AFRICAN UNIVERSITY OF SCIENCE AND TECHNOLOGY

PRESSURE TRANSIENT ANALYSIS IN ANISOTROPIC COMPOSITE RESERVOIRS – A THREE DIMENSIONAL SEMI-ANALYTICAL APPROACH

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PRESSURE TRANSIENT ANALYSIS IN ANISOTROPIC COMPOSITE RESERVOIRS – A THREE DIMENSIONAL SEMI-ANALYTICAL APPROACH

A DISSERTATION APPROVED FOR

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BY

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Dedication

This is to my family; east or west, home is best. Dad, for being hard hearted, yet not hating. Mum, for giving without taking. For cheering and believing; Siblings, you all are worth loving.

"Having found by experience that where someone else failed, another has succeeded; what was unknown in one century, the next has discovered; and that Sciences and Arts are not cast in a mold, but rather by little and little formed and shaped by often handling and polishing them over: even as bears fashion their young whelps by often licking them:

What my own strength has not been able to uncover, I cease not from working at and trying out and, by reshaping and solidifying this new material, in molding and heating it, I bequeath to him who follows, some facility and make it more supple and malleable for him. The second will do for the third, which is why difficulty does not make me despair or my inability…"

- *Montaigne, The Essays, Book II, Chapter XII*

Acknowledgement

I acknowledge the grace and inspiration that Almighty God has bestowed upon my life; each step has never been a dull moment.

This is an opportunity to document the patience and openhandedness of my supervisor, Dr. Alpheus Igbokoyi; for sharing his office, laptop and personal desktop computer even when I had no right to such luxury, for “wasting” precious hours analyzing issues with me, and above all, for believing in me at every stage of this work.

Members of my thesis committee – Prof. D. Tiab, for leaving the comfort of your office to listen to a novice “lecture” you on well testing; Prof. D. Ogbe for holding no grudges when I changed supervisor.

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The financial support from the NNPC/ESSO national post-graduate scholarship board is highly appreciated; it kept my mind focused on the task at hand.

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Abstract

A composite reservoir is one with multiple lateral compartments; each having distinct rock and fluid properties separated by geologic discontinuities. Geologically, composite reservoirs separated by leaky faults are rife but literature containing detailed three dimensional analyses of such anisotropic systems is few.

For the first time using transform methods, a general three dimensional point source semi-analytical solution for the drawdown response in composite clastic and naturally fractured systems separated by a leaky fault is developed. This solution is then converted to line source for horizontal and vertical wells and plane source for hydraulically fractured wells, while treating the leaky fault as a thin finite conductivity fracture separating two reservoirs with different rock properties; arbitrary orientation is adopted for both horizontal and hydraulically fractured wells to obtain a general solution in an anisotropic system. Implementation of this solution is achieved using a simple computer program which can generate type curves in a computationally cheap and effective manner.

Sensitivity analysis conducted on the pressure signature of horizontal, vertical and hydraulic fractured wells with respect to anisotropy, partial penetration, angle of orientation, well position, reservoir dimensions, dimensionless fault conductivity and mobility ratios in both clastic and naturally fractured reservoir systems reveals that the drawdown response has a convoluted dependence on reservoir properties of both compartments and separating semi-permeable medium. Similar conclusion holds true for interference testing and for the first time, it is documented that the intersection time between the long-time approximation of both observer pressure drawdown and observer pressure derivative is unique; from which producer hydraulic diffusivity could be obtained.

Commercial simulator and three field data used to validate this semi-analytical model yields comparable results and direct synthesis of reservoir parameters from the resulting equation for various wells under different boundary conditions is examined.

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Chapter 1

General

1.1 Introduction

Reservoir heterogeneity is the reality and has been studied under various contexts including composite systems, multilayered systems with or without cross-flow, dual porosity systems, dual permeability systems, dual porosity-dual permeability systems, triple porosity systems; each with its corresponding flow anisotropy.

A composite reservoir is one with multiple compartments, each having distinct rock or (and) fluid properties separated by geologic discontinuities, faults (sealing or leaking) being common. Bense et al. (2006) showed how fault zones can behave as strongly anisotropic hydraulic conduit-barriers due to clay smearing, drag of sand, grain re-orientation and lateral segmentation of fault plane and in composite systems; and these faults could be modeled as thin lines of skin across which fluid seepage occurs or as zones of finite widths and thicknesses with distinct rock properties, hence treated as separate rock/flow units.

The composite compartments could form actual composite geometries where flow is moderated by separating discontinuity or virtual composite geometries, in which different flow units are conceptually observed on pressure signatures in the absence of any governing physical boundaries (de Swaan 1998). Interestingly, this conceptual composite reservoir model has found application in many situations including burning front detection in thermal recovery processes, wells intercepting zones with different rock properties, flood front and abrupt facie change detection. Kuchuk et al. (1997) noted that this model could result from depositional and tectonic factors like in reservoirs with edge-water encroachment and meandering point bar reservoirs.

Composite systems occur in clastic and naturally fractured reservoirs; clastic reservoirs are those with negligible fracture networks while naturally fractured reservoirs have fracture networks that control or contribute to net reservoir flow behaviour; a fracture being a macroscopic planar discontinuity that results from stresses that exceed the rupture strength of the rock along which there is physical separation (Aguilera 2003). These natural fractures could enhance, reduce or have negligible effect on fluid flow.

Nelson (2001) classified naturally fractured reservoirs (NFR) based on the fracture network behaviour into four broad types where Type 1 fractures provide the main reservoir porosity and permeability, Type 2 fractures provide the major portion of reservoir permeability, Type 3 fractures assist permeability in an already producible reservoir, and Type 4 fractures create significant reservoir anisotropy due to mineralization or other barrier in the fracture spaces leaving the matrix as the sole porosity and permeability contributor to the reservoir.

Apart from the geological abundance of composite reservoir systems, the importance of studying them has an economic angle, like accounting for the degree of communication between the compartments to determine well placement and optimization.

Much work has been done on various aspects composite reservoirs well testing using diverse approaches including one-dimensional, two-dimensional and elliptical approaches (numerical, analytical, boundary element or semi-analytical) under various boundary conditions. The computational expense of numerical methods for higher order pressure transient application accuracy is often exorbitant, while analytical approach might become infeasible due to the complexity of the governing fluid flow equations. The semi-analytical approach is a middle ground between these two especially when a complex problem cannot be solved rigorously and must be approximated by dividing the solution into steps; this has received much attention in the recent past (Medeiros et al. 2006; Kamal 2009).

Despite the importance of composite reservoir systems, literature with detailed three dimensional analyses of such anisotropic systems is few or unavailable and this work achieves for the first time a customizable three dimensional semi-analytical approach to well testing in such systems under various boundary conditions. A sensitivity analysis of key parameters affecting fluid flow during pressure drawdown and interference testing in both composite clastic and naturally fractured reservoir systems is discussed using type curves. These parameters include dimensionless fault conductivity, mobility ratio, angle of well inclination, permeability anisotropy, distance of well to fault, reservoir dimensions, well/ fracture length, storativity ratio contrast, inter-porosity ratio contrast, and partial completion ratio.

1.2 Literature Review

Carslaw et al. (1959) studied heat conduction in infinite composite circular or hollow circular cylinders using the Laplace transformation and noted that the results could be very complicated. In petroleum literature, Bixel et al. (1963) published one of the first composite reservoir studies using a two dimensional point source analysis and modeling the system as lateral compartments separated by a linear discontinuity which has no distinct property. The result provided a method of estimating the distance to a discontinuity using an overlay technique to match an experimental curve with one of the theoretical curves and noted the possibility of modeling fluid-fluid contacts and facies change as linear discontinuities.

Kazemi (1966) confirmed this by modeling enhanced oil recovery where the burning front has infinitesimal thickness, noting that the distance to the burning front could be calculated from a pressure fall-off test on an injection well of a forward combustion project; Carter (1966) documented the importance of a composite system with no-flow boundaries to reservoir limit testing, while the demarcation between different temperature zones in a non-isothermal reservoir as permeability barriers was modeled by Mangold et al. (1981) using numerical methods to examine such effects in buildup, drawdown and injection well tests.

de Swaan (1976) noted that the Warren et al. (1963) NFR procedure is less convenient because despite obtaining two adjusting parameters without direct physical meaning, it still requires a third parameter (shape factor) to arrive at the probable values of the reservoir properties. The study showed how to analyze NFR's based exclusively on reservoir fluid flow parameters that may be obtained by well-logging and core analysis but also documented that the proposed method does not lead to an analytical description of the transition between the two straight lines of the fractured-reservoir well pressure plot. The author also demonstrated how to extend this method to composite reservoirs with blocks of varying properties uniformly distributed in a fracture medium with definite permeability and porosity.

Satman (1985) studied interference testing in composite reservoirs, conducting sensitivity of hydraulic diffusivity, distance to radial discontinuity, mobility ratio, distance between wells, skin and wellbore storage at the active well on the composite system; producing type curves using different combinations of these parameters.

Stanislav et al. (1987) investigated the infinitesimal thickness skin model of infinite conductivity vertical fractures in a composite system using elliptical flow geometry and suggested using the pressure derivative form of model solutions to reduce the problem of type cures non-uniqueness.

The model studied by Bixel et al. (1963) was also considered by Yaxley (1987) who treated the linear discontinuity as a semi-permeable barrier with flow resistance effect; a departure from the prevailing infinitesimal thickness skin model. He found that the drawdown and buildup behaviors of the active well resembled inverted forms of the characteristic behavior of a well in a naturally fractured reservoir and went ahead to generate interference type curves for the special case of a fault perpendicular to a line of intersection joining the active and observation wells. The possibility of generating type curves for other fault orientation and unequal formation thickness on opposite sides of the fault was also documented.

(Obut et al. 1987; Olarewaju et al. 1989) also modeled composite systems using the infinitesimal thickness skin approach, the later included wellbore storage effects in a finite reservoir, while the former using elliptical flow co-ordinates on vertically fractured injection wells showed how to locate flood front position in enhanced oil recovery processes.

Ambastha et al. (1987) studied a composite system consisting of an aquifer separated from a reservoir by a partially communicating fault treated as a boundary skin of negligible thickness with no storage while (Ambastha 1988; Ambastha et al. 1989) looked at the composite system in more detail under different boundary conditions. He also noted that the analysis of interference tests in composite reservoirs could be complicated since the pressure signature observed depends on property contrasts between the compartments and location of observation well.

Kikani et al. (1991) extended the composite reservoir model to naturally fractured reservoir undergoing water flooding, steam injection or acid stimulation by assuming a composite reservoir system with matrix skin.

Abbaszadeh et al. (1995) noted that the existing solutions for a leaky fault are not valid when the fault permeability is substantially larger than the formation permeability due to limitation of the infinitesimal thickness skin zone model to account for fluid flow along the fault plane and not only across it; another departure from the infinitesimal thickness skin model. Although this observation was taken into account, the reuse and patching up of existing solutions where necessary put the results of this work under doubt considering the modification intended to be incorporated into the composite model.

Kuchuk et al. (1997) documented the pressure behavior in laterally composite reservoir by modeling the zone interfaces as infinitesimal thickness skin which enabled the analysis of n-zone composite reservoir model. Although the solution to n-zone composite system is welcome, the flow model for such generalization is not robust enough since the skin zone model approach is a far cry from reality.

Ambastha et al. (1998) conducted three-dimensional numerical investigation of well test analysis in composite reservoirs mimicking thermal recovery situations in a closed, box-shaped reservoir due to unavailability of analytical solution for pressure transient tests under composite reservoir situations with complex swept region; the result indicated that the computed swept volume from pseudo-steady state analysis is independent of the swept region shape.

More recently, (Boussila et al. 2003; Anisur Rahman et al. 2003) did two dimensional analysis on composite reservoirs. Although the former supported the inverted forms of the pressure derivative signature of an active well near a single leaky fault documented by Yaxley (1987), there was no striking addition to existing literature. The later went ahead to propose an interesting way of incorporating the flow effect along the leaky fault into the composite model, documenting a detailed solution procedure. However, the physical validity of the generated plots is questionable; this might be computational in nature.

All these studies used the instantaneous steady point source approach for either one dimensional radial or rectangular, two dimensional rectangular or radial co-ordinate flow system, modeling the composite system as lateral compartments separated by faults which can be infinitesimal skin regions or distinct flow units.

1.3 Problem Statement

Since no well is produced from one perforation, there arises the need to tackle the composite model using a instantaneous steady line or plane source approach as the case may be instead of the instantaneous steady point source approach common in literature; this should allow a realistic study of the behavior of vertical, horizontal and hydraulically fractured wells in composite reservoir systems.

Reservoirs are not cylindrical, and wells are rarely fully penetrating or perforated, hence there is need to incorporate these geometric components in building the composite model. Also, the two dimensional co-ordinate system common in composite reservoir literature which does not account for these geometric effects has to be modified.

Geologically, fault zones have finite thicknesses and could be highly porous or (and) permeable or otherwise; hence, modeling the fault zones in composite reservoirs as distinct zones with unique rock properties which allows fluid flow across and along its plane is necessary.

Reservoir permeability anisotropy which is believed to coincide with principal stress axes is a key component which is missing in many existing composite reservoir models found in literature.

Similarly, accounting for the deviation of wells from the principal permeability axis along the horizontal plane in composite reservoir literature is non-existent and well performance under several boundary conditions in such system is also unavailable.

This work proposes a customizable three dimensional well test analysis of arbitrarily oriented horizontal wells, vertical wells and, planar arbitrarily oriented hydraulically fractured wells in composite clastic and naturally fractured reservoirs infinite in one direction, infinite in two directions and no-flow boundary condition.

It is important to note that this is by no means an elimination of all the assumptions found in the composite model literature; this work proposes a customizable three-dimensional semi-analytical approach to pressure transient testing in anisotropic composite clastic and naturally fractured reservoirs in a computationally cheap and effective manner.

1.4 Overview

The second chapter outlines the mathematical model to be used in analyzing the composite system in both clastic and naturally fractured reservoirs, some assumptions inherent in the system description and the solution methodology.

Chapters three, four and five discusses the drawdown response of horizontal well, hydraulically fractured well and vertical well producers respectively in various reservoir boundary conditions.

Chapter six focuses chiefly on interference testing of different wells under various boundary conditions with emphasis on horizontal well analysis.

In the last two paragraphs, type curves are used for drawdown response sensitivity analysis on various reservoir parameters while direct syntheses are developed from the asymptotic analysis of equation obtained from the model.

The appendices contain point-source solutions under various boundary conditions, productivity indices, pseudo skin factors, simulation and field data.

Chapter 2

Model Formulations

2.1 System Description

The reservoir has rectangular co-ordinates and is cuboid comprising two compartments separated by a thin semi-permeable barrier where compartment I has a point source producer and compartment II a point source observer as shown below.

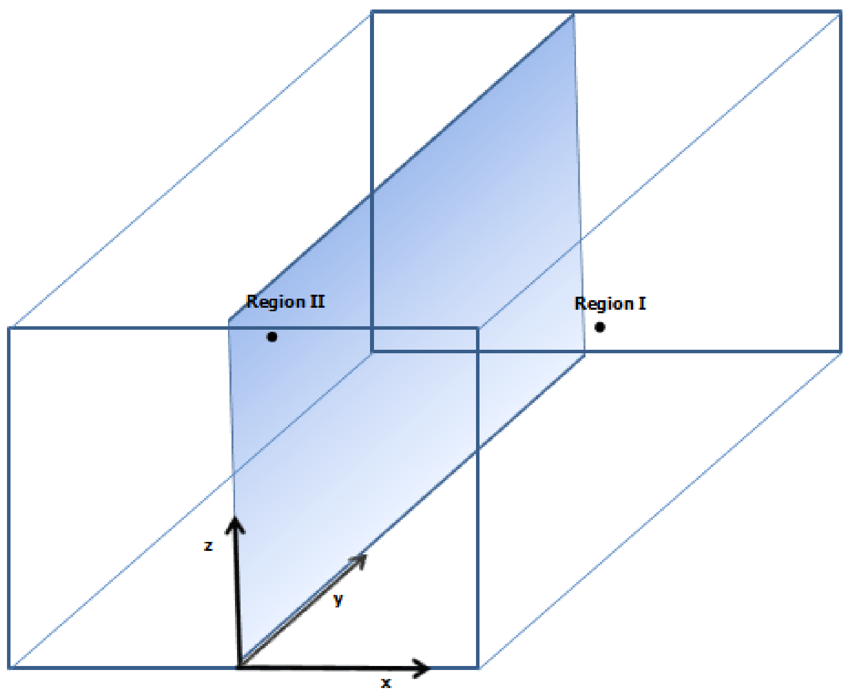

Fig.2.1.1 Composite reservoir system showing point sources (producer and observer)

Below are some of the properties of this system;

- Compartment I and II are anisotropic in terms of permeability
- The reservoir is heterogeneous in porosity, rock compressibility and other rock properties
- The semi-permeable barrier is of finite thickness, finite capacity and allows reasonable fluid flow along and across its plane

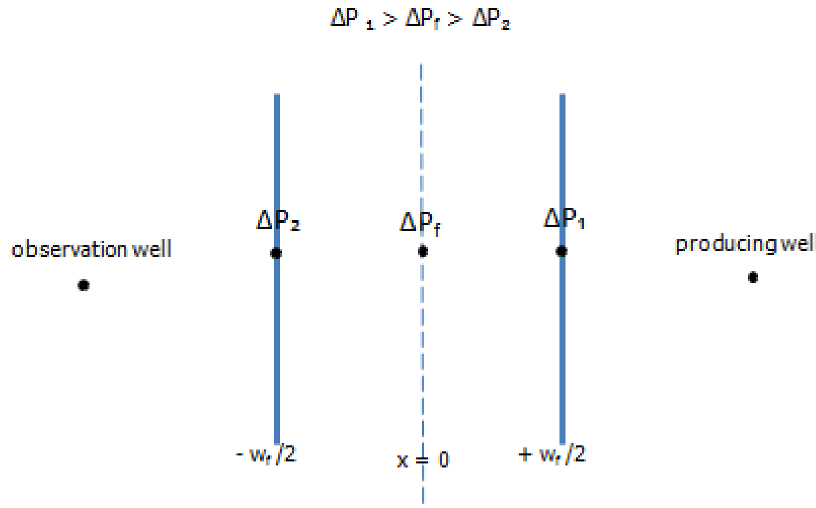

Fig.2.1.2 Semi-permeable barrier as fault zone

The semi-permeable barrier shown in Fig. 2.1.2 above as a fault can be modeled using the fluid leakage relationship below.

$$\frac{k_f A}{\mu_f} \frac{(\Delta P_1 - \Delta P_f)}{\frac{w_f}{2}}\Bigg|_{x=0} = \frac{k_{x1} A}{\mu_1} \frac{\partial \Delta P_1}{\partial x}\Big|_{x=0} \tag{2.1.1}$$

$$\frac{k_f A}{\mu_f} \frac{(\Delta P_f - \Delta P_2)}{\frac{w_f}{2}}\Bigg|_{x=0} = \frac{k_{x2} A}{\mu_2} \frac{\partial \Delta P_2}{\partial x}\Big|_{x=0} \tag{2.1.2}$$

From (2.1.1) and (2.1.2),

$$[\Delta P_1 - \Delta P_f]_{x=0} = \left(\frac{\frac{k_{x_1}}{\mu_1}}{\frac{k_f}{\mu_f}}\right)\left(\frac{w_f}{2}\right)\frac{\partial \Delta P_1}{\partial x}\Big|_{x=0} \tag{2.1.3}$$

$$[\Delta P_f - \Delta P_2]_{x=0} = \left(\frac{\frac{k_{x_2}}{\mu_2}}{\frac{k_f}{\mu_f}}\right)\left(\frac{w_f}{2}\right)\frac{\partial \Delta P_2}{\partial x}\Big|_{x=0} \tag{2.1.4}$$

From (2.1.3) and (2.1.4),

$$\left[\Delta P_1 - \Delta P_f\right]_{x=0} = \frac{w_f}{2} M_1 \frac{\partial \Delta P_1}{\partial x}\bigg|_{x=0} \tag{2.1.5}$$

$$\left[\Delta P_f - \Delta P_2\right]_{x=0} = \frac{w_f}{2} M_2 \frac{\partial \Delta P_2}{\partial x}\bigg|_{x=0} \tag{2.1.6}$$

The fluid flow equations governing the different regions of this composite system are given below.

- **Clastic Reservoirs**

$$k_{x_1} \frac{\partial^2 \Delta P_1}{\partial x^2} + k_{y_1} \frac{\partial^2 \Delta P_1}{\partial y^2} + k_{z_1} \frac{\partial^2 \Delta P_1}{\partial z^2} + q\mu_1 \delta(x - x_W)\delta(y - y_W)\delta(z - z_W) = \emptyset_1 C_{t_1} \mu_1 \frac{\partial \Delta P_1}{\partial t} \quad x > 0 \tag{2.1.7}$$

$$k_{x_2} \frac{\partial^2 \Delta P_2}{\partial x^2} + k_{y_2} \frac{\partial^2 \Delta P_2}{\partial y^2} + k_{z_2} \frac{\partial^2 \Delta P_2}{\partial z^2} = \emptyset_2 C_{t_2} \mu_2 \frac{\partial \Delta P_2}{\partial t} \qquad x < 0 \tag{2.1.8}$$

$$k_f \frac{\partial^2 \Delta P_f}{\partial y^2} + k_f \frac{\partial^2 \Delta P_f}{\partial z^2} + \frac{2}{w_f}\left[k_{x_1} \frac{\partial \Delta P_1}{\partial x} - k_{x_2} \frac{\partial \Delta P_2}{\partial x}\right]_{x=0} = \emptyset_f C_{tf} \mu_f \frac{\partial \Delta P_f}{\partial t} \tag{2.1.9}$$

- **Naturally Fractured Reservoirs**

$$k_{x_1}\frac{\partial^2 \Delta P_1}{\partial x^2} + k_{y_1}\frac{\partial^2 \Delta P_1}{\partial y^2} + k_{z_1}\frac{\partial^2 \Delta P_1}{\partial z^2} + q\mu_1 \delta(x - x_W)\delta(y - y_W)\delta(z - z_W)$$
$$= \emptyset_1 C_{t_1}\mu_1 \frac{\partial \Delta P_1}{\partial t} + \emptyset_{m_1} C_{t_{m_1}}\mu_{m_1}\frac{\partial \Delta P_{m_1}}{\partial t} \qquad x > 0 \qquad (2.1.10)$$

$$k_{x_2}\frac{\partial^2 \Delta P_2}{\partial x^2} + k_{y_2}\frac{\partial^2 \Delta P_2}{\partial y^2} + k_{z_2}\frac{\partial^2 \Delta P_2}{\partial z^2} = \emptyset_2 C_{t_2}\mu_2 \frac{\partial \Delta P_2}{\partial t} + \emptyset_{m_2} C_{t_{m_2}}\mu_{m_2}\frac{\partial \Delta P_{m_2}}{\partial t} \qquad x < 0 \qquad (2.1.11)$$

$$k_f\frac{\partial^2 \Delta P_f}{\partial y^2} + k_f\frac{\partial^2 \Delta P_f}{\partial z^2} + \frac{2}{w_f}\left[k_{x_1}\frac{\partial \Delta P_1}{\partial x} - k_{x_2}\frac{\partial \Delta P_2}{\partial x}\right]_{x=0} = \emptyset_f C_{tf}\mu_f \frac{\partial \Delta P_f}{\partial t} \qquad (2.1.12)$$

$$\emptyset_{m_1} C_{t_{m_1}}\frac{\partial \Delta P_{m_1}}{\partial t} = \frac{\psi_1 k_{m_1}(\Delta P_1 - \Delta P_{m_1})}{\mu_{m_1}} \qquad (2.1.13)$$

$$\emptyset_{m_2} C_{t_{m_2}}\frac{\partial \Delta P_{m_2}}{\partial t} = \frac{\psi_2 k_{m_2}(\Delta P_2 - \Delta P_{m_2})}{\mu_{m_2}} \qquad (2.1.14)$$

The reservoir boundary and initial conditions in this study include;

- **Infinite in two directions**

  Domain: $-\infty \leq x \leq \infty$; $-\infty \leq y \leq \infty$; $0 \leq z \leq h_Z$

x_W = distance between well and fault along x direction

y_W = distance from center of fault plane to point source along y direction

z_W = distance between well and lower reservoir boundary along z direction

Initial Condition

$$\Delta P_1(x,y,z,0) = \Delta P_2(x,y,z,0) = \Delta P_f(y,z,0) = 0 \qquad (2.1.15)$$

Boundary Condition

$$\Delta P_1(x \to \infty, y, z, t) = \Delta P_2(x \to -\infty, y, z, t) =$$
$$\Delta P_1(x, y \to \pm\infty, z, t) = \Delta P_2(x, y \to \pm\infty, z, t) = \Delta P_f(y \to \pm\infty, z, t) = 0 \qquad (2.1.16)$$

Other Conditions

$$\Delta P_1{}'|_{z=0,h_z} = \Delta P_2{}'|_{z=0,h_z} = \Delta P_f{}'\big|_{z=0,h_z} = 0 \qquad \forall\, x > 0, x < 0, x = 0 \qquad (2.1.17)$$

- **Finite in two directions**

Domain: $-\infty \leq x \leq \infty$; $0 \leq y \leq h_y$; $0 \leq z \leq h_z$

x_W = distance between well and fault along x direction

y_W = distance from point source to lower edge of reservoir along y direction

z_W = distance between well and lower reservoir boundary along z direction

Initial Condition

$$\Delta P_1(x, y, z, 0) = \Delta P_2(x, y, z, 0) = \Delta P_f(y, z, 0) = 0 \qquad (2.1.18)$$

Boundary Condition

$$\Delta P_1(x \to \infty, y, z, t) = \Delta P_2(x \to -\infty, y, z, t) = 0 \qquad (2.1.19)$$

Other Conditions

$$\Delta P_1{}'|_{z=0,h_z} = \Delta P_2{}'|_{z=0,h_z} = \Delta P_f{}'\big|_{z=0,h_z} = 0 \qquad \forall\, x > 0, x < 0, x = 0 \qquad (2.1.20)$$

$$\Delta P_1{}'|_{y=0,h_y} = \Delta P_2{}'|_{y=0,h_y} = \Delta P_f{}'\big|_{y=0,h_y} = 0 \qquad \forall\, x > 0, x < 0, x = 0 \qquad (2.1.21)$$

- **Finite in three directions**

Domain: $-h_x \le x \le h_x;\ 0 \le y \le h_y;\quad 0 \le z \le h_z$

x_W = distance between well and fault along x direction

y_W = distance from point source to lower edge of reservoir along y direction

z_W = distance between well and lower reservoir boundary along z direction

Initial Condition

$$\Delta P_1(x, y, z, 0) = \Delta P_2(x, y, z, 0) = \Delta P_f(y, z, 0) = 0 \tag{2.1.22}$$

Other Conditions

$$\Delta P_1{}'|_{x=h_x} = \Delta P_2{}'|_{x=-h_x} = 0 \tag{2.1.23}$$

$$\Delta P_1{}'|_{z=0,h_z} = \Delta P_2{}'|_{z=0,h_z} = \Delta P_f{}'\big|_{z=0,h_z} = 0 \qquad \forall\, x > 0, x < 0, x = 0 \tag{2.1.24}$$

$$\Delta P_1{}'|_{y=0,h_y} = \Delta P_2{}'|_{y=0,h_y} = \Delta P_f{}'\big|_{y=0,h_y} = 0 \qquad \forall\, x > 0, x < 0, x = 0 \tag{2.1.25}$$

2.2 General Assumptions

These are some of the assumption governing the formulation of the fluid flow equations both in clastic and naturally fractured reservoir systems

- Instantaneous steady point source
- Slightly compressible single-phase reservoir fluid of constant viscosity
- Fluid flow is Darcy
- Isothermal reservoir of uniform formation thickness
- The principal permeability direction is the x direction
- Negligible stress effect on reservoir properties
- The semi-permeable barrier is vertical, of limited width and isotropic in permeability
- The semi-permeable barrier is entirely matrix in NFR case
- Pseudo-steady matrix flow system for NFR
- Fractures mainly conduct fluid to the wellbore in NFR

In general the NFR system is based on the type 2 fracture system proposed by Warren et al. (1963) where two parameters (storativity ratio and inter-porosity flow) are sufficient to characterize the deviation of the behavior of a double-porosity medium from that of a homogeneous, porous medium. It is important to note that Darcy units are used in the course of this study.

2.3 Solution Methodology

Instantaneous point source and Green's function methods rank high among the documented tools used to solve heat and fluid flow problems over the years.

Gringarten et al. (1973) produced tables of instantaneous Green's and source functions which could be used with the Newman's product method to generate solutions for different reservoir flow problems, while Clonts et al. (1986) used these tables to generate the instantaneous source function for drain hole-reservoir configuration. However, the challenge with the Green's function method is finding the appropriate Green's function suited for the reservoir flow configuration under study.

(Cinco Ley 1974; Ozkan 1988) combined the instantaneous point source method with integration to analyze the productivity of horizontal wells; pressure behavior of slanted wells and inclined fractures in anisotropic reservoirs.

In this study, the flow equations are made dimensionless using reservoir or well parameters which are known to a reasonable degree, which allows the dimensionless solution obtained in this study to be compared to documented solutions or type curves; and considering the novelty of the proposed approach, the source method is used since it allows the solution of problems from first principles.

Additionally, Fourier and Laplace transforms are used in the solution process based on the limits of the spatial and temporal domains of the model (Jolley 1961; Tranter 1966; Davies 2001; Duffy 2004).

The point source solutions are converted to equivalent line source solutions by integrating as seen in (Cinco Ley 1974; Ozkan 1988): this line source solution is left in Laplace space to be inverted numerically using the Gaver-Stefhest algorithm (Davies 2001); the main reason being that the radius of investigation is a primary function of time. The method proposed by (Argawal et al. 1970; Ambastha et al. 1989) is used to incorporate Skin and Wellbore storage effect in Laplace space.

Different methods used in analyzing pressure data include type-curve matching, essentially a trial-and-error procedure used especially when there is insufficient data; pressure derivative (Tiab et al. 1980; Bourdet et al. 1989; Mattar 1999), a more sensitive technique to reservoir properties through amplification of heterogeneities hardly visible on conventional plots, consolidation of separate characteristics that would otherwise require different plots in a single graph, flow regime pattern recognition Tarek (2005); the TDS technique introduced by Tiab (1993) as an alternative to type curve matching.

Although series solutions of differential equations most times have slow convergence, the pressure derivative obtained from this solution is a signature since it converges quickly after a reasonable number of terms; hence it is used for analysis in this work.

The type curves produced from the corresponding line and plane source solutions are obtained with MATLAB® R2007b and Microsoft® Excel 2007. The figure below summarizes the work flow in the course of this study.

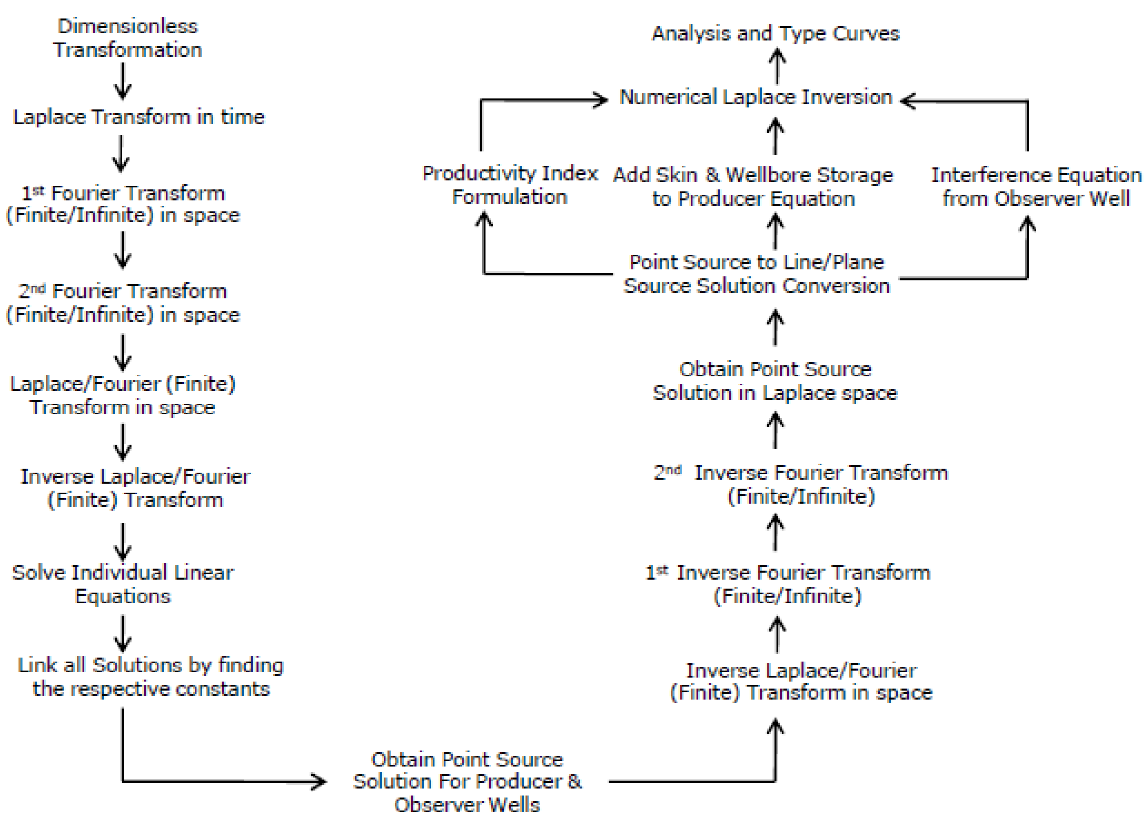

Fig. 2.3.1 Solution Methodology Workflow

2.4 Transforms for Reservoir Infinite in Two Directions

$$\mathcal{L}\{P(x_{D,}y_D,z_D,t_D)\} = \bar{P}(x_{D,}y_D,z_D,s) = \int_0^\infty P(x_{D,}y_D,z_D,t_D)\,e^{-st_D}dt_D \tag{2.4.1}$$

$$\mathcal{L}^{-1}\{\bar{P}(x_{D,}y_D,z_D,s)\} = P(x_{D,}y_D,z_D,t_D) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st_D}\bar{P}(x_{D,}y_D,z_D,s)ds \tag{2.4.2}$$

$$\mathcal{F}\{\bar{P}(x_{D,}y_D,z_D,s)\} = \dot{\bar{P}}\,(x_{D,}y_D,k,s) = \frac{2}{\mathrm{h}_{zD}}\int_0^{\mathrm{h}_{zD}}\bar{P}(x_{D,}y_D,z_D,s)\cos(\frac{n\pi z_D}{\mathrm{h}_{zD}})\,dz_D \tag{2.4.3}$$

$$\mathcal{F}^{-1}\left\{\dot{\bar{P}}\,(x_{D,}y_D,k,s)\right\} = \bar{P}(x_{D,}y_D,z_D,s) = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(\frac{n\pi z_D}{\mathrm{h}_{zD}}) \tag{2.4.4}$$

$$\mathcal{F}\left\{\dot{\bar{P}}\,(x_{D,}y_D,k,s)\right\} = \hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\dot{\bar{P}}\,(x_{D,}y_D,k,s)\,e^{-i\omega y_D}dy_D \tag{2.4.5}$$

$$\mathcal{F}^{-1}\left\{\hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s)\right\} = \dot{\bar{P}}\,(x_{D,}y_D,k,s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s)\,e^{i\omega y_D}d\omega \tag{2.4.6}$$

$$\mathcal{L}\left\{\hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s)\right\} = \hat{\dot{\bar{\bar{P}}}}\,(\acute{s},\omega,k,s) = \int_0^\infty\hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s)\,e^{-\acute{s}x_D}dx_D \tag{2.4.7}$$

$$\mathcal{L}^{-1}\left\{\hat{\dot{\bar{\bar{P}}}}\,(\acute{s},\omega,k,s)\right\} = \hat{\dot{\bar{P}}}\,(x_{D,}\omega,k,s) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\acute{s}x_D}\,\hat{\dot{\bar{\bar{P}}}}\,(\acute{s},\omega,k,s)d\acute{s} \tag{2.4.8}$$

2.5 Transforms for Reservoir Finite in Two Directions

$$\mathcal{L}\{P(x_{D,}y_D, z_D, t_D)\} = \bar{P}(x_{D,}y_D, z_D, s) = \int_0^{\infty} P(x_{D,}y_D, z_D, t_D)\, e^{-st_D} dt_D \tag{2.5.1}$$

$$\mathcal{L}^{-1}\{\bar{P}(x_{D,}y_D, z_D, s)\} = P(x_{D,}y_D, z_D, t_D) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st_D}\bar{P}(x_{D,}y_D, z_D, s) ds \tag{2.5.2}$$

$$\mathcal{F}\{\bar{P}(x_{D,}y_D, z_D, s)\} = \dot{\bar{P}}\,(x_{D,}y_D, k, s) = \frac{2}{\mathrm{h}_{zD}}\int_0^{\mathrm{h}_{zD}} \bar{P}(x_{D,}y_D, z_D, s)\cos(\frac{n\pi z_D}{\mathrm{h}_{zD}})\, dz_D \tag{2.5.3}$$

$$\mathcal{F}^{-1}\left\{\dot{\bar{P}}\,(x_{D,}y_D, k, s)\right\} = \bar{P}(x_{D,}y_D, z_D, s) = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(\frac{n\pi z_D}{\mathrm{h}_{zD}}) \tag{2.5.4}$$

$$\mathcal{F}\left\{\dot{\bar{P}}\,(x_{D,}y_D, k, s)\right\} = \ddot{\bar{P}}\,(x_{D,}l, k, s) = \frac{2}{\mathrm{h}_{yD}}\int_0^{\mathrm{h}_{yD}} \dot{\bar{P}}\,(x_{D,}y_D, z_D, s)\cos(\frac{m\pi y_D}{\mathrm{h}_{yD}})\, dy_D \tag{2.5.5}$$

$$\mathcal{F}^{-1}\left\{\ddot{\bar{P}}\,(x_{D,}l, k, s)\right\} = \dot{\bar{P}}\,(x_{D,}y_D, k, s) = \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty} \mathcal{F}_m \cos\left(\frac{m\pi y_D}{\mathrm{h}_{yD}}\right) \tag{2.5.6}$$

$$\mathcal{L}\left\{\ddot{\bar{P}}\,(x_{D,}l, k, s)\right\} = \ddot{\bar{\bar{P}}}\,(\acute{s}, l, k, s) = \int_0^{\infty} \ddot{\bar{P}}\,(x_{D,}l, k, s)\, e^{-\acute{s}x_D} dx_D \tag{2.5.7}$$

$$\mathcal{L}^{-1}\left\{\ddot{\bar{\bar{P}}}\,(\acute{s}, l, k, s)\right\} = \ddot{\bar{P}}\,(x_{D,}l, k, s) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\acute{s}x_D}\,\ddot{\bar{\bar{P}}}\,(\acute{s}, l, k, s) d\acute{s} \tag{2.5.8}$$

2.6 Transforms for Reservoir Finite in Three Directions

$$\mathcal{L}\{P(x_{D,}y_D, z_D, t_D)\} = \bar{P}(x_{D,}y_D, z_D, s) = \int_0^\infty P(x_{D,}y_D, z_D, t_D)\, e^{-st_D} dt_D \qquad (2.6.1)$$

$$\mathcal{L}^{-1}\{\bar{P}(x_{D,}y_D, z_D, s)\} = P(x_{D,}y_D, z_D, t_D) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st_D}\bar{P}(x_{D,}y_D, z_D, s) ds \qquad (2.6.2)$$

$$\mathcal{F}\{\bar{P}(x_{D,}y_D, z_D, s)\} = \dot{\bar{P}}\,(x_{D,}y_D, k, s) = \frac{2}{h_{zD}}\int_0^{\mathrm{h}_{zD}} \bar{P}(x_{D,}y_D, z_D, s)\cos(\frac{n\pi z_D}{h_{zD}})\, dz_D \qquad (2.6.3)$$

$$\mathcal{F}^{-1}\left\{\dot{\bar{P}}\,(x_{D,}y_D, k, s)\right\} = \bar{P}(x_{D,}y_D, z_D, s) = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(\frac{n\pi z_D}{h_{zD}}) \qquad (2.6.4)$$

$$\mathcal{F}\left\{\dot{\bar{P}}\,(x_{D,}y_D, k, s)\right\} = \ddot{\bar{P}}\,(x_{D,}l, k, s) = \frac{2}{h_{yD}}\int_0^{h_{yD}} \dot{\bar{P}}\,(x_{D,}y_D, z_D, s)\cos(\frac{m\pi y_D}{h_{yD}})\, dy_D \qquad (2.6.5)$$

$$\mathcal{F}^{-1}\left\{\ddot{\bar{P}}\,(x_{D,}l, k, s)\right\} = \dot{\bar{P}}\,(x_{D,}y_D, k, s) = \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty} \mathcal{F}_m \cos\left(\frac{m\pi y_D}{h_{yD}}\right) \qquad (2.6.6)$$

$$\mathcal{F}\left\{\ddot{\bar{P}}\,(x_{D,}l, k, s)\right\} = \dddot{\bar{P}}\,(\omega, l, k, s) = \frac{2}{h_{xD}}\int_0^{h_{xD}} \ddot{\bar{P}}\,(x_{D,}l, k, s)\cos(\frac{p\pi x_D}{h_{xD}})\, dx_D \qquad (2.6.7)$$

$$\mathcal{F}^{-1}\left\{\dddot{\bar{P}}\,(\omega, l, k, s)\right\} = \ddot{\bar{P}}\,(x_{D,}l, k, s) = \frac{\mathcal{F}_0^p}{2} + \sum_{p=1}^{\infty} \mathcal{F}_p \cos\left(\frac{p\pi x_D}{h_{xD}}\right) \qquad (2.6.8)$$

Chapter 3

Arbitrary Oriented Horizontal Wells

Although hydraulic fracturing (HF) has been a potential rival to horizontal drilling, horizontal well (HW) has proven its worth time after time in situations of high coning tendencies where HF has not been feasible. Clonts et al. (1986) also noted that HW supersedes HF and vertical well when multiple targets and tight formation with vertical wells is involved. Moreover, it is easier to expose greater portion of the formation to longer producing interval lengths in HW than HF since we can easily monitor and keep control of the HW path with the prevailing technology.

3.1 Dimensionless Transformation

These dimensionless transformations are used in obtaining subsequent point source solutions.

$$x_D = \frac{x}{L} \tag{3.1.1}$$

$$y_D = \frac{y}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{3.1.2}$$

$$z_D = \frac{z}{L}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{3.1.3}$$

$$r_{WD} = \frac{r_W}{L} \tag{3.1.4}$$

$$w_D = \frac{w_f}{2L} \tag{3.1.5}$$

$$h_{xD} = \frac{h_x}{L} \tag{3.1.6}$$

$$h_{yD} = \frac{h_y}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{3.1.7}$$

$$h_{zD} = \frac{h_z}{L}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{3.1.8}$$

$$t_D = \frac{k_{x_1} t}{\emptyset_1 \mu_1 C_{t_1} L^2} \tag{3.1.9}$$

$$C_D = \frac{C}{2\pi {r_w}^2 \emptyset_1 L C_{t_1}} \tag{3.1.10}$$

$$P_{D_{n,f}} = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}}\Delta P_{n,f}}{\mu_1 q} \quad \Delta P_{n,f} = P_i - P_{n,f} \tag{3.1.11}$$

$$F_{CD} = \frac{w_f k_f}{2k_{x_1} L} \tag{3.1.12}$$

$$x_D{}' = \frac{Lcos\theta}{L} \tag{3.1.13}$$

$$y_D{}' = \frac{Lsin\theta}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{3.1.14}$$

$$\theta' = \arctan\left(\sqrt{\frac{k_{x_1}}{k_{y_1}}} tan\theta\right) \tag{3.1.15}$$

$$dy_D = dL_{yD} sin\theta \tag{3.1.16}$$

$$L_{yD} = \frac{1}{2}\sqrt{\left(x_D{}'^2 + y_D{}'^2 \right)} \tag{3.1.17}$$

$$L_{yD} = \frac{1}{2}\sqrt{\cos^2\theta + \frac{k_{x_1}}{k_{y_1}}\sin^2\theta} = \frac{1}{2}\sqrt{\cos^2\theta + v_y \sin^2\theta} \tag{3.1.18}$$

Converting to the y plane, L_{yD} $becomes$ $L_{yD} sin\theta'$

This might be used interchangeably for convenience purposes

3.2 Line source solution for Clastic Reservoirs

- Infinite in two directions

The point source solution for a reservoir infinite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin}$$
$$= \frac{1}{\mathrm{sh}_{zD}}\left[\begin{array}{l} \displaystyle\int_{-\infty}^{\infty} \frac{(1+\mathrm{RS_w})}{2R}\left\{1 + e^{-2Rx_{WD}}\frac{\left(RD_p - D_p^{'}\right)}{\left(RD_p + D_p^{'}\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \\ \displaystyle + \sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\left\{1 + e^{-2Qx_{WD}}\frac{\left(QC_p - C_p^{'}\right)}{\left(QC_p + C_p^{'}\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \end{array}\right]$$

(3.2.1)

Converting the point source solution to line source solution by integrating Eq. 3.2.1 with respect to (w.r.t.) L_{yD} from Y_{WD} - $L_{yD}\sin\theta^{/}$ to Y_{WD} + $L_{yD}\sin\theta^{/}$. Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{2}{\mathrm{sh}_{zD}\,sin\theta}(\alpha + \beta)$$

$$\alpha + \beta = \left[\begin{array}{l} \displaystyle\int_{-\infty}^{\infty} \frac{1}{\omega}\frac{(1+\mathrm{RS_w})}{2R}\left\{1 + e^{-2Rx_{WD}}\frac{\left(RD_p - D_p^{'}\right)}{\left(RD_p + D_p^{'}\right)}\right\}\sin\left(\omega L_{yD}\right)d\omega \\ \displaystyle + \sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\omega}\frac{(1+\mathrm{QS_w})}{Q}\cos^2(\,kz_{WD})\left\{1 + e^{-2Qx_{WD}}\frac{\left(QC_p - C_p^{'}\right)}{\left(QC_p + C_p^{'}\right)}\right\}\sin\left(\omega L_{yD}\right)d\omega \end{array}\right] \quad (3.2.2)$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin\,and\,wellbore\,storage}$$
$$= \frac{\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin}}{1 + s^2 C_D\,\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin}} \quad (3.2.3)$$

- **Finite in two directions**

The point source solution for a reservoir finite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD},y_D,z_D,s)_{with\,skin} = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha+\beta) \tag{3.2.4}$$

$$\alpha = \left[\begin{array}{c} \dfrac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{4\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\dfrac{\left(\sqrt{\mathrm{s}}E_p-E_p{'}\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p{'}\right)}\right\} \\ +\displaystyle\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{'}\right)}{\left(TF_p+F_p{'}\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p{'}\right)}{\left(RD_p+D_p{'}\right)}\right\} \\ +\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p{'}\right)}{\left(QC_p+C_p{'}\right)}\right\} \end{array}\right]$$

$$\alpha = \mathcal{F}_{00}+\mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n}+\mathcal{F}_{mn}$$

Converting the point source solution to line source solution by integrating Eq. 3.2.4 w.r.t. L_{yD} from Y_{WD} - $L_{yD}\sin\theta^{/}$ to Y_{WD} + $L_{yD}\sin\theta^{/}$. Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{WD}\,(x_{WD},y_{WD},z_{WD},s)_{with\,skin} = \frac{4\pi}{sh_{yD}h_{zD}sin\theta}(\alpha+\beta) \tag{3.2.5}$$

$$\alpha = \left[\begin{array}{c} \dfrac{L_{yD}\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{2\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\dfrac{\left(\sqrt{\mathrm{s}}E_p-E_p{'}\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p{'}\right)}\right\} \\ +\displaystyle\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{Tl}\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{'}\right)}{\left(TF_p+F_p{'}\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{L_{yD}(1 + \mathrm{RS_w})}{R} \cos^2(kz_{WD}) \left\{ 1 + e^{-2Rx_{WD}} \frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)} \right\} \\ + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2(1 + \mathrm{QS_w})}{Ql} \cos^2(kz_{WD}) \sin l\left(L_{yD}\right) \cos^2(ly_{WD}) \left\{ 1 + e^{-2Qx_{WD}} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)} \right\} \end{array} \right]$$

Dimensionless pressure at the wellbore becomes;

$$\begin{aligned} &\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage} \\ &= \frac{\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2 C_D\ \overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \end{aligned} \qquad (3.2.6)$$

- **Finite in three directions**

The point source solution for a reservoir finite in three directions is given below.

$$\overline{P}_{WD}\ (x_{WD}, y_D, z_D, s)_{with\ skin}\ = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{3.2.7}$$

$$\mathcal{F}_{00} = \left[\frac{1}{4\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{matrix}\cosh\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + S_w\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\sinh2\sqrt{\mathrm{s}}(h_{xD} - x_{WD}) + 1 + \cosh2\sqrt{\mathrm{s}}(h_{xD} - x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})}\left\{\begin{matrix}\cosh\mathrm{T}(h_{xD} - 2x_{WD}) + S_w\mathrm{T}\sinh\mathrm{T}(h_{xD} - 2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w\mathrm{T}\sinh2\mathrm{T}(h_{xD} - x_{WD}) + 1 + \cosh2\mathrm{T}(h_{xD} - x_{WD})]}{TF_p\sinh\mathrm{T}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)}{2R\sinh R(h_{xD})}\left\{\begin{matrix}\cosh R(h_{xD} - 2x_{WD}) + S_w\mathrm{R}\sinh R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w\mathrm{R}\sinh2R(h_{xD} - x_{WD}) + 1 + \cosh2R(h_{xD} - x_{WD})]}{RD_p\sinh R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{matrix}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})}\times \\ \left\{\begin{matrix}\cosh Q(h_{xD} - 2x_{WD}) + S_w\mathrm{Q}\sinh Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w\mathrm{Q}\sinh2Q(h_{xD} - x_{WD}) + 1 + \cosh2Q(h_{xD} - x_{WD})]}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{matrix}\right\}\end{matrix}\right]$$

Converting the point source solution to line source solution by integrating Eq. 3.2.7 w.r.t. L_{yD} from Y_{WD} - $L_{yD}\sin\theta^{/}$ to Y_{WD} + $L_{yD}\sin\theta^{/}$

Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{4\pi}{\mathrm{s}\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta} (\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{3.2.8}$$

$$\mathcal{F}_{00} = \left[\frac{L_{yD}}{2\sqrt{\mathrm{s}}\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD})} \left\{ \begin{matrix} \mathrm{cosh}\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + S_w\sqrt{\mathrm{s}}\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right) \\ - \dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\mathrm{sinh}2\sqrt{\mathrm{s}}(h_{xD} - x_{WD}) + 1 + \mathrm{cosh}2\sqrt{\mathrm{s}}(h_{xD} - x_{WD})\right]}{\sqrt{\mathrm{s}}E_p \,\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD}) + E_p{}' \cosh\sqrt{\mathrm{s}}(h_{xD})} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty} \frac{\sin l\left(L_{yD}\right)\cos^2(\,l y_{WD})}{Tl\sinh T(h_{xD})} \left\{ \begin{matrix} \mathrm{coshT}(h_{xD} - 2x_{WD}) + S_w\mathrm{TsinhT}(h_{xD} - 2x_{WD}) + \cosh(Th_{xD}) \\ - \dfrac{F_p{}'[S_w\mathrm{Tsinh2T}(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2T}(h_{xD} - x_{WD})]}{TF_p\,\mathrm{sinhT}(h_{xD}) + F_p{}'\cosh T(h_{xD})} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty} \frac{L_{yD}\cos^2(\,k z_{WD})}{R\sinh R(h_{xD})} \left\{ \begin{matrix} \mathrm{cosh}R(h_{xD} - 2x_{WD}) + S_w\mathrm{Rsinh}R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) \\ - \dfrac{D_p{}'[S_w\mathrm{Rsinh2}R(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2}R(h_{xD} - x_{WD})]}{RD_p\,\mathrm{sinh}R(h_{xD}) + D_p{}'\cosh R(h_{xD})]} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{mn} = \left[\begin{matrix} \displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos^2(\,k z_{WD})\sin l\left(L_{yD}\right)\cos^2(\,l y_{WD})}{Ql\sinh Q(h_{xD})} \times \\ \left\{ \begin{matrix} \mathrm{cosh}Q(h_{xD} - 2x_{WD}) + S_w\mathrm{Qsinh}Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) \\ - \dfrac{C_p{}'[S_w\mathrm{Qsinh2}Q(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2}Q(h_{xD} - x_{WD})]}{QC_p\,\mathrm{sinh}Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})} \end{matrix} \right\} \end{matrix} \right]$$

Dimensionless pressure at the wellbore becomes;

$$\begin{aligned} &\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage} \\ &= \frac{\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2 C_D\,\overline{P}_{WD}\ (x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \end{aligned} \tag{3.2.9}$$

3.3 Line source solution for Naturally Fractured Reservoirs

Everything is the same as section 3.2 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

3.4 Type Curves

This subsection is devoted to the generated producer drawdown response type curves under various boundary conditions depicting key features.

