

APPLICATION OF MINIMUM CURVATURE METHOD TO  
WELLPATH CALCULATIONS

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By

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APPLICATION OF MINIMUM CURVATURE METHOD TO  
WELLPATH CALCULATIONS

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## **ABSTRACT**

A major drawback of directional and horizontal well drilling is the numerous complex computations required to be done while planning a well. These computations are very stressful and time consuming especially when done manually.

One of the objectives of this study was to develop a user friendly Excel Spreadsheet program that would make the computations of these well trajectory parameters easier, faster and accurate.

An Excel Spreadsheet program was developed employing the Minimum Curvature method (and for other five methods) for wellpath design and planning. This would help increase the usage of these trajectory methods especially the Minimum Curvature method.

The program is able to provide pictorial views both in the vertical and horizontal plane of the trajectory of the drilling bit's position in the wellbore. This would therefore help to minimize risk and uncertainty surrounding hitting predetermined target. This is possible because deviations can easily be detected and the necessary directional corrections or adjustment be initiated to re-orient the drilling bit to the right course before (planning process) and during the drilling operations.

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I would also like to thank all my foes for making me to become stronger and more fortified in the Lord God Almighty.

## **DEDICATION**

I would like to dedicate the entire project to my God Almighty who is my shield, provider, strength, joy, redeemer, maker, doctor, teacher, comforter, hope, my all in all, for His divine grace, favour, protection, good health, and ways that have always beaten my imaginations not forgetting how He always rescues me out from any dilemmas and establishes my foot on the hills.

## **DECLARATION**

I declare that this work is my own. It is being submitted for the Degree of Master of Science in Petroleum Engineering at the African University of Science and Technology, Abuja-Nigeria. It has not been submitted for any degree or examination in any other university.

# **CHAPTER 1**

## **FORMULATION OF THE PROBLEM**

### **1.1 Introduction**

Directional drilling is the science and art of deviating a wellbore along a planned course to a subsurface target whose location is a given lateral distance and direction from the vertical (Bourgoyne et al, 1991). Directional drilling and horizontal wells represent an efficient way to achieve or hitting special targets that may or are very difficult to reach using vertical wells (Tarek, 2000). Directional drilling is relatively done to increase production rates, control water and gas conning, control sand production and increase recovery rate (Bourgoyne et al, 1991). There are many reservoirs which can not be tapped by vertical wells or would be uneconomical to exploit with vertical wells (Bourgoyne et al, 1991). Other reservoirs are also characterized by vertical permeability or the pay zones may be very thin and producing with vertical wells would require quiet a number of them which would make vertical wells to be very uneconomical in such situations. The application of vertical wells in such formations could also result in lower ultimate recovery (Tarek, 2000). In such low permeability formations, the only way out is to use directional and horizontal well technology which has over the years proved to enhance ultimate recovery (Bourgoyne et al, 1991). Due to the more economical nature of directional and horizontal wells over vertical wells in most cases, they are mostly the preferred technology in offshore drilling technology (Bourgoyne et al, 1991).

### **1.2 Problem Definition**

A major drawback of directional and horizontal well drilling is the numerous complex computations required to be done ahead of time before drilling resumes and also during drilling operations (Sawaryn, 2005). These computations become very stressful and more complex when done manually. The programs available in the market used for these computations are usually very expensive to acquire, but the



development of a user-friendly Excel Spreadsheet program which employs the Minimum Curvature method for wellpath planning would help minimize the stress and time in executing these complex computations. More importantly, an Excel Spreadsheet program is very flexible and can easily be modified or updated at any point in time to meet the needs of the industry.

This study seeks to use the Minimum Curvature Method to design well path using Excel Spreadsheet to determine the optimum well path parameters. This would help reduce risks and uncertainty or prevent deviation from target, minimise non-productive time and reduce drilling cost. This would therefore enhance planning process and maximizing return on investment.

### **1.3 Objectives of Study**

The objectives of this study are:

1. To develop a user friendly Excel Spreadsheet program that will make the computations of well trajectory parameters easier, faster and accurate.
2. Use the Excel Spreadsheet program to evaluate and compare some of the survey calculation method available for the industry. These methods are the Tangential, Averaging Angle, Balanced Tangential, Mercury, Radius of Curvature and the Minimum Curvature methods.

### **1.4 Methodology and Scope of Study.**

Some of the survey trajectory computation methods available for the industry would be reviewed and a user friendly Excel Spreadsheet program would be developed to calculate the well trajectory parameters. A comparison analysis of each of the computed results will be evaluated and analysed for accuracy purposes. The programme would be validated using available software and results from literature.

### **1.5 Organization of the Study**

Chapter Two reviews the theoretical background of Directional Drilling, Basic

Concepts, Equipments and Types of Survey perform during directional drilling operations. It would also review some of the methods for calculating wellbore trajectory available to the industry with much emphasis on the Minimum Curvature Method

Chapter Three describes the derivation of equations that were used for the development of the Excel Spreadsheet program

Chapters Four describes the development of the Excel-Spreadsheet program including computational and graphical presentation of results. Also included are data used, analysis of results, and comparison of the various methods.

Chapter Five contains summary, conclusions and recommendations

## **CHAPTER 2**

### **THEORETICAL BACKGROUND ON DIRECTIONAL DRILLING**

#### **2.1 Directional Drilling**

Directional drilling is the science and art of deviating a well bore along a planned course to a subsurface target whose location is a given lateral distance and direction from the vertical (Osisanya, 2009). Directional wells are drilled with intentional control to hit a pre-determined target with the aid of controlling inclination (angle) and azimuth (direction). Directional drilling is drilling in three dimensions (3-D).

##### **2.1.1 History of Directional Drilling**

Much attention was not given to this technology until the late 1920s when there were several lawsuits alleging that wells drilled from a rig on one property had crossed its concession and were penetrating a reservoir on an adjacent property. Horizontal and Multilateral well technologies are very special oriented wells of directional drilled within the late three decades (Osisanya, 2009). Horizontal wells are normally new wells drilled from the surface vertically to a particular depth and horizontal to additional 1,000-ft or more. Horizontal drilling is potentially the most important new completion technique since hydraulic fracturing. A multilateral well is a well in which there is more than one horizontal or near horizontal lateral well drilled from a single main bore and connected back to the same main bore. Multilateral wells are not a new concept but their successful application has increased over the last decade. Multilaterals represent an alternative well construction strategy to complement vertical, inclined, horizontal and extended reach well trajectories (Osisanya, 2009).

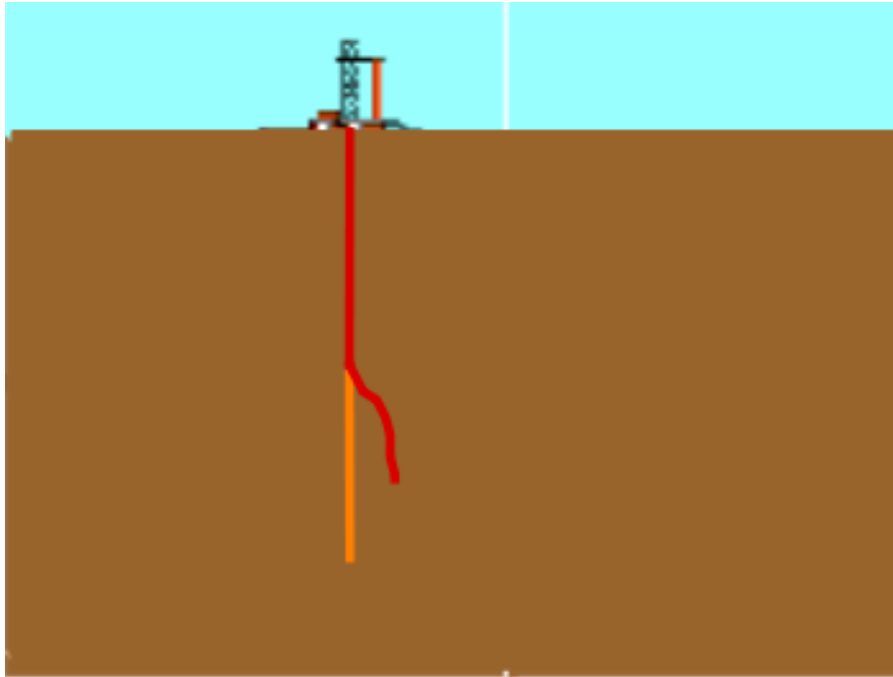
Another school of thought thinks the early directional drilling was really motivated by economics (Bourgoyne et al, 1991). An example is California's offshore oil field which is believed to be the spawning ground for directional drilling and equipment due to economical reasons. Oil and gas discoveries in the Gulf of Mexico

and in other countries expanded the application of the technologies thereafter

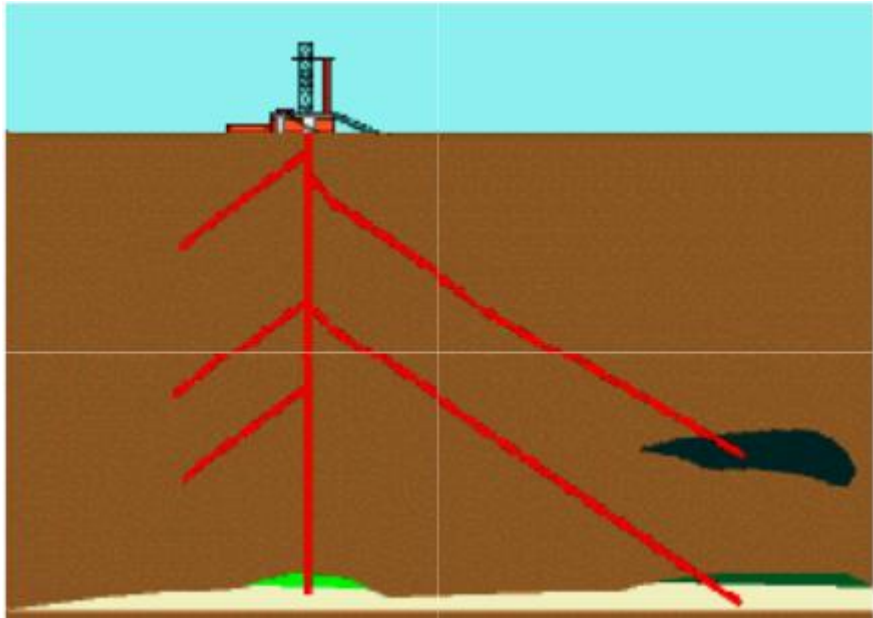
### 2.1.2 Reasons for Directional Drilling

Directional drilling has proven technically and economically feasible in a broad range of geologic settings, including tight gas, heavy oil, and coalbed methane (Molvar, 2003). This method is proven to substantially increase producible reserves of oil and gas. Because the increased productivity of directional drilling compensates for additional costs, directional drilling is often more profitable than vertical drilling (Molvar, 2003). Some of the reasons for or applications of directional drilling which are shown from Figures 2.1a to 2.1h, are listed below (Osisanya, 2009);

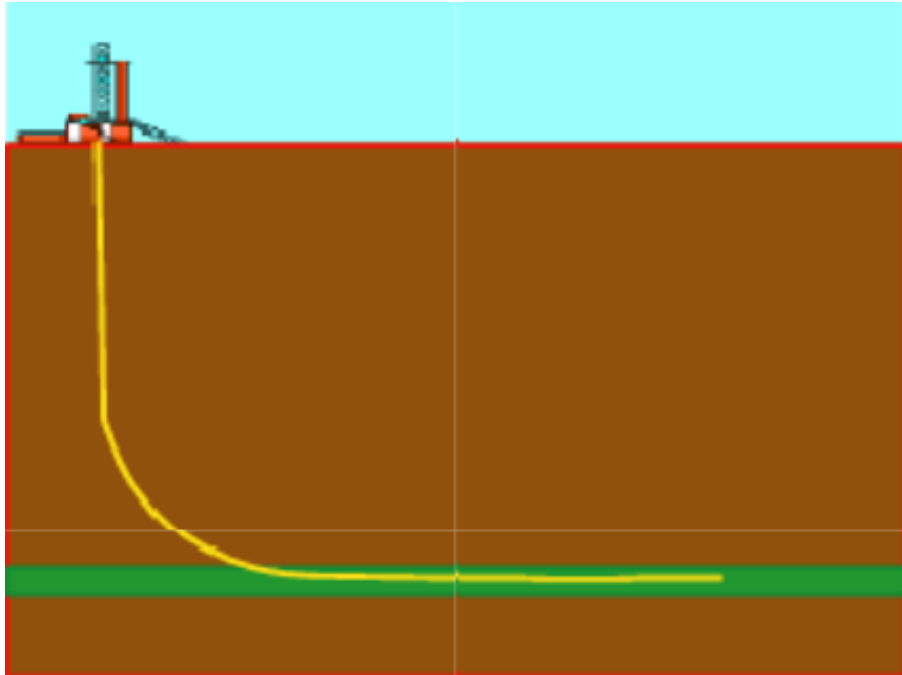
- For Economics/Environmental Issues
- To drill multiple wells from artificial structures, field development offshore in deep waters or remote locations
- To sidetrack any obstruction ('junk') in the original wellbore
- To explore for additional producing horizons in adjacent sectors of the field
- To re-drill well
- To put out fire resulting from blowout (relief wells)
- To drill to reservoirs avoiding inaccessible locations
- For Salt Dome drilling
- For Fault controlling



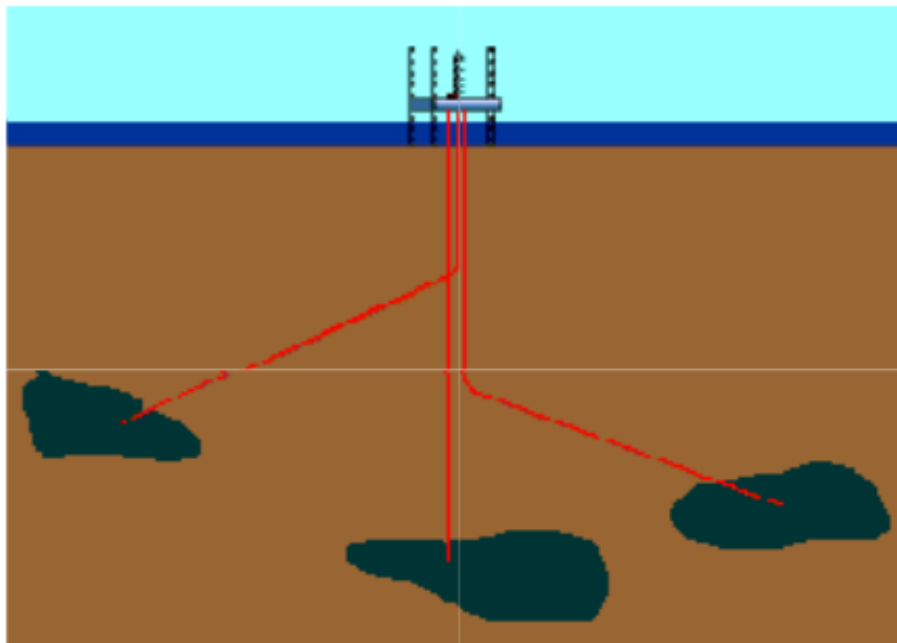
**Figure 2.1a- A Sidetrack Drilling (Osisanya, 2009)**



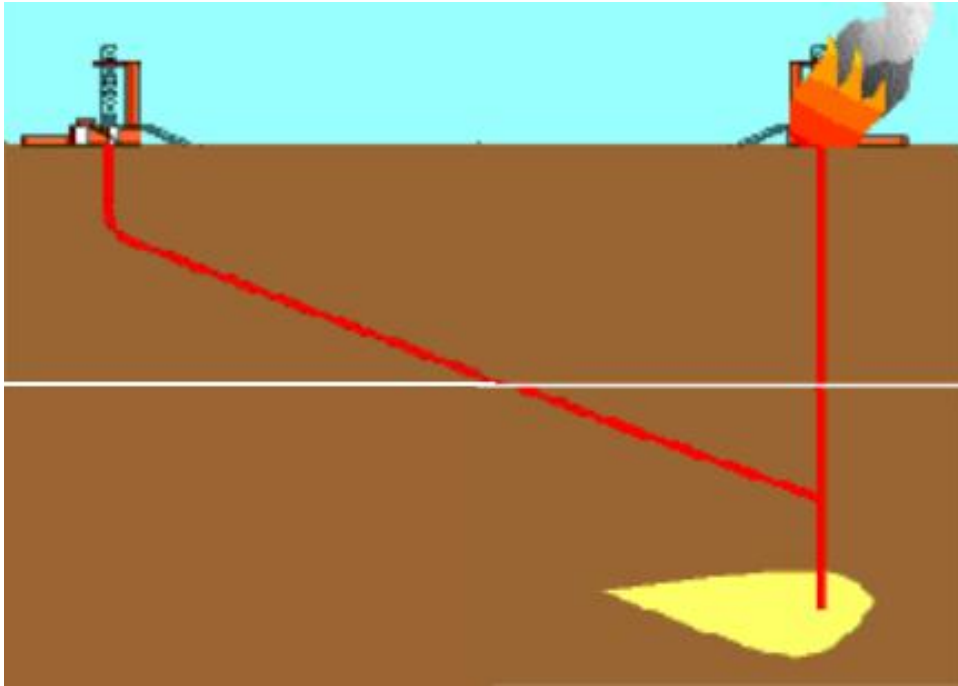
**Figure 2.1b- A multilateral well (Osisanya, 2009)**



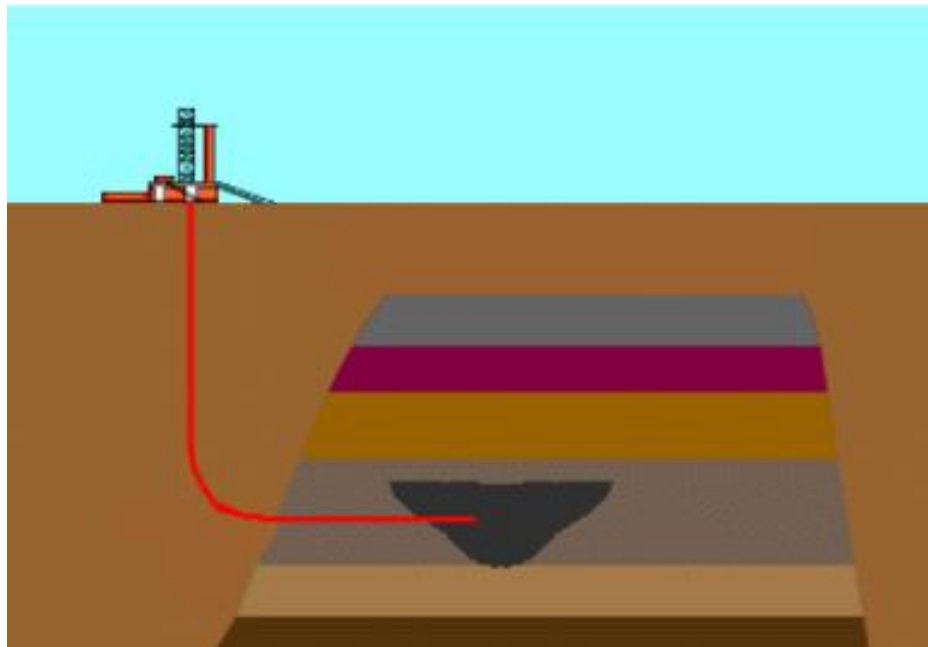
**Figure 2.1c- A Re-drill/Drainage well (Osisanya, 2009)**



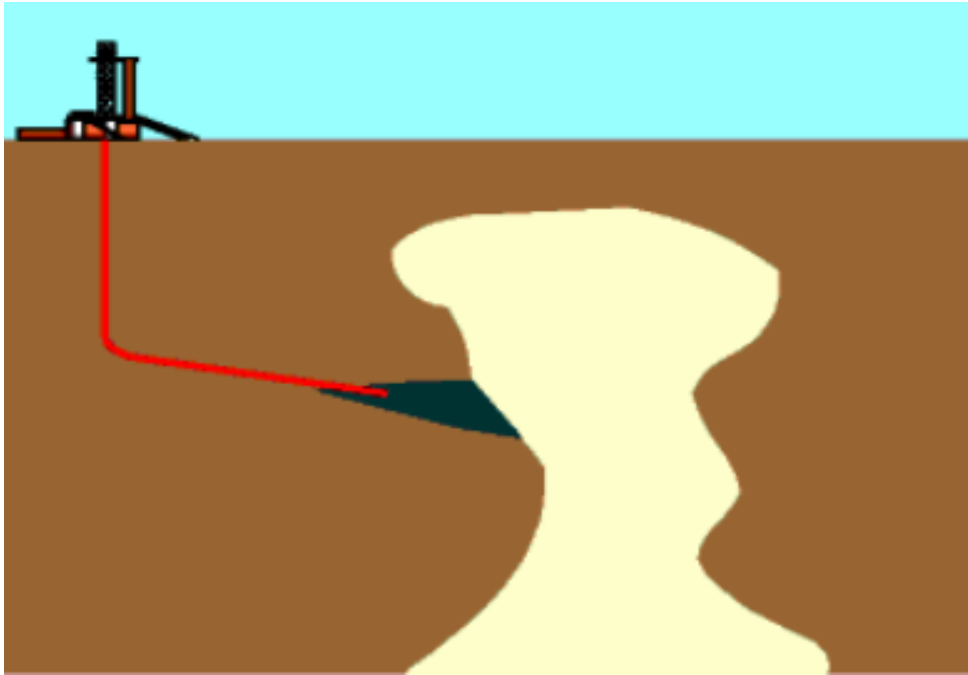
**Figure 2.1d- Drilling multi-reservoirs from a single platform (Osisanya, 2009)**



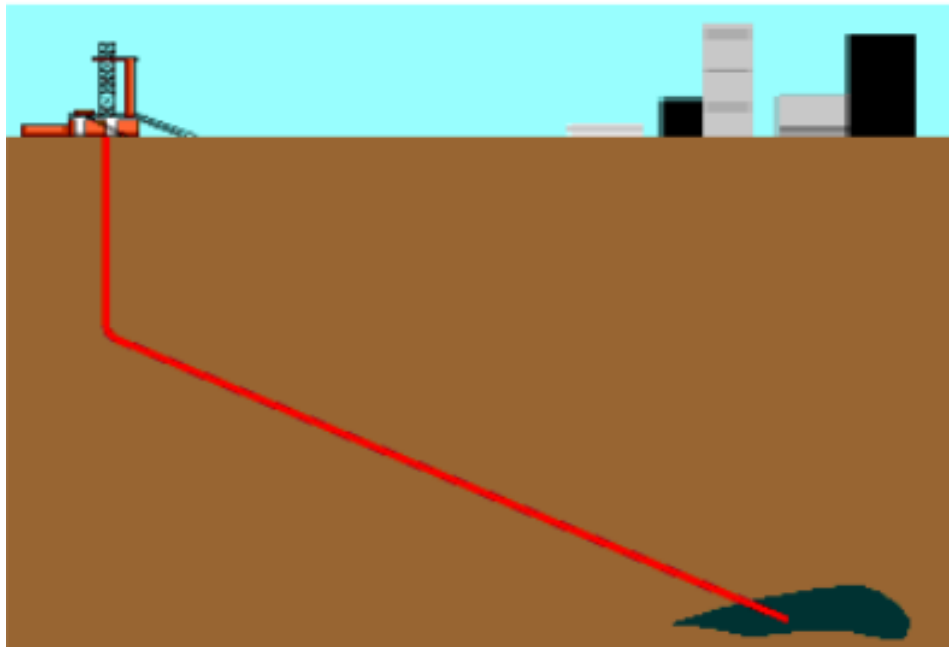
**Figure 2.1e- A Relief well (Osisanya, 2009)**



**Figure 2.1f- Controlling a Fault (Osisanya, 2009)**



**Figure 2.1g- A Salt Dome drilling (Osisanya, 2009)**



**Figure 2.1h- Avoiding Inaccessible location (Osisanya, 2009)**



## **2.2 Fundamental Concepts / Basis of Directional Drilling**

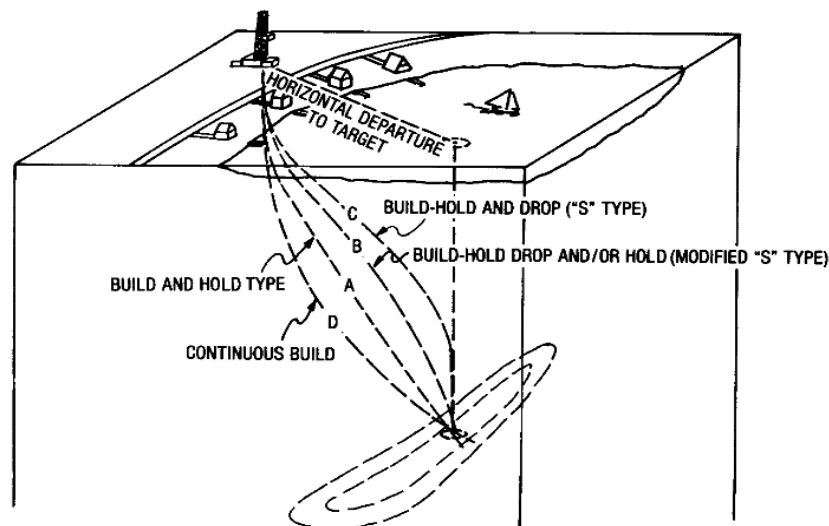
For any directional drilling, three components are measured at any given point in the wellbore in order to determine its position. The technique of measurement of these three components is termed a survey. The depth, drift angle (inclination) and azimuth are measured (Osisanya, 2009). There are various tools that are used in taking measurements in the wellbore. These tools include;

- Totco - measure only the angle of inclination. It does not give an azimuth reading
- Magnetic Single Shot
- Magnetic Multi Shot
- Gyro Multi Shot
- Measurement While Drilling

In most cases, before the commencement of the drilling operation, periodic surveys are taken with a downhole camera instrument, single shot cameras to provide survey data such as inclination and azimuth of the well bore (Bourgoyne et al, 1991). The Single Shot and Multi Shot tools are units run inside a barrel, like a Totco (Osisanya, 2009). It contains a timer, camera, angle unit with compass and a single film disk. The timer controls the camera, takes picture of angle unit & compass. The film is developed on surface and examined with magnifier. The Multishot takes a sequence of images on a film strip at preset time or real-time intervals. The Gyro tool uses gyro instead of magnetic compass. When Magnetic tools are in use, they must be positioned inside non-magnetic BHA components or else the compass will not read accurately. An example of non-magnetic tools is an alloy called Monel. Monel is a Nickel alloy which does not distort magnetic fields. Measurement While Drilling tool is a set of triaxial magnetometers and inclinometers measuring tools which measures azimuth, inclination and tool face azimuth. It makes use of mud pulse telemetry which transmits data to the surface in real time, where surface equipment decodes the pulses and displays the data such as optional information (pressure, temperature) (Osisanya, 2009).

## 2.3 Well Trajectory Planning

The accurate determination of a wellbore position is very critical to well placement, collision avoidance, reservoir modelling and equity determination (Sawaryn, 2005). Careful planning is especially critical in directional drilling and is always an important factor in minimizing well costs. The starting point in planning a directional well is to design the wellbore path, or survey parameters to intersect a given target, thus, establishing the target coordinates with respect to the surface location to hit the target (Bourgoyne et al, 1991). The next stage is to consider the effect of geology on BHA that would be employed and finally, the factors that would influence the final wellbore trajectory (Bourgoyne et al, 1991). A well path can follow four different paths as shown in (Figure 2.2). Below are the four different paths the wellbore could follow.



**Figure 2.2-** The four general paths the wellbore could follow (Bourgoyne et al, 1991).

### 2.3.1 Type A: Build and Hold

This pattern employs a shallow initial deflection and a straight-angle approach to the target. The wellbore penetrates the target at an angle equal to the maximum build-up angle. It requires the lowest inclination angle to hit the target. The build and hold model is mostly used for the following;

1. For moderate depth

2. Single zone, no intermediate casing
3. In deeper wells without large lateral displacement.

#### 2.3.2 Type B: Build, Hold and Drop

Type B is a modified 'S' after type 'C' (Bourgoyne et al, 1991). After a relatively shallow deflection, this pattern holds angle until the well has reached most of its required horizontal displacement. At this point, the angle is reduced or brought back to vertical to reach the target. The wellbore penetrates the target at an inclination angle less than the maximum inclination angle in the holds section. The build, hold and drop requires more inclination angle than the build and hold to hit the target. The Type 'B', modified 'S' pattern may present certain problems but is most applicable to the following (Bourgoyne et al, 1991);

1. For wells exposing multiple pay zones
2. For wells with lease or target limitations

#### 2.3.3 Type C: Build, Hold and Drop or 'S' shape Trajectory

The 'S' shape requires more inclination angle than the modified 'S' to hit the target (Bourgoyne et al, 1991). For the 'S' shape, the wellbore trajectory penetrates the target vertically.

#### 2.3.4 Type D: Continuous Build

Unlike the Type A and B patterns, this trajectory has a relatively deep initial deflection. This requires the highest inclination angle of all the trajectory types to hit the target (Bourgoyne et al, 1991). In this case, the inclinations keep increasing right up to or through the target. The continuous build pattern is well-suited for the following;

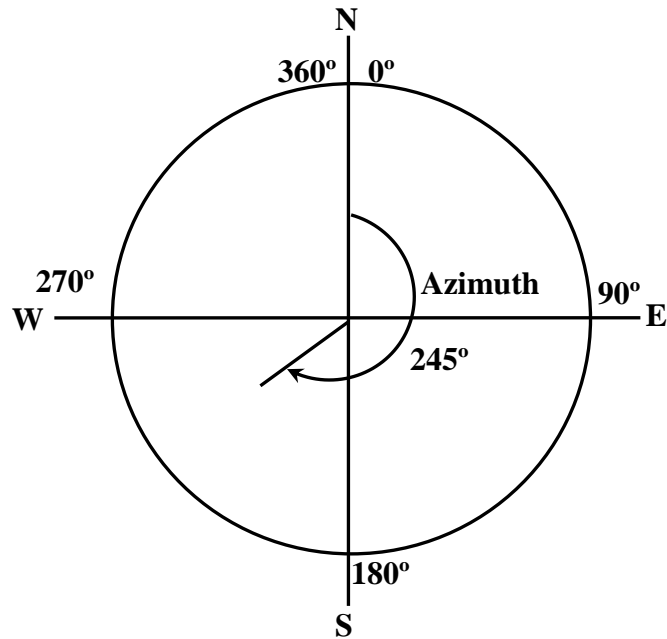
1. Salt-Dome drilling,
2. Fault drilling,
3. For re-drills or Sidetrack

### **2.4. Planning the Well Survey Parameters**

In general, directional wells are supposed to be visualized in 3-D, but this is not

feasible on papers. Hence the 3-D is generally divided into 2-D plans. That is the vertical and the horizontal plans. The vertical plan defines the true vertical depth, drift angle and the horizontal departure. The horizontal plan defines the coordinates and the azimuth. In directional drilling, 90°-quadrant scheme as shown in Figure 2.3 is used to cite directions and the degrees are always read from north to east or west (N-E or N-W), and from S-E or SW. The directional angle is always read clockwise from true north. It is this 90°-quadrant scheme that is used to determine the horizontal departure. In planning the well trajectory, the following steps are; followed;

- If the basic well profile is established, the next step is to plan the trajectory in detail, starting with the target location. The target location is expressed in terms of its true vertical depth (TVD) and horizontal departure.
- Establishment of target depth, number of targets, target radius, and horizontal departure to target.
- Selecting a kick-off point that seems appropriate. The initial deflection and direction are essentials at the start of a directional well (Kick off point). The bottomhole target diameter or radius dictates how much control one has to exercise over the well trajectory. Generally, the greater the degree of control, the higher the drilling costs. Largest target should therefore be established to ensure meeting most if not all well objectives.



**Figure 2.3- Directional Quadrant and Compass Measurement (Modified after Baker Hughes, 1995).**

#### 2.4.1 Planning the Kick-off and Trajectory Change

Kicking off a well involves the setting of the lead angle, inclination and course of the next station from the KOP. The determination of tools for the deflection primarily depends on the type of formations encountered at the kick-off point. There are several methods for kicking off the directional wells. Figure 2.4a and 2.4b show two different cases for a build-hold-and drop for which the rate of build differs from the kick off point. These give rise to different radius of curvatures. Some of the tools or equipments used in kicking off a well are;

- Whipstock
- Badgering or Jetting
- Rotary stabilized BHA, fixed or adjustable stabilizers
- PDM with bent sub above
- PDM with bent housing (steerable system)
- Rotary steerable systems, dynamically controlled (POWER DRIVE)

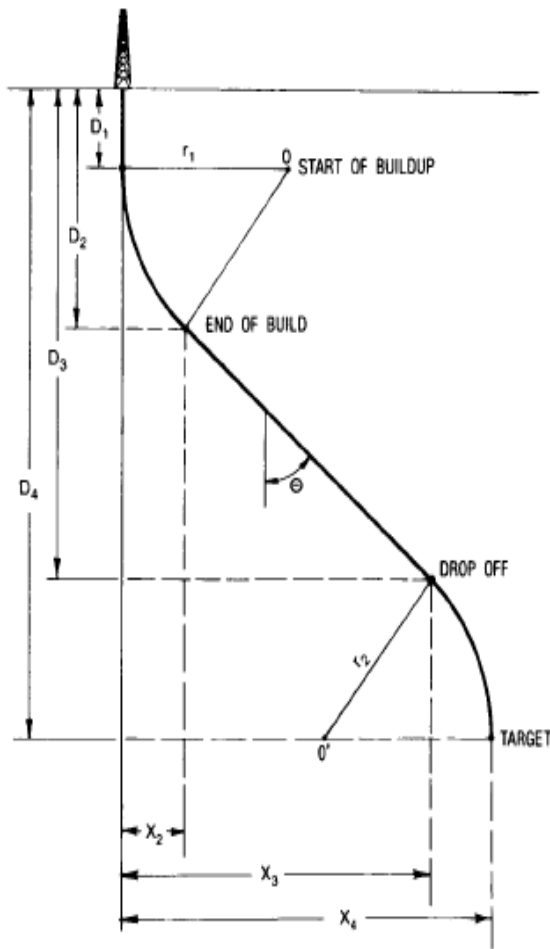


Figure 2.4a-Build-hold-and drop for case  
for where  $r_1 < x_3$  and  $r_1 + r_2 < x_4$

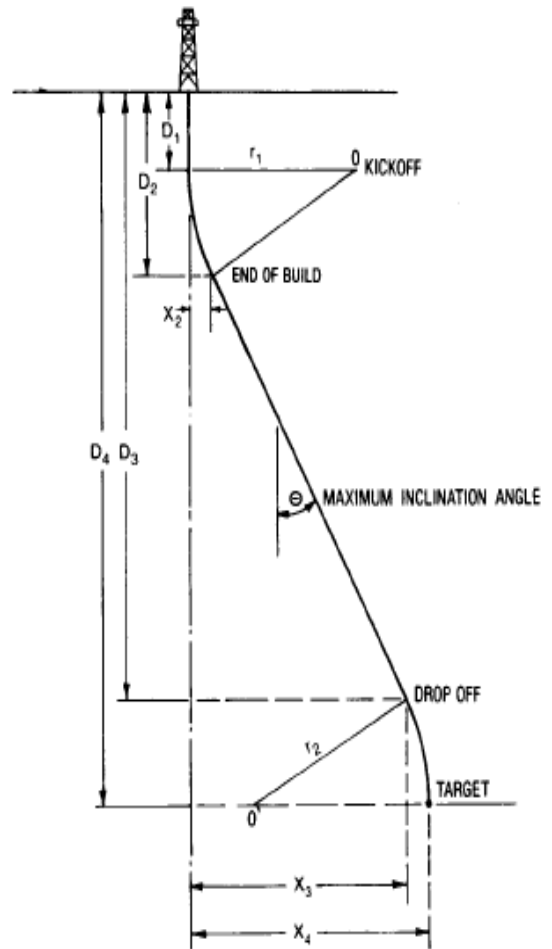


Figure 2.4b--uild-hold-and drop  
case where  $r_1 < x_3$  and  $r_1 + r_2 > x_4$

(Bourgoyne et al, 1991).

## 2.4.2 Directional Surveying

Directional surveying is used in locating a wellbore. It permits the determination of the location of the bottomhole.

