive laboratory study that resulted in liquid holdup and pressure gradient measurement. The research involved about 4000 multiphase flow tests conducted in an 185ft vertical flow system. The flow string ranged from 1.26 inch to 5.60 in ID and included two annulus configurations. Near atmospheric conditions were used for most of the test with gas phase and hydrocarbon or water as the liquid phase. The radioactive-tracer technique was used for the measurement of liquid holdup. The equipment has a transparent section that allowed for the visual observation of a pattern. Three flow regimes were identified and correlations were developed from slip-velocity and friction factor from which liquid holdup (void fraction) can be computed.

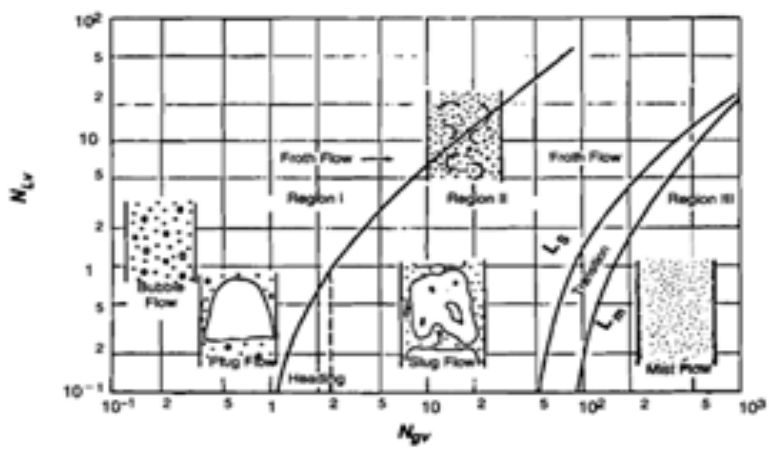


Figure 2.4 **Duns and Ros (1963) Flow-pattern map**[3]

Bubble/slug boundary:

$$N_{GVB/S} = L_1 + L_2 N_{Lv} \qquad 2.66$$

Where L_1 and L_2 are functions of N_d

$$N_d = d\sqrt{\frac{\rho_L g}{\sigma_L}}$$

2.67

Slug/transition boundary:

$$N_{GVS/Tr} = 50 + 36 N_{LV} \qquad 2.68$$

Transition/mist boundary:

$$N_{GVTr/M} = 75 + 84 N_{Lv}^{0.75} \qquad 2.69$$

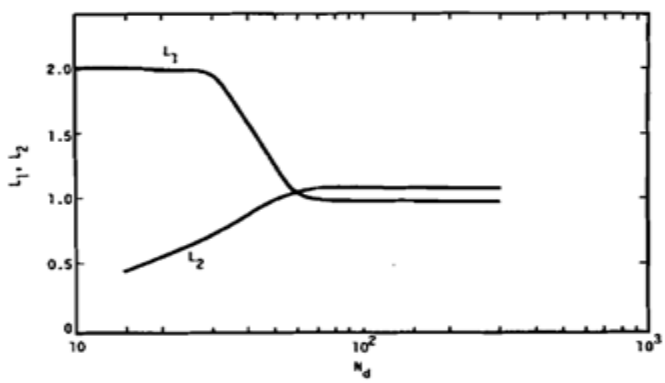


Figure 2.5- **Duns and Ros (1963) bubble/slug transition parameters**[3]

Liquid holdup prediction

Duns and Ros (1963), developed empirical correlations for the dimensionless slip-velocity number, S, instead of attempting to directly compute the liquid holdup (void fraction). The dimensionless slip number is defined in the same way as the gas velocity number N_{LV} liquid velocity number $N_{GV.}$

$$s = v_s \sqrt[4]{\frac{\rho_L}{g\sigma_L}} \qquad 2.70$$

In field units:

$$s = 1.938\, v_s \sqrt[4]{\frac{\rho_L}{g\sigma_L}} \qquad 2.71$$

$$v_s = v_g - v_L = \frac{v_{sg}}{1-H_L} - \frac{v_{sL}}{H_L} \qquad 2.72$$

v_s is the slip-velocity or

$$H_L = \frac{v_s - v_m + \sqrt{(v_m - v_s)^2 + 4\, v_s v_{sL}}}{2\, v_s} \qquad 2.73$$

Gravitational pressure gradient prediction

The following procedure is used to calculate the gravitational component of the pressure gradient:

- Step one: Calculate the dimensionless slip-velocity, S, using appropriate correlation
- Step two: Solve equation 2.63 for the slip-velocity
- Step three: Calculate the liquid holdup from Equation 2.65, or
- Step four: Calculate the slip density

  $$\rho_s = \rho_L H_L + \rho_g (1 - H_L) \qquad 2.74$$

- Step five: Calculate the gravitational component of the pressure gradient by first identifying the flow regime:

Bubble Flow: Bubble flow exists if $N_{gv} < N_{gv\,B/S}$, for bubble flow the dimensionless slip-velocity number is given by

$$s = F_1 + F_2 N_{Lv} + F_3' \left(\frac{N_{gv}}{1 - N_{Lv}} \right)^2 \qquad 2.75$$

Where F_1 and F_2 are given in Fig. 2.6. They are functions of the liquid velocity number, $N_{Lv.}$

F_3' *can be obtaind ¿*

$$F_3' = F_3 - \frac{F_4}{N_d} \qquad 2.76$$

The friction pressure-gradient component for bubble flow is given by:

$$\left(\frac{dP}{dL} \right)_f = \left(\frac{2 f\, \rho_L v_{sL} v_m}{d} \right)$$

2.77

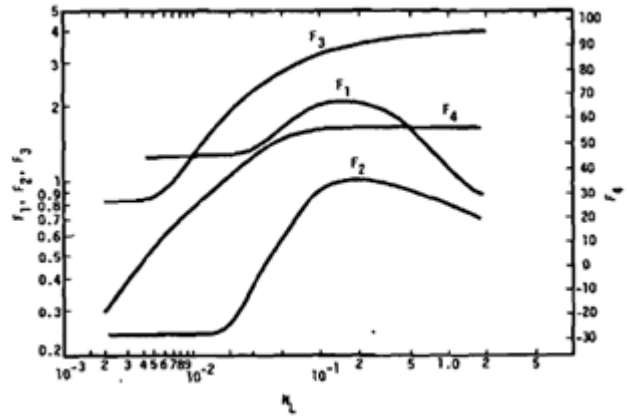


Figure 2.6 - **Duns and Ros (1963) Bubble-flow, slip-velocity parameter**[3]

From experimental data, Duns and Ros (1963) developed this equation for *f*:

$$f = f_1 \frac{f_2}{f_3} \qquad 2.78$$

f_1 , is the main friction factor, and its value can be obtained from a Moody diagram as a function of Reynolds number for the liquid phase.

$$N_{\Re_L} = \frac{\rho_L v_{SL} d}{\mu_L} \qquad 2.79$$

Equation 2.70 gives f_2 which is the friction factor correction term for the in-situ gas/liquid ratio and is given in Fig.2.6. Duns and Ros (1963) define the factor f_3 as a second-order correction factor for both liquid viscosity and in-situ gas/liquid ratio. f_3 is important for kinematic viscosities greater than approximately 50cp is given by:

$$f_3 = 1 + \frac{f_1}{4}\sqrt{\frac{v_{sg}}{50\, v_{SL}}} \qquad 2.80$$

The acceleration component of the pressure gradient was considered to be negligible for bubble flow by the Duns and Ros (1963) Model.

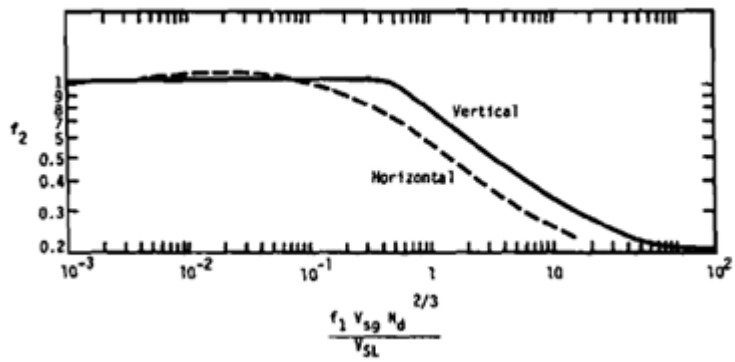


Figure 2.7 – **Duns and Ros (1963) Bubble-flow friction factor parameters**[3]

Slug flow: Slug flow exists if $N_{GVB/S} < N_{GV} < N_{GVS/Tr}$.

Then the dimensionless slip-velocity number is estimated from the correlation:

$$s = (1+f_5)\frac{N_{gv}^{0.982} + f_6^{'}}{(1+f_7 N_{Lv})^2} \qquad 2.81$$

Where f_5, f_6 and f_7 are given in Fig. 2.8 as functions of the liquid viscosity number, N_L and

$$f_6^{'} = 0.029\, N_d + f_6 \qquad 2.82$$

Slug flow friction pressure-gradient component is calculated exactly the same way as for bubble flow. Again,the Duns and Ros (1963) model, considered the acceleration component of the total pressure gradient to be negligible.

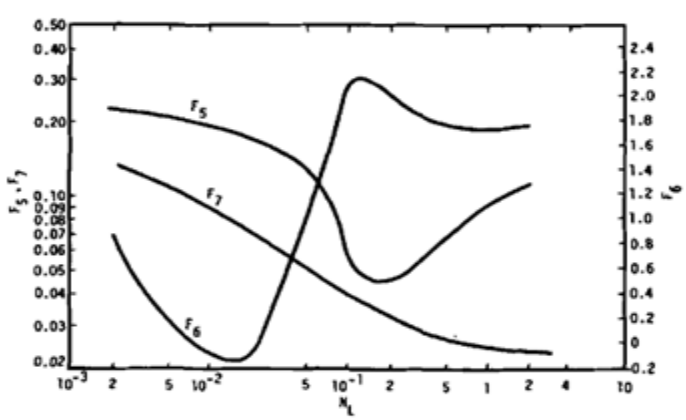


Figure 2.8 – **Duns and Ros (1963) Slug-flow, slip-velocity parameters**[3]

Mist Flow: Mist flow exists if $N_{GV} > N_{GVTr/M}$.

The Duns and Ros (1963) model assumed that, at high gas rates, the entrained liquid is transported mainly as small droplets in a gas continuum. The result is nearly a no-slip condition between the phases. Thus:

$s=0, v_s=0, H_L=\lambda_L$.

Friction in the mist flow pattern originates from the shear stress between the gas and the pipe wall. Thus, the friction component of the pressure gradient is determined from:

$$\left(\frac{df}{dz}\right)_f = \frac{f\,\rho_g v_{sg}^2}{2d} \qquad 2.83$$

The friction factor is obtained from the Moody diagram as a function of Reynold number for the gas phase since a no-slip assumption is made.

$$N_{\Re_g} = \frac{\rho_g v_{sg} d}{\mu_g} \qquad 2.84$$

The Duns and Ros (1963) model noted that the wall roughness for mist flow is the thickness of the liquid film that covers the pipe wall. There is an increased shear between the gas and the film that is caused by waves on the film, this, in turn, can cause the greatest part of the pressure gradient. The drag of the gas deforming the film in opposition to the surface is responsible for the waves. This process is affected by liquid viscosity and is also governed by a form of the Weber number.

$$N_{We} = \frac{\rho_g v_{sg}^2 \epsilon}{\sigma_L} \qquad 2.85$$

This influence was accounted for by making N_{We} a function of a dimnsionlss number containing liquid viscosity.

$$N_\mu = \frac{\mu_L^2}{\rho_L \sigma_L \in} \qquad 2.86$$

The value of roughness ϵ, may be very small, but the relative roughness ϵ/d, never becomes smaller than the value for the pipe itself. At the flow pattern transition to slug, the amplitude of the waves of the film may become large with crest of opposite wavefronts touching and forming liquid bridges. The relative rougness, ϵ/d approaches 0.5. Between these limits, ϵ/d can be obtained from equations developed from Fig. 2.8.

$$N_{We}N_{\mu} \leq 0.005\,;\ \frac{\epsilon}{d} = \frac{0.07749\,\sigma_L}{\rho_g v_{sg}^2 d} \qquad 2.87$$

And

$$N_{We}N_{\mu} > 0.005\,;\ \frac{\epsilon}{d} = \frac{0.3713\,\sigma_L}{\rho_g v_{sg}^2 d}\left(N_{We}N_{\mu}\right)^{0.302} \qquad 2.88$$

Where d is the ID in ft, v_{sg} is the superficial velocity in ft/s, ρ_g is the gas phase density in Ibft^{-3} and σ_L surface tension in dynes/cm.