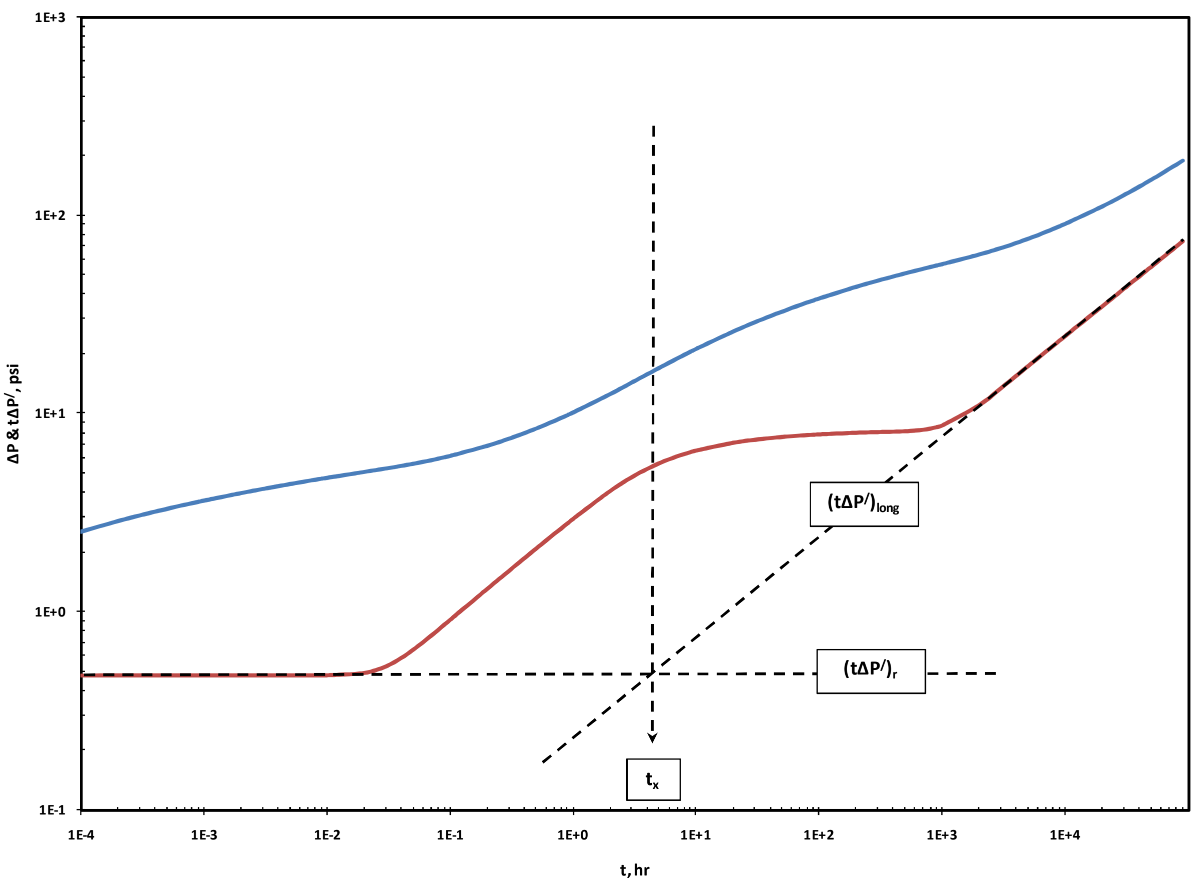

Fig. 3.4.1 Representation showing key interpretation features

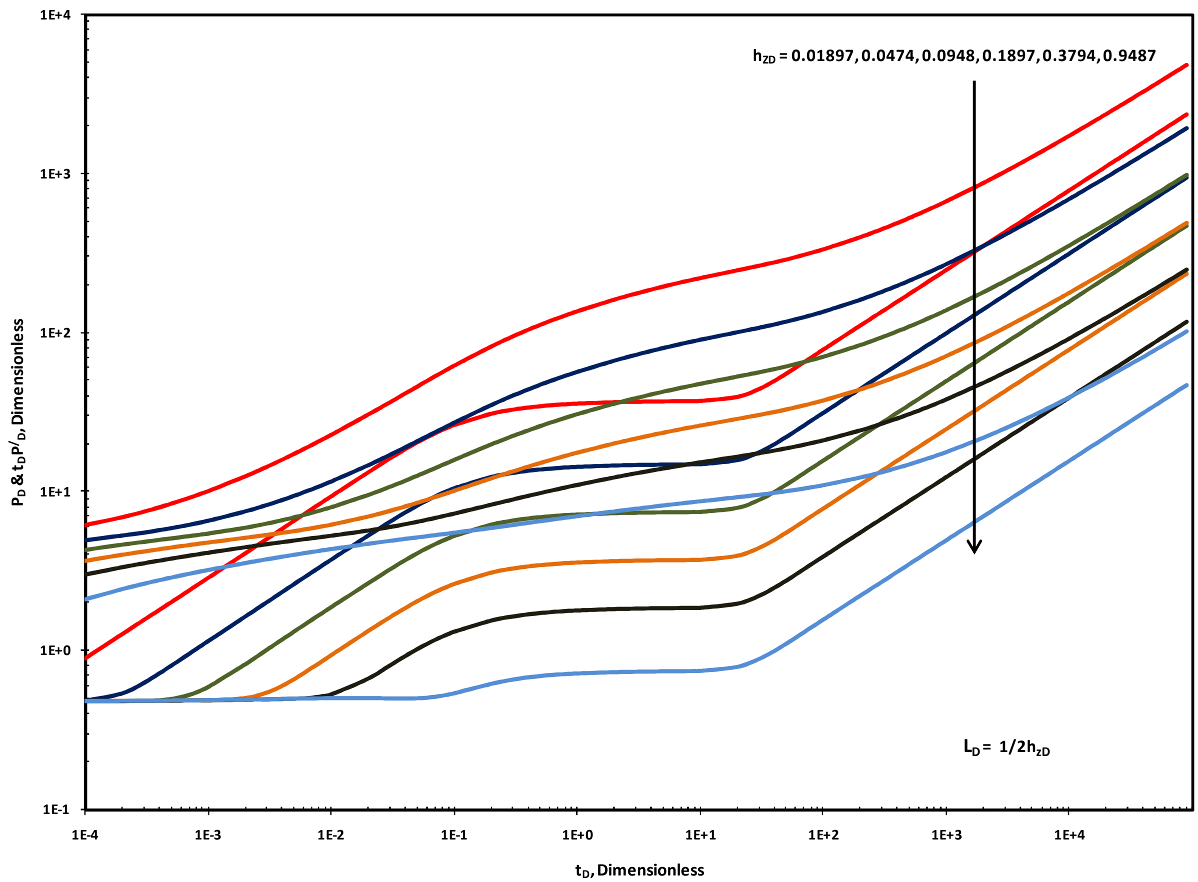

Fig. 3.4.2 Effect of characteristic horizontal well length

The characteristic horizontal well length is inversely proportional to the dimensionless reservoir height given in the figure above. The figure shows that as the characteristic horizontal well length increases, it exposes more formation to flow into the well from the vertical radial direction and there exists a cutoff for which the early radial flow regime is almost non-existent causing the loss of that initial production from the vertical plane. This means that for the same reservoir height and well length, the reservoir's vertical permeability must be comparatively good with respect to the horizontal permeability for horizontal wells to effectively drain the reservoir.

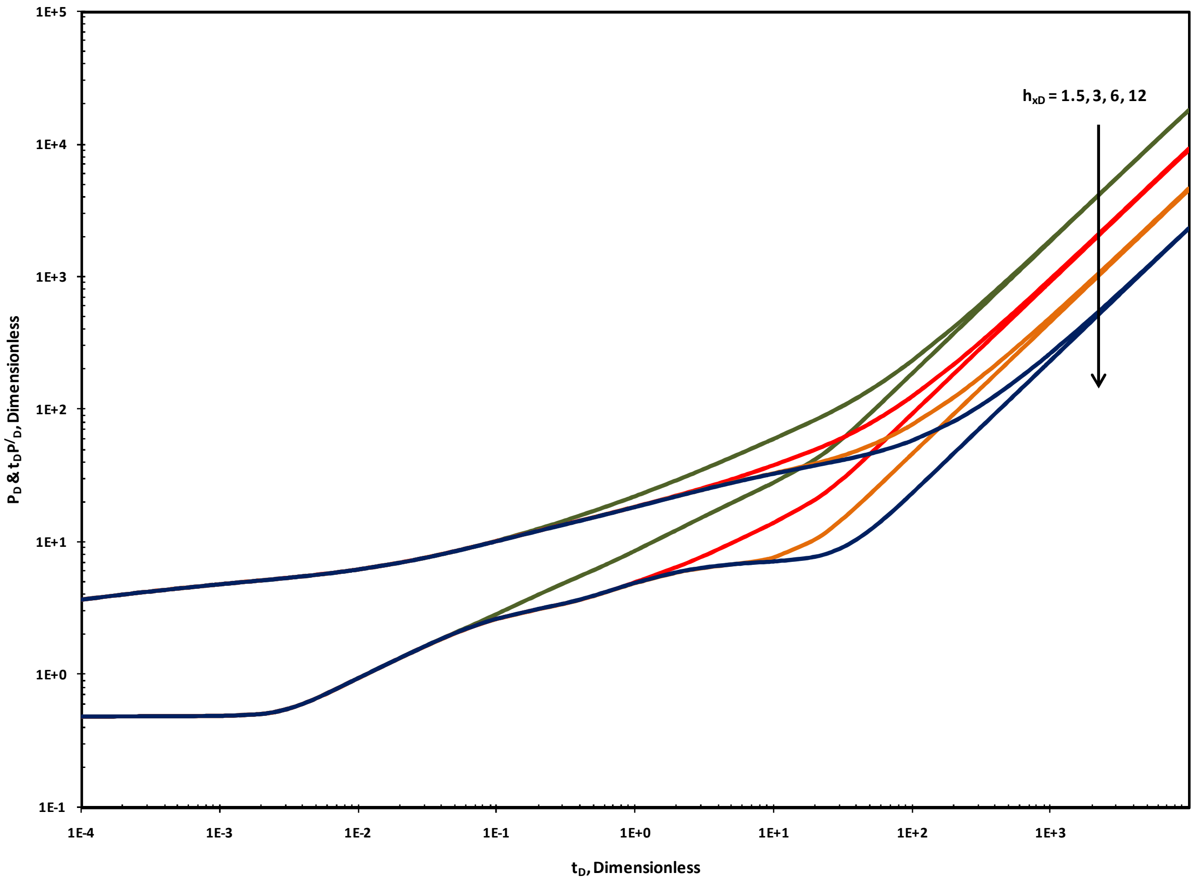

Fig. 3.4.3 Effect of reservoir size along x axis

The figure depicts that provided the reservoir is long enough in the x-direction, the duration of the pseudo-radial flow (which is along the horizontal plane) is extended until fluid-flow gets to the reservoir boundaries.

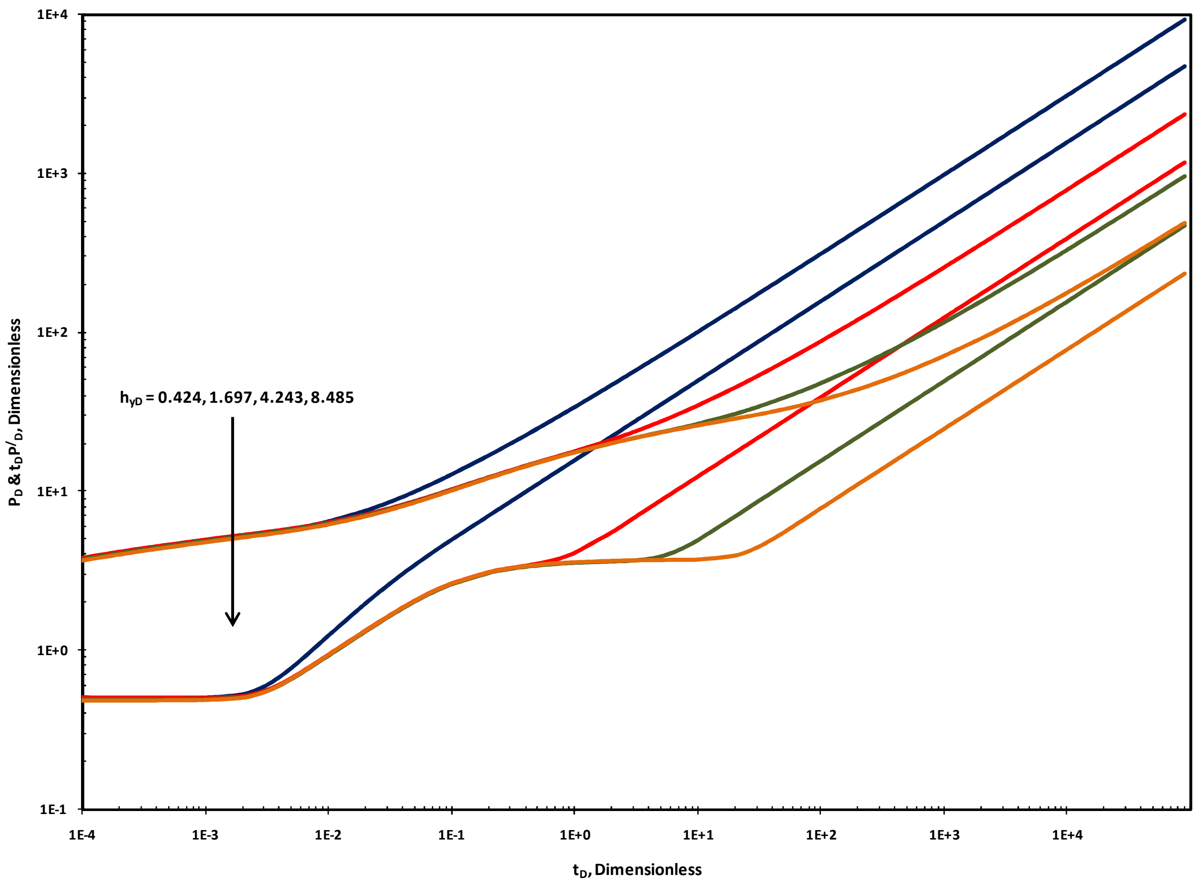

Fig. 3.4.4 Effect of reservoir size along y axis

The figure also depicts that provided the reservoir is long enough in the y-direction, the duration of the pseudo-radial flow (which is along the horizontal plane) is extended until fluid-flow gets to the reservoir boundaries just as should be expected for reservoir increase in x-direction.

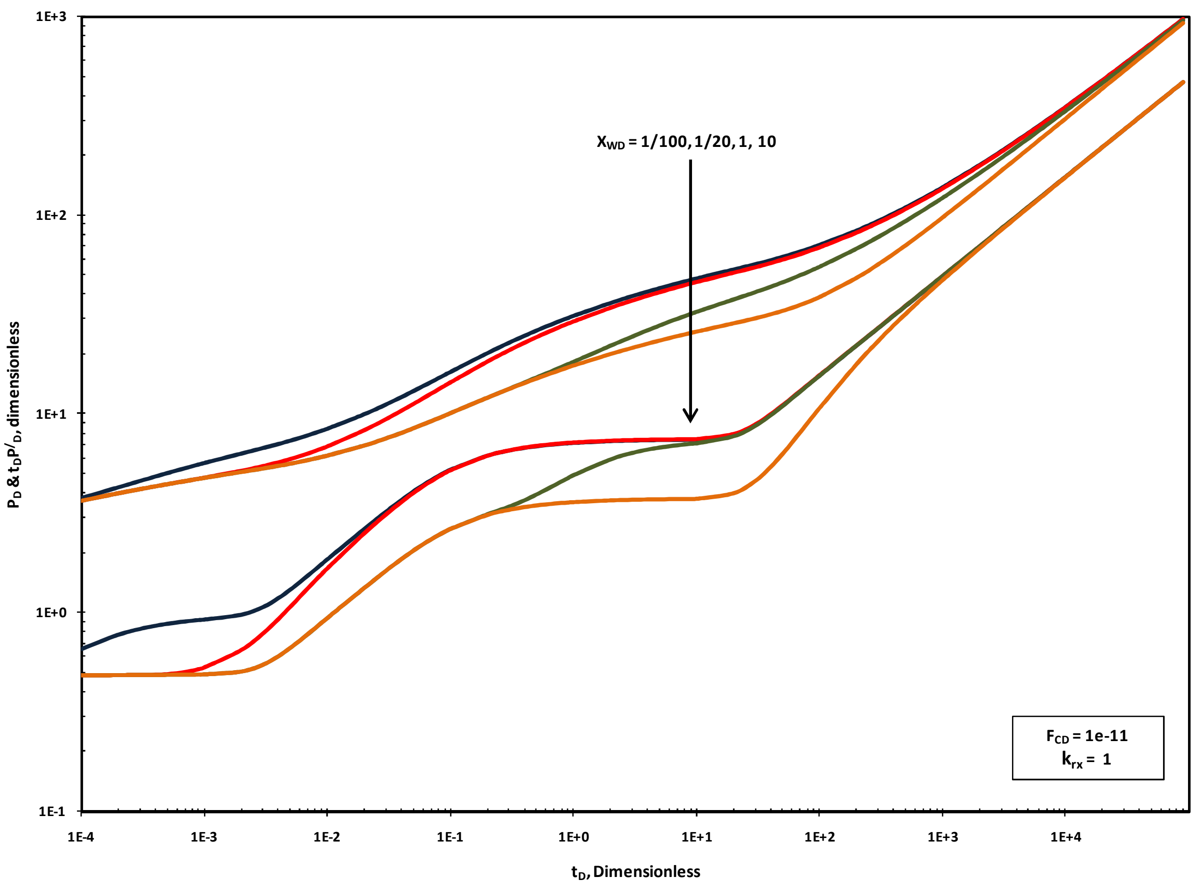

Fig. 3.4.5 Effect of horizontal well distance to fault

The figure above shows that the closer the producer is to the fault, the shorter and distorted the early radial flow regime becomes since the time to reach to the fault would vary with distance to fault. Additionally, depending on the relative position of the producer from other sealing boundaries around in the reservoir, slope-doubling might be experienced as can be seen for X_{WD}=1.

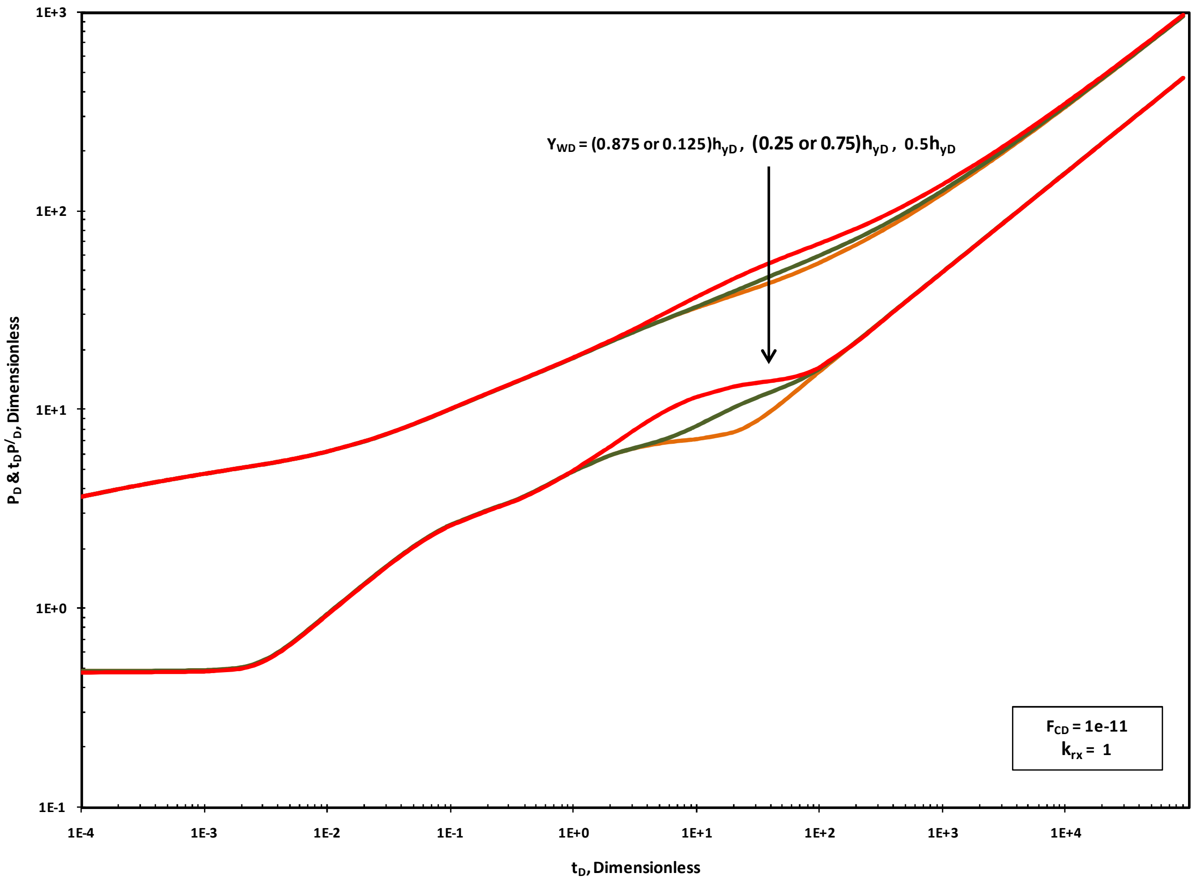

Fig. 3.4.6 Effect of horizontal well distance to reservoir boundary along y-axis

The figure above illustrates the effect of pressure support from other parts of the reservoir. The closer the producer is to a reservoir boundary, the higher the pressure drop experienced since it will take the fluid coming from other parts of the reservoir a longer time to get to the well and since the parameter under discussion is the distance to the reservoir boundary along y-axis, the effect would be felt during the pseudo-radial flow regime.

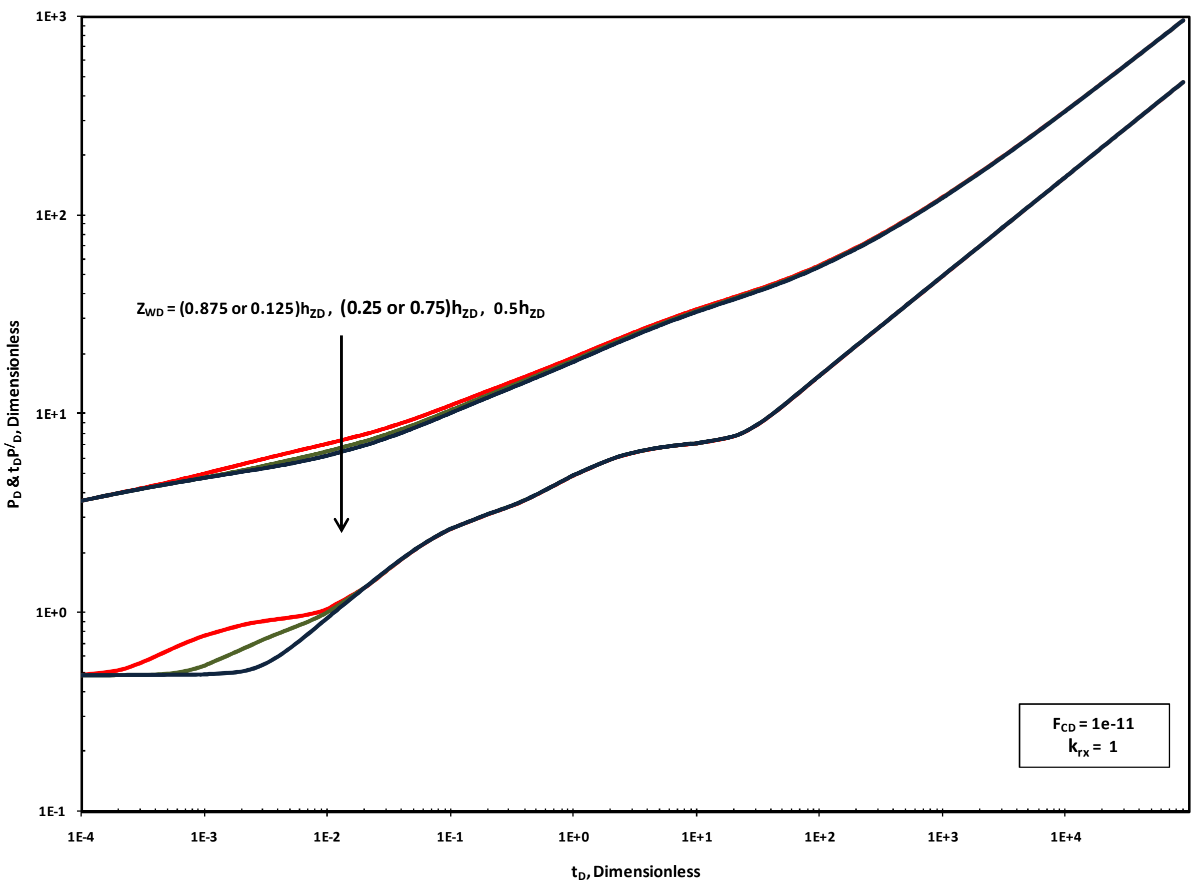

Fig. 3.4.7 Effect of horizontal well distance to reservoir boundary along z-axis

The figure above shows a similar effect to that observed in Fig. 3.4.6 but now this phenomenon is experienced during the early radial flow regime which is along the vertical plane.

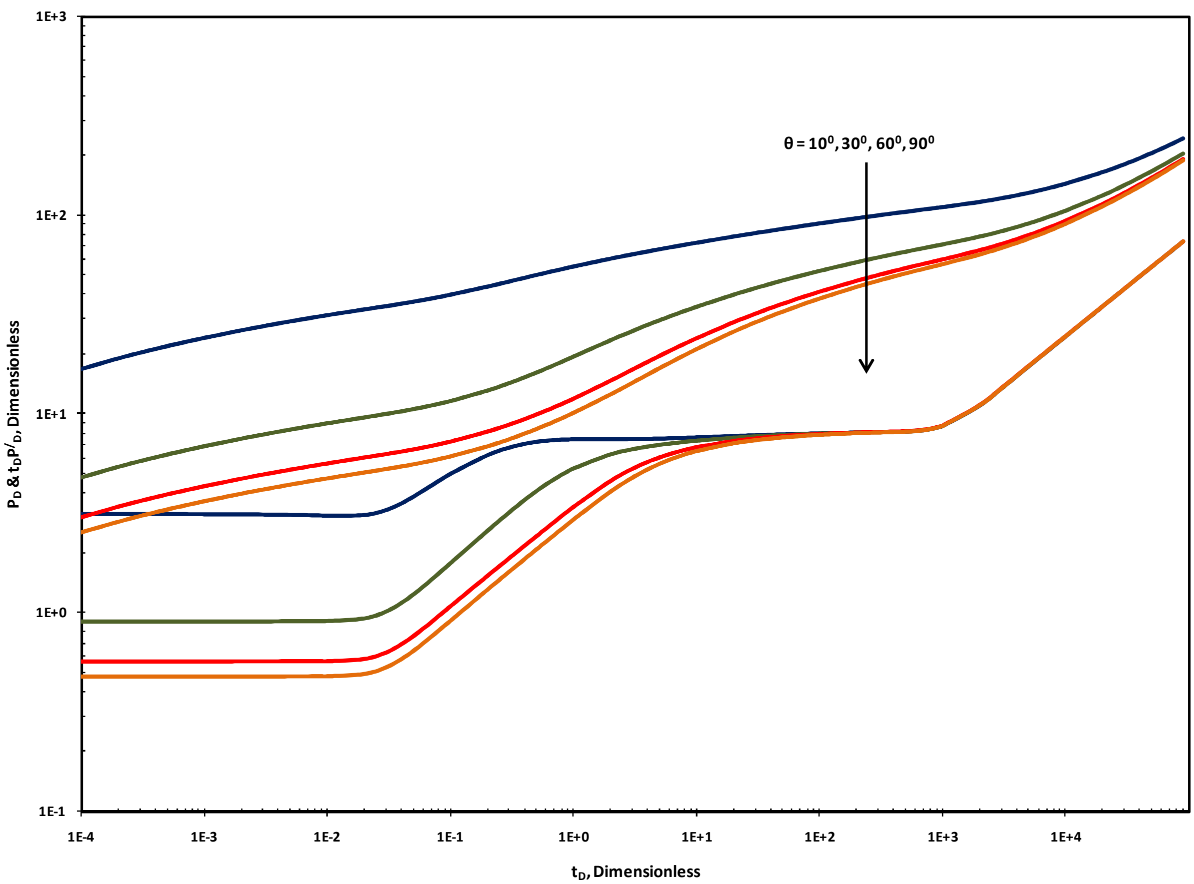

Fig. 3.4.8 Effect of horizontal well angle in reservoir with high permeability anisotropy

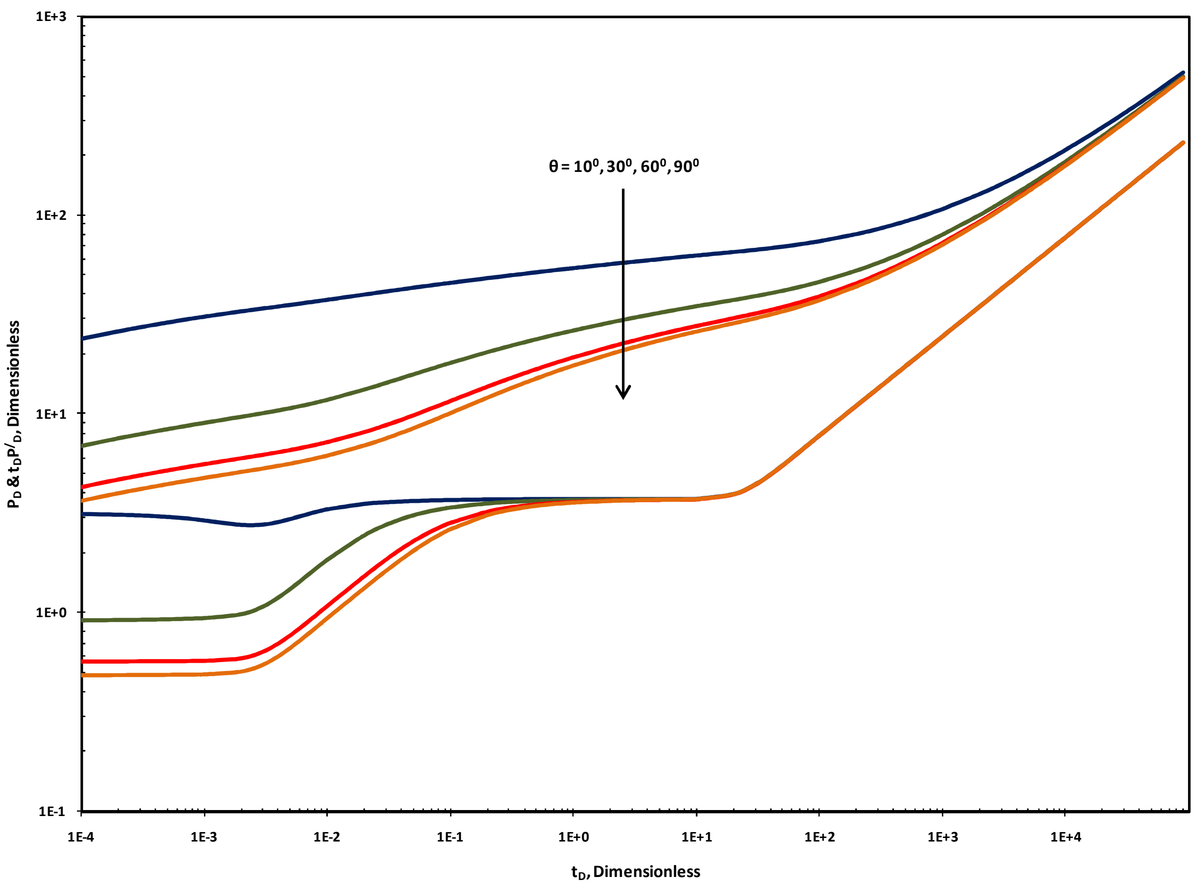

Fig. 3.4.9 Effect of horizontal well angle in reservoir with low permeability anisotropy

Figs. 3.4.8 and 3.4.9 show that the early pressure behavior is greatly impacted by the well orientation to the maximum permeability, which is in x-direction in study. As the angle of deviation decreases, the infinite conductivity effect of the horizontal well decreases. However the late behavior is the same for all angles of deviation from the maximum permeability. The well-orientation angle does not affect the start and end of the early radial flow regime irrespective of the degree of areal permeability anisotropy as noted by (Yildiz et al. 1997; Spivey et al. 1999). As the pressure response continues, pseudo-radial flow develops and its start time is affected by the inclination angle; the smaller the angle, the earlier it starts and vice versa. As the horizontal well becomes parallel to the maximum permeability direction, the early linear flow disappears. This is because the impact of the maximum permeability to the early flow regimes has been greatly reduced. Also, the lesser the areal permeability anisotropy, the earlier the pseudo-radial flow regime starts and vice versa. This helps plan how long a well test should be conducted to attain certain flow regimes that can help provide information about target flow or reservoir parameters.

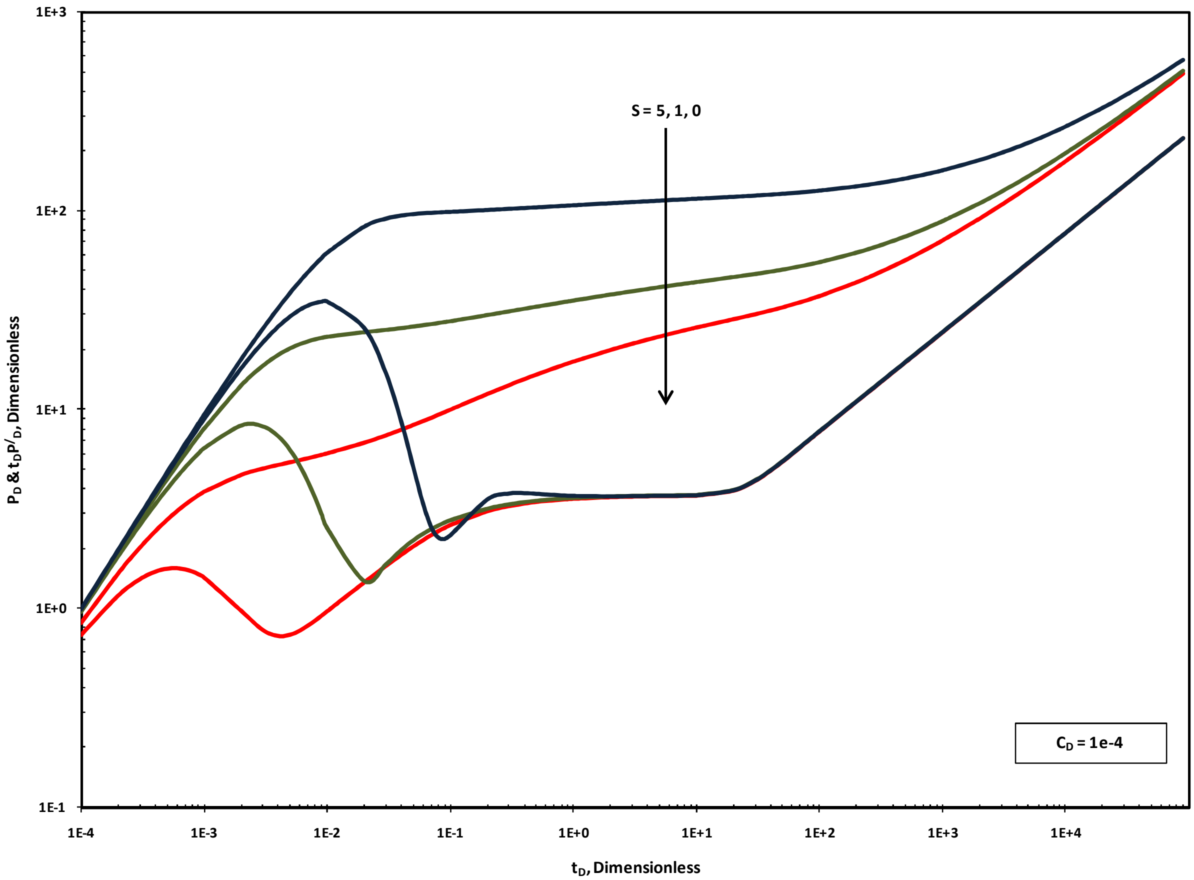

Fig. 3.4.10 Skin effect on horizontal well drawdown response

Fig. 3.4.10 shows the effect of skin and wellbore storage on the pressure response. Normal response as expected is observed.

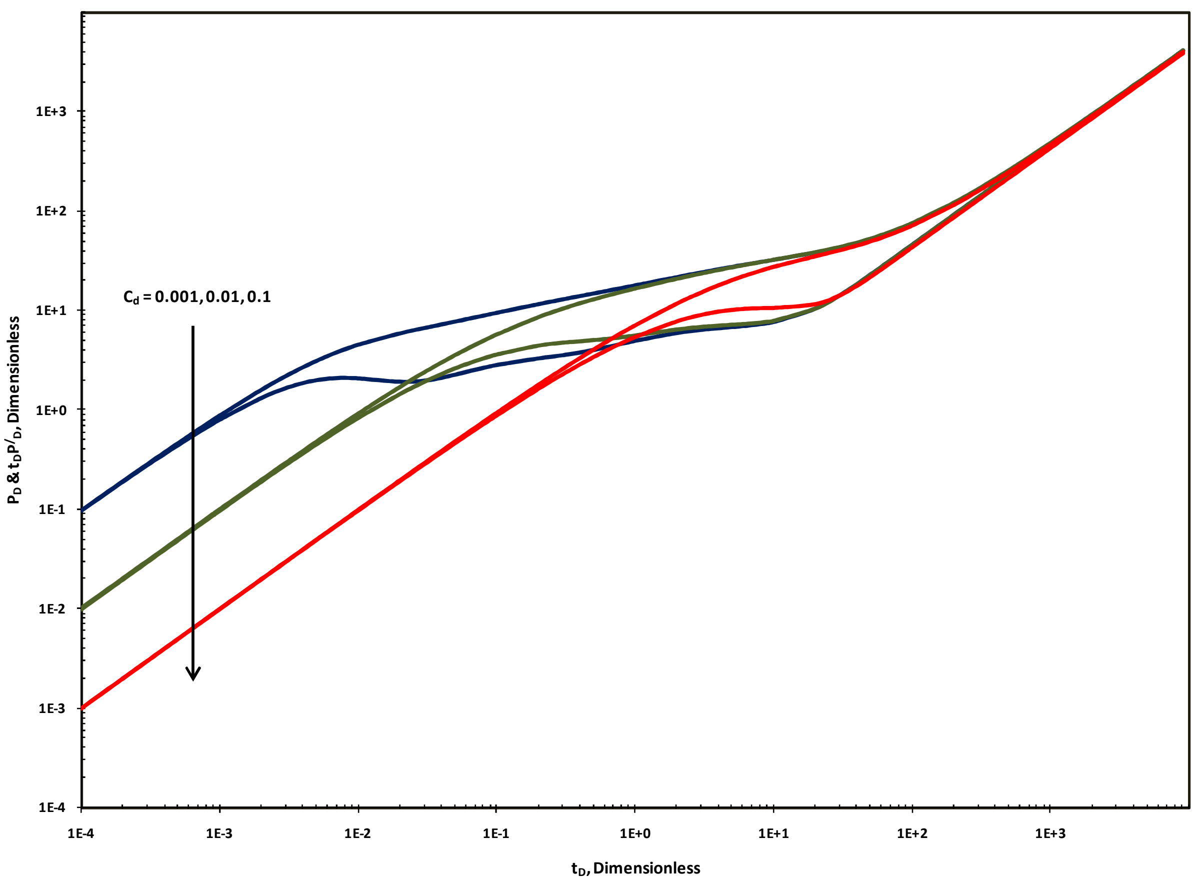

Fig. 3.4.11 Wellbore storage effect on horizontal well drawdown response

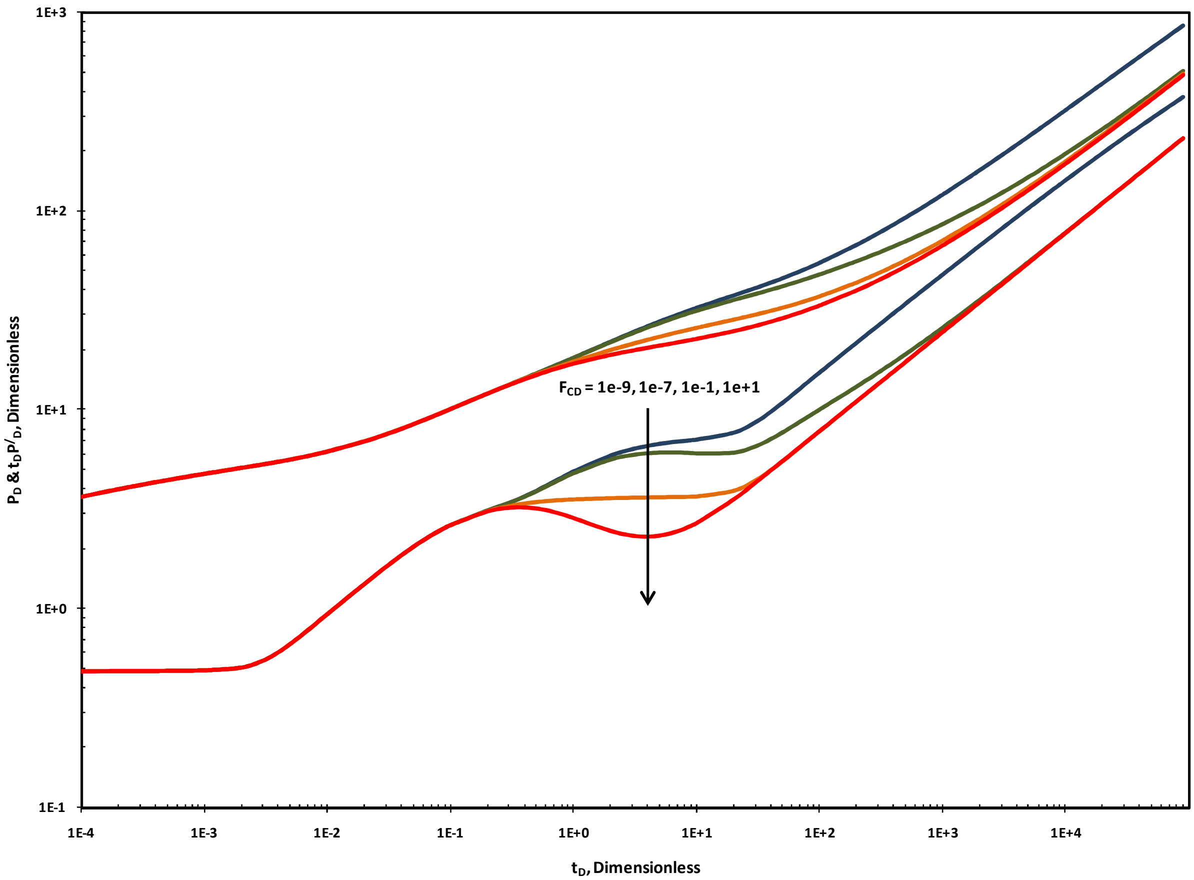

Fig. 3.4.12 Dimensionless fault conductivity effect on horizontal well response

Fig. 3.4.12 shows the effect of fault conductivity on flow contribution from the second compartment. The higher the fault conductivity, the easier it is for compartment II to support the producer, hence a lower pressure drop is experienced as a result of flow contribution from compartment II. When the reverse is the case, the whole reservoir will behave as if it were smaller in size causing a higher pressure drop – the fault behaving like a seal.