### 2.4.2.1 Reference Systems and Coordinates.

All survey systems measure inclination and azimuth at a particular measured depth (depths measured "along hole"). These measurements are tied to some fixed reference systems so that the course of the borehole can be calculated and recorded (Baker Hughes, 1995). These reference systems include depth, inclination and azimuth.

- *Depth References:* Measured Depth (MD) and True Vertical Depth (TVD) are the two kinds of depths measured during the course of a directional well. This depth can be measured referenced to pipe tallying, wireline depth counter, or mud loggers depth counter while the VD is always calculated from the deviation survey data. In most drilling operations the rotary table elevation is used as the working depth reference. Below Rotary Table (BRT) and Rotary Kelly Bushing (RKB) are used to indicate depths measured from the rotary table which can also be referred to as Derrick Floor Elevation (DFE). For floating drilling rigs the rotary table elevation is not fixed and hence a mean rotary table elevation has to be used. In order to compare individual wells within the same field, a common depth reference must be defined and referred to (Baker Hughes, 1995).
- *Inclination References:* The inclination of a well-bore is the angle (in degrees) between the vertical and the well bore axis at a particular point. The vertical reference is the direction of the local gravity vector and could be indicated by a plumb bob (Baker Hughes, 1995) or a float on a liquid.
- *Azimuth References.* There are three azimuth reference systems for all directional surveying (Baker Hughes, 1995) which includes;
  - Magnetic North
  - True (Geographic) North
  - Grid North

Some tools used in the determination of directions are;

- Magnetic survey instrument; this requires a non-magnetic drill collar. To nullify the magnetic effect of the drill string non-magnetic drill collar is made of special nickel alloy. Magnetic survey methods include;
  - Single shot; this has three basic units such as timing device or motion units, camera section (light/film), and angle unit (magnetic compass and plus bob line). The assembly is run on wireline or drop on a dull bit. All survey assembly must be aligned parallel to the axis of the wellbore
  - Multishot survey; run at the end of the well before running casing. Is loaded with 16mm film which can record several readings at required depth intervals.

- Gyroscope survey; this can be run without non-magnetic drill collar. The gyro unit is first oriented to a known direction.
  - Single shot is used to orient deflection tools in area of magnetic influence where the magnetic survey cannot be used. Is used to determine drift and directional of the wellbore.
  - Multishot is used for cased or uncased bore and used 10mm film. This tool is very sensitive and must be handle with extreme care.

Other survey instruments include;

1. Drift indicator or Totco which is used in straight holes just to record drift angles.
2. Measurement while drilling
3. DOT – downhole orientation tool
4. Steering tool

#### *2.4.2.2 Lead Angle*

It is a normal practice to allow a “lead angle” when kicking off a well because roller cone bits used with rotary assemblies tend to “walk to the right”, of the target. The wells are generally kicked off in a direction several degrees to the left of the target direction. In extreme cases the lead angles could be as large as 20°. The use of steerable motors and Polycrystalline Diamond Compact bits for rotary drilling have drastically reduced the need for wells to be given a “lead angle” thus many wells of late are deliberately kicked off with no lead angle (i.e. in the target direction).

### **2.5 Methods for Calculating Wellbore Trajectories.**

There are over eighteen methods available for calculating or determining the trajectory of a wellbore (Bourgoyne et al, 1991). The main difference in all the techniques is that one group uses straight lines approximations and the other assumes the wellbore is more of a curve and is approximated with curved segments. Listed below are six of the methods in ascending order of preference and also complexity of the techniques;

1. Tangential method



2. Balanced tangential method
3. Mercury method
4. Angle Averaging method
5. Radius of curvature
6. Minimum radius of curvature

Figures 2.5, 2.6 and 2.7 show the Vertical Profile of a Wellbore Trajectory, Horizontal Profile of a Wellbore Trajectory and a Three dimensional view of a wellbore showing components that comprise the X, Y, and Z parts of the trajectory.

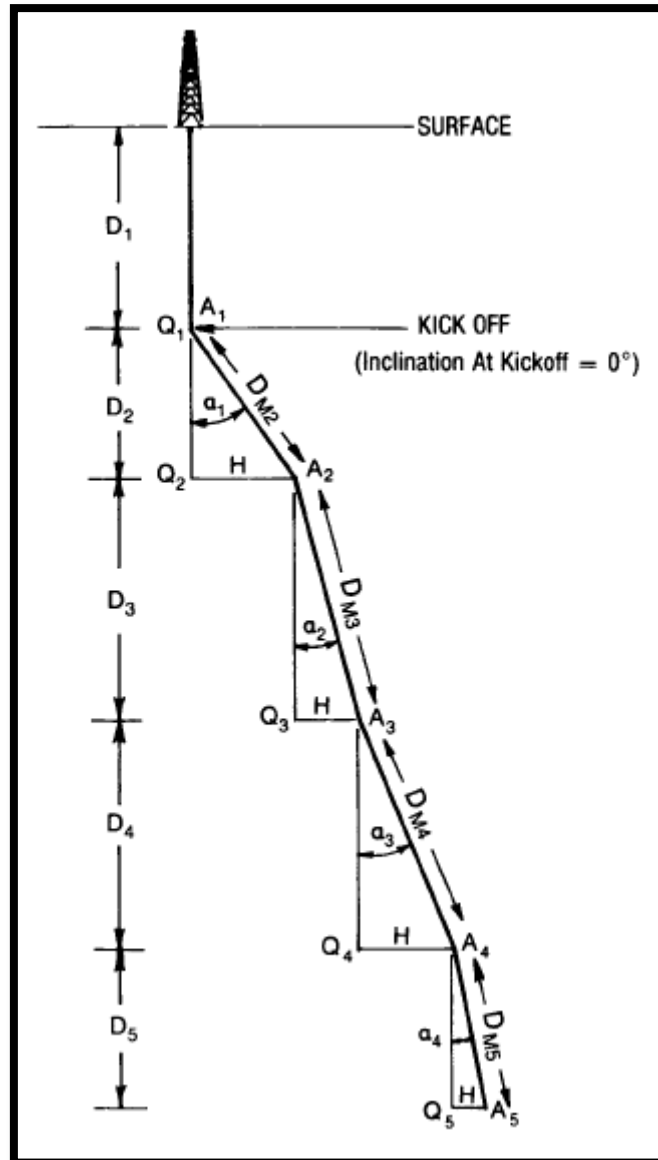


Figure 2.5- A Vertical Profile of a Wellbore Trajectory (Bourgoyne et al, 1991).

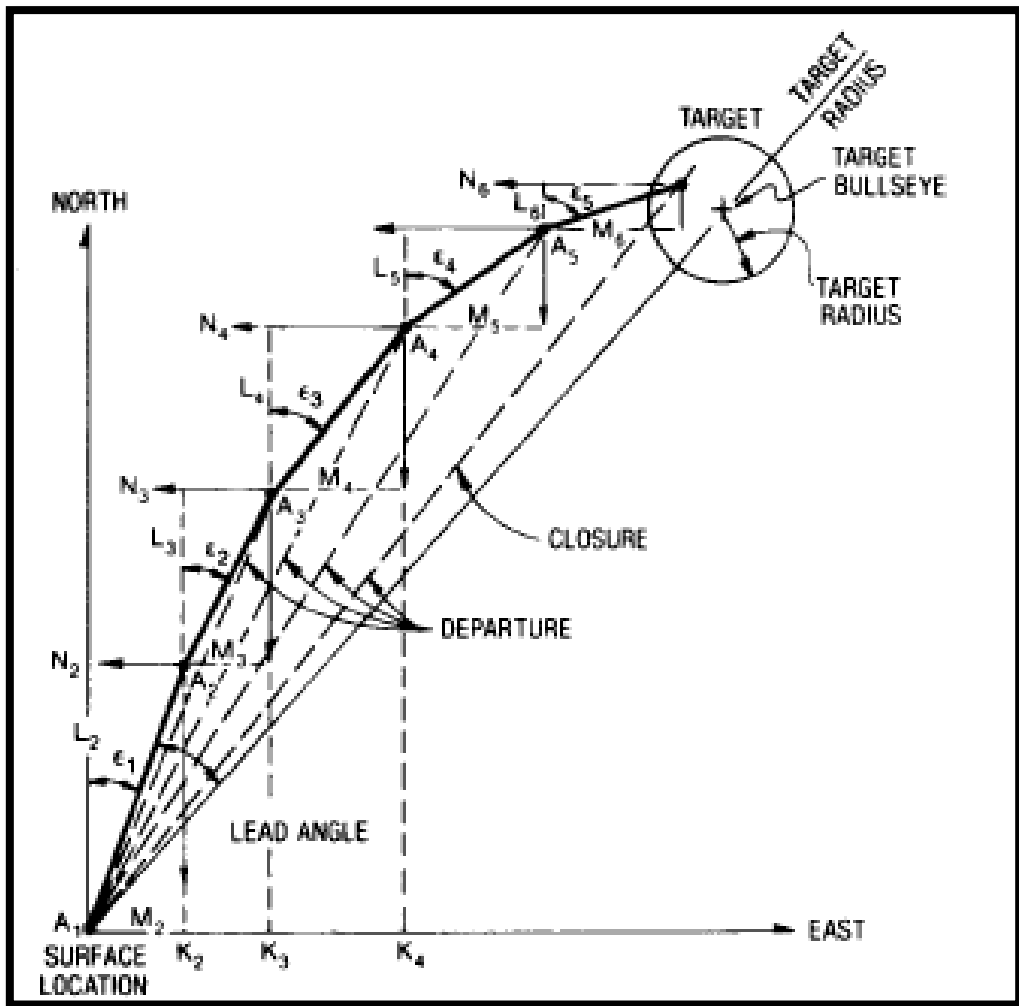


Figure 2.6- A Horizontal Profile of a Wellbore Trajectory (Bourgoyne et al, 1991).

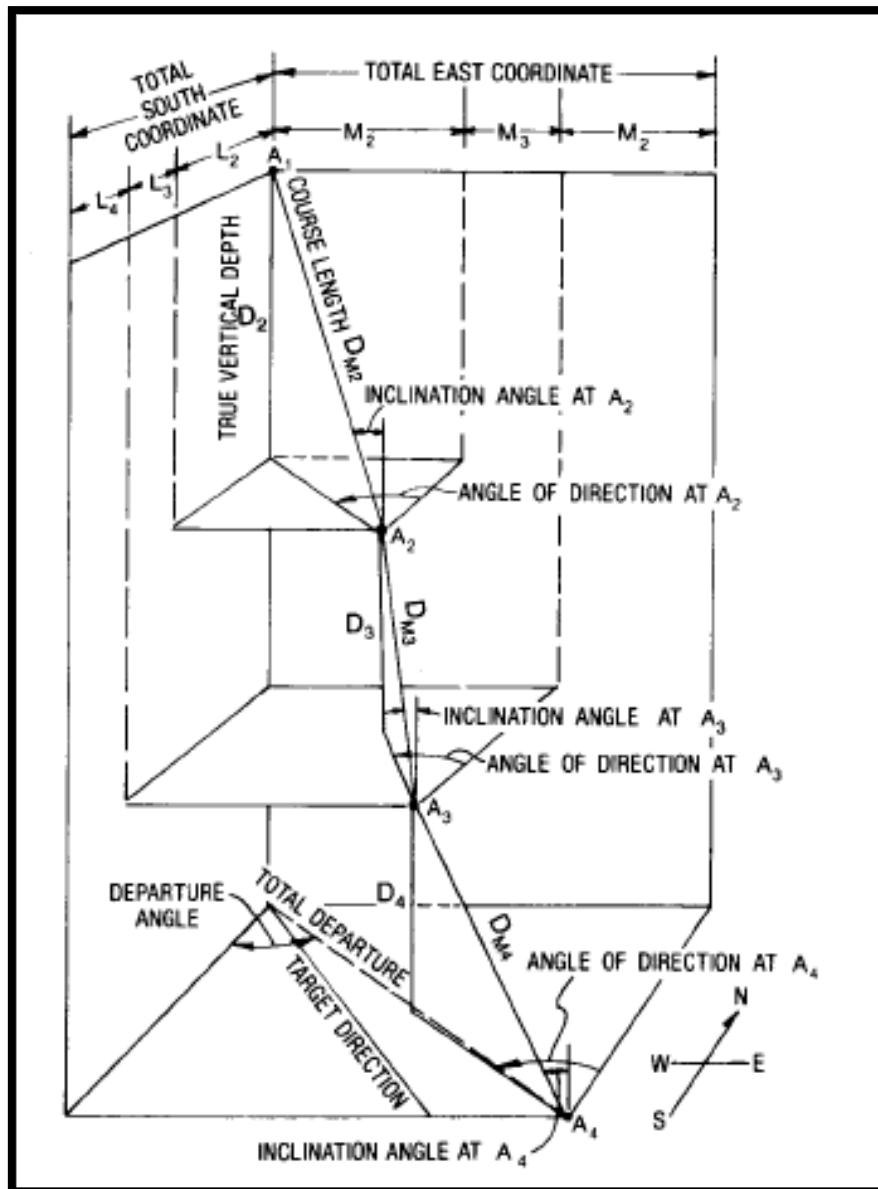


Figure 2.7- Three dimensional view of a wellbore showing components that comprise the X, Y, and Z parts of the trajectory, (Bourgoyne et al, 1991).

#### 2.5.1. Tangential Method (also known as backward station or terminal angle method)

It is the simplest and old method used for years (Bourgoyne et al, 1991). Its original derivation or presentation to industry is unknown. The tangential method assumes that the hole would maintain the same drift and course between each station, thus projected angles remain constant over the preceding course length from the previous station. This method is very inaccurate based on its assumption, especially in build and hold configuration where it shows less vertical and more horizontal displacement than there actually is as well as in turn and hold configuration where it shows more vertical and less horizontal displacement than is present (Bourgoyne et al, 1991). The tangential method makes the well appear too shallow and the lateral displacements are also too large.

#### 2.5.2. Balanced Tangential Method

The balanced tangential method uses the inclination and direction angles at the top and bottom of the course length to tangentially balance the two sets of measured angles. This method combines the trigonometric functions to provide the average inclination and direction angles which are used in standard computational procedures. This technique provides a smoother curve which should more closely approximate the actual wellbore between surveys. The longer the distance between survey stations, the greater the possibility of error.

#### 2.5.3. Mercury Method

The mercury method is a combination of the tangential and the balanced tangential method. It assumes that portion of the measured course defined by the length of the measuring tool in a straight line (tangentially) and the remainder of the measured course in a balanced tangential manner. The name of the mercury method originated from its common usage at the Mercury, Nevada test site by the United State Government.

#### 2.5.4. The Averaging Method (also known as angle averaging technique)

The averaging method considers the average of the angles over a course length increment in its calculations (Bourgoyne et al, 1991). It is based on the assumption

that the wellbore is parallel to the simple average of both the drift and course angles between two stations. This method is simple enough and mostly adopted for field use since computations can be done with a non-programmable calculator. but is rather difficult to justify theoretically.

#### 2.5.5 Radius of Curvature Method

The radius or curvature method assumes that the wellbore is a smooth arc between surveys. The wellbore follows a smooth spherical arc between surveys points and passes through the measured angles at both ends, which is theoretically sound and very accurate. However, this method involves very complex calculations hence requires a programmable calculator or computer to do the computations involve. The assumption that the wellbore is a smooth curve between surveys makes this method less sensitive to placement and distances between the survey points than other methods (Tarek, 2000). It becomes an unsatisfactory method when data are closely spaced, as the subtractions in the equation may create either dividing by zero errors or an incorrect TVD when the borehole is a straight line but deviated.

#### 2.5.6 Minimum Curvature Method

The minimum curvature method has emerged as the accepted industry standard for the calculation of 3D directional surveys. The well's trajectory is represented by a series of circular arcs and straight lines (Sawaryn, 2005). Collections of other points, lines, and planes can be used to represent features such as adjacent wells, lease lines, geological targets, and faults (Sawaryn, 2005). The minimum curvature assumes that the hole is a spherical arc with a minimum curvature or a maximum radius of curvature between stations (Bourgoyne et al, 1991). That is the wellbore follows a smoothest possible circular arc between stations. This is essentially the balanced tangential method, with each result multiplied by a ratio factor (RF). This method involves very complex calculations but with the advent of computers and programmable hand calculators, it has become the most common and acceptable method for the industry.

### 2.5.7 Comparison of the Six Methods

Table 2.1 shows the comparison of the six methods listed above.

**Table 2.1: Comparison of accuracy of the six methods (Bourgoyne et al, 1991)**

<b>Method</b>	<b>TVD</b>	<b>Diff. From Actual (ft)</b>	<b>North Displacement</b>	<b>Diff. From Actual (ft)</b>
Tangential	1628.61	-25.38	998.02	43.09
Balanced Tangential	1653.61	-0.38	954.72	-0.21
Mercury	1653.62	-0.37	954.89	0.04
Angle -Averaging	1654.18	0.19	955.04	0.11
Radius of Curvature	1653.99	0	954.93	0
Minimum Curvature	1653.99	0	954.93	0

The tangential method shows considerable error for the northing, easting and elevation which makes it no longer preferred in the industry (Bourgoyne et al, 1991). The differences among the average angle, balanced tangential, radius of curvature and minimum curvature are very small and any of the methods could be used for calculating the trajectory. Because the Minimum Curvature method is the most widely preferred method in the oil industry, more emphasis would be laid on in the next section than the other methods.

The Tangential, Balanced Tangential, Mercury and Angle averaging, are applicable to wellbore trajectory which follows straight line course whiles the Radius of Curvature is strictly applicable to a wellbore trajectory that follows a curved segment. The Minimum Curvature method is applicable to any trajectory path. The tangential method is mostly applicable to formations with large area extent (larger target radius; greater degree of control). The Minimum Curvature method is applicable to any target radius.

## 2.6 The Minimum Curvature Method

### 2.6.1 Introduction

The first approach to applying the Minimum Curvature method is credited to

Mason and Taylor in 1971 (Sawaryn, 2005). Zaremba also submitted an identical approach which he termed the *circular arc method* that very year. In all the Minimum Curvature methods, two adjacent survey points are assumed to lie on a circular arc. This arc is located in a plane and the orientation of which is defined by known inclination and direction angles at the ends of the arc (Bourgoyne et al, 1991). In 1985, the Minimum Curvature method was recognized by the industry as one of the most accurate methods, but was regarded as cumbersome for hand calculation. The emergence of well-trajectory planning packages to help manage directional work in dense well clusters increased its popularity. With the application of the Minimum Curvature method, toolface, interpolation, intersection with a target plane, minimum and maximum true vertical depth (TVD) in a horizontal section, point closest to a circular arc. Survey station to a target position with and without the direction defined, nudges, and steering runs can be determined. Figures 2.8a and 2.8b show the geometry of the minimum curvature method.



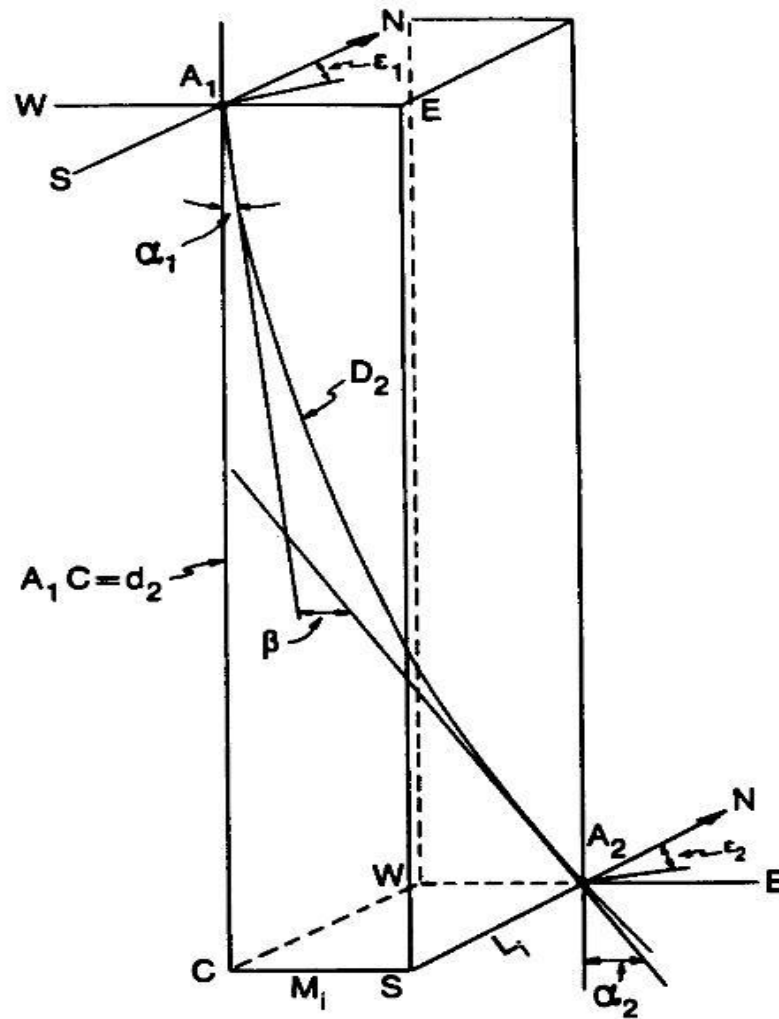


Figure 2.8a- A curve representation of a wellbore between survey stations  $A_1$  and  $A_2$  (Bourgoyne et al, 1991)

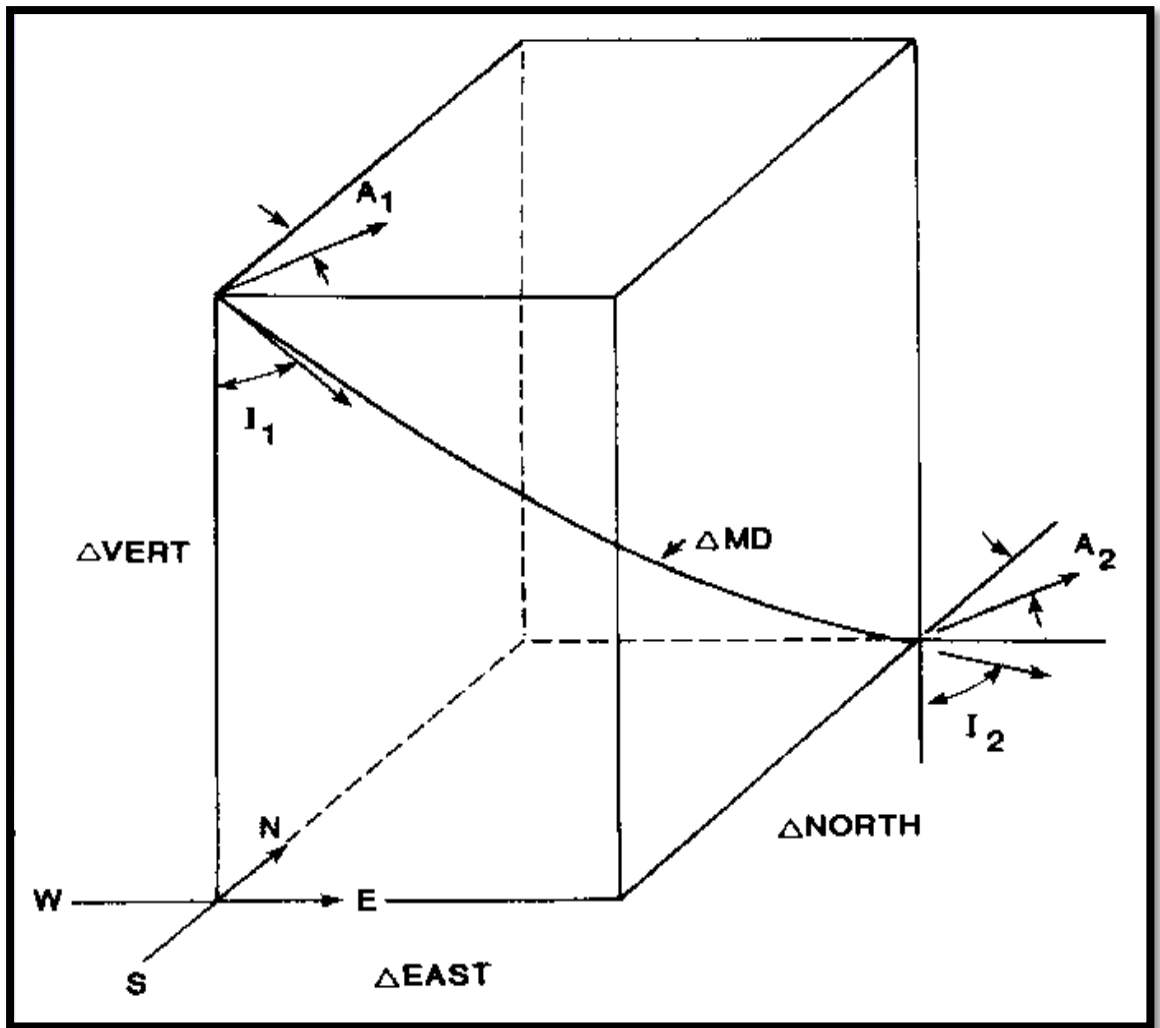


Figure 2.8b- A 3D view of the Minimum Curvature Geometry

## **CHAPTER 3**

### **PROGRAM DEVELOPMENT IN EXCEL SPREADSHEET**

#### **3.1 Introduction**

Excel is an electronic spreadsheet program that can be used for storing, organizing and manipulating data. With the application of excel spreadsheet, some of the survey calculation methods available to the Oil industry would be reviewed and a user friendly Excel Spreadsheet program would be developed to calculate the wellpath trajectory parameters using the Tangential, Angle Averaging, Balanced Tangential, Mercury, Radius of Curvature and the Minimum Curvature methods. All the methods listed above would have a common data input interface in the excel spreadsheet such that once the necessary input data are entered, the program would automatically compute the wellpath trajectory coordinates employing all the six methods.

#### **3.2 Data Input Interface**

The data input interface in the Excel Spreadsheet contains three sections which are;

1. The Well Description Input Data section
  2. The Adjustment/Correction Input Data section
  3. The Spatial and Meta Data Input section
- 
1. The Well description Input Data section requires the input of description data such as Company Name, Field Name, Well Name, Reservoirs / Fluid type and name of the Drilling Engineer in charge of operations.
  2. The Adjustment/Correction Input section requires the input of adjustment or correctional data values such as the;
    - Lead Angle and or
    - The Magnetic Declination adjustment value(s)

These values are needed for the adjustment of the spatial data. The lead and magnetic declination angles must be in degrees. The directions for both the lead and magnetic declination angle must be specified. If the bit walk is to the

right, the word 'Right' should be selected from the drop-down menu otherwise the word 'Left' should be selected for left walk of bit to make the necessary adjustments. The same approach is applicable to the magnetic declination adjustment. If the magnetic declination is to the West, the word 'West' must be selected otherwise the 'East' should be selected for East magnetic declination.

3. The Spatial and Meta Input Data section. This is where data such as the;
  1. Measured Depths
  2. Inclination angles
  3. Measured Bearings or unadjusted Azimuths
  4. Spatial data of the reference station and other meta data of other stations are entered.

### **3.3 The Computational Procedure for the Wellpath Trajectory Design**

#### **3.3.1 Data Input Interface**

The use of magnetic compass in bearing reading requires a magnetic declination correction or adjustment to be effected to the reading for attaining the correct azimuth which is a requisite to the computation of the wellbore coordinates. The directions for the lead angle must also be specified. If the bit walk and magnetic declination adjustment are specified, the necessary computations could then be done. The true azimuth computation (corrected for azimuth and lead angle) is given by;

$$\phi^{\circ} = \phi^{\circ}(\text{read}) \pm \text{Magnetic Declination Angle} \pm \text{Lead Angle} \dots\dots\dots 3.1$$

The signs of the corrections are determined by the direction of the magnetic declination and the lead angle. Thus, if the lead angle is to the right, the correction is negative otherwise it is positive, this is shown in Figure 3.1. Also, if the magnetic declination is to westward, the sign is positive otherwise it is negative.

Lead Angle (Bit Walk)	$\phi^\circ$ 0.00	Bit walk Right	Select (Right) for right bit-walk or (Left) for Left bit walk  Select (West) for west declination or (East) for East Declination
Magnetic Declination Adjustment Value	$\phi^\circ$ 0.00	Magnetic Declination East	

Bit Walk Direction  
Select the word 'Right' for right bit-walk or 'Left' for left bit-walk

Figure 3.1- Correctional (Bit-walk and Magnetic Declination) data input interface for Wellpath Trajectory Computation.

Target Coordinates	Feet	Units of Measurement
Station ID	Chem1	Feet
Northing	954.930	
Easting	0.000	
Elevation	1653.987	
MD	2000.000	

Starting/Initial Coordinates	Feet	Units of Measurement
Station ID	Bron 7	Feet
Northing	0.000	
Easting	0.000	
Elevation	0.000	
Survey Tool Length (STL)	15.000	For Mercury Method

Drop-down icon

Select (Feet) for units in feet or (Metre) for units in metres

Select (Feet) for units in feet or (Metre) for units in metres

Figure 3.2- The Target, Starting/initial coordinates and unit of measurement data input interface for Wellpath Trajectory Computation.

The target, initial or starting coordinates may be entered if needed, or else, zeros must be entered as the starting and or the same for the target coordinates. The unit of measurement of the coordinates or measured distances must be selected under the 'units' column as shown in Figure 3.2. The word 'Feet' must be selected for units in feet or 'Metre' for units in metres.

The final data needed to be entered are the Measured distances (MD), Inclination angles (I) and Measure Bearings (A) with their corresponding descriptive data such as the station ID. Both the inclination and azimuth angles must be entered in degrees. An example of the data input interface is shown in Table 3.1. The whole circle bearing reading approached must be adhered to when entering the azimuth angles.

**Table 3.1- The Final Data Input Interface for Wellpath Trajectory Computation.**

Station ID	Measured distance MD	Inclination $\Theta^\circ$ (I)	Azimuth $\Phi^\circ$ (A)
KOP	0.00	0.00	0
1	100.00	3.00	0
2	200.00	6.00	0
3	300.00	9.00	0
4	400.00	12.00	0
5	500.00	15.00	0
6	600.00	18.00	0
7	700.00	21.00	0
8	800.00	24.00	0
9	900.00	27.00	0
10	1000.00	30.00	0
11	1100.00	33.00	0
12	1200.00	36.00	0
13	1300.00	39.00	0
14	1400.00	42.00	0
15	1500.00	45.00	0
16	1600.00	48.00	0
17	1700.00	51.00	0
18	1800.00	54.00	0
19	1900.00	57.00	0
20	2000.00	60.00	0

Enter only Whole Circle Reading bearings Values

### **3.4 Derivation of Associated Equations for the various Wellpath Trajectory Methods used for the development of the Excel Spreadsheet Program**

1. The Tangential method is the simplest and old method used for years. Its original derivation or presentation to the industry is unknown. The tangential method assumes that the hole would maintain the same drift and course between each station, thus projected angles remain constant over the preceding course length from the previous station as shown in Figure 3.3. This method is very inaccurate based on its assumption, especially in build and hold configurations where it shows less vertical and more horizontal displacement than the actual data (Bourgoyne et al, 1991).

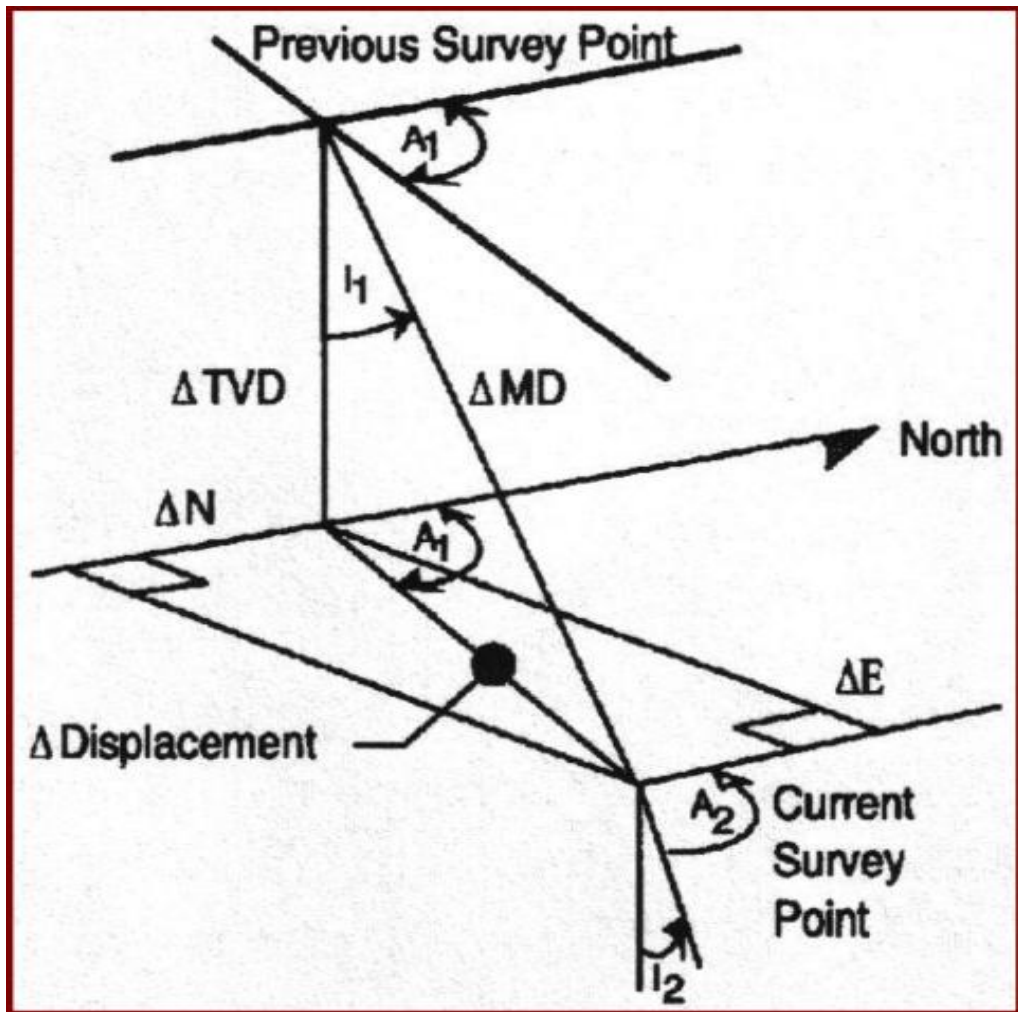


Figure. 3.3- 3D view of the Tangential Method (Osisanya, 2009)



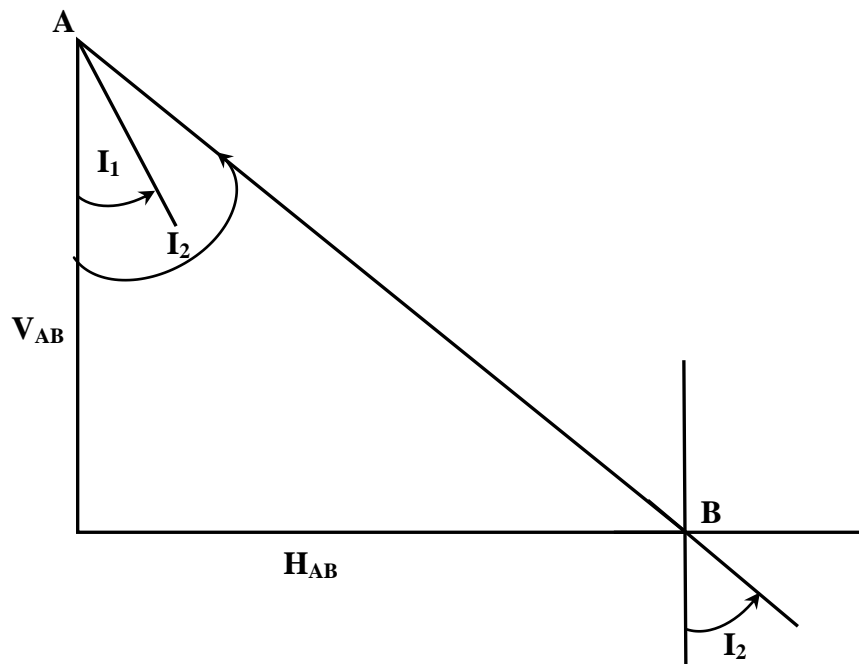


Figure 3.4- Vertical plane view of the Tangential method

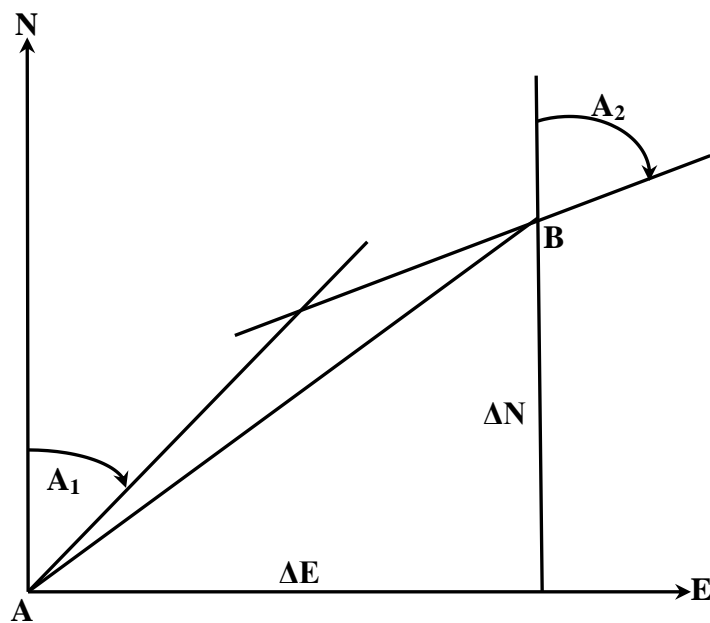


Figure 3.5- Horizontal plane view of the Tangential method

For the following parameters known, A, |AB| (measured distance, ΔMD), Inclination Angles I<sub>1</sub> and I<sub>2</sub>, Azimuth A<sub>1</sub> and A<sub>2</sub>, the changes in the vertical and horizontal plane can be determined as well as the North, East and Elevation coordinates through the following equations;

*From Figures 3.4 and 3.5*

$$V_{AB} = AB \cos(I_2) \dots\dots\dots 3.2$$

$$H_{AB} = AB \sin(I_2) \dots\dots\dots 3.3$$

$$\Delta E = H_{AB} \sin(A_2) \dots\dots\dots 3.4$$

$$\Delta N = H_{AB} \cos(A_2) \dots\dots\dots 3.5$$

$$\Delta V = V_{AB} \dots\dots\dots 3.6$$

*Substituting eqn 3.2 into eqn 3.6 and eqn 3.3 into eqn 3.4 and eqn 3.5 give*

$$\Delta E = \Delta MD \sin(I_2) \sin(A_2) \dots\dots\dots 3.7$$

$$\Delta N = \Delta MD \sin(I_2) \cos(A_2) \dots\dots\dots 3.8$$

$$\Delta V = \Delta MD \cos(I_2) \dots\dots\dots 3.9$$

The total East/West, North/South and Elevation/Depth are calculated as;

$$E_n = \sum_i^n E_i \dots\dots\dots 3.10$$

$$N_n = \sum_i^n N_i \dots\dots\dots 3.11$$

$$V_n = \sum_i^n V_i \dots\dots\dots 3.12$$

2. The Balanced Tangential method uses the inclination and direction angles at the top and bottom of the course length to tangentially balance the two sets of measured angles as shown in Figure 3.6. This method combines the trigonometric functions to provide the average inclination and direction angles which are used in standard computational procedures. This technique provides a smoother curve which should more closely approximate the actual wellbore between surveys. The longer the distance between survey stations, the greater the possibility of error.