Values of the friction factor, *f,* for the mist flow regime can be found for $\frac{\epsilon}{d} > 0.05$ by extrapolation on the Moody diagram.

$$f = 4\left\{\frac{1}{\left[4\log_{10}\left(0.27\frac{\epsilon}{d}\right)\right]^2} + 0.067\left(\frac{\epsilon}{d}\right)^{1.73}\right\} \qquad 2.89$$

The actual area available for gas decreases as the wave height of the film on the pipe wall increases, because the diameter open to the flow of gas is now, $d-\epsilon$ ¿). Duns and Ros (1963) refined the friction component of the pressure by suggesting a replacement of d with $(d-\epsilon)$ and v_{sg} with $v_{sg}d^2/(d-\epsilon)^2$ throughout the calculations. This modification resulted in a trial-and-error procedure to determine the value of ϵ .

In mist flow regime, the acceleration component of the pressure gradient cannot be neglected as it was in bubble and slug flow. The acceleration component of the pressure gradient can be approximated by equation 2.90 as provided by Beggs and Brill (1973).

$$\left(\frac{dp}{dz}\right)_{acc} = \frac{v_m v_{sg} \rho_n}{P}\left(\frac{dp}{dz}\right) \qquad 2.90$$

$$E_k = \frac{v_m v_{sg} \rho_n}{P} \qquad 2.91$$

Where $E_{k,}$ is the dimensionless kinetic energy.

The total pressure gradient can be calculated from the equation:

$$\left(\frac{dp}{dz}\right)_T = \frac{\left(\frac{dp}{dz}\right)_{el} + \left(\frac{dp}{dz}\right)_f}{1 - E_k}$$

2.92

Transition Region: The condition for transition zone is defined as $N_{GVS/Tr} < N_{GV} < N_{GVTr/M,}$ .If this region is predicted, linear interpolation between the flow pattern boundaries: $N_{GVS/Tr} \wedge N_{GVTr/M}$, was suggested by the Duns and Ros (1963) model to obtain the pressure gradient. This means that to obtain the transition pressure gradient, a calculation of pressure gradients with both slug flow and mist flow correlations are required. The pressure gradient in the transition regime then is calculated from the equation:

$$\left(\frac{dp}{dz}\right)_l = A\left(\frac{dp}{dz}\right)_{slug} + (1 - A)\left(\frac{dp}{dz}\right)_{mist} \qquad 2.93$$

Where

$$A = \frac{N_{gvTr/M} - N_{gv}}{N_{gvTr/M} - N_{gvS/M}}$$

2.94

In the mist flow pressure gradient calculation, an increased accuracy was claimed in the region by modifying the gas phase density used as:

$$\rho_g' = \frac{\rho_g N_{gv}}{N_{gvTr/M}} \qquad 2.95$$

Duns and Ros (1963) model – summary.

The Duns and Ros (1963), correlation is developed for vertical flow of gas and liquid mixtures in wells. This correlation is valid for a wide range of oil and gas mixtures and flow segments. Although the correlation is intended for use with "dry" oil gas mixture, it can also be applicable to wet mixtures with a suitable correction. For water contents less than 10%, the Duns and Ros(1963) correlation (with a correction factor) has been reported to work well in the bubble, slug (plug) and froth region. The pressure profile prediction performance of the Duns and Ros (1963) method is outlined below in relation to the vertical flow variables considered.

- Tubing Size: In general, the pressure drop is seen to be over-predicted for a range of tubing diameters between 1 and 3 inches.
- Oil Gravity: Good predictions of the pressure profile are obtained for a broad range of oil gravities (13 – 56°API).
- Gas-Liquid Ratio (GLR): The pressure drop is over-predicted for a wide range of GLR. The errors become especially larger (>20%) for GLR greater than 5000.
- Water-Cut: The Duns and Ros model is not applicable for multiphase flow mixtures of oil, water, and gas. However, the correlation can be used with a suitable correction factor as mentioned above.

CHAPTER THREE

METHODOLOGY

3.1 Correlation selection

Selection of the candidate correlation for the determination of void fraction was based on three criteria: (1) the original development of the correlation to include factors such as the type of multiphase fluids used, flow stream orientation, pipe inclination, and pipe characteristics; (2) statistical results of the correlation primarily on the extent of deviation of predicted values from the base case (measured value); (3) the time of development of the correlation. The correlations used were carefully selected to span half a century to bridge any likelihood of knowledge gap.

The void fraction estimation is done in a chronological order of correlations to include: Hughmark (1962), Nicklin (1962), Neal and Bankoff (1965), Grescovich and Cooper (1975), Jowitt (1981), Clark and Flemmer (1985), Hassan and Kabir (1988), Kokal and Stanislav (1989), Toshiba (1989) and Hassan (1995). Statistical analysis of correlation performance was carried out. The computation of the void fraction by the ten selected correlations was carried out using Microsoft Excel 2010. This software offers a variety of options regarding statistical data analysis tools that may be employed in the performance comparison.

3.2 Correlation Performance Evaluation

To compare the performance of the correlations, the agreement between the measured value and predicted values was tested using the following statistical tools: tables, maximum values, minimum values, relative error, step plots, cross plots, percentage absolute average error, average percentage absolute error, bar charts, and multiple bar charts.

3.3 Identification of Flow Regimes

The next step was a classification of the flow stream into flow regimes or flow maps. For the purpose of this work, the Duns and Ros (1963) method was adopted for the flow

characterization, since the method was originally developed for a two-phase system in vertical pipes. To achieve the characterization:

- I computed the flow velocity numbers which include: liquid velocity number, gas velocity number, diameter number and liquid viscosity number, using equations 2.47, 2.48, 2.49 and 2.50.
- Flow constants and transition parameters were obtained using the Duns and Ros (1963) model flow transition charts. See Figures 2.5, 2.6, 2.7 and 2.8.
- Finally, the flow stream was characterized using the conditionality of equation 2.58 for bubble/slug boundary confirmation, equation 2.59 for slug/transition boundary and equation 2.61 for transition/mist boundary.

3.4 Pressure Gradient Prediction Using "Slip Consideration, No Flow Pattern" Model

The void fraction predicted by each of the ten correlations was used to compute the mixture density and mixture viscosity using equations 3.1 and 3.2 respectively.

The effect of the different correlations used in predicting void fraction was felt in the determination of the mixture density and the mixture viscosity as follows:

$$\rho_m = (1-\varepsilon)\rho_L + \varepsilon \rho_G \qquad 3.1$$

$$\mu_m = (1-\varepsilon)\mu_L + \varepsilon \mu_G \qquad 3.2$$

Pressure gradient prediction using "slip consideration, flow pattern consideration" Duns and Ros (1963) Model

The flow regime identified earlier in this study was leveraged upon at this stage. So there was no need to repeat the process. The gravitational pressure gradient and frictional pressure gradients were computed as follows:

- Bubble Flow: Bubble flow exists if $N_{gv} < N_{gv\,B/S}$,
  The dimensionless slip-velocity number was calculated from equation 2.67 and the flow constants obtained from Figure 2.6 and equation 2.68. Slip-density was calculated from equation 2.66. Then the gravitational pressure gradient was obtained using a modified form of equation 5.6 by replacing the mixture density with now calculated slip-density. The friction pressure-gradient component for bubble flow is given by equation 2.69.

- Slug flow: Slug flow exists if $N_{GVB/S} < N_{GV} < N_{GVS/Tr}$

Then the dimensionless slip-velocity number was estimated from the equation 2.73, the flow constants from Figure 2.8 and equation 2.74. Also, gravitational pressure gradient was obtained by replacing mixture density by slip-density to form a modified form of equation 5.6. Slug flow friction pressure-gradient component was calculated exactly the same way as for bubble flow. The acceleration pressure drop was considered negligible for both bubble flow and slug flow hence it was not computed. This agrees with the Duns and Ros (1963) Model.

3.5 Ten Selected Correlations

Ten correlations were used for the prediction of void fraction. The selection was done in chronological order to span developments in the field of multiphase for the last century. The majority of the correlation used the drift flux correlations.

1. Hughmark correlation (1962)

$$\varepsilon = 0.82\left(\frac{U_{SG}}{U_{SG}+U_{SL}}\right)$$

3.3

2. Nicklin et al. (1962)

$$\varepsilon = \frac{U_{SG}}{1.2\,U_M + 0.35\sqrt{gD}}$$

3.4

3. Neal and Bankoff (1965)

$$\varepsilon = 1.25\left(\frac{U_{SG}}{U_M}\right)^{1.88}\left(\frac{U_{SL}^2}{gD}\right)^{0.2} \qquad 3.5$$

4. Greskovich and Cooper (1975)

$$\varepsilon = \frac{U_{SG}}{1.0\,U_M + 0.671\sqrt{gD\left(sin\theta\right)^{0.263}}}$$

3.6

5. Clark and Flemmer (1985)

$$\varepsilon = \frac{U_{SG}}{1.07\,U_M + 1.53\left(g\sigma\left(\frac{\rho_L - \rho_G}{\rho_L^2}\right)\right)^{0.25}}$$

3.7

6. Jowitt (1981)

$$\varepsilon = \frac{U_{SG}}{\left(\left[1 + 0.796\exp\left(-0.061\sqrt{\frac{\rho_L}{\rho_G}}\right)\right]U_M + 0.034\left(\sqrt{\frac{\rho_L}{\rho_G}} - 1\right)\right)}$$

3.8

7. Hassan and Kabir (1988)

$$\varepsilon = \frac{U_{SG}}{1.2\,U_M + V_s}$$

3.92

$$v_s = 0.35\left[\frac{gD\left(\rho_L - \rho_G\right)}{\rho_L}\right]^{0.5} \qquad 3.9b$$

8. Kokal and Stanislav (1989)

$$\varepsilon = \frac{U_{SG}}{1.2\,U_M + 0.345\sqrt{gD\left(1 - \frac{\rho_G}{\rho_L}\right)}}$$

3.10

9. Toshiba (1989)

$$\varepsilon = \frac{U_{SG}}{1.08\,U_M + 0.45} \qquad 3.11$$

10. Hassan (1995)

$$\varepsilon = \frac{U_{SG}}{1.12\,U_M + 0.345\sqrt{gD\left(1-\frac{\rho_G}{\rho_L}\right)}}$$

3.12

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Results Analysis

Results from the ten selected correlations was done using Microsoft Excel for computation. The multiphase system was silicone oil (viscosity = 0.00525 kg/ms) and air (viscosity = 0.000018kg/ms). Table 5.1 gives the void prediction by each of the ten correlations under the same flow conditions.