However, it is important to note that there is a minimum and maximum cutoff for fault conductivity, beyond which the fault conductivity effect remains the same. In this study, the minimum and maximum cutoff dimensionless conductivity are about 10^{-9} and 10 respectively which correspond to sealing and infinite conductivity behavior in the fault.

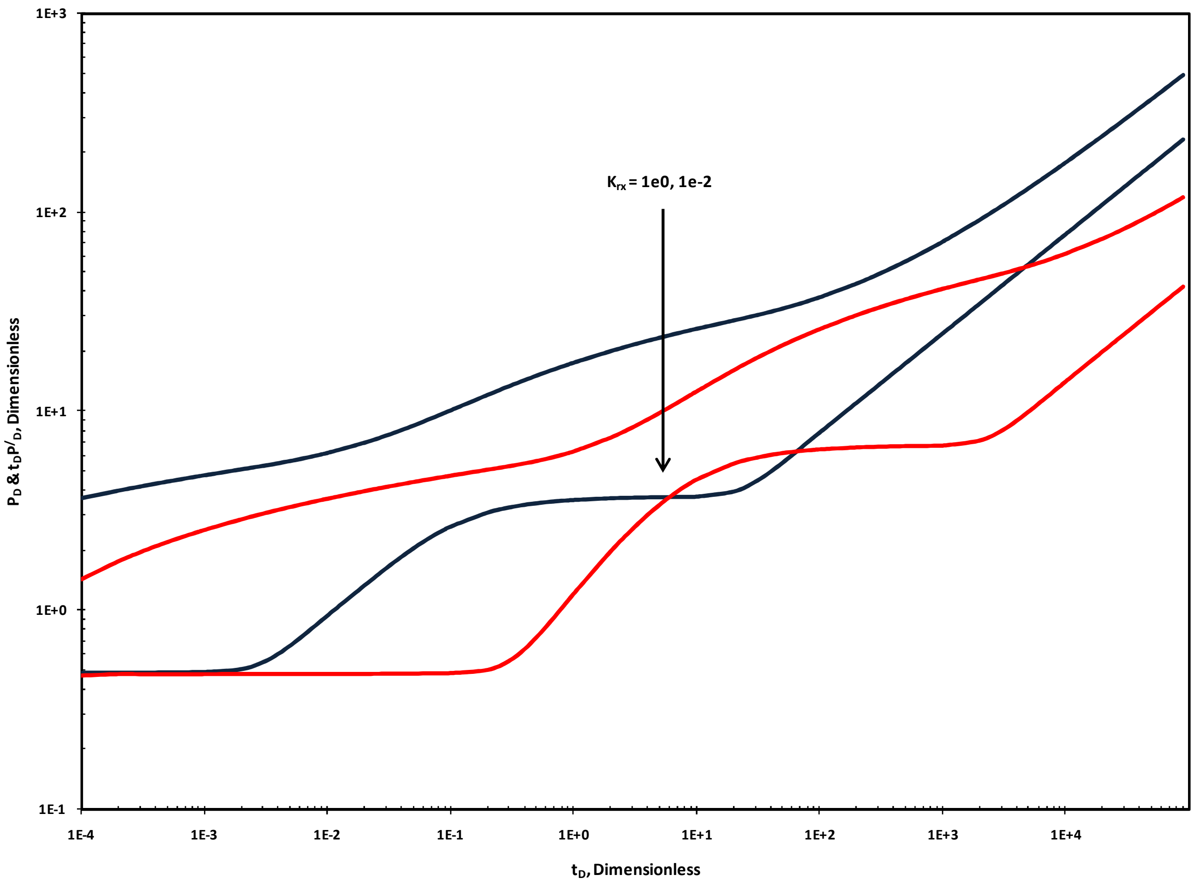

Fig. 3.4.13 Drawdown response when x-permeability in compartment I is higher than II

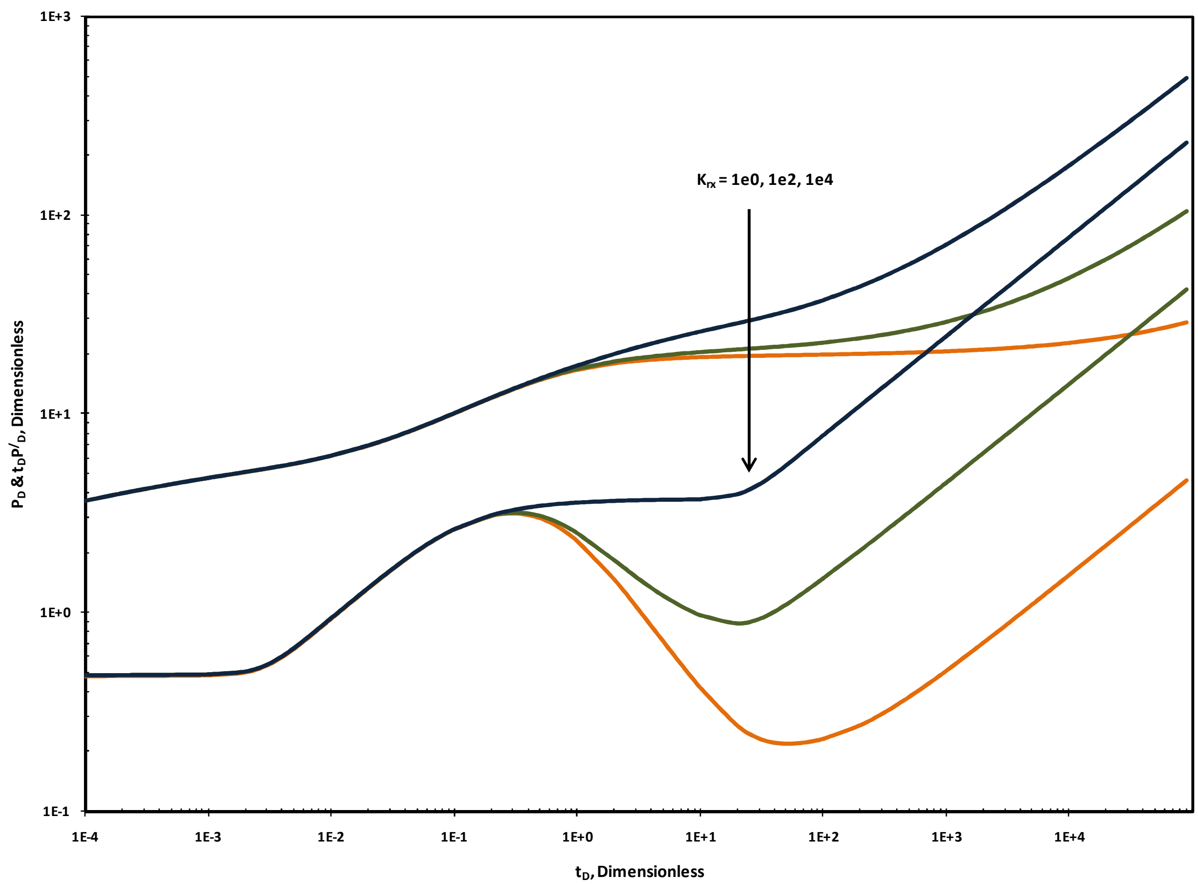

Fig. 3.4.14 Drawdown response when x-permeability in compartment II is higher than I

Fig. 3.4.14 shows that the higher the x-permeability contrast, the higher the downward rotation of the pseudo-radial flow regime. This observation assumes that the fault separating both compartments is conductive. The speed at which depletion is supported by compartment II is high enough to reduce the pressure drop drastically, hence the rotation observed. This behavior portraits such compartmentalized clastic reservoir as a naturally fractured reservoir.

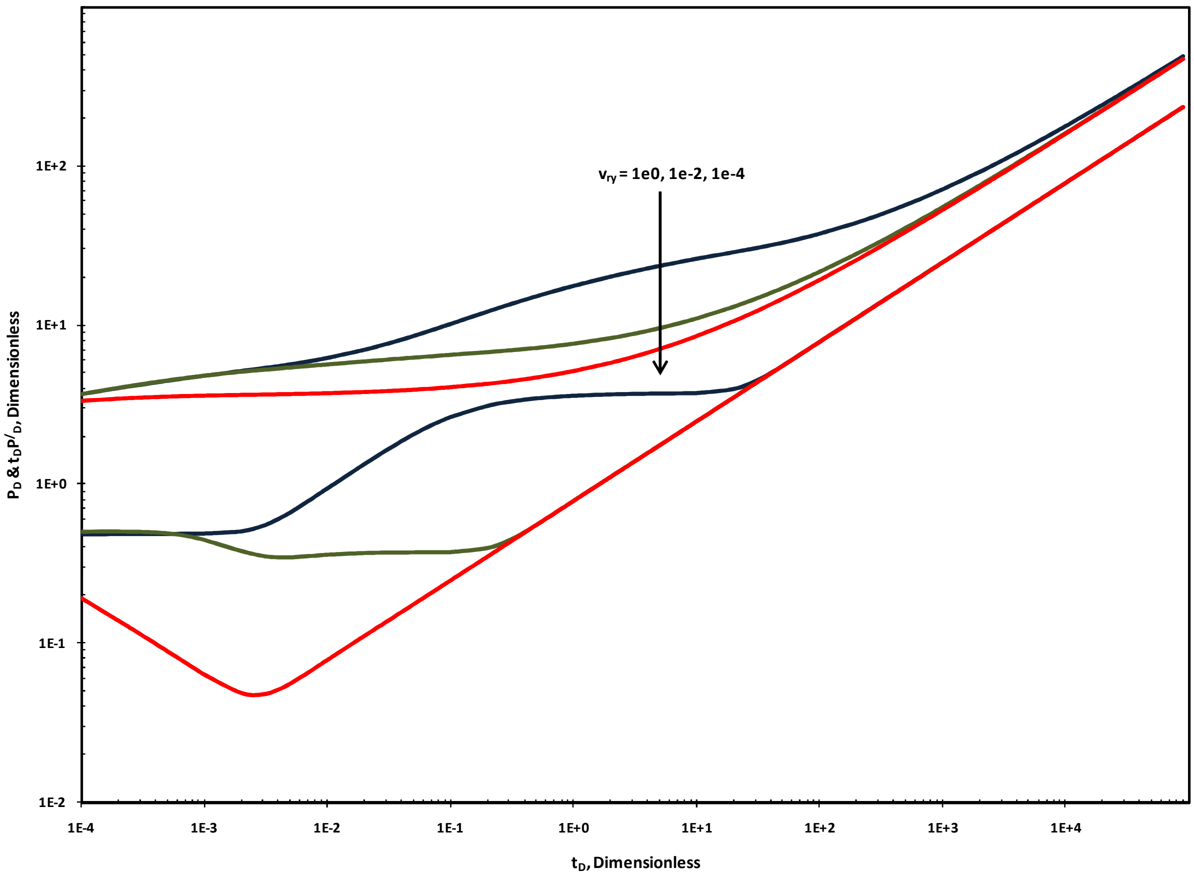

Fig. 3.4.15 Drawdown response when y-permeability in compartment I is higher than II

The inverse peaking of the pressure derivative in Fig. 3.4.15 as the y-permeability contrast increases in favour of compartment I shows that provided the principal permeability is in the x-direction, the early linear and pseudo-radial flow regimes that would have taken place before the higher y-permeability effect in compartment II gets to the producer in compartment I might be lost – the reason being that flow to the well is not properly supported in early times by flow from compartment II causing the well to behave like a point source (as seen by the -1/2 slope on the curve above).

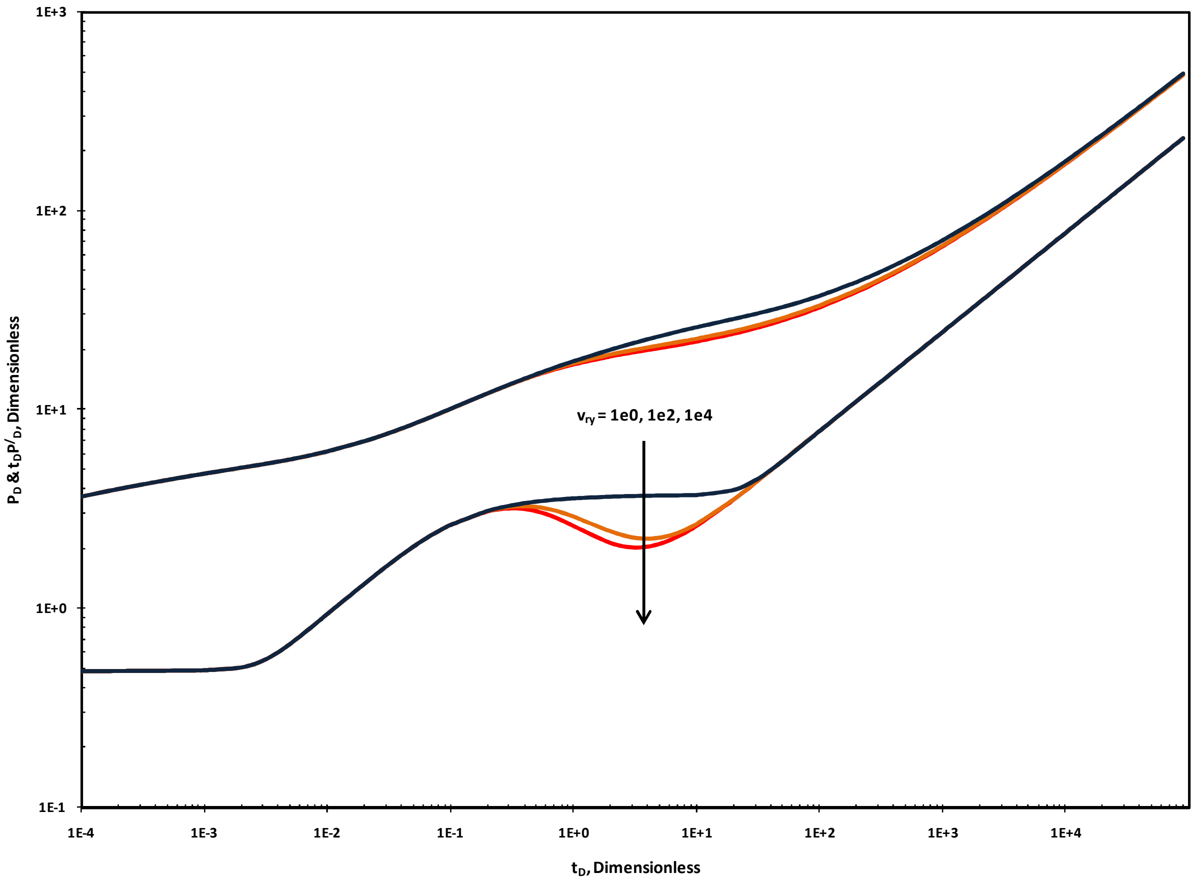

Fig. 3.4.16 Drawdown response when y-permeability in compartment II is higher than I

The inverse peaking of the pressure derivative in Fig. 3.4.16 as a result of this y-permeability contrast type shows that compartment II behaves like a pressure support during the pseudo-radial flow regime thereby causing a lower pressure drop than would have been expected.

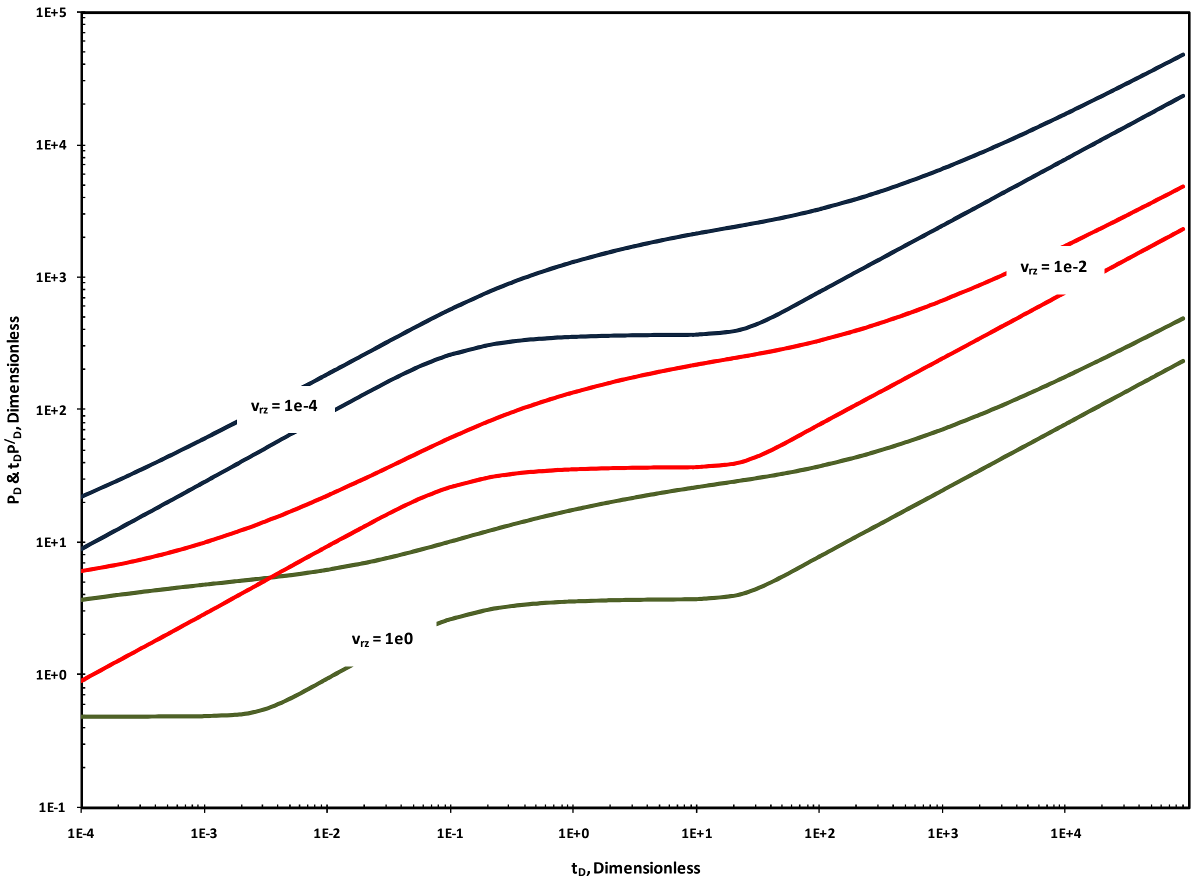

Fig. 3.4.17 Drawdown response when z-permeability in compartment I is higher than II

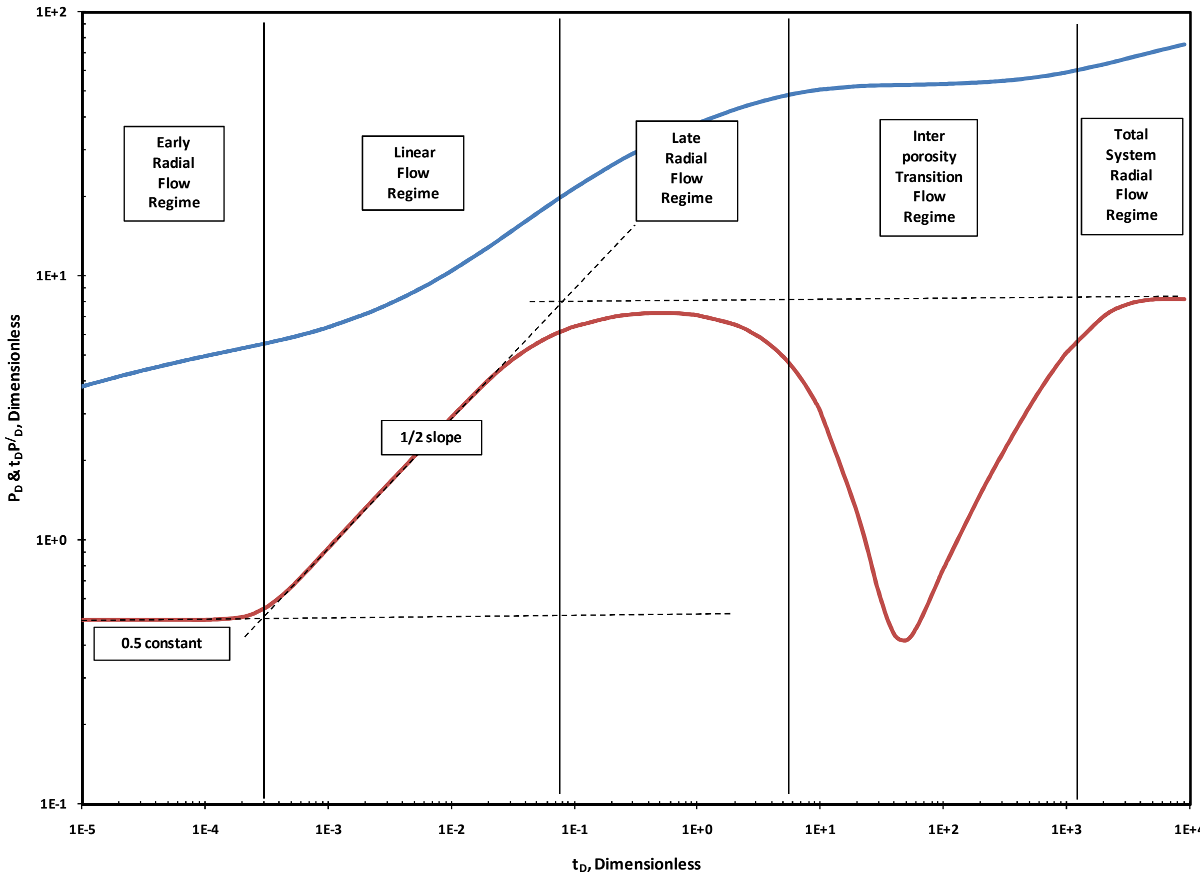

Fig. 3.4.18 Representative flow regimes associated with horizontal wells in NFR

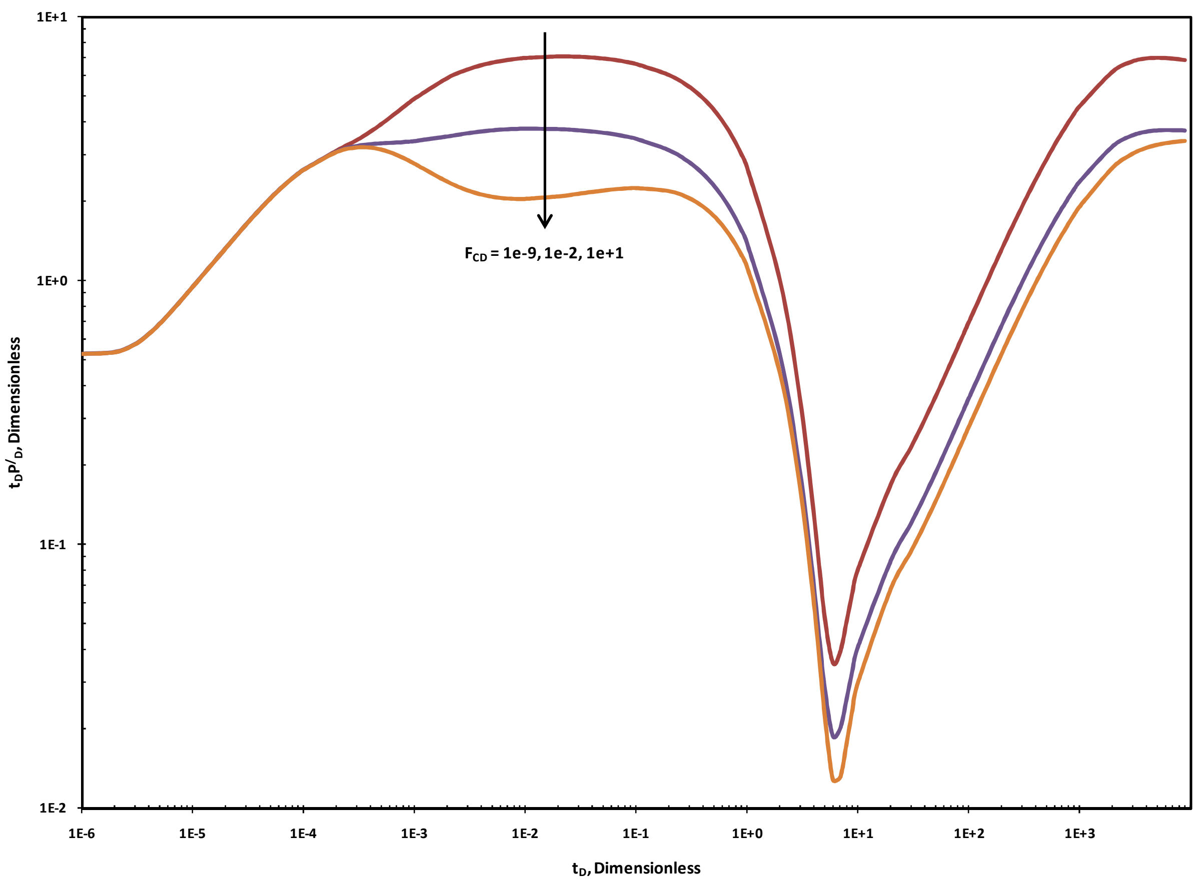

Fig. 3.4.19 Dimensionless fault conductivity and horizontal well response in NFR

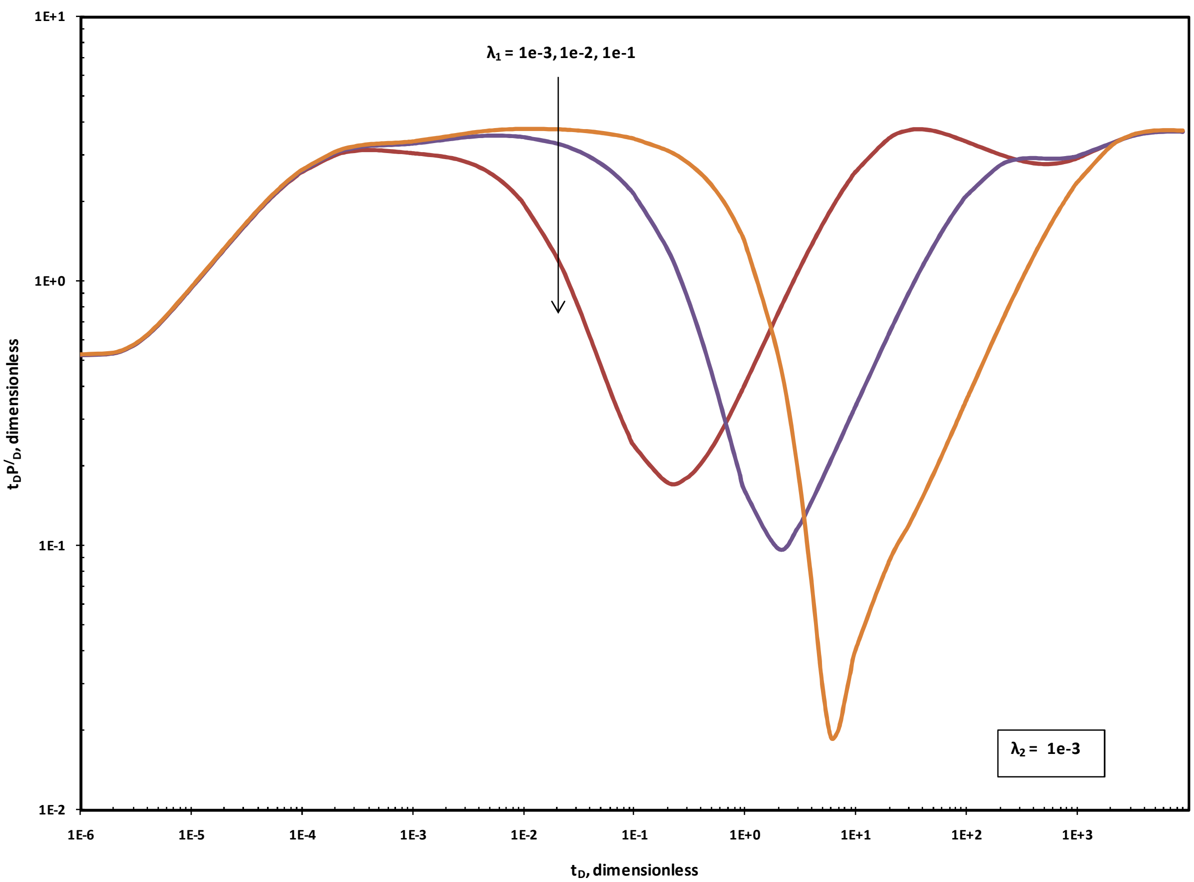

Fig. 3.4.20 Effect of inter-porosity parameter contrast on HW response in NFR (I > II)

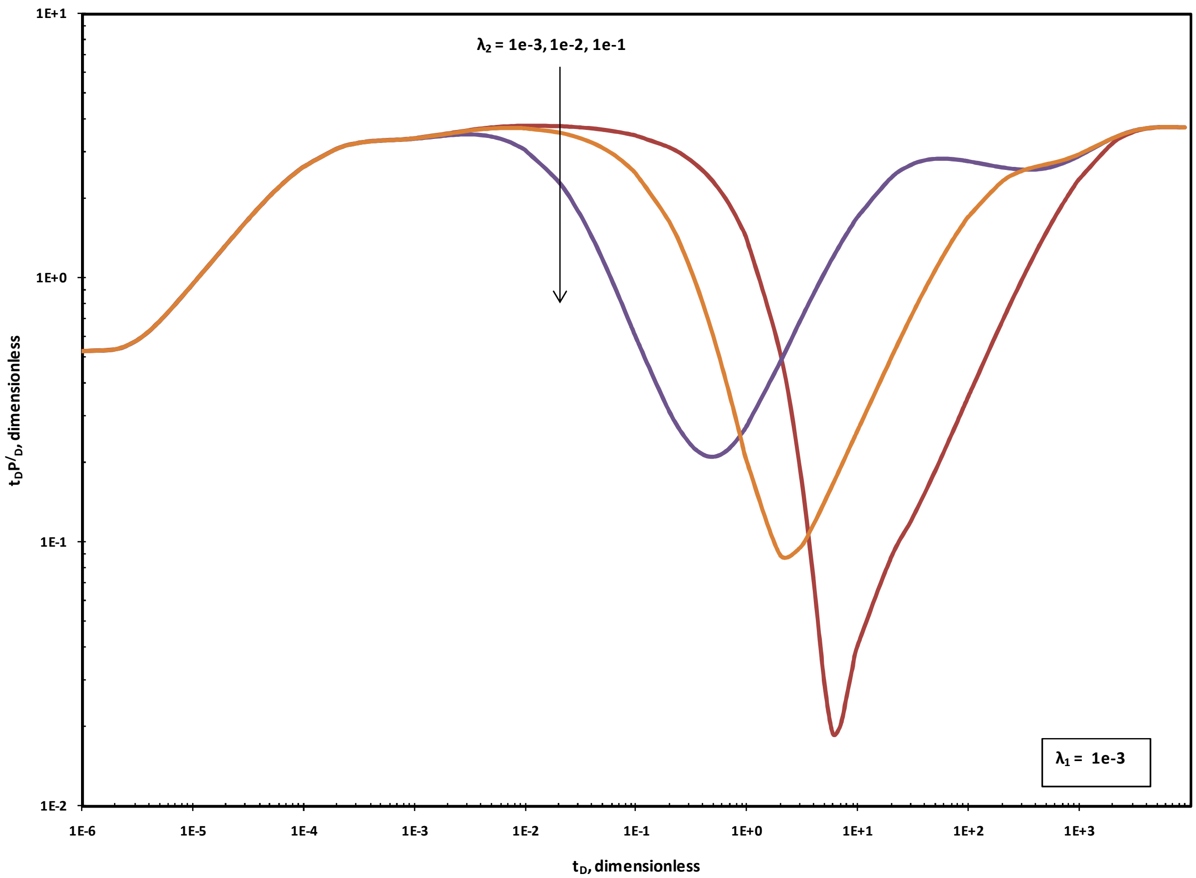

Fig. 3.4.21 Effect of inter-porosity parameter contrast on HW response in NFR (II > I)

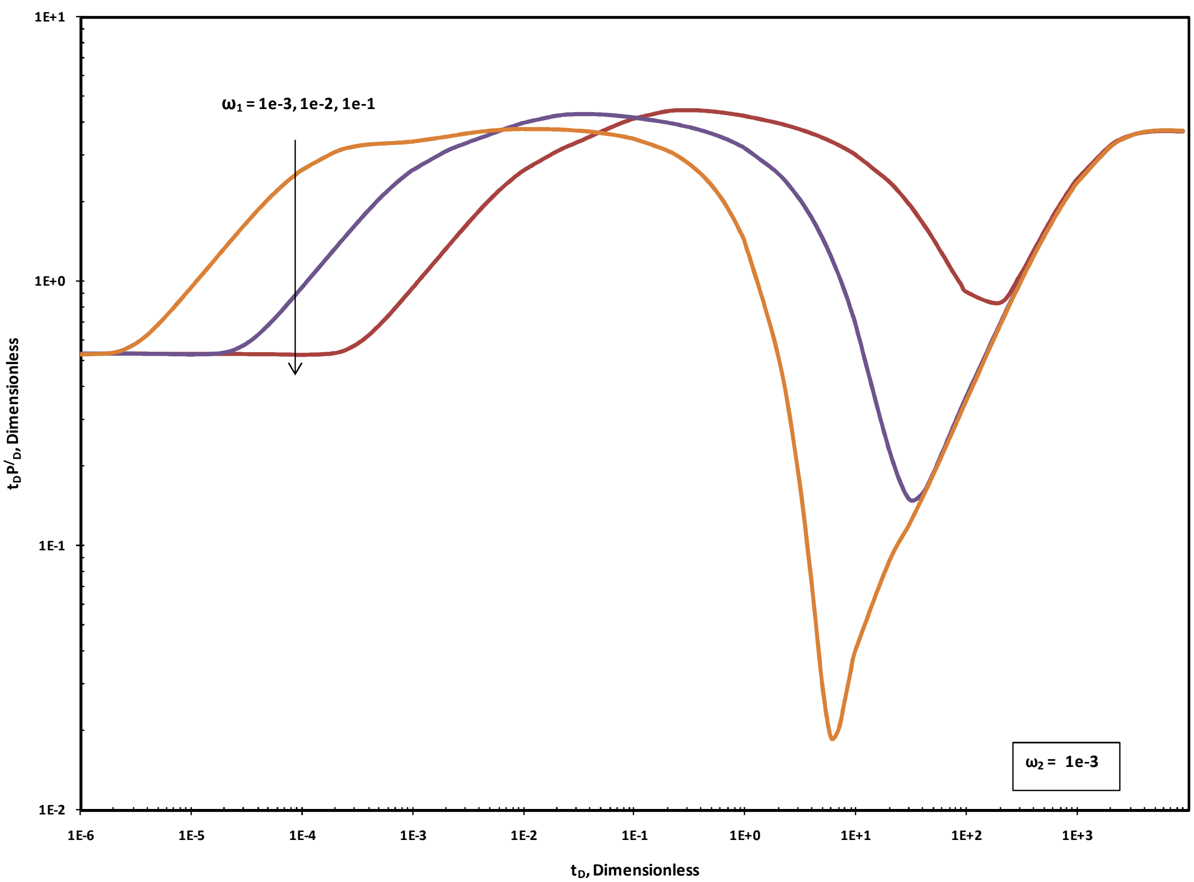

Fig. 3.4.22 Effect of storativity ratio contrast on HW response in NFR (I > II)

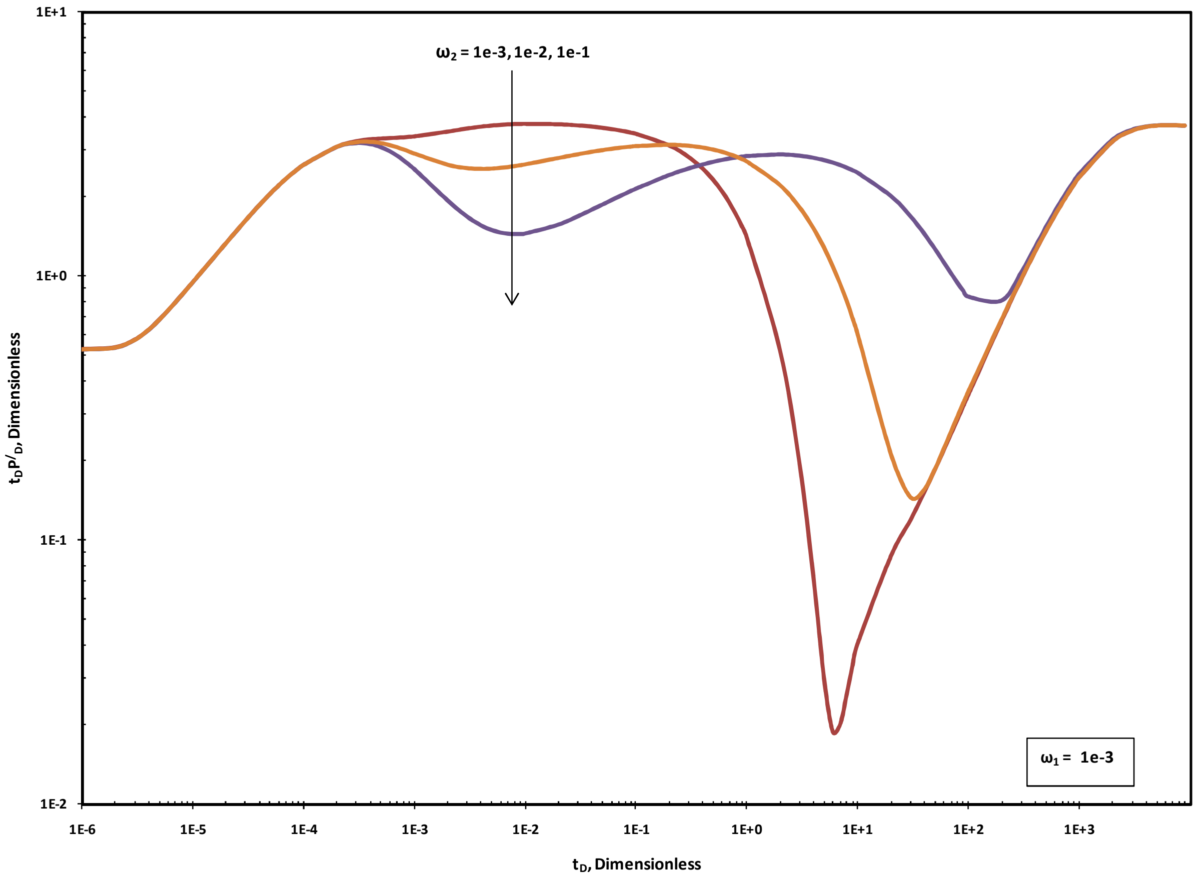

Fig. 3.4.23 Effect of storativity ratio contrast on HW response in NFR (II > I)

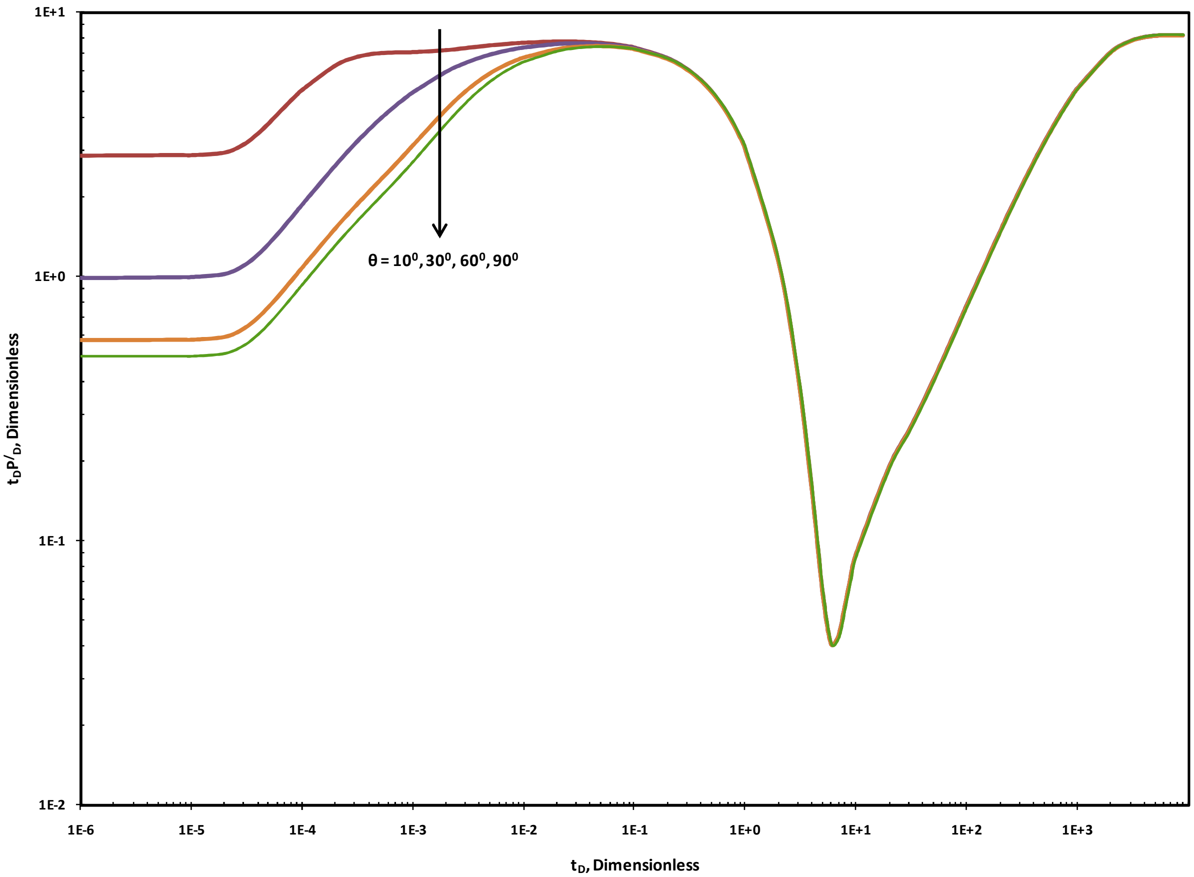

Fig. 3.4.24 Effect of HW orientation in high areal permeability contrast NFR

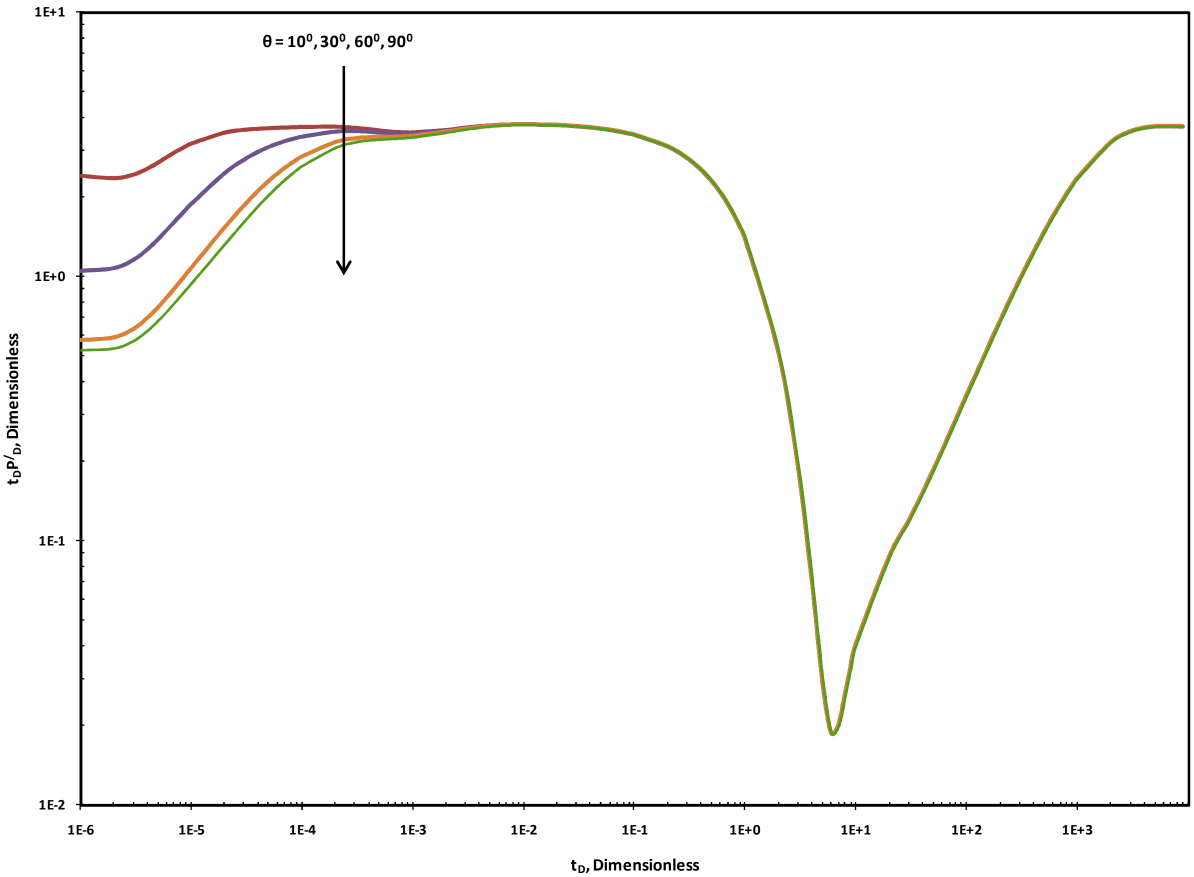

Fig. 3.4.25 Effect of HW orientation in low areal permeability contrast NFR

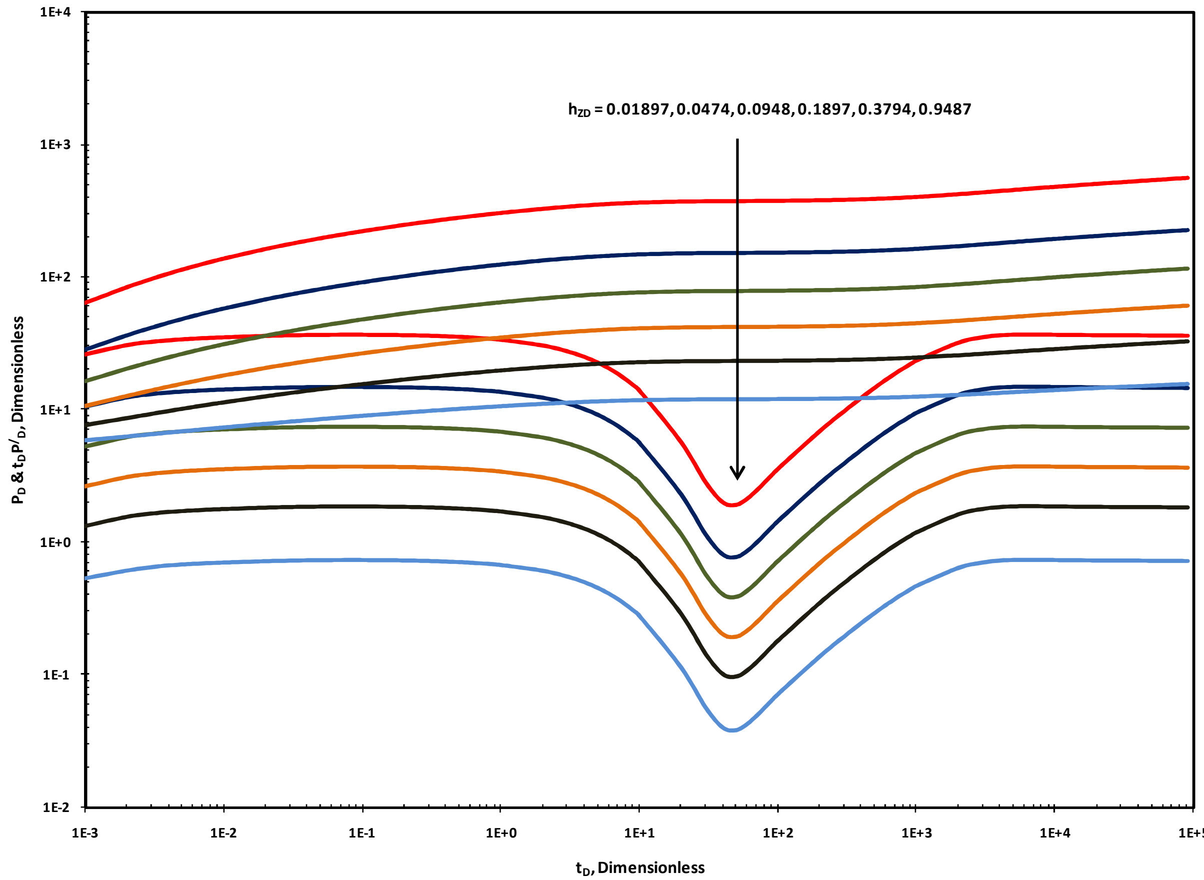

Fig. 3.4.26 Effect of HW characteristic length in NFR

3.5 Interpretation Development

Interpretation for long time approximation of the wellbore pressure will be made for the case where Skin and wellbore storage effects are absent.