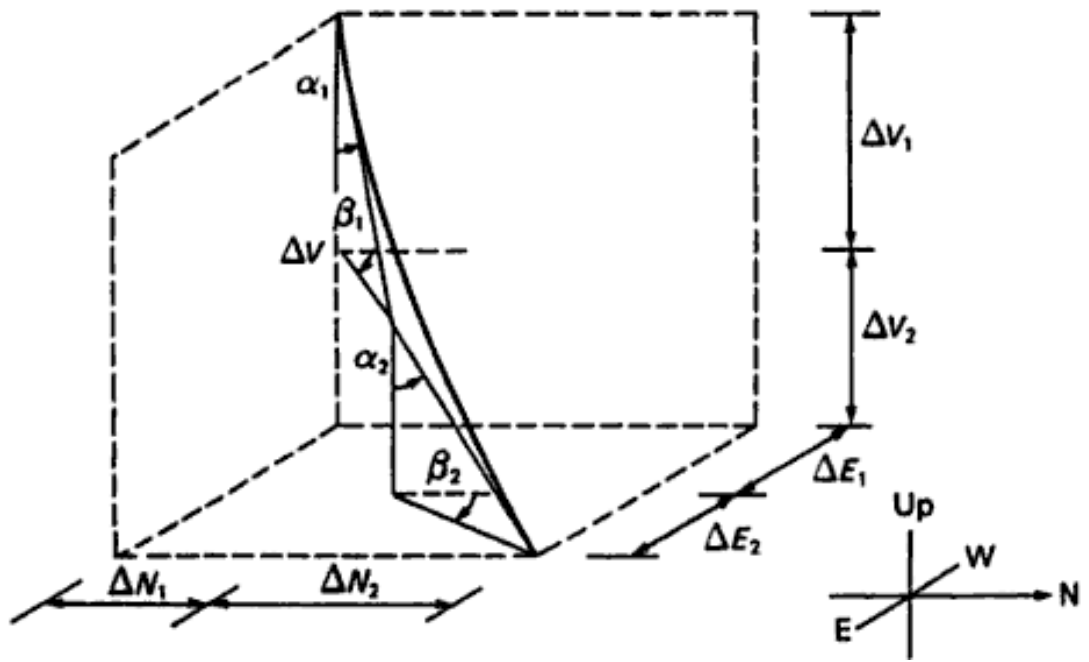


Figure 3.6-A 3D view of the Balanced Tangential Method (Inglis, 1987)

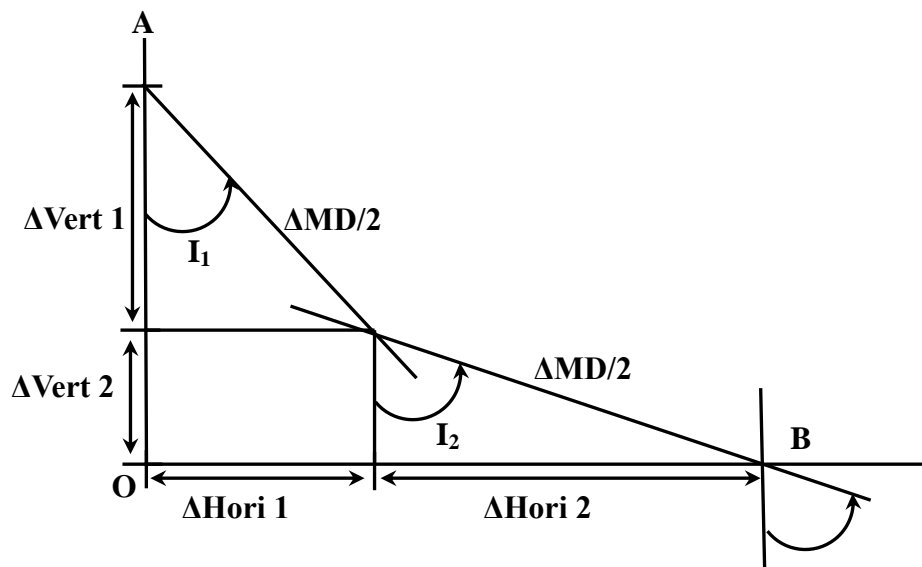
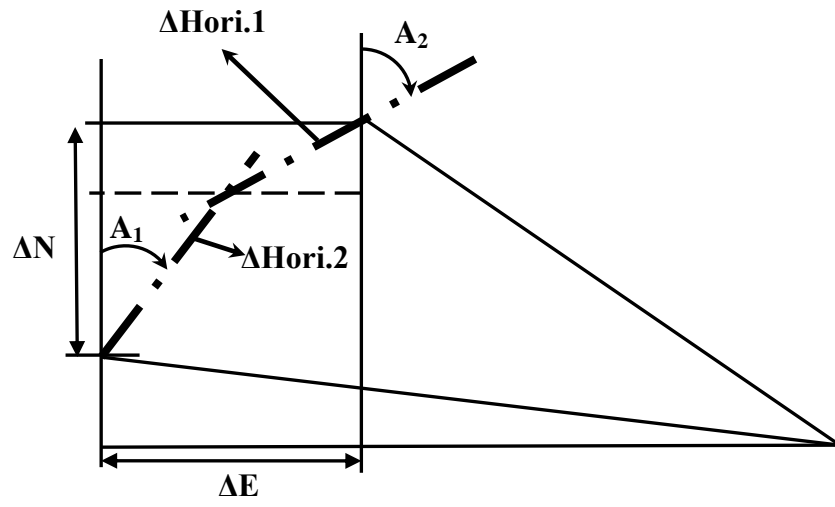


Figure 3.7- Vertical plane of the Balanced Tangential method



**Figure 3.8- Horizontal plane of the Balanced Tangential method**

For the following parameters known, A, measured distances,  $\Delta MD/2$ , Inclination Angles  $I_1$  and  $I_2$ , Azimuth  $A_1$  and  $A_2$ , the changes in the vertical and horizontal plane can be determined as well as the Northing, Easting and Elevation coordinates through the following equations;

From Figures 3.7 and 3.8;

$$\Delta V_{AB} = \Delta \text{vert } 1 + \Delta \text{vert } 2 \dots\dots\dots 3.13$$

$$\Delta H_{AB} = \Delta \text{Hori } 1 + \Delta \text{Hori } 2 \dots\dots\dots 3.14$$

where

$$\Delta \text{vert } 1 = \Delta MD/2 \cos I_1 \dots\dots\dots 3.15$$

$$\Delta \text{vert } 2 = \Delta MD/2 \cos I_2 \dots\dots\dots 3.16$$

$$\Delta \text{Hori } 1 = \Delta MD/2 \sin I_1 \dots\dots\dots 3.17$$

$$\Delta \text{Hori } 2 = \Delta MD/2 \sin I_2 \dots\dots\dots 3.18$$

Therefore substituting equation 3.15 and equation 3.16 into equation 3.13 and also equation 3.17 and equation 3.18 into equation 3.14 give

$$\Delta V_{AB} = \Delta MD/2 \cos(I_1) + \Delta MD/2 \cos(I_2) \dots\dots\dots 3.19$$

$$\Delta H_{AB} = \Delta MD/2 \sin(I_1) + \Delta MD/2 \sin(I_2) \dots\dots\dots 3.20$$

From Figure 3.8;

$$\Delta E = (\Delta \text{Hori } 1) \sin A_1 + (\Delta \text{Hori } 2) \sin A_2 \dots\dots\dots 3.21$$

$$\Delta N = (\Delta \text{Hori } 1) \cos A_1 + (\Delta \text{Hori } 2) \cos A_2 \dots\dots\dots 3.22$$

Therefore substituting eqn 3.17 and eqn 3.18 into eqn 3.21 and eqn 3.22 and factorizing, give

$$\Delta E = \Delta MD/2 [\sin I_1 \sin A_1 + \sin I_2 \sin A_2] \dots\dots\dots 3.23$$

$$\Delta N = \Delta MD/2 [\sin I_1 \cos A_1 + \sin I_2 \cos A_2] \dots\dots\dots 3.24$$

$$\Delta V = \Delta MD/2 [\cos I_1 + \cos I_2] \dots\dots\dots 3.25$$

The total East/West, North/South and Elevation/Depth are calculated using equations 3.10, 3.11 and 3.12.

3. The Mercury method is a combination of the tangential and the balanced tangential method. It assumes that portion of the measured course defined by the length of the measuring tool in a straight line (tangentially) and the

remainder of the measured course in a balanced tangential manner. The name of the mercury method originated from its common usage at the Mercury, Nevada test site by the United State Government.

The coordinates of the Mercury method is computed using the following equations;

$$\Delta E = \left[ \frac{\Delta MD - STL}{2} \right] [\sin I_1 \sin A_1 + \sin I_2 \sin A_2] + (STL) \sin I_2 \sin A_2 \dots\dots\dots 3.26$$

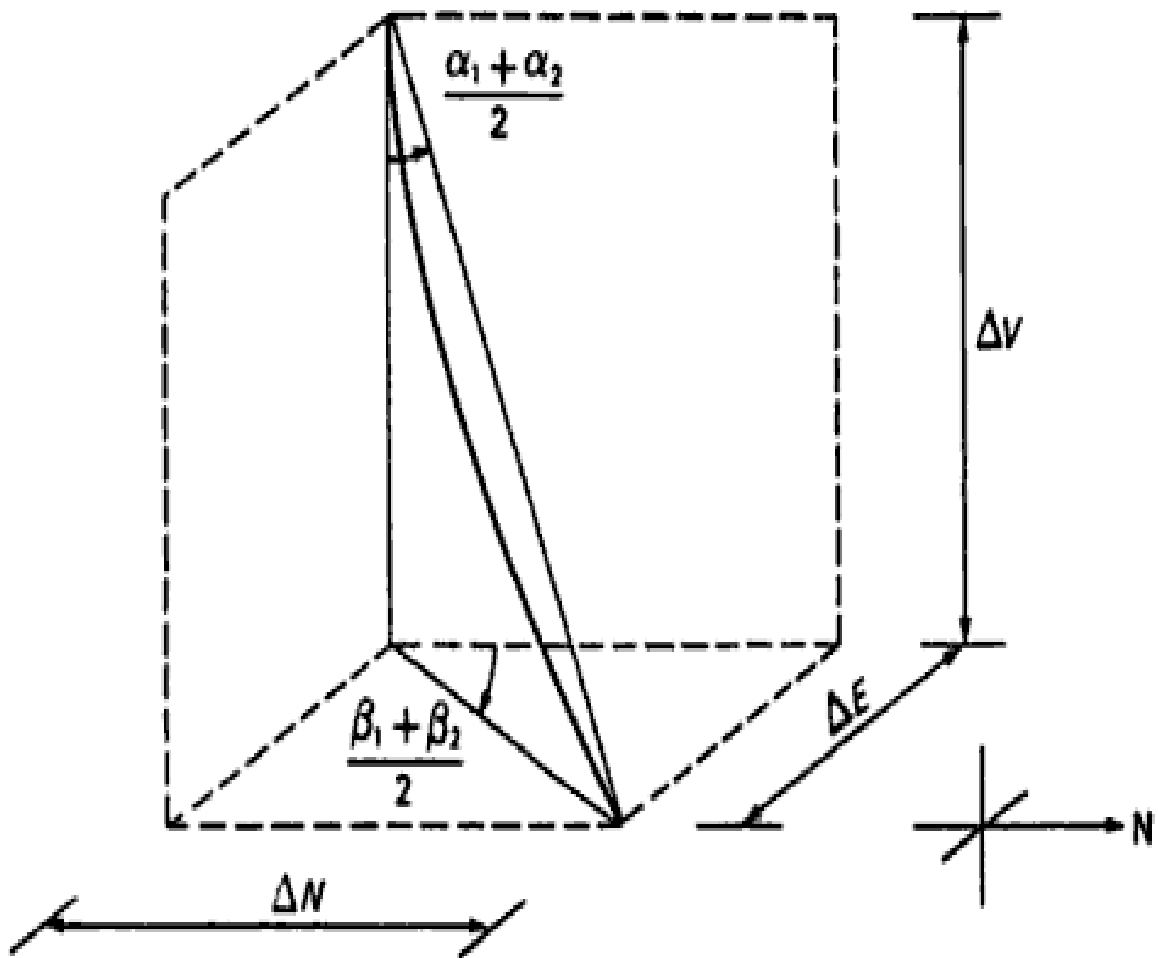
$$\Delta N = \left[ \frac{\Delta MD - STL}{2} \right] [\sin I_1 \cos A_1 + \sin I_2 \cos A_2] + (STL) \sin I_2 \cos A_2 \dots\dots\dots 3.27$$

$$\Delta V = \left[ \frac{\Delta MD - STL}{2} \right] [\cos I_1 + \cos I_2] + (STL) \cos I_2 \dots\dots\dots 3.28$$

where *STL* is the length of the survey tool used for the measurements.

The total East/West, North/South and Elevation/Depth are calculated using equations 3.10, 3.11 and 3.12.

4. The Averaging method considers the average of the angles over a course length increment in its calculations (Bourgoyne et al, 1991) as shown in Figure 3.9. It is based on the assumption that the wellbore is parallel to the simple average of both the drift and course angles between two stations. This method is simple enough and mostly adopted for field use since computations can be done with a non-programmable calculator. but is rather difficult to justify theoretically.



**Figure 3.9- A 3D view of the Angle Averaging Method (Inglis, 1987)**

Where  $\alpha_1$  and  $\alpha_2$  are the Inclination angles ( $I_1$  and  $I_2$ ), and  $\beta_1$  and  $\beta_2$  are the Azimuths ( $A_1$  and  $A_2$ ).

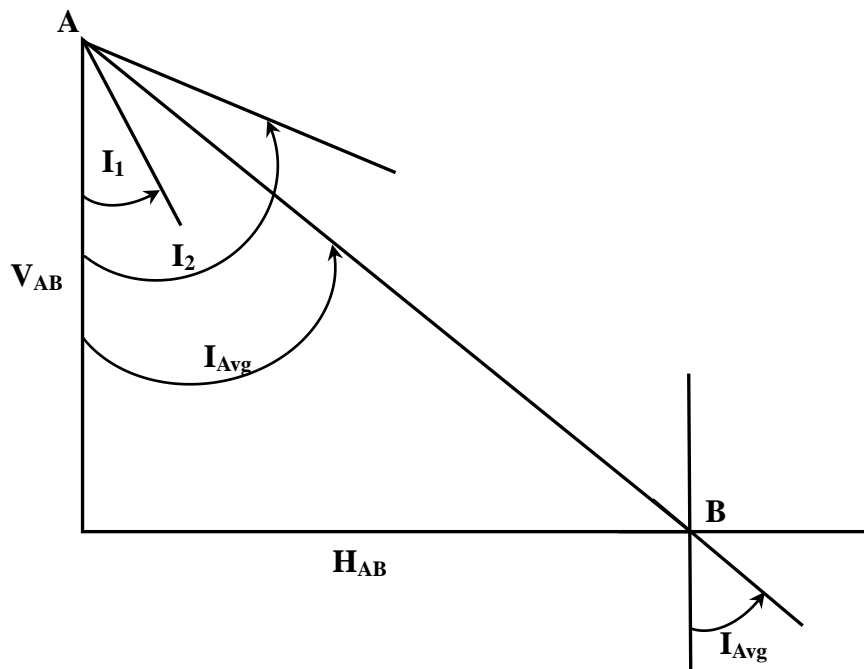


Figure 3.10- Vertical plane view of the Angle Averaging method

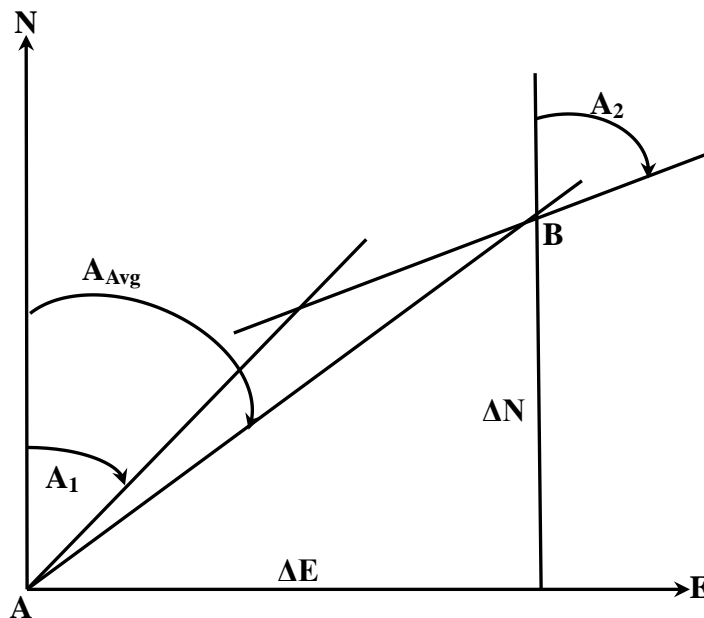


Figure 3.11- Horizontal plane of the Angle Averaging



For the following parameters known, A, |AB| (measured distance, ΔMD), Inclination Angles I<sub>1</sub> and I<sub>2</sub>, Azimuth A<sub>1</sub> and A<sub>2</sub>, the changes in the vertical and horizontal plane can be determined as well as the Northing, Easting and Elevation coordinates through the following equations;

From Figures 3.10 .and 3.11;

$$A_{Avg} = (A_1 + A_2)/2 \dots\dots\dots 3.29a$$

$$I_{Avg} = (I_1 + I_2)/2 \dots\dots\dots 3.29b$$

$$V_{AB} = AB\cos(I_{Avg}) \dots\dots\dots 3.30$$

$$H_{AB} = AB\sin(I_{Avg}) \dots\dots\dots 3.31$$

$$\Delta E = H_{AB}\sin(A_{Avg}) \dots\dots\dots 3.32$$

$$\Delta N = H_{AB}\cos(A_{Avg}) \dots\dots\dots 3.33$$

$$\Delta V = V_{AB} \dots\dots\dots 3.34$$

Substituting eqn 3.30 into eqn 3.34 and eqn 3.31 into eqn 3.32, and eqn 3.33, give

$$\Delta E = \Delta MD\sin(I_{Avg})\sin(A_{Avg}) \dots\dots\dots 3.35a$$

$$\Delta N = \Delta MD\sin(I_{Avg})\cos(A_{Avg}) \dots\dots\dots 3.35b$$

$$\Delta V = \Delta MD\cos(I_{Avg}) \dots\dots\dots 3.35c$$

The total East/West, North/South and Elevation/Depth are calculated using equations 3.10, 3.11 and 3.12.

5. The Radius of curvature method assumes that the wellbore is a smooth arc between surveys. The wellbore follows a smooth spherical arc between surveys points and passes through the measured angles at both ends, which is theoretically sound and very accurate. However, this method involves very complex calculations hence requires a programmable calculator or computer to do the computations involve. The assumption that the wellbore is a smooth curve between surveys makes this method less sensitive to placement and distances between the survey points than other methods (Tarek, 2000). It becomes an unsatisfactory method when data is closely spaced, as the subtractions in the equation may create either dividing by zero errors or an incorrect TVD when the borehole is a straight line but deviated

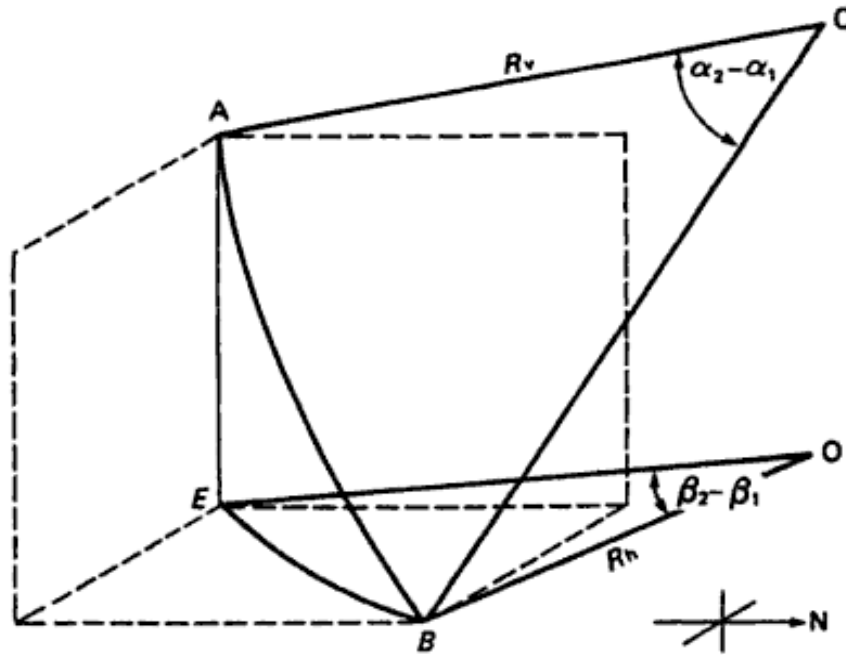


Figure 3.12- A 3D view of the Radius of Curvature Method (Inglis, 1987)

Where  $\beta$  is the azimuth,  $\alpha$  is the inclination angle,  $R_v$  and  $R_h$  are the radii of the curvatures

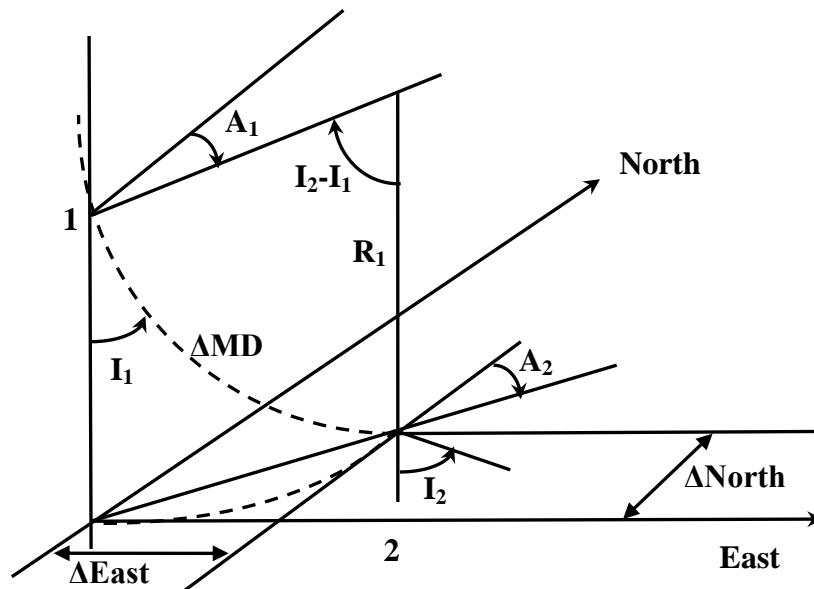


Figure 3.13- A 3D view of the Radius of Curvature method

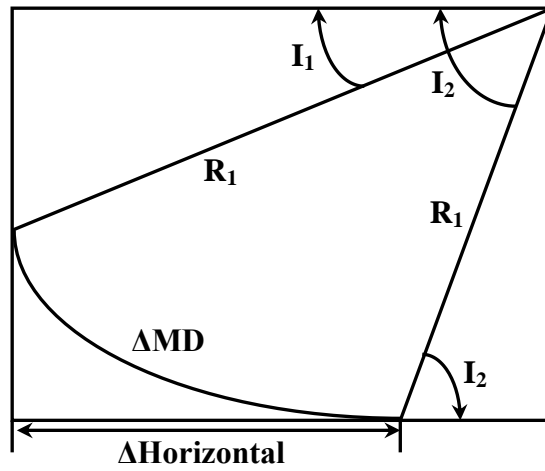


Figure 3.14- Vertical plane of the Radius of Curvature method

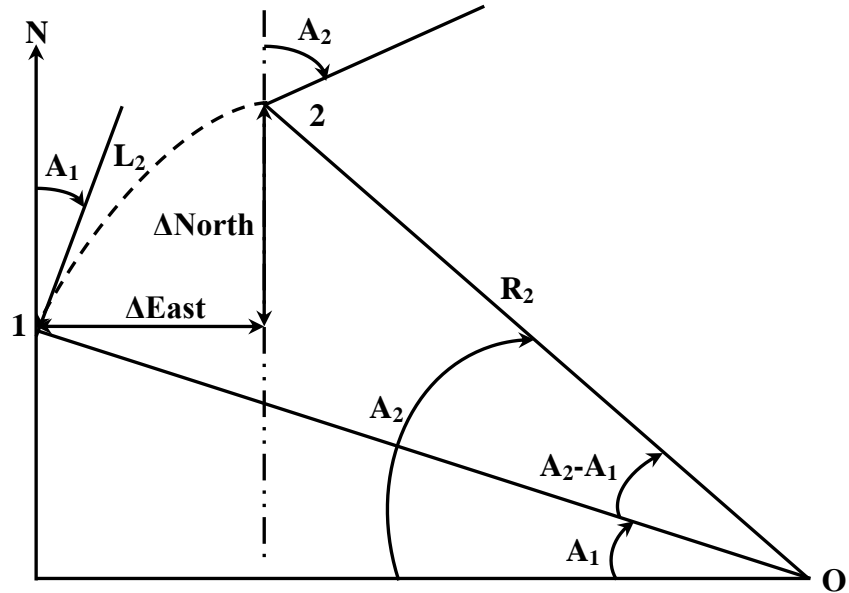


Figure 3.15- Horizontal plane of the Radius of Curvature method

For the following parameters known, A, measured distances (MD), Inclination Angles  $I_1$  and  $I_2$ , Azimuth  $A_1$  and  $A_2$ , the changes in the vertical and horizontal plane can be determined as well as the Northing, Easting and Elevation coordinates through the following equations;

From Figure 3.13; the length of the arc,

$$\Delta MD = R_1(I_2 - I_1)(\text{Angle in radians}) \dots\dots\dots 3.36a$$

$$\Delta MD = R_1(I_2 - I_1)\left(\frac{\pi}{180}\right)(\text{Angle in deg}) \dots\dots\dots 3.36b$$

$$\therefore R_1 = [\Delta MD / (I_2 - I_1)]\left(\frac{180}{\pi}\right)(\text{deg}) \dots\dots\dots 3.37$$

From Figures 3.14 and 3.15;

$$\Delta V = R_1 \sin I_2 - R_1 \sin I_1 \dots\dots\dots 3.38$$

$$\Delta H = R_1 \cos I_1 - R_1 \cos I_2 \dots\dots\dots 3.39$$

Substituting eqn 3.37 into eqn 3.38 and eqn 3.39 and factorizing, give

$$\Delta V = [\Delta MD / (I_2 - I_1)]\left(\frac{180}{\pi}\right)(\sin I_2 - \sin I_1) \dots\dots\dots 3.40$$

$$\Delta H = [\Delta MD / (I_2 - I_1)]\left(\frac{180}{\pi}\right)(\cos I_1 - \cos I_2) \dots\dots\dots 3.41$$

$$\text{Also, } L_2 = R_2(A_2 - A_1)(\text{rad}) \dots\dots\dots 3.42a$$

$$L_2 = R_2(A_2 - A_1)\left(\frac{\pi}{180}\right)(\text{deg}) \dots\dots\dots 3.42b$$

in the vertical plane

$$L_2 = [\Delta MD / (I_2 - I_1)]\left(\frac{180}{\pi}\right)(\cos I_1 - \cos I_2) \dots\dots\dots 3.42c$$

$$\therefore R_2 = \left[\frac{L_2}{A_2 - A_1}\right](180/\pi)(\text{deg}) \dots\dots\dots 3.43$$

$$\Delta E = R_2 \cos A_1 - R_2 \cos A_2 \dots\dots\dots 3.44a$$

$$\Delta E = R_2(\cos A_1 - \cos A_2) \dots\dots\dots 3.44b$$

$$\Delta N = R_2(\sin A_2 - \sin A_1) \dots\dots\dots 3.45$$

Substituting equation 3.42c into equation 3.43, and equation 3.43 into equations 3.44b and 3.45 and factorizing with all angles in radians, give for normal cases (case 0)

$$\Delta E = \frac{\Delta MD(\cos I_1 - \cos I_2)(\cos A_1 - \cos A_2)}{[(I_2 - I_1)(A_2 - A_1)]} \dots\dots\dots 3.46a$$

$$\Delta N = \frac{\Delta MD(\cos I_1 - \cos I_2)(\sin A_2 - \sin A_1)}{[(I_2 - I_1)(A_2 - A_1)]} \dots\dots\dots 3.47a$$

$$\Delta V = \frac{\Delta MD(\sin I_2 - \sin I_1)}{(I_2 - I_1)} \dots\dots\dots 3.48a$$

For special cases.

Case 1, if  $I_1 = I_2$

$$\Delta E = \frac{\Delta MD \sin I_1 (\cos A_1 - \cos A_2)}{[(A_2 - A_1)]} \dots\dots\dots 3.46b$$

$$\Delta N = \frac{\Delta MD \sin I_1 (\sin A_2 - \sin A_1)}{[(A_2 - A_1)]} \dots\dots\dots 3.47b$$

$$\Delta V = \Delta MD \cos I_1 \dots\dots\dots 3.48b$$

Case 2 if  $I_1 \neq I_2$  and  $A_1 = A_2$

$$\Delta E = \frac{\Delta MD \sin A_1 (\cos I_1 - \cos I_2)}{[(I_2 - I_1)]} \dots\dots\dots 3.46c$$

$$\Delta N = \frac{\Delta MD \cos A_1 (\cos I_1 - \cos I_2)}{[(I_2 - I_1)]} \dots\dots\dots 3.47c$$

$$\Delta V = \frac{\Delta MD (\sin I_2 - \sin I_1)}{[(I_2 - I_1)]} \dots\dots\dots 3.48c$$

Case 3, if  $I_1 = I_2$  and  $A_1 \neq A_2$

$$\Delta E = \Delta MD (\sin I_1 \sin A_1) \dots\dots\dots 3.46d$$

$$\Delta N = \Delta MD (\sin I_1 \cos A_1) \dots\dots\dots 3.47d$$

$$\Delta V = \Delta MD (\cos I_1) \dots\dots\dots 3.48d$$

The total East/West, North/South and Elevation/Depth are calculated using equations 3.10, 3.11 and 3.12.

6. The Minimum curvature method has emerged as the accepted industry standard for the calculation of 3D directional surveys, the well's trajectory is represented by a series of circular arcs and straight lines (Sawaryn, 2005). Collections of other points, lines, and planes can be used to represent features such as adjacent wells, lease lines, geological targets, and faults. The minimum curvature assumes that the hole is a spherical arc with a minimum curvature or a maximum radius of curvature between stations. That is the wellbore follows a smoothest possible circular arc between stations. This methods involves very complex calculations but with the advent of computers and programmable hand calculators, it has become the most common and acceptable method for the industry. Figures 3.16a and 3.16b show the geometry of the minimum curvature method and Figure 3.17 shows the effect of dogleg severity on ratio factor.

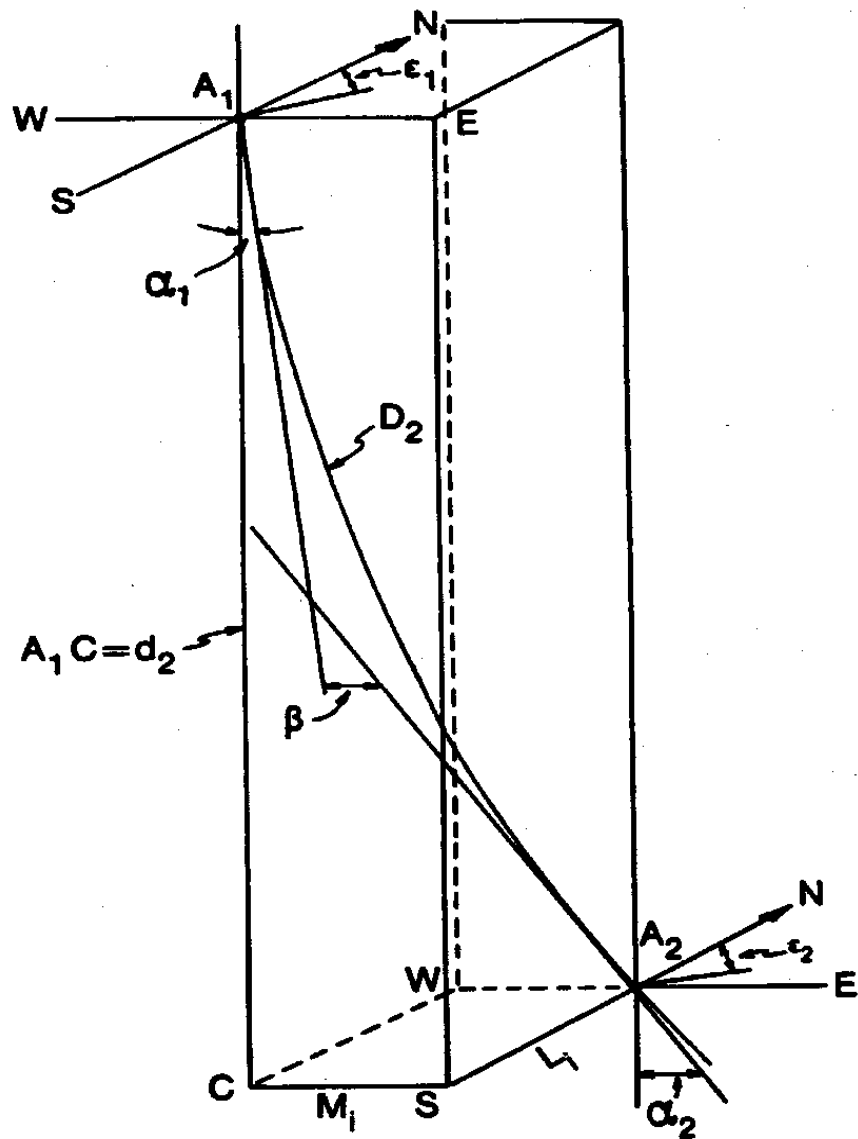


Figure 3.16a- A 3D view of the Minimum Curvature method (Bourgoyne et al, 1991).