Table 4.1 **Void fraction prediction from the ten selected correlations**

Void fraction	Base case	Hughmark 1962	Nicklin 1962	Neal &Bankoff 1965	Greskovich& Cooper 1975	Jowitt 1981	Clark &Femmer 1985	Hassan &Kabir 1988	Kokal& Stanislav 1989	Toshiba 1989	Hassan 1995
Average	0.503	0.690	0.537	0.350	0.523	0.402	0.591	0.537	0.539	0.523	0.569
Max.	0.896	0.812	0.786	0.511	0.889	0.74	0.819	0.786	0.787	0.843	0.84
Min.	0.064	0.271	0.104	0.066	0.069	0.044	0.134	0.104	0.104	0.078	0.107

Table 4.1 gives the void fraction obtained from all the ten selected correlations for the air- silicon oil multiphase system in a riser. Using this data, the effect of gas superficial velocity was investigated at a constant liquid velocity to see how each of the correlation responded to the variations in gas superficial velocity. The graphs below depict the performance of the correlations with varying gas rates.

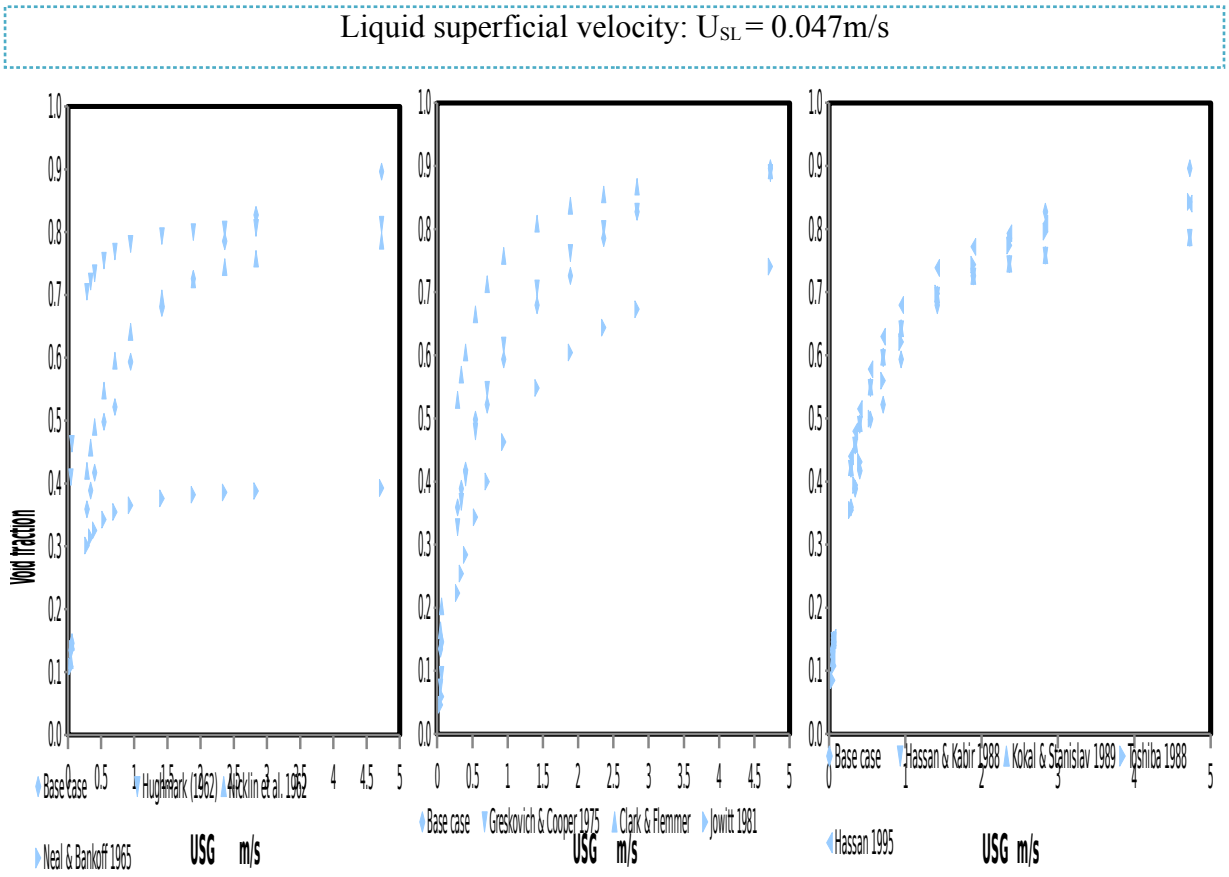


Figure 4.1- **Void fraction prediction at constant liquid velocity: Liquid superficial velocity U_{SL} = 0.047m/s.**

From Figure 4.1A:

Nicklin et al. (1962) gave a good early match of the base case void fraction but starts to show some instability as the gas superficial velocity increases at constant liquid velocity.

Neal and Bankoff (1965), gave a very good match early only for a small range of void fraction and thereafter remarkably under predicted the void fraction as the gas superficial velocity increased. The likely explanation to this may be because the correlation itself was not a drift flux model that is; it did not consider the possibility of the two components of the flow stream having

different velocities at any instant, thereby failing to factor into its derivation the relative velocity component of the flow.

From Figure 4.1B:

Greskovich and Cooper (1975) correlation gave an excellent match with the base case void fraction throughout the gas superficial velocity considered.

Clark and Flemmer (1985) gave a similar profile as the base case but over-predicted its value at all the values of gas superficial velocity considered. Since Clark and Flemmer's correlation is drift flux model based, the possible explanation to this over-prediction of void fraction could be that the distributive coefficient (C = 1.17) in the correlation was too small.

From Figure 4.1C:

Toshiba (1989) gace a very good prediction of the void fraction from the start of the flow but gradually under predicted as the U_{SG} increased above 2 m/s. According to Govier & Aziz (1972), increasing gas rates at a constant liquid rate will eventually create cap-shaped bubbles which almost bridge the pipe, heralding the onset of slug flow in vertical upflow.

Liquid superficial velocity: U_{SL} = 0.071m/s

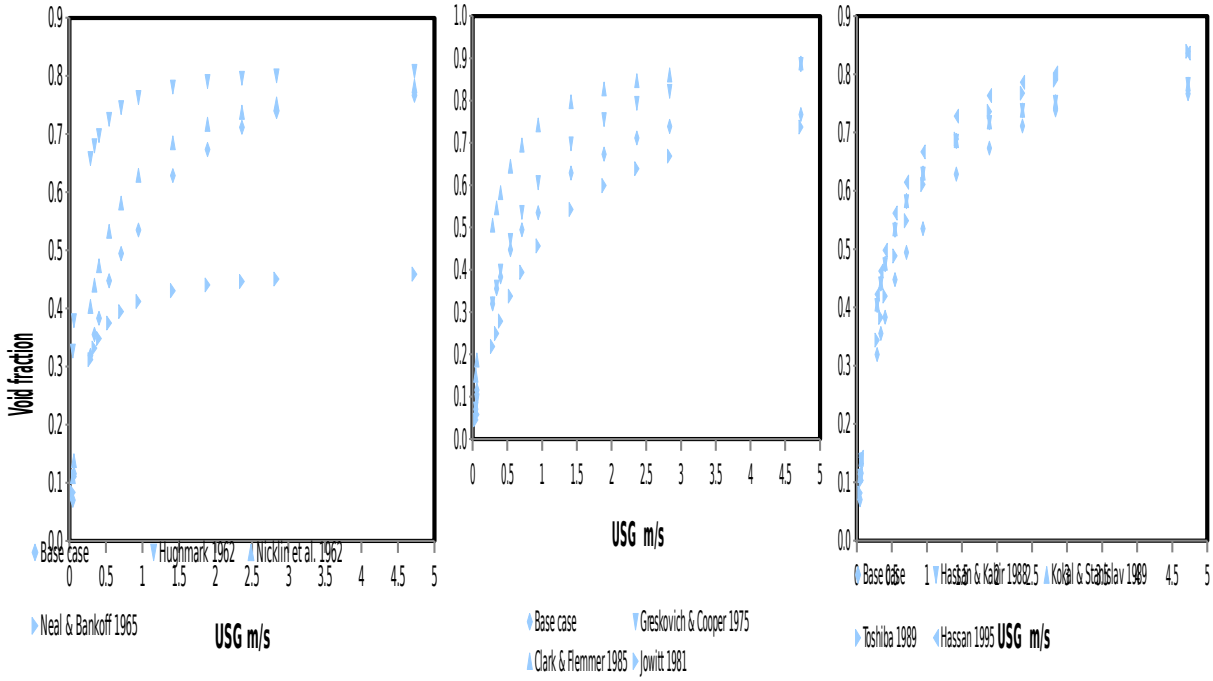


Figure 4.2 - **Void fraction prediction at constant liquid velocity: Liquid superficial velocity U_{SL} = 0.071m/s**

From Figure 4.2A:

Nicklin et al. (1962) gave a similar profile to the base case void fraction but over-predicted the values. This behavior shows a clear continuation of the instability experienced by the same correlation in Fig. 4.1A as the gas rate increased. A clear indication that this correlation performs better for liquid dominated flow as indicated by Spedding et al. (1990): That the Nicklin et al. (1962) correlation gave an acceptable prediction for bubble and slug flows for all inclination angles.

From Figure 4.2B

The Clark and Flemmer (1985) correlation, showed the same behavior (over prediction) as in Fig. 5.1A even at an increased liquid superficial velocity from 0.047m/s to 0.071 m/s.

Greskovich and Cooper (1975) continued to show excellent prediction even as the liquid velocity increased from 0.047 m/s to 0.071m/s, but gradually starts over predicting as the gas velocity increases above 1m/s.

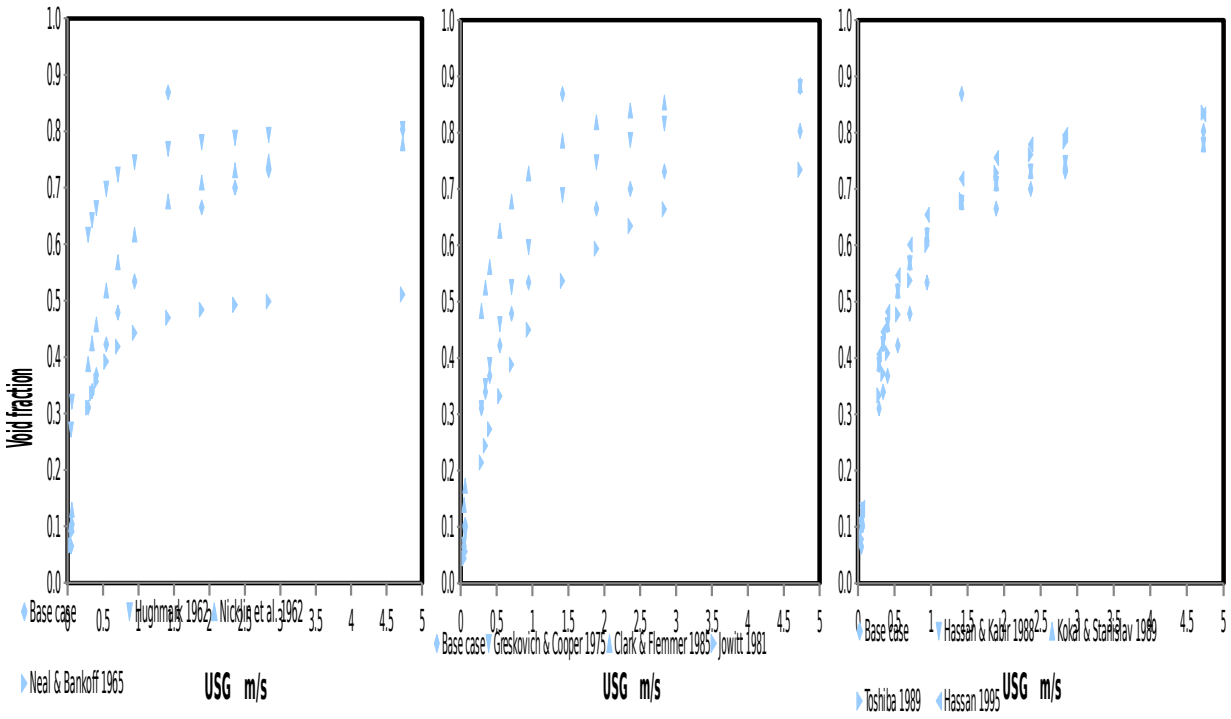
From Figure 4.2C

Toshiba (1989) gave a good match only to gas velocity as high as 0.6 m/s. but as the velocity increased, the correlation failed.