Generally, for reservoirs infinite in two directions, finite in two and three directions, the following applies to them.

Common Radial Flow Regime Interpretation

From Fig. 3.4.1,

1.

$$(t_D P_D')_r = \frac{0.5}{sin\theta} \tag{3.5.1}$$

$$(t_D P_D')_r = (t\Delta P')_r \times \frac{2\pi\sqrt{k_{y1}k_{z1}}L}{q\mu_1} \tag{3.5.2}$$

Combining Eq. 3.5.1 and Eq. 3.5.2, we get

$$\left(\sqrt{k_{y1}k_{z1}}\right)(Lsin\theta) = \frac{q\mu_1}{4\pi \times (t\Delta P')_r} \tag{3.5.3}$$

2.

From Eq. 3.5.1,

$$P_{D_r} = \frac{0.5}{sin\theta} lnt_{D_r} \tag{3.5.4}$$

Dividing Eq. 3.5.4 by Eq. 3.5.1,

$$lnt_{D_r} = \frac{P_{D_r}}{(t_D P_D')_r} \tag{3.5.5}$$

$$ln\left(\frac{k_{x1}}{\emptyset_1\mu_1 C_{t1}L^2}\right) = \frac{\Delta P_r}{(t\Delta P')_r} - lnt_r \tag{3.5.6}$$

$$\frac{k_{x1}}{\emptyset_1\mu_1 C_{t1}L^2} = exp\left(\frac{\Delta P_r}{(t\Delta P')_r}\right) + \frac{1}{t_r} \tag{3.5.7}$$

Specific Long time Interpretation

From Fig. 3.4.1,

- Reservoir Finite in Two Directions

At long times,

$$\overline{P}_{WD} \rightarrow \frac{4\pi}{sh_{yD}h_{zD}sin\theta} \times \frac{L_{yD}}{\sqrt{s}(1 + k_{rx}\sqrt{\eta_D})} \tag{3.5.8}$$

Taking Inverse Laplace transform,

$$t_D P_D' \rightarrow \frac{4\sqrt{\pi}L_{yD}}{h_{yD}h_{zD}sin\theta} \times \frac{\sqrt{t_D}}{(1 + k_{rx}\sqrt{\eta_D})} \tag{3.5.9}$$

Considering the point of intersection between the pressure derivative (boundary effects) and the early radial flow line by equating (3.5.1) and (3.5.9),

$$\sqrt{\frac{k_{x2}}{k_{x1}} \times \frac{\emptyset_2\mu_2 C_{t2}}{\emptyset_1\mu_1 C_{t1}}} = \frac{8LL_{yD}}{h_y h_z} \times \sqrt{\frac{\pi k_{y1}k_{z1}t_x}{k_{x1}\emptyset_1\mu_1 C_{t1}}} - 1 \tag{3.5.10}$$

$$\frac{\sqrt{k_{x2}\emptyset_2\mu_2 C_{t2}}}{L_{yD}} = \frac{8L}{h_y h_z} \times \sqrt{\pi k_{y1}k_{z1}t_x} - \sqrt{k_{x1}\emptyset_1\mu_1 C_{t1}} \tag{3.5.11}$$

- **Reservoir Finite in Three Directions**

At long times,

$$\mathcal{F}_{m0} \to 0; \quad \mathcal{F}_{0n} \to 0; \quad \mathcal{F}_{mn} \to 0 \ \ and \ \ \overline{P}_{WD} \ \ \to \frac{4\pi L_{yD}}{sh_{yD}h_{zD}sin\theta} \times \frac{\cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}\sinh\sqrt{s}(h_{xD})} \qquad (3.5.12)$$

Taking Inverse Laplace transform,

$$t_D P_D' \to \frac{4\pi L_{yD}}{h_{yD}h_{zD}sin\theta} \times \frac{t_D}{h_{xD}} \qquad (3.5.13)$$

Considering the point of intersection between the pressure derivative (boundary effects) and the early radial flow line by equating (3.5.1) and (3.5.13),

$$L_{yD} = \frac{h_y h_z}{L\sqrt{k_{y1}k_{z1}t_x}} \times \frac{\emptyset_1 \mu_1 C_{t1}}{8\pi} \qquad (3.5.14)$$

In addition to these direct syntheses, traditional techniques like type curve matching are also applicable across all reservoir boundary types. The equations 3.2.3, 3.2.6 and 3.2.9 can be used to generate customizable type curves and the analysis of clastic reservoir systems above can be extended to NFR.

3.6 Application

Fig. 3.6.1 shows a match between this model and Eclipse 2010.1; the discrepancies should be from the permeability distribution across the cells which were not considered during the formulation of the 3D semi-analytical model.

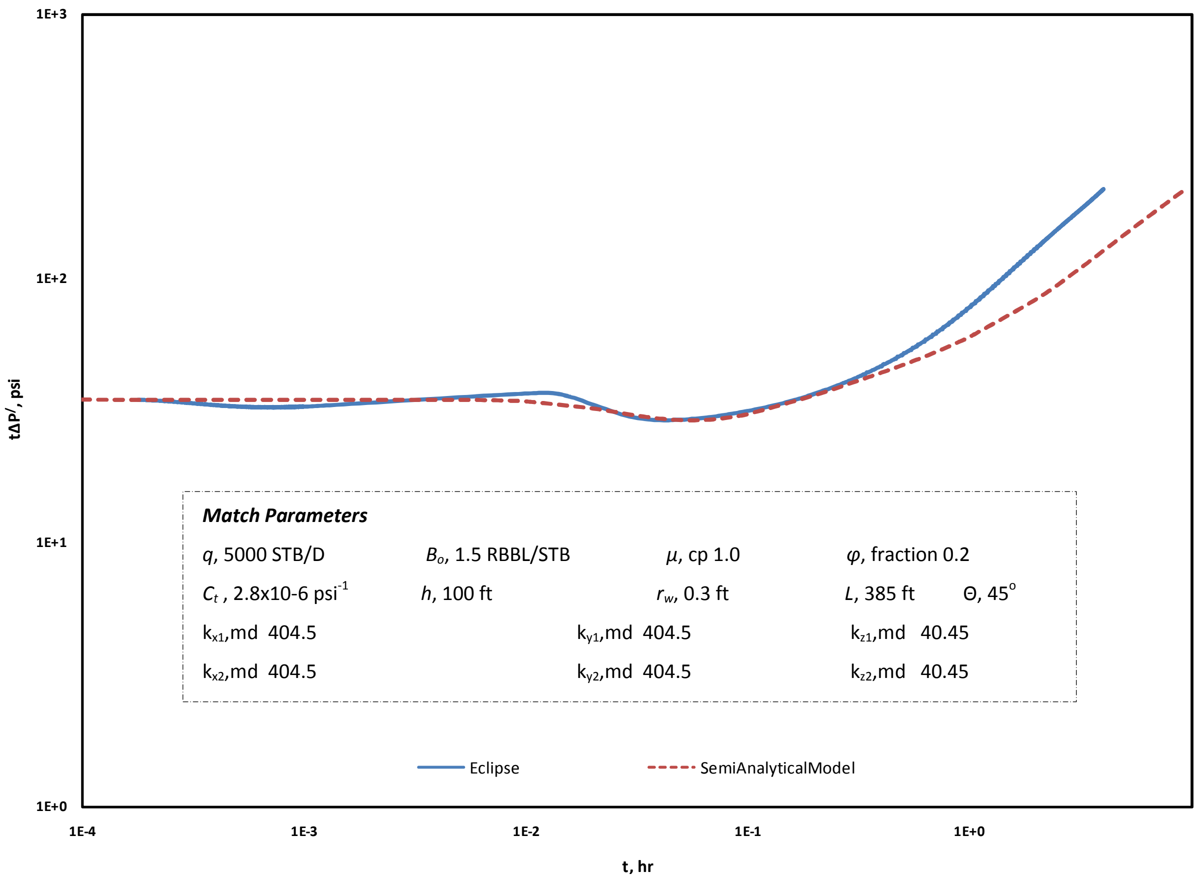

Fig. 3.6.1 Curve match between this model and Eclipse 2010.1 drawdown response

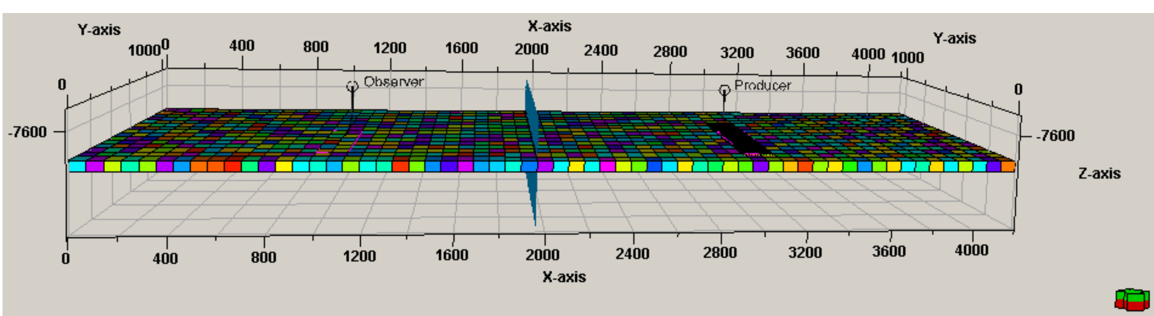

Fig. 3.6.2 Grid used in this study which was built and exported from petrel 2010

Three field examples are considered to validate the semi-analytical solution shown in Eq. 3.2.6. In the three examples, the reservoirs are bounded by two parallel faults. Sand to sand juxtaposition has been suspected in the last two examples. Figs. 3.6.3 – 3.6.5 show the results of the type curve match. The horizontal wells are drilled parallel to the faults hence the orientation angle used to obtain the match is 0.5 degree. The distance between the two parallel faults are measured from the structural maps. The permeability contrast as noted in the result of examples 2 and 3 in Appendix M is used to model the suspected sand to sand juxtaposition. The reservoir performance in examples 2 and 3 actually shows that the initial oil in place might be bigger than mapped. The perfect match obtained in the three examples thus validates the semi-analytical model.

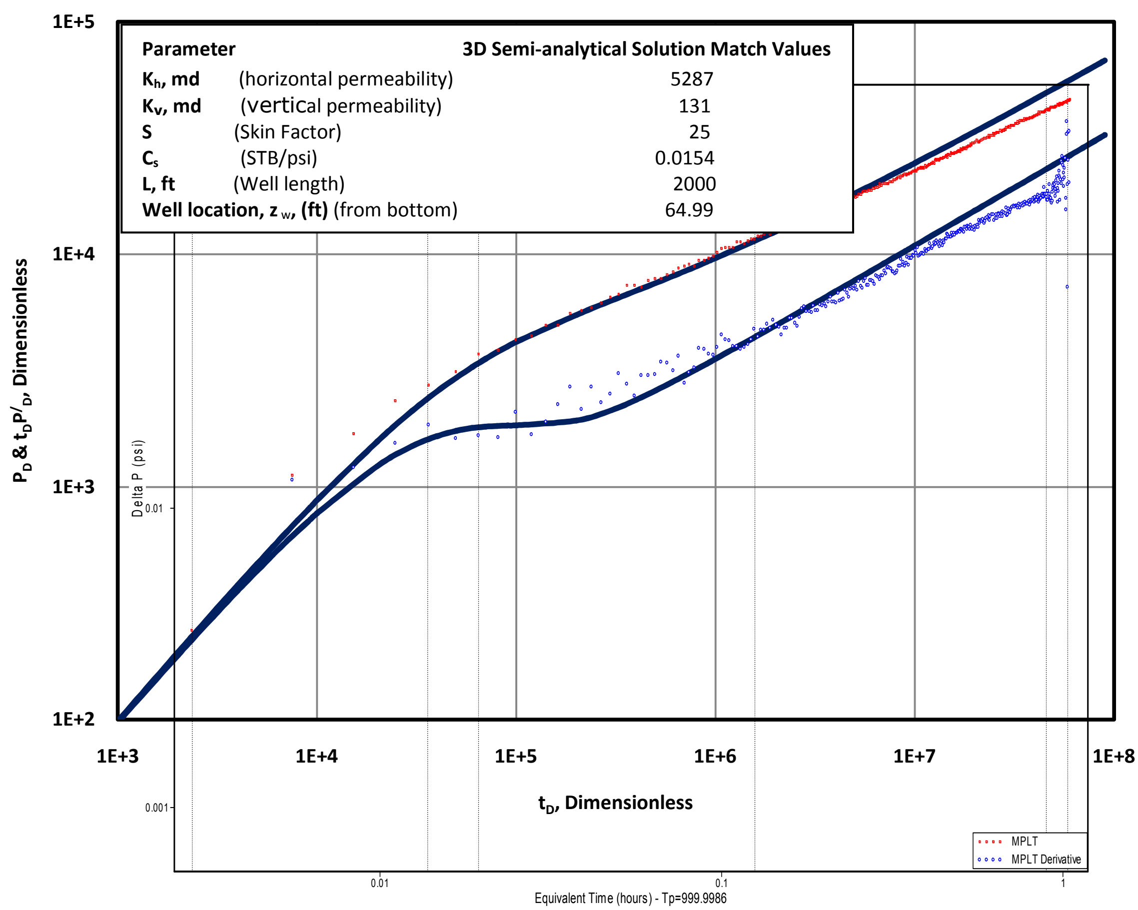

Fig. 3.6.3 Curve match between well AX-15 and 3D semi-analytical model: Example 1

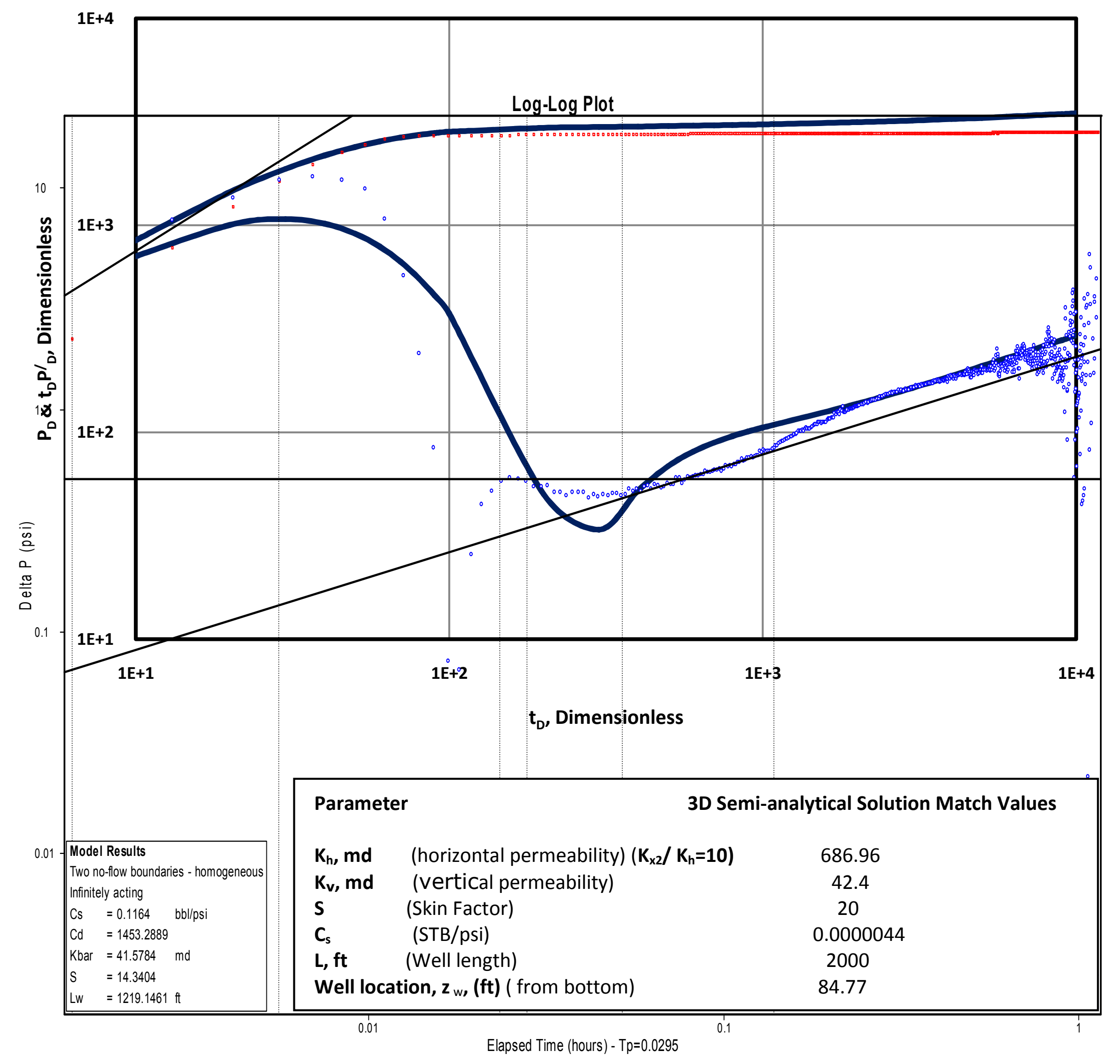

Fig. 3.6.4 Curve match between well AX-20 and 3D semi-analytical model: Example 2

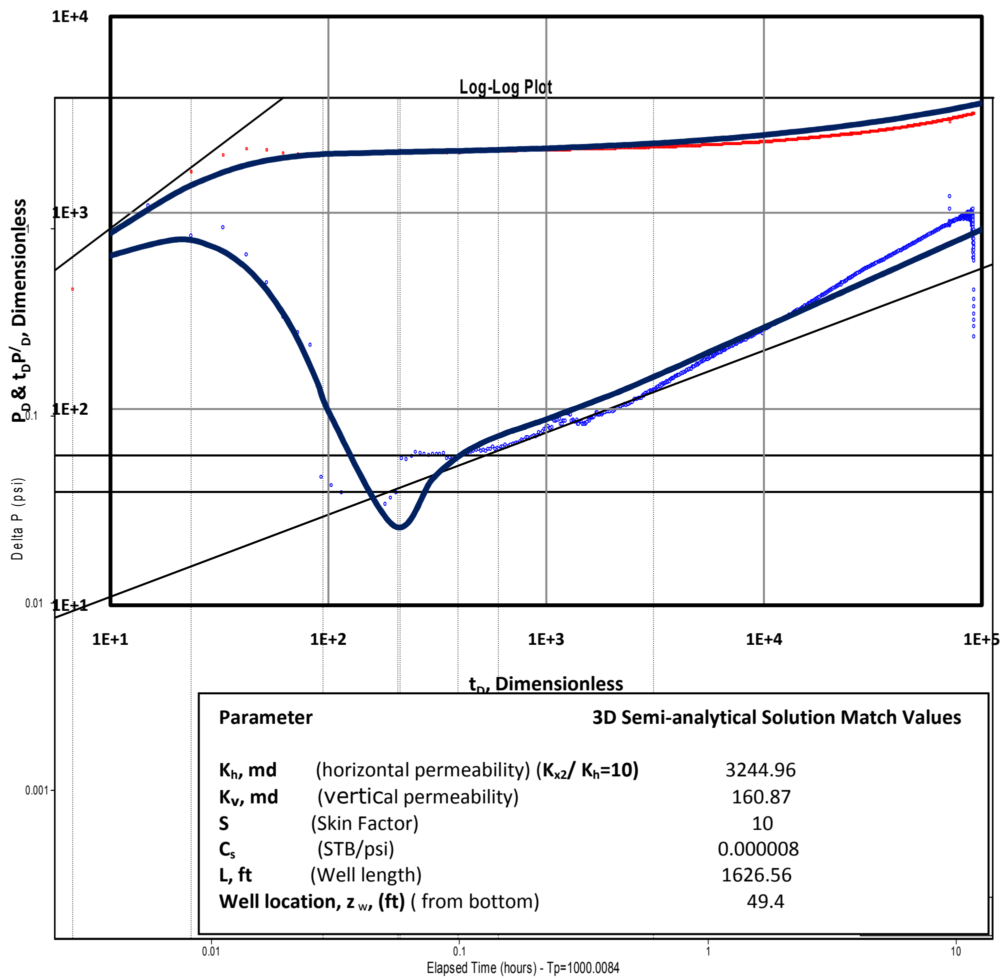

Fig. 3.6.5 Curve match between well AX-23 and 3D semi-analytical model: Example 3

Generally for Figs. 3.6.4 and 3.6.5, there exists uncertainty in some of the parameters used for the curve matches. For Skin, the range falls between ±5 while the Wellbore Storage coefficient (WBS) falls within $\pm 5x10^{-5}$ and a factor of 5-10 exists for the x-permeability contrast ratio (k_{x2}/k_{x1}). For Fig. 3.6.3, due to the amount of eclipsing caused by WBS, the main parameters with uncertainty are the Skin and WBS and they fell within a very narrow range.

3.7 Discussion

- Clastic Reservoir System

Clonts et al. (1986) noted that as far as the characteristic length exceeds at least 10, there exists an instantaneous end of the initial radial flow for all practical purposes causing the HW to behave like a general uniform flux hydraulically fractured vertical well; otherwise, the flow starts with the initial radial flow perpendicular to drain-hole axes.

The equivalent characteristic length in this study is given by the equation below.

$$L_D = \frac{L}{2h_z}\sqrt{\frac{k_{z_1}}{k_{x_1}}} = \frac{1}{2h_{zD}} \tag{3.7.1}$$

Hence, from Fig. 3.4.2 it can be seen that approximately above L_D of 10, there practically exists no early radial flow signature, confirming what (Clonts et al. 1986) documented The incorporation of this data during optimum HW length selection is very important as this among other factors helps predict the performance of the HW since this characteristic length contains key flow and reservoir parameters.

Figs. 3.4.3 and 3.4.4 depict other effects reservoir dimensions can have on the pressure signature obtained during a test. Below a critical reservoir size, the flow transitions straight to boundary effects skipping the pseudo-radial flow regime indicating the absence of space for the missing flow regimes to have been observed.

The closer the HW is to the fault, the greater the distortion of the early radial, early linear and pseudo-radial flow regimes as shown in Fig. 3.4.5. Also, the greater the HW is to any of the reservoir boundaries, the greater the pressure drop observed (see Figs. 3.4.6 and 3.4.7). However, it is worthy of note that Y_{WD} affects the pseudo-radial flow regime alone while Z_{WD} only affects the early radial. The smaller Z_{WD} is, hemi-radial flow begins to develop with its characteristic doubling of the early radial flow regime slope. Similarly, the smaller X_{WD} is, there exists a doubling of the early and late radial flow regime slopes. Note that the effect of Y_{WD} and Z_{WD} is the same no matter the boundary (top or bottom; right or left) provided that the distances are similar.

Considering Figs. 3.4.8 and 3.4.9 the well-orientation angle does not affect the start and end of the early radial flow regime irrespective of the degree of areal permeability anisotropy as noted by (Yildiz et al. 1997; Spivey et al. 1999). However, small angle deviations from the principal permeability direction substantially increase pressure drops and these pressure drops become negligible as the angles increase. As drawdown continues, pseudo-radial flow develops and its start time is affected by the inclination angle; the smaller the angle, the earlier it starts and vice versa. Also, the lesser the areal permeability anisotropy, the earlier this flow regime starts and vice versa.

The preceding paragraph helps plan how long a well test should be conducted to attain certain flow regimes that can help provide information about target flow or reservoir parameters.

The importance of drilling HW with minimum mechanical damage is emphasized in Fig. 3.4.10 which shows that the greater skin present, the greater the pressure drop experienced, the greater the obscuring of early flow regimes (early radial and linear) and vice versa. Also, by observing the pressure derivative signature as shown in Fig. 3.4.11, it is apparent that the peak of the hump caused by Wellbore Storage (WBS) is controlled by the skin factor (Tiab 1993).

The deviation of the pressure derivative pseudo-radial flow signature from horizontal depends on the dimensionless fault conductivity. Fig. 3.4.12 shows that the easier it is for the fault to conduct fluid across, the lower the pressure drop experienced during the pseudo-radial flow regime and vice versa. However, it is important to note that there is a minimum and maximum cutoff for fault conductivity, beyond which the fault conductivity effect will not register on the pressure and pressure derivative curve and this poses the question, "Why do type curves for composite reservoirs generated in literature hardly experienced in the field?" A reasonable answer is because these type curves were generated without taking into account the physics behind fault conductivity effect. Variation of parameters like length, permeability and fault width (parameters affecting fault conductivity) must be done proportionally to other dimensionless variables when generating these type curves.

Considering the permeability contrast in the x direction on both compartments of the composite reservoir, it is apparent that the when $k_{x1} > k_{x2}$ *(constant* k_{x2}*)*, although the duration of each flow regime is the same, the beginning of all flow regimes after early linear flow is delayed compared to when there is no permeability contrast in x direction. This is reasonable, since (Davlau et al. 1988) suggests that early linear flow is a result of flow when the early radial flow is forced to linear (x-y plane) after hitting the top and bottom boundaries (see Fig. 3.4.13).

Similarly when $k_{x2} > k_{x1}$ *(constant* k_{x1}*)*, there exists a downward rotation of the pseudo-radial flow regime since the pseudo-radial is believed to occur when flow has gone far into the reservoir such that the HW behaves as point (see Fig. 3.4.14); the speed at which depletion is supported by compartment II is high enough to reduce the pressure drop drastically, hence the rotation observed. Another important observation is that the intersection of the pseudo-radial line and the boundary effect line is constant no matter how $k_{x2} > k_{x1}$ *(constant* k_{x1}*)*.

The higher the contrast of when $k_{y1} > k_{y2}$ *(constant* k_{y2}*)*, the more the early-radial and linear flow regimes become eclipsed by spherical flow regime; this indicates that the higher this contrast, the less effective the well length becomes since it starts behaving more or less as a point source (see Fig. 3.4.15). Continuing with permeability contrast in the y direction, from Fig. 3.4.16, when $k_{y2} > k_{y1}$ *(constant* k_{y1}*)*, there exists an inverse peaking of the pseudo radial flow regime due to a similar process as discussed in the previous paragraph.

When $k_{z2} > k_{z1}$ *(constant* k_{z1}*)*, there exists no observable change in the pressure profile observed but when $k_{z1} > k_{z2}$ *(constant* k_{z2}*)*, the profile resembles more or less that of the characteristic length variation discussed previously (see Fig. 3.4.17). This shows that it is z-permeability in the producing compartment that controls well performance in composite reservoirs.

In practice, the early radial flow will often be obscured by wellbore storage effect; hence it will be necessary to perform a HW test before decision making to make sure a pseudo-radial flow exists for the test to be interpretable.

- Naturally Fractured Reservoir System

Fig. 3.4.18 displays the typical observed flow regimes as noted by (Igbokoyi 2008) which include:

1. Early radial flow
2. Linear flow
3. Late radial flow
4. Inter-porosity transition flow
5. Total system radial flow

Since our model is type-2 fracture, the first two flow regimes obtained from the HW test will be as a result of fluid already stored in the fracture – early radial and linear flows; meanwhile, the matrix is trying to replenish the fracture to keep pace with the depletion by the well.

However, once the matrix is not able to match this depletion pace, a “delay” occurs which shows up as an inter-porosity flow regime. Obviously, the greater the inter-porosity flow parameter (λ), the earlier this transition occurs and vice versa. This pseudo-steady transition flow regime is marked by a unit slope signature.

Once a pseudo-steady state rate of transfer of fluid among matrix, fracture and HW has been achieved, a total system radial flow is then recorded before any reservoir boundary effects.

From Fig. 3.4.19, the easier it is for the fault to allow communication between both reservoir compartments, the lower the pressure drop experienced.

Normally, the greater λ is, the earlier the occurrence of the transition flow regime and the lower its trough. However, the greater the λ contrast between the reservoir compartments, the earlier the transition flow regime occurs but the higher the trough observed compared to when there is no contrast in λ. The pressure drop experience for the same λ contrast is almost the same no matter the compartment with the higher λ value (see Figs. 3.4.20 and 3.4.21).

Considering storativity ratio contrast (ω), when $\omega_1 > \omega_2$, the greater this contrast, the later the fracture linear flow starts, the absence of the late fracture radial flow and the lower the transition trough. Also when $\omega_2 > \omega_1$, the greater this contrast, the lower the transition trough and the conversion of the late fracture radial flow into another trough. These behaviors are shown in Figs. 3.4.22 and 3.4.23.

Similar areal permeability and well inclination angle relationship exists as those discussed for clastic reservoirs above except in NFR, the pseudo-radial flow is now divided by an inter-porosity flow period (see Figs. 3.4.24 and 3.4.25).

The effect of characteristic length on the pressure signature in NFR is similar to that of clastic reservoirs. Fig. 3.4.26 indicates that for all practical purpose, the higher this length, the more the reservoir behaves as if it has a lower storativity ratio i.e. it performs less.

Chapter 4

Arbitrarily Oriented Hydraulically Fractured Vertical Wells

Hydraulic fractured wells (HF) play an important role in exploiting tight formation and are usually not fully penetrating in reservoirs with gas cap or aquifer close by even when intended to since there is need to avoid early water or gas breakthrough (Igbokoyi et al. 2008).

Much work has been done on HF in clastic reservoir systems from (Cinco Ley 1974) to (Anh et al. 2010) but little has been done for composite reservoir systems.

The method used for obtaining the source solution in this work was adapted after (Raghavan et al. 1978) as given below.

$$P_{D_{partial\_penetration}} = \frac{\mathrm{h_z}}{L_{zf}} \times P_{D_{full\_penetration}} \tag{4.1}$$

4.1 Dimensionless Transformation

These dimensionless transformations are used in obtaining subsequent point source solutions.

$$x_D = \frac{x}{L} \tag{4.1.1}$$

$$y_D = \frac{y}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{4.1.2}$$

$$z_D = \frac{z}{L}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{4.1.3}$$

$$r_{WD} = \frac{r_W}{L} \tag{4.1.4}$$

$$w_D = \frac{w_f}{2L} \tag{4.1.5}$$

$$h_{xD} = \frac{h_x}{L} \tag{4.1.6}$$

$$h_{yD} = \frac{h_y}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{4.1.7}$$

$$h_{zD} = \frac{h_z}{L}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{4.1.8}$$

$$t_D = \frac{k_{x_1} t}{\emptyset_1 \mu_1 C_{t_1} L^2} \tag{4.1.9}$$

$$C_D = \frac{C}{2\pi {r_w}^2 \emptyset_1 L C_{t_1}} \tag{4.1.10}$$

$$P_{D_{n,f}} = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}}\Delta P_{n,f}}{\mu_1 q} \quad \Delta P_{n,f} = P_i - P_{n,f} \tag{4.1.11}$$

$$F_{CD} = \frac{w_f k_f}{2k_{x_1} L} \tag{4.1.12}$$

$$x_D{}' = \frac{Lcos\theta}{L} \tag{4.1.13}$$

$$y_D{}' = \frac{Lsin\theta}{L}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{4.1.14}$$

$$\theta' = \arctan\left(\sqrt{\frac{k_{x_1}}{k_{y_1}}} tan\theta\right) \tag{4.1.15}$$

$$dy_D = dL_{yD} sin\theta \tag{4.1.16}$$

$$L_{fD} = \sqrt{\left(x_D{}'^2 + y_D{}'^2 \right)} \tag{4.1.17}$$

$$L_{fD} = \sqrt{\cos^2\theta + \frac{k_{x_1}}{k_{y_1}}\sin^2\theta} = \sqrt{\cos^2\theta + v_y \sin^2\theta} \tag{4.1.18}$$

$$L_{zD} = \frac{L_{zf}}{2L}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{4.1.19}$$

Converting to the y plane, L_{fD} *becomes* $L_{fD} sin\theta'$

This might be used interchangeably for convenience purposes

4.2 Line source solution for Clastic Reservoirs

- Infinite in two directions

The point source solution for a reservoir infinite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin}$$
$$= \frac{1}{\mathrm{sh}_{zD}}\left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \\ \displaystyle +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \end{array}\right] \quad (4.2.1)$$

Converting the point source solution to line source solution by integrating Eq. 4.2.1 with respect to (w.r.t.) (L_{fD} and L_{zD}) from Y_{WD} - $L_{fD}\sin\theta^{/}$ to Y_{WD} + $L_{fD}\sin\theta^{/}$ and Z_{WD} – L_{zD} to Z_{WD} + L_{zD} respectively.

Converting the anisotropic system to an equivalent isotropic system and taking account of the effect of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{h_{zD}}{2L_{zD}} \times \frac{4}{sh_{zD}sin\theta}(\alpha+\beta)$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{2}{sL_{zD}sin\theta}(\alpha+\beta) \quad (4.2.2)$$

$$\alpha+\beta = \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{L_{zD}}{\omega}\frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right\}\sin\left(\omega L_{fD}\right)d\omega \\ \displaystyle +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\omega}\frac{(1+\mathrm{QS_w})}{Qk}\cos^2(\,kz_{WD})\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\}\sin\left(\omega L_{fD}\right)\sin(kL_{zD})\,d\omega \end{array}\right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin\,and\,wellbore\,storage}$$
$$= \frac{\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin}}{1 + s^2 C_D\,\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin}} \tag{4.2.3}$$

- **Finite in two directions**

The point source solution for a reservoir finite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha+\beta) \tag{4.2.4}$$

$$\alpha = \left[\begin{array}{c} \dfrac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{4\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\dfrac{\left(\sqrt{\mathrm{s}}E_p - E_p{}'\right)}{\left(\sqrt{\mathrm{s}}E_p + E_p{}'\right)}\right\} \\ +\displaystyle\sum_{m=1}^{\infty}\dfrac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\dfrac{\left(TF_p - F_p{}'\right)}{\left(TF_p + F_p{}'\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\dfrac{(1+\mathrm{RS_w})}{2R}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Rx_{WD}}\dfrac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right\} \\ +\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\dfrac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\dfrac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\} \end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

Converting the point source solution to line source solution by integrating Eq. 4.2.4 w.r.t. (L_{fD} and L_{zD}) from Y_{WD} - $L_{fD}\sin\theta^{/}$ to Y_{WD} + $L_{fD}\sin\theta^{/}$ and Z_{WD} – L_{zD} to Z_{WD} + L_{zD} respectively

Converting the anisotropic system to an equivalent isotropic system and taking account of the effect of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}sin\theta}(\alpha+\beta)$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}sin\theta}(\alpha+\beta) \tag{4.2.5}$$

$$\alpha = \left[\begin{array}{c} \dfrac{L_{zD}L_{fD}\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\dfrac{\left(\sqrt{\mathrm{s}}E_p-E_p{'}\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p{'}\right)}\right\} \\ +\displaystyle\sum_{m=1}^{\infty}\dfrac{2L_{zD}(1+\mathrm{TS_w})}{Tl}\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})\left\{1+e^{-2Tx_{WD}}\dfrac{\left(TF_p-F_p{'}\right)}{\left(TF_p+F_p{'}\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\dfrac{2L_{fD}(1+\mathrm{RS_w})}{Rk}\sin k(L_{zD})\cos^2(\,kz_{WD})\left\{1+e^{-2Rx_{WD}}\dfrac{\left(RD_p-D_p{'}\right)}{\left(RD_p+D_p{'}\right)}\right\} \\ +\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\dfrac{4(1+\mathrm{QS_w})}{Qkl}\cos^2(\,kz_{WD})\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})\left\{1+e^{-2Qx_{WD}}\dfrac{\left(QC_p-C_p{'}\right)}{\left(QC_p+C_p{'}\right)}\right\} \end{array}\right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD},y_{WD},z_{WD},s)_{with\ skin\ and\ wellbore\ storage} = \frac{\overline{P}_{WD}\,(x_{WD},y_{WD},z_{WD},s)_{with\ skin}}{1+s^2C_D\,\overline{P}_{WD}\,(x_{WD},y_{WD},z_{WD},s)_{with\ skin}} \tag{4.2.6}$$

- **Finite in three directions**

The point source solution for a reservoir finite in three directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{4\pi}{\mathrm{s}h_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{4.2.7}$$

$$\mathcal{F}_{00} = \left[\frac{1}{4\sqrt{\mathrm{s}}\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{c}\mathrm{cosh}\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + S_w\sqrt{\mathrm{s}}\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\mathrm{sinh}2\sqrt{\mathrm{s}}(h_{xD} - x_{WD}) + 1 + \mathrm{cosh}2\sqrt{\mathrm{s}}(h_{xD} - x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\,\mathrm{sinh}\sqrt{\mathrm{s}}(h_{xD}) + E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})}\left\{\begin{array}{c}\mathrm{coshT}(h_{xD} - 2x_{WD}) + S_w\mathrm{TsinhT}(h_{xD} - 2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'\left[S_w\mathrm{Tsinh2T}(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2T}(h_{xD} - x_{WD})\right]}{TF_p\,\mathrm{sinhT}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)}{2R\sinh R(h_{xD})}\left\{\begin{array}{c}\mathrm{cosh}R(h_{xD} - 2x_{WD}) + S_w\mathrm{Rsinh}R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'\left[S_w\mathrm{Rsinh2}R(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2}R(h_{xD} - x_{WD})\right]}{RD_p\,\mathrm{sinh}R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})} \times \\ \left\{\begin{array}{c}\mathrm{cosh}Q(h_{xD} - 2x_{WD}) + S_w\mathrm{Qsinh}Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'\left[S_w\mathrm{Qsinh2}Q(h_{xD} - x_{WD}) + 1 + \mathrm{cosh2}Q(h_{xD} - x_{WD})\right]}{QC_p\,\mathrm{sinh}Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{array}\right\}\end{array}\right]$$

Converting the point source solution to line source solution by integrating Eq. 4.2.7 w.r.t. (L_{fD} and L_{zD}) from Y_{WD} - $L_{fD}\sin\theta^{/}$ to Y_{WD} + $L_{fD}\sin\theta^{/}$ and Z_{WD} – L_{zD} to Z_{WD} + L_{zD} respectively

Converting the anisotropic system to an equivalent isotropic system and taking account of the effect of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{h_{zD}}{2L_{zD}} \times \frac{4\pi}{sh_{yD}h_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{2\pi}{sh_{yD}L_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{4.2.8}$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}L_{fD}}{\sqrt{s}\mathrm{sinh}\sqrt{s}(h_{xD})}\left\{\begin{matrix}\cosh\sqrt{s}(h_{xD} - 2x_{WD}) + S_w\sqrt{s}\mathrm{sinh}\sqrt{s}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{s}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{s}\mathrm{sinh}2\sqrt{s}(h_{xD} - x_{WD}) + 1 + \cosh 2\sqrt{s}(h_{xD} - x_{WD})\right]}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{}'\cosh\sqrt{s}(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{matrix}\displaystyle\sum_{m=1}^{\infty}\frac{2L_{zD}\sin l\left(L_{fD}\right)\cos^2(ly_{WD})}{Tl\sinh T(h_{xD})} \times \\ \left\{\begin{matrix}\cosh\mathrm{T}(h_{xD} - 2x_{WD}) + S_w\mathrm{T}\mathrm{sinh}\mathrm{T}(h_{xD} - 2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w\mathrm{T}\mathrm{sinh}2\mathrm{T}(h_{xD} - x_{WD}) + 1 + \cosh 2\mathrm{T}(h_{xD} - x_{WD})]}{TF_p\,\mathrm{sinh}\mathrm{T}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{matrix}\right\}\end{matrix}\right]$$

$$\mathcal{F}_{0n} = \left[\begin{matrix}\displaystyle\sum_{n=1}^{\infty}\frac{2L_{fD}\sin k(L_{zD})\cos^2(kz_{WD})}{Rk\sinh R(h_{xD})} \times \\ \left\{\begin{matrix}\cosh R(h_{xD} - 2x_{WD}) + S_w\mathrm{R}\mathrm{sinh}R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w\mathrm{R}\mathrm{sinh}2R(h_{xD} - x_{WD}) + 1 + \cosh 2R(h_{xD} - x_{WD})]}{RD_p\,\mathrm{sinh}R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{matrix}\right\}\end{matrix}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{matrix}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4\cos^2(kz_{WD})\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos^2(ly_{WD})}{Qkl\sinh Q(h_{xD})} \times \\ \left\{\begin{matrix}\cosh Q(h_{xD} - 2x_{WD}) + S_w\mathrm{Q}\mathrm{sinh}Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w\mathrm{Q}\mathrm{sinh}2Q(h_{xD} - x_{WD}) + 1 + \cosh 2Q(h_{xD} - x_{WD})]}{QC_p\,\mathrm{sinh}Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{matrix}\right\}\end{matrix}\right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage} = \frac{\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2C_D\,\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \tag{4.2.9}$$

4.3 Line source solution for Naturally Fractured Reservoirs

Everything is the same as section 4.2 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

4.4 Type Curves

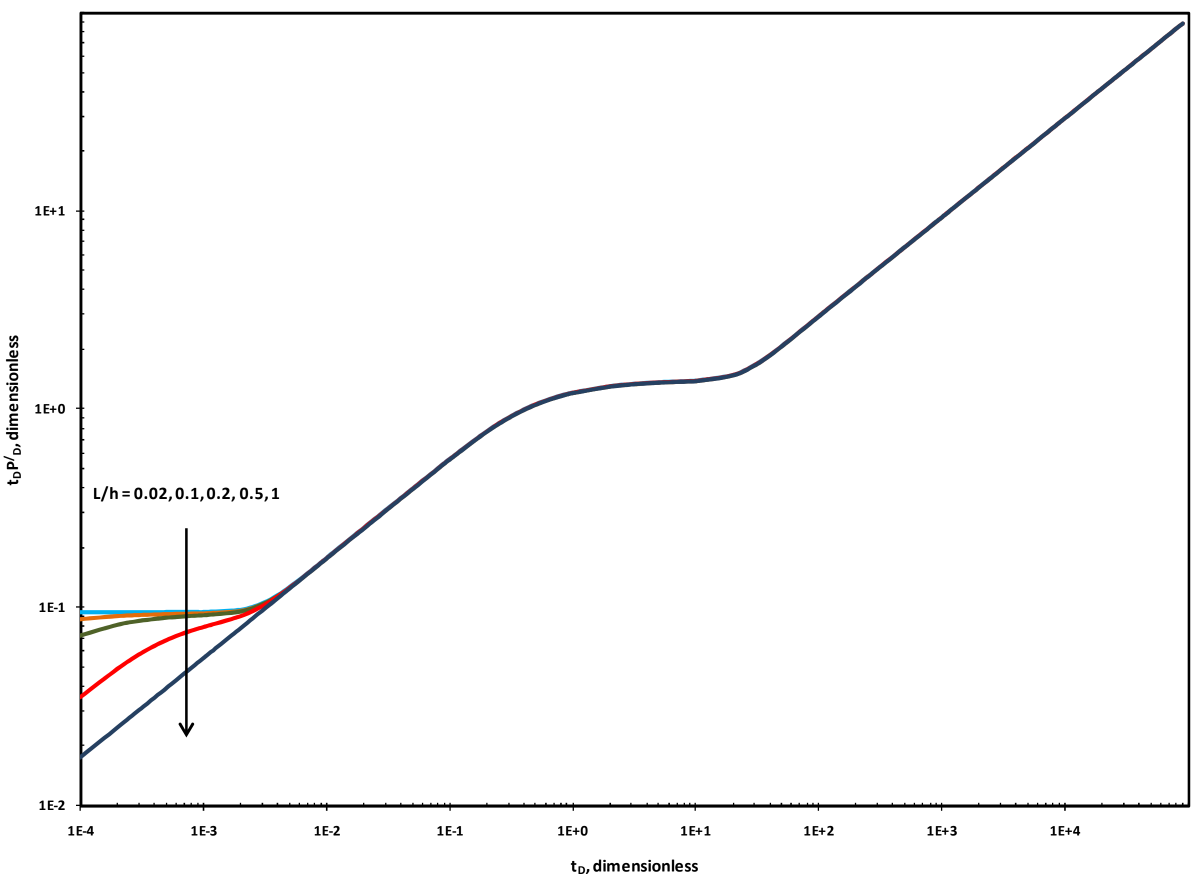

Fig. 4.4.1 Effect of partial completion on HF drawdown response in clastic reservoir