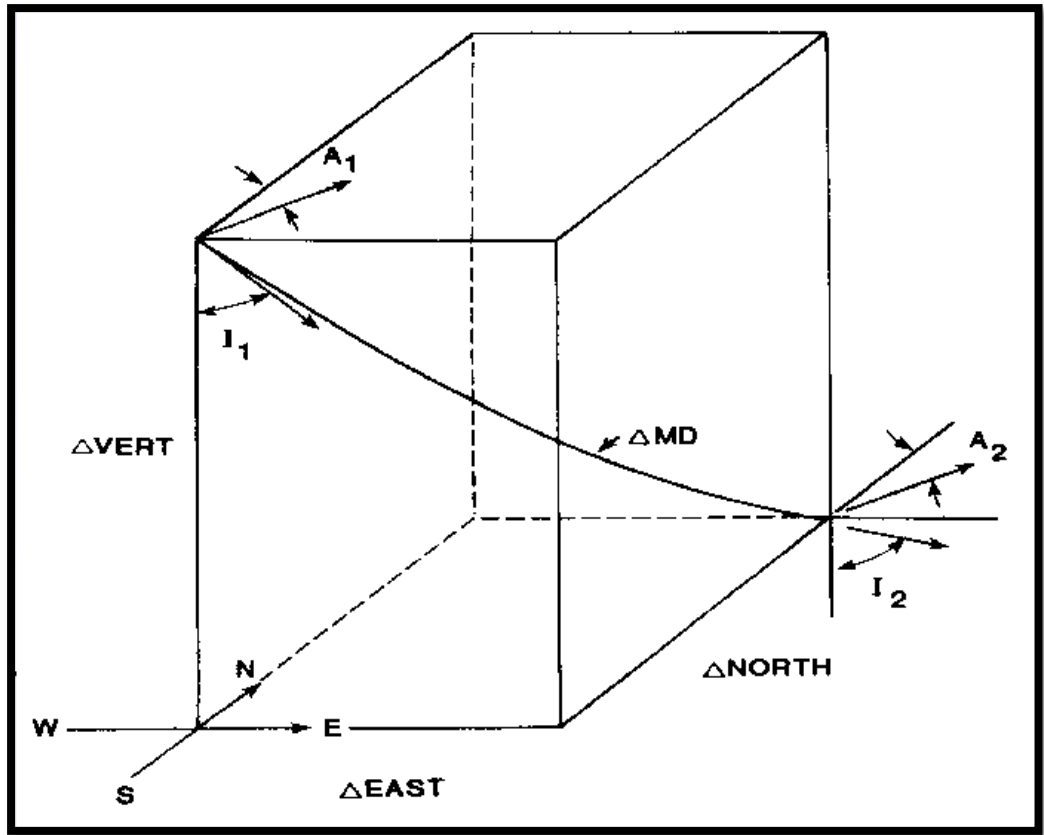


Figure 3.16b- A 3D view of the Minimum Curvature Geometry

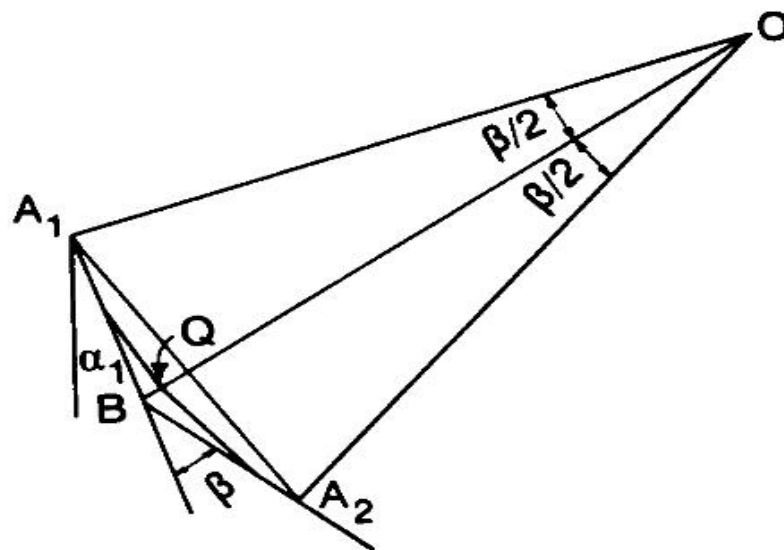


Figure 3.17- A representation of the Minimum Curvature ratio factor (RF) (Bourgoyne et al, 1991).

Where in figure 3.16a, Figures 3.16b and 3.17,  $\beta$  is the dogleg severity,  $\varepsilon$  is the azimuth,  $\alpha$  is the inclination angle. The doglegs in directional holes bend the casing and induce added axial stress (Chukwu, 2008). Except for large casings this is not critical for most loads actually encountered when designing for this effect on tension (Chukwu, 2008).

This is essentially the Balanced Tangential Method, with each result multiplied by a ratio factor (RF) as follows:

$$\Delta E = \Delta MD / 2 [\sin I_1 \sin A_1 + \sin I_2 \sin A_2] RF \dots\dots\dots 3.49$$

$$\Delta N = \Delta MD / 2 [\sin I_1 \cos A_1 + \sin I_2 \cos A_2] RF \dots\dots\dots 3.50$$

$$\Delta V = \Delta MD / 2 [\cos I_1 + \cos I_2] RF \dots\dots\dots 3.51$$

RF can be derived from Figure 3.17 as follows;

The straight line segments  $A_1B + BA_2$  adjoin the segment  $A_1Q + QA_2$  at points  $A_1$  and  $A_2$ . It can then be shown that

$$A_1Q = OA_1 \cdot \beta / 2 \dots\dots\dots 3.52a$$

$$QA_2 = OA_2 \cdot \beta / 2 \dots\dots\dots 3.52b$$

$$A_1B = OA_1 \cdot \tan (\beta / 2) \dots\dots\dots 3.52c$$

$$BA_2 = OA_2 \cdot \tan(\beta / 2) \dots\dots\dots 3.52d$$

The ratio of dividing the straight line section (eqns 3.52c and 3.52d) with the curved section (eqns 3.52a and 3.52b) respectively, defines the ratio factor, RF

$$RF = A_1B/A_1Q = BA_2/QA_2 = \tan (\beta / 2) / \beta / 2 \dots\dots\dots 3.53a$$

$$RF = A_1B/A_1Q = BA_2/QA_2 = \tan (\beta / 2) / \beta / 2 \dots\dots\dots 3.53a$$

$$RF = 2/\beta_i \tan (\beta_i / 2) \dots\dots\dots 3.53b$$

Where the dogleg angle  $\beta$  is the overall angle change of the drill pipe between any two stations is computed as;

$$\beta_i = \cos^{-1}[\cos(I_2 - I_1) - \sin I_1 \sin I_2 [1 - \cos(A_2 - A_1)]] \dots\dots\dots 3.54a$$

or

$$\beta = \cos^{-1}[\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos (A_2 - A_1)] \dots\dots\dots 3.54b$$



The condition of applying this ratio factor is that if the dogleg angle is less than 0.25 radians, then it reasonable to set the ratio factor to one (1) else the computation holds (Bourgoyne et al, 1991) This is usually done to avoid singularity in straight hole. If the  $\beta$  is approximately less than  $15^\circ$ , then the resultant error will be less than 1 part in  $10^9$  (BP Amoco, 1999). The RF can also be calculated using equations if the dogleg angle is less than approximately  $15^\circ$ , this is a truncated series expansion given by the form (BP Amoco, 1999)

$$RF = 1 + \beta^2/12 + \beta^4/120 + 17\beta^6/20160 \dots\dots\dots 3.55$$

Once the  $\beta$  and RF are determined, the spatial coordinates (easting, northing and elevation coordinates) can be computed.

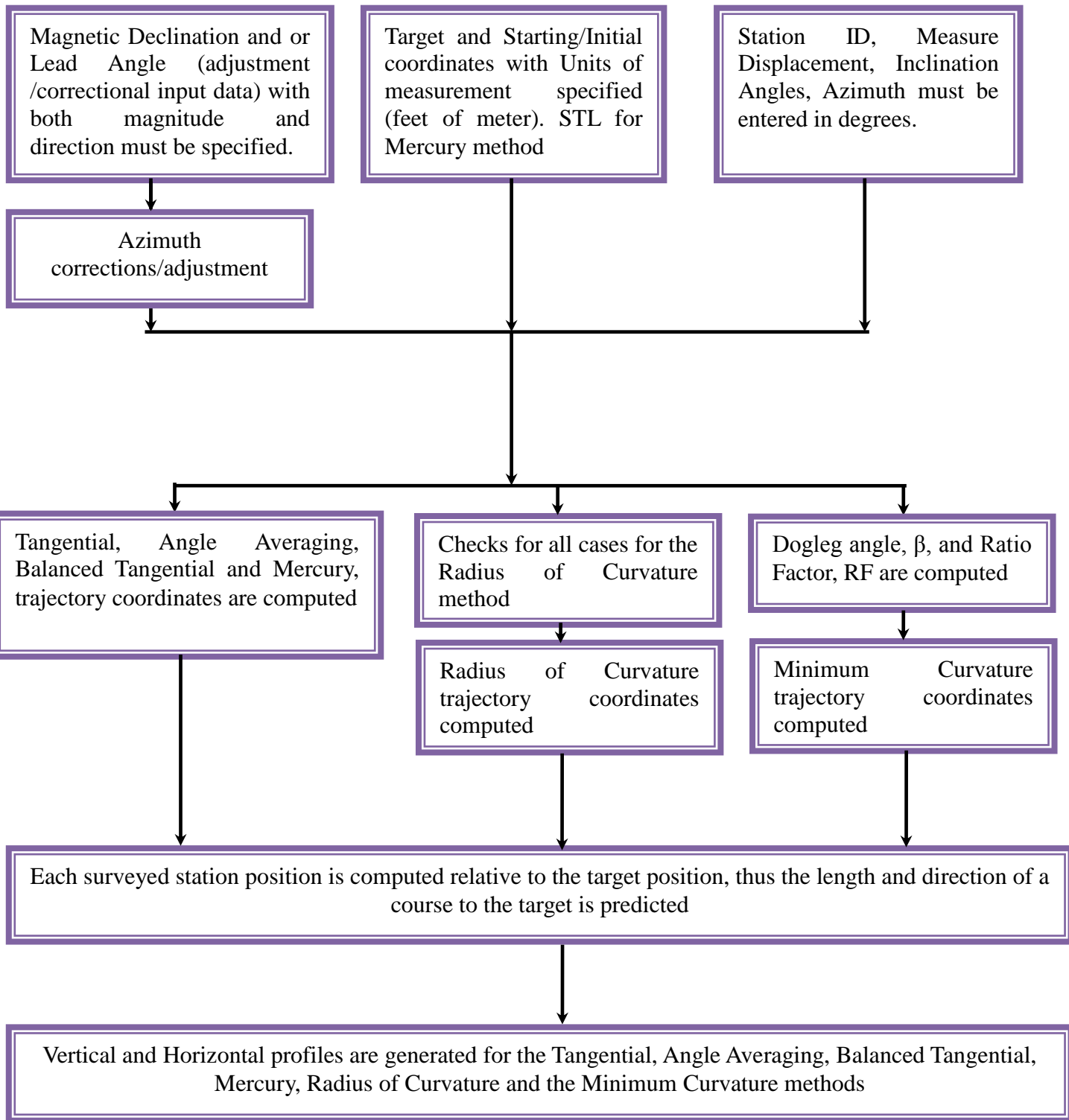
In summary, Tables 3.2 and 3.3 summarizes all the equations that were used to generate the excel spreadsheet program to compute the wellpath trajectory coordinates employing the six different methods listed above. Figure 3.18 shows the summarized flow chart for the Excel Spreadsheet program.

**Table 3.2-Summary of all the equations used to generate the excel spreadsheet program**

<b>Method</b>	<b>Equations</b>
Tangential	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \Delta MD \sin(I_2) \sin(A_2)</math></li> <li>2. <math>\Delta N = \Delta MD \sin(I_2) \cos(A_2)</math></li> <li>3. <math>\Delta V = \Delta MD \cos(I_2)</math></li> </ol>
Balanced Tangential	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \Delta MD / 2 [\sin I_1 \sin A_1 + \sin I_2 \sin A_2]</math></li> <li>2. <math>\Delta N = \Delta MD / 2 [\sin I_1 \cos A_1 + \sin I_2 \cos A_2]</math></li> <li>3. <math>\Delta V = \Delta MD / 2 [\cos I_1 + \cos I_2]</math></li> </ol>
Mercury	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \left[ \frac{\Delta MD - STL}{2} \right] [\sin I_1 \sin A_1 + \sin I_2 \sin A_2] + (STL) \sin I_2 \sin A_2</math></li> <li>2. <math>\Delta N = \left[ \frac{\Delta MD - STL}{2} \right] [\sin I_1 \cos A_1 + \sin I_2 \cos A_2] + (STL) \sin I_2 \cos A_2</math></li> <li>3. <math>\Delta V = \left[ \frac{\Delta MD - STL}{2} \right] [\cos I_1 + \cos I_2] + (STL) \cos I_2</math></li> </ol>
Angle Averaging	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \Delta MD \sin(I_{Avg}) \sin(A_{Avg})</math></li> <li>2. <math>\Delta N = \Delta MD \sin(I_{Avg}) \cos(A_{Avg})</math></li> <li>3. <math>\Delta V = \Delta MD \cos(I_{Avg})</math></li> </ol>
Radius of Curvature	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \frac{\Delta MD (\cos I_1 - \cos I_2) (\cos A_1 - \cos A_2)}{[(I_2 - I_1)(A_2 - A_1)]}</math></li> <li>2. <math>\Delta N = \frac{\Delta MD (\cos I_1 - \cos I_2) (\sin A_2 - \sin A_1)}{[(I_2 - I_1)(A_2 - A_1)]}</math></li> <li>3. <math>\Delta V = \frac{\Delta MD (\sin I_2 - \sin I_1)}{(I_2 - I_1)}</math></li> </ol>
Minimum Curvature	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \Delta MD / 2 [\sin I_1 \sin A_1 + \sin I_2 \sin A_2] RF</math></li> <li>2. <math>\Delta N = \Delta MD / 2 [\sin I_1 \cos A_1 + \sin I_2 \cos A_2] RF</math></li> <li>3. <math>\Delta V = \Delta MD / 2 [\cos I_1 + \cos I_2] RF</math></li> </ol> <p style="text-align: center;">Where</p> $RF = 2 / \beta_i \tan(\beta_i / 2) \text{ and}$ $\beta_i = \cos^{-1} [\cos(I_2 - I_1) - \sin I_1 \sin I_2 [1 - \cos(A_2 - A_1)]]$

**Table 3.3- Equations for Special cases for the Radius of Curvature method**

Special Case	Radius of Curvature method
<p style="text-align: center;"><i>Case 1</i>  , if <math>I_1 = I_2</math></p>	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \frac{\Delta MD \sin I_1 (\cos A_1 - \cos A_2)}{[A_2 - A_1]}</math></li> <li>2. <math>\Delta N = \frac{\Delta MD \sin I_1 (\sin A_2 - \sin A_1)}{[A_2 - A_1]}</math></li> <li>3. <math>\Delta V = \Delta MD \cos I_1</math></li> </ol>
<p style="text-align: center;"><i>Case 2</i>  if and <math>A_1 = A_1</math></p>	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \frac{\Delta MD \sin A_1 (\cos I_1 - \cos I_2)}{[I_2 - I_1]}</math></li> <li>2. <math>\Delta N = \frac{\Delta MD \cos A_1 (\cos I_1 - \cos I_2)}{[I_2 - I_1]}</math></li> <li>3. <math>\Delta V = \frac{\Delta MD (\sin I_2 - \sin I_1)}{[I_2 - I_1]}</math></li> </ol>
<p style="text-align: center;"><i>Case 3,</i>  if <math>I_1 = I_2</math> and <math>A_1 = A_1</math></p>	<ol style="list-style-type: none"> <li>1. <math>\Delta E = \Delta MD (\sin I_1 \sin A_1)</math></li> <li>2. <math>\Delta N = \Delta MD (\sin I_1 \cos A_1)</math></li> <li>3. <math>\Delta V = \Delta MD (\cos I_1)</math></li> </ol>



**Figure 3.18- A Summarized Flow Chart for the Excel Spreadsheet Program**

## **CHAPTER 4**

### **COMPUTATION, COMPARISON AND ANALYSIS OF RESULTS**

#### **4.1 Introduction**

The objectives of this project were to develop a user friendly Excel Spreadsheet program that would make the computations of well trajectory parameters easier, faster and accurate. Furthermore, to use the Excel Spreadsheet program to evaluate and compare results from the survey calculation methods the Tangential, Averaging Angle, Balanced Tangential, Mercury, Radius of Curvature and the Minimum Curvature methods available for the industry. In order to minimize risk and uncertainty surrounding hitting predetermined target, pictorial views of the trajectory, position of the drilling bit in the well would be generated for directional and correctional or adjustment purposes during the drilling operations.

Chapter three describes the excel program that was developed to do the computations of the wellpath trajectory coordinates. This chapter presents the results of the various computations using the six trajectory methods data available from Adams, (1985) and Bourgoyne et al, (1991). Comparison and analysis of the results generated by the excel spreadsheet program would be reviewed. Figure 4.1 shows the generalised interface of the excel spreadsheet data input section for all the six method developed in Chapter three.

Company	AmoBron		
Field	G21		
Well	7A		
Reservoir/Fluid Type	Light Oil		
Engineer	Amorin Richard		
Date	Monday, 31 August 2009		

Bit Walk Direction  
Select the word 'Right' for right bit-walk or 'Left' for left bit-walk

Lead Angle (Bit Walk)	0.00	Bit walk	Right
Magnetic Declination Adjustment Value	0.00	Magnetic Declination	East

Select (Right) for right bit-walk or (Left) for Left bit-walk

Select (West) for west declination or (East) for East Declination

Target Coordinates	Feet	Units of Measurement
Station ID	Chem1	
Northing	954.930	
Easting	0.000	Feet
Elevation	1653.987	
MD	2000.000	

Select (Feet) for units in feet or (Metre) for units in metres

Starting/Initial Coordinates	Feet	Units of Measurement
Station ID	Bron 7	
Northing	0.000	
Easting	0.000	Feet
Elevation	0.000	
Survey Tool Length (STL)	15.000	For Mercury Method

Select (Feet) for units in feet or (Metre) for units in metres

Station ID	Measured distance MD	Inclination $\theta^\circ$ (I)	Azimuth $\phi^\circ$ (A)
KOP	0.00	0.00	0
1	100.00	3.00	0
2	200.00	6.00	0
3	300.00	9.00	0
4	400.00	12.00	0
5	500.00	15.00	0
6	600.00	18.00	0
7	700.00	21.00	0
8	800.00	24.00	0
9	900.00	27.00	0
10	1000.00	30.00	0
11	1100.00	33.00	0
12	1200.00	36.00	0
13	1300.00	39.00	0
14	1400.00	42.00	0
15	1500.00	45.00	0
16	1600.00	48.00	0
17	1700.00	51.00	0
18	1800.00	54.00	0
19	1900.00	57.00	0
20	2000.00	60.00	0

Enter only Whole Circle Reading bearings Values

Figure 4.1- A General Interface of the Excel Spreadsheet Data Input Section.

#### 4.2 Validation of the Excel Spreadsheet Program

Two literature data were used in validating the excel spreadsheet program. The first was the data used by Adams, (1985) and the second was from Bourgoyne et al, (1991). Adams, (1985) used the survey data given in Table 4.1 to compute the wellbore trajectory using Tangential, Angle Averaging and Radius of Curvature methods only and came out with the results in Table 4.2. Adams, (1985) used an Adams and Rountree computer program in his trajectory computations.

**Table 4.1- Survey Data Obtained from Adams, (1985)**

Measured Depth, (ft)	Hole Angle (°)	Azimuth (°)
3,000	2	N28E
3,300	4	N10E
3,600	8	N35E
3,900	12	N25E
5,000	15	N30E
6,000	16	N28E
7,000	17	N50E
8,000	17	N20E
9,000	17	N30E
10,000	17	N25E

**Table 4.2- Wellbore Trajectory Results from Adams, (1985)\***

Measured Depth	<u>Tangential Method</u>			<u>Average Method</u>			<u>Radius of Curvature</u>		
	TVD	North/ -South	East/ -West	TVD	North/ -South	East/ -West	TVD	North/ -South	East/ -West
3,000.0	3,000.0	0.0	0.0	3,000.0	0.0	0.0	3,000.0	0.0	0.0
3,300.0	3,299.0	20.6	3.6	3,299.0	14.8	5.1	3,299.0	14.8	5.1
3,600.0	3,596.0	54.8	27.6	3,597.0	43.8	17.1	3,597.0	43.5	17.0
3,900.0	3,889.0	113.3	53.9	3,893.0	88.9	43.2	3,893.0	88.6	43.0
5,000.0	4,952.0	357.9	196.3	4,963.0	316.7	161.7	4,962.0	316.2	161.5
6,000.0	5,913.0	601.3	325.7	5,926.0	550.4	291.3	5,926.0	550.0	291.1
7,000.0	6,869.0	789.2	549.7	6,885.0	771.2	476.0	6,885.0	769.3	468.7
8,000.0	7,826.0	1,063.9	649.7	7,841.0	1,010.7	673.7	7,841.0	1,006.1	634.5
9,000.0	8,782.0	1,317.1	795.8	8,798.0	1,275.6	761.3	8,797.0	1,270.7	757.9
10,000.0	9,738.0	1,582.1	919.4	9,754.0	1,535.0	896.3	9,754.0	1,530.0	892.9

\*All measurements in feet and computed with the Adams and Rountree computer system

Using the same data from table 4.1 the following trajectory results shown in table 4.3 were obtained using the Excel Spreadsheet Program

**Table 4.3- Wellbore Trajectory Results from the Excel Spreadsheet Program employing the Six Trajectory Methods**

<b>Tangential</b>			<b>Angle Averaging</b>			<b>Balanced Tangential</b>			<b>Mercury</b>			<b>Radius of Curvature</b>			<b>Minimum Curvature</b>		
<b>N</b>	<b>E</b>	<b>Z (TVD)</b>	<b>N</b>	<b>E</b>	<b>Z (TVD)</b>	<b>N</b>	<b>E</b>	<b>Z (TVD)</b>	<b>N</b>	<b>E</b>	<b>Z (TVD)</b>	<b>N</b>	<b>E</b>	<b>Z (TVD)</b>	<b>N</b>	<b>E</b>	<b>Z (TVD)</b>
<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>	<b>Feet</b>
0.00	0.00	3000.00	0.00	0.00	3000.00	0.00	0.00	3000.00	0.00	0.00	3000.00	0.00	0.00	3000.00	0.00	0.00	3000.00
20.61	3.63	3299.27	15.46	2.73	3299.59	14.93	4.27	3299.54	14.93	4.27	3299.54	14.78	5.09	3299.57	14.93	4.28	3299.58
54.81	27.58	3596.35	44.43	14.73	3597.95	42.33	18.07	3597.72	42.33	18.07	3597.72	43.52	16.99	3597.87	42.35	18.07	3597.92
111.34	53.94	3889.79	89.55	40.77	3893.39	87.70	43.22	3892.98	87.70	43.22	3892.98	88.57	43.00	3893.25	87.74	43.24	3893.32
357.90	196.29	4952.31	317.32	159.35	4962.99	314.61	162.72	4962.22	314.61	162.72	4962.22	316.25	161.52	4962.74	314.71	162.77	4962.85
601.27	325.70	5913.57	551.06	288.91	5926.63	548.37	292.13	5925.81	548.37	292.13	5925.81	549.96	291.08	5926.35	548.48	292.18	5926.47
789.20	549.67	6869.88	771.78	467.64	6885.44	764.03	468.82	6884.60	764.03	468.82	6884.60	769.33	468.71	6885.16	764.35	469.05	6886.22
1063.94	649.66	7826.18	1011.27	635.34	7841.75	995.36	630.80	7840.90	995.36	630.80	7840.90	1006.10	634.50	7841.47	996.13	631.34	7844.36
1317.15	795.85	8782.49	1276.25	758.90	8798.05	1259.33	753.89	8797.21	1259.33	753.89	8797.21	1270.74	757.91	8797.77	1260.16	754.46	8800.87
<b>1582.12</b>	<b>919.41</b>	<b>9738.79</b>	<b>1535.59</b>	<b>893.90</b>	<b>9754.36</b>	<b>1518.42</b>	<b>888.76</b>	<b>9753.51</b>	<b>1518.42</b>	<b>888.76</b>	<b>9753.51</b>	<b>1530.00</b>	<b>892.87</b>	<b>9754.08</b>	<b>1519.26</b>	<b>889.34</b>	<b>9757.22</b>

**Table 4.4- A Comparative Wellbore Trajectory Results Summary between the Adams, (1985) and the Excel Spreadsheet Program**

	<u>Tangential Method</u>				<u>Average Method</u>				<u>Radius of Curvature</u>	
	Measured		North/	East/		North/	East/		North/	East/
	Depth	TVD	-South	-West	TVD	-South	-West	TVD	-South	-West
Adams (1985)	10,000.00	9,738.00	1,582.10	919.4	9,754.00	1,535.00	896.3	9,754.00	1,530.00	892.9
Excel	10,000.00	9,738.79	1,582.12	919.41	9,754.36	1,535.59	893.9	9,754.08	1,530.00	892.87
Difference		-0.79	-0.02	-0.01	-0.36	-0.59	2.4	-0.08	0	0.03



Figures 4.2 to 4.9 show the vertical and horizontal plots of the combined Excel Spreadsheet Program and Adams results.

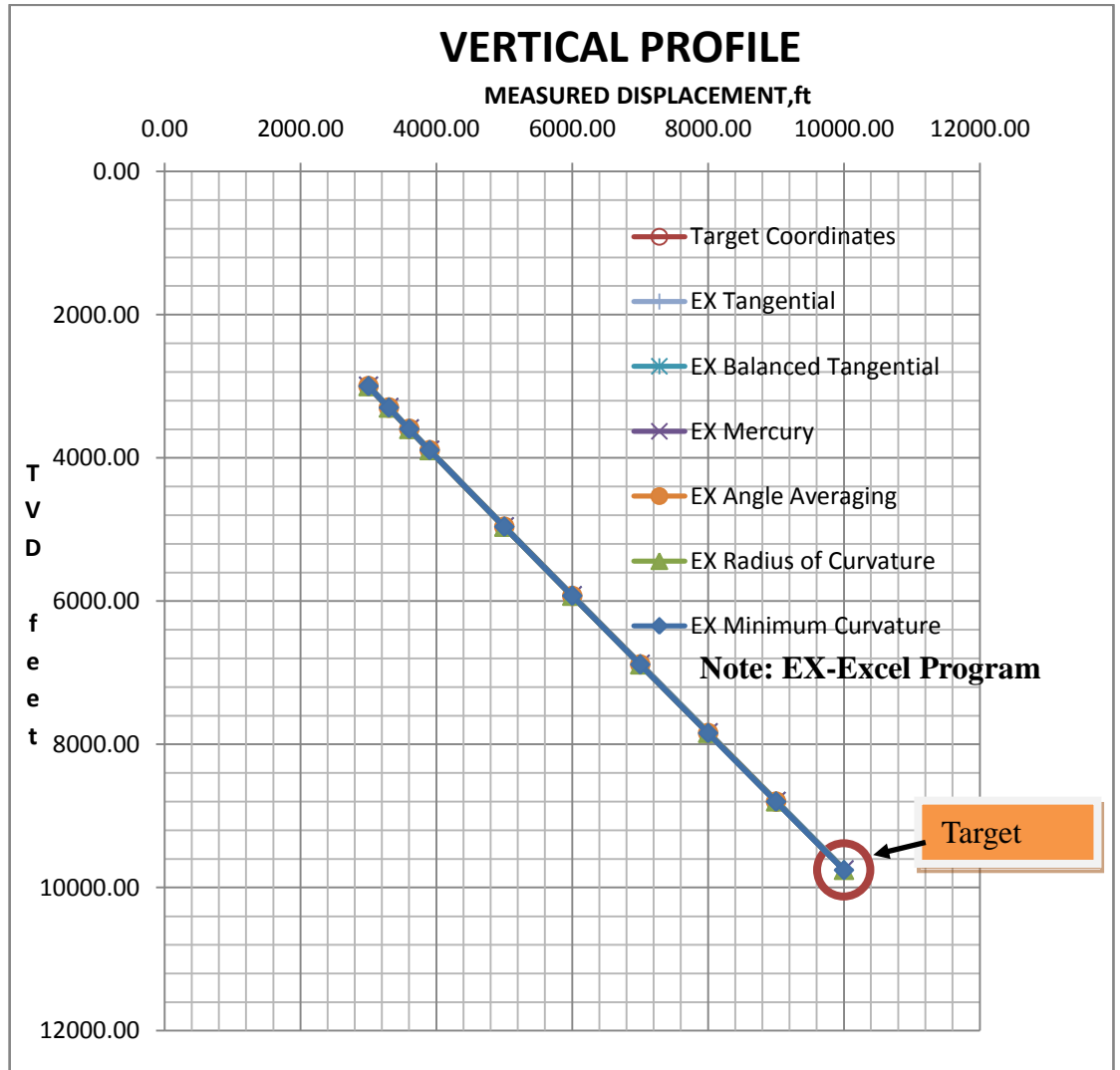
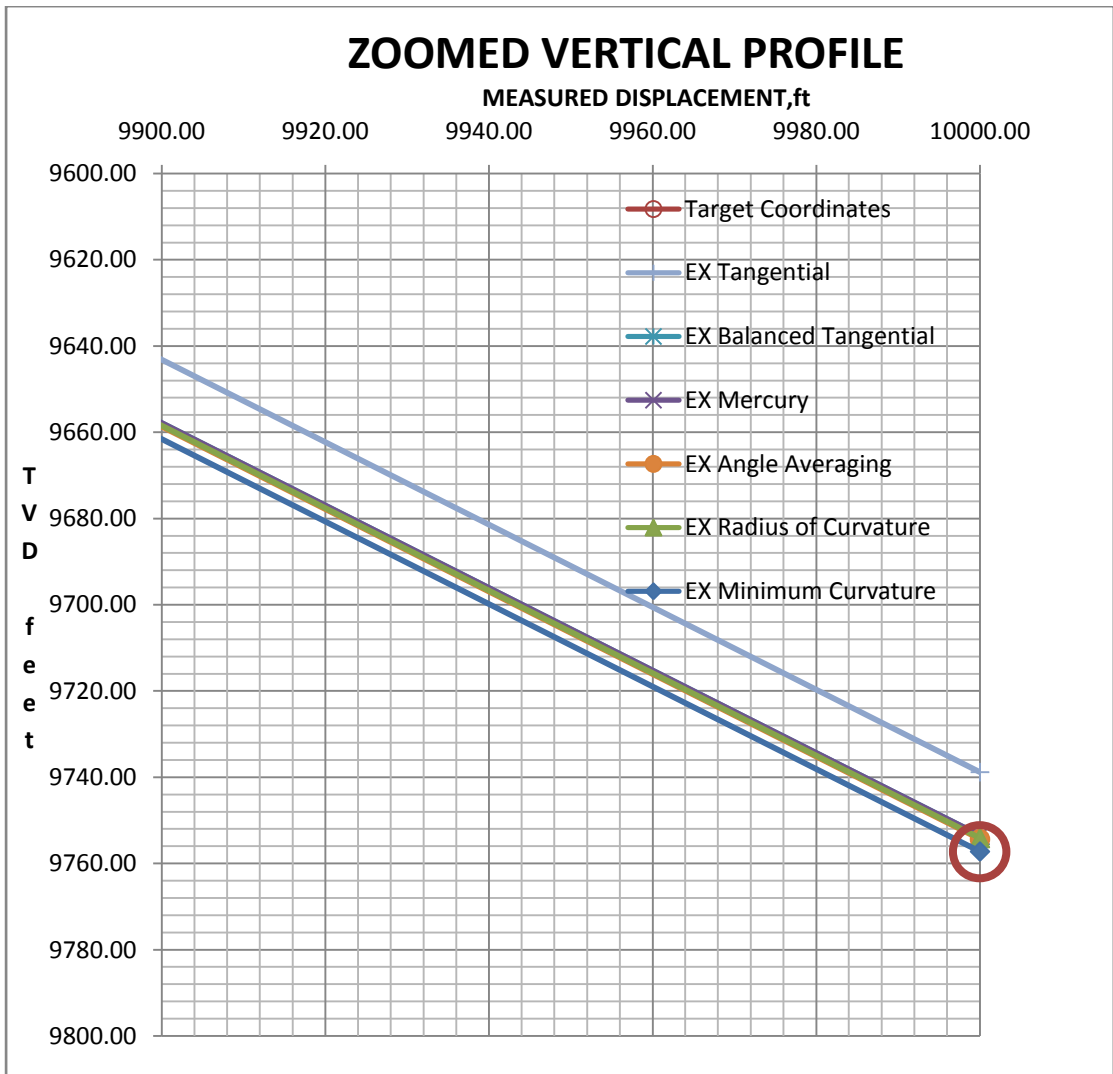
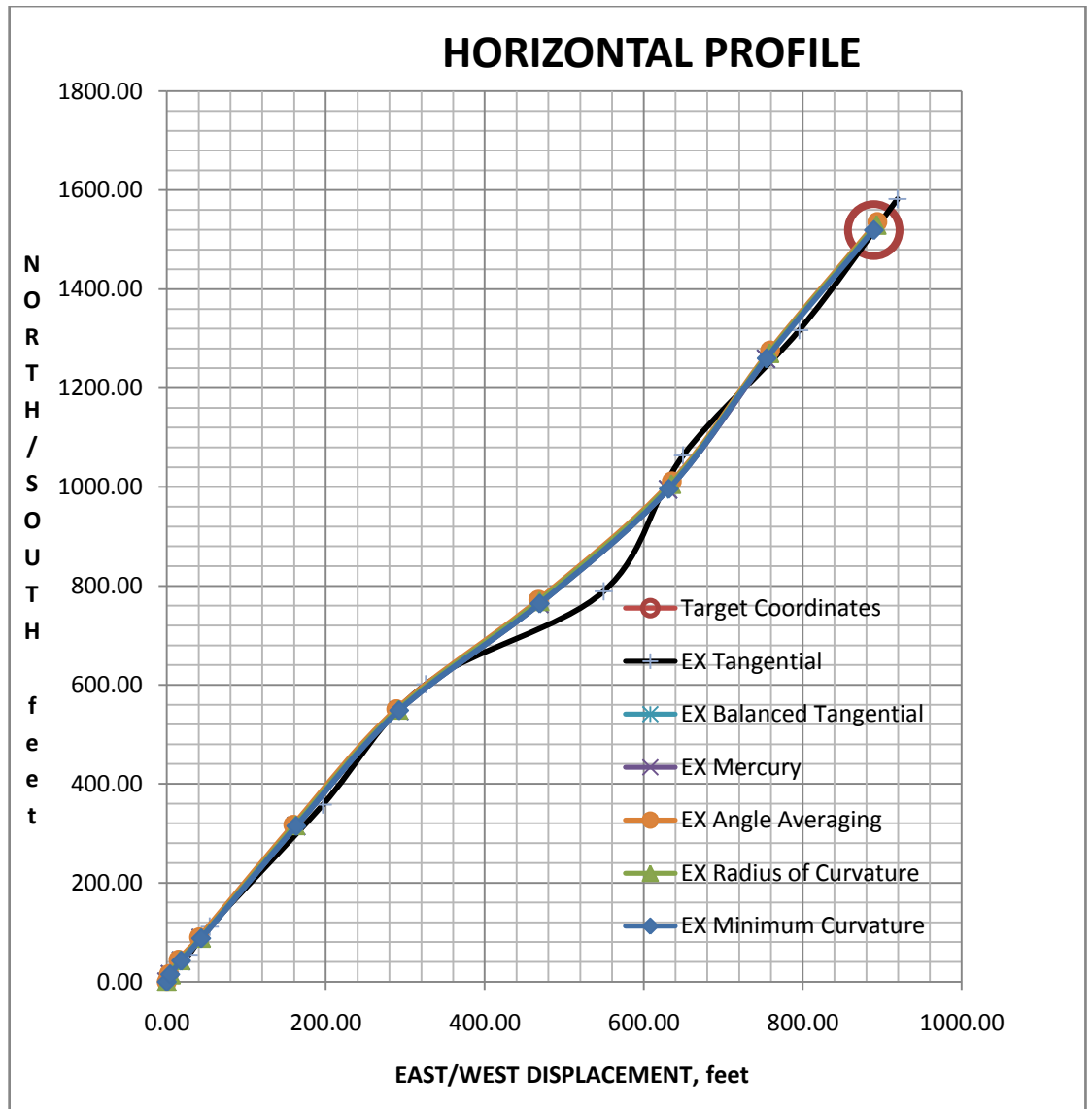