Kokal and Stanislav (1989) correlation gave a good match of the base case void fraction at higher gas superficial velocities above 2 m/s and over prediction at lower velocities. A likely explanation to this could be drawn from the fact that the correlation was originally developed for horizontal and $\mp 9^o$ near horizontal pipes.

Neal and Bankoff (1965), remarkably under predicted the void fraction as the gas superficial velocity increased. A likely explanation to this may be because the correlation itself was not a drift flux model that is; it did not consider the possibility of the two components of the flow stream having different velocities at any instant, thereby failing to factor into its derivation the relative velocity component of the flow.

Liquid superficial velocity: $U_{SL} = 0.095$m/s



F

igure 4.3 - **Void fraction prediction at constant liquid velocity: Liquid superficial velocity U_{SL} = 0.095m/s.**

From Figure 4.3A

Nicklin et al (1962) gave a similar profile to the base case void fraction but over-predicted the values. This behavior shows a clear continuation of the instability experienced by the same correlation in Fig. 4.1A as the gas rate is increasing.

From Figure 4.3B

The Clark and Flemmer (1985) correlation, shows the same behavior (over prediction) as in Fig. 4.1A even at increased liquid superficial velocity from 0.047m/s to 0.071 m/s.

Greskovich and Cooper (1975) continued to show excellent prediction even as the liquid velocity increased from 0.047 m/s to 0.095m/s, but gradually start over-predicting as the gas velocity increases above 1m/s. This behavior indicates that the correlation is performed better for low gas rate flow regimes.

From Figure 4.3C

The Kokal and Stanislav (1989) correlation gave a good match of the base case void fraction at higher gas superficial velocities above 2 m/s and over prediction at lower velocities. Considering the performance of the correlation from the three liquid velocities considered it can be deducted that Kokal and Stanislav (1989) gives the best match at high gas rates hence good for gas dominated flows.

Toshiba (1989) gave a good match only to gas velocity as high as 0.6 m/s. but as the velocity increased, the correlation fails.

Neal and Bankoff (1965), remarkably under predicted the void fraction as the gas superficial velocity is increased. A likely explanation to this may be because the correlation itself was not a drift flux model.

Cross plot of predicted void fraction against base case void fraction

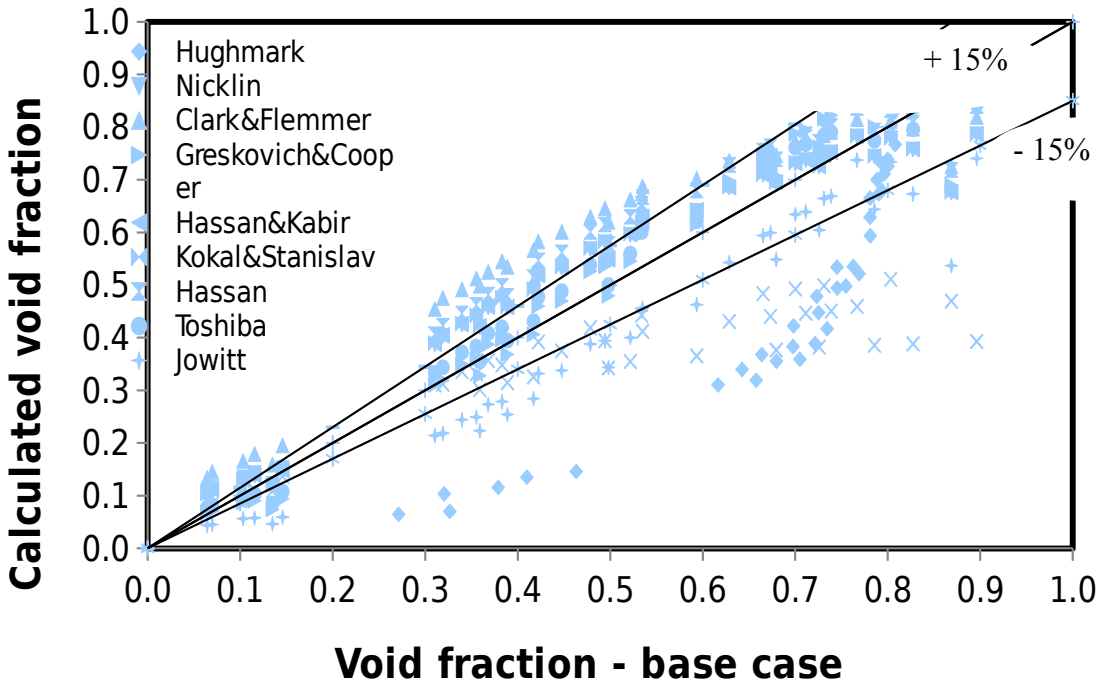


Figure 4.4 **Comparison of selected correlations with no flow pattern consideration using error limit.**

Figure 4.4 is the cross plot of the void fraction predicted by the ten selected correlations against the base case void fraction band.

At void fraction less than 0.2, Neal and Bankoff (1965) fall within the acceptable limits but completely fall outside the -15% width as the void fraction increases above 0.4. This is because the correlation did not account for drift velocity.

The Kokal and Stanislav (1989) correlation gave an excellent prediction at higher values of void fraction greater than 0.5, indicating its good performance for gas dominated flows.

The Toshiba (1989) correlation gave values of void fraction within +15% band for all void fraction above 0.3.

Greskovich and Cooper (1975) continued to show excellent prediction within the +15% limit

4.2 Result of Total Pressure Gradient Prediction

Cross plot for the predicted pressure gradient vs base case

+30% +15%

15%

-30%

Figure 4.5 **Implication of void fraction estimation on pressure gradient prediction**

From Figure 4.5, Greskovich and Cooper (1975) correlation gave the best pressure match for a pressure gradient range of 4000 Pa/m to 6000 Pa/m but under-predicted at lower pressure gradients. Also from Fig. 4.5, it can be seen that Greskovich and Cooper (1975) slightly over-predicted at higher pressure gradients with fewer points. It can, therefore, be deduced that this correlation gives its best performance for slug flow regime.

Hassan (1995) correlation continuously under predicted the pressure gradient as seen from Fig. 4.5, but gives an excellent pressure march at higher pressure gradient between 7500 Pa/m to 8500 Pa/m. this behavior can be corroborated with the trend shown by Hassan (1995) in Figure

4.1A – C, the correlation consistently gave a good match at a void fraction less than 0.1, and a near single-liquid-phase flow.

Nicklin et al (1962) gave a good match from 2000 Pa/m to 4000 Pa/m then under predicted and again gave a good prediction at a much higher pressure gradient from 7500 Pa/m and above. This behavior of Nicklin et al. (1962) can be attributed to the drift flux velocity term in the equation which depends solely on the pipe diameter only without considering the fluid characteristics. This is why the performance of the correlation remains fairly constant showing little sensitivity to flow pattern variation.

4.3 Result Comparison

The result comparison was carried out using statistical tools of Average absolute error, Absolute average error relative error.

Table 4.2 **Error Estimation**

	AVE. Void Fraction	%ABS.AVE. Error	AVE.%ABS Error
Base case	0.503		
Hughmark (1962)	0.690	27.188	33.051
Nicklin et al.(1962)	0.537	6.445	13.026
Neal &Bankoff(1965)	0.350	43.749	38.693
Greskovich& Cooper (1975)	0.523	3.920	10.768
Jowitt(1981)	0.402	25.073	39.576
Clark &Flemmer(1985)	0.591	14.897	20.724
Hassan &Kabir(1988)	0.537	6.457	13.036
Kokal&Stanislav(1989)	0.539	6.720	13.273
Toshiba (1989)	0.523	3.938	9.851
Hassan (1995)	0.569	11.735	16.109

Figure 4.6 **Comparison of selected correlations with no flow pattern consideration**

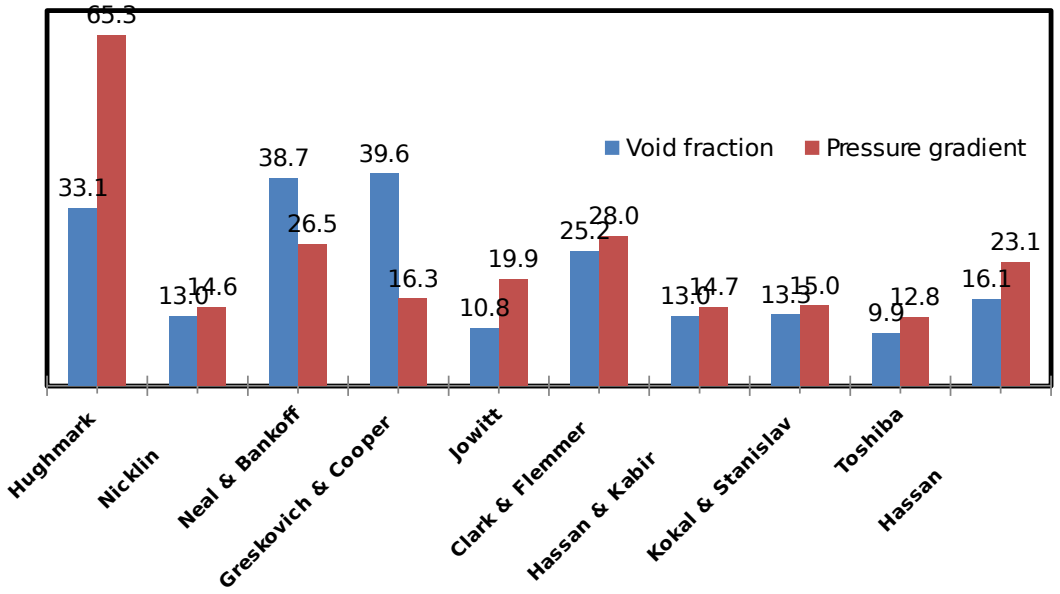


Figure 4.7 – **Implication of void fraction estimation on pressure gradient prediction**

Figure 4.7 shows the sensitivity of void fraction estimation on overall pressure gradient determination. From the chart above, it can be seen that on the average, the higher the magnitude of the error in void estimation, the higher the error estimation in the pressure gradient. Hughmark (1962) correlation gave the highest error in pressure determination 65.3% which corresponds to about 33.1% average percentage error in the void fraction prediction, while Toshiba (1989) correlation gave the minimum error in pressure estimation 9.9% which corresponds to 12.8% average absolute error. Greskovich and Cooper (1975) though they gave a high average error, they still gave a good prediction. This is because the majority of the void fraction correctly predicted pressure gradient, falls in one flow zone (slug flow), making the error in the pressure prediction to be relatively low.