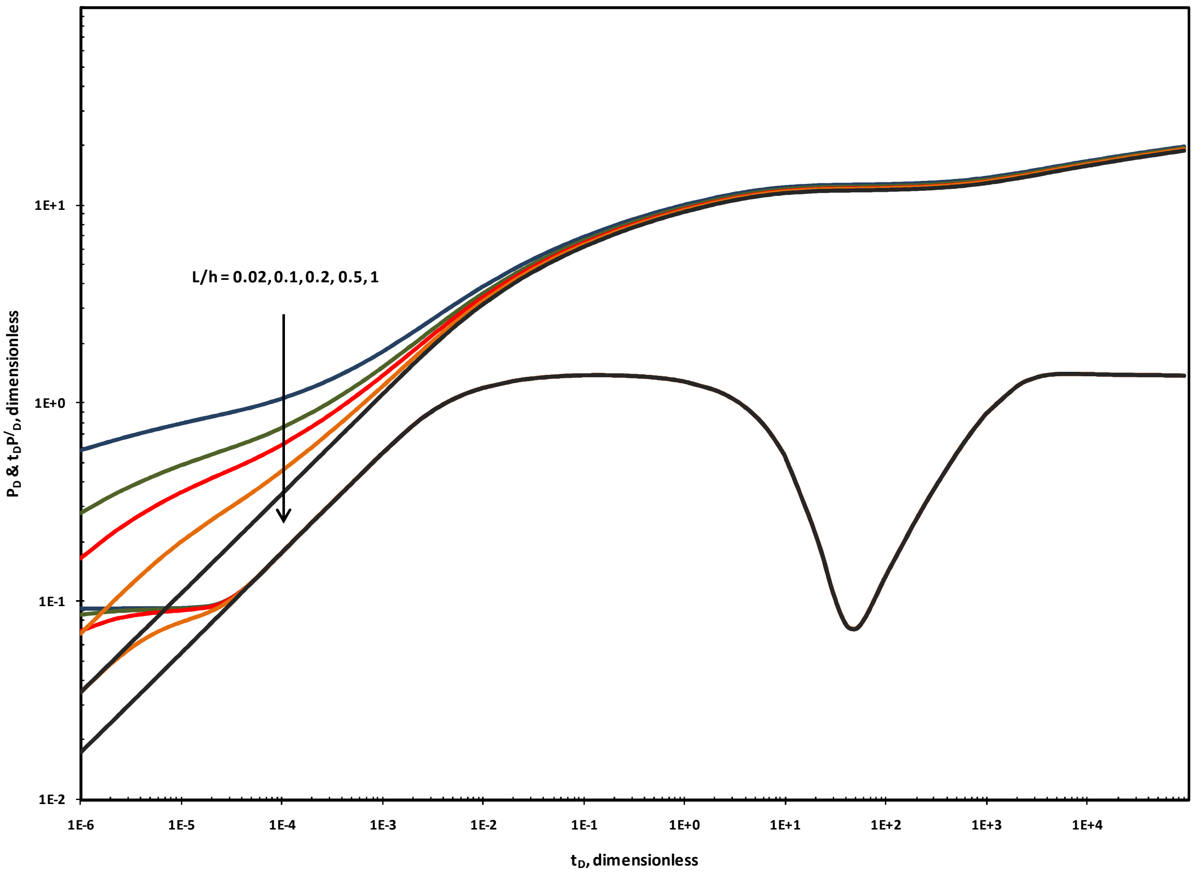

Fig. 4.4.2 Effect of partial completion on HF drawdown response in NFR

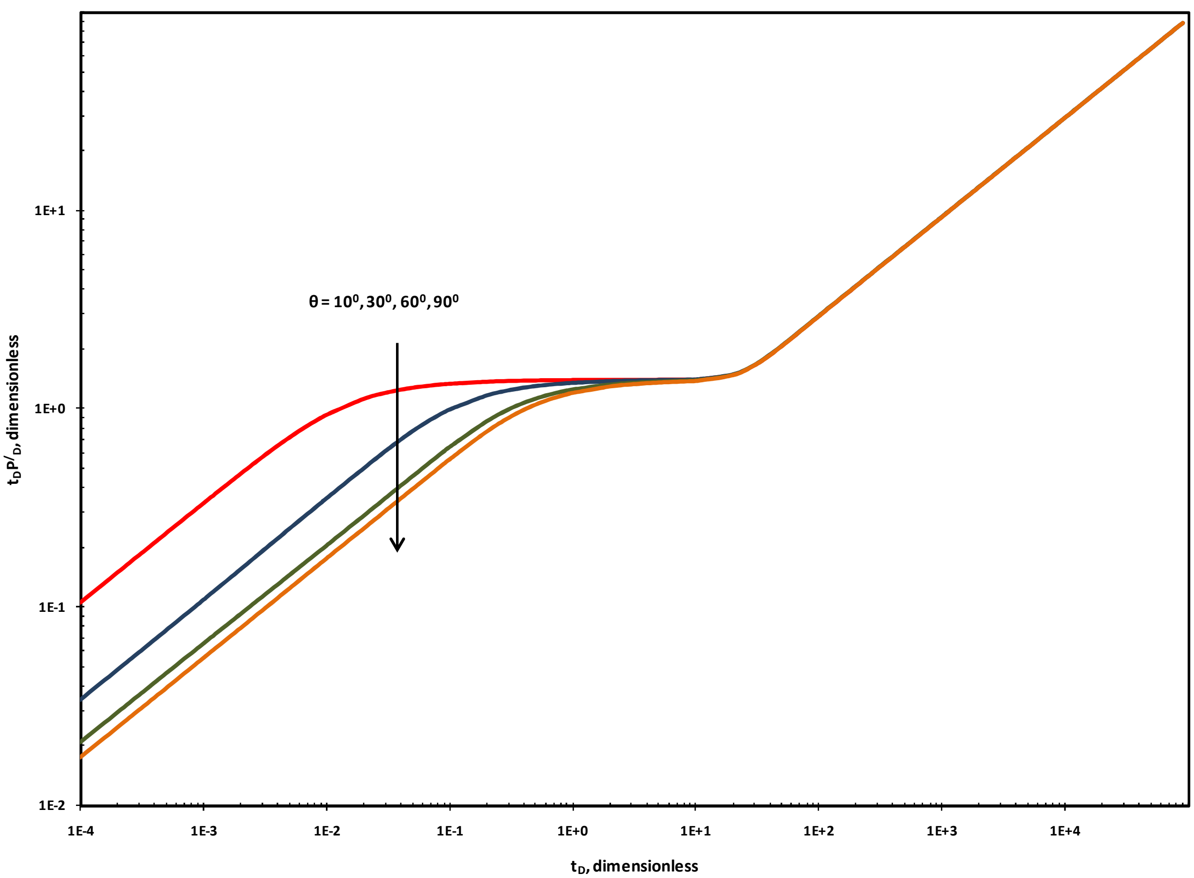

Fig. 4.4.3 Effect of low areal permeability anisotropy on HF drawdown response

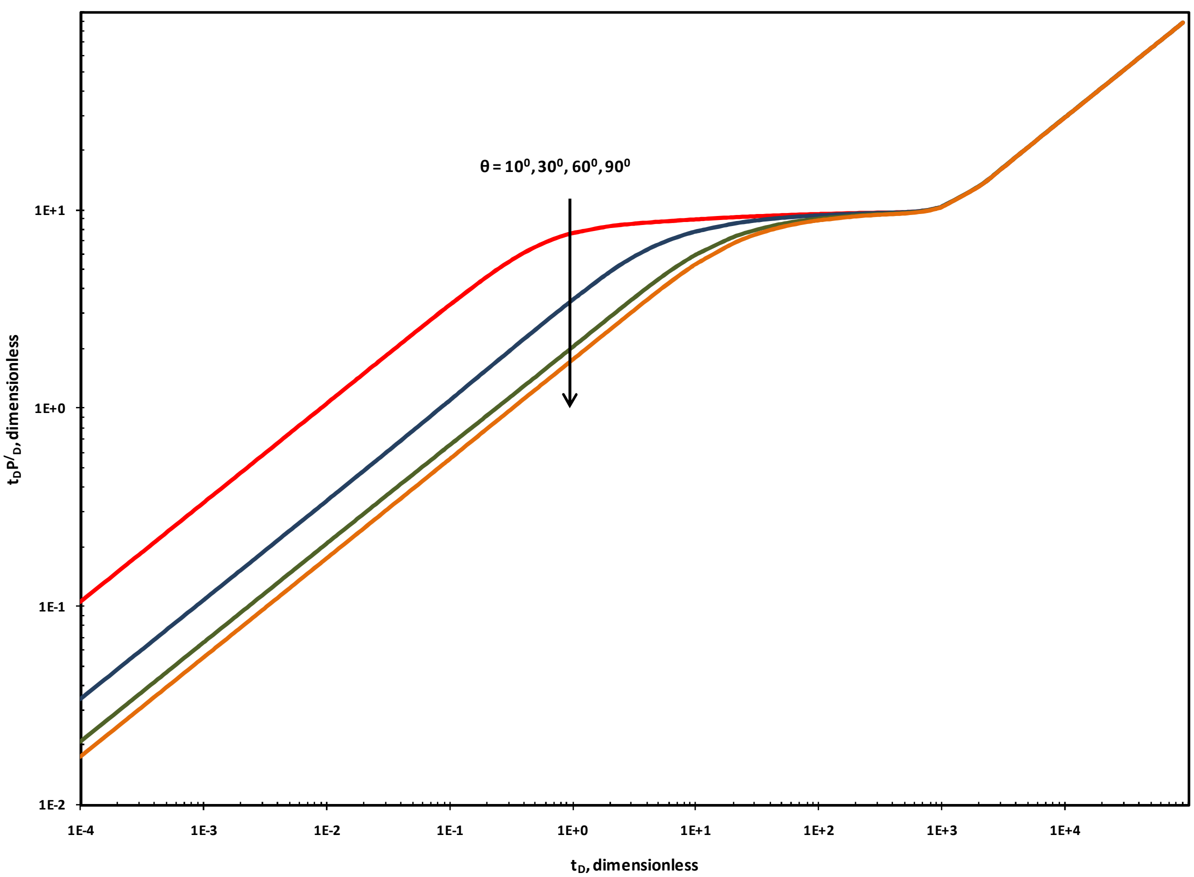

Fig. 4.4.4 Effect of high areal permeability anisotropy on HF drawdown response

4.5 Interpretation Development

Interpretation for long time approximation of the wellbore pressure will be made for the case where Skin and wellbore storage effects are absent.

Specific Long time Interpretation

- **Reservoir Finite in Two Directions**

At long times,

$$\overline{P}_{WD} \rightarrow \frac{4\pi}{sh_{yD}sin\theta} \times \frac{L_{fD}}{\sqrt{s}(1 + k_{rx}\sqrt{\eta_D})} \qquad (4.5.1)$$

Taking Inverse Laplace transform,

$$t_D P'_D \rightarrow \frac{4\sqrt{\pi}L_{fD}}{h_{yD}sin\theta} \times \frac{\sqrt{t_D}}{(1 + k_{rx}\sqrt{\eta_D})} \qquad (4.5.2)$$

- **Reservoir Finite in Three Directions**

At long times,

$$\mathcal{F}_{m0} \to 0; \quad \mathcal{F}_{0n} \to 0; \quad \mathcal{F}_{mn} \to 0 \ \ and \ \ \overline{P}_{WD} \quad \to \frac{4\pi L_{fD}}{sh_{yD}sin\theta} \times \frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})} \tag{4.5.3}$$

Taking Inverse Laplace transform,

$$t_D P_D' \to \frac{4\pi L_{fD}}{h_{yD}sin\theta} \times \frac{t_D}{h_{xD}} \tag{4.5.4}$$

In addition to these direct syntheses, traditional techniques like type curve matching are also applicable across all reservoir boundary types and the equations 4.2.3, 4.2.6 and 4.2.9 can be used to generate customizable type curves.

The analysis of clastic reservoir systems above can be extended to NFR.

4.6 Discussion

The important flow regimes associated with HF include:

1. Fracture linear flow regime (½ slope)
2. Bilinear flow regime (¼ slope)
3. Formation linear flow regime (½ slope)
4. Elliptical flow regime (0.36 slope)
5. Pseudo-radial flow regime (0.5 constant value)

For all practical purposes, the fracture linear flow regime is usually too short to be observed on a pressure test or masked by wellbore storage effects. Moreover, the elliptical (biradial) flow regime is not always experienced depending on the HF size to reservoir dimensions.

Similar deductions apply here as in the case of HW in the previous chapter with respect to well inclination angle, areal permeability anisotropy, reservoir dimension, well position, fault conductivity and mobility ratio contrast.

As the portion of the fractured well open to flow decreases, the ½ slope (fracture linear flow regime) and ¼ slope (bilinear flow regime) transitions might be experienced but when this fracture height open to flow is very limited, both the fracture linear and bilinear flow regimes disappear, giving rise to early radial flow regime (constant slope). This indicates that the hydraulically fractured vertical well can behave as an equivalent HW (see Figs. 4.4.1 and 4.4.2)

Chapter 5

Vertical Wells

Studies of the effect of partial penetration on the pressure signature produced by wells in clastic and NFRs abound in literature (Raghavan et al. 1978; Bui et al. 2000) but little exist for the case of composite reservoirs.

5.1 Dimensionless Transformation

These dimensionless transformations are used in obtaining subsequent point source solutions.

$$x_D = \frac{x}{x_W} \tag{5.1.1}$$

$$y_D = \frac{y}{x_W}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{5.1.2}$$

$$z_D = \frac{z}{x_W}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{5.1.3}$$

$$r_{WD} = \frac{r_W}{x_W} \tag{5.1.4}$$

$$w_D = \frac{w_f}{2x_W} \tag{5.1.5}$$

$$h_{xD} = \frac{h_x}{x_W} \tag{5.1.6}$$

$$h_{yD} = \frac{h_y}{x_W}\sqrt{\frac{k_{x_1}}{k_{y_1}}} \tag{5.1.7}$$

$$h_{zD} = \frac{h_z}{x_W}\sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{5.1.8}$$

$$t_D = \frac{k_{x_1} t}{\emptyset_1 \mu_1 C_{t_1} {x_W}^2} \tag{5.1.9}$$

$$C_D = \frac{C}{2\pi {r_W}^2 \emptyset_1 L C_{t_1}} \tag{5.1.10}$$

$$P_{D_{n,f}} = \frac{2\pi x_W \sqrt{k_{y_1} k_{z_1}} \Delta P_{n,f}}{\mu_1 q} \quad \Delta P_{n,f} = P_i - P_{n,f} \tag{5.1.11}$$

$$F_{CD} = \frac{w_f k_f}{2 k_{x_1} x_w} \tag{5.1.12}$$

$$L_{zD} = \frac{L}{2x_W} \sqrt{\frac{k_{x_1}}{k_{z_1}}} \tag{5.1.13}$$

5.2 Line source solution for Clastic Reservoirs

- **Infinite in two directions**

The point source solution for a reservoir infinite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin}$$
$$= \frac{1}{s\mathrm{h}_{zD}}\left[\begin{array}{l} \displaystyle\int_{-\infty}^{\infty} \frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \\ \displaystyle+\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} \frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\} e^{i\omega(y_D - y_{WD})}\,d\omega \end{array}\right] \tag{5.2.1}$$

Converting the point source solution to line source solution by integrating Eq. 5.2.1 with respect to (w.r.t.) L_{zD} from Z_{wD}-L_{zD} to $Z_{wD} + L_{zD.}$ Taking account of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{2}{s\mathrm{h}_{zD}}(\alpha + \beta)$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{1}{sL_{zD}}(\alpha + \beta) \tag{5.2.2}$$

$$\alpha + \beta = \left[\begin{array}{l} \displaystyle\int_{-\infty}^{\infty} \frac{L_{zD}(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right\} d\omega \\ \displaystyle+\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} \frac{(1+\mathrm{QS_w})}{Qk}\cos^2(kz_{WD})\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\}\sin(kL_{zD})\,d\omega \end{array}\right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage}$$
$$= \frac{\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2 C_D\,\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \tag{5.2.3}$$

- **Finite in two directions**

The point source solution for a reservoir finite in two directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin} = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha + \beta) \tag{5.2.4}$$

$$\alpha = \left[\begin{array}{c} \frac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{4\sqrt{\mathrm{s}}}\left\{1 + e^{-2\sqrt{\mathrm{s}}x_{WD}}\frac{\left(\sqrt{\mathrm{s}}E_p - E_p{'}\right)}{\left(\sqrt{\mathrm{s}}E_p + E_p{'}\right)}\right\} \\ + \sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1 + e^{-2Tx_{WD}}\frac{\left(TF_p - F_p{'}\right)}{\left(TF_p + F_p{'}\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\cos(kz_{WD})\cos(kz_D)\left\{1 + e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{'}\right)}{\left(RD_p + D_p{'}\right)}\right\} \\ + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)\left\{1 + e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{'}\right)}{\left(QC_p + C_p{'}\right)}\right\} \end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

Converting the point source solution to line source solution by integrating Eq. 5.2.4 with respect to (w.r.t.) L_{zD} from Z_{wD}-L_{zD} to $Z_{wD} + L_{zD}$. Taking account of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha + \beta)$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}}(\alpha + \beta) \tag{5.2.5}$$

$$\alpha = \left[\begin{array}{c} \frac{L_{zD}\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{2\sqrt{\mathrm{s}}}\left\{1 + e^{-2\sqrt{\mathrm{s}}x_{WD}}\frac{\left(\sqrt{\mathrm{s}}E_p - E_p{'}\right)}{\left(\sqrt{\mathrm{s}}E_p + E_p{'}\right)}\right\} \\ + \sum_{m=1}^{\infty}\frac{L_{zD}(1+\mathrm{TS_w})}{T}\cos^2(ly_{WD})\left\{1 + e^{-2Tx_{WD}}\frac{\left(TF_p - F_p{'}\right)}{\left(TF_p + F_p{'}\right)}\right\} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{(1+\mathrm{RS_w})}{Rk} \sin k(L_{zD}) \cos^2(kz_{WD}) \left\{ 1 + e^{-2Rx_{WD}} \frac{\left(RD_p - {D_p}'\right)}{\left(RD_p + {D_p}'\right)} \right\} \\ + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2(1+\mathrm{QS_w})}{Qk} \cos^2(kz_{WD}) \sin k(L_{zD}) \cos^2(ly_{WD}) \left\{ 1 + e^{-2Qx_{WD}} \frac{\left(QC_p - {C_p}'\right)}{\left(QC_p + {C_p}'\right)} \right\} \end{array} \right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P_{WD}}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage} = \frac{\overline{P_{WD}}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2 C_D\,\overline{P_{WD}}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \quad (5.2.6)$$

- **Finite in three directions**

The point source solution for a reservoir finite in three directions is given below.

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{5.2.7}$$

$$\mathcal{F}_{00} = \left[\frac{1}{4\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{c}\cosh\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + S_w\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\sinh 2\sqrt{\mathrm{s}}(h_{xD} - x_{WD}) + 1 + \cosh 2\sqrt{\mathrm{s}}(h_{xD} - x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})}\left\{\begin{array}{c}\cosh\mathrm{T}(h_{xD} - 2x_{WD}) + S_w\mathrm{T}\sinh\mathrm{T}(h_{xD} - 2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w\mathrm{T}\sinh 2\mathrm{T}(h_{xD} - x_{WD}) + 1 + \cosh 2\mathrm{T}(h_{xD} - x_{WD})]}{TF_p\sinh\mathrm{T}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)}{2R\sinh R(h_{xD})}\left\{\begin{array}{c}\cosh R(h_{xD} - 2x_{WD}) + S_w\mathrm{R}\sinh R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w\mathrm{R}\sinh 2R(h_{xD} - x_{WD}) + 1 + \cosh 2R(h_{xD} - x_{WD})]}{RD_p\sinh R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})}\times \\ \left\{\begin{array}{c}\cosh Q(h_{xD} - 2x_{WD}) + S_w\mathrm{Q}\sinh Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w\mathrm{Q}\sinh 2Q(h_{xD} - x_{WD}) + 1 + \cosh 2Q(h_{xD} - x_{WD})]}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{array}\right\}\end{array}\right]$$

Converting the point source solution to line source solution by integrating Eq. 5.2.7 with respect to (w.r.t.) L_{zD} from Z_{wD}-L_{zD} to Z_{wD} + $L_{zD.}$ Taking account of partial penetration of the well in this system,

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\,skin} = \frac{\mathrm{h}_{zD}}{2L_{zD}}\times\frac{4\pi}{sh_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin} = \frac{2\pi}{s h_{yD} L_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{5.2.8}$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}}{2\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{\begin{matrix}\cosh\sqrt{s}(h_{xD} - 2x_{WD}) + S_w\sqrt{s}\sinh\sqrt{s}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{s}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{s}\sinh 2\sqrt{s}(h_{xD} - x_{WD}) + 1 + \cosh 2\sqrt{s}(h_{xD} - x_{WD})\right]}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{}'\cosh\sqrt{s}(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{L_{zD}\cos^2(l y_{WD})}{T\sinh T(h_{xD})}\left\{\begin{matrix}\cosh T(h_{xD} - 2x_{WD}) + S_w T\sinh T(h_{xD} - 2x_{WD}) + \cosh(T h_{xD}) \\ -\dfrac{F_p{}'[S_w T\sinh 2T(h_{xD} - x_{WD}) + 1 + \cosh 2T(h_{xD} - x_{WD})]}{T F_p\sinh T(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{\cos^2(k z_{WD})\sin k(L_{zD})}{Rk\sinh R(h_{xD})}\left\{\begin{matrix}\cosh R(h_{xD} - 2x_{WD}) + S_w R\sinh R(h_{xD} - 2x_{WD}) + \cosh(R h_{xD}) \\ -\dfrac{D_p{}'[S_w R\sinh 2R(h_{xD} - x_{WD}) + 1 + \cosh 2R(h_{xD} - x_{WD})]}{R D_p\sinh R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{matrix}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{matrix}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(k z_{WD})\sin k(L_{zD})\cos^2(l y_{WD})}{Qk\sinh Q(h_{xD})} \times \\ \left\{\begin{matrix}\cosh Q(h_{xD} - 2x_{WD}) + S_w Q\sinh Q(h_{xD} - 2x_{WD}) + \cosh(Q h_{xD}) \\ -\dfrac{C_p{}'[S_w Q\sinh 2Q(h_{xD} - x_{WD}) + 1 + \cosh 2Q(h_{xD} - x_{WD})]}{Q C_p\sinh Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{matrix}\right\}\end{matrix}\right]$$

Dimensionless pressure at the wellbore becomes;

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin\ and\ wellbore\ storage} = \frac{\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}}{1 + s^2 C_D\,\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s)_{with\ skin}} \tag{5.2.9}$$

5.3 Line source solution for Naturally Fractured Reservoirs

Everything is the same as section 5.2 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

5.4 Type Curves

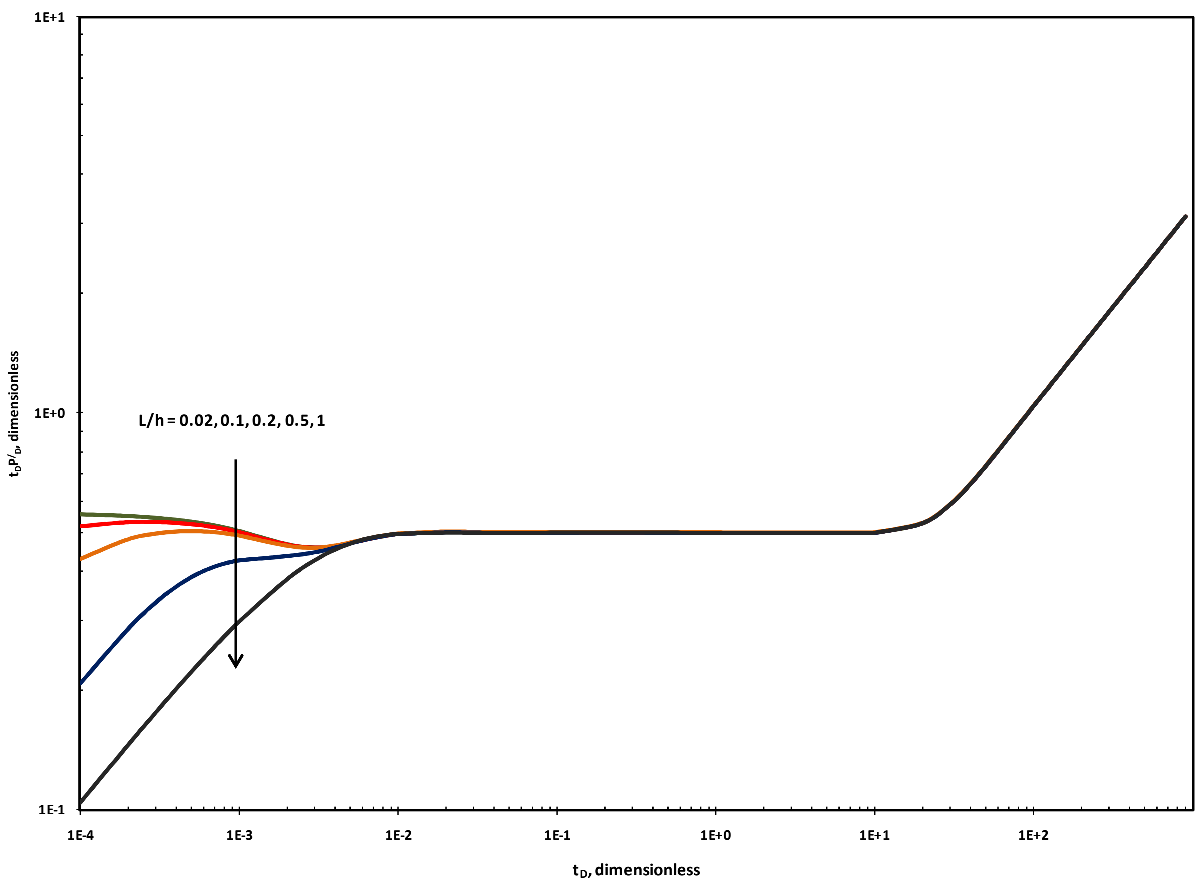

Fig. 5.4.1 Effect of partial completion on vertical well drawdown response

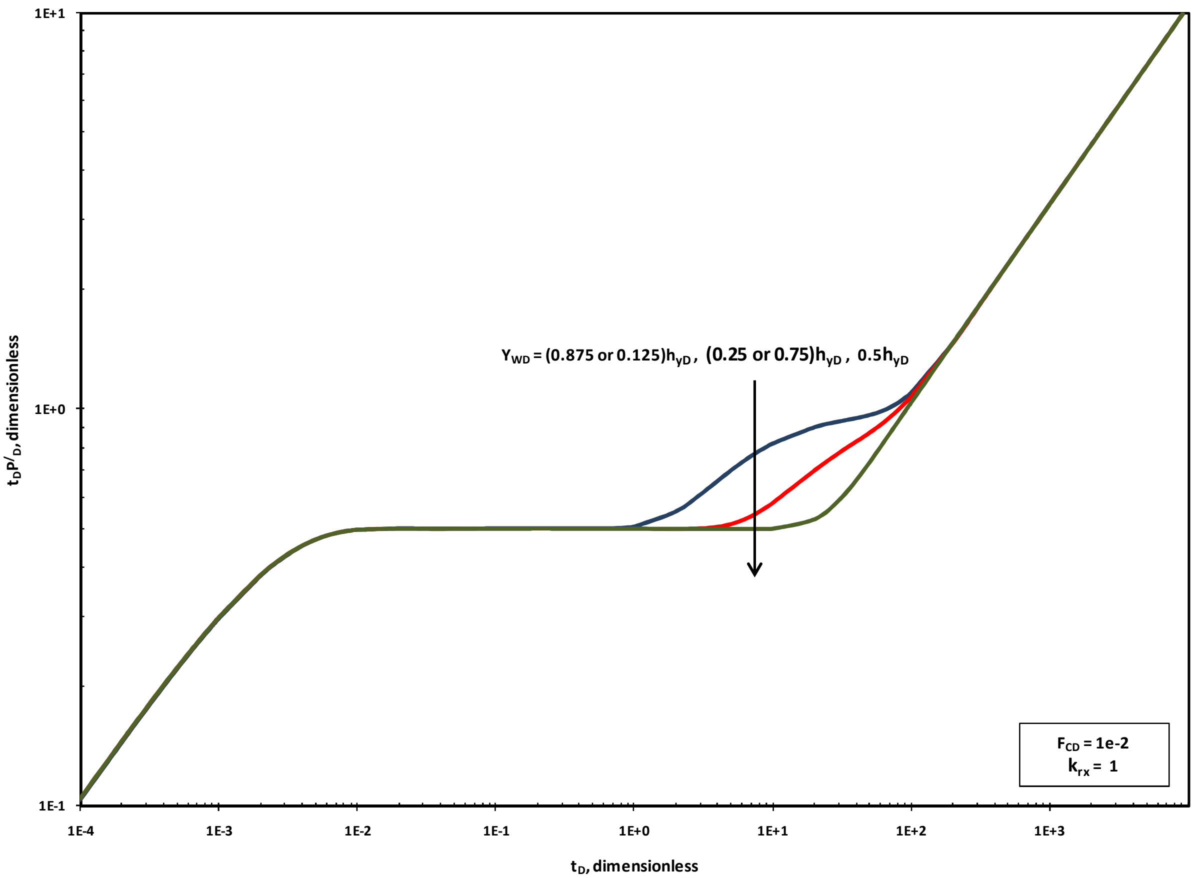

Fig. 5.4.2 Effect of y-distance of vertical well to boundary on drawdown response

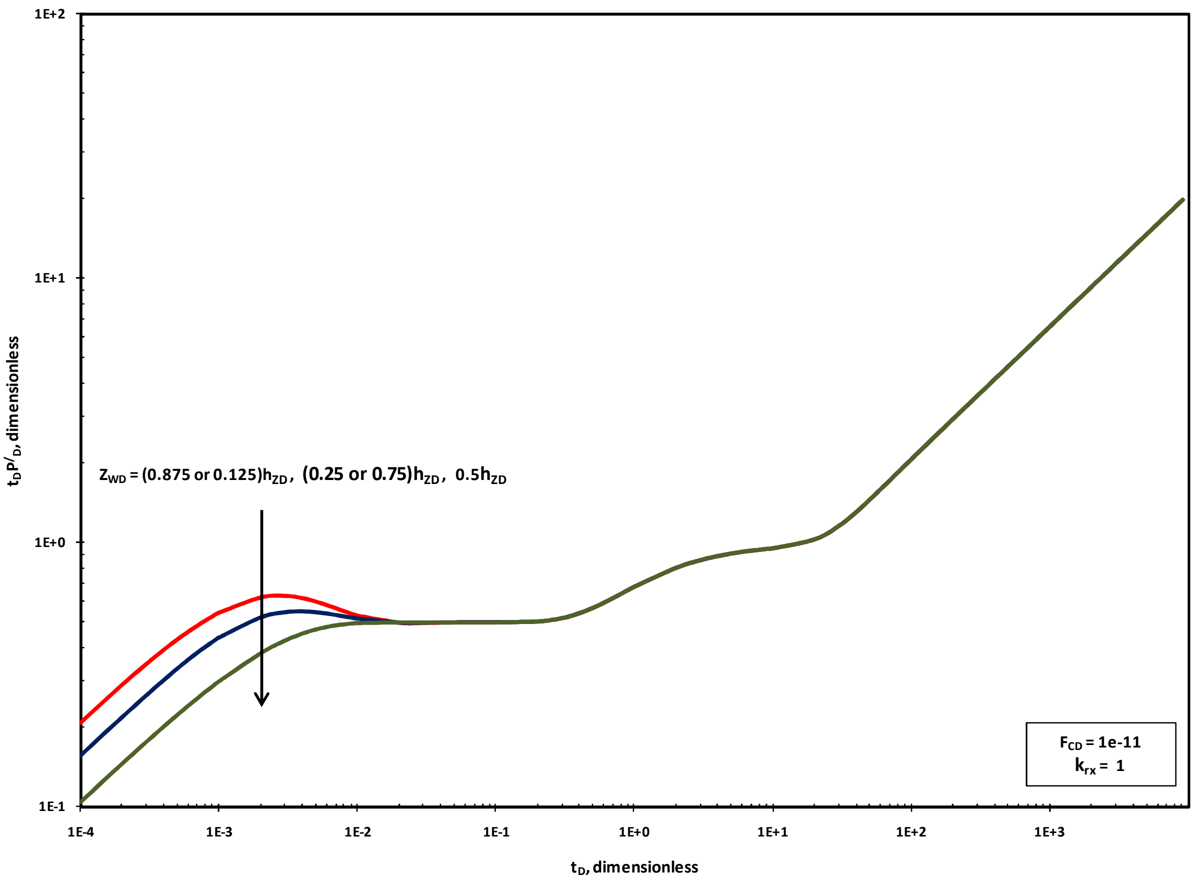

Fig. 5.4.3 Effect of z-distance of vertical well to boundary on drawdown response

5.5 Interpretation Development

Interpretation for long time approximation of the wellbore pressure will be made for the case where Skin and wellbore storage effects are absent.

Specific Long time Interpretation

- **Reservoir Finite in Two Directions**

At long times,

$$\overline{P}_{WD} \rightarrow \frac{2\pi}{sh_{yD}} \times \frac{1}{\sqrt{s}(1 + k_{rx}\sqrt{\eta_D})} \tag{5.5.1}$$

Taking Inverse Laplace transform,

$$t_D P_D' \rightarrow \frac{2\sqrt{\pi}}{h_{yD}} \times \frac{\sqrt{t_D}}{(1 + k_{rx}\sqrt{\eta_D})} \tag{5.5.2}$$

- **Reservoir Finite in Three Directions**

At long times,

$$\mathcal{F}_{m0} \rightarrow 0; \quad \mathcal{F}_{0n} \rightarrow 0; \; \mathcal{F}_{mn} \rightarrow 0 \; and \; \overline{P}_{WD} \rightarrow \frac{2\pi}{sh_{yD}} \times \frac{\cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}\sinh\sqrt{s}(h_{xD})} \tag{5.5.3}$$

Taking Inverse Laplace transform,

$$t_D P_D' \rightarrow \frac{2\pi}{h_{yD}} \times \frac{t_D}{h_{xD}} \tag{5.5.4}$$

In addition to these syntheses, traditional techniques like type curve matching are also applicable across all reservoir boundary types. The equations 5.2.3, 5.2.6 and 5.2.9 can be used to generate customizable type curves and the analysis of clastic reservoir systems above can be extended to NFR.

5.6 Discussion

Apart from the flow regimes unique to vertical wells, it has similar pressure signature to that of HF. The smaller the portion of the well open to flow, the more the early linear flow regime (½ slope) deforms to spherical flow (-½ slope) since the producing interval now behaves like a point source (see Fig. 5.4.1).

Chapter 6

Interference Testing

Interference test has proven to be a good reservoir characterization tool from which the average areal transmissivity, storativity, and degree of communication between wells could be obtained.

Despite the documented success of horizontal wells as producers, little is known about its behaviour as observers, let alone in composite reservoir systems.

Chen et al. (1984), Malekzadeh et al. (1991), Brown et al. (1991), Malekzadeh (1992), Al-Khamis et al. (2001), Houali et al. (2005), Al-Khamis et al. (2005) and Awotunde et al. (2008) are among the few who have researched interference testing both in clastic and NFRs.

Hence, this study shall not re-invent the wheel but state in few words outstanding observations made that has never been documented before. In this study, the following dimensionless quantity is given below

$$L_{yD} = L_{fD} = \frac{L_{observer}}{2L_{producer}}\sqrt{\cos^2\theta + \frac{k_{x_1}}{k_{y_1}}\sin^2\theta} = \frac{1}{2}\sqrt{\cos^2\theta + v_y\sin^2\theta} \tag{6.1}$$

Converting to the y plane, $L_{yD} or\ L_{fD}\ becomes\ L_{yD} sin\theta' or\ L_{fD} sin\theta'$

This might be used interchangeably for convenience purposes

6.1 Line source solution for Clastic Reservoirs

- Infinite in two directions

The point source solution for a reservoir infinite in two directions is given below.

$$\overline{P}_{D2}\,(x_D, y_D, k, s)$$

$$= \frac{1}{\mathrm{sh}_{zD}} \left[\begin{array}{c} \int\limits_{-\infty}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \\ + \sum\limits_{n=1}^{\infty} \int\limits_{-\infty}^{\infty} 2\cos(kz_D)\cos(kz_{WD}) \frac{e^{Q_1(x_D - x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \end{array} \right] \tag{6.1.1}$$

(i) Horizontal Wells

Converting the point source solution to line source solution by integrating (6.1.1) with respect to (w.r.t.) L_{yD} from $Y^{/}_{WD}$ - $L_{yD}\sin\theta^{/}$ to $Y^{/}_{WD}$ + $L_{yD}\sin\theta^{/}$

Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) = \frac{2}{\mathrm{sh}_{zD} sin\theta}(\alpha + \beta)$$

$$\alpha + \beta = \left[\begin{array}{c} \int\limits_{-\infty}^{\infty} \frac{1}{\omega} \frac{e^{R_1(x_D - x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y^{/}_{WD} - y_{WD})} \sin(\omega L_{yD})\, d\omega \\ + \sum\limits_{n=1}^{\infty} \int\limits_{-\infty}^{\infty} \frac{2}{\omega} \cos(kz_D)\cos(kz_{WD}) \frac{e^{Q_1(x_D - x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y^{/}_{WD} - y_{WD})} \sin(\omega L_{yD})\, d\omega \end{array} \right]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{2}{\text{sh}_{zD} sin\theta}(\alpha+\beta)$$

$$\alpha+\beta = \left[\begin{array}{c} \int_{-\infty}^{\infty} \frac{1}{\omega} \frac{e^{-R_1(x_{WD}^{/}+x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_{WD}^{/}-y_{WD})} \sin\left(\omega L_{yD}\right) d\omega \\ + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{2}{\omega} \cos\left(kz_{WD}^{/}\right) \cos(kz_{WD}) \frac{e^{-Q_1(x_{WD}^{/}+x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_{WD}^{/}-y_{WD})} \sin\left(\omega L_{yD}\right) d\omega \end{array}\right] \tag{6.1.2}$$

(ii) Hydraulically Fractured Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.1) w.r.t. (L_{fD} and L_{zD}) from $Y^{/}_{WD} - L_{fD}\sin\theta^{/}$ to $Y^{/}_{WD} + L_{fD}\sin\theta^{/}$ and $Z^{/}_{WD} - L_{zD}$ to $Z^{/}_{WD} + L_{zD}$ respectively.

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{\text{h}_{zD}}{2L_{zD}} \times \frac{4}{\text{sh}_{zD} sin\theta}(\alpha+\beta)$$

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{2}{sL_{zD} sin\theta}(\alpha+\beta)$$

$$\alpha+\beta = \left[\begin{array}{c} \int_{-\infty}^{\infty} \frac{L_{zD}}{\omega} \frac{e^{R_1(x_D-x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_{WD}^{/}-y_{WD})} \sin\left(\omega L_{fD}\right) d\omega \\ + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{2}{\omega k} \frac{e^{Q_1(x_D-x_{WD})}}{QC_p + C_p^{'}} \cos(kz_{WD}) \cos\left(kz_{WD}^{/}\right) e^{i\omega(y_{WD}^{/}-y_{WD})} \sin\left(\omega L_{fD}\right) \sin(kL_{zD}) d\omega \end{array}\right]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{2}{sL_{zD}sin\theta}(\alpha+\beta) \qquad (6.1.3)$$

$$\alpha+\beta = \left[\begin{array}{c} \int_{-\infty}^{\infty} \frac{L_{zD}}{\omega}\frac{e^{-R_1(x_{WD}^{/}+x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_{WD}^{/}-y_{WD})}\sin\left(\omega L_{fD}\right)d\omega \\ +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{2}{\omega k}\frac{e^{-Q_1(x_{WD}^{/}+x_{WD})}}{QC_p + C_p^{'}}\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)e^{i\omega(y_{WD}^{/}-y_{WD})}\sin\left(\omega L_{fD}\right)\sin(kL_{zD})\,d\omega \end{array}\right]$$

(iii) Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.1) with respect to (w.r.t.) L_{zD} from $Z^{/}_{WD}$-L_{zD} to $Z^{/}_{WD}$ + L_{zD}

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{2}{s\mathrm{h}_{zD}}(\alpha+\beta)$$

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{1}{sL_{zD}}(\alpha+\beta)$$

$$\alpha+\beta = \left[\begin{array}{c} \int_{-\infty}^{\infty} 2L_{zD}\frac{e^{R_1(x_D-x_{WD})}}{RD_p + D_p^{'}}e^{i\omega(y_D-y_{WD})}d\omega \\ +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty} 4\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD})\frac{e^{Q_1(x_D-x_{WD})}}{k[QC_p + C_p^{'}]}e^{i\omega(y_D-y_{WD})}d\omega \end{array}\right]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{1}{sL_{zD}}(\alpha + \beta) \tag{6.1.4}$$

$$\alpha + \beta = \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} 2L_{zD} \frac{e^{-R_1(x_{WD}^{/} + x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_{WD}^{/} - y_{WD})} d\omega \\ \displaystyle + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} 4\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD}) \frac{e^{-Q_1(x_{WD}^{/} + x_{WD})}}{k[QC_p + C_p^{'}]} e^{i\omega(y_{WD}^{/} - y_{WD})} d\omega \end{array}\right]$$

- **Finite in two directions**

The point source solution for a reservoir finite in two directions is given below.

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha+\beta) \tag{6.1.5}$$

$$\alpha = \left[\frac{e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{2\left(\sqrt{\mathrm{s}}E_p + E_p^{'}\right)} + \sum_{m=1}^{\infty}\frac{e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(TF_p + F_p^{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\frac{e^{R_1(x_D - x_{WD})}}{\left(RD_p + D_p^{'}\right)}\cos(kz_D)\cos(kz_{WD}) \\ \displaystyle + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2e^{Q_1(x_D - x_{WD})}}{\left(QC_p + C_p^{'}\right)}\cos(kz_D)\cos(kz_{WD})\cos(ly_{WD})\cos(ly_D) \end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad\qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

(i) Horizontal Wells

Converting the point source solution to line source solution by integrating (6.1.5) with respect to (w.r.t.) L_{yD} from $Y^{/}_{WD}$ - $L_{yD}\sin\theta^{/}$ to $Y^{/}_{WD}$ + $L_{yD}\sin\theta^{/}$

Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}sin\theta}(\alpha+\beta)$$

$$\alpha = \left[\frac{L_{yD}e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{\left(\sqrt{\mathrm{s}}E_p + E_p^{'}\right)} + \sum_{m=1}^{\infty}\frac{2e^{T_1(x_D - x_{WD})}\sin l\left(L_{yD}\right)\cos(ly_{WD})\cos\left(ly^{/}_{WD}\right)}{l\left(TF_p + F_p^{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{2L_{yD} e^{R_1 (x_D - x_{WD})}}{\left(RD_p + D_p{}'\right)} \cos(kz_D) \cos(kz_{WD}) \\ + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4 e^{Q_1 (x_D - x_{WD})}}{l\left(QC_p + C_p{}'\right)} \cos(kz_D) \cos(kz_{WD}) \sin l\left(L_{yD}\right) \cos(ly_{WD}) \cos\left(ly'_{WD}\right) \end{array} \right]$$

At the Observation well,

$$\overline{P}_{D2}\left(x'_{WD}, y'_{WD}, z'_{WD}, s\right) = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta} (\alpha + \beta) \tag{6.1.6}$$

$$\alpha = \left[\frac{L_{yD} e^{-\sqrt{s\eta_D}(x'_{WD} + x_{WD})}}{\left(\sqrt{s}E_p + E_p{}'\right)} + \sum_{m=1}^{\infty} \frac{2 e^{-T_1 (x'_{WD} + x_{WD})} \sin l\left(L_{yD}\right) \cos(ly_{WD}) \cos\left(ly'_{WD}\right)}{l\left(TF_p + F_p{}'\right)} \right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{2L_{yD} e^{-R_1 (x'_{WD} + x_{WD})}}{\left(RD_p + D_p{}'\right)} \cos\left(kz'_{WD}\right) \cos(kz_{WD}) \\ + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4 e^{-Q_1 (x'_{WD} + x_{WD})}}{l\left(QC_p + C_p{}'\right)} \cos\left(kz'_{WD}\right) \cos(kz_{WD}) \sin l\left(L_{yD}\right) \cos(ly_{WD}) \cos\left(ly'_{WD}\right) \end{array} \right]$$

(ii) Hydraulically Fractured Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.5) w.r.t. (L_{fD} and L_{zD}) from $Y'_{WD} - L_{fD}\sin\theta'$ to $Y'_{WD} + L_{fD}\sin\theta'$ and $Z'_{WD} - L_{zD}$ to $Z'_{WD} + L_{zD}$ respectively.

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}\left(x_D, y_D, z_D, s\right) = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta} (\alpha + \beta)$$

$$\overline{P}_{D2}\left(x_D, y_D, z_D, s\right) = \frac{2\pi}{s\mathrm{h}_{yD} L_{zD} sin\theta} (\alpha + \beta)$$

$$\alpha = \left[\frac{2L_{zD}L_{fD}e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{\left(\sqrt{s}E_p + E_p^{'}\right)} + \sum_{m=1}^{\infty}\frac{4L_{zD}\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)e^{T_1(x_D - x_{WD})}}{l\left(TF_p + F_p^{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c}\sum_{n=1}^{\infty}4L_{fD}\sin k(L_{zD})\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\frac{e^{R_1(x_D - x_{WD})}}{k\left(RD_p + D_p^{'}\right)} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}8\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\frac{e^{Q_1(x_D - x_{WD})}}{kl\left(QC_p + C_p^{'}\right)}\end{array}\right]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{2\pi}{\mathrm{s}h_{yD}L_{zD}sin\theta}(\alpha + \beta) \tag{6.1.7}$$

$$\alpha = \left[\frac{2L_{zD}L_{fD}e^{-\sqrt{s\eta_D}(x_{WD}^{/} + x_{WD})}}{\left(\sqrt{s}E_p + E_p^{'}\right)} + \sum_{m=1}^{\infty}\frac{4L_{zD}\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)e^{-T_1(x_{WD}^{/} + x_{WD})}}{l\left(TF_p + F_p^{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c}\sum_{n=1}^{\infty}4L_{fD}\sin k(L_{zD})\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\frac{e^{-R_1(x_{WD}^{/} + x_{WD})}}{k\left(RD_p + D_p^{'}\right)} + \\ \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}8\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\frac{e^{-Q_1(x_{WD}^{/} + x_{WD})}}{kl\left(QC_p + C_p^{'}\right)}\end{array}\right]$$

(iii) Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.5) with respect to (w.r.t.) L_{zD} from $Z^{/}_{WD}$ - L_{zD} to $Z^{/}_{WD}$ + L_{zD}

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\,(\alpha+\beta)$$

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}}\,(\alpha+\beta)$$

$$\alpha = \left[\frac{L_{zD}e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{\left(\sqrt{s}E_p + E_p{'}\right)} + \sum_{m=1}^{\infty}\frac{2L_{zD}e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(TF_p + F_p{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\frac{2e^{R_1(x_D - x_{WD})}}{k\left(RD_p + D_p{'}\right)}\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD}) \\ \displaystyle +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4e^{Q_1(x_D - x_{WD})}}{k\left(QC_p + C_p{'}\right)}\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\cos(ly_{WD})\cos(ly_D)\sin(kL_{zD}) \end{array}\right]$$

At the Observation well,

$$\overline{P}_{D2}\,\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}}\,(\alpha+\beta) \tag{6.1.8}$$

$$\alpha = \left[\frac{L_{zD}e^{-\sqrt{s\eta_D}(x_{WD}^{/} + x_{WD})}}{\left(\sqrt{s}E_p + E_p{'}\right)} + \sum_{m=1}^{\infty}\frac{2L_{zD}e^{-T_1(x_{WD}^{/} + x_{WD})}\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)}{\left(TF_p + F_p{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\frac{2e^{-R_1(x_{WD}^{/} + x_{WD})}}{k\left(RD_p + D_p{'}\right)}\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD}) \\ \displaystyle +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4e^{-Q_1(x_{WD}^{/} + x_{WD})}}{k\left(QC_p + C_p{'}\right)}\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\sin(kL_{zD}) \end{array}\right]$$

- **Finite in three directions**

The point source solution for a reservoir finite in three directions is given below.