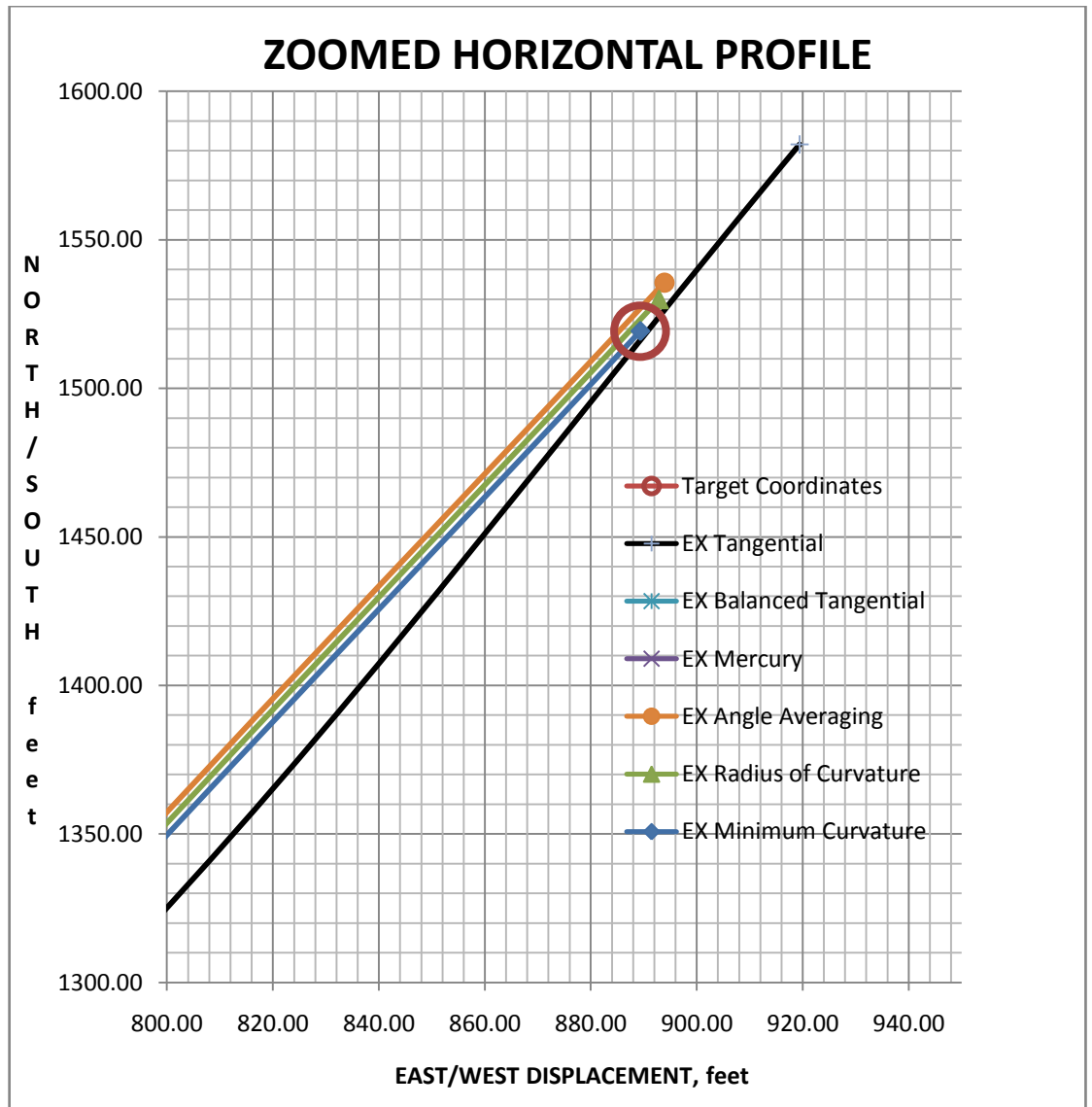
Figure 4.2 – A vertical plot of the Excel Spreadsheet Program results using Adams data



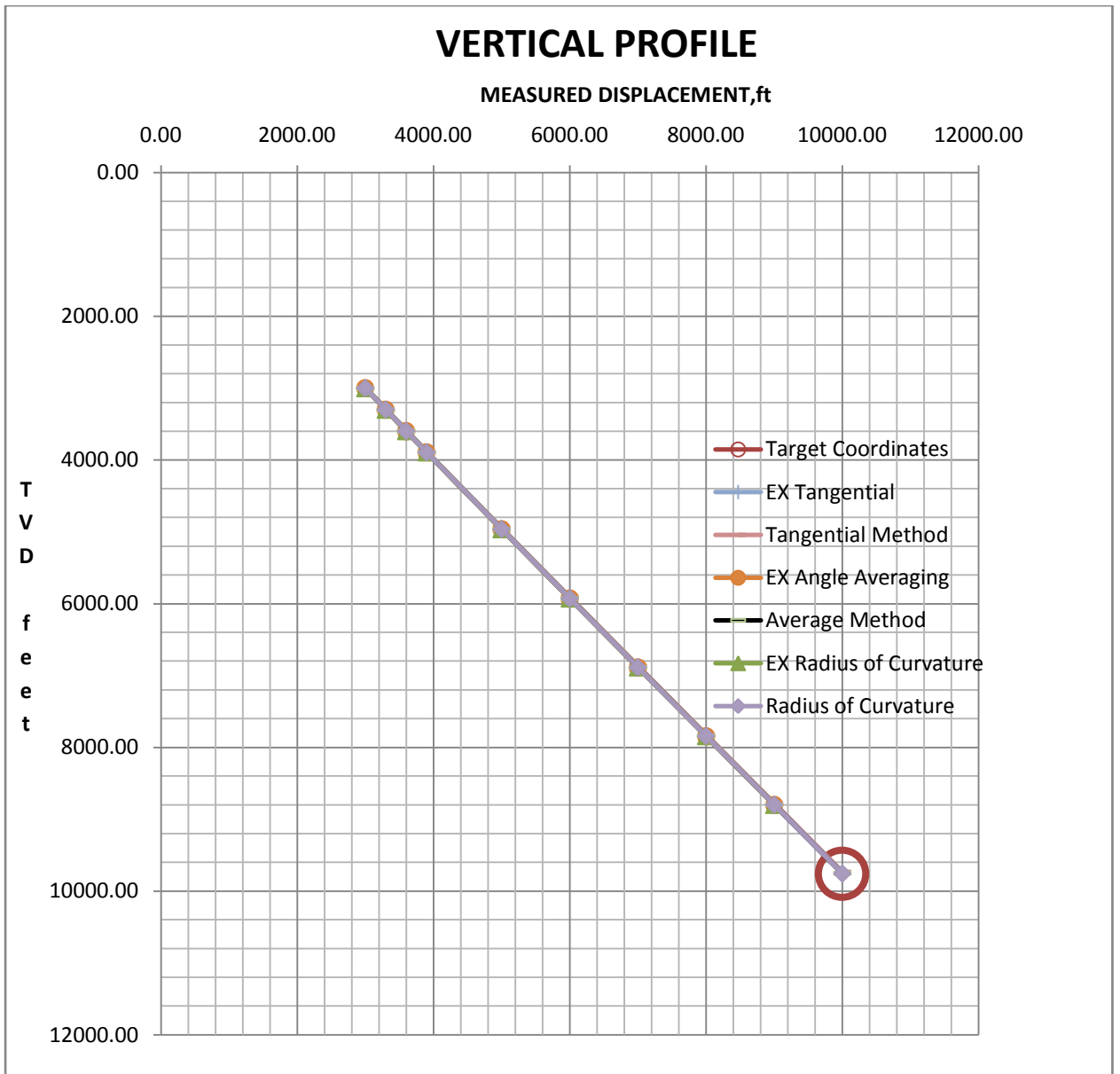
**Figure 4.3 – A zoomed vertical plot of the Excel Spreadsheet Program results using Adams data**



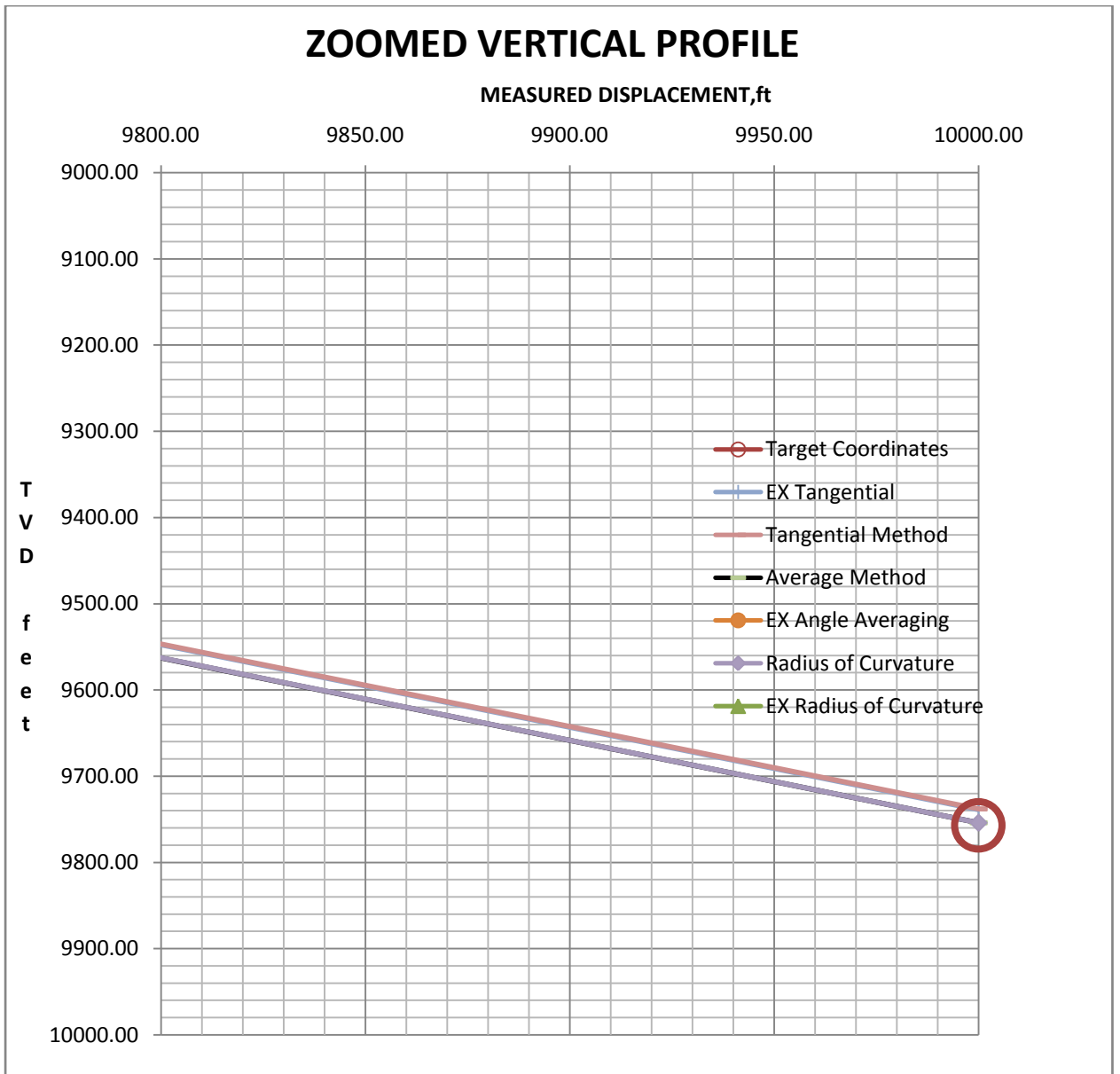
**Figure 4.4 – A horizontal plot of the Excel Spreadsheet Program results using Adams data**



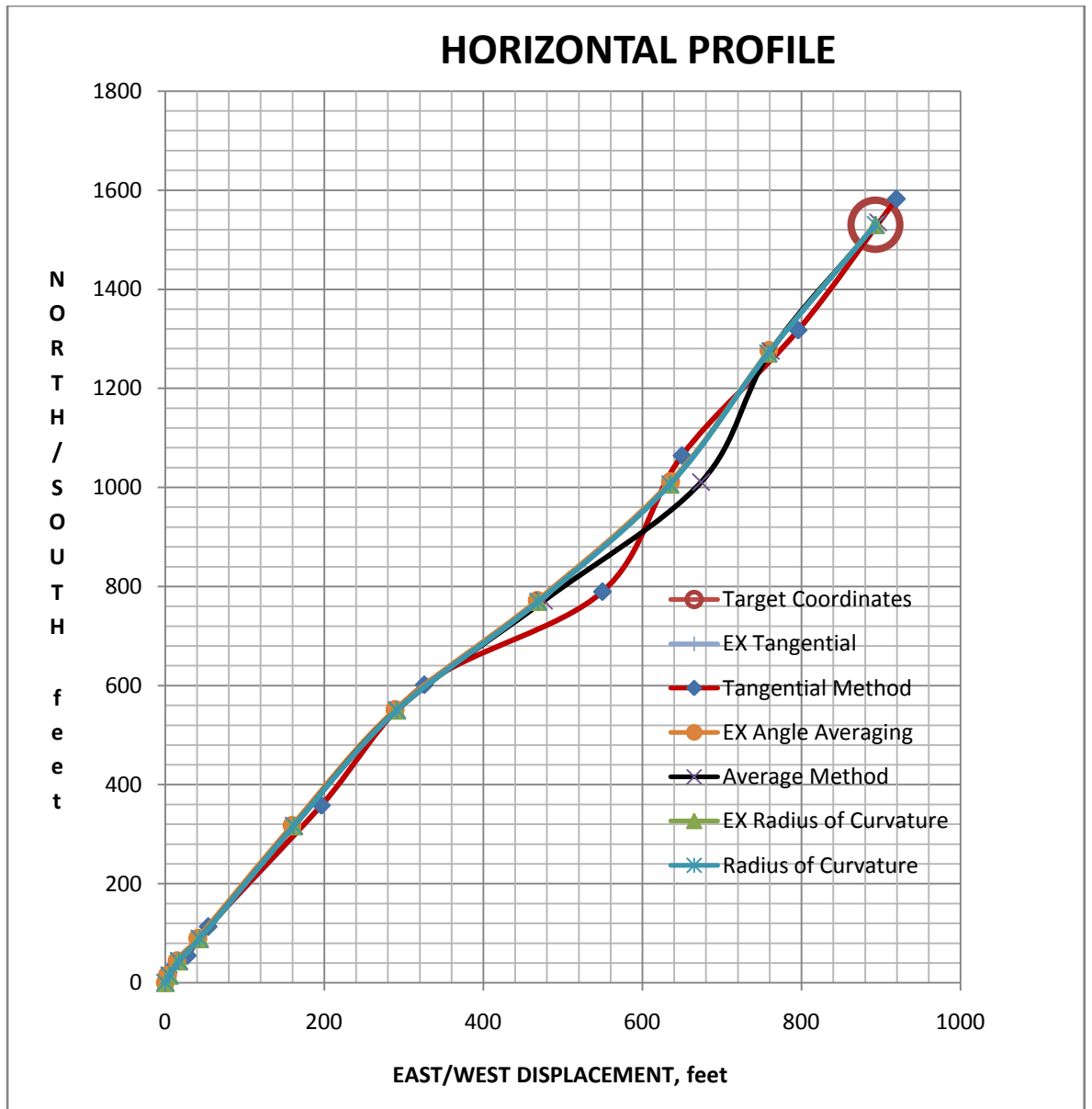
**Figure 4.5 – A zoomed horizontal plot of the Excel Spreadsheet Program results using Adams data**



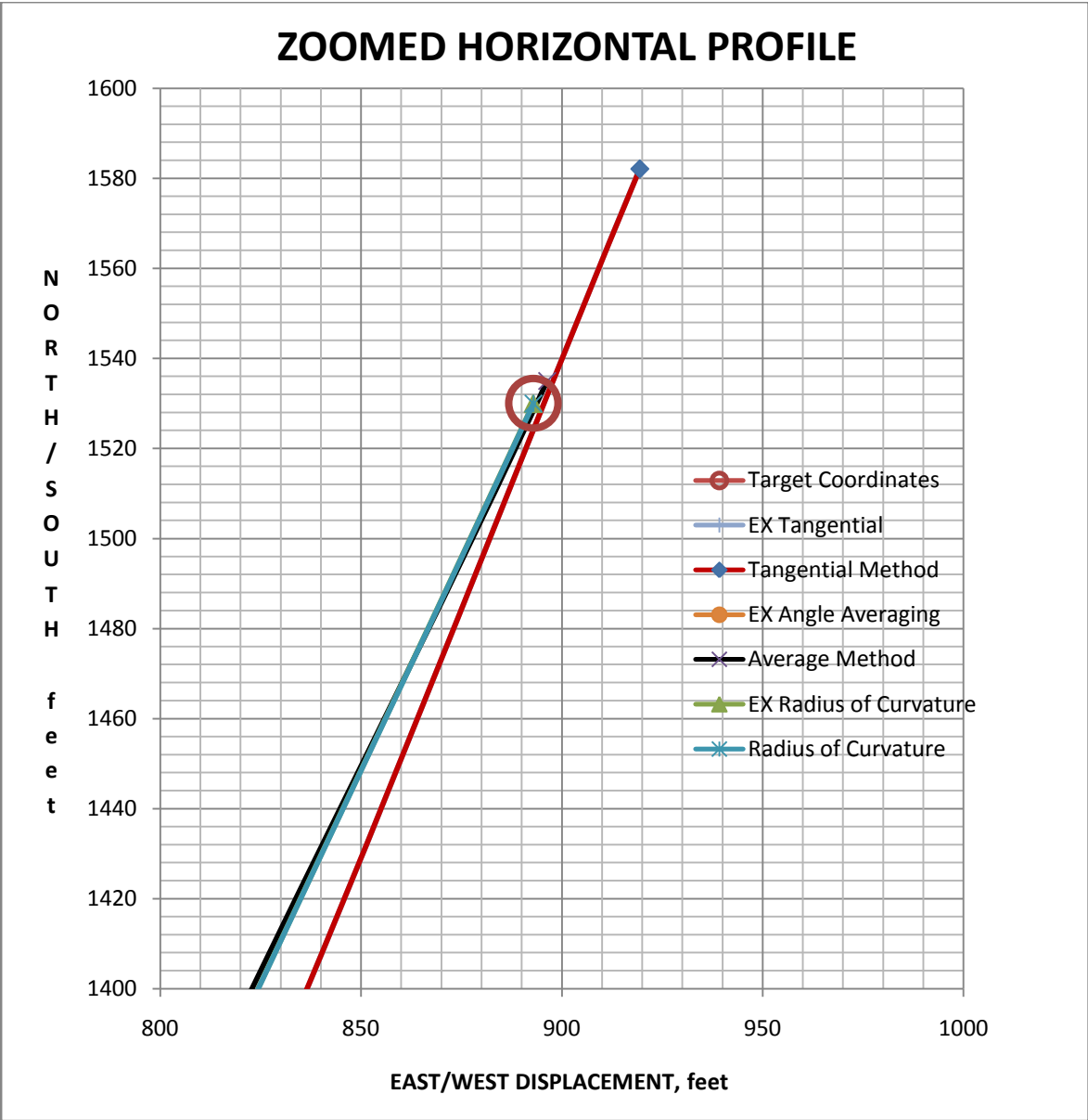
**Figure 4.6 – A vertical plot of both Adams and Excel Spreadsheet Program results**



**Figure 4.7 – A zoomed vertical plot of both Adams and Excel Spreadsheet Program results**



**Figure 4.8 – A horizontal plot of both Adams and Excel Spreadsheet Program results**



**Figure 4.9 – A zoomed horizontal plot of both Adams and Excel Spreadsheet Program results**



Table 4.5 gives a summary of the results obtained using the excel spreadsheet program using the Adams, (1985) data. The Target coordinates was assumed to be from the Minimum Curvature method computations with; Northing = 1519.26ft, Easting = 889.34ft, TVD = 9757.22ft, MD = 10,000.00ft. The vertical and horizontal plots of each individual plots of Table 4.3 are shown in appendix B.

**Table 4.5- A Comparative Wellbore Trajectory Results Summary from the Excel Spreadsheet Program using the Adams, (1985) Data**

<b>Trajectory Methods</b>	<b>True Vertical Depth</b>	<b>Difference from Actual (ft)</b>	<b>Total Displacement</b>	<b>Difference from Actual (ft)</b>
<b>Tangential</b>	1582.12	62.86	1829.87	69.45
<b>Balanced Tangential</b>	1518.42	-0.84	1759.41	-1.01
<b>Angle-Averaging</b>	1,535.59	16.33	1776.82	16.40
<b>Radius of Curvature</b>	1530.00	10.74	1771.47	11.05
<b>Minimum Curvature</b>	1519.26	0.00	1760.42	0.00
<b>Mercury</b>	1518.42	-0.84	1759.41	-1.01

**4.3 Summary of Observations and Analysis of Results using the Adams, (1985) Data.**

The following observations and analysis can be drawn from the Tables 4.3, 4.4 and 4.5.

- The results obtained from the Excel Spreadsheet are close to the results obtained from the Adams, (1985) computations.
- The slight difference in results might be due to the difference in approximation (number of decimal places used, round –off).
- The tangential method shows considerable error for the North, East and Elevation followed by the radius of curvature method.
- The considerable error in the radius of curvature method might be due to portion of the wellbore data being much closed to each other. This may cause subtractions in the equation creating either dividing by zero errors or an incorrect TVD when the borehole is a straight line but deviated.

Secondly, Bourgoyne et al, (1991) survey data and summary of final computed results given in Table 4.6, was used for further validation of the excel spreadsheet program. Bourgoyne et al, (1991) considered all the six wellbore trajectory methods in his computations.

**Table 4.6- Survey Data and Summary of Computed Wellbore Trajectory Results Obtained from Bourgoyne et al, (1991).**

Direction:	Due North			
Survey Interval:	100 ft			
Rate of build:	3°/100 ft			
Total Inclination:	60° at 2,000 ft			
	Total Vertical Depth Difference From		North Displacement Difference From	
<b><u>Calculation Method</u></b>	<b><u>Actual (ft)</u></b>		<b><u>Actual (ft)</u></b>	
Tangential	1,628.61	-25.4	998.02	43.09
Balanced Tangential	1,653.61	-0.38	954.72	-0.21
Angle Averaging	1,654.18	0.19	955.04	0.11
Radius of Curvature	1,653.99	0	954.93	0
Minimum Curvature	1,653.99	0	954.93	0
Mercury*	1,153.62	-0.37	954.89	0.04

\*A fifteen foot survey tool was used for the computation of the Mercury Method.

Using the same data from Table 4.6 the following trajectory results shown in Table 4.7 were obtained using the Excel Spreadsheet Program employing the six methods. The target coordinates were assumed to be from the Minimum Curvature method computations with; North = 954.93, East 0.00, TVD = 1653.99, MD = 2000.00 all measurement were in feet.

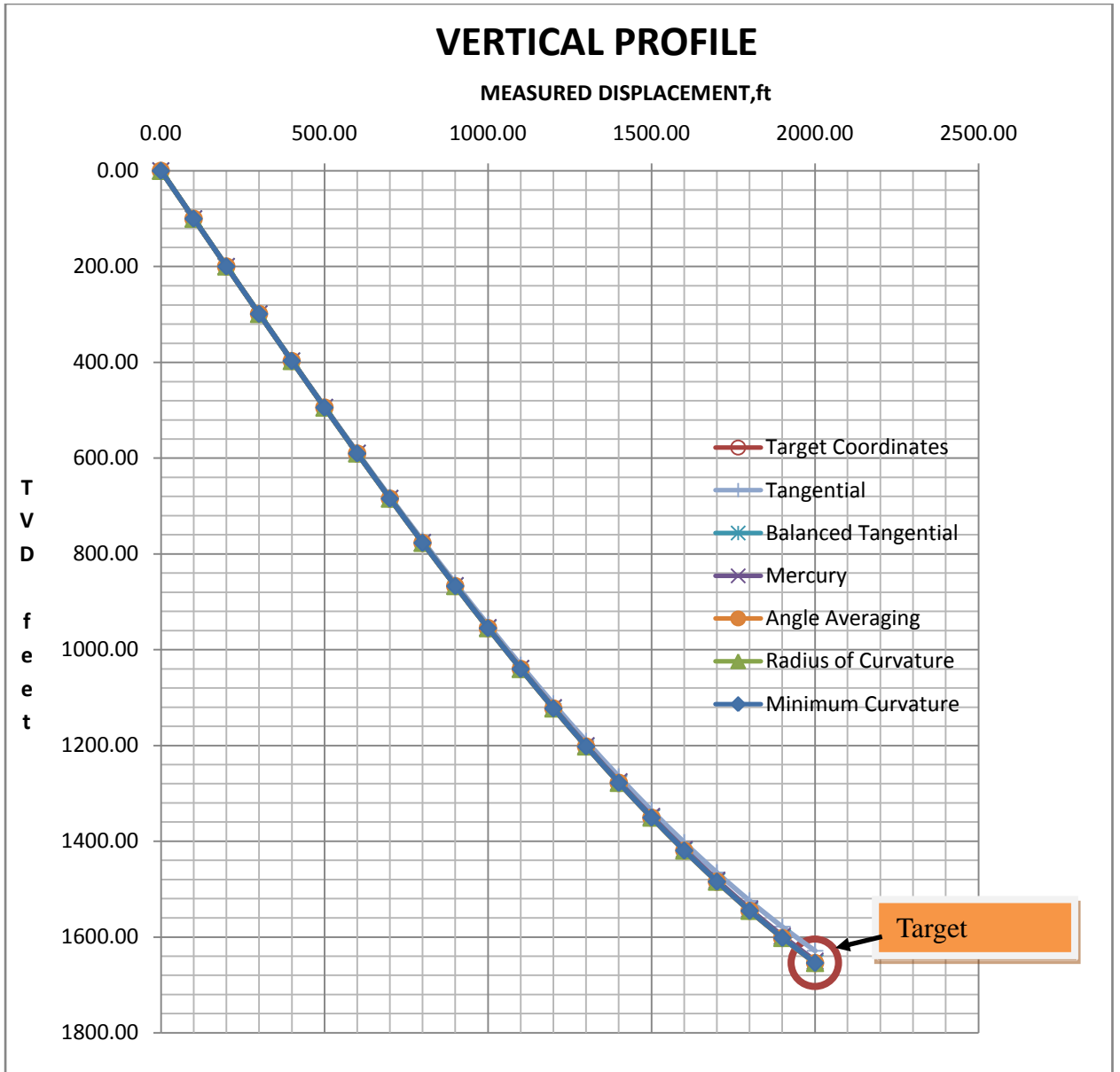
**Table 4.7- Wellbore Trajectory Results Obtained from the Excel Spreadsheet Program using Bourgoyne et al, (1991) Data**

Tangential			Angle Averaging			Balanced Tangential			Mercury			Radius of Curvature			Minimum Curvature		
N	E	Z (TVD)	N	E	Z (TVD)	N	E	Z (TVD)	N	E	Z (TVD)	N	E	Z (TVD)	N	E	Z (TVD)
Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5.234	0.000	99.863	2.618	0.000	99.966	2.617	0.000	99.931	3.009	0.000	99.921	2.617	0.000	99.954	2.617	0.000	99.954
15.686	0.000	199.315	10.464	0.000	199.657	10.460	0.000	199.589	11.244	0.000	199.548	10.462	0.000	199.635	10.462	0.000	199.635
31.330	0.000	298.084	23.516	0.000	298.802	23.508	0.000	298.700	24.681	0.000	298.607	23.514	0.000	298.768	23.514	0.000	298.768
52.121	0.000	395.899	41.740	0.000	397.127	41.725	0.000	396.991	43.285	0.000	396.827	41.735	0.000	397.082	41.735	0.000	397.082
78.003	0.000	492.491	65.084	0.000	494.364	65.062	0.000	494.195	67.003	0.000	493.939	65.077	0.000	494.308	65.077	0.000	494.308
108.905	0.000	587.597	93.486	0.000	590.246	93.454	0.000	590.044	95.771	0.000	589.677	93.475	0.000	590.179	93.475	0.000	590.179
144.741	0.000	680.955	126.867	0.000	684.511	126.823	0.000	684.276	129.511	0.000	683.778	126.852	0.000	684.432	126.852	0.000	684.432
185.415	0.000	772.310	165.135	0.000	776.899	165.078	0.000	776.632	168.129	0.000	775.984	165.116	0.000	776.810	165.116	0.000	776.810
230.814	0.000	861.410	208.186	0.000	867.157	208.115	0.000	866.860	211.520	0.000	866.042	208.162	0.000	867.058	208.162	0.000	867.058
280.814	0.000	948.013	255.902	0.000	955.039	255.814	0.000	954.711	259.564	0.000	953.707	255.873	0.000	954.930	255.873	0.000	954.930
335.278	0.000	1031.880	308.152	0.000	1040.303	308.046	0.000	1039.946	312.131	0.000	1038.736	308.117	0.000	1040.184	308.117	0.000	1040.184
394.057	0.000	1112.782	364.792	0.000	1122.715	364.667	0.000	1122.331	369.076	0.000	1120.898	364.751	0.000	1122.587	364.751	0.000	1122.587
456.989	0.000	1190.496	425.668	0.000	1202.051	425.523	0.000	1201.639	430.243	0.000	1199.967	425.620	0.000	1201.913	425.620	0.000	1201.913
523.902	0.000	1264.811	490.613	0.000	1278.091	490.445	0.000	1277.653	495.464	0.000	1275.727	490.557	0.000	1277.945	490.557	0.000	1277.945
594.612	0.000	1335.521	559.449	0.000	1350.629	559.257	0.000	1350.166	564.560	0.000	1347.969	559.385	0.000	1350.474	559.385	0.000	1350.474
668.927	0.000	1402.434	631.986	0.000	1419.464	631.770	0.000	1418.978	637.343	0.000	1416.496	631.914	0.000	1419.302	631.914	0.000	1419.302
746.641	0.000	1465.366	708.027	0.000	1484.409	707.784	0.000	1483.900	713.613	0.000	1481.120	707.946	0.000	1484.239	707.946	0.000	1484.239
827.543	0.000	1524.145	787.362	0.000	1545.285	787.092	0.000	1544.756	793.160	0.000	1541.664	787.272	0.000	1545.109	787.272	0.000	1545.109
911.410	0.000	1578.609	869.775	0.000	1601.926	869.477	0.000	1601.377	875.767	0.000	1597.962	869.675	0.000	1601.743	869.675	0.000	1601.743
<b>998.013</b>	<b>0.000</b>	<b>1628.609</b>	<b>955.039</b>	<b>0.000</b>	<b>1654.176</b>	<b>954.711</b>	<b>0.000</b>	<b>1653.609</b>	<b>961.207</b>	<b>0.000</b>	<b>1649.859</b>	<b>954.930</b>	<b>0.000</b>	<b>1653.987</b>	<b>954.930</b>	<b>0.000</b>	<b>1653.987</b>

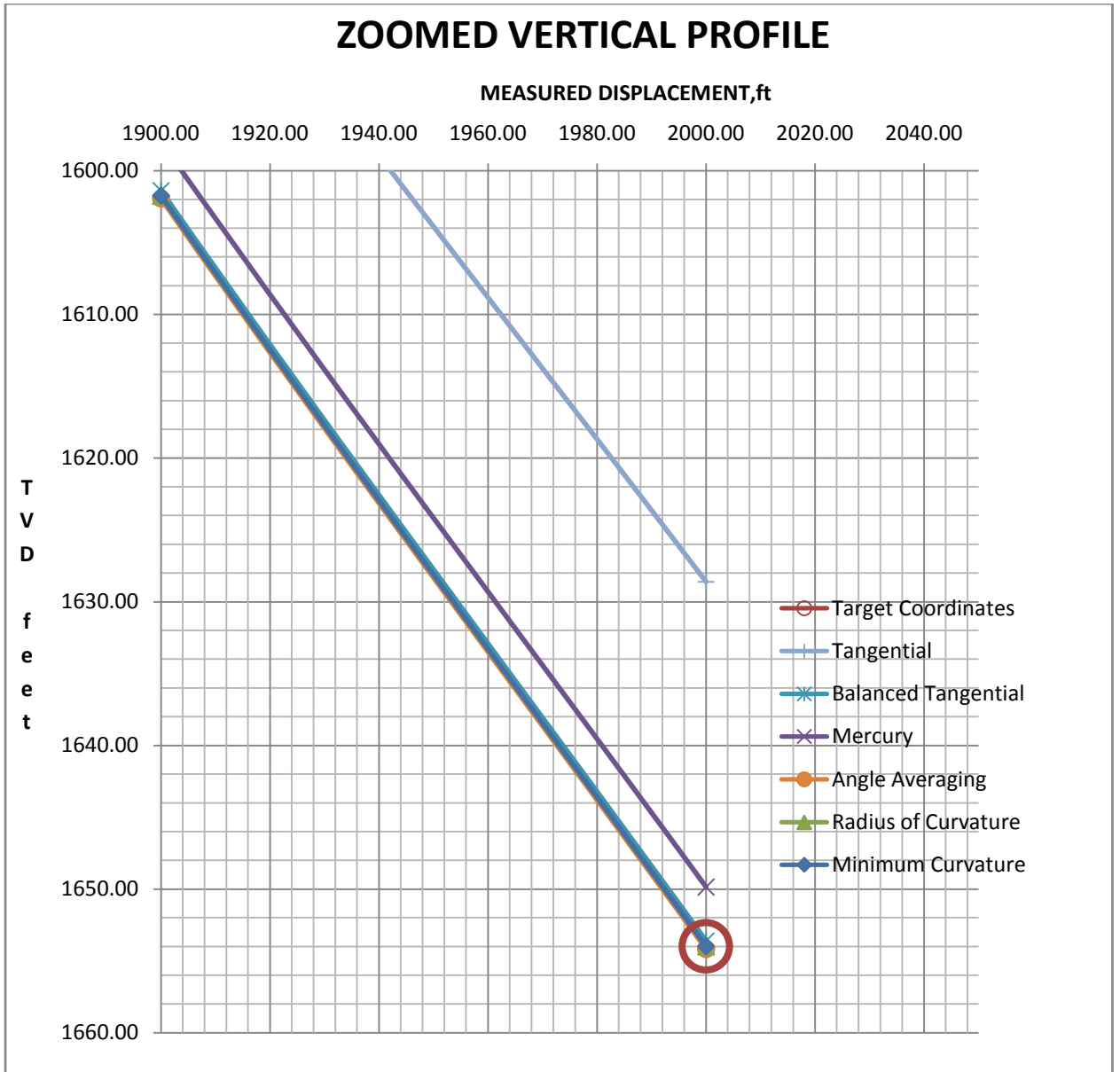
**Table 4.8- A Comparative Wellbore Trajectory Results Summary between the Bourgoyne et al, (1991) and the Excel Spreadsheet Program**

Trajectory Methods	True Vertical Depth (TVD) (ft)		Difference from Actual (TVD) (ft)		North Displacement (ND) (ft)		Difference from Actual (ND) (ft)	
	Bourgoyne	Excel	Bourgoyne	Excel	Bourgoyne	Excel	Bourgoyne	Excel
<b>Tangential</b>	1,628.61	1628.61	-25.38	-25.38	998.02	998.01	43.09	43.08
<b>Balanced Tangential</b>	1,653.61	1653.61	-0.38	-0.38	954.72	954.71	-0.21	-0.22
<b>Angle-Averaging</b>	1,654.18	1654.18	0.19	0.19	955.04	955.04	0.11	0.11
<b>Radius of Curvature</b>	1,653.99	1653.99	0.00	0.00	954.93	954.93	0	0.00
<b>Minimum Curvature</b>	1,653.99	1653.99	0.00	0.00	954.93	954.93	0	0.00
<b>Mercury</b>	1.153.62	1649.86	-0.37	-4.13	954.89	961.21	0.04	6.28

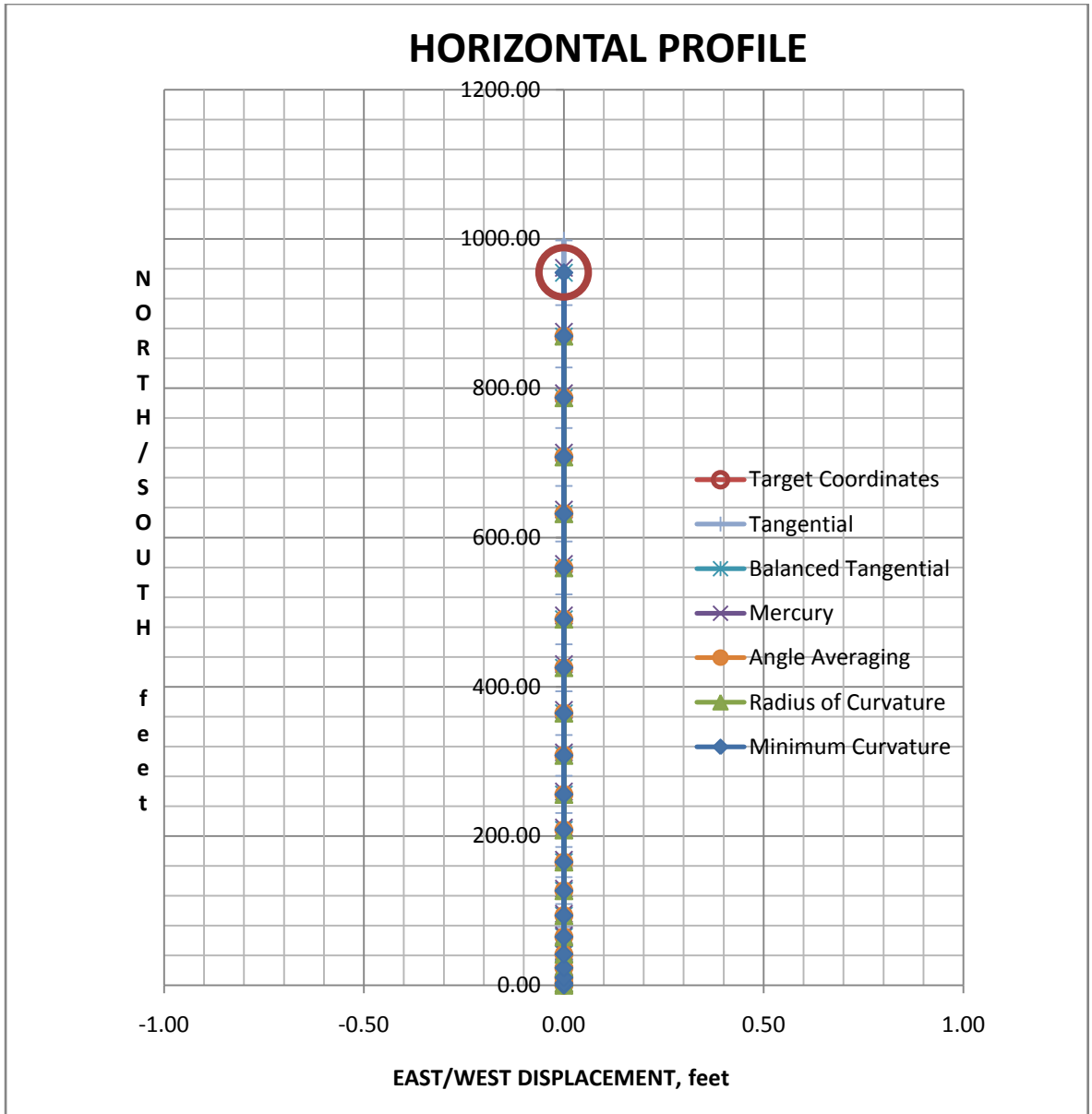
The vertical and horizontal plots of Table 4.7 are shown in Figures 4.10 to 4.19b.