4.4 Comparison with Respect to Flow Regime

An important aspect of multiphase-phase flow in pipes is the geometric distribution of the system under varying flow conditions. This multiphase component distribution is termed flow regime or pattern. The flow regime in any multiphase system is sensitive to the pipe orientation, the direction of flow, and fluid characteristics. Published works on flow map prediction can be classified into empirical and analytical categories. Empirical flow pattern maps are those based on experiment whereby data are collected and plots of independent variables generated. The quality of such a flow map depends on the accuracy of the experimental design. A possible setback of this type of flow map is its subjectivity as the final results are affected by the perception of flow patterns by the experiment[5]. Analytical flow regime classification, on the other hand, is based on the mathematical modeling of the physical description of both the pipe properties and the fluid characteristics[6]. Conservation laws, constitutive equations and dimension analysis are the common tools used here.

For the purpose of this study, the Duns and Ros (1963) model (category “c”) was used to determine the overall pressure gradient, since this model was originally designed for vertical upflow systems.

4.4.1 Results for Duns and Ros (1963) model

Flow pattern identification – the first step was to characterize the multiphase flow stream into flow regimes.

Table 4.3A - **Bubble flow identification: Determination of friction parameters**

N_d	N_L	L_1	L_2	f_1	f_2	f_3	f_4	f'_3
44.49	0.03	1.40	0.80	1.50	0.30	2.00	40.00	1.101

(See appendix for calculations and charts)

Table 4.3A shows the bubble flow identification data and the flow parameters needed for the computation of slip-velocity number.

L_1 and L_2 are geometric variables obtained as functions of the pipe diameter number N_d, using Figure 2.5. The 1.40 and 0.80 values were imputed into equation 2.66 to obtain the bubble/slug boundary data.$f_1$and f_2 are functions of the liquid viscosity number N_L and were obtained from Figure 2.6, and f_3 from equation 2.76. The flow constants were used to estimate the slip-velocity number using equation 2.75.The actual slip-velocity of the flow was determined from equation 2.70 and the bubble flow liquid hold up and void fraction from the solution of equation 2.73.

Table 4.3B -**Bubble flow identification: Determination of transition velocity numbers**

Average values							
U_{SL} (m/s)	U_{SG} (m/s)	U_m (m/s)	N_{LV}	N_{GV}	$N_{GV(B/S)}$	$N_{GV((S/Tr)}$	$N_{GV(Tr/M)}$
0.071	0.054	0.125	0.584	0.445	1.868	71.041	130.737

(See appendix for calculations and charts)

Table 4.3B shows the average values of the transitional velocity values obtained from equation 2.66, 2.68 and 2.69. The gas velocity number data for the bubble-slug boundary was plotted as a function of gas velocity number data as shown below.

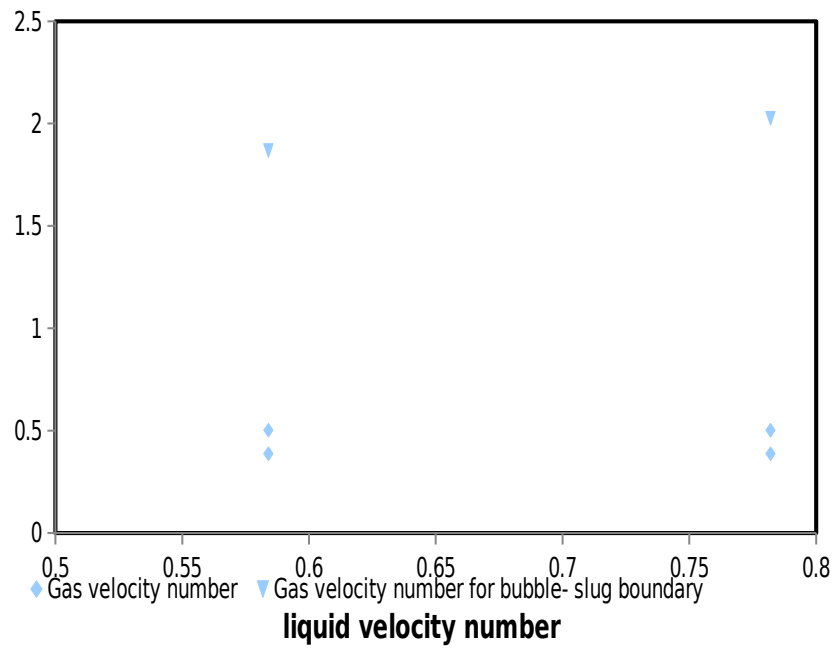


Figure 4.8 **Bubble flow identified**

Figure 4.8 shows identification of bubble flow. The basis for this was from Duns and Ros (1963) model that stated that, if at any instant gas velocity number is less than its value for bubble-slug transition, then the flow is considered to be bubble i.e. ($N_{GV} < N_{GV(B/S)}$). From the graph, at all values of the liquid velocity, the gas velocity number was less than its value at the transition boundary.

Table 4.4A -**Slug flow identification: Determination of flow constants**

N_d	N_L	f_5	f_6	f_7	f'_6
44.49	0.03	0.16	0.54	0.06	1.83

(See appendix for calculations and charts)

Table 4.4A gives the pipe diameter number, liquid viscosity number and the flow parameters needed to compute the slug regime slip number. f_5, f_6 and f_7 were all obtained as functions of the liquid viscosity number given in Figure 2.6 and f'_6 from equation 2.82. The dimensionless slip-velocity number was computed from equation 2.81. The actual slip-velocity and the slug

flow liquid hold up and void fraction were obtained from the solutions of equations 2.70 and 2.73.

Table 4.4B **-Slug flow identification: Determination of transition velocity number**

Average values							
U_{SL} (m/s)	U_{SG} (m/s)	U_m (m/s)	N_{LV}	N_{GV}	$N_{GV(B/S)}$	$N_{GV((S/T)}$	$N_{GV(T/M)}$
0.071	0.515	0.586	0.584	4.240	1.868	71.041	130.713

(See appendix for calculations and charts)

Table 4.4B shows the average values of the transitional velocity values obtained from equation 2.66, 2.68 and 2.69. The gas velocity number data for bubble/slug boundary and slug/transition were plotted as functions of gas velocity number data as shown below.

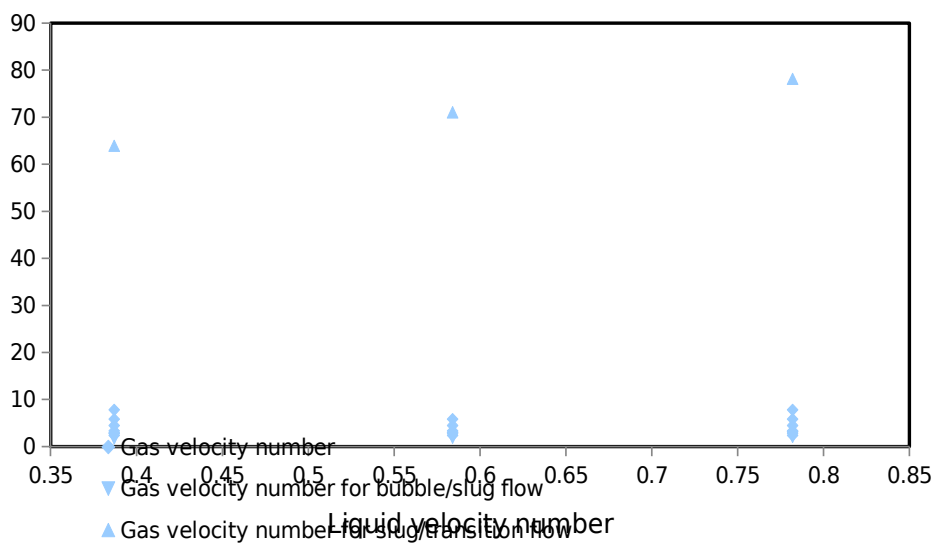


Figure 4.9 **Slug flow identified**

Figure 4.9 shows identification of slug flow. The basis for this was from the Duns and Ros (1963) model that states that, if at any instant the gas velocity number is greater than its value for bubble-slug transition and less than its value for slug-transition, then the flow is considered to be in the slug regime i.e ($N_{GV(B/S)} < N_{GV} < N_{GV(S/T)}$).

Table 4.5 **Churn flow identified $N_{GV} < N_{GV(S/T)} < N_{GV(T/M)}$**

Average values							
U_{SL} (m/s)	U_{SG} (m/s)	U_M (m/s)	N_{LV}	N_{GV}	$N_{GV(B/S)}$	$N_{GV(S/T)}$	$N_{GV(T/M)}$
0.071	2.541	2.612	0.584	20.914	1.868	71.041	130.763

Table 4.5 shows the average values of the transitional velocity values obtained from equation 2.66, 2.68 and 2.69. The gas velocity number data for slug/transition flow boundary and transition/mist were plotted as functions of gas velocity number data as shown below.

The Duns and Ros (1963) model assumed that, at high gas rates, the entrained liquid is transported mainly as small droplets in a gas continuum. The result is nearly a no-slip condition between the phases. Thus,

$$s=0, v_s=0, H_L=\lambda_L \quad .$$

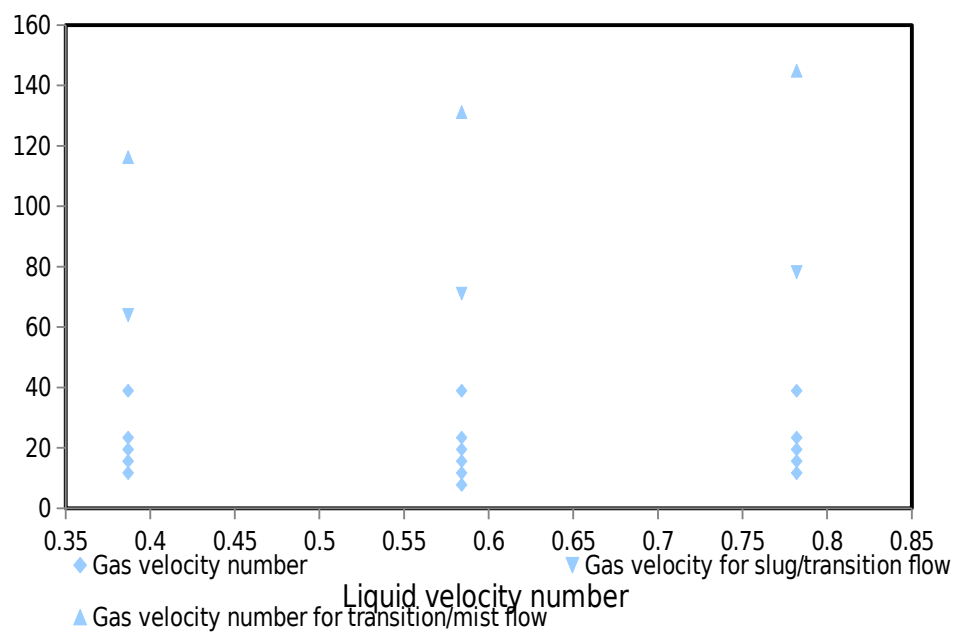


Figure 4.10 **Churn flow identified**

Figure 4.10 shows identification of churn flow. The basis for this was from the Duns and Ros (1963) method that states that, if at any instant gas velocity number is less than its value for slug-transition and less than its value for slug-mist, then the flow is considered to be in the slug regime i.e $N_{GV} < N_{gv((S/T)} < N_{gv(T/M)}$,

Table 4.6 **Performance of correlations based on flow regime**

	Bubble			Slug			Churn		
	ε	E_1	E_2	ε	E_1	E_2	ε	E_1	E_2
Base case	0.106			0.425			0.734		
Hughmark (1962)	0.362	70.785	71.282	0.709	40.047	40.460	0.793	7.507	10.841
Nicklin et al. (1962)	0.125	15.196	20.977	0.503	15.471	15.903	0.728	0.744	6.987
Neal & Bankoff (1965)	0.099	6.268	11.396	0.354	20.107	19.847	0.439	67.215	68.954
Greskovich & Cooper (1975)	0.081	30.942	32.168	0.438	2.977	4.838	0.779	5.838	9.043
Jowitt (1981)	0.051	105.314	103.671	0.315	35.221	37.433	0.626	17.180	17.816

Clark & Flemmer (1985)	0.163	35.147	36.181	0.572	25.607	26.165	0.771	4.852	9.145
Hassan &Kabir (1988)	0.125	15.232	20.995	0.503	15.487	15.920	0.728	0.738	6.988
Kokal & Stanislav (1989)	0.126	16.028	21.398	0.505	15.843	16.284	0.729	0.601	7.026
Toshiba (1989)	0.092	14.528	23.706	0.457	6.988	6.939	0.755	2.822	7.751
Hassan (1995)	0.129	17.968	22.562	0.530	19.818	20.190	0.776	5.477	9.353

ε = Void Fraction Prediction

E_1 =Percentage Absolute Average Error

E_2 = Average Percentage Absolute Error

$$\Delta P = -8825\,\varepsilon + 8826$$

Figure 4.11 **Bubble flow pressure gradient prediction**

From Figure 4.11, all the curves show pressure decline as the void fraction increases. This behavior was expected since the system under study was a vertical pipe; the primary contributor to the overall pressure gradient was the gravitational pressure which depended on the density of the column. Since air (the gas phase in the study) is lighter than silicone oil (the liquid phase in this study), it is normal for the hydrostatic head to decline with increasing gas rate.