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{6.1.9}$$

$$\mathcal{F}_{00} = \frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{2[\sqrt{\mathrm{s}}E_p \sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}' \cosh(\sqrt{\mathrm{s}}h_{xD})]}[e^{\sqrt{\mathrm{s}\eta_D}x_D} + e^{-\sqrt{\mathrm{s}\eta_D}(x_D+2h_{xD})}]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})}\;[e^{T_1x_D} + e^{-T_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{\cos(kz_D)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh(Rh_{xD})}[e^{R_1x_D} + e^{-R_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cos(kz_D)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \times [e^{Q_1x_D} + e^{-Q_1(x_D+2h_{xD})}]$$

(i) Horizontal Wells

Converting the point source solution to line source solution by integrating (6.1.9) with respect to (w.r.t.) L_{yD} from Y'_{WD} - $L_{yD}\sin\theta'$ to Y'_{WD} + $L_{yD}\sin\theta'$

Converting the anisotropic system to an equivalent isotropic system,

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\mathcal{F}_{00} = \frac{L_{yD}\cosh\sqrt{s}(h_{xD} - x_{WD})}{[\sqrt{s}E_p \sinh\sqrt{s}(h_{xD}) + E_p{}' \cosh(\sqrt{s}h_{xD})]}[e^{\sqrt{s\eta_D}x_D} + e^{-\sqrt{s\eta_D}(x_D+2h_{xD})}]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{2\sin l(L_{yD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cosh T(h_{xD} - x_{WD})}{l[TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})]}\ [e^{T_1 x_D} + e^{-T_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{2L_{yD}\cos(kz_D)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh(Rh_{xD})}[e^{R_1 x_D} + e^{-R_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\sin l(L_{yD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cos(kz_D)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{l[QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})]} \times [e^{Q_1 x_D} + e^{-Q_1(x_D+2h_{xD})}]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{4\pi}{s h_{yD} h_{zD} \sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (6.1.10)$$

$$\mathcal{F}_{00} = \frac{L_{yD}\cosh\sqrt{s}(h_{xD} - x_{WD})}{[\sqrt{s}E_p \sinh\sqrt{s}(h_{xD}) + E_p{}' \cosh(\sqrt{s}h_{xD})]}[e^{-\sqrt{s\eta_D}x_{WD}^{/}} + e^{-\sqrt{s\eta_D}\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{2\sin l(L_{yD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cosh T(h_{xD} - x_{WD})}{l[TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})]}\ [e^{-T_1 x_{WD}^{/}} + e^{-T_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{2L_{yD}\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh(Rh_{xD})}[e^{-R_1 x_{WD}^{/}} + e^{-R_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\sin l(L_{yD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{l[QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})]} \times [e^{-Q_1 x_{WD}^{/}} + e^{-Q_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

(ii) Hydraulically Fractured Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.9) w.r.t. (L_{fD} and L_{zD}) from $Y'_{WD} - L_{fD}\sin\theta'$ to $Y'_{WD} + L_{fD}\sin\theta'$ and $Z'_{WD} - L_{zD}$ to $Z'_{WD} + L_{zD}$ respectively.

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s)\ = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{2\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}sin\theta}\,(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s)\ = \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}sin\theta}\,(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\mathcal{F}_{00} = \frac{2L_{zD}L_{fD}\cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{}'\cosh\left(\sqrt{s}h_{xD}\right)}\,[e^{\sqrt{s\eta_D}x_D} + e^{-\sqrt{s\eta_D}(x_D+2h_{xD})}]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty}\frac{4L_{zD}\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly'_{WD}\right)\cosh T(h_{xD} - x_{WD})}{l[TF_p\sinh T(h_{xD}) + F_p{}'\cosh(Th_{xD})]}\ [e^{T_1x_D} + e^{-T_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty}\frac{4L_{fD}\sin k(L_{zD})\cos(kz_{WD})\cos\left(kz'_{WD}\right)\cosh R(h_{xD} - x_{WD})}{k[RD_p\sinh R(h_{xD}) + D_p{}'\cosh(Rh_{xD})]}\,[e^{R_1x_D} + e^{-R_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{8\cos(kz_{WD})\cos\left(kz'_{WD}\right)\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos(ly_{WD})\cos\left(ly'_{WD}\right)\cosh Q(h_{xD} - x_{WD})}{kl[QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})]} \times [e^{Q_1x_D} + e^{-Q_1(x_D+2h_{xD})}]$$

At the Observation well,

$$\overline{P}_{D2}\left(x'_{WD}, y'_{WD}, z'_{WD}, s\right)\ = \frac{2\pi}{s\mathrm{h}_{yD}L_{zD}sin\theta}\,(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (6.1.11)$$

$$\mathcal{F}_{00} = \frac{2L_{zD}L_{fD}\cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{}'\cosh\left(\sqrt{s}h_{xD}\right)}\,[e^{-\sqrt{s\eta_D}x'_{WD}} + e^{-\sqrt{s\eta_D}\left(2h_{xD} - x'_{WD}\right)}]$$

$$\mathcal{F}_{m0}=\sum_{m=1}^{\infty}\frac{4L_{zD}\sin l(L_{fD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cosh T(h_{xD}-x_{WD})}{l[TF_p\sinh T(h_{xD})+F_p{}^{'}\cosh(Th_{xD})]}[e^{-T_1x_{WD}^{/}}+e^{-T_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{0n}=\sum_{n=1}^{\infty}\frac{4L_{fD}\sin k(L_{zD})\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\cosh R(h_{xD}-x_{WD})}{k[RD_p\sinh R(h_{xD})+D_p{}^{'}\cosh(Rh_{xD})]}[e^{-R_1x_{WD}^{/}}+e^{-R_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{mn}=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{8\cos(kz_{WD})\cos\left(kz_{WD}^{/}\right)\sin k(L_{zD})\sin l(L_{fD})\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cosh Q(h_{xD}-x_{WD})}{kl[QC_p\sinh Q(h_{xD})+C_p{}^{'}\cosh(Qh_{xD})]}\times[e^{-Q_1x_{WD}^{/}}+e^{-Q_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

(iii) Vertical Wells

Converting the point source solution to line source solution by integrating (6.1.5) with respect to (w.r.t.) L_{zD} from $Z^{/}_{WD}-L_{zD}$ to $Z^{/}_{WD}+L_{zD}$

Taking account the effect of partial penetration of the well in this system,

$$\overline{P}_{D2}(x_D,y_D,z_D,s)=\frac{\mathrm{h}_{zD}}{2L_{zD}}\times\frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00}+\mathcal{F}_{m0}+\mathcal{F}_{0n}+\mathcal{F}_{mn})$$

$$\overline{P}_{D2}(x_D,y_D,z_D,s)=\frac{2\pi}{s\mathrm{h}_{yD}L_{zD}}(\mathcal{F}_{00}+\mathcal{F}_{m0}+\mathcal{F}_{0n}+\mathcal{F}_{mn})$$

$$\mathcal{F}_{00}=\frac{L_{zD}\cosh\sqrt{\mathrm{s}}(h_{xD}-x_{WD})}{[\sqrt{s}E_p\sinh\sqrt{\mathrm{s}}(h_{xD})+E_p{}^{'}\cosh\left(\sqrt{s}h_{xD}\right)]}[e^{\sqrt{\mathrm{s}\eta_D}x_D}+e^{-\sqrt{\mathrm{s}\eta_D}(x_D+2h_{xD})}]$$

$$\mathcal{F}_{m0}=\sum_{m=1}^{\infty}\frac{2L_{zD}\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD}-x_{WD})}{TF_p\sinh T(h_{xD})+F_p{}^{'}\cosh(Th_{xD})}[e^{T_1x_D}+e^{-T_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{2\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD})\cosh R(h_{xD}-x_{WD})}{k[RD_p \sinh R(h_{xD}) + D_p{}^{'} \cosh(Rh_{xD})]} [e^{R_1 x_D} + e^{-R_1(x_D+2h_{xD})}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\cos(ly_{WD})\cos(ly_D)\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD})\cosh Q(h_{xD}-x_{WD})}{k[QC_p \sinh Q(h_{xD}) + C_p{}^{'} \cosh(Qh_{xD})]} \times [e^{Q_1 x_D} + e^{-Q_1(x_D+2h_{xD})}]$$

At the Observation well,

$$\overline{P}_{D2}\left(x_{WD}^{/}, y_{WD}^{/}, z_{WD}^{/}, s\right) = \frac{2\pi}{s h_{yD} L_{zD}} (\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \tag{6.1.12}$$

$$\mathcal{F}_{00} = \frac{L_{zD}\cosh\sqrt{s}(h_{xD}-x_{WD})}{[\sqrt{s}E_p \sinh\sqrt{s}(h_{xD}) + E_p{}^{'} \cosh(\sqrt{s}h_{xD})]} [e^{-\sqrt{s\eta_D}x_{WD}^{/}} + e^{-\sqrt{s\eta_D}\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{2L_{zD}\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cosh T(h_{xD}-x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}^{'} \cosh(Th_{xD})} [e^{-T_1 x_{WD}^{/}} + e^{-T_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{2\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD})\cosh R(h_{xD}-x_{WD})}{k[RD_p \sinh R(h_{xD}) + D_p{}^{'} \cosh(Rh_{xD})]} [e^{-R_1 x_{WD}^{/}} + e^{-R_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\cos(ly_{WD})\cos\left(ly_{WD}^{/}\right)\cos\left(kz_{WD}^{/}\right)\cos(kz_{WD})\sin(kL_{zD})\cosh Q(h_{xD}-x_{WD})}{k[QC_p \sinh Q(h_{xD}) + C_p{}^{'} \cosh(Qh_{xD})]} \times [e^{-Q_1 x_{WD}^{/}} + e^{-Q_1\left(2h_{xD}-x_{WD}^{/}\right)}]$$

6.2 Line source solution for Naturally Fractured Reservoirs

Everything is the same as section 6.1 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

6.3 Type Curves

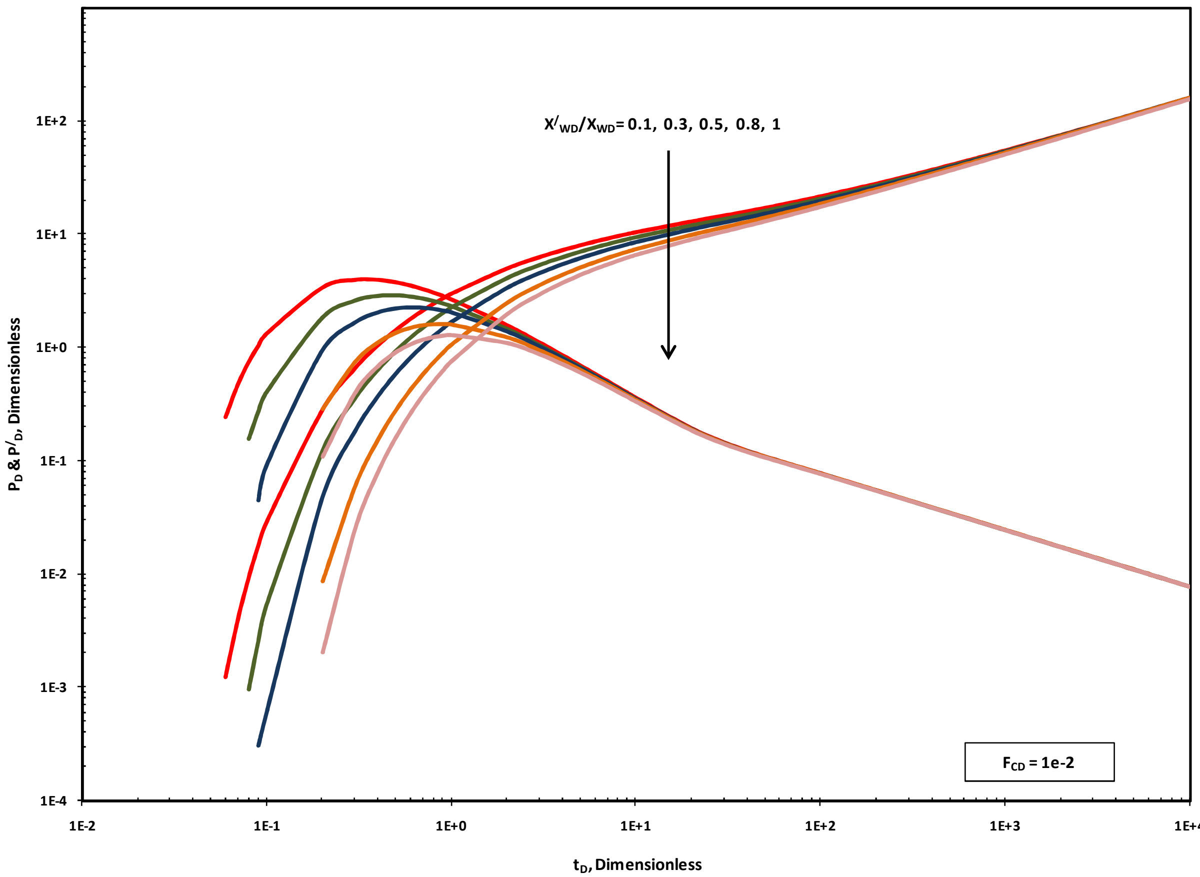

Fig. 6.3.1 Effect of observer distance from fault on observer drawdown response

The figure above shows the effect of changing the observer distance to the fault while keeping the producer distance to the fault constant. The result makes sense because the closer the producer is to the observer, the greater the drawdown experienced by the observer.

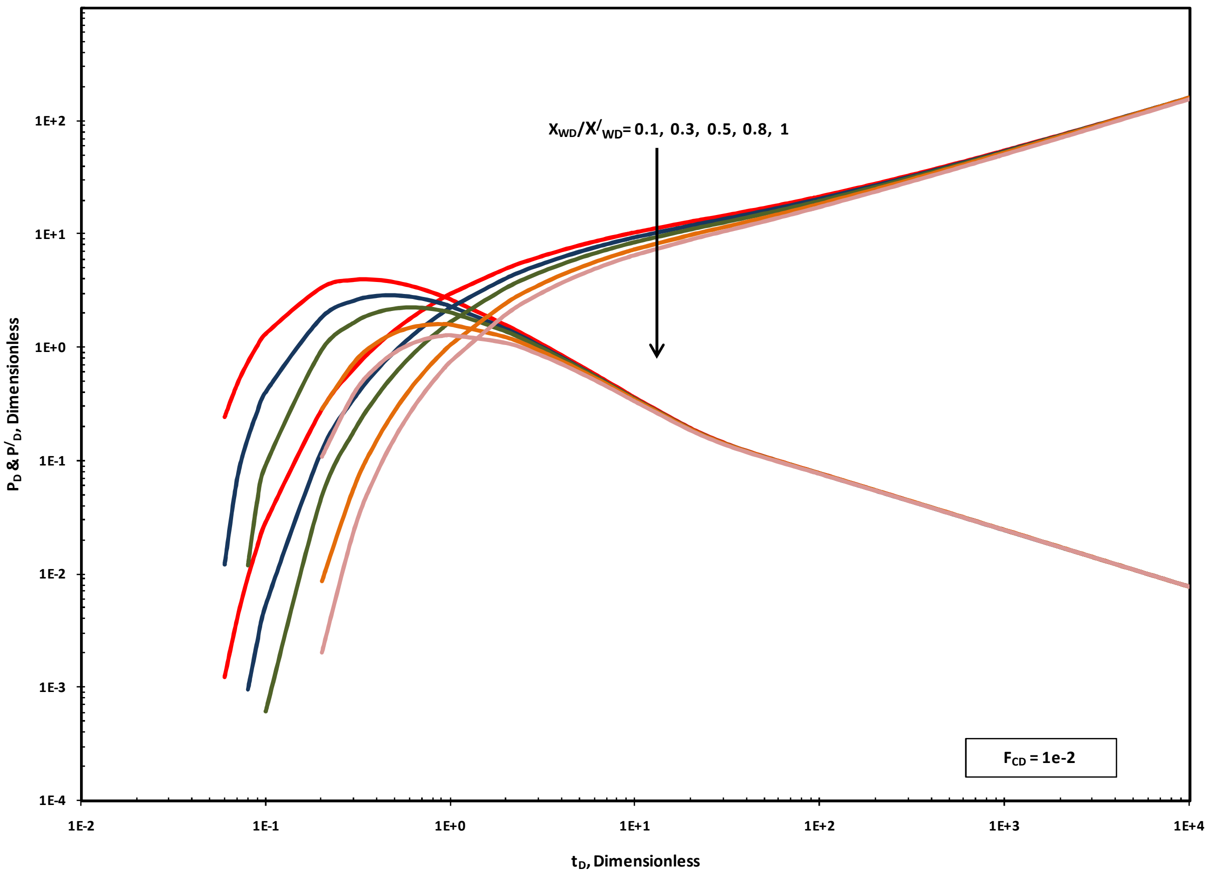

Fig. 6.3.2 Effect of producer distance from fault on observer drawdown response

The figure above shows the effect of changing the producer distance to the fault while keeping the observer distance to the fault constant. A similar phenomenon is observed here as that of the previous figure.

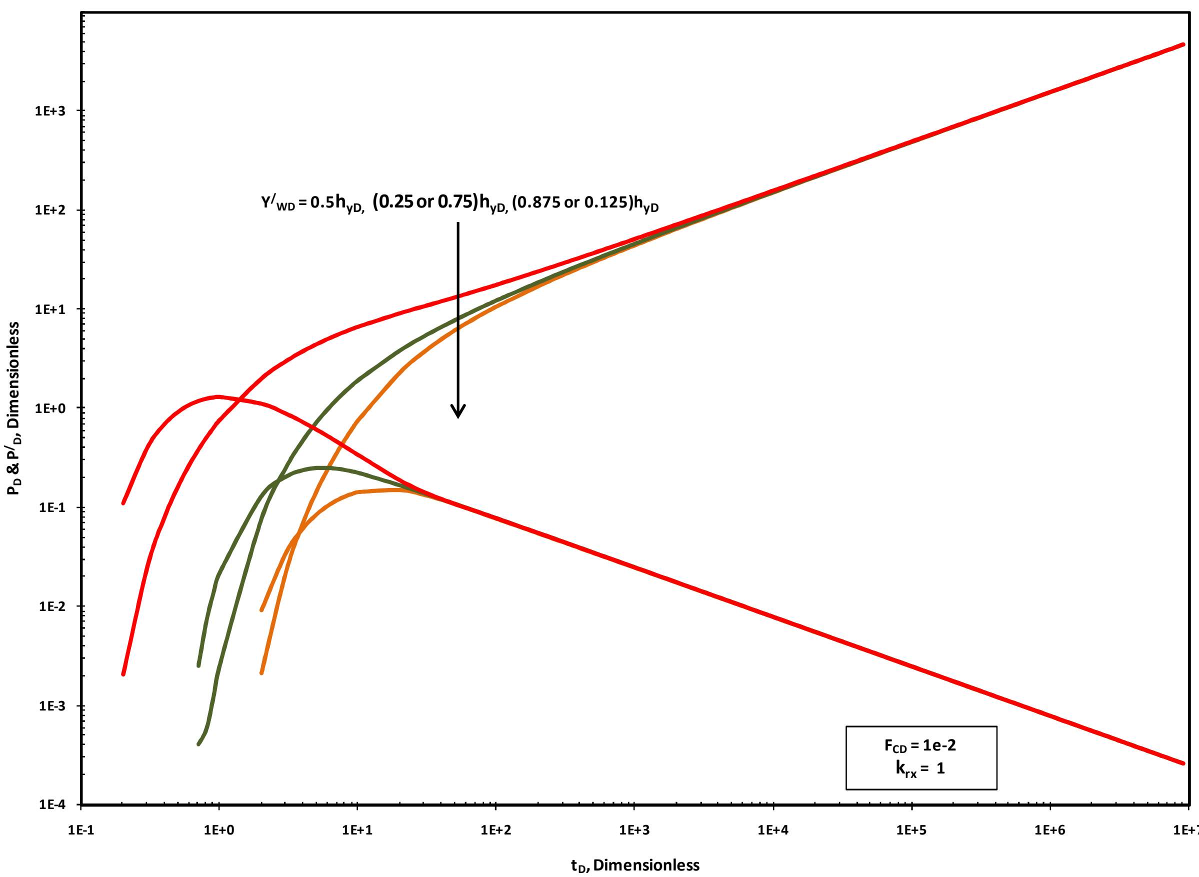

Fig. 6.3.3 Observer y-distance from boundary on observer drawdown response

This figure depicts the effect of observer distance to reservoir boundary along y-direction while keeping the producer at the middle of the reservoir vertically. The result is that the farther the observer is from the middle of the reservoir the lower pressure drop it experiences because it will take a longer time for the disturbance caused by the producer to be propagated to the observer.

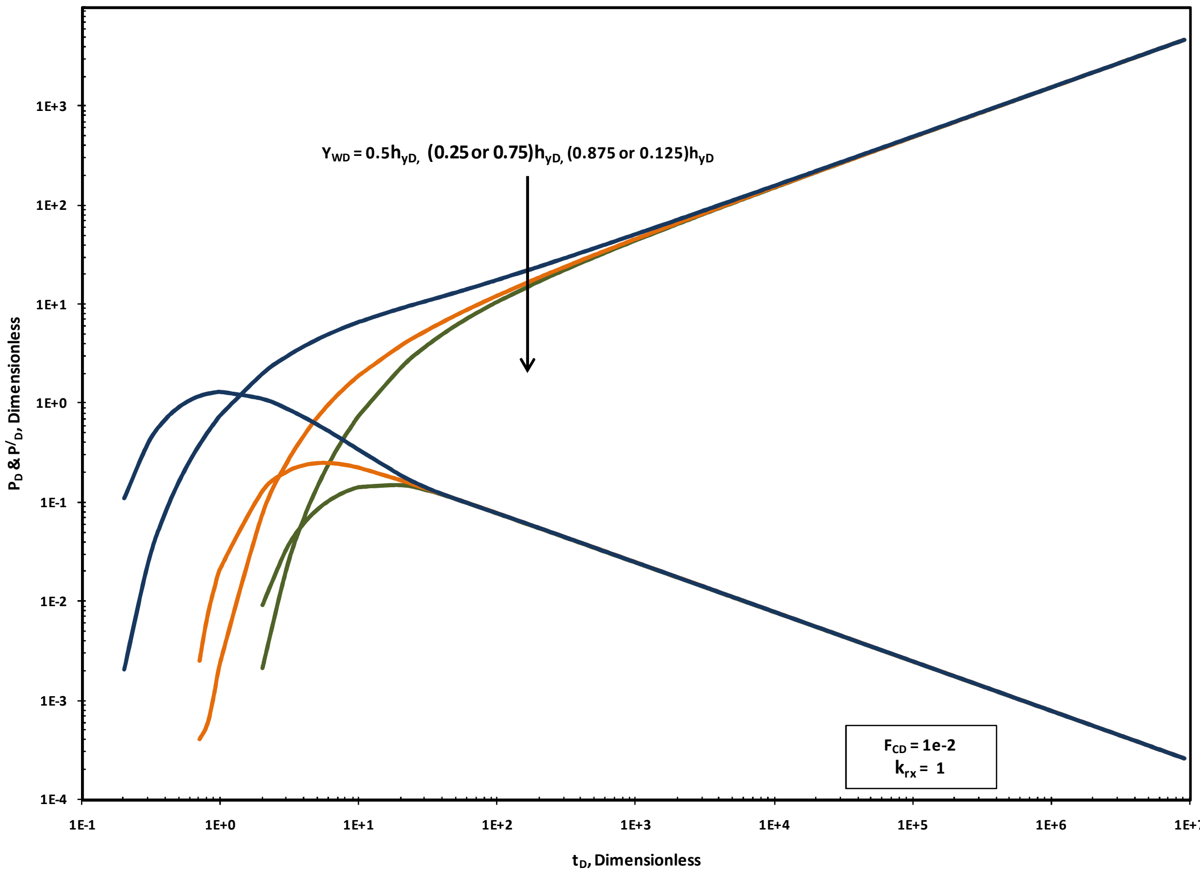

Fig. 6.3.4 Producer y-distance from boundary on observer drawdown response

The figure above illustrates the effect of producer distance to reservoir boundary along y-direction while keeping the observer at the middle of the reservoir vertically. The result is similar to that of Fig.6.3.3.

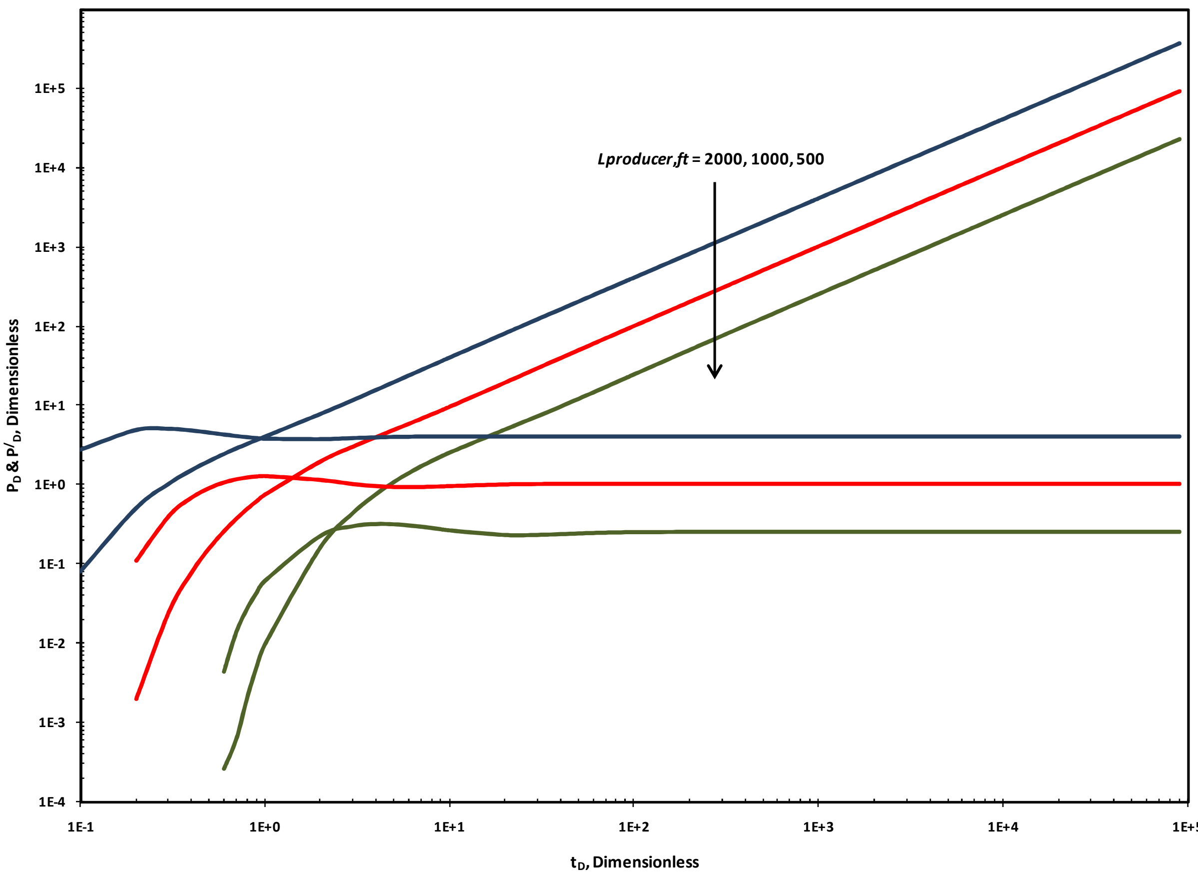

Fig. 6.3.5 Producer length effect on observer response in closed reservoir system

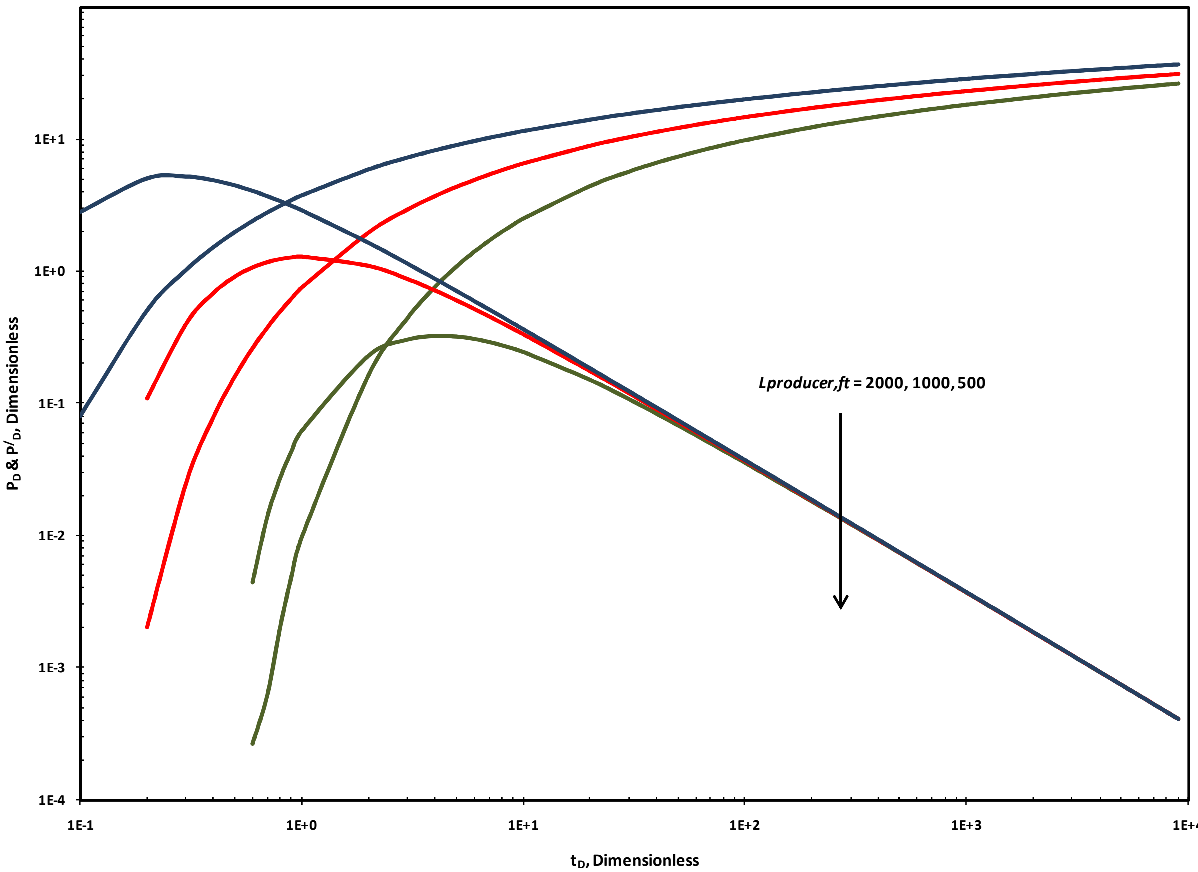

Fig. 6.3.6 Producer length effect on observer response in infinite reservoir system

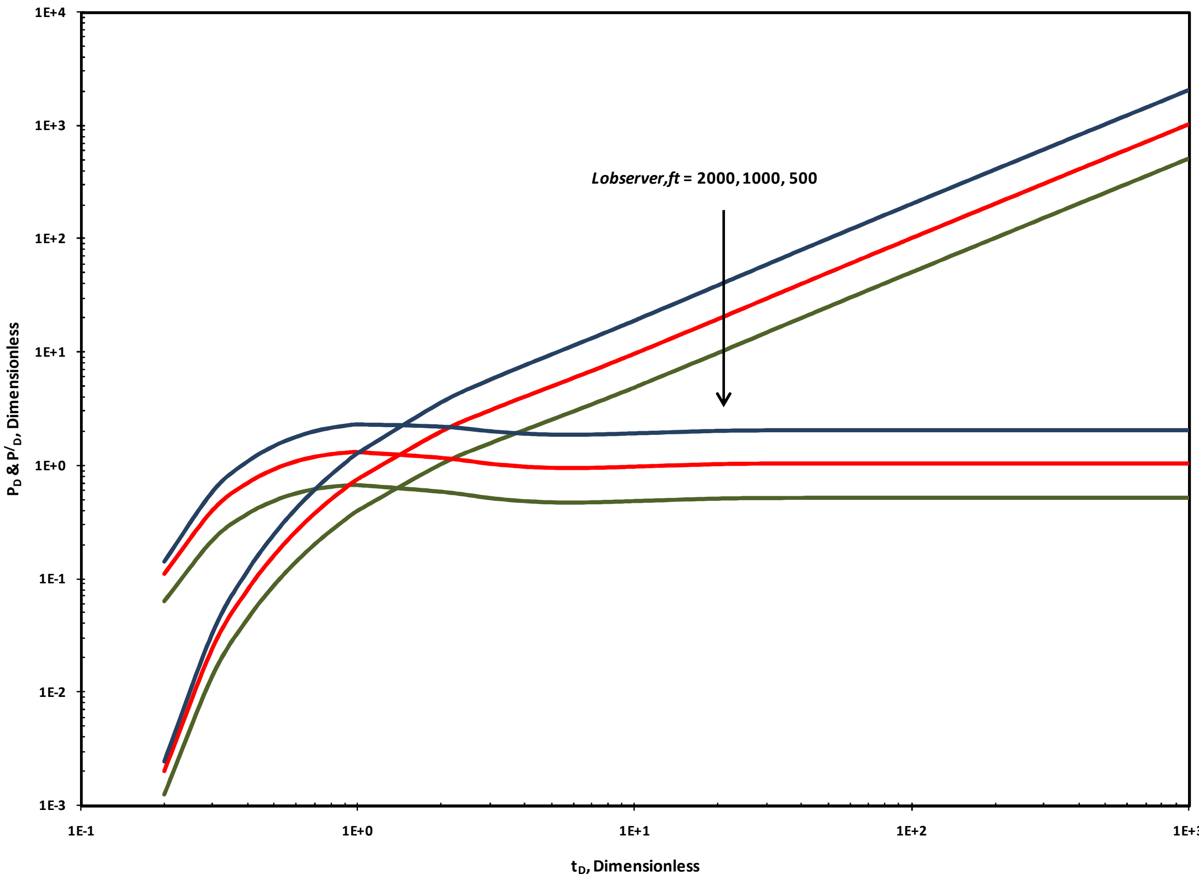

Fig. 6.3.7 Observer length effect on observer response in closed reservoir system

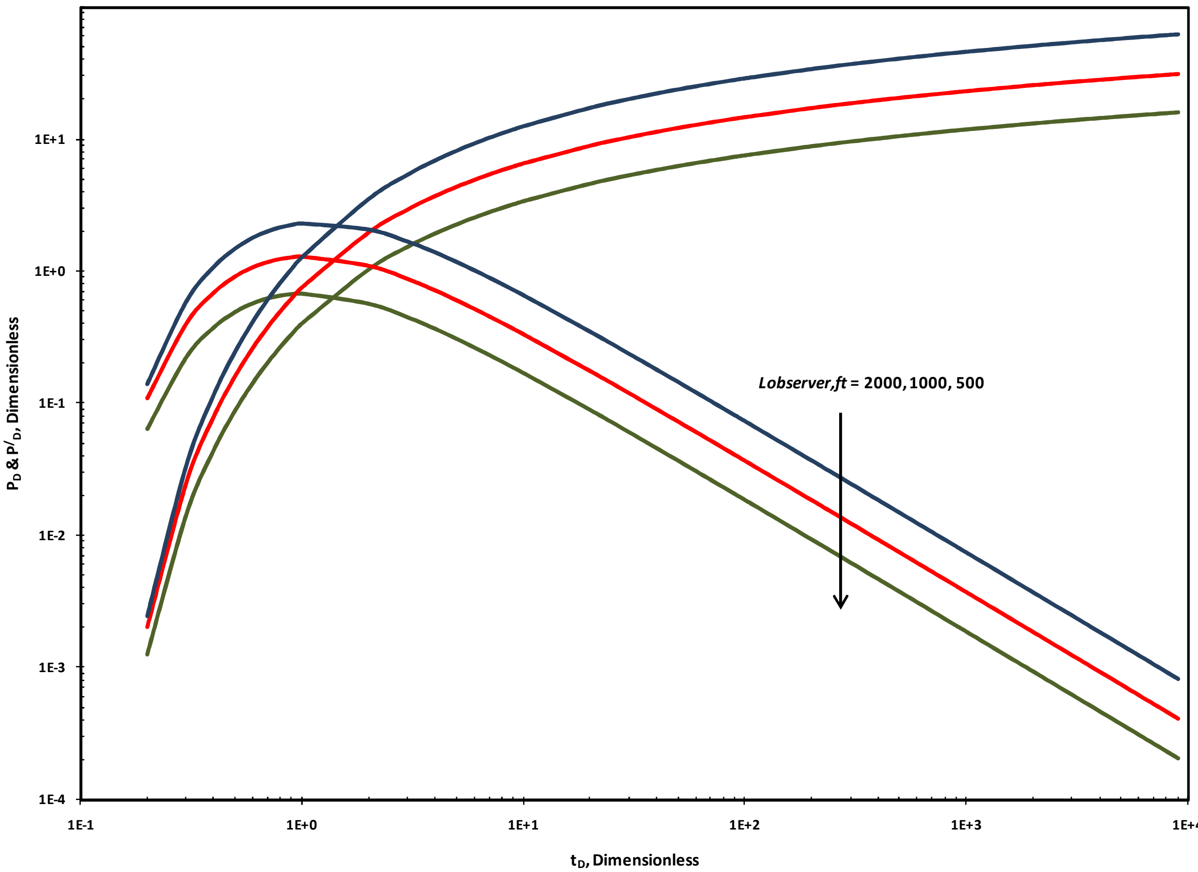

Fig. 6.3.8 Observer length effect on observer response in infinite reservoir system

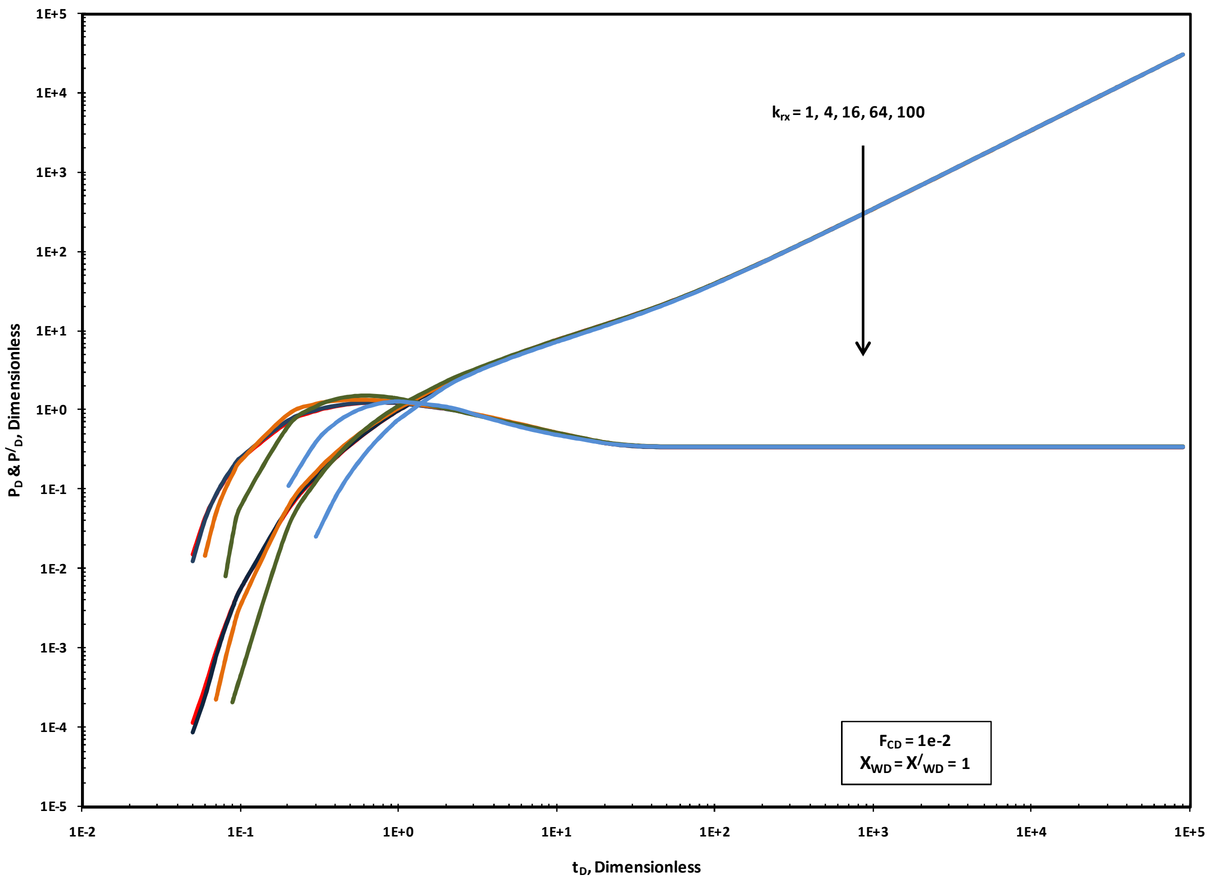

Fig. 6.3.9 Effect of k_x contrast in closed reservoir system (II > I)

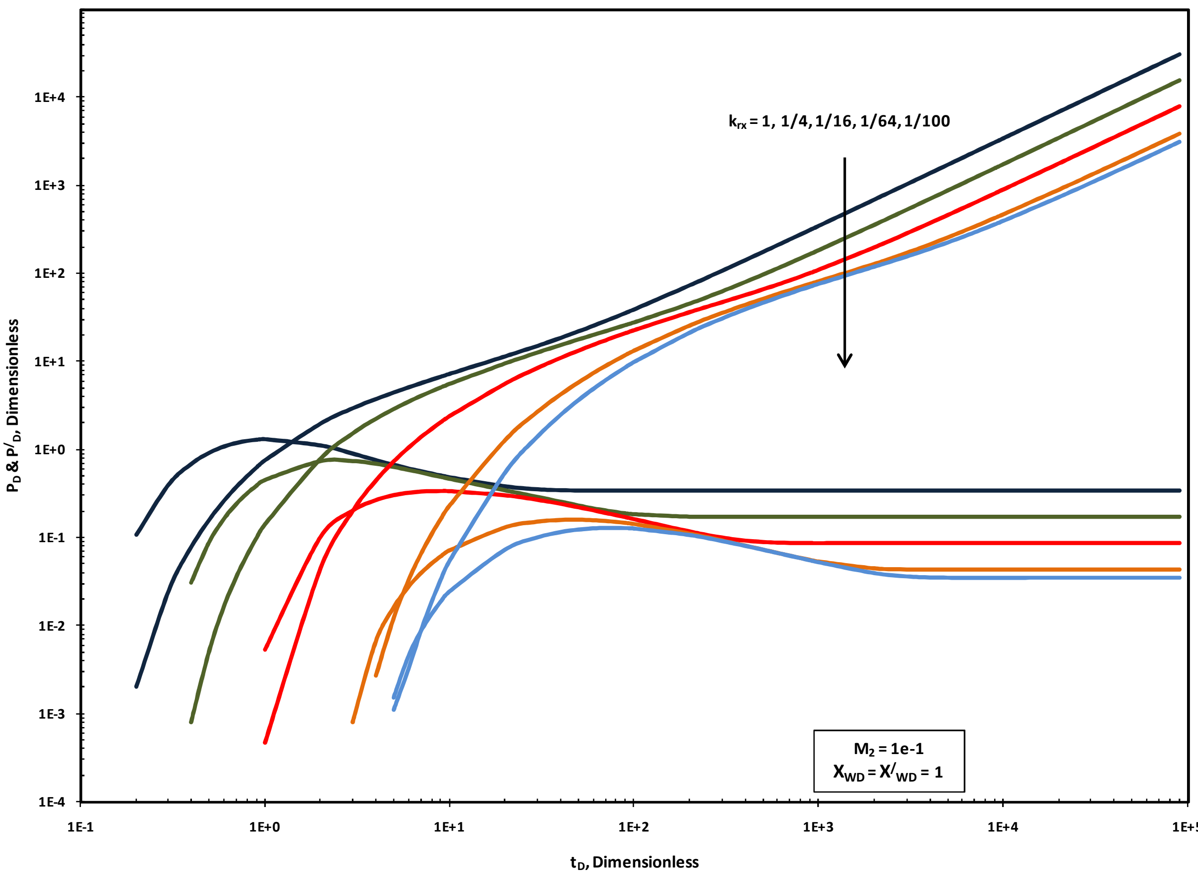

Fig. 6.3.10 Effect of k_x contrast in closed reservoir system (I > II)

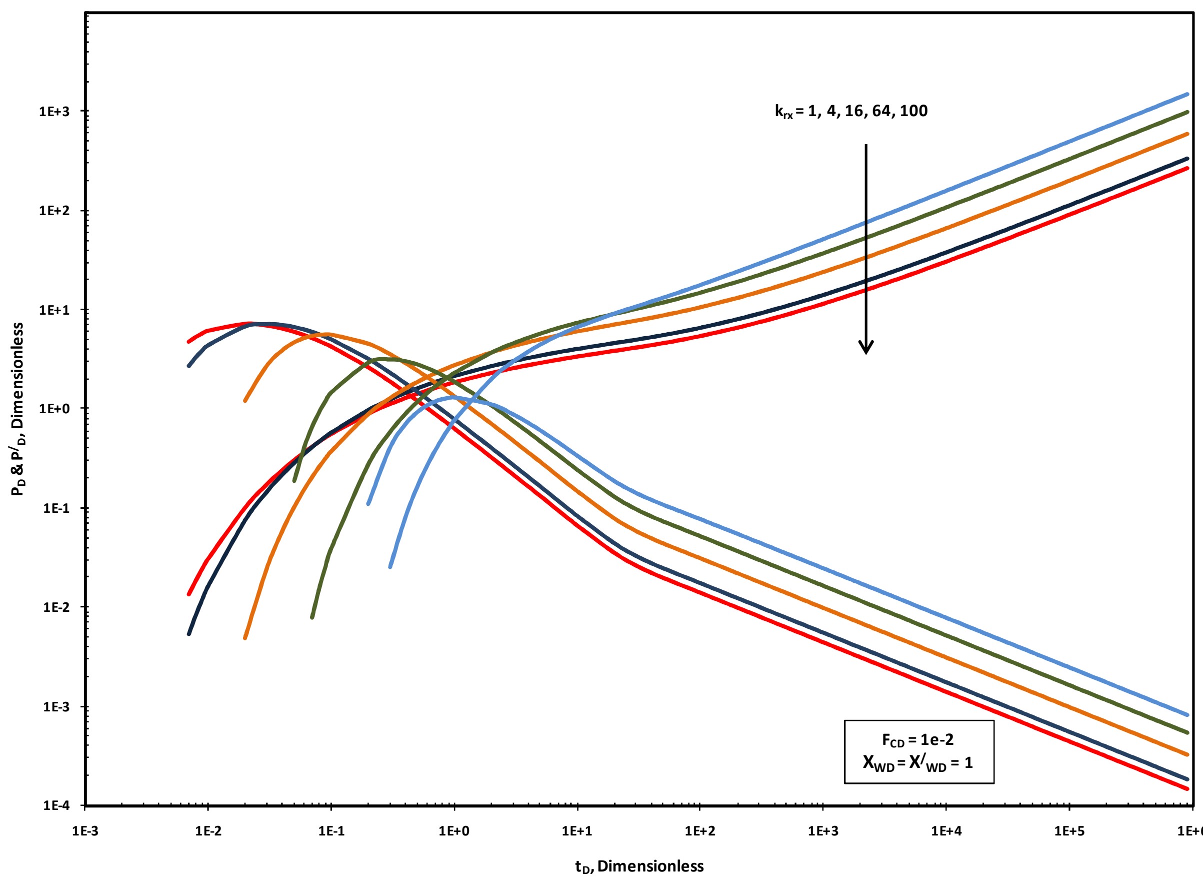

Fig. 6.3.11 Effect of k_x contrast in reservoir finite in two directions (II > I)