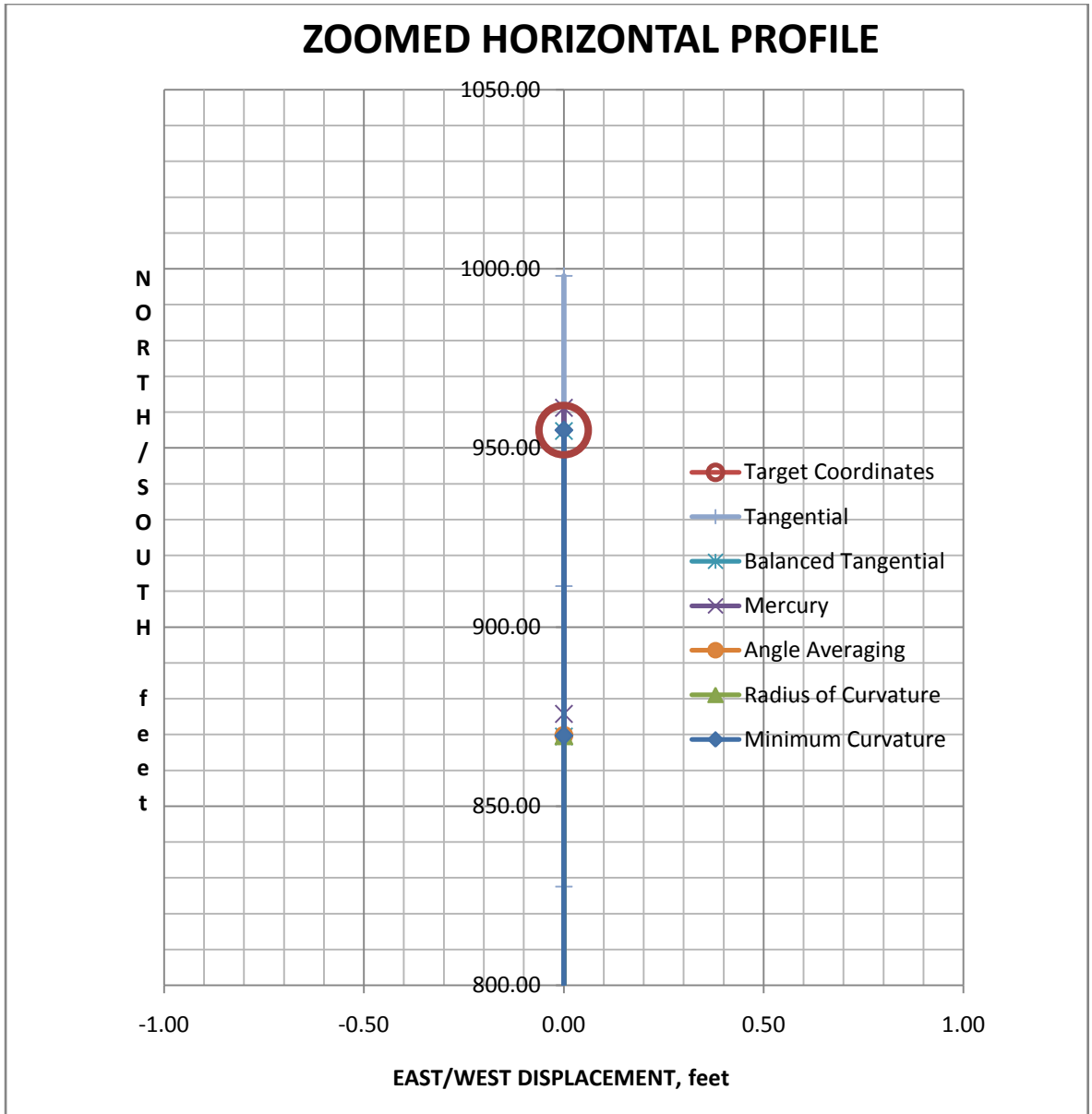
**Figure 4.10 – A vertical plot of the Excel Spreadsheet Program results using Bourgoyne data**



**Figure 4.11 – A zoomed vertical plot of the Excel Spreadsheet Program results using Bourgoyne data**

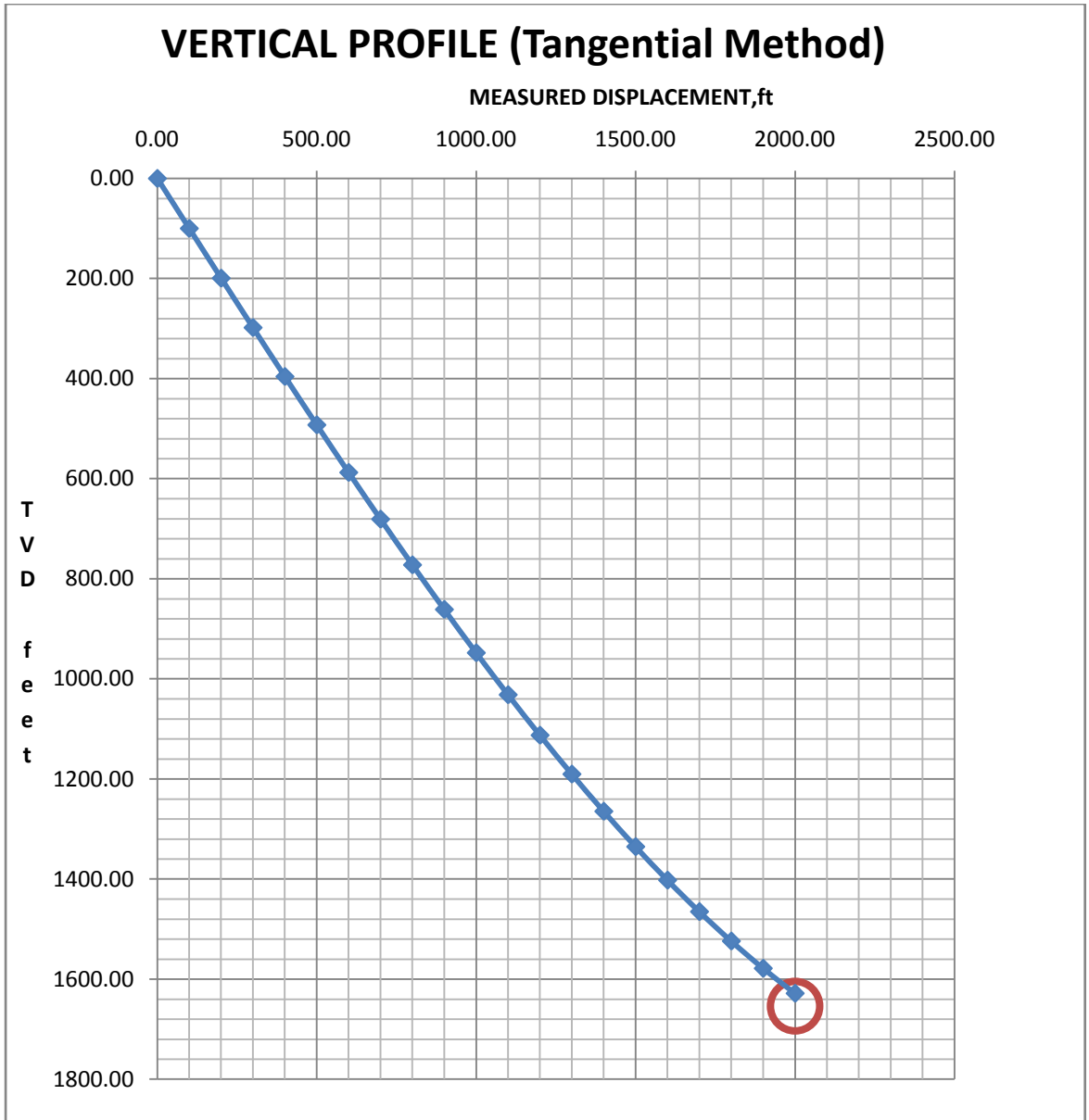


**Figure 4.12 – A horizontal plot of the Excel Spreadsheet Program results using Bourgoyne data**

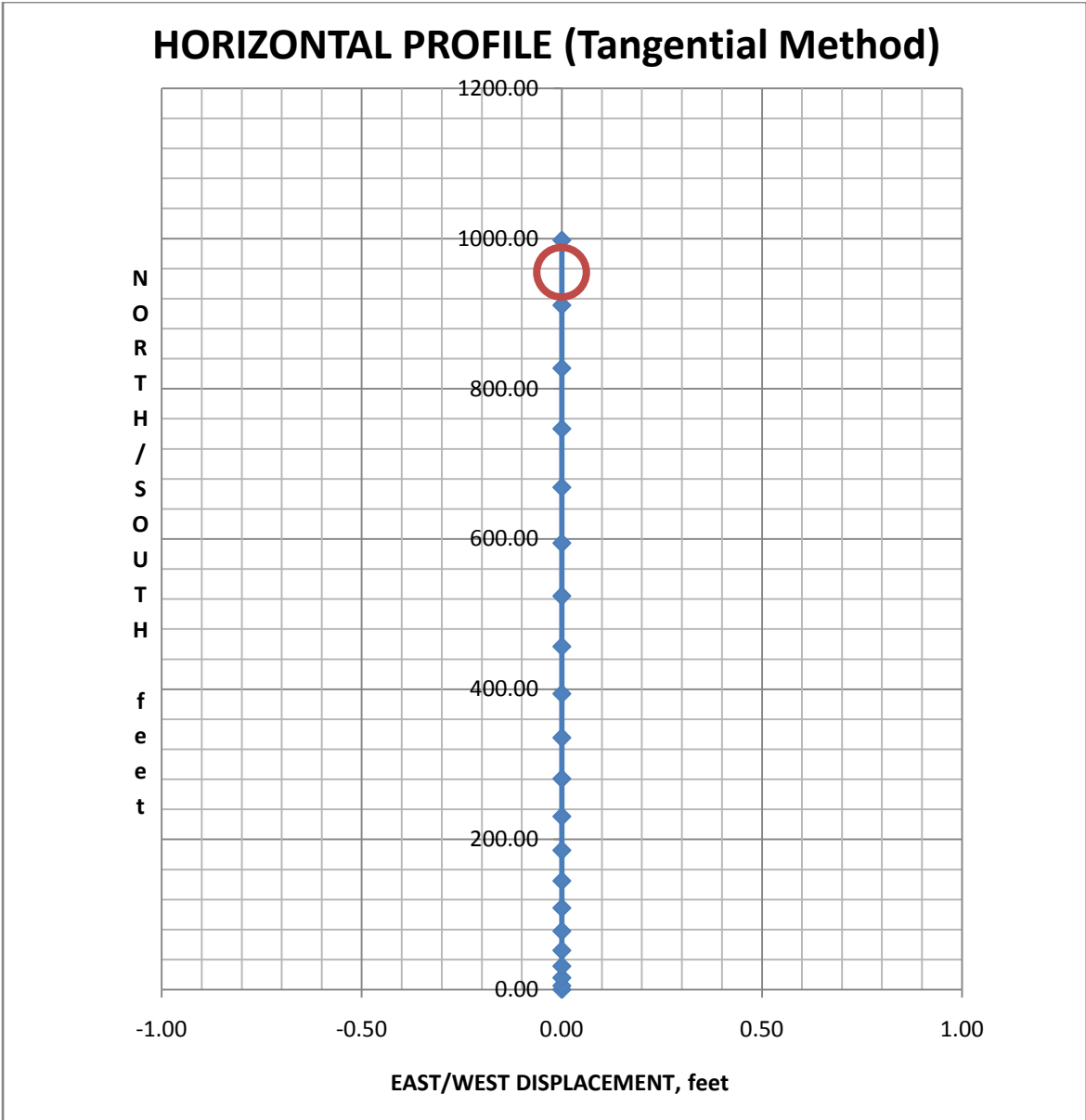


**Figure 4.13 – A zoomed horizontal plot of the Excel Spreadsheet Program results using Bourgoyne data**

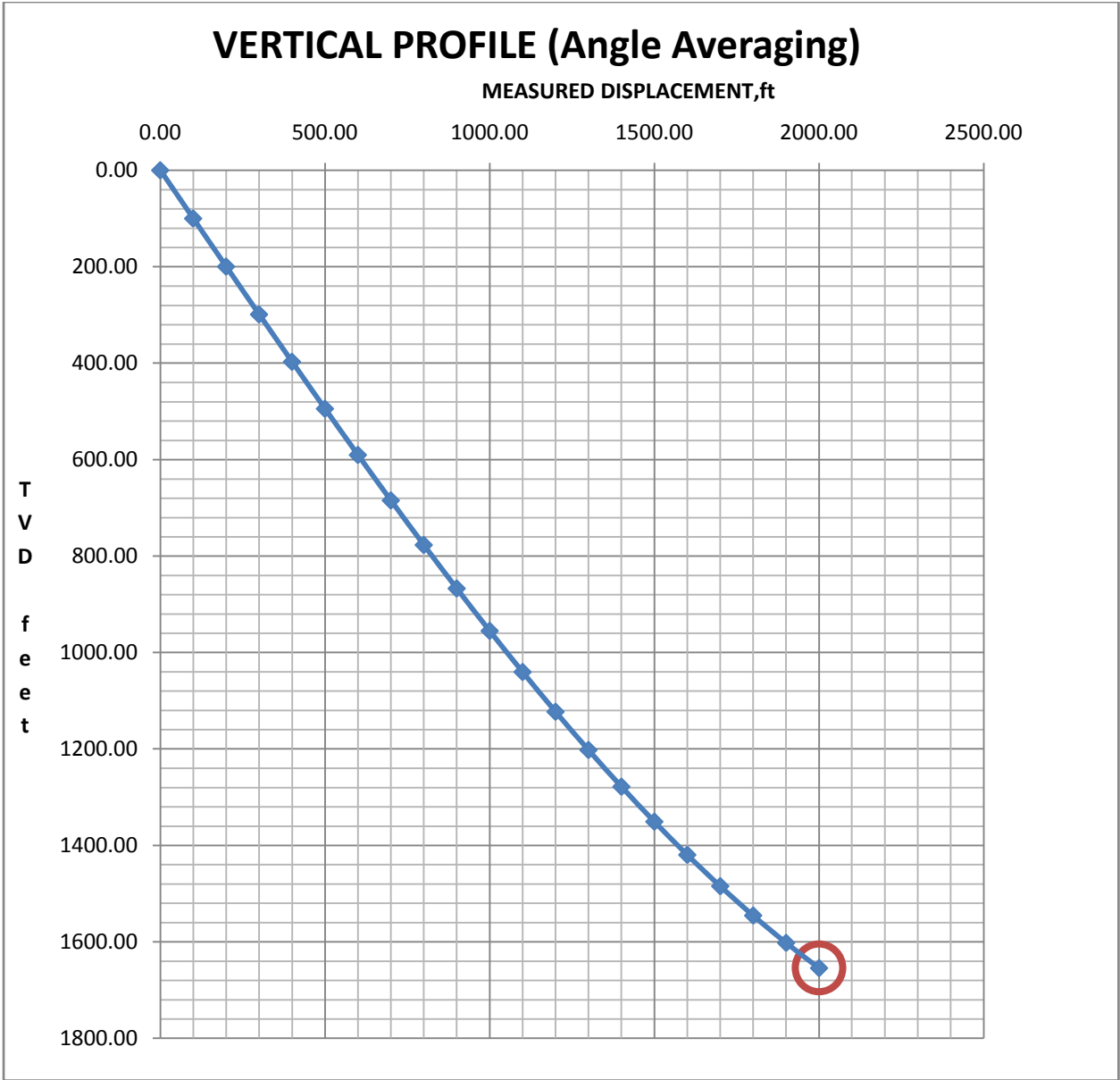




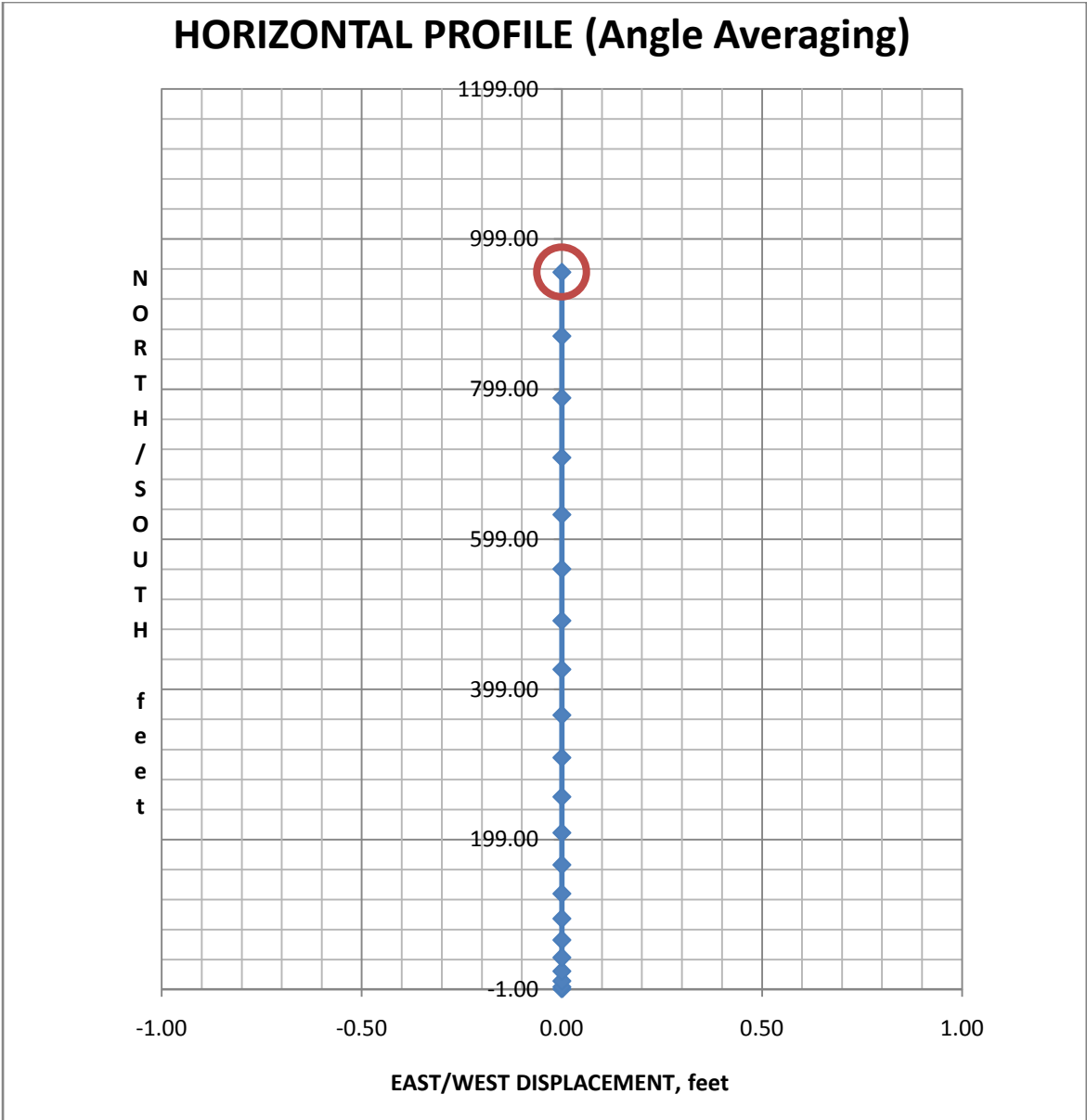
**Figure 4.14a- Vertical Profile of the Tangential Method for results in Table 4.6**



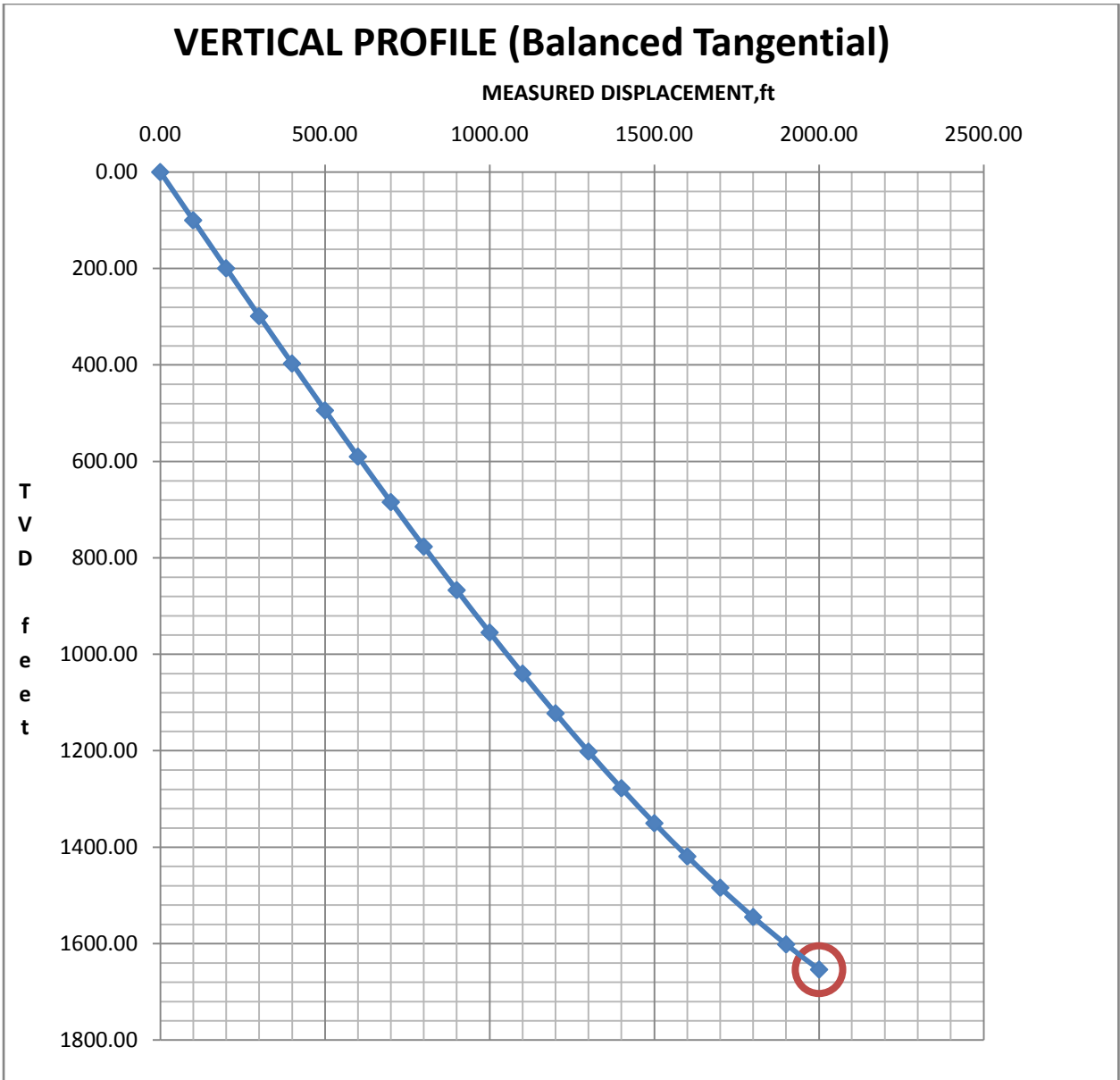
**Figure 4.14b- Horizontal Profile of the Tangential Method for results in Table 4.7**



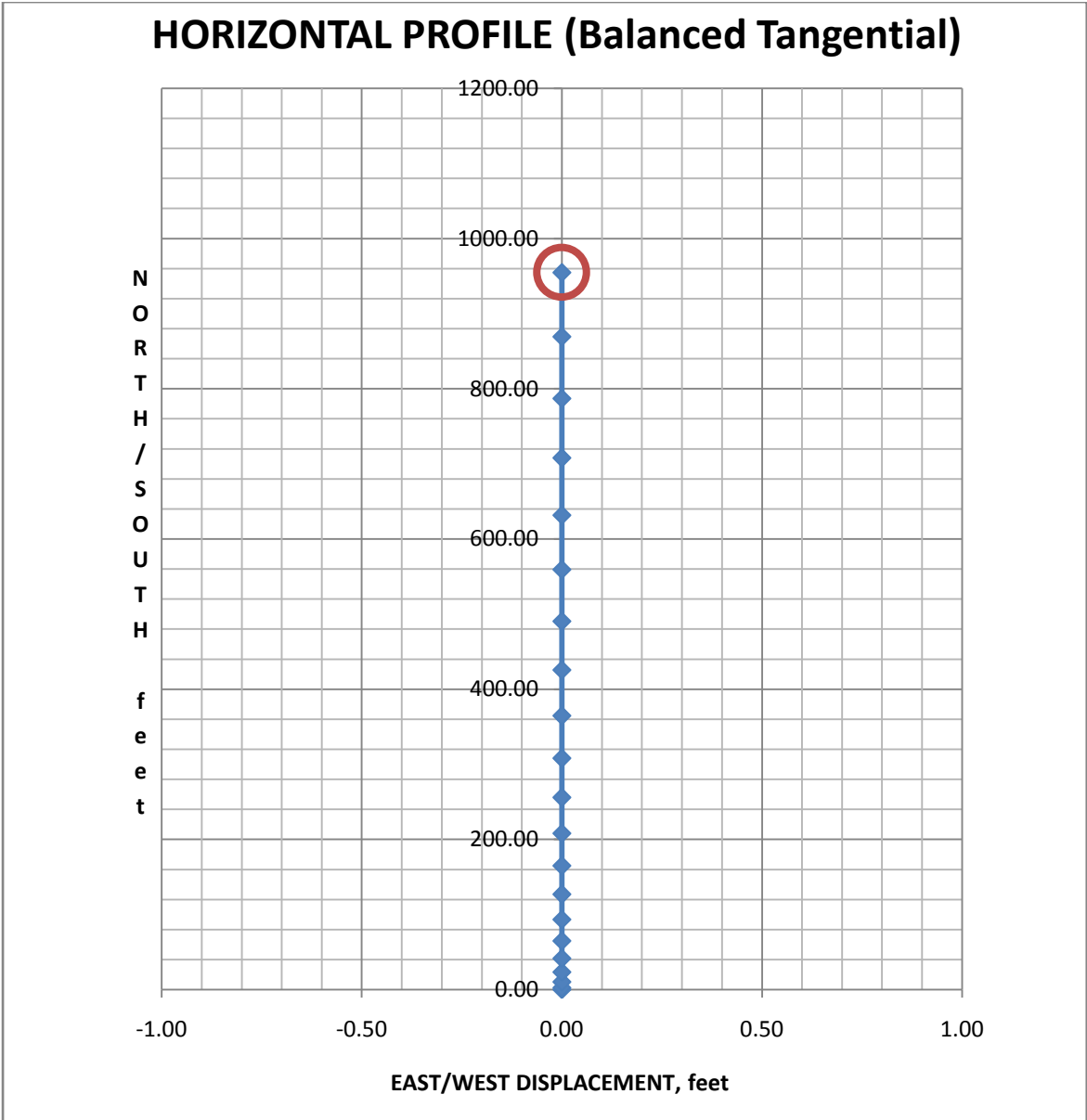
**Figure 4.15a- Vertical Profile of the Angle Averaging Method for results in Table 4.6**



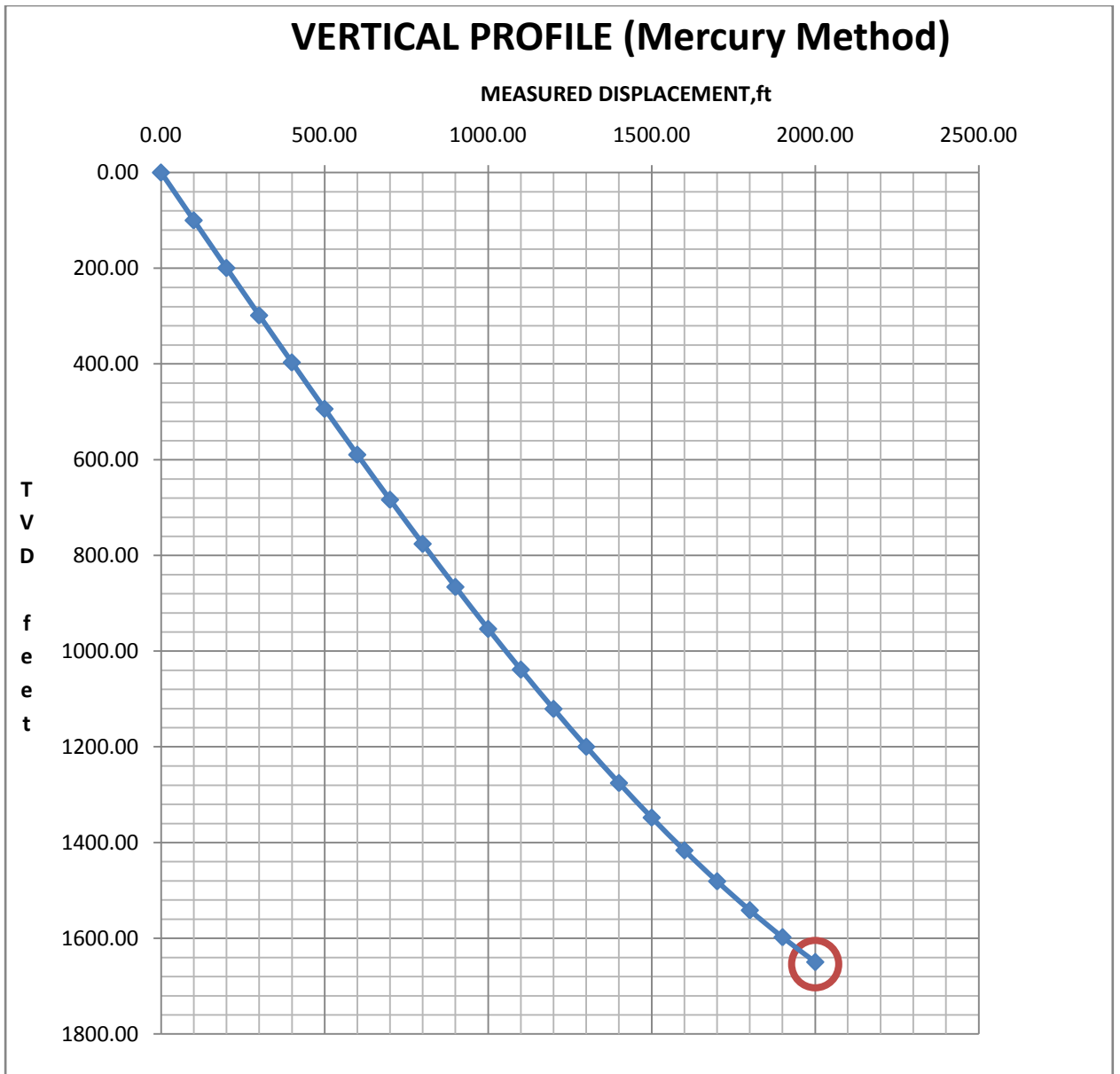
**Figure 4.15b- Horizontal Profile of the Angle Averaging Method for results in Table 4.7**