Nicklin et al (1962), Hassan and Kabir, Kokal and Stanislav, and Hassan all under predicted the pressure gradient throughout the flow regime. But Toshiba (1989) correlation intersects the base case pressure gradient at about 8000 Pa/m and progressively over-predicted the pressure gradient as the void fraction increases.

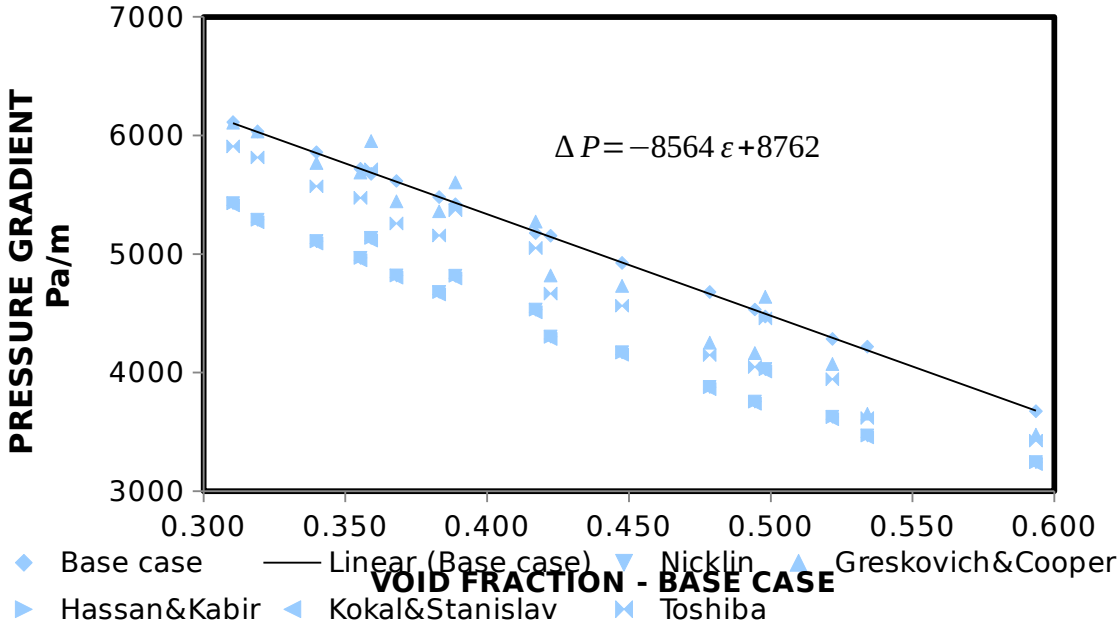


Figure 4.12 – **Slug flow pressure gradient prediction**

From Figure 4.12, the slope of the decline was 8564 Pa/m which was lesser than the slope of Figure 4.11, 8825 Pa/m. This reduction in pressure was expected as the multiphase flow moves from predominantly liquid dominated flow (bubble) to a flow with increasing gas population (slug flow).

All the correlations performed fairly well in this regime as they all showed the same trend in the pressure decline (the curves approximate parallel lines). But Greskovich and Cooper (1975) correlation gave the best match to the base case pressure gradient.

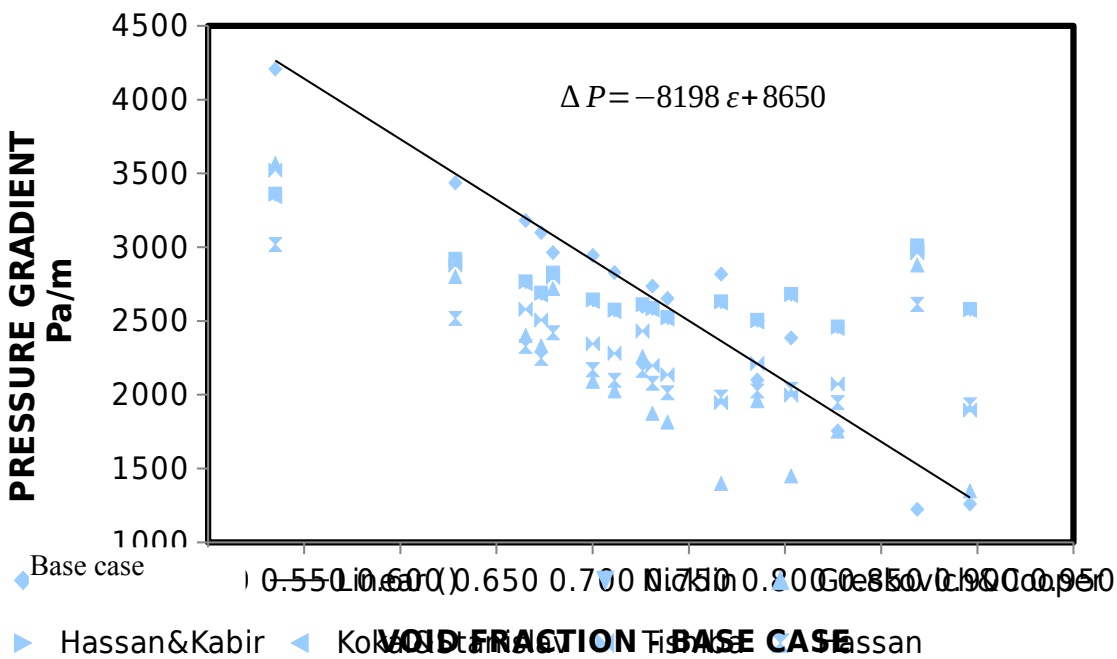


Figure 4.13 – **Churn flow pressure gradient prediction**

From Figure 4.13, there was a further decline in pressure (slope = 8198 Pa/m) this was because the gas population increased rapidly at the expense of the liquid phase, hence reduction in the flow pressure. It can be seen also from the Figure that the graphs do not give perfect straight lines as in Figure 4.11 and Figure 4.12. This deviation from straight line relation is an indication of a change from predominantly liquid dominated flow to gas dominated flow. The clustering of the points without a well-defined pattern show that the presence of uncertainties and instabilities a clear indication of the presence of churn flow. Considering the distribution of the points around the base case one can infer that Nicklin et al. (1962) may be suitable for the pressure prediction.

Table 4.7 **Bubble Flow: Average Total Pressure Gradient**

Bubble Flow: Average Total Pressure Gradient Pa/m					
Model: Slip considered, flow pattern not considered					
Base case	Hassan & Kabir	Kokal & Stanislav	Toshiba	Hassan	Nicklin et al.
7893.18	7725.77	7715.37	8011.33	7689.12	7726.25
%ABS. AVE Error	2.17	2.3	1.47	2.65	2.16

Relative Error	0.20	0.21	0.14	0.25	0.20
Duns and Ros (1963) Model					
Duns & Ros	Hassan & Kabir	Kokal &Stanislav	Toshiba	Hassan	Nicklin et al.
7234.668	7725.77	7715.37	8011.33	7689.12	7726.25
%ABS. AVE. Error	6.36	6.23	9.69	5.91	6.36
Relative Error	0.18	0.18	0.30	0.17	0.18

Table 4.8 **Slug Flow: Average Total Pressure Gradient**

Slug Flow: Average Total Pressure Gradient Pa/m						
Model: Slip considered, flow pattern not considered						
Base case	Neal & Bankoff	Greskovich & Cooper	Hassan & Kabir	Kokal & Stanislav	Toshiba	Hassan
5119.59	5755.28	5002.43	4426.30	4407.38	4834.62	4184.17
%ABS. AVE. Error	11.05	2.34	15.66	16.16	5.89	22.36
Relative Error	0.15	0.03	0.21	0.22	0.08	0.30
Duns and Ros (1963) Model						

Duns & Ros	Neal & Bankoff	Greskovich& Cooper	Hassan & Kabir	Kokal & Stanislav	Toshiba	Hassan
4812.25	5755.28	5002.43	4426.30	4407.38	4834.62	4184.17
%ABS. AVE. Error	16.39	3.80	8.72	9.19	0.46	15.01
Relative Error	0.31	0.07	0.16	0.17	0.01	0.28

4.5 Flow Assurance Scheme Developed

Based on the results of this study and the data used to produce these results, the following conclusions were drawn:

- Nicklin et al. (1962) drift flux correlation gives the best void fraction (Figure 4.1). The prediction from this correlation shows a fairly constant average absolute error of about 20.98% (Table 4.6) for low gas rate flow (bubble), but over-predicted the void fraction as the gas rate increases (Figure 4.1A, 4.2A and 4.3A). Nicklin et al. (1962) achieved 14.18% absolute error in predicting pressure gradient when compared to the pressure obtained from the Duns and Ros (1963) model (Table 4.7).
- Toshiba (1989), gives a very good void prediction but gradually over predicts as the liquid velocity is increased (Figure 4.1C, 4.2C, and 4.3C). Also considering Figure 4.4, Toshiba (1989) correlation gives values of a void fraction within a +15% band for all void fraction above 0.3. Toshiba (1989) achieved a relative error of 0.01 in predicting pressure gradient when compared to the pressure obtained from Duns and Ros model (Table 4.7). Hence a good correlation for slug flows in vertical upflow.
- Greskovich and Cooper (1975) give the best prediction for a void fraction in slug flow regime with about 4.84% average absolute error (Figure 4.4), and an under-predicted void fraction as the gas rate increases (Figure 4.1). Also within the slug flow regime, Greskovich and Cooper (1975) give a very good fit for pressure prediction with only

about 3.80% average absolute error when compared to the pressure obtained from the Duns and Ros (1963) Model (Table 4.7).

- Hassan and Kabir (1989), show progressively higher accuracy and stability in the direction of increasing gas rate with an average absolute error of 6.99% in the churn flow regime. Hence a good correlation for transitional flow region.

At the end of this research work, within the limits of data available and span of correlations considered, the following flow assurance scheme was developed for multiphase systems with fluid characteristics similar to those used for this work.