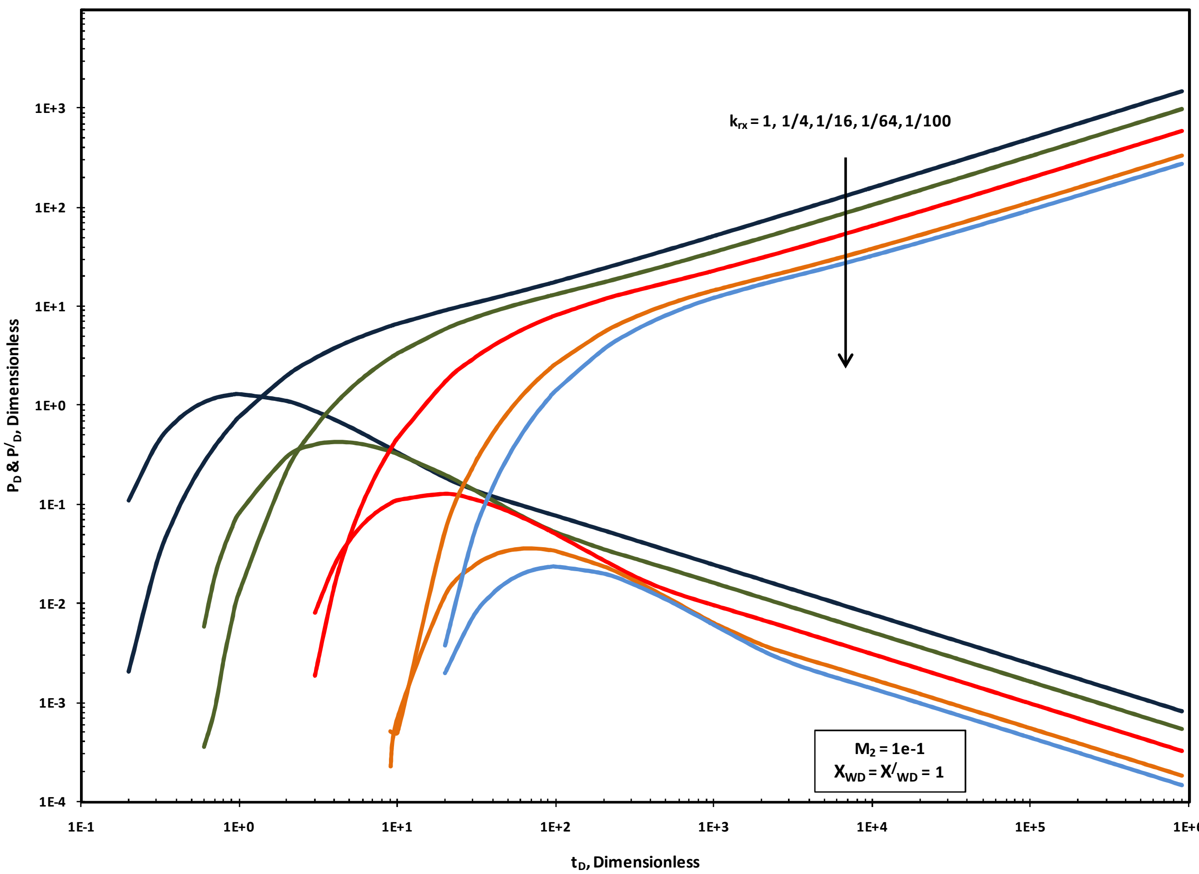

Fig. 6.3.12 Effect of k_x contrast in reservoir finite in two directions (I > II)

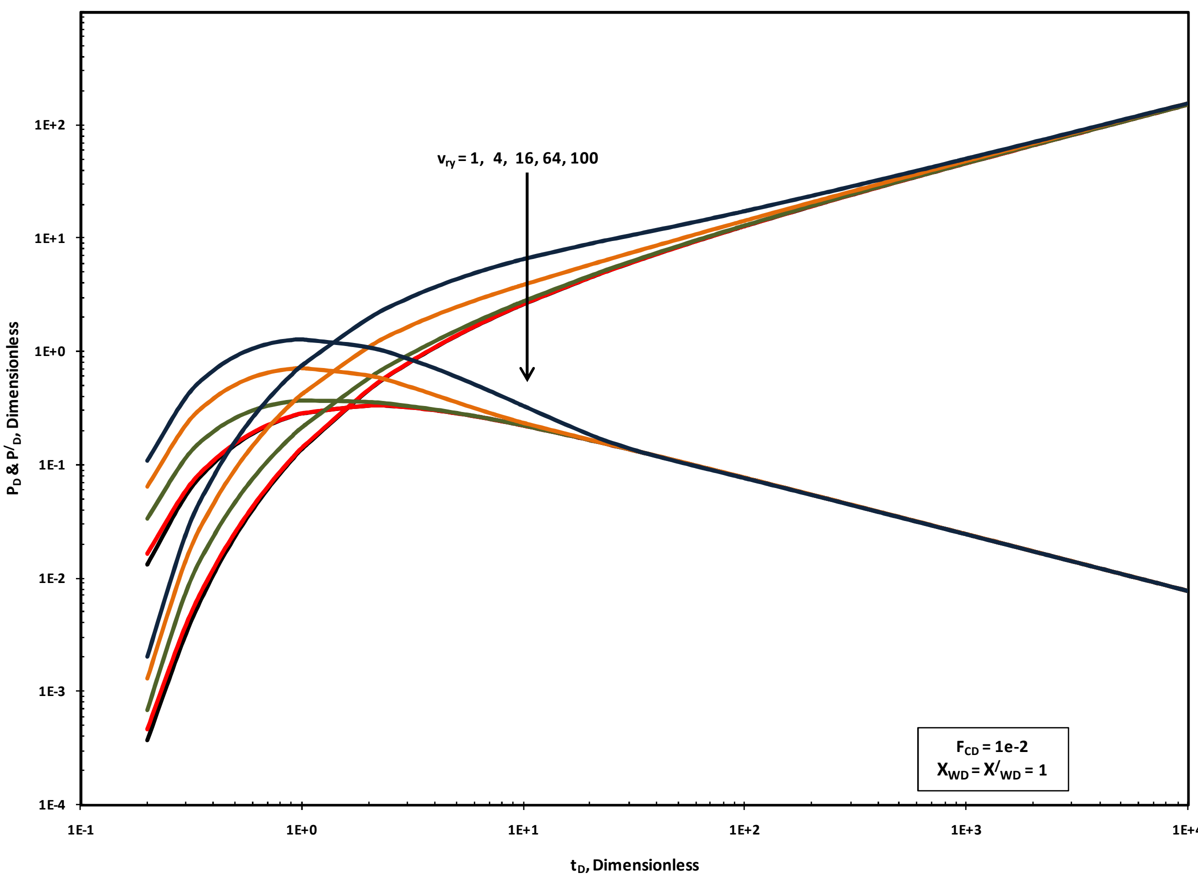

Fig. 6.3.13 Effect of k_y contrast in reservoir finite in two directions (II > I)

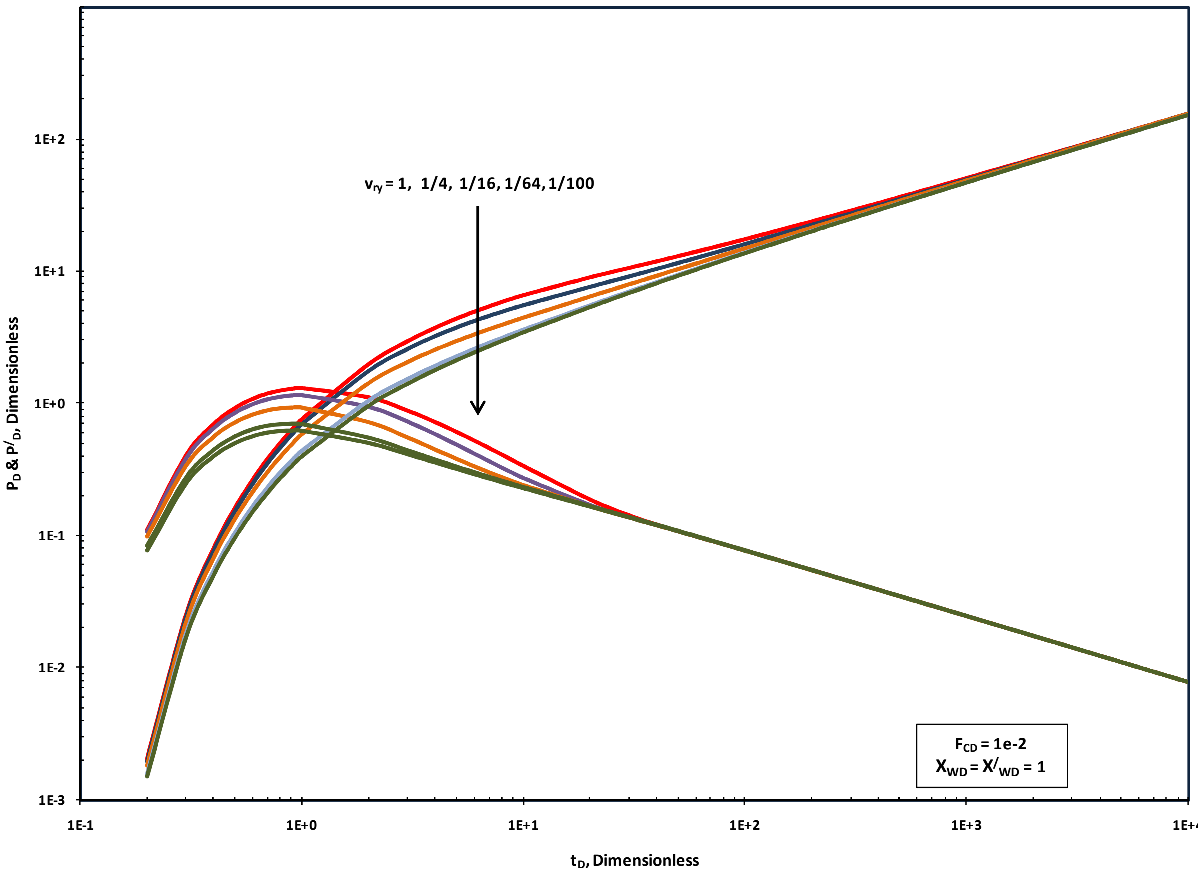

Fig. 6.3.14 Effect of k_y contrast in reservoir finite in two directions (I > II)

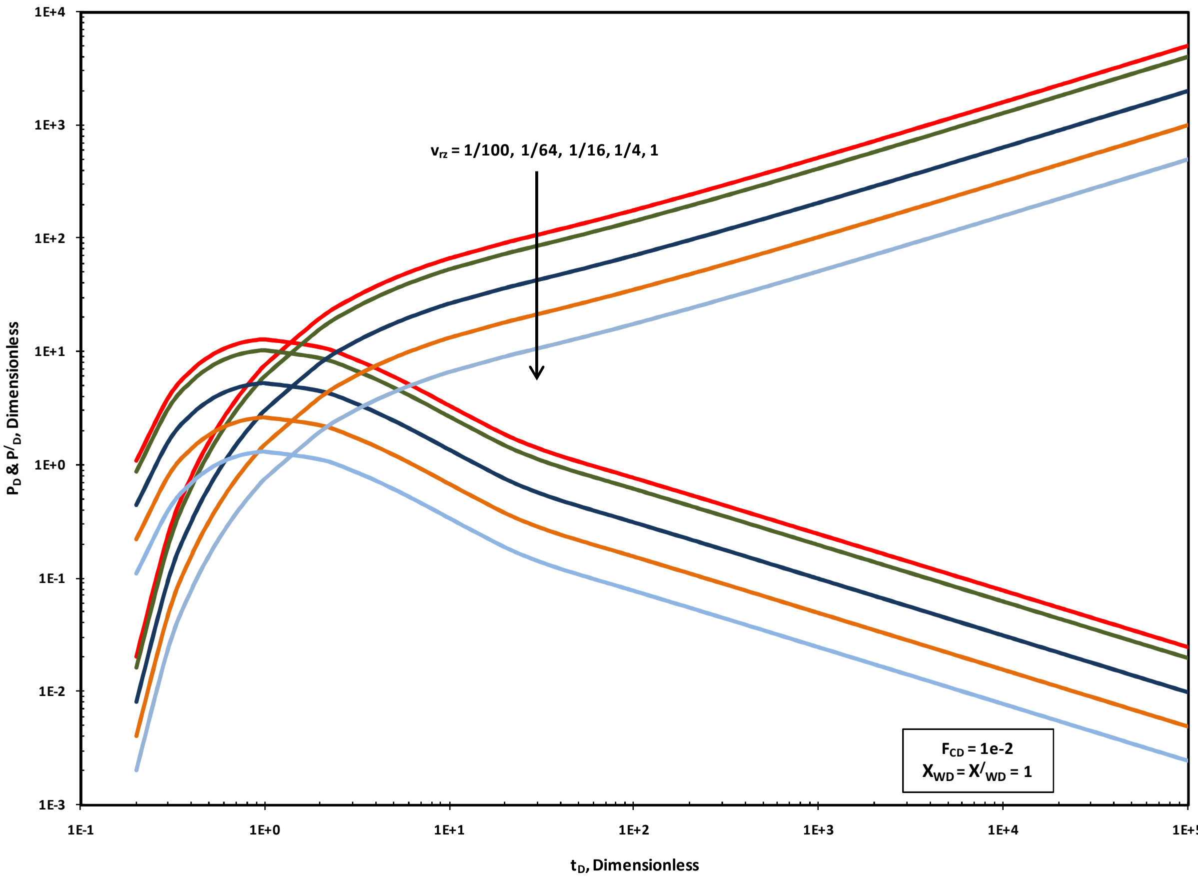

Fig. 6.3.15 Effect of k_z contrast in reservoir finite in two directions (I > II)

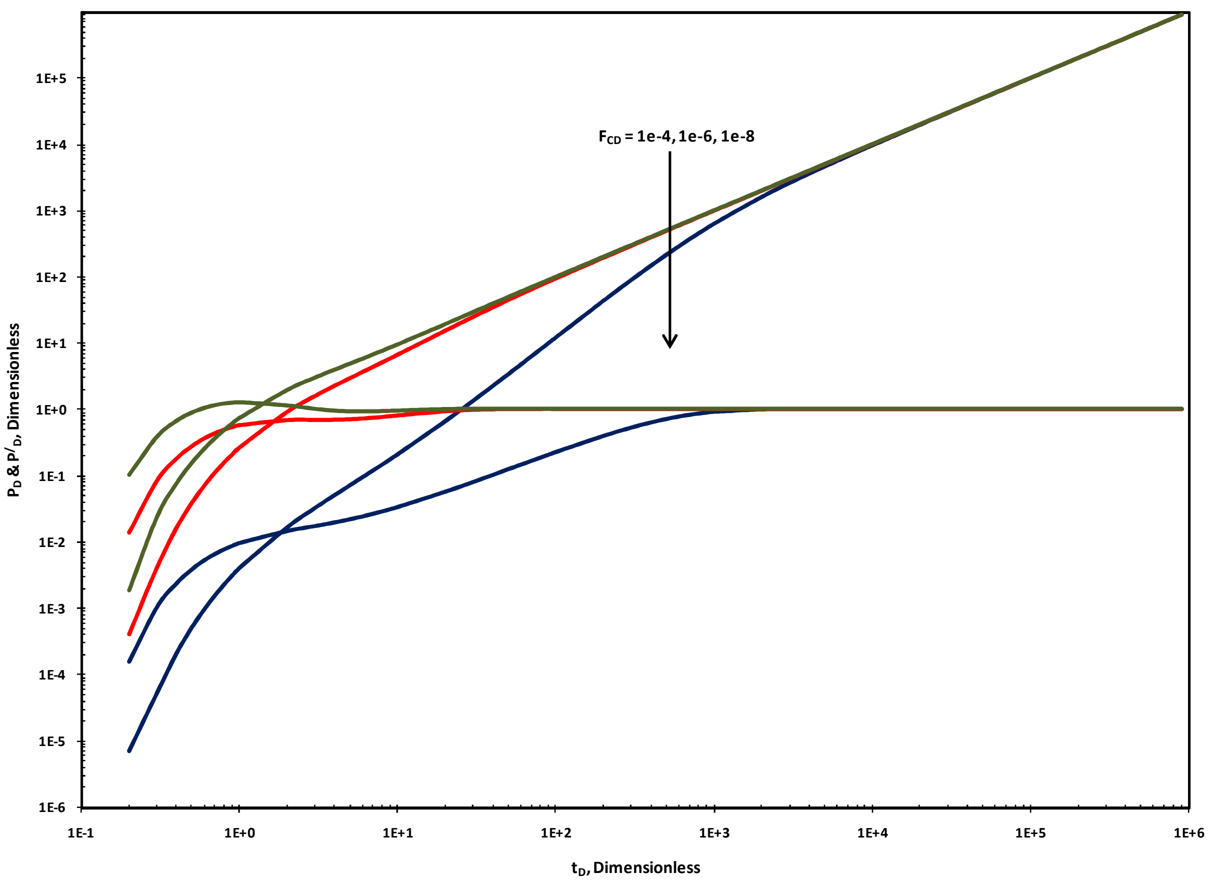

Fig. 6.3.16 Effect of partial communicating fault in closed reservoir system

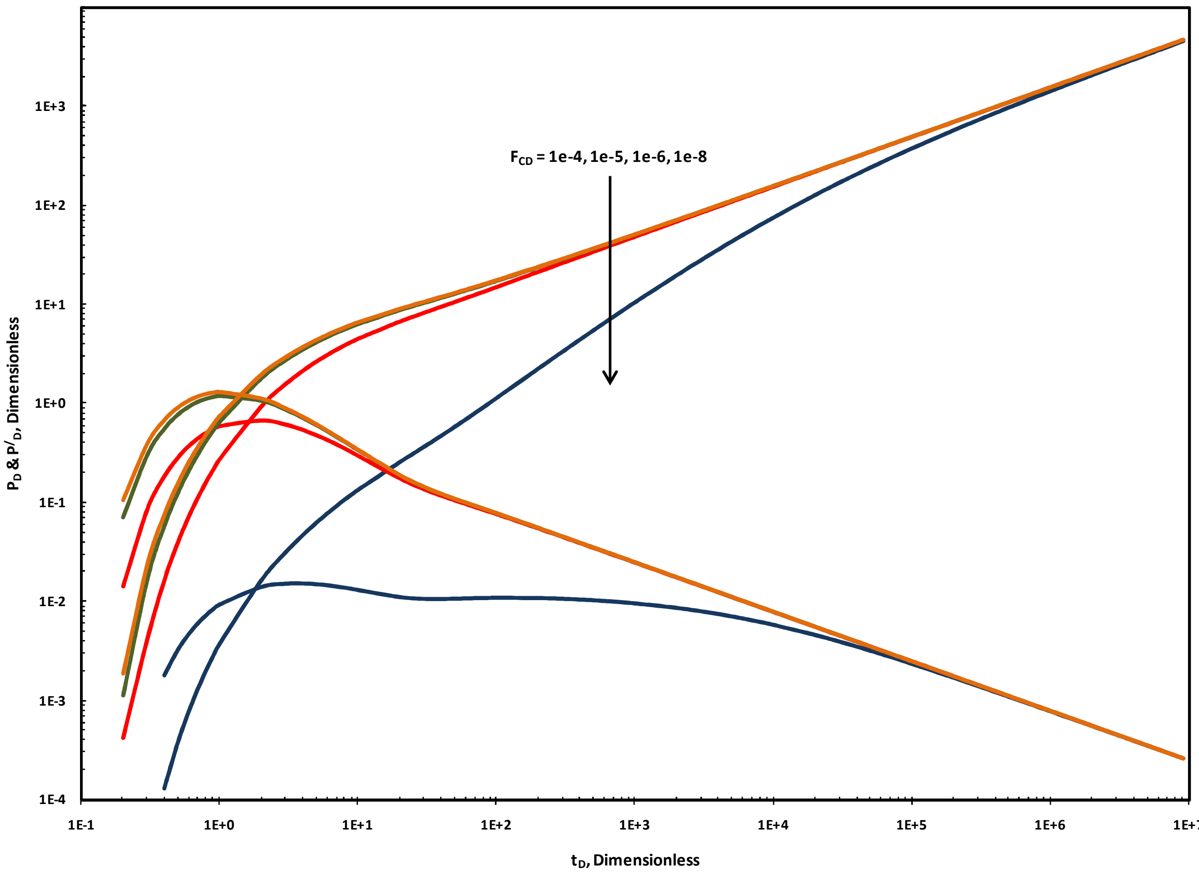

Fig. 6.3.17 Effect of partial communicating fault in reservoir finite in two directions

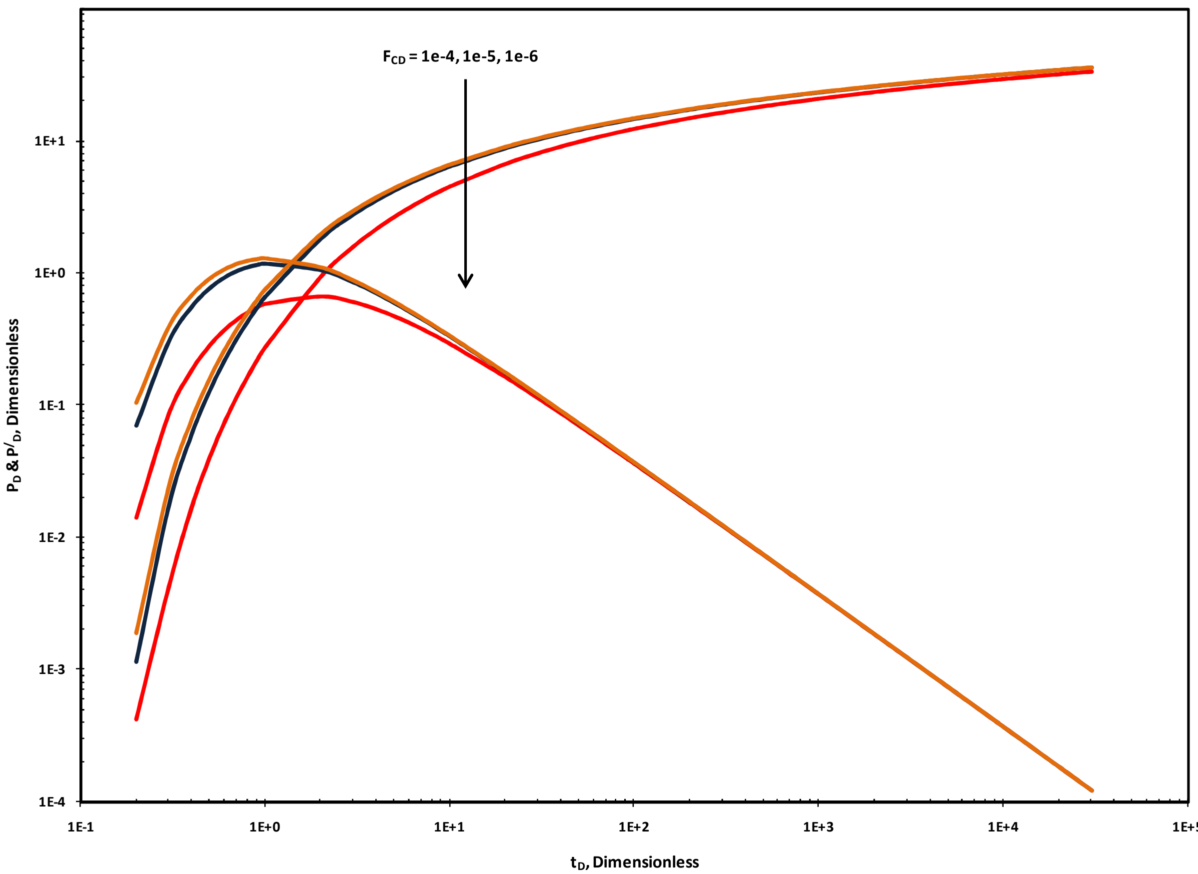

Fig. 6.3.18 Effect of partial communicating fault in reservoir infinite in two directions

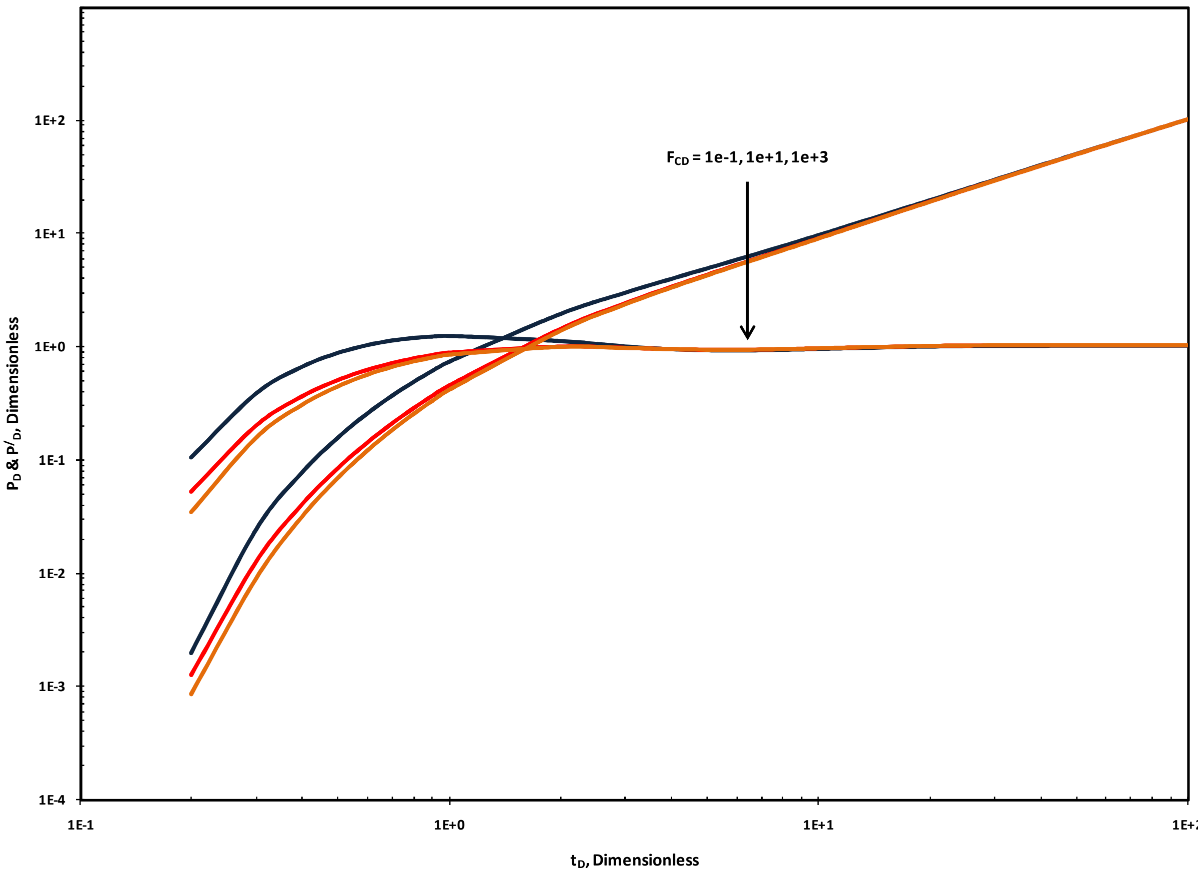

Fig. 6.3.19 Effect of superconducting fault in closed reservoir system

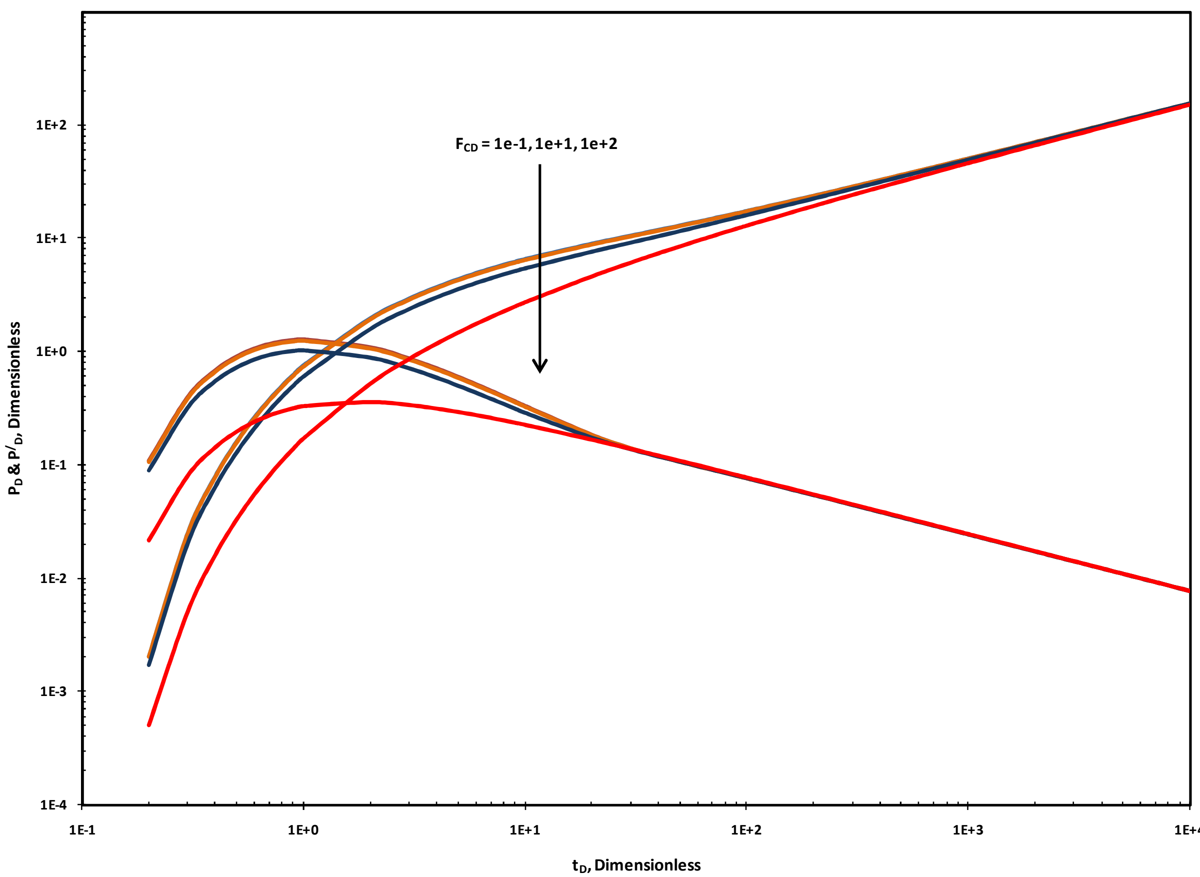

Fig. 6.3.20 Effect of superconducting fault in reservoir finite in two directions

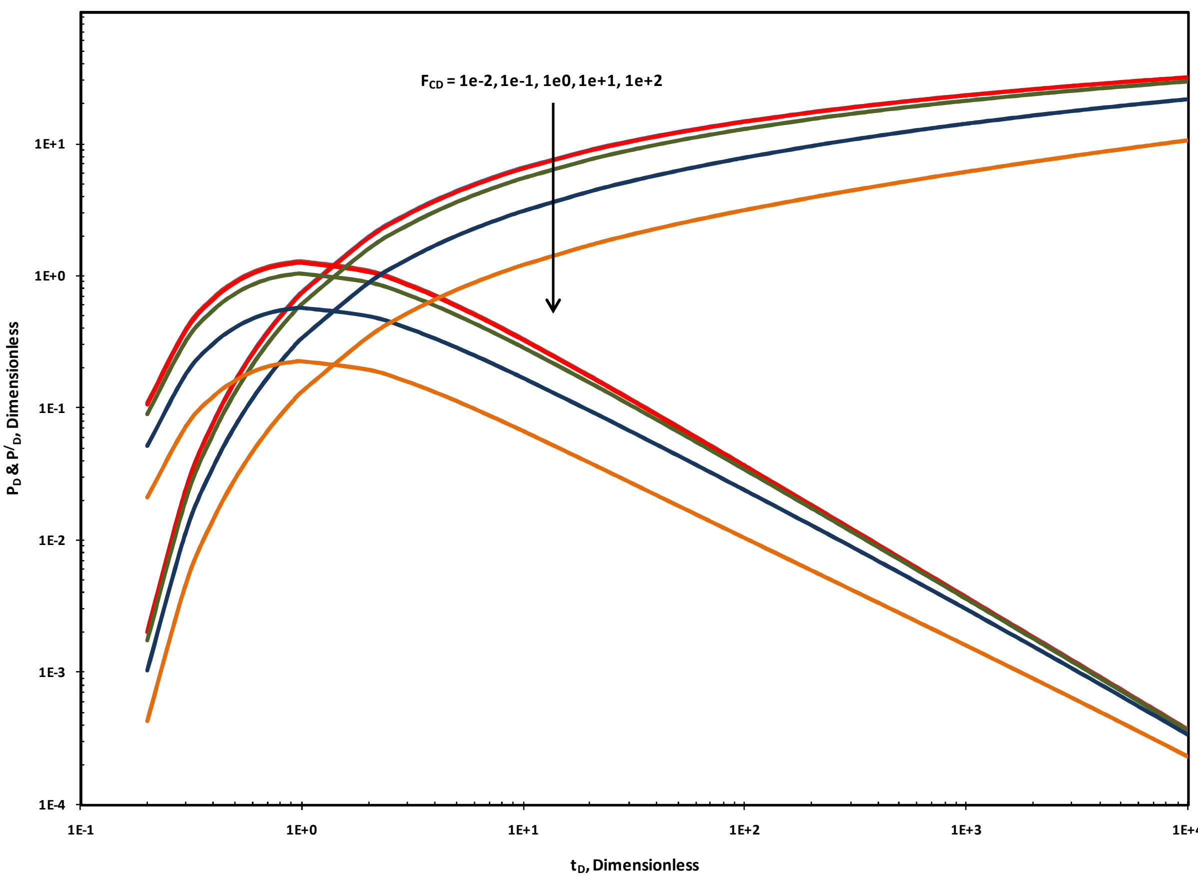

Fig. 6.3.21 Effect of superconducting fault in reservoir infinite in two directions

6.4 Interpretation Development

Interpretation for long time approximation of the wellbore pressure will be made for the case where Skin and wellbore storage effects are absent and horizontal wells shall be the focus in this development.

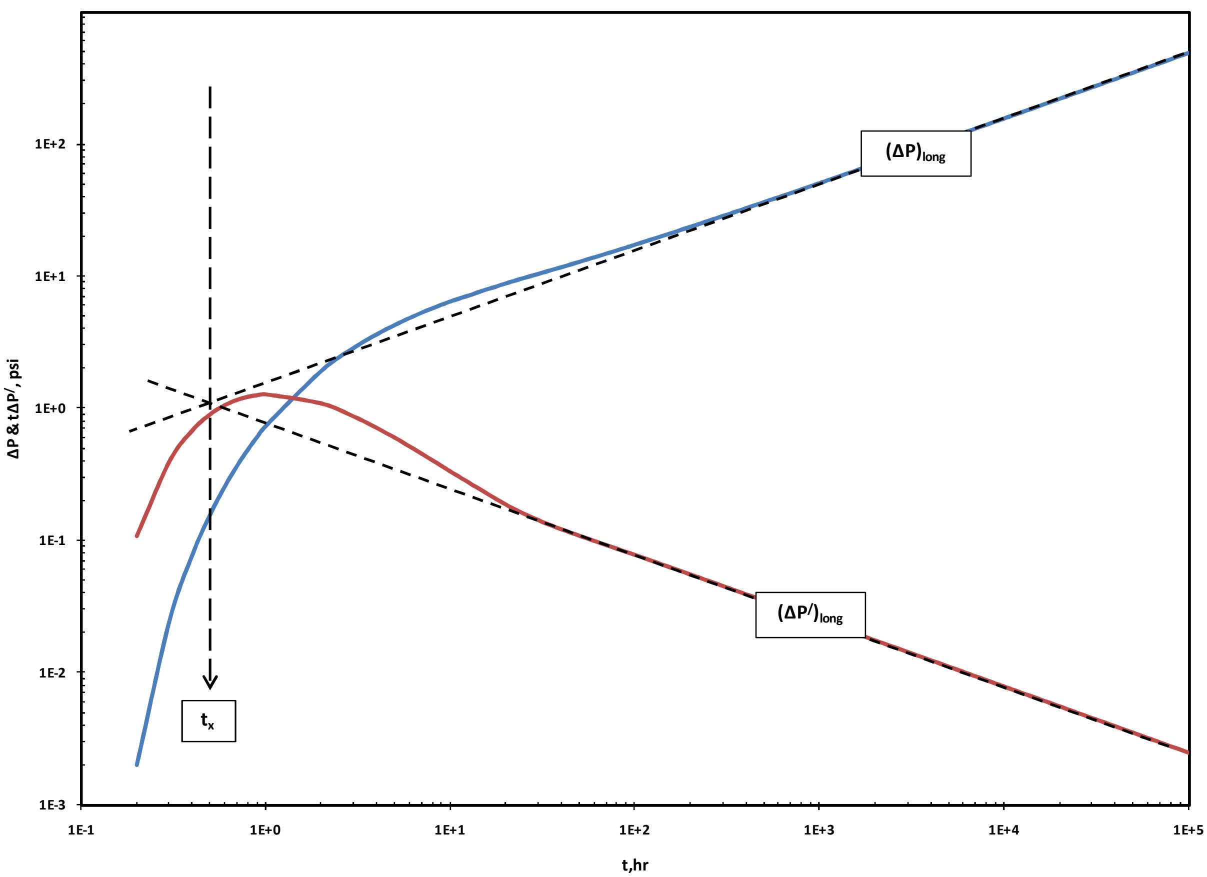

Fig. 6.4.1 Schematic interpretation curve for reservoir infinite in two directions

Specific Long time Interpretation

From Fig. 6.4.1

- Reservoir Finite in Two Directions

At long times,

$$\overline{P}_{D2} \rightarrow \frac{2\pi}{\mathrm{sh}_{yD} sin\theta} \times \frac{L_{yD}}{\sqrt{s}(1 + k_{rx}\sqrt{\eta_D})} \tag{6.4.1}$$

Taking Inverse Laplace transform,

$$P_D' \rightarrow \frac{4\sqrt{\pi}L_{yD}}{h_{yD}h_{zD} sin\theta} \times \frac{1}{\sqrt{t_D}(1 + k_{rx}\sqrt{\eta_D})} \tag{6.4.2}$$

$$P_D \rightarrow \frac{8\sqrt{\pi}L_{yD}}{h_{yD}h_{zD} sin\theta} \times \frac{\sqrt{t_D}}{(1 + k_{rx}\sqrt{\eta_D})} \tag{6.4.3}$$

Considering the point of intersection between the long time pressure derivative and pressure lines by equating (6.4.2) and (6.4.3),

$$t_{Dx} = \frac{1}{2} \tag{6.4.4}$$

From Eq. 6.4.4,

$$\frac{k_{x1}}{\emptyset_1 \mu_1 C_{t1}} = \frac{L^2}{2t_x} \qquad where\ L = Length\ of\ producer \tag{6.4.5}$$

From Eq. 6.4.3, the pressure at the point of intersection becomes

$$P_{Dx} \rightarrow \frac{8\sqrt{\pi}L_{yD}}{h_{yD}h_{zD} sin\theta} \times \frac{1}{\sqrt{2}(1 + k_{rx}\sqrt{\eta_D})} \tag{6.4.6}$$

From the equation above,

$$\sqrt{\frac{k_{x2}}{k_{x1}} \times \frac{\emptyset_2 \mu_2 C_{t2}}{\emptyset_1 \mu_1 C_{t1}}} = \frac{LL_{yD} q \mu_1}{h_y h_z k_{x1} sin\theta (\Delta P_x)} \times \sqrt{\frac{8}{\pi}} - 1 \tag{6.4.7}$$

- **Reservoir Finite in Three Directions**

At long times,

$$\overline{P}_{D2} \rightarrow \frac{2\pi L_{yD}}{s h_{yD} h_{zD} sin\theta} \times \frac{\cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}\,\sinh\sqrt{s}(h_{xD})} \tag{6.4.8}$$

Taking Inverse Laplace transform,

$$P_D' \rightarrow \frac{2\pi L_{yD}}{s h_{xD} h_{yD} h_{zD} sin\theta} \tag{6.4.9}$$

$$P_D \rightarrow \frac{2\pi L_{yD} t_D}{s h_{xD} h_{yD} h_{zD} sin\theta} \tag{6.4.10}$$

Considering the point of intersection between the long time pressure derivative and pressure lines by equating (6.4.9) and (6.4.10),

$$t_{Dx} = 1 \tag{6.4.11}$$

From Eq. 6.4.11,

$$\frac{k_{x1}}{\emptyset_1 \mu_1 C_{t1}} = \frac{L^2}{t_x} \qquad where\ L = Length\ of\ producer \tag{6.4.12}$$

From Eq. 6.4.9,

$$\frac{\emptyset_1 C_{t1} sin\theta}{L_{yD}} = \frac{q}{h_x h_y h_z (\Delta P^{/}_{long})} \tag{6.4.13}$$

In addition to these direct syntheses, traditional techniques like type curve matching are also applicable across all reservoir boundary types and the analysis of clastic reservoir systems above can be extended to NFR.

However, it is important to note that the main drawback to using the unique point of intersection between the long time approximations of the drawdown and derivative responses is that the interference test should at least be conducted long enough till the boundary effects are felt by the observer.

6.5 Discussion

As noted by Al-Khamis et al. (2001), horizontal well interference test introduce features and issues requiring specific procedures for evaluation and analysis. It is important to note that the observer's first flow regime does not necessarily have to be pseudo-radial but depends on the reservoir size and worthy of note is that for all practical purpose, the observer drawdown is independent of well orientation angle.

From Figs. 6.3.1 and 6.3.2, the observer drawdown response depends solely on the producer distance from observer along x-direction and not on the distance of either of them from the fault. Yaxley (1987) who noted that observer drawdown response depended solely on producer distance from fault and not on fault position did not investigate the effect of observer distance to fault. Similarly, the observer pressure response is only affected by the distance between both wells along the y-direction (see Figs. 6.3.3 and 6.3.4). However, it is noted that the position of either well vertically does not affect the observer drawdown response – this being as a result of the time required for pressure wave caused by producer to get to observer, since the first flow in horizontal wells is in the vertical plane.

From the ongoing discussion, one can confidently assert that the time to reach the peak of the observer pressure derivative profile is independent of individual well position and rather depends on the distance between both observer and producer.

Considering the effect of well length, it is observed that the observer drawdown response is dependent on both observer and producer lengths (see Figs. 6.3.5, 6.3.6, 6.3.7 and 6.3.8). This is expected since a longer producer causes more drawdown response on the producer and a similar thing happens for a longer observer since this means that more of the disturbance created by the producer will now be felt by the observer.

However, there exists a distinction in the effect of both length increases. Increasing the observer length does not affect the flow regimes but increasing the producer length hastens observer pseudo-radial flow regime where this is the first flow regime encountered. This is in partial agreement with that of Awotunde et al. (2008) since they noted that increasing observer length delays observer pseudo-radial flow regime.

Although k_x, k_y, and k_z contrast in both compartments affect observer pressure response, not all affect the flow regimes (see Figs. 6.3.9, 6.3.10, 6.3.11, 6.3.12, 6.3.13, 6.3.14 and 6.3.15). When k_{z2} is greater than k_{z1}, there is no change in observer drawdown response; however, the greater the k_{z1} compared to k_{z2}, there is a higher observer drawdown response and as the k_y contrast between both compartments increase, there is a lower observer drawdown response. It is only k_x contrast that affects flow regimes; the more k_{x1} is greater than k_{x2}, the later the time to reach peak of derivative plot and vice versa.

Yaxley (1987) showed that partial communicating fault attenuates observer drawdown response relative to the no-fault case meaning that less of the fluid in observer compartment gets across the fault to the producer; hence, an apparent smaller reservoir size and this is confirmed in Figs. 6.3.16, 6.3.17 and 6.3.18. However, from Figs. 6.3.19, 6.3.20 and 6.3.21 as the fault conducts more fluid across, there exists a decrease in drawdown response relative to the no-fault case.

Chapter 7

Conclusion and Recommendation

7.1 Conclusions

A composite reservoir is one with multiple lateral compartments; each having distinct rock and fluid properties separated by geologic discontinuities. Point source solutions in this reservoir system are important especially in Repeat Formation Testing or Modular Dual Testing and for the first time, point source solutions for producers and observers under various reservoir boundary conditions in composite reservoir systems are documented as shown in Tables 7.1.1 and 7.1.2.

In this study, a customizable 3D point source semi-analytical solution for drawdown response in composite systems separated by a leaky fault is developed; this solution is then converted to a line source for horizontal and vertical wells and plane source for hydraulically fractured wells by integration, while treating the leaky fault as a thin finite conductivity fracture separating two reservoirs with different rock properties. This customizable solution is implemented using a simple computer program, which can generate type curves in a computationally cheap and effective manner.

The pressure signatures obtained reveal that the drawdown response is a convoluted relationship between rock and fluid properties from both compartments and the semi-permeable medium. Similar conclusion holds for interference testing and for the first time, it is noted that the intersection time between the long-time approximation of both observer pressure drawdown and observer pressure derivative is unique; from which producer hydraulic diffusivity could be obtained.

Simulation and field data validates this model and for the first time, direct synthesis of reservoir parameters from the resulting 3D semi-analytical solution under various boundary conditions is obtained – most of which are iterative dependants on other parameters or themselves. From the resulting type curves, it is observed that there exists a parameter cutoff that would register on the pressure signature observed.