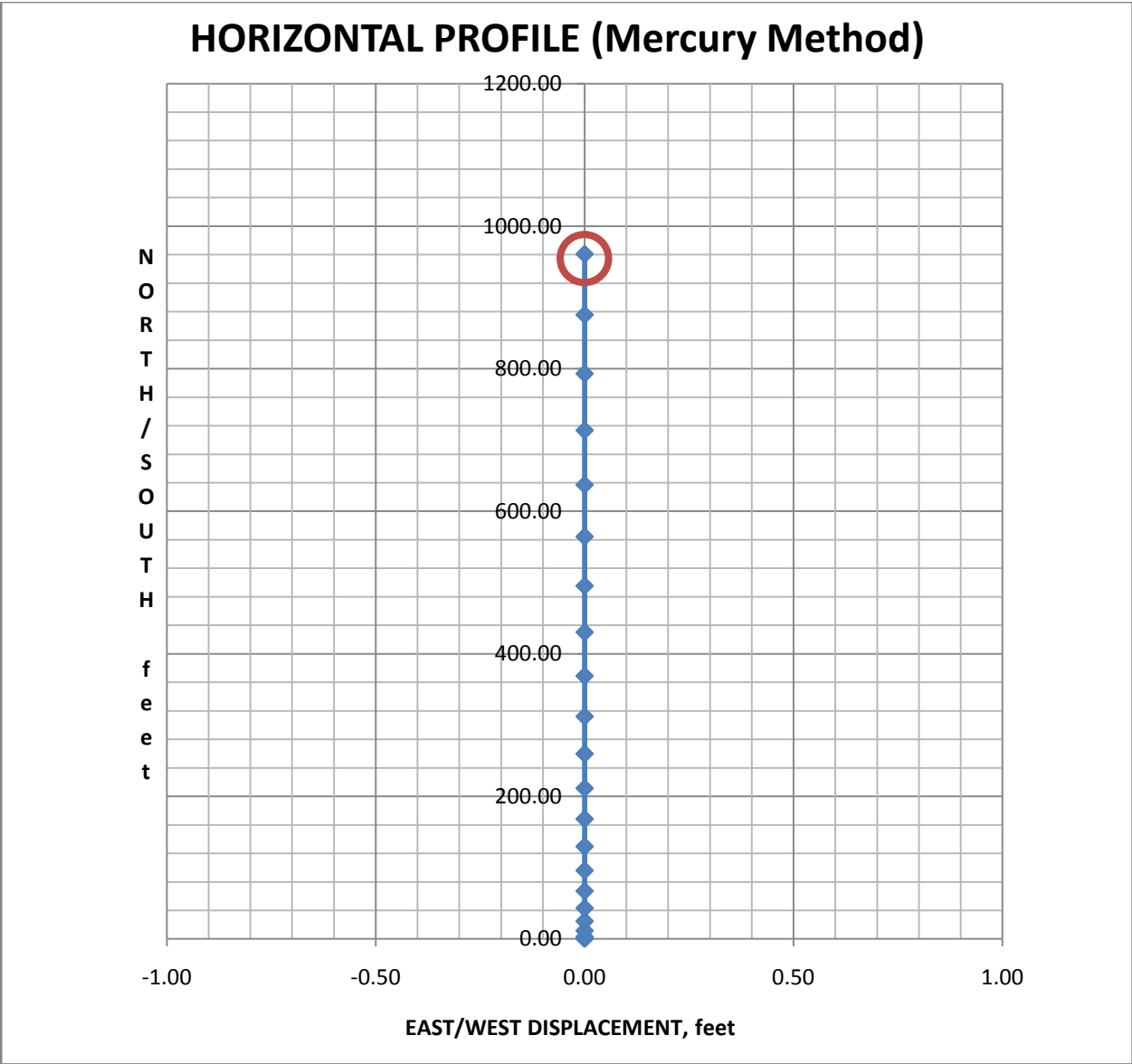
**Figure 4.16a- Vertical Profile of the Balanced Tangential Method for results in Table 4.7**



**Figure 4.16b- Horizontal Profile of the Balanced Tangential Method for results in Table 4.7**

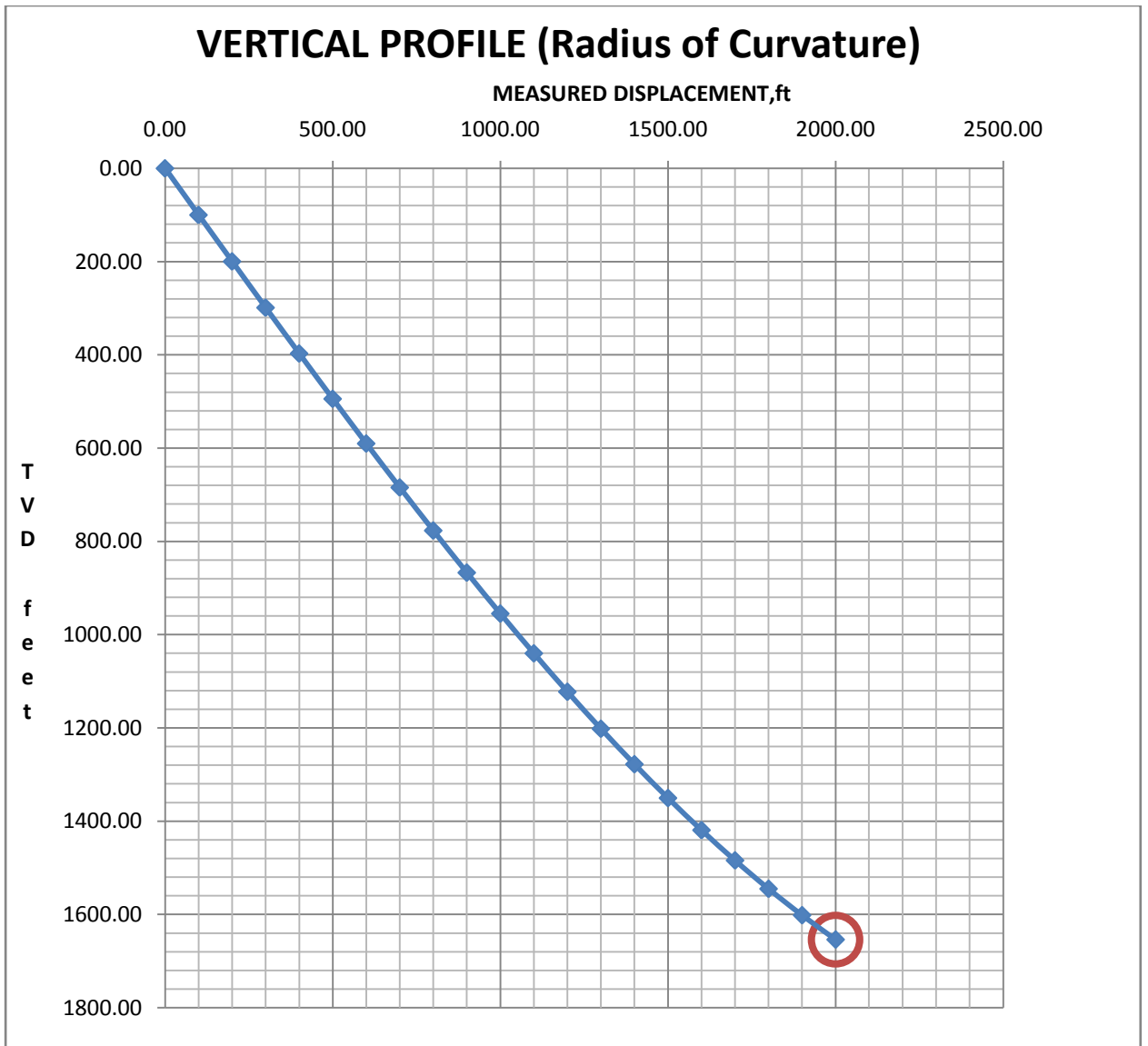


**Figure 4.17a- Vertical Profile of the Mercury Method for results in Table 4.7**

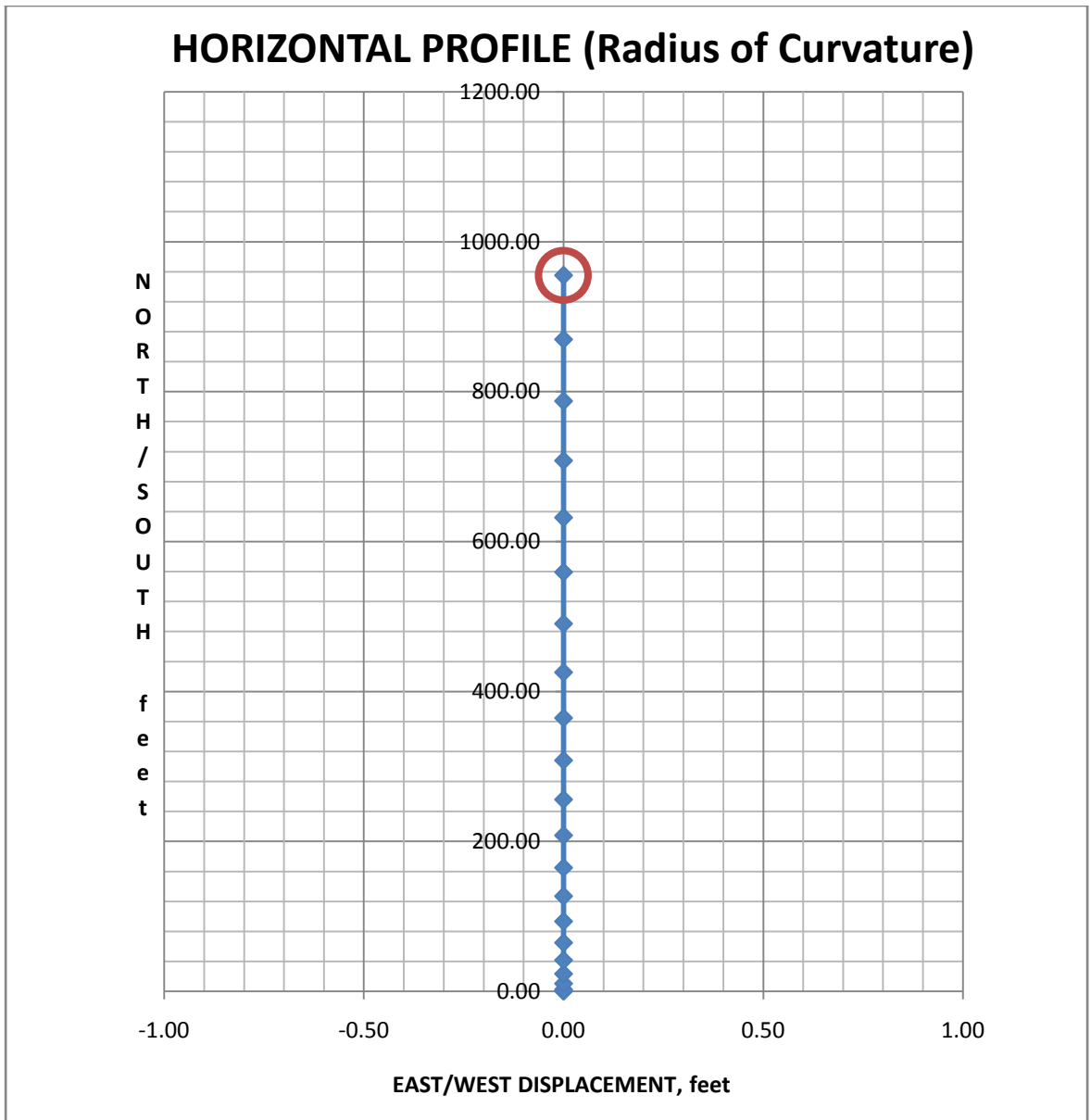


**Figure 4.17b- Horizontal Profile of the Mercury Method for results in Table 4.7**

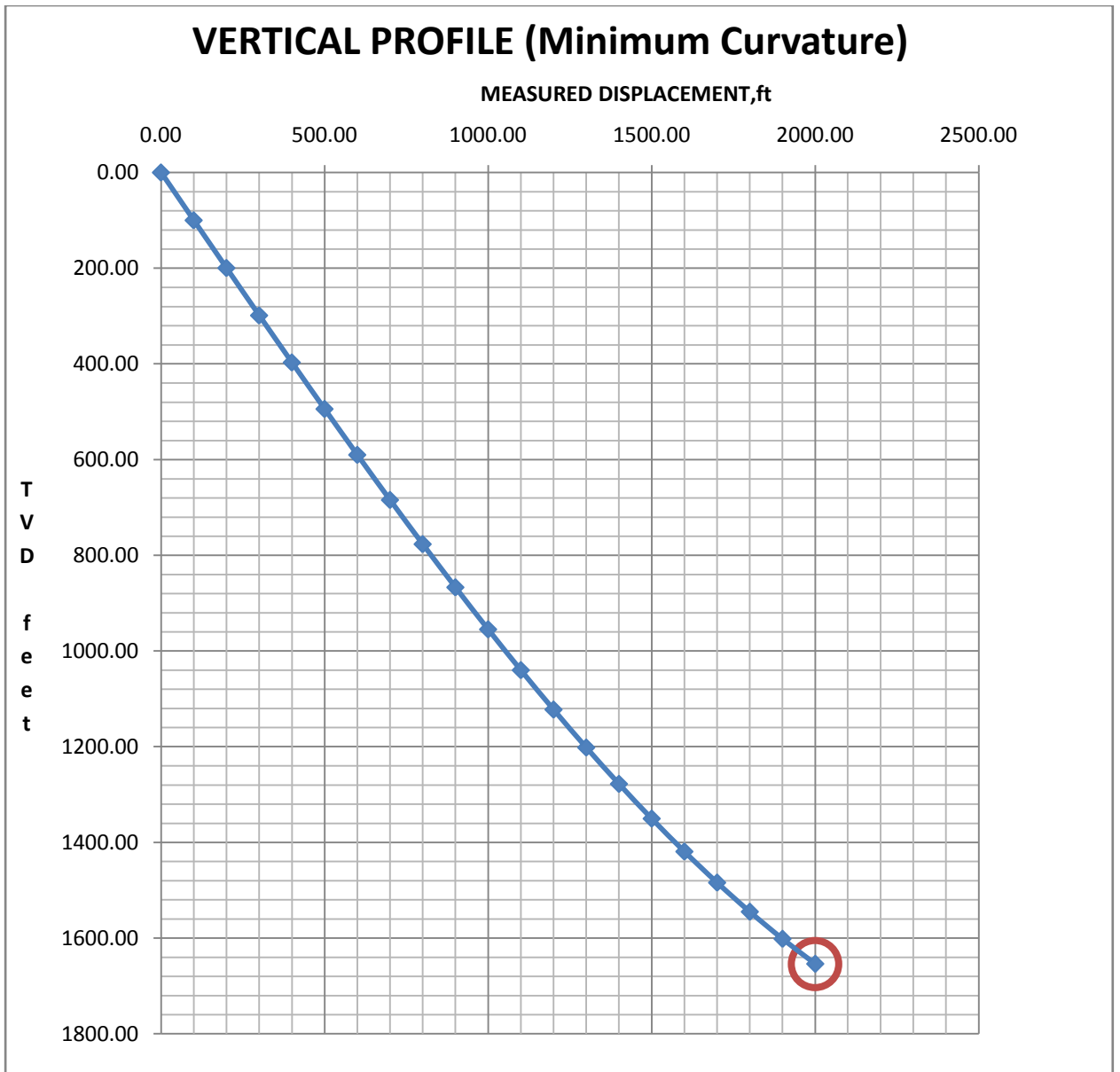




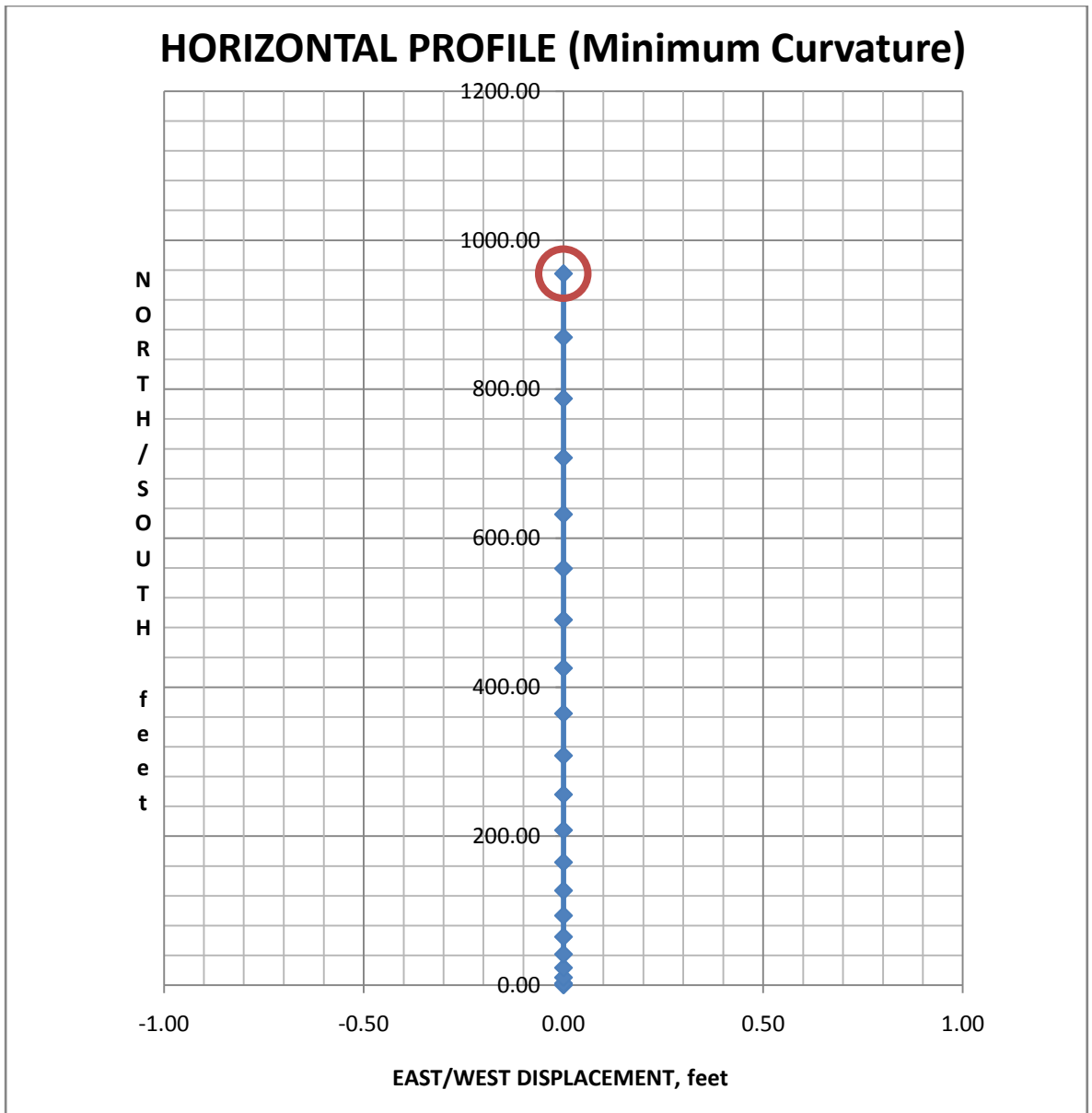
**Figure 4.18a- Vertical Profile of the Radius of Curvature Method for results in Table 4.7**



**Figure 4.18b- Horizontal Profile of the Radius of Curvature Method for results in Table 4.7**



**Figure 4.19a- Vertical Profile of the Minimum Curvature Method for results in Table 4.7**



**Figure 4.19b- Horizontal Profile of the Minimum Curvature Method for results in Table 4.7**

#### **4.4 Summary of Observations and Analysis of Results using the Bourgoyne et al, (1991) Data**

The following observations and analysis can be drawn from Tables 4.6, 4.7 and 4.8.

- The results obtained from the Excel Spreadsheet are exactly the same results obtained from Bourgoyne et al, (1991) computations with the exception of the Mercury method.
- The tangential method shows considerable error for the North, East and TVD followed by the mercury method. With the exception of the tangential and mercury methods, the differences between the average angle, balanced tangential, radius of curvature and minimum curvature are very small.

#### **4.5 General Discussion**

The advantages of using the Excel Spreadsheet compared to other available commercial software packages are that;

- The Excel Spreadsheet program is cheaper
- It is also user-friendly (friendly data input interfaces)
- It is faster since it consumes lesser computer memory
- It can be easily modified to suit the needs of any individual operator
- Produces the same results as the ones obtained from the commercial software packages.

From this study the Minimum Curvature method is the best method recommended for the calculating wellbore trajectory paths because it is applicable to any trajectory path. This method is particularly useful when planning trajectory paths for drilling relief wells.

## **CHAPTER 5**

### **SUMMARY, CONCLUSIONS AND RECOMMENDATIONS**

#### **5.1 Summary**

The major drawback of drilling directional wells is the complex computations that have to be done while planning the well. These computations become more complex when done manually. A new approach has been proposed which minimises the computational time and errors. The application of the developed user-friendly Excel Spreadsheet program employing the Tangential, Angle Averaging, Balanced Tangential, Mercury, Radius of Curvature and the Minimum Curvature methods for well-trajectory computations will help increase the usage of these methods. This application will therefore enhance the accuracy during both planning and drilling operations. Further, the Excel Spreadsheet program can easily be modified or updated at any point in time to meet the needs of the industry.

#### **5.2 Conclusions**

- 1 A user friendly Excel Spreadsheet program was developed that incorporated the Tangential, Angle Averaging, Balanced Tangential, Mercury, Radius of Curvature and the Minimum Curvature methods for the computation of well trajectory from survey data.
- 2 The developed user-friendly Excel Spreadsheet was validated using two data from the literature. Results obtained were fairly the same as obtained and very accurate.
- 3 The program provides pictorial views both in the vertical and horizontal plane of the trajectory position of the drilling bit in the wellbore. These help to minimize risk and uncertainty surrounding hitting predetermined target since deviations can easily be detected and the necessary directional corrections or adjustment initiated with less ease. The program computes the position at each survey station and therefore be able to predict the length and direction from a survey station relative to the target position.

- 4 The differences in results obtained using the average angle, balanced tangential, mercury, radius of curvature and minimum curvature method are very small hence any of the methods can be used for calculating the well trajectory.
- 5 The sensitivity of deviation from hitting a target of each of the methods may differ for different operators. This may also be related to the area extent of the target and that the accuracy of each method is very relative.

### **5.3 Recommendations**

- 1 Field data should be used to do more validation of the excel spreadsheet program.
- 2 The program should be constantly up-dated/graded to fully meet the dynamic requirements of the industry should the need be.
- 3 The Minimum Curvature Method should be embedded in survey calculations to enhance accuracy during planning process.

## NOMENCLATURE

<b>Symbol/Acronym</b>	<b>Description</b>
$A_i, \varepsilon, \text{ or } \Phi$	Azimuth
BHA	Bottom Hole Assembly
BRT	Below Rotary Table
DFE	Derrick Floor Elevation
$D_i, D_{Mi}, \text{ TVD}$	True Vertical Distances
DOT	Downhole Orientation Tool
$I, \alpha_i, \Theta$	Inclination Angles
LA	Lead Angle
MWD	Measurement While Drilling
ND	North Displacement
$R_i, r_i$	Radius of Curvature
RKB	Rotary Kelly Bushing
STL	Survey Tool Length
TOD	Turn Off Depth
$X_i \text{ or MD}$	Measured Displacement
$\beta, DL$	Dogleg Severity Angle
$\Delta E, M_i$	Change in Easting
$\Delta N, L_i$	Change in Northing
$\Delta V$	Change in Elevation



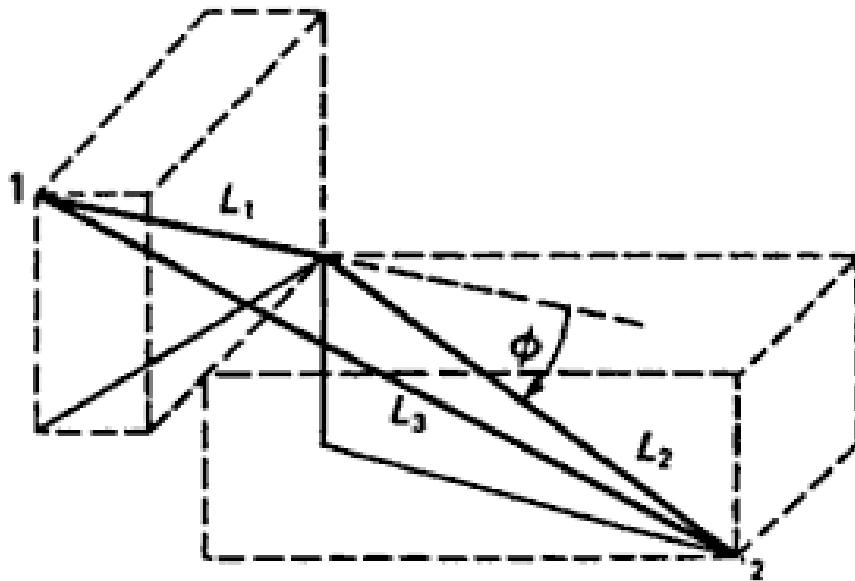
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## APPENDIX A

### DERIVATION OF DOGLEG SEVERITY ANGLE ( $\beta$ )

Assuming that at the upper station inclination and azimuth have been measured as  $I_1$  and  $A_1$  and the lower station the corresponding angles are  $I_2$  and  $A_2$ , the two straight line segments whose lengths are  $L_1$  and  $L_2$  are defined by the angles. The change in total angle ( $\Phi = \beta$ ) between these two segments are shown in figure A. The angle  $\beta$  can be determined by considering the triangle bounded by the lines  $L_1$ ,  $L_2$  and  $L_3$ . The true length of  $L_3$  can be determined by considering the vertical depth and the horizontal displacement between the stations 1 and 2. (Inglis, 1987).



**Fig. A. A diagram illustrating the dogleg severity**

$$\Delta V = L_1 \cos I_1 + L_2 \cos I_2 \dots\dots\dots A1$$

$\Delta H$  can be derived from the horizontal projection of  $L_1$  and  $L_2$  by applying substituting for  $\Delta V$  for  $\Delta H$ ;

$$(L_3)^2 = (L_1)^2 + (L_2)^2 + 2L_1L_2[\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos (\Delta A)] \dots\dots\dots A2$$

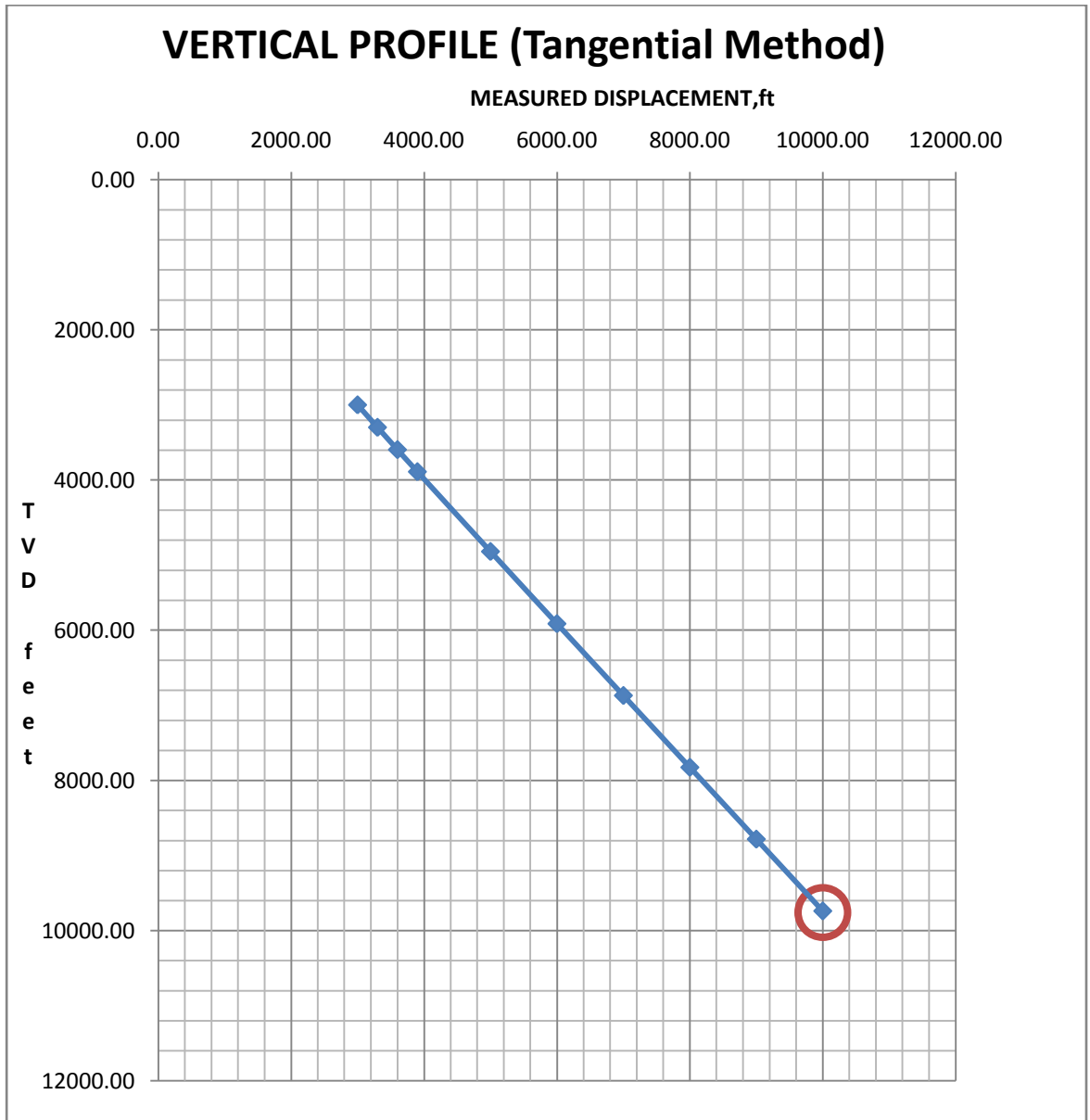
Applying the cosine rule to the triangle bounded by lines  $L_1$ ,  $L_2$  and  $L_3$ ;

$$\cos(180 - \beta) = \frac{(L_1)^2 + (L_2)^2 - (L_3)^2}{2L_1L_2} = -[\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos (\Delta A)] \dots\dots\dots A3$$

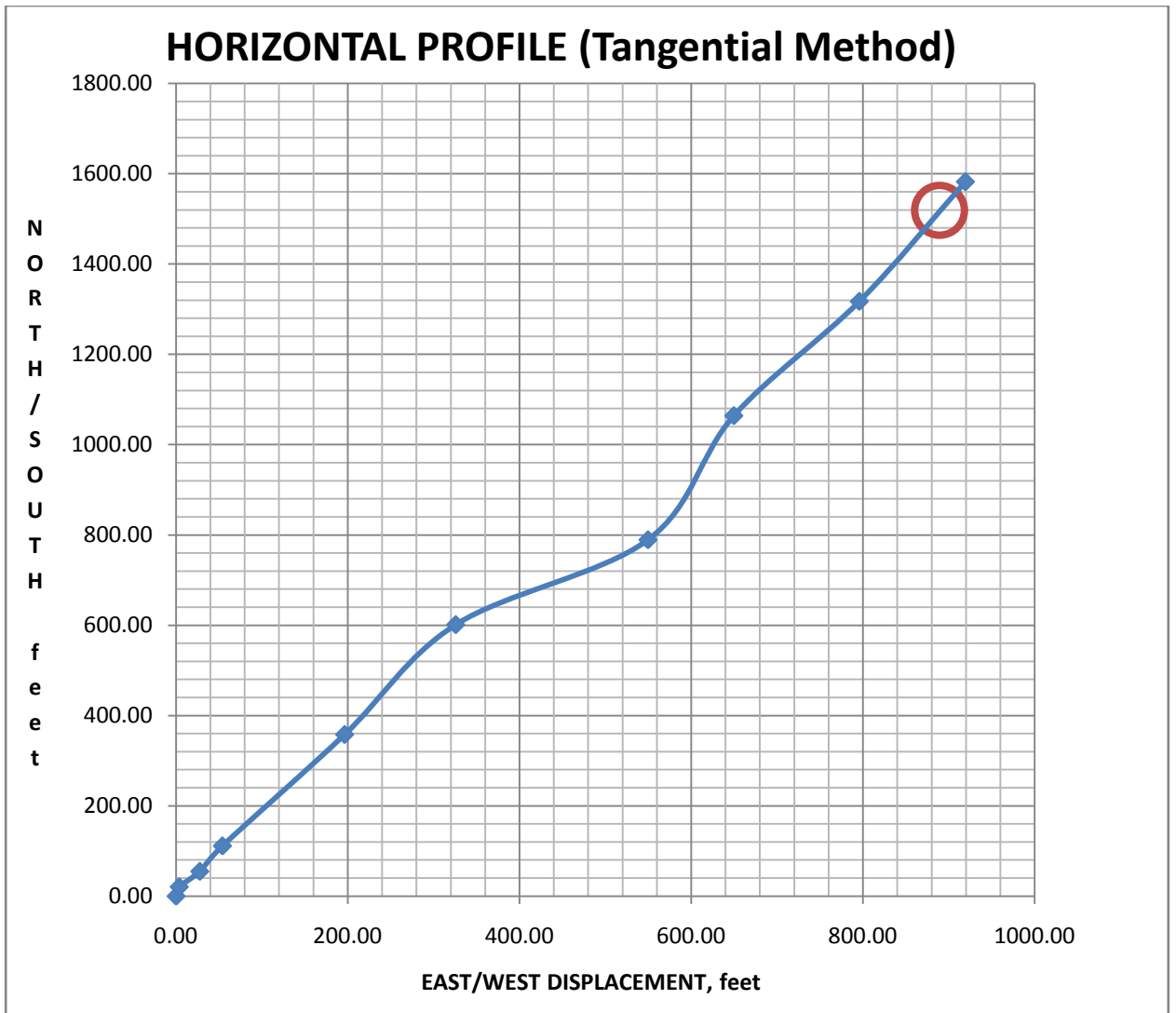
$$\cos(\beta) = [\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos (\Delta A)] \dots\dots\dots A4$$

$$\therefore \beta = \cos^{-1}[\cos I_1 \cos I_2 + \sin I_1 \sin I_2 \cos (A_2 - A_1)] \text{ (Inglis, 1987)} \dots\dots\dots A5$$

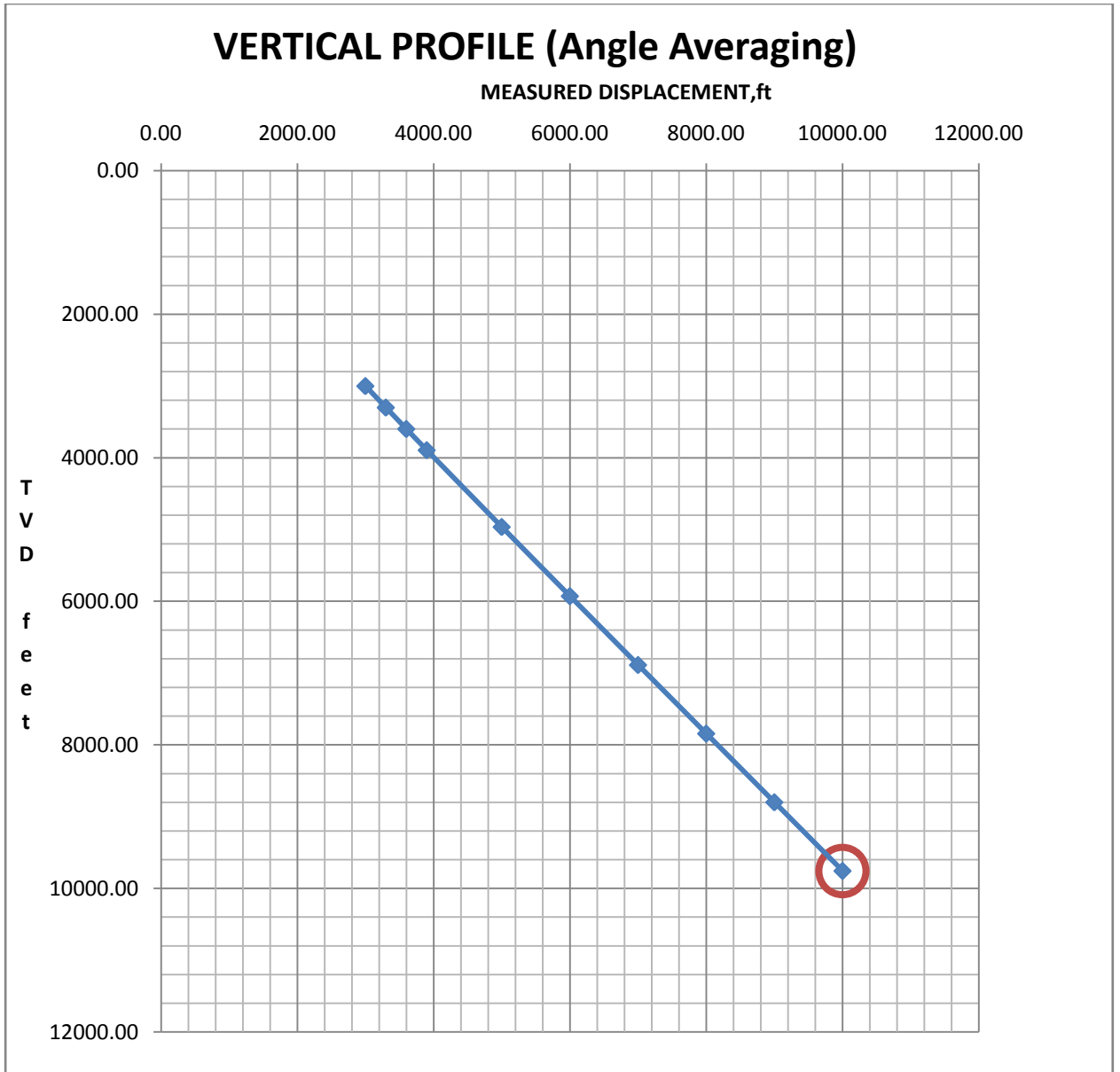
**APPENDIX B**  
**INDIVIDUAL VERTICAL AND HORIZONTAL PLOTS FOR**  
**TABLE 4.3**