Table 4.9 **Flow scheme developed for vertical upflow in 67mm ID pipe**

	Percentage Average Absolute Error Expected	
Flow regime/The two best correlations	Void fraction	Pressure Gradient (Duns & Ros (1963) Model
Bubble		
Nicklin et al (1962)	20.98	6.36
Kokal & Stanislav (1989)	21.40	6.23
Slug		
Greskovich & Cooper (1975)	4.84	3.80
Toshiba	6.94	0.46

(1989)		
Churn		
Hassan &Kabir (1988)	6.99	
Kokal & Stanislav (1989)	7.03	

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Based on the results of this study and the data used to produce these results, the following conclusions were drawn:

- The Nicklin et al. (1962) drift flux correlation gives the best void fraction for bubble flow. The prediction from this correlation shows a fairly constant average absolute error of about 20.98% for low gas rate flow (bubble).
- Greskovich and Cooper (1975) give the best prediction for void fraction in slug flow regime with about 4.84% average absolute error in void fraction prediction. Also within the slug flow regime, Greskovich and Cooper (1975) give a very good fit for pressure prediction with only about 3.80% average absolute error when compared to the pressure obtained from Duns and Ros (1963) Model.
- Hassan and Kabir (1989), show a progressively higher accuracy and stability in the direction of increasing gas rate with an average absolute error of 6.99% in the churn flow regime. Hence a good correlation for transitional flow region.

5.2 Recommendations

- Within the limits of data available, the Nicklin et al (1962) shows a significantly good performance for bubble flow and slug flow. Resultantly, an improvement in this correlation is possible with the modification of the distribution parameter that will hopefully make it possible to be used for gas dominated flows.
- A comparison study using laboratory data depends to a large extent on the quality and range of data used. It is suggested that the degree of generality of the conclusions reached be extended by carrying out further testing with hydrocarbon data.

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APPENDIX A: Void fraction prediction by the ten selected correlations

Base case	Hughmark (1962)	Nicklin (1962)	Neal &Bankoff (1965)	Greskovich& Cooper (1975)	Jowitt (1981)	Clark &Flemmer (1985)	Hassan &Kabir (1988)	Kokal&Stanislav (1989)	Toshiba (1989)	Hassan (1995)
0.135	0.410	0.119	0.109	0.074	0.046	0.159	0.119	0.120	0.085	0.122
0.146	0.463	0.148	0.137	0.094	0.059	0.195	0.148	0.149	0.108	0.152
0.359	0.705	0.420	0.301	0.328	0.223	0.498	0.420	0.423	0.355	0.440
0.389	0.721	0.457	0.315	0.368	0.254	0.535	0.457	0.460	0.394	0.480
0.417	0.735	0.490	0.325	0.406	0.284	0.566	0.490	0.492	0.431	0.515
0.498	0.755	0.548	0.342	0.479	0.344	0.620	0.548	0.550	0.500	0.578
0.522	0.769	0.595	0.355	0.546	0.400	0.662	0.595	0.598	0.560	0.630
0.594	0.781	0.641	0.365	0.615	0.462	0.702	0.641	0.643	0.621	0.680
0.679	0.794	0.695	0.376	0.706	0.548	0.746	0.695	0.696	0.698	0.738
0.726	0.800	0.725	0.382	0.762	0.604	0.771	0.725	0.726	0.744	0.772
0.786	0.804	0.744	0.386	0.800	0.644	0.786	0.744	0.745	0.774	0.793
0.827	0.807	0.758	0.388	0.828	0.673	0.797	0.758	0.758	0.796	0.808
0.896	0.812	0.786	0.393	0.889	0.740	0.819	0.786	0.787	0.843	0.840
0.070	0.327	0.111	0.084	0.071	0.045	0.145	0.111	0.112	0.081	0.114
0.116	0.379	0.138	0.111	0.090	0.058	0.179	0.138	0.139	0.103	0.143
0.319	0.658	0.403	0.312	0.319	0.219	0.475	0.403	0.406	0.344	0.423

APPENDIX A: Void Fraction Prediction by the Ten Selected Correlations (Cont'd)

0.355	0.680	0.440	0.332	0.359	0.249	0.512	0.440	0.443	0.383	0.462

0.383	0.697	0.473	0.348	0.397	0.279	0.545	0.473	0.476	0.420	0.498
0.448	0.725	0.532	0.375	0.469	0.338	0.601	0.533	0.535	0.488	0.562
0.494	0.745	0.581	0.394	0.536	0.394	0.645	0.581	0.583	0.549	0.615
0.535	0.763	0.629	0.412	0.606	0.456	0.688	0.629	0.631	0.611	0.667
0.629	0.781	0.685	0.431	0.698	0.542	0.736	0.685	0.686	0.689	0.728
0.673	0.790	0.717	0.440	0.755	0.599	0.762	0.717	0.718	0.736	0.763
0.711	0.796	0.737	0.446	0.794	0.639	0.779	0.737	0.738	0.768	0.786
0.739	0.800	0.752	0.451	0.822	0.669	0.791	0.752	0.753	0.790	0.802
0.767	0.808	0.782	0.459	0.885	0.737	0.815	0.782	0.783	0.839	0.836
0.064	0.271	0.104	0.066	0.069	0.044	0.134	0.104	0.104	0.078	0.107

0.103	0.321	0.130	0.091	0.087	0.056	0.166	0.130	0.131	0.099	0.134
0.310	0.617	0.388	0.310	0.311	0.214	0.454	0.388	0.390	0.333	0.407
0.340	0.643	0.424	0.335	0.350	0.244	0.492	0.425	0.427	0.372	0.446
0.368	0.664	0.458	0.357	0.387	0.273	0.525	0.458	0.460	0.409	0.482
0.422	0.698	0.518	0.392	0.460	0.332	0.583	0.518	0.520	0.477	0.547
0.478	0.723	0.568	0.419	0.526	0.388	0.629	0.568	0.570	0.538	0.601
0.534	0.745	0.617	0.443	0.597	0.450	0.674	0.617	0.619	0.601	0.654
0.869	0.769	0.675	0.469	0.689	0.537	0.725	0.676	0.677	0.680	0.718
0.665	0.781	0.709	0.484	0.748	0.594	0.754	0.709	0.710	0.729	0.755
0.700	0.788	0.731	0.492	0.787	0.634	0.772	0.731	0.732	0.761	0.779

0.731	0.793	0.746	0.498	0.816	0.664	0.784	0.746	0.747	0.784	0.796
0.803	0.804	0.779	0.511	0.881	0.734	0.811	0.779	0.779	0.835	0.832

APPENDIX B: Calculation of Pressure gradient by ten selected void fraction correlations with no flow regime

Base case	Hughmark	Nicklin et al.	Neal & Bankoff	Greskovic & Cooper	Jowitt	Clark & Flemmer	Hassan & Kabir	Kokal & Stanislav	Toshiba	Hassan
7635	5211	7779	7865	8174	8415	7423	7778	7767	8073	7747
7537	4743	7523	7619	7999	8302	7102	7523	7510	7875	7481
8208	5946	7850	8087	8198	8426	7545	7850	7840	8107	7818
7806	5485	7609	7850	8029	8316	7245	7608	7597	7918	7567
8257	6433	7913	8241	8221	8437	7648	7913	7904	8139	7880
7915	6000	7684	8026	8057	8330	7366	7683	7673	7956	7642
5676	2619	5137	6188	5953	6876	4444	5136	5114	5715	4961
5420	2476	4816	6076	5603	6611	4128	4815	4794	5370	4616
5176	2363	4532	5988	5273	6355	3854	4531	4509	5051	4309

4474	2192	4031	5857	4640	5847	3389	4030	4010	4458	3764
4285	2075	3626	5777	4072	5372	3028	3626	3608	3944	3321
3675	1984	3246	5731	3478	4855	2698	3245	3229	3425	2898
6033	3037	5289	6096	6033	6920	4651	5289	5268	5814	5117
5718	2847	4968	5928	5688	6659	4328	4967	4947	5474	4772
5480	2694	4680	5790	5360	6406	4047	4680	4660	5157	4462
4926	2455	4171	5573	4731	5903	3562	4170	4151	4564	3909
4533	2289	3755	5426	4164	5431	3182	3754	3737	4048	3455
6112	3403	5430	6113	6109	6963	4839	5429	5410	5908	5261
5859	3178	5109	5899	5768	6706	4512	5108	5089	5572	4917
5616	2993	4820	5719	5444	6456	4225	4819	4801	5257	4606
5154	2699	4303	5425	4819	5958	3726	4303	4285	4666	4047
4679	2489	3878	5214	4253	5489	3328	3877	3861	4148	3583

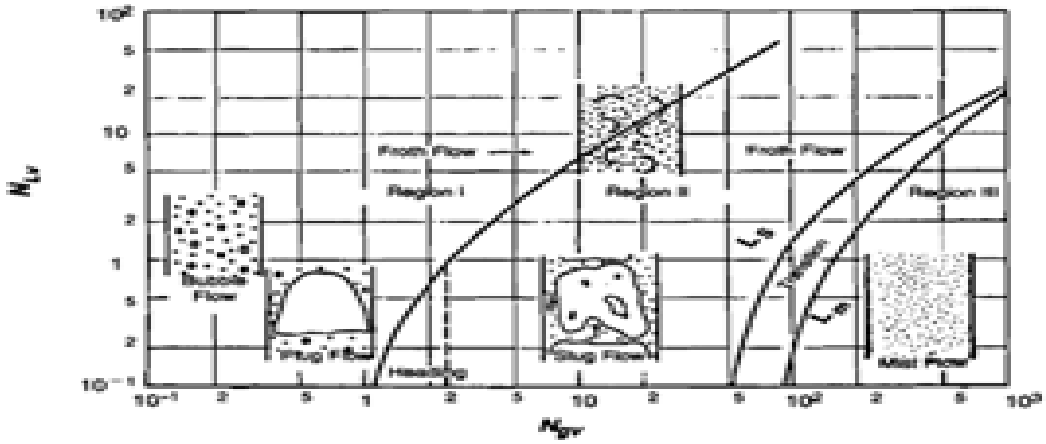
APPENDIX B: Calculation of Pressure gradient by ten selected void fraction correlations with no flow regime (Cont'd)

4219	2313	3470	5041	3653	4974	2957	3469	3454	3617	3132
2964	1912	2825	5754	2720	4172	2348	2824	2811	2795	2420
2599	1898	2609	5841	2258	3746	2175	2609	2598	2432	2165
2100	1921	2504	5994	1960	3482	2094	2504	2495	2213	2025
1755	1964	2459	6187	1753	3312	2063	2458	2450	2073	1947
1261	2270	2578	7290	1348	3126	2185	2578	2571	1895	1932
4206	2152	3360	5316	3566	4916	2830	3359	3344	3523	3017
3437	2032	2917	5262	2800	4230	2451	2917	2904	2880	2517

3100	1994	2688	5300	2331	3801	2261	2688	2677	2506	2247
2828	2002	2574	5411	2026	3534	2169	2573	2564	2280	2098
2653	2035	2522	5567	1814	3362	2130	2521	2513	2136	2013
2817	2324	2629	6514	1399	3172	2238	2629	2623	1946	1984
1224	2149	3008	4910	2879	4288	2551	3007	2995	2962	2612
3181	2087	2765	4899	2402	3856	2345	2765	2755	2579	2328
2943	2081	2642	4971	2092	3586	2242	2642	2633	2347	2169
2737	2106	2584	5092	1875	3413	2196	2584	2576	2197	2078
2386	2378	2680	5905	1450	3218	2290	2680	2674	1997	2037

APPENDIX C: Calculation of Pressure gradient by Duns and Ros (1963) model

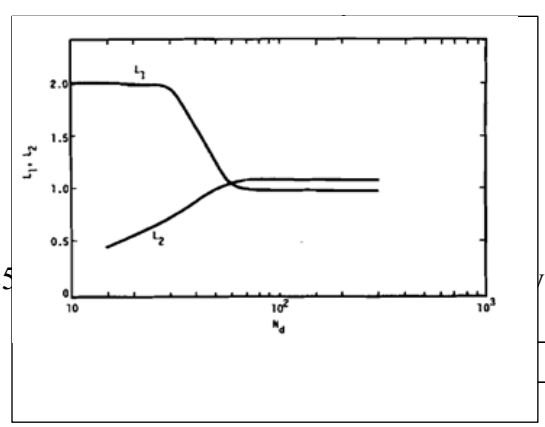
C1 Duns and Ros (1963) flow-pattern map[3]



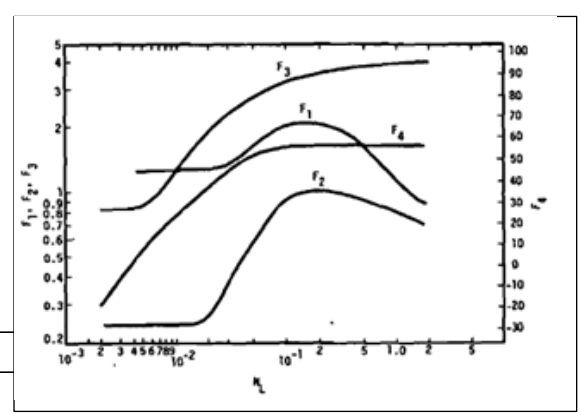
C2 Bubble flow identification: flow constants

s	v_s	H_L	ε	$\Re_L$	f_1	f'_2	f_2	f_3	f
2340.885	0.510	0.916	0.084	539.829	0.030	0.377	1.000	1.001	0.030
2340.885	0.510	0.892	0.108	539.829	0.030	0.489	1.000	1.001	0.030
2214.935	0.482	0.916	0.084	815.486	0.020	0.165	1.000	1.001	0.020
2214.935	0.482	0.891	0.109	815.486	0.020	0.214	1.000	1.001	0.020
2119.340	0.462	0.917	0.083	1091.143	0.015	0.092	1.000	1.000	0.015
2119.340	0.462	0.893	0.107	1091.143	0.015	0.120	1.000	1.000	0.015

C3 Duns and Ros (1963) bubble/slug



C4 Duns and Ros (1973) bubble-

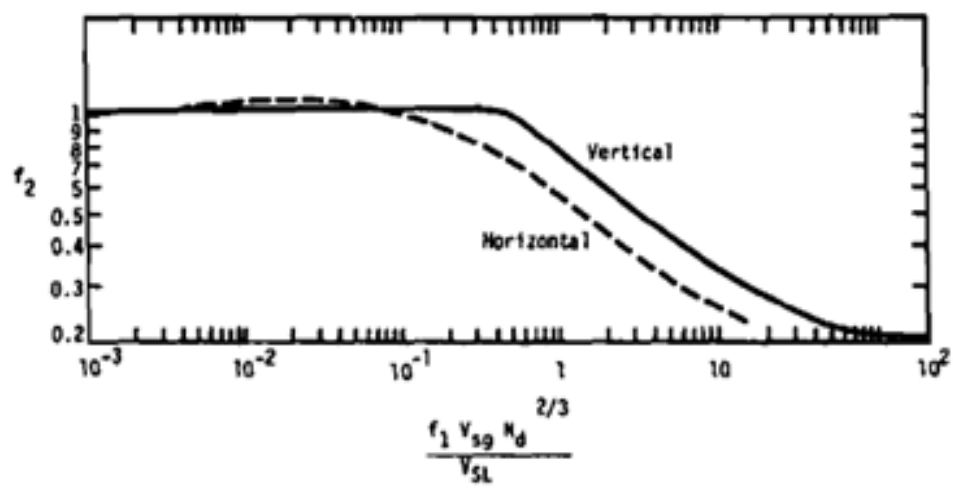


C5 [illegible] numbers

	N_{GV}

(m/s)	(m/s)	(m/s)					
0.047	0.047	0.094	0.387	0.387	1.710	63.928	116.207
0.047	0.061	0.108	0.387	0.502	1.710	63.928	116.207
0.071	0.047	0.118	0.584	0.387	1.868	71.041	131.149
0.071	0.061	0.132	0.584	0.502	1.868	71.041	131.149
0.095	0.047	0.142	0.782	0.387	2.026	78.153	144.855
0.095	0.061	0.156	0.782	0.502	2.026	78.153	144.855

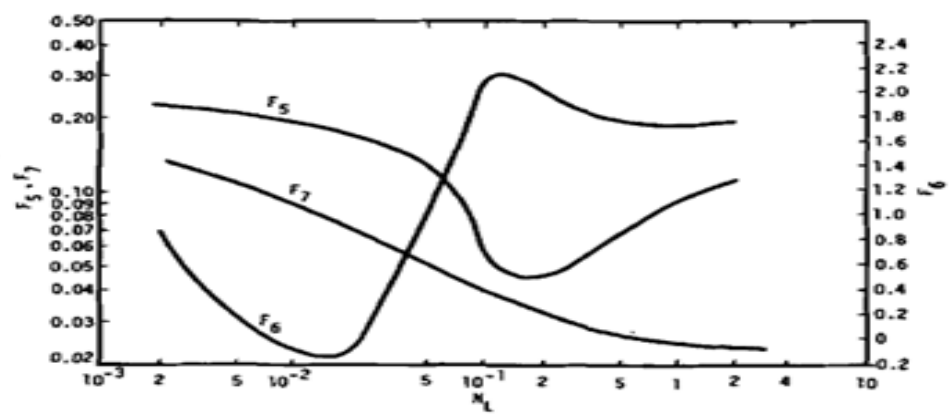
C6 Duns and Ros (1973) bubble-flow friction-factor parameters[3]



C7 Bubble flow - Total pressure gradient

ΔP_f N/m^3	ΔP_g N/m^3	ΔP_T N/m^3
3.514	7251.307	7254.821
4.037	6861.098	6865.135
4.414	7420.895	7425.309
4.937	7062.088	7067.025
5.312	7557.501	7562.813
5.836	7227.068	7232.904

C8 Duns and Ros (1973) slug-flow, slip-velocity parameters.[3]



C9 Slug flow identification: flow constants

s	v_s	H_L	ε	Re_L	f1	f'2	f2	f3	f
4.614	0.561	0.578	0.422	539.829	0.030	2.309	3.000	1.003	0.089
5.107	0.620	0.561	0.439	539.829	0.030	2.759	0.500	1.003	0.015
5.634	0.684	0.546	0.454	539.829	0.030	3.240	0.490	1.003	0.014
6.857	0.833	0.520	0.480	539.829	0.030	4.362	0.450	1.004	0.013
8.292	1.007	0.501	0.499	539.829	0.030	5.686	0.400	1.004	0.012
10.334	1.255	0.484	0.516	539.829	0.030	7.578	0.380	1.005	0.011
4.509	0.548	0.588	0.412	815.486	0.020	1.012	7.800	1.001	0.153
4.991	0.606	0.568	0.432	815.486	0.020	1.209	7.200	1.002	0.141
5.505	0.669	0.550	0.450	815.486	0.020	1.420	6.400	1.002	0.125
6.701	0.814	0.519	0.481	815.486	0.020	1.912	5.000	1.002	0.098
8.103	0.984	0.492	0.508	815.486	0.020	2.491	2.750	1.002	0.054
4.408	0.535	0.597	0.403	1091.143	0.015	0.565	9.000	1.001	0.132
4.878	0.593	0.574	0.426	1091.143	0.015	0.675	8.800	1.001	0.129
5.382	0.654	0.554	0.446	1091.143	0.015	0.793	8.200	1.001	0.120
6.550	0.796	0.516	0.484	1091.143	0.015	1.068	7.700	1.001	0.113
7.921	0.962	0.483	0.517	1091.143	0.015	1.392	6.800	1.001	0.100
9.872	1.199	0.448	0.552	1091.143	0.015	1.855	5.200	1.002	0.076

C10 Slug flow identification: Velocity numbers

U_{SL} (m/s)	U_{SG} (m/s)	U_m (m/s)	N_{LV}	N_{GV}	$N_{Gv(B/S)}$	$N_{GV((S/T)}$	$N_{GV(T/M)}$
0.047	0.288	0.335	0.387	2.371	1.710	63.928	116.207
0.047	0.344	0.391	0.387	2.832	1.710	63.928	116.207
0.047	0.404	0.451	0.387	3.326	1.710	63.928	116.207
0.047	0.544	0.591	0.387	4.478	1.710	63.928	116.207
0.047	0.709	0.756	0.387	5.836	1.710	63.928	116.207
0.047	0.945	0.992	0.387	7.779	1.710	63.928	116.207
0.071	0.288	0.359	0.584	2.371	1.868	71.041	131.149
0.071	0.344	0.415	0.584	2.832	1.868	71.041	131.149
0.071	0.404	0.475	0.584	3.326	1.868	71.041	131.149
0.071	0.544	0.615	0.584	4.478	1.868	71.041	131.149
0.071	0.709	0.780	0.584	5.836	1.868	71.041	131.149
0.095	0.288	0.383	0.782	2.371	2.026	78.153	144.855
0.095	0.344	0.439	0.782	2.832	2.026	78.153	144.855
0.095	0.404	0.499	0.782	3.326	2.026	78.153	144.855
0.095	0.544	0.639	0.782	4.478	2.026	78.153	144.855
0.095	0.709	0.804	0.782	5.836	2.026	78.153	144.855
0.095	0.945	1.040	0.782	7.779	2.026	78.153	144.855

C11 Slug flow - Total pressure gradient

ΔP_f N/m^3	ΔP_g N/m^3	ΔP_T N/m^3
37.515	5103.846	5141.361
7.296	4952.968	4960.264
8.245	4821.278	4829.523

9.918	4596.054	4605.972
11.271	4422.697	4433.968
14.042	4270.602	4284.644
104.651	5189.962	5294.613
111.655	5015.671	5127.326
113.583	4859.613	4973.197
114.861	4580.468	4695.329
80.101	4348.243	4428.344
128.887	5270.621	5399.508
144.437	5071.740	5216.177
152.971	4890.536	5043.507
183.913	4556.874	4740.787
204.320	4266.280	4470.600
202.063	3961.122	4163.185

C12 Churn flow identification: Velocity numbers

Usl (m/s)	Usg (m/s)	U_m (m/s)	N_{LV}	N_{GV}	$N_{gv(B/S)}$	$N_{gv((S/T)}$	$N_{gv(T/M)}$
0.047	1.418	1.465	0.387	11.673	1.710	63.928	116.207
0.047	1.891	1.938	0.387	15.566	1.710	63.928	116.207
0.047	2.363	2.41	0.387	19.452	1.710	63.928	116.207
0.047	2.836	2.883	0.387	23.345	1.710	63.928	116.207
0.047	4.727	4.774	0.387	38.912	1.710	63.928	116.207
0.071	0.945	1.016	0.584	7.779	1.868	71.041	131.149
0.071	1.418	1.489	0.584	11.673	1.868	71.041	131.149
0.071	1.891	1.962	0.584	15.566	1.868	71.041	131.149
0.071	2.363	2.434	0.584	19.452	1.868	71.041	131.149
0.071	2.836	2.907	0.584	23.345	1.868	71.041	131.149

0.071	4.727	4.798	0.584	38.912	1.868	71.041	131.149
0.095	1.418	1.513	0.782	11.673	2.026	78.153	144.855
0.095	1.891	1.986	0.782	15.566	2.026	78.153	144.855
0.095	2.363	2.458	0.782	19.452	2.026	78.153	144.855
0.095	2.836	2.931	0.782	23.345	2.026	78.153	144.855
0.095	4.727	4.822	0.782	38.912	2.026	78.153	144.855