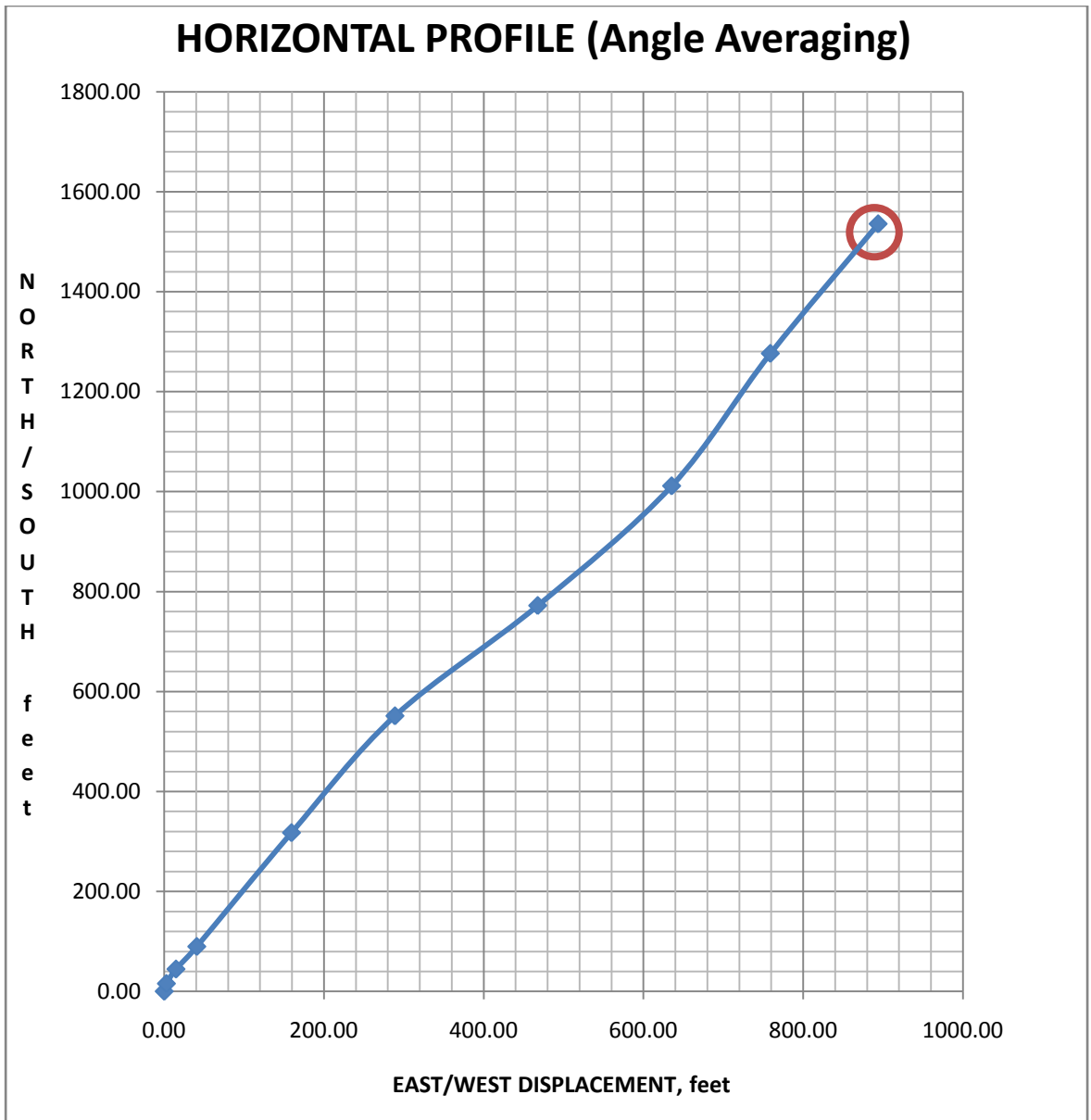
**Figure B.1a- Vertical Profile of the Tangential Method for results in Table 4.3**



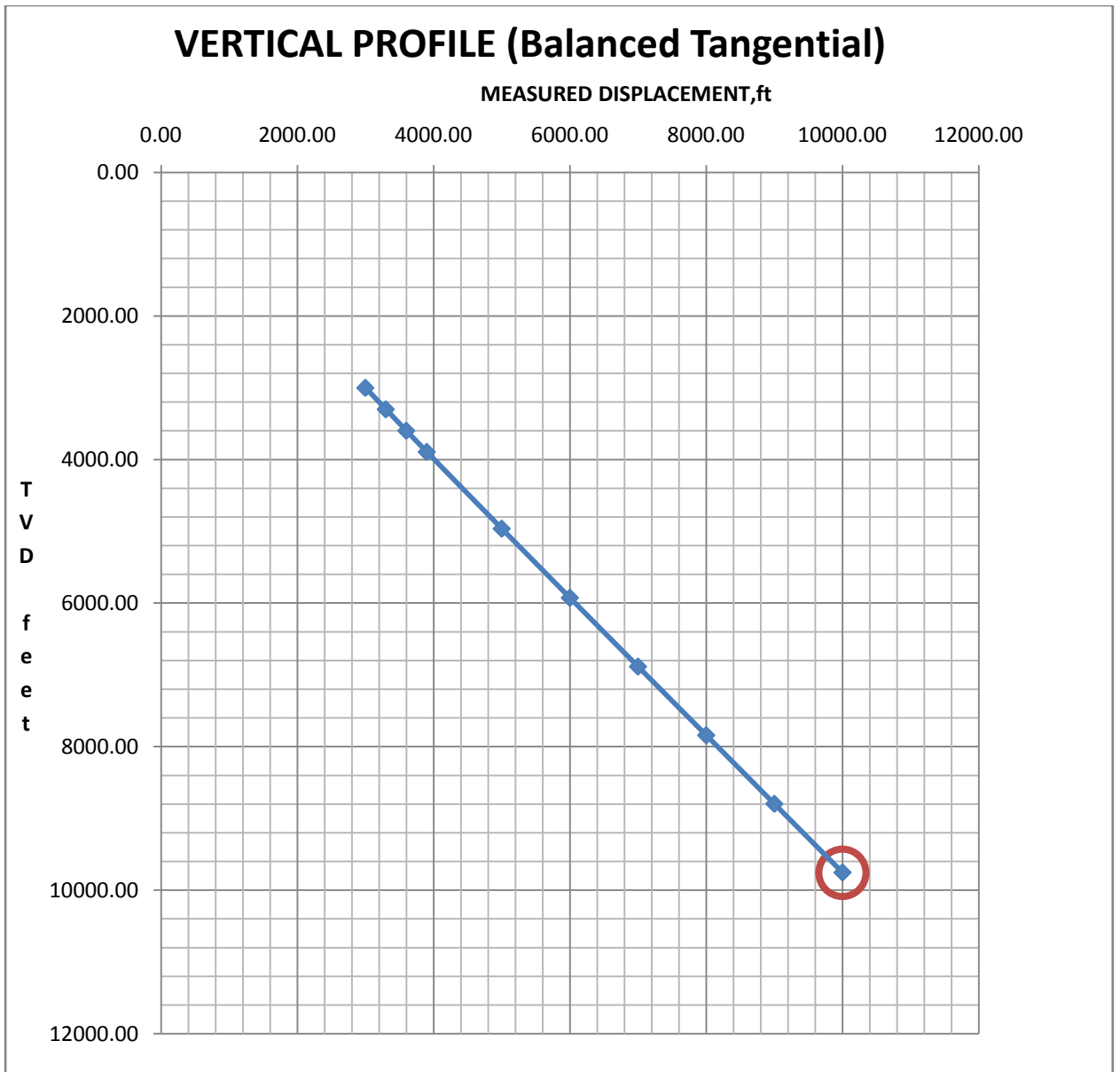
**Figure B.2a- Horizontal Profile of the Tangential Method for results in Table 4.3**



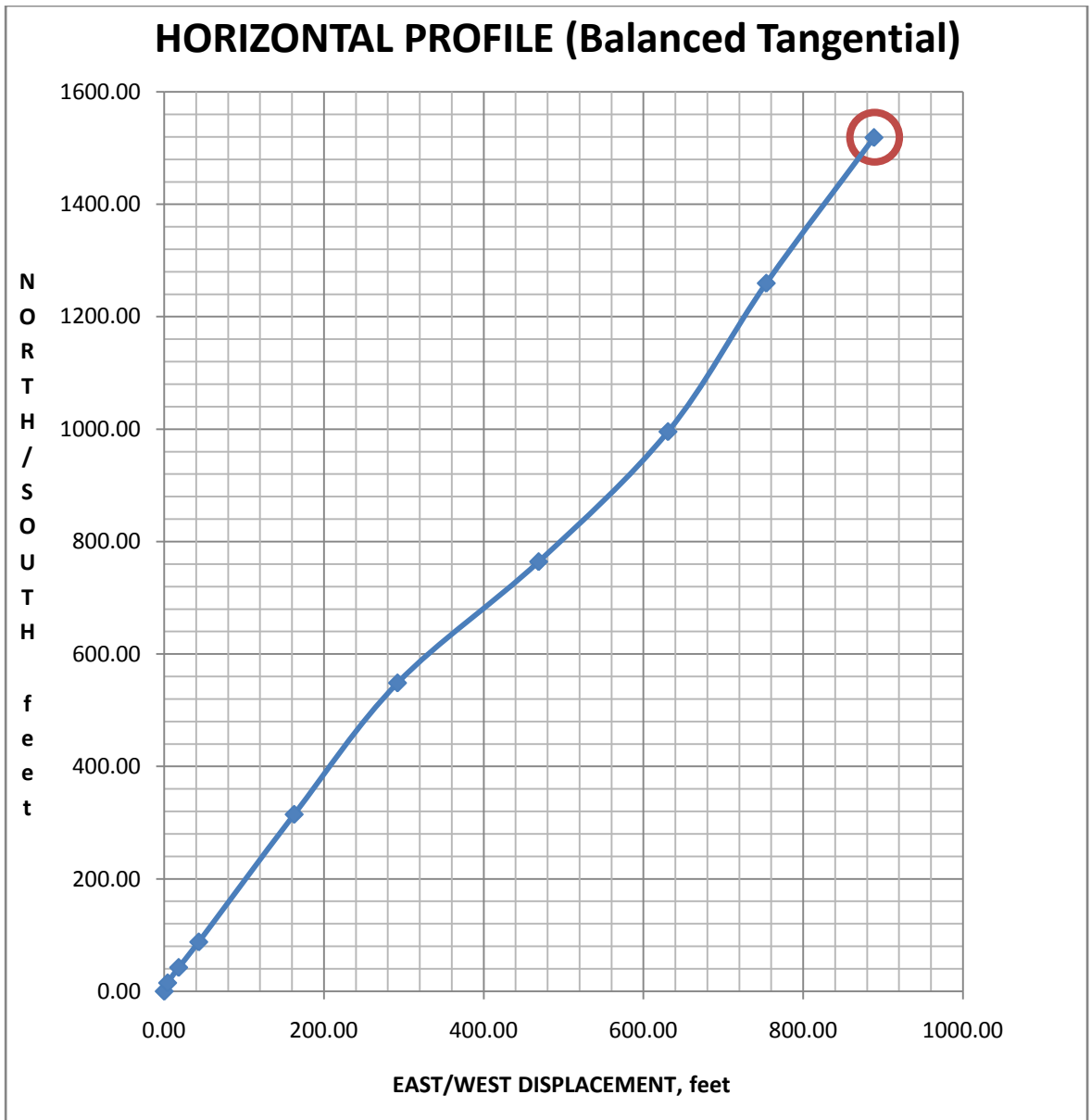
**Figure B.2a- Vertical Profile of the Angle Averaging Method for results in Table 4.3**



**Figure B.2a- Horizontal Profile of the Angle Averaging Method for results in Table 4.3**

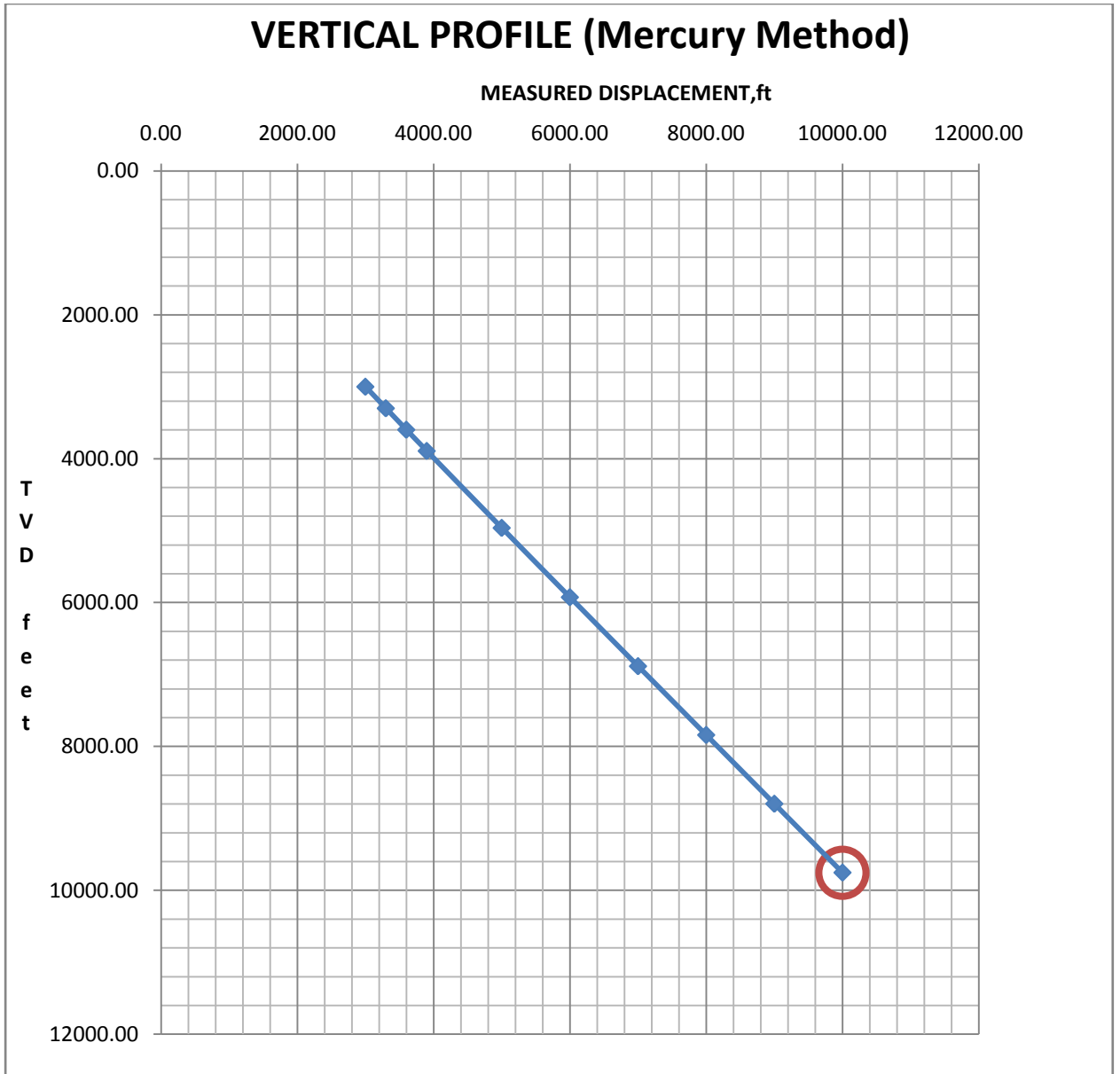


**Figure B.3a- Vertical Profile of the Balanced Tangential Method for results in Table 4.3**

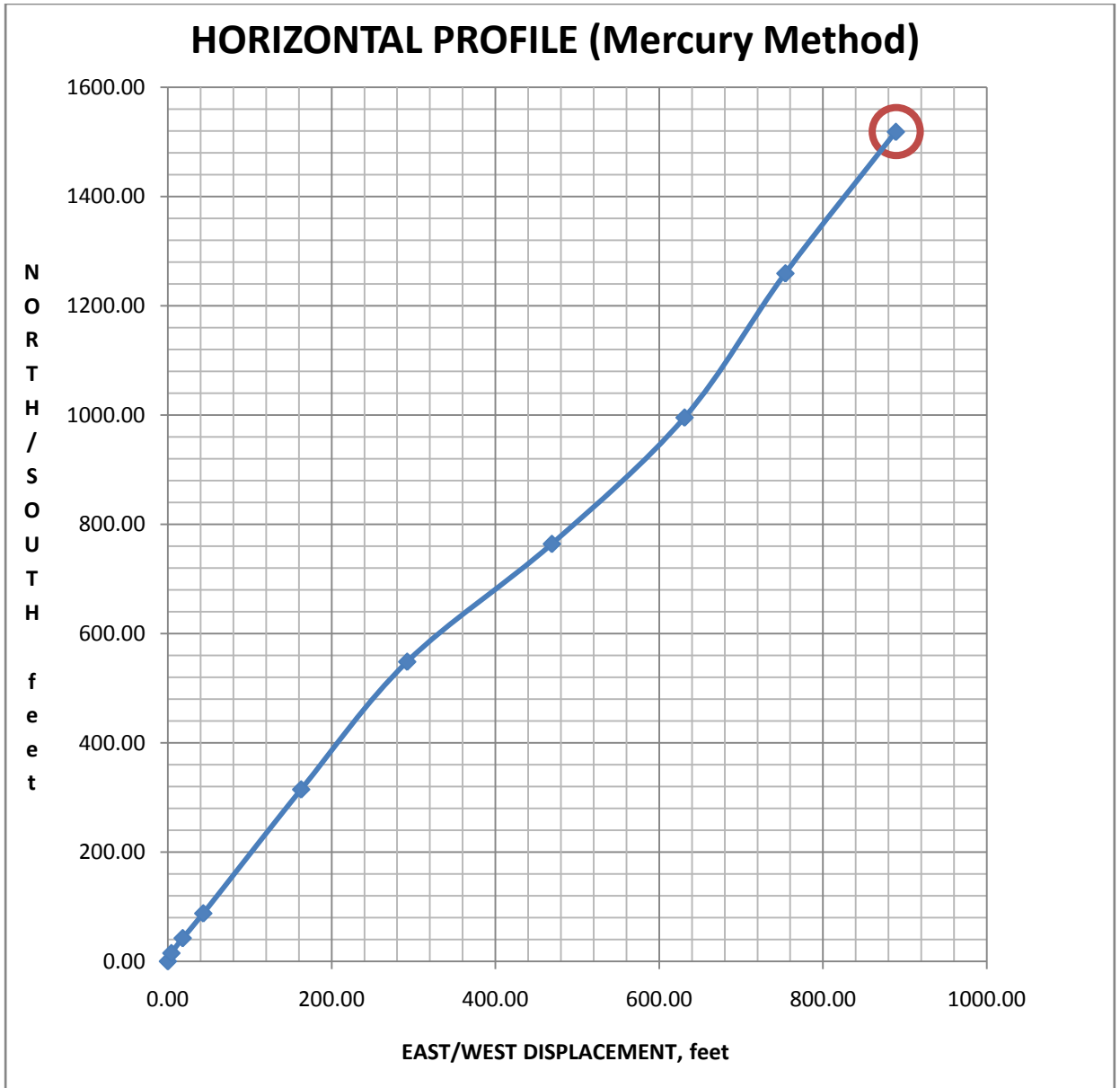


**Figure B.3b- Horizontal Profile of the Balanced Tangential Method for results in Table 4.3**

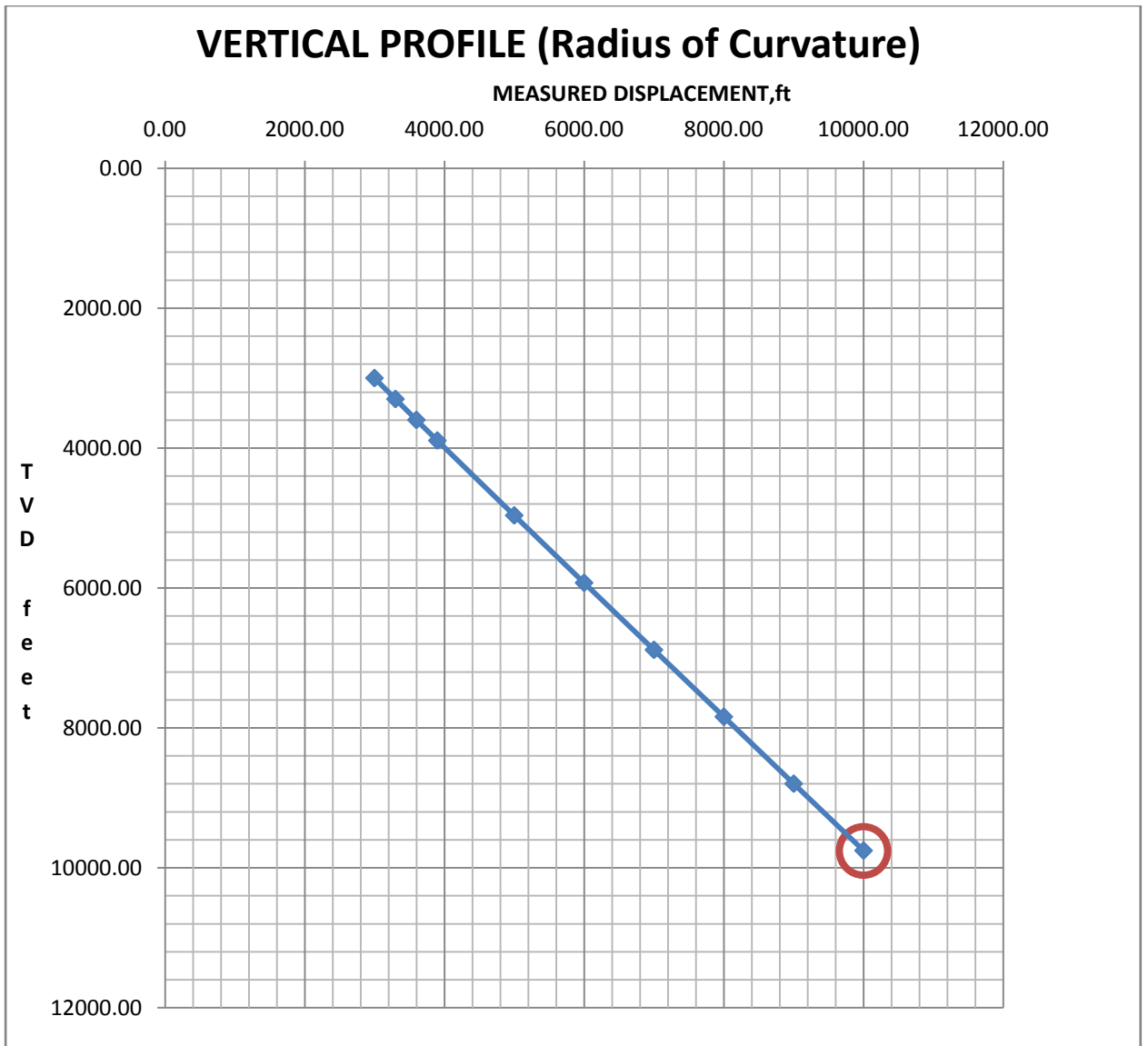




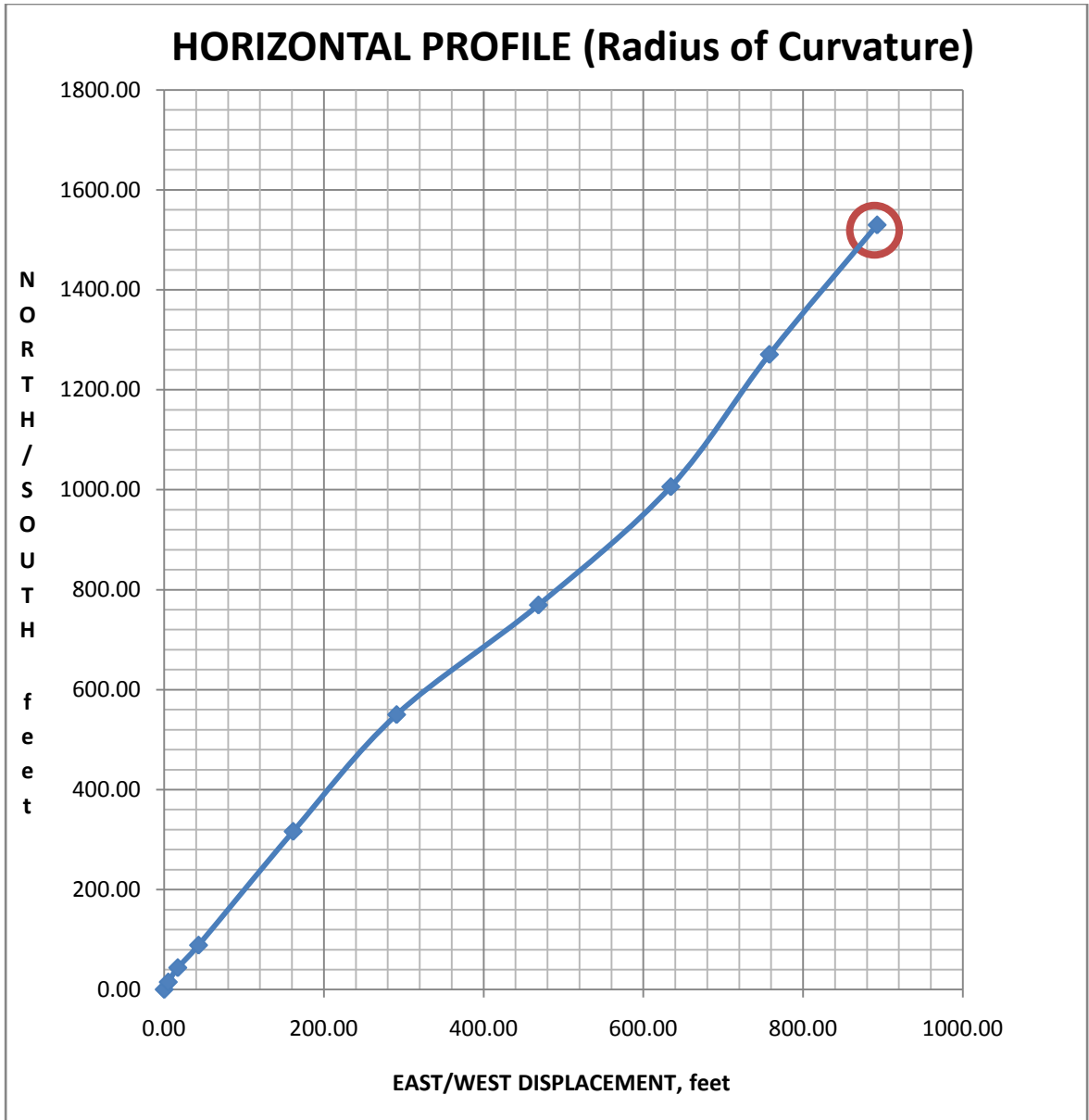
**Figure B.4a- Vertical Profile of the Mercury Method for results in Table 4.3**



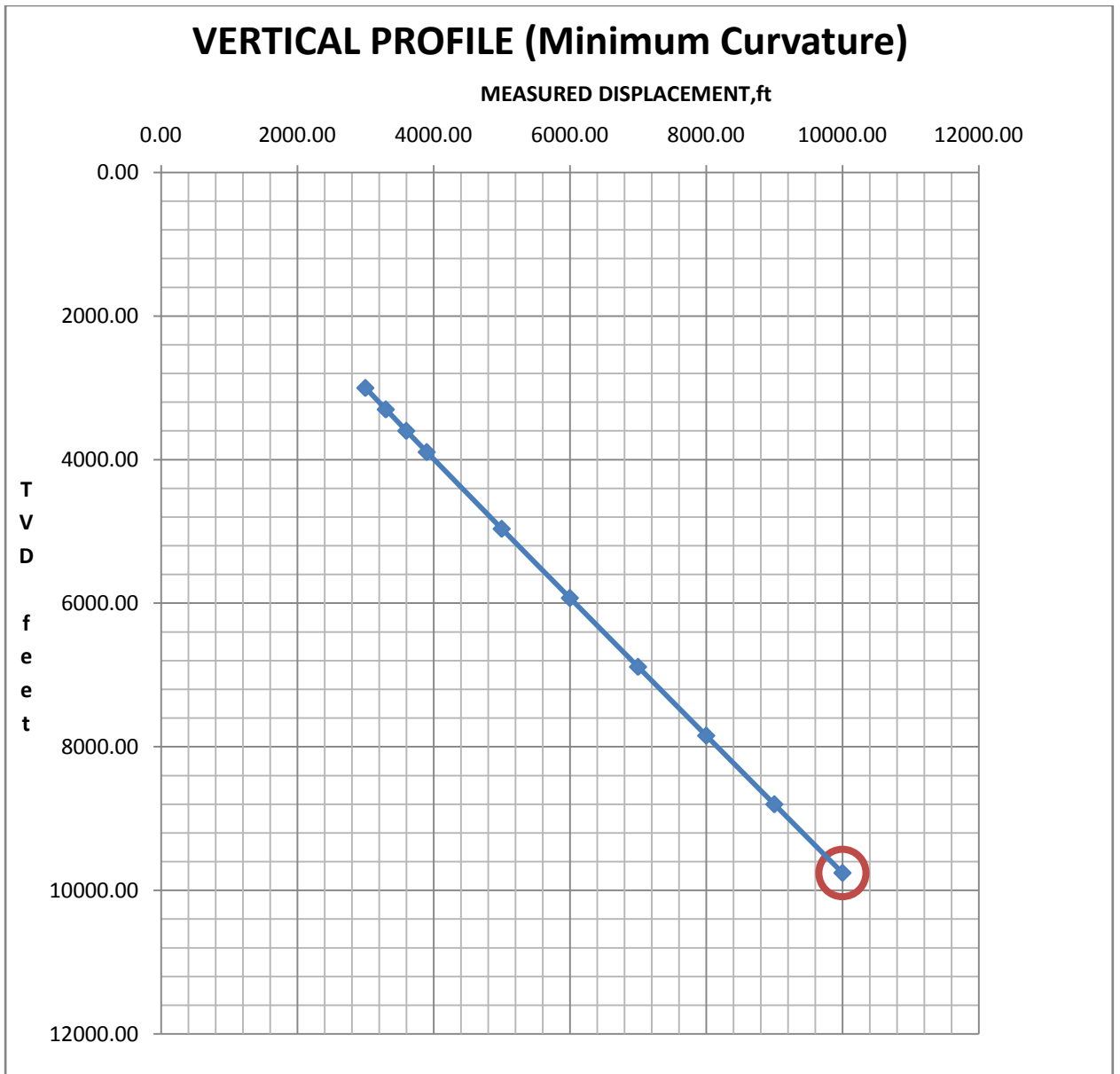
**Figure B.4b- Horizontal Profile of the Mercury Method for results in Table 4.3**



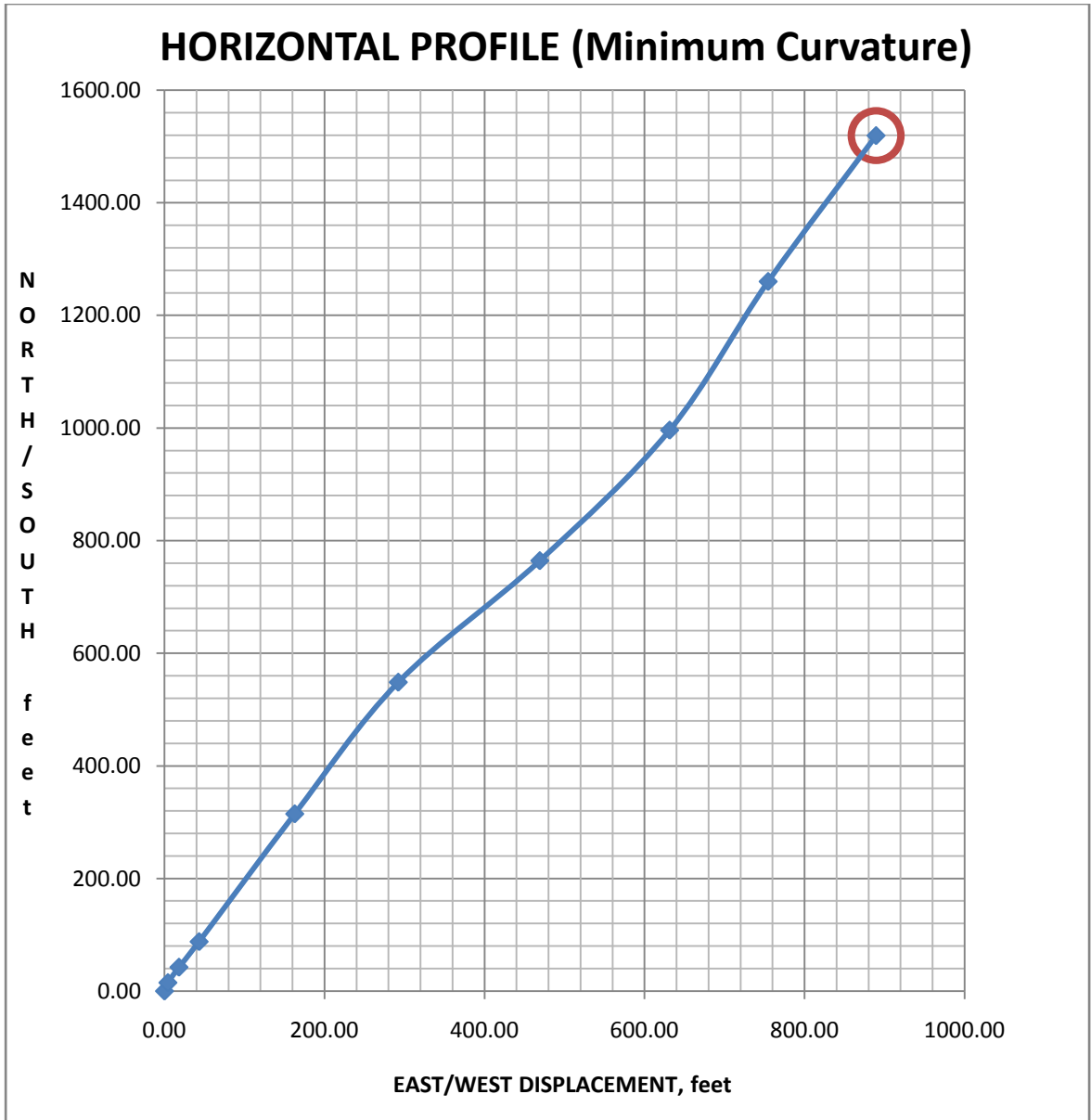
**Figure B.5a- Vertical Profile of the Radius of Curvature Method for results in Table 4.3**



**Figure B.5b- Horizontal Profile of the Radius of Curvature Method for results in Table 4.3**



**Figure B.6a- Vertical Profile of the Minimum Curvature Method for results in Table 4.3**



**Figure B.6b- Horizontal Profile of the Minimum Curvature Method for results in Table 4.3**