Table 7.1.1 Producer point source solution for composite clastic reservoir

BOUNDARY	POINT SOURCE SOLUTION
Infinite in two directions	$\overline{P}_{WD}\,(x_{WD},y_D,z_D,s)_{with\,skin}$ $= \frac{1}{\mathrm{sh}_{zD}}\left[\begin{array}{c}\int_{-\infty}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p^{'}\right)}{\left(RD_p+D_p^{'}\right)}\right\}e^{i\omega(y_D-y_{WD})}\,d\omega \\ +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos^2(kz_{WD})\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p^{'}\right)}{\left(QC_p+C_p^{'}\right)}\right\}e^{i\omega(y_D-y_{WD})}d\omega\end{array}\right]$
Finite in two directions	$\overline{P}_{WD}\,(x_{WD},y_D,z_D,s)_{with\,skin} = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\alpha+\beta)$ $\alpha = \left[\begin{array}{c}\frac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{4\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\frac{\left(\sqrt{\mathrm{s}}E_p-E_p^{'}\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p^{'}\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p^{'}\right)}{\left(TF_p+F_p^{'}\right)}\right\}\end{array}\right]$ $\beta = \left[\begin{array}{c}\sum_{n=1}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\cos^2(kz_{WD})\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p^{'}\right)}{\left(RD_p+D_p^{'}\right)}\right\} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos^2(kz_{WD})\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p^{'}\right)}{\left(QC_p+C_p^{'}\right)}\right\}\end{array}\right]$
Finite in three directions	$\overline{P}_{WD}\,(x_{WD},y_D,z_D,s)_{with\,skin} = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00}+\mathcal{F}_{m0}+\mathcal{F}_{0n}+\mathcal{F}_{mn})$ $\mathcal{F}_{00} = \left[\frac{1}{4\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{c}\cosh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD})+S_w\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD})+\cosh\left(\sqrt{\mathrm{s}}h_{xD}\right) \\ -\frac{E_p^{'}\left[S_w\sqrt{\mathrm{s}}\sinh2\sqrt{\mathrm{s}}(h_{xD}-x_{WD})+1+\cosh2\sqrt{\mathrm{s}}(h_{xD}-x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD})+E_p^{'}\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\}\right]$ $\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})}\left\{\begin{array}{c}\cosh\mathrm{T}(h_{xD}-2x_{WD})+S_w\mathrm{T}\sinh\mathrm{T}(h_{xD}-2x_{WD})+\cosh(Th_{xD}) \\ -\frac{F_p^{'}\left[S_w\mathrm{T}\sinh2\mathrm{T}(h_{xD}-x_{WD})+1+\cosh2\mathrm{T}(h_{xD}-x_{WD})\right]}{TF_p\sinh\mathrm{T}(h_{xD})+F_p^{'}\cosh T(h_{xD})}\end{array}\right\}\right]$ $\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{\cos^2(kz_{WD})}{2R\sinh R(h_{xD})}\left\{\begin{array}{c}\cosh R(h_{xD}-2x_{WD})+S_w\mathrm{R}\sinh R(h_{xD}-2x_{WD})+\cosh(Rh_{xD}) \\ -\frac{D_p^{'}\left[S_w\mathrm{R}\sinh2R(h_{xD}-x_{WD})+1+\cosh2R(h_{xD}-x_{WD})\right]}{RD_p\sinh R(h_{xD})+D_p^{'}\cosh R(h_{xD})}\end{array}\right\}\right]$ $\mathcal{F}_{mn} = \left[\begin{array}{c}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos^2(kz_{WD})\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})}\times \\ \left\{\begin{array}{c}\cosh Q(h_{xD}-2x_{WD})+S_w\mathrm{Q}\sinh Q(h_{xD}-2x_{WD})+\cosh(Qh_{xD}) \\ -\frac{C_p^{'}\left[S_w\mathrm{Q}\sinh2Q(h_{xD}-x_{WD})+1+\cosh2Q(h_{xD}-x_{WD})\right]}{QC_p\sinh Q(h_{xD})+C_p^{'}\cosh Q(h_{xD})}\end{array}\right\}\end{array}\right]$

Table 7.1.2 Observer point source solution for composite clastic reservoir

BOUNDARY	POINT SOURCE SOLUTION
Infinite in two directions	$\overline{P}_{D2}(x_D, y_D, k, s) = \frac{1}{\mathrm{sh}_{zD}}\left[\begin{array}{c} \int_{-\infty}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{RD_p + D_p{'}} e^{i\omega(y_D - y_{WD})} d\omega \\ + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} 2\cos(kz_D)\cos(kz_{WD}) \frac{e^{Q_1(x_D - x_{WD})}}{QC_p + C_p{'}} e^{i\omega(y_D - y_{WD})} d\omega \end{array}\right]$
Finite in two directions	$\overline{P}_{D2}(x_D, y_D, k, s) = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\alpha + \beta)$ $\alpha = \left[\frac{e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{2(\sqrt{s}E_p + E_p{'})} + \sum_{m=1}^{\infty} \frac{e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{(TF_p + F_p{'})}\right]$ $\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{(RD_p + D_p{'})}\cos(kz_D)\cos(kz_{WD}) \\ + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2e^{Q_1(x_D - x_{WD})}}{(QC_p + C_p{'})}\cos(kz_D)\cos(kz_{WD})\cos(ly_{WD})\cos(ly_D) \end{array}\right]$
Finite in three directions	$\overline{P}_{D2}(x_D, y_D, k, s) = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$ $\mathcal{F}_{00} = \frac{\cosh\sqrt{s}(h_{xD} - x_{WD})}{2[\sqrt{s}E_p \sinh\sqrt{s}(h_{xD}) + E_p{'}\cosh(\sqrt{s}h_{xD})]}[e^{\sqrt{s\eta_D}x_D} + e^{-\sqrt{s\eta_D}(x_D + 2h_{xD})}]$ $\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{'}\cosh(Th_{xD})}[e^{T_1 x_D} + e^{-T_1(x_D + 2h_{xD})}]$ $\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{\cos(kz_D)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{'}\cosh(Rh_{xD})}[e^{R_1 x_D} + e^{-R_1(x_D + 2h_{xD})}]$ $\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cos(kz_D)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{'}\cosh(Qh_{xD})} \times [e^{Q_1 x_D} + e^{-Q_1(x_D + 2h_{xD})}]$

For NFR, everything is the same as in the tables above except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

7.2 Recommendations

- **Updating Boundary Conditions**

  - ✓ Well performance and pressure signatures of these wells under infinite-acting, constant pressure, three-sealing and other boundary-types in composite reservoirs could be considered

- **Updating Reservoir Model**

  - ✓ This two-compartment solution could be extended to n-compartments since adjacent compartments are coupled by the semi-permeable barrier's solution
  - ✓ This solution concept could be extended to multilayer reservoir systems

- **Updating Solution Model**

  - ✓ The instantaneous point source solution could be converted to a continuous source instead of a steady source by integrating the flow rate function with time
  - ✓ The line or plane source solution along the vertical plane should be extended to account for deviated wells

- **Updating Analyses of Semi-analytical Solution**

  - ✓ If possible, more flow regimes should be analyzed analytically from the equations obtained as this would make interpretation more unique.
  - ✓ Also, direct synthesis of reservoir parameters for isotropic system for real situations should be considered for situations where there is very little areal permeability anisotropy
  - ✓ More field examples and simulation results should be interpreted using this model to validate its robustness

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APPENDIX A – General

A –1 Nomenclature

δ = Dirac delta function (unit impulse function)

η = hydraulic diffusivity, ft^2/hr [m^2/s]

Θ = angle of inclination of the horizontal well to x-axis [degree]

= angle of inclination of hydraulic fractures to x-axis [degree]

λ = inter-porosity ratio, dimensionless

μ = oil viscosity, cp [Pa.s]

Φ = porosity, fraction

ψ_1, ψ_2 = geometric parameter for heterogeneous region, ft^{-2} [m^{-2}]

ω = storativity ratio, dimensionless

ΔP = pressure drawdown, psi [Pa]

h_x = half length of reservoir, ft [m]

h_y = width of reservoir, ft [m]

h_z = height of reservoir, ft [m]

k_{ij} = directional permeability, md

l_1, l_2 = characteristic dimension of heterogeneous region, ft [m]

m = number of normal sets of fractures, dimensionless

q = production rate of point source, RB/D [m^3/d]

r_W = well bore radius ft [m]

s = Laplace transform parameter

t = time, hr [s]

w = width of fault, ft [m]

x_W, y_W, z_W = co-ordinates relative to fault, ft [m]

C = total wellbore storage, bbl/psi [cm/Pa]

C_t = total compressibility, psi^{-1} [Pa^{-1}]

F_{CD} = fault dimensionless conductivity

L = length of perforated interval in vertical well, ft [m]

= half length of fracture, ft [m]

= length of perforated interval in horizontal well, ft [m]

L_{zf} = height of fracture open to flow, ft [m]

M = Mobility ratio, dimensionless

P = reservoir pressure, psi [Pa]

S = near wellbore skin factor, dimensionless

S_w = specific skin factor, dimensionless

$\mathcal{F}$, $\mathcal{F}^{-1}$= Fourier and inverse Fourier operators

$\mathcal{L}$, $\mathcal{L}^{-1}$= Laplace and inverse Laplace operators

A –2 Subscripts and Superscripts

$_1$ = region I (producer)

$_2$ = region II (observer)

$_f$ = fault

$_i$ = initial

$_{r,x,y,z,\mu}$ = ratio

$_t$ = total

$_D$ = dimensionless

$_{WD}$ = dimensionless at wellbore

$_{SD}$ = dimensionless pseudo

$'$ = differential

$^-$ = Laplace

$\cdot$ = finite Fourier cosine

^ = infinite Fourier

A–3 Parameters

$$\eta_1 = \frac{k_{x_1}}{(\phi c_t)_{1+m_1}\mu_1} \text{ (for NFR); } = \frac{k_{x_1}}{\phi_1\mu_1 c_{t_1}} \text{ (for Clastic);}$$

$$\eta_2 = \frac{k_{x_2}}{\phi_2\mu_2 c_{t_2}}$$

$$\eta_f = \frac{k_f}{\phi_f\mu_f c_{t_f}}$$

$$\eta_m = \frac{k_{x_2}}{\phi_{m_2}\mu_{m_2} c_{t_{m2}}}$$

$$\eta_D = \frac{\eta_1}{\eta_2} \quad \text{for NFR;} \quad = \frac{\omega_t\omega_2}{k_{rx}}$$

$$\eta_{Df} = \frac{\eta_1}{\eta_f}$$

$$\eta_{Dm} = \frac{\eta_1}{\eta_m} = \frac{\omega_t(1-\omega_2)}{k_{rx}}$$

$$\lambda_1 = \frac{\psi_1 L^2 k_{m1}}{4k_{x1}}$$

$$\lambda_2 = \frac{\psi_2 L^2 k_{m2}}{4k_{x1}}$$

$$\psi_1 = \frac{4m(m+2)}{l_1^2}$$

$$\psi_2 = \frac{4m(m+2)}{l_2^2}$$

$$\omega_1 = \frac{(\phi c_t)_1}{(\phi c_t)_{1+m1}}$$

$$\omega_2 = \frac{(\phi c_t)_2}{(\phi c_t)_{2+m2}}$$

$$\omega_t = \frac{(\phi c_t)_{2+m2}}{(\phi c_t)_{1+m1}}$$

$$f_1(s) = \left\{\frac{\omega_1 s(1-\omega_1)+\lambda_1}{s(1-\omega_1)+\lambda_1}\right\}$$

$$f_2(s) = \frac{1}{k_{rx}}\left\{\frac{\omega_t \omega_2 s(1-\omega_2)+\lambda_2}{s(1-\omega_2)+(\lambda_2/\omega_t)}\right\}$$

$$k_{rj} = \frac{k_{j_2}}{k_{j_1}}$$

$$v_j = \frac{k_{x_1}}{k_{j_1}}$$

$$v_{rj} = \frac{k_{rj}}{k_{rx}}$$

$$M_1 = \frac{k_{x_1}}{\mu_1} \Big/ \frac{k_f}{\mu_f}$$

$$M_2 = \frac{k_{x_2}}{\mu_2} \Big/ \frac{k_f}{\mu_f}$$

APPENDIX B – Reservoir Boundary-Specific Symbols

B –1 Infinite in Two Directions

ω = infinite Fourier transforms parameter in y_D

$k = n\pi/h_{zD}$, finite cosines Fourier transform parameter in z_D

$C_p = 1 + w_D\left(M_1 C_p' + M_2 Q_1\right)$

$C_p' = k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)$

$D_p = 1 + w_D (M_1 D_p' + M_2 R_1)$

$D_p' = k_{rx} R_1 + F_{CD} R_2 (1 + w_D M_2 R_1)$

$Q = \sqrt{\omega^2 + k^2 + s}$

$Q_1 = \sqrt{\omega^2 v_{ry} + k^2 v_{rz} + \eta_D s}$

$Q_2 = \omega^2 v_y + k^2 v_z + \eta_{Df} s$

$R = \sqrt{\omega^2 + s}$

$R_1 = \sqrt{\omega^2 v_{ry} + \eta_D s}$

$R_2 = \omega^2 v_y + \eta_{Df} s$

B –2 Finite in Two Directions

$k = n\pi/h_{zD}$, finite cosines Fourier transform parameter in z_D

$l = m\pi/h_{yD}$, finite cosines Fourier transform parameter in y_D

$C_p = 1 + w_D\left(M_1 C_p' + M_2 Q_1\right)$

$C_p' = k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)$

$D_p = 1 + w_D (M_1 D_p' + M_2 R_1)$

$D_p' = k_{rx} R_1 + F_{CD} R_2 (1 + w_D M_2 R_1)$

$E_p = 1 + w_D (M_1 E_p' + M_2 \sqrt{\eta_D s})$

$E_p' = k_{rx} \sqrt{\eta_D s} + \eta_{Df} s F_{CD}\left(1 + w_D M_2 \sqrt{\eta_D s}\right)$

$F_p = 1 + w_D (M_1 F_p' + M_2 T_1)$

$F_p' = k_{rx}T_1 + F_{CD}T_2(1 + w_D M_2 T_1)$

$Q = \sqrt{l^2 + k^2 + s}$

$Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + \eta_D s}$

$Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$

$R = \sqrt{k^2 + s}$

$R_1 = \sqrt{k^2 v_{rz} + \eta_D s}$

$R_2 = k^2 v_z + \eta_{Df} s$

$T = \sqrt{l^2 + s}$

$T_1 = \sqrt{l^2 v_{ry} + \eta_D s}$

$T_2 = l^2 v_y + \eta_{Df} s$

B –3 Finite in Three Directions

$\omega = p\pi/h_{xD}$, finite cosines Fourier transform parameter in x_D

$k = n\pi/h_{zD}$, finite cosines Fourier transform parameter in z_D

$l = m\pi/h_{yD}$, finite cosines Fourier transform parameter in y_D

$C_p = e^{-2Q_1 \mathrm{h}_{x\mathrm{D}}}(1 - w_D M_2 Q_1) + 1 + w_D(M_1 C_p' + M_2 Q_1)$

$C_p' = Q_1\left(1 - e^{-2Q_1 \mathrm{h}_{x\mathrm{D}}}\right)(w_D M_2 Q_1 F_{CD} + k_{rx}) + Q_2 F_{CD}\left(e^{-2Q_1 \mathrm{h}_{x\mathrm{D}}} + 1\right)$

$D_p = e^{-2R_1 \mathrm{h}_{x\mathrm{D}}}(1 - w_D M_2 R_1) + 1 + w_D(M_1 D_p' + M_2 R_1)$

$D_p' = R_1\left(1 - e^{-2R_1 \mathrm{h}_{x\mathrm{D}}}\right)(w_D M_2 R_1 F_{CD} + k_{rx}) + R_2 F_{CD}\left(e^{-2R_1 \mathrm{h}_{x\mathrm{D}}} + 1\right)$

$E_p = e^{-2\sqrt{\eta_D s}\mathrm{h}_{x\mathrm{D}}}\left(1 - w_D M_2\sqrt{\eta_D s}\right) + 1 + w_D(M_1 E_p' + M_2\sqrt{\eta_D s})$

$E_p' = \sqrt{\eta_D s}\left(1 - e^{-2\sqrt{\eta_D s}\mathrm{h}_{x\mathrm{D}}}\right)(w_D M_2\sqrt{\eta_D s} F_{CD} + k_{rx}) + \eta_{Df} s F_{CD}\left(e^{-2\sqrt{\eta_D s}\mathrm{h}_{x\mathrm{D}}} + 1\right)$

$F_p = e^{-2T_1 \mathrm{h}_{x\mathrm{D}}}(1 - w_D M_2 T_1) + 1 + w_D(M_1 F_p' + M_2 T_1)$

$F_p' = T_1\left(1 - e^{-2T_1 \mathrm{h}_{x\mathrm{D}}}\right)(w_D M_2 T_1 F_{CD} + k_{rx}) + T_2 F_{CD}\left(e^{-2T_1 \mathrm{h}_{x\mathrm{D}}} + 1\right)$

$Q = \sqrt{l^2 + k^2 + s}$

$Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + \eta_D s}$

$Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$

$R = \sqrt{k^2 + s}$

$R_1 = \sqrt{k^2 v_{rz} + \eta_D s}$

$R_2 = k^2 v_z + \eta_{Df} s$

$T = \sqrt{l^2 + s}$

$T_1 = \sqrt{l^2 v_{ry} + \eta_D s}$

$T_2 = l^2 v_y + \eta_{Df} s$

Everything for Clastic reservoirs here is the same for Naturally Fractured reservoirs by replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

APPENDIX C – Point Source Solution for Producer in Reservoir Infinite in Two Directions

C-1 Clastic Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2} + \frac{\partial^2 P_{D1}}{\partial {y_D}^2} + \frac{\partial^2 P_{D1}}{\partial {z_D}^2} + 2\pi\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = \frac{\partial P_{D1}}{\partial t_D} \quad x_D > 0 \quad (C-1-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2} = \eta_D \frac{\partial P_{D2}}{\partial t_D} \qquad x_D < 0 \qquad (C-1-2)$$

$$v_y\frac{\partial^2 P_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 P_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D} - k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (C-1-3)$$

<u>IC</u>: $P_{D1}(x_D, y_D, z_D, 0) = P_{D2}(x_D, y_D, z_D, 0) = P_{Df}(y_D, z_D, 0) = 0 \qquad (C-1-4)$

<u>BC</u>: $P_{D1}(x_D \to \infty, y_D, z_D, t_D) = P_{D2}(x_D \to -\infty, y_D, z_D, t_D) \qquad (C-1-5)$

$P_{D1}(x_D, y_D \to \pm\infty, z_D, t_D) = P_{D2}(x_D, y_D \to \pm\infty, z_D, t_D) = P_{Df}(y_D \to \pm\infty, z_D, t_D) = 0$

<u>OTHER CONDITIONS</u>

$$P_{D1}'|_{z_D=0,h_{z_D}} = P_{D2}'|_{z_D=0,h_{z_D}} = P_{Df}'\Big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (C-1-6)$$

$$\left[P_{D1} - P_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial P_{D1}}{\partial x_D}\Big|_{x_D=0} \qquad (C-1-7)$$

$$\left[P_{Df} - P_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial P_{D2}}{\partial x_D}\Big|_{x_D=0} \qquad (C-1-8)$$

(A) Laplace transform of (C-1-1),(C-1-2),(C-1-3),(C-1-5),(C-1-6),(C-1-7),(C-1-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial x_D{}^2}+\frac{\partial^2 \bar{P}_{D1}}{\partial y_D{}^2}+\frac{\partial^2 \bar{P}_{D1}}{\partial z_D{}^2}+\frac{2\pi}{s}\delta(x_D-x_{WD})\delta(y_D-y_{WD})\delta(z_D-z_{WD})= s\bar{P}_{D1} \quad x_D>0 \quad (C-1-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial x_D{}^2}+v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial y_D{}^2}+v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial z_D{}^2}=\eta_D s\bar{P}_{D2} \qquad x_D<0 \qquad (C-1-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial y_D{}^2}+v_z\frac{\partial^2 \bar{P}_{Df}}{\partial z_D{}^2}+\frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D}-k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0}=\eta_{Df}s\bar{P}_{Df} \qquad (C-1-11)$$

(C-1-9), (C-1-10), (C-1-11) were gotten using (C-1-4)

BC: $\bar{P}_{D1}(x_D\to\infty,y_D,z_D,s)=\bar{P}_{D2}(x_D\to-\infty,y_D,z_D,s)$ $\qquad (C-1-12)$

$$\bar{P}_{D1}(x_D,y_D\to\pm\infty,z_D,s)=\bar{P}_{D2}(x_D,y_D\to\pm\infty,z_D,s)=\bar{P}_{Df}(y_D\to\pm\infty,z_D,s)=0$$

OTHER CONDITIONS

$$\bar{P}_{D1}{}'\big|_{z_D=0,h_{z_D}}=\bar{P}_{D2}{}'\big|_{z_D=0,h_{z_D}}=\bar{P}_{Df}{}'\big|_{z_D=0,h_{z_D}}=0 \qquad \forall\, x_D>0, x_D<0, x_D=0 \qquad (C-1-13)$$

$$\left[\bar{P}_{D1}-\bar{P}_{Df}\right]_{x_D=0}=w_D M_1\frac{\partial \bar{P}_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (C-1-14)$$

$$\left[\bar{P}_{Df}-\bar{P}_{D2}\right]_{x_D=0}=w_D M_2\frac{\partial \bar{P}_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (C-1-15)$$

(B) Finite cosine fourier transform of (C-1-9) through (C-1-15) w.r.t. (z_D)

$$\frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {y_D}^2} - (k^2 + s)\, \dot{\bar{P}}_{D1} = -\frac{4\pi}{s\mathrm{h}_{zD}} \cos(kz_{WD})\, \delta(x_D - x_{WD})\delta(y_D - y_{WD}) \;\; x_D > 0 \quad (C-1-16)$$

$$\frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {x_D}^2} + v_{ry} \frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {y_D}^2} - (k^2 v_{rz} + \eta_D s)\, \dot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (C-1-17)$$

$$v_y \frac{\partial^2 \dot{\bar{P}}_{Df}}{\partial {x_D}^2} + \frac{1}{F_{CD}} \left[\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} = \left(k^2 v_z + \eta_{Df} s\right) \dot{\bar{P}}_{Df} \qquad (C-1-18)$$

(C-1-16), (C-1-17), (C-1-18) were gotten using (C-1-13)

<u>BC</u>:

$$\dot{\bar{P}}_{D1}(x_D \to \infty, y_D, k, s) = \dot{\bar{P}}_{D2}(x_D \to -\infty, y_D, k, s) \qquad (C-1-19)$$

$$\dot{\bar{P}}_{D1}(x_D, y_D \to \pm\infty, k, s) = \dot{\bar{P}}_{D2}(x_D, y_D \to \pm\infty, k, s) = \dot{\bar{P}}_{Df}(y_D \to \pm\infty, k, s) = 0$$

<u>OTHER CONDITIONS</u>

$$\left[\dot{\bar{P}}_{D1} - \dot{\bar{P}}_{Df} \right]_{x_D=0} = w_D M_1 \left. \frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} \right|_{x_D=0} \qquad (C-1-20)$$

$$\left[\dot{\bar{P}}_{Df} - \dot{\bar{P}}_{D2} \right]_{x_D=0} = w_D M_2 \left. \frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D} \right|_{x_D=0} \qquad (C-1-21)$$

(C) Infinite fourier transform from (C-1-16) through to (C-1-21) w.r.t.(y_D)

$$\frac{\partial^2 \bar{\hat{\bar{P}}}_{D1}}{\partial {x_D}^2} - (l^2 + k^2 + s)\,\bar{\hat{\bar{P}}}_{D1} = -\frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{s\mathrm{h}_{zD}}\delta(x_D - x_{WD}) \qquad x_D > 0 \qquad (C-1-22)$$

$$\frac{\partial^2 \bar{\hat{\bar{P}}}_{D2}}{\partial {x_D}^2} - (\omega^2 v_{ry} + k^2 v_{rz} + \eta_D s)\,\bar{\hat{\bar{P}}}_{D2} = 0 \qquad x_D < 0 \qquad (C-1-23)$$

$$\left(\omega^2 v_y + k^2 v_z + \eta_{Df} s\right)\bar{\hat{\bar{P}}}_{Df} = \frac{1}{F_{CD}}\left[\frac{\partial \bar{\hat{\bar{P}}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{\hat{\bar{P}}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (C-1-24)$$

(C-1-22), (C-1-23), (C-1-24) were gotten using (C-1-19)

<u>BC</u>: $\bar{\hat{\bar{P}}}_{D1}(x_D \to \infty, \omega, k, s) = \bar{\hat{\bar{P}}}_{D2}(x_D \to -\infty, \omega, k, s) = 0 \qquad (C-1-25)$

<u>OTHER CONDITIONS</u>

$$\left[\bar{\hat{\bar{P}}}_{D1} - \bar{\hat{\bar{P}}}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \bar{\hat{\bar{P}}}_{D1}}{\partial x_D}\Bigg|_{x_D=0} \qquad (C-1-26)$$

$$\left[\bar{\hat{\bar{P}}}_{Df} - \bar{\hat{\bar{P}}}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \bar{\hat{\bar{P}}}_{D2}}{\partial x_D}\Bigg|_{x_D=0} \qquad (C-1-27)$$

(D) Eliminate $\hat{\dot{\bar{P}}}_{Df}$ from the pair of equations (C-1-24) & (C-1-27), (C-1-26) & (C-1-27)

$$\left(\omega^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\hat{\dot{\bar{P}}}_{D2} + w_D M_2 \frac{\partial \hat{\dot{\bar{P}}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \hat{\dot{\bar{P}}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \hat{\dot{\bar{P}}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (C-1-28)$$

$$\left[\hat{\dot{\bar{P}}}_{D1} - \hat{\dot{\bar{P}}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \hat{\dot{\bar{P}}}_{D1}}{\partial x_D} + M_2 \frac{\partial \hat{\dot{\bar{P}}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (C-1-29)$$

but (C-1-22) is still non-homogenous in x_D on the R.H.S so,

(E)Laplace transform w.r.t.(x_D) of (C-1-22)

$$\hat{\dot{\bar{\bar{P}}}}_{D1}\left[\acute{s}^2 - (\omega^2 + k^2 + s)\right] - \hat{\dot{\bar{P}}}'_{D1}(x_D = 0) - \acute{s}\,\hat{\dot{\bar{P}}}_{D1}(x_D = 0) = -\frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{sh_{zD}}\,e^{-\acute{s}x_{WD}}$$

$$\hat{\dot{\bar{\bar{P}}}}_{D1}(\acute{s},\omega,k,s) = \frac{\acute{s}\,\hat{\dot{\bar{P}}}_{D1}(0,\omega,k,s)}{\acute{s}^2 - (\omega^2 + k^2 + s)} + \frac{\hat{\dot{\bar{P}}}'_{D1}(0,\omega,k,s)}{\acute{s}^2 - (\omega^2 + k^2 + s)} - \frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}e^{-\acute{s}x_{WD}}}{\mathrm{sh}_{zD}\left[\acute{s}^2 - (\omega^2 + k^2 + s)\right]}$$

$$(C-1-30)$$

(F)Inverse Laplace transform of (C-1-30) w.r.t. ($\acute{s}$)

$$\hat{\dot{\bar{P}}}_{D1}(x_D,\omega,k,s) = \hat{\dot{\bar{P}}}_{D1}(0,\omega,k,s)\cosh(Qx_D) + \frac{\hat{\dot{\bar{P}}}'_{D1}(0,\omega,k,s)\sinh(Qx_D)}{Q}$$

$$-\frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{sh_{zD}}\frac{\sinh Q(x_D - x_{WD})}{Q} \qquad (C-1-31)$$

where $Q = \sqrt{\omega^2 + k^2 + s}$

Applying (C-1-25) in (C-1-31)

$$\hat{\dot{\bar{P}}}_{D1}(0,\omega,k,s) + \frac{\hat{\dot{\bar{P}}}'_{D1}(0,\omega,k,s)}{Q} = \frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}e^{-Qx_{WD}}}{sh_{zD}Q} \qquad (C-1-32)$$

(G) Consider general solution of (C-1-23) Region II honoring respective B.C

$$\hat{\bar{P}}_{D2}(x_D, \omega, k, s) = A_1 e^{Q_1 x_D} + A_2 e^{-Q_1 x_D} \qquad where \ Q_1 = \sqrt{\omega^2 v_{ry} + k^2 v_{rz} + \eta_D s}$$

using $(C-1-26)$, as $x_D \to -\infty$ and dividing both sides by $e^{-Q_1 x_D}$

$$0 = A_2 + 0, \qquad so\ that\ \hat{\bar{P}}_{D2}(x_D, \omega, k, s) = A_1 e^{Q_1 x_D} \qquad (C-1-33)$$

(H) Substitute (C-1-33) into (C-1-28)

If $Q_2 = \omega^2 v_y + k^2 v_z + \eta_{Df} s$; $\qquad Q_2 A_1 (1 + w_D M_2 Q_1) = \frac{1}{F_{CD}} \left[\hat{\bar{P}}'_{D1} \Big|_{x_D=0} - k_{rx} A_1 Q_1 \right]$

$$\hat{\bar{P}}'_{D1} \Big|_{x_D=0} = A_1 [k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)] = A_1 C_p{}' \qquad (C-1-34)$$

where $C_p{}' = k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)$ $\qquad (C-1-35)$

(I) Substitute (C-1-34)&(C-1-33) into (C-1-29)

$$\hat{\bar{P}}_{D1} \Big|_{x_D=0} - A_1 = w_D [M_1 A_1 C_p{}' + M_2 A_1 Q_1]$$

$$\hat{\bar{P}}_{D1} \Big|_{x_D=0} = A_1 [1 + w_D (M_1 C_p{}' + M_2 Q_1)] = A_1 C_p \qquad (C-1-36)$$

where $C_p = 1 + w_D (M_1 C_p{}' + M_2 Q_1)$ $\qquad (C-1-37)$

(J) Solve (C-1-32), (C-1-34), (C-1-36) simultaneously

$$\frac{A_1 C_p{}'}{Q} + A_1 C_p = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{sh_{zD} Q}$$

$$So \qquad A_1 = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{\mathrm{sh}_{zD}(QC_p + C_p{}')} \qquad (C-1-38)$$

(K) Substitute (C-1-34) & (C-1-36) into (C-1-31)

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = A_1 C_p \cosh(Qx_D) + \frac{A_1 C_p{}' \sinh(Qx_D)}{Q} - \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} \frac{\sinh Q(x_D - x_{WD})}{Q}$$

Substituting for A_1,

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{2\mathrm{sh}_{zD}\left(QC_p + C_p{}'\right)} [C_p(e^{Qx_D} + e^{-Qx_D}) + \frac{C_p{}'}{Q}(e^{Qx_D} - e^{-Qx_D})] - \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{2sh_{zD} Q} [\left(e^{Q(x_D - x_{WD})} - e^{-Q(x_D - x_{WD})}\right)]$$

Collecting like terms,

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} [e^{Q(x_D - x_{WD})} \frac{\left(QC_p + C_p{}'\right)}{\left(QC_p + C_p{}'\right)} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)} + \left(e^{-Q(x_D - x_{WD})} - e^{Q(x_D - x_{WD})}\right)]$$

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} \left[e^{-Q(x_D - x_{WD})} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right] \qquad (C-1-39)$$

(L) Considering near wellbore specific Skin factor (S_w) and pressure at wellbore,

$$from\ Darcy's\ law \quad \frac{\partial \Delta P}{\partial x} = \frac{q\mu}{2\pi r_W L k_{x1}}$$

$$Also \quad \Delta P_s = \frac{q\mu S}{2\pi L k_{xz}}\ where\ k_{xz} = \sqrt{k_{x1}k_{z1}}$$

$$After\ relevant\ subtitution, \quad \Delta P_s = r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial \Delta P}{\partial x}$$

$$Applying\ dimensionless\ transformation, \quad P_{Ds} = \frac{1}{L} r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial P_D}{\partial x_D}$$

$$So \quad P_{WD} = P_D - P_{Ds}\ at\ x_{D=}x_{WD}\ where\ S_w = r_{WD}S\sqrt{\frac{k_{x1}}{k_{z1}}}$$

After taking relevant double fourier and laplace transform,

$$\bar{\hat{\hat{P}}}_{WD}(x_{WD},\omega,k,s)_{with\ skin} = \left[\bar{\hat{\hat{P}}}_{D1}(x_D,\omega,k,s) - S_w\frac{\partial\,\bar{\hat{\hat{P}}}_{D1}(x_D,\omega,k,s)}{\partial x_D}\right]_{x_{D=}x_{WD}} \qquad (C-1-40)$$

$$\bar{\hat{\hat{P}}}_{WD}(x_{WD},\omega,k,s)_{with\ skin} = \frac{\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}(1+\mathrm{QS_w})}{\mathrm{sh}_{zD}Q}\left[1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right] \qquad (C-1-41)$$

(M) Inverse Infinite Fourier Transform w.r.t. (ω)

$$\bar{\dot{P}}_{WD}(x_{WD},y_D,k,s)_{with\ skin} = \frac{\cos(kz_{WD})}{\mathrm{sh}_{zD}}\int_{-\infty}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right\}e^{i\omega(y_D - y_{WD})}d\omega$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} \;=\; \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}\Big|_{n=0} \quad and\ \ \mathcal{F}_n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}$$

$$\mathcal{F}_0^n = \frac{1}{\mathrm{sh}_{zD}} \int_{-\infty}^{\infty} \frac{(1 + \mathrm{RS_w})}{R} \left\{ 1 + e^{-2Rx_{WD}} \frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)} \right\} e^{i\omega(y_D - y_{WD})}\, d\omega$$

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin}$$

$$= \frac{1}{\mathrm{sh}_{zD}} \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{(1 + \mathrm{RS_w})}{2R} \left\{ 1 + e^{-2Rx_{WD}} \frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)} \right\} e^{i\omega(y_D - y_{WD})}\, d\omega \\ \displaystyle + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{(1 + \mathrm{QS_w})}{Q} \cos(kz_{WD}) \cos(kz_D) \left\{ 1 + e^{-2Qx_{WD}} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)} \right\} e^{i\omega(y_D - y_{WD})} d\omega \end{array} \right]$$

$$(C - 1 - 42)$$

C –2 Naturally Fractured Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2}+\frac{\partial^2 P_{D1}}{\partial {y_D}^2}+\frac{\partial^2 P_{D1}}{\partial {z_D}^2}+2\pi\delta(x_D-x_{WD})\delta(y_D-y_{WD})\delta(z_D-z_{WD})$$
$$=\omega_1\frac{\partial P_{D1}}{\partial t_D}+(1-\omega_1)\frac{\partial P_{D_{m_1}}}{\partial t_D}\qquad x_D>0 \qquad (C-2-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2}+v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2}+v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2}=\eta_D\frac{\partial P_{D2}}{\partial t_D}+\eta_{Dm}\frac{\partial P_{D_{m_2}}}{\partial t_D}\qquad x_D<0 \qquad (C-2-2)$$

$$v_y\frac{\partial^2 P_{Df}}{\partial {y_D}^2}+v_z\frac{\partial^2 P_{Df}}{\partial {z_D}^2}+\frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D}-k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0}=\eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (C-2-3a)$$

$$(1-\omega_1)\frac{\partial P_{Dm1}}{\partial t_D}=\lambda_1\left(P_{D1}-P_{Dm1}\right) \qquad (C-2-3b)$$

$$\omega_t(1-\omega_2)\frac{\partial P_{Dm2}}{\partial t_D}=\lambda_2\left(P_{D2}-P_{Dm2}\right) \qquad (C-2-3c)$$

<u>IC</u>: $P_{D1}(x_D,y_D,z_D,0)=P_{D2}(x_D,y_D,z_D,0)=P_{Df}(y_D,z_D,0)=0$ $\qquad (C-2-4)$

<u>BC</u>: $P_{D1}(x_D\to\infty,y_D,z_D,t_D)=P_{D2}(x_D\to-\infty,y_D,z_D,t_D)$ $\qquad (C-2-5)$

$P_{D1}(x_D,y_D\to\pm\infty,z_D,t_D)=P_{D2}(x_D,y_D\to\pm\infty,z_D,t_D)=P_{Df}(y_D\to\pm\infty,z_D,t_D)=0$

<u>OTHER CONDITIONS</u>

$$P_{D1}'|_{z_D=0,h_{z_D}}=P_{D2}'|_{z_D=0,h_{z_D}}=P_{Df}'\Big|_{z_D=0,h_{z_D}}=0 \qquad \forall\, x_D>0, x_D<0, x_D=0 \qquad (C-2-6)$$

$$\left[P_{D1}-P_{Df}\right]_{x_D=0}=w_DM_1\frac{\partial P_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (C-2-7)$$

$$\left[P_{Df}-P_{D2}\right]_{x_D=0}=w_DM_2\frac{\partial P_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (C-2-8)$$

(D) Laplace transform of (C-2-1),(C-2-2),(C-2-3a),(C-2-3b),(C-2-3c), (C-2-5),(C-2-6),(C-2-7), (C-2-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {y_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {z_D}^2} + \frac{2\pi}{s}\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = sf_1(s)\bar{P}_{D1} \qquad x_D > 0$$

$$(C-2-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial {z_D}^2} = sf_2(s)\bar{P}_{D2} \qquad x_D < 0 \qquad (C-2-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 \bar{P}_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}s\bar{P}_{Df} \qquad (C-2-11)$$

(C-2-9), (C-2-10), (C-2-11) were gotten using (C-2-3b), (C-2-3c), (C-2-4)

BC: $\bar{P}_{D1}(x_D \to \infty, y_D, z_D, s) = \bar{P}_{D2}(x_D \to -\infty, y_D, z_D, s)$ $\qquad (C-2-12)$

$\bar{P}_{D1}(x_D, y_D \to \pm\infty, z_D, s) = \bar{P}_{D2}(x_D, y_D \to \pm\infty, z_D, s) = \bar{P}_{Df}(y_D \to \pm\infty, z_D, s) = 0$

OTHER CONDITIONS

$$\bar{P}_{D1}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{D2}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{Df}{}'\Big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (C-2-13)$$

$$\left[\bar{P}_{D1} - \bar{P}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \bar{P}_{D1}}{\partial x_D}\Big|_{x_D=0} \qquad (C-2-14)$$

$$\left[\bar{P}_{Df} - \bar{P}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \bar{P}_{D2}}{\partial x_D}\Big|_{x_D=0} \qquad (C-2-15)$$

(E) Finite cosine fourier transform of (C-2-9) through (C-2-15) w.r.t. (z_D)

$$\frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {y_D}^2} - \left(k^2 + sf_1(s)\right) \dot{\bar{P}}_{D1} = -\frac{4\pi}{\mathrm{s h}_{zD}} \cos(kz_{WD})\, \delta(x_D - x_{WD})\delta(y_D - y_{WD}) \qquad x_D > 0 \tag{C-2-16}$$

$$\frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {x_D}^2} + v_{ry} \frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {y_D}^2} - (k^2 v_{rz} + sf_2(s))\, \dot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \tag{C-2-17}$$

$$v_y \frac{\partial^2 \dot{\bar{P}}_{Df}}{\partial {x_D}^2} + \frac{1}{F_{CD}} \left[\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} = \left(k^2 v_z + \eta_{Df} s\right) \dot{\bar{P}}_{Df} \tag{C-2-18}$$

(C-2-16), (C-2-17), (C-2-18) were gotten using (C-2-13)

<u>BC</u>: $\dot{\bar{P}}_{D1}(x_D \to \infty, y_D, k, s) = \dot{\bar{P}}_{D2}(x_D \to -\infty, y_D, k, s)$ $\quad (C-2-19)$

$$\dot{\bar{P}}_{D1}(x_D, y_D \to \pm\infty, k, s) = \dot{\bar{P}}_{D2}(x_D, y_D \to \pm\infty, k, s) = \dot{\bar{P}}_{Df}(y_D \to \pm\infty, k, s) = 0$$

<u>OTHER CONDITIONS</u>

$$\left[\dot{\bar{P}}_{D1} - \dot{\bar{P}}_{Df} \right]_{x_D=0} = w_D M_1 \left. \frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} \right|_{x_D=0} \tag{C-2-20}$$

$$\left[\dot{\bar{P}}_{Df} - \dot{\bar{P}}_{D2} \right]_{x_D=0} = w_D M_2 \left. \frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D} \right|_{x_D=0} \tag{C-2-21}$$

(F) Infinite fourier transform from (C-2-16) through to (C-2-21) w.r.t.(y_D)

$$\frac{\partial^2 \hat{\bar{P}}_{D1}}{\partial {x_D}^2} - \left(l^2 + k^2 + sf_1(s)\right) \hat{\bar{P}}_{D1} = -\frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{s\mathrm{h}_{zD}} \delta(x_D - x_{WD}) \quad x_D > 0 \quad (C-2-22)$$

$$\frac{\partial^2 \hat{\bar{P}}_{D2}}{\partial {x_D}^2} - \left(\omega^2 v_{ry} + k^2 v_{rz} + sf_2(s)\right) \hat{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (C-2-23)$$

$$\left(\omega^2 v_y + k^2 v_z + \eta_{Df} s\right) \hat{\bar{P}}_{Df} = \frac{1}{F_{CD}} \left[\frac{\partial \hat{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \hat{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} \qquad (C-2-24)$$

(C-2-22), (C-2-23), (C-2-24) were gotten using (C-2-19)

BC: $\hat{\bar{P}}_{D1}(x_D \to \infty, \omega, k, s) = \hat{\bar{P}}_{D2}(x_D \to -\infty, \omega, k, s) = 0 \qquad (C-2-25)$

OTHER CONDITIONS

$$\left[\hat{\bar{P}}_{D1} - \hat{\bar{P}}_{Df}\right]_{x_D=0} = w_D M_1 \left.\frac{\partial \hat{\bar{P}}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (C-2-26)$$

$$\left[\hat{\bar{P}}_{Df} - \hat{\bar{P}}_{D2}\right]_{x_D=0} = w_D M_2 \left.\frac{\partial \hat{\bar{P}}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (C-2-27)$$

(D) Eliminate $\hat{\bar{P}}_{Df}$ from the pair of equations (C-2-24) & (C-2-27), (C-2-26) & (C-2-27)

$$\left(\omega^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\hat{\bar{P}}_{D2} + w_D M_2 \frac{\partial \hat{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \hat{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \hat{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (C-2-28)$$

$$\left[\hat{\bar{P}}_{D1} - \hat{\bar{P}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \hat{\bar{P}}_{D1}}{\partial x_D} + M_2 \frac{\partial \hat{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (C-2-29)$$

but (C-2-22) is still non-homogenous in x_D on the R.H.S so,

(G) Laplace transform w.r.t.(x_D) of (C-2-22)

$$\bar{\hat{\bar{P}}}_{D1}\left[\acute{s}^2 - \left(\omega^2 + k^2 + sf_1(s)\right)\right] - \hat{\bar{P}}'_{D1}(x_D = 0) - \acute{s}\,\hat{\bar{P}}_{D1}(x_D = 0) = -\frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD}}\,e^{-\acute{s}x_{WD}}$$

$$\bar{\hat{\bar{P}}}_{D1}(\acute{s},\omega,k,s) = \frac{\acute{s}\,\hat{\bar{P}}_{D1}(0,\omega,k,s)}{\acute{s}^2 - \left(\omega^2 + k^2 + sf_1(s)\right)} + \frac{\hat{\bar{P}}'_{D1}(0,\omega,k,s)}{\acute{s}^2 - \left(\omega^2 + k^2 + sf_1(s)\right)} - \frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}e^{-\acute{s}x_{WD}}}{\mathrm{sh}_{zD}[\acute{s}^2 - (\omega^2 + k^2 + sf_1(s))]} \qquad (C-2-30)$$

(F)Inverse Laplace transform of (C-2-30) w.r.t. ($\acute{s}$)

$$\hat{\bar{P}}_{D1}(x_D,\omega,k,s) = \hat{\bar{P}}_{D1}(0,\omega,k,s)\cosh(Qx_D) + \frac{\hat{\bar{P}}'_{D1}(0,\omega,k,s)\sinh(Qx_D)}{Q} - \frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD}}\frac{\sinh Q(x_D - x_{WD})}{Q} \qquad (C-2-31)$$

$$\text{where } Q = \sqrt{\omega^2 + k^2 + sf_1(s)}$$

Applying (C-2-25) in (C-2-31)

$$\hat{\bar{P}}_{D1}(0,\omega,k,s)+\frac{\hat{\bar{P}}'_{D1}(0,\omega,k,s)}{Q}=\frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}e^{-Qx_{WD}}}{s\mathrm{h}_{zD}Q} \qquad (C-2-32)$$

(G) Consider general solution of (C-2-23) Region II honoring respective B.C

$$\hat{\bar{P}}_{D2}(x_D,\omega,k,s)=A_1e^{Q_1x_D}+A_2e^{-Q_1x_D} \qquad \textit{where} \text{ where } Q_1=\sqrt{\omega^2v_{ry}+k^2v_{rz}+sf_2(s)}$$

using $(C-2-26)$, as $x_D\to-\infty$ and dividing both sides by $\mathrm{e}^{-Q_1x_D}$

$$0 \;=\; A_2+0, \qquad \textit{so that}\; \hat{\bar{P}}_{D2}(x_D,\omega,k,s)=A_1e^{Q_1x_D} \qquad (C-2-33)$$

(H) Substitute (C-2-33) into (C-2-28)

If $Q_2=\omega^2v_y+k^2v_z+\eta_{Df}s$; $\qquad Q_2A_1(1+w_DM_2Q_1)=\frac{1}{F_{CD}}\left[\hat{\bar{P}}'_{D1}\Big|_{x_D=0}-k_{rx}A_1Q_1\right]$

$$\hat{\bar{P}}'_{D1}\Big|_{x_D=0}=A_1[k_{rx}Q_1+F_{CD}Q_2(1+w_DM_2Q_1)]=A_1C_p{'} \qquad (C-2-34)$$

where $C_p{'}=k_{rx}Q_1+F_{CD}Q_2(1+w_DM_2Q_1)$ $\qquad (C-2-35)$

(II) Substitute (C-2-34)&(C-2-33) into (C-2-29)

$$\hat{\bar{P}}_{D1}\Big|_{x_D=0}-A_1=w_D\left[M_1A_1C_p{'}+M_2A_1Q_1\right]$$

$$\hat{\bar{P}}_{D1}\Big|_{x_D=0}=A_1\left[1+w_D(M_1C_p{'}+M_2Q_1)\right]=A_1C_p \qquad (C-2-36)$$

where $C_p=1+w_D(M_1C_p{'}+M_2Q_1)$ $\qquad (C-2-37)$

(J) Solve (C-2-32), (C-2-34), (C-2-36) simultaneously

$$\frac{A_1 C_p{}'}{Q} + A_1 C_p = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{\mathrm{sh}_{zD} Q}$$

$$So \qquad A_1 = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{\mathrm{sh}_{zD}(QC_p + C_p{}')} \qquad (C-2-38)$$

(K) Substitute (C-2-34) & (C-2-36) into (C-2-31)

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = A_1 C_p \cosh(Qx_D) + \frac{A_1 C_p{}' \sinh(Qx_D)}{Q} - \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} \frac{\sinh Q(x_D - x_{WD})}{Q}$$

Substituting for A_1,

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}} e^{-Qx_{WD}}}{2\mathrm{sh}_{zD}\left(QC_p + C_p{}'\right)} [C_p(e^{Qx_D} + e^{-Qx_D}) + \frac{C_p{}'}{Q}(e^{Qx_D} - e^{-Qx_D})] - \frac{8\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{2s\mathrm{h}_{zD} Q} [\left(e^{Q(x_D - x_{WD})} - e^{-Q(x_D - x_{WD})}\right)]$$

Collecting like terms,

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} [e^{Q(x_D - x_{WD})} \frac{\left(QC_p + C_p{}'\right)}{\left(QC_p + C_p{}'\right)} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)} + \left(e^{-Q(x_D - x_{WD})} - e^{Q(x_D - x_{WD})}\right)]$$

$$\hat{\bar{P}}_{D1}(x_D, \omega, k, s) = \frac{\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD} Q} \left[e^{-Q(x_D - x_{WD})} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right] \qquad (C-2-39)$$

Steps (L), (M) and (N) are similar to those in Clastic Reservoirs above

APPENDIX D – Point Source Solution for Producer in Reservoir Finite in Two Directions

D-1 Clastic Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2}+\frac{\partial^2 P_{D1}}{\partial {y_D}^2}+\frac{\partial^2 P_{D1}}{\partial {z_D}^2}+2\pi\delta(x_D-x_{WD})\delta(y_D-y_{WD})\delta(z_D-z_{WD})=\frac{\partial P_{D1}}{\partial t_D}\quad x_D>0 \quad (D-1-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2}+v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2}+v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2}=\eta_D\frac{\partial P_{D2}}{\partial t_D}\qquad x_D<0 \quad (D-1-2)$$

$$v_y\frac{\partial^2 P_{Df}}{\partial {y_D}^2}+v_z\frac{\partial^2 P_{Df}}{\partial {z_D}^2}+\frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D}-k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0}=\eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (D-1-3)$$

<u>IC</u>: $P_{D1}(x_D,y_D,z_D,0)=P_{D2}(x_D,y_D,z_D,0)=P_{Df}(y_D,z_D,0)=0 \qquad (D-1-4)$

<u>BC</u>: $P_{D1}(x_D\to\infty,y_D,z_D,t_D)=P_{D2}(x_D\to-\infty,y_D,z_D,t_D)=0 \qquad (D-1-5)$

<u>OTHER CONDITIONS</u>

$$P_{D1}'|_{z_D=0,h_{z_D}}=P_{D2}'|_{z_D=0,h_{z_D}}=P_{Df}'\big|_{z_D=0,h_{z_D}}=0\qquad \forall\, x_D>0,x_D<0,x_D=0 \qquad (D-1-6)$$

$$P_{D1}'|_{y_D=0,h_{y_D}}=P_{D2}'|_{y_D=0,h_{y_D}}=P_{Df}'\big|_{y_D=0,h_{y_D}}=0\qquad \forall\, x_D>0,x_D<0,x_D=0$$

$$\left[P_{D1}-P_{Df}\right]_{x_D=0}=w_DM_1\frac{\partial P_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-1-7)$$

$$\left[P_{Df}-P_{D2}\right]_{x_D=0}=w_DM_2\frac{\partial P_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-1-8)$$

(H) Laplace transform of (D-1-1),(D-1-2),(D-1-3),(D-1-5),(D-1-6),(D-1-7),(D-1-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {y_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {z_D}^2} + \frac{2\pi}{s}\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = s\bar{P}_{D1} \quad x_D > 0 \quad (D-1-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial {z_D}^2} = \eta_D s\bar{P}_{D2} \qquad x_D < 0 \qquad (D-1-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 \bar{P}_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df} s\bar{P}_{Df} \qquad (D-1-11)$$

(D-1-9), (D-1-10), (D-1-11) were gotten using (D-1-4)

<u>BC</u>: $\bar{P}_{D1}(x_D \to \infty, y_D, z_D, s) = \bar{P}_{D2}(x_D \to -\infty, y_D, z_D, s) = 0 \qquad (D-1-12)$

<u>OTHER CONDITIONS</u>

$$\bar{P}_{D1}'\big|_{z_D=0,h_{z_D}} = \bar{P}_{D2}'\big|_{z_D=0,h_{z_D}} = \bar{P}_{Df}'\big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (D-1-13)$$

$$\bar{P}_{D1}'\big|_{y_D=0,h_{y_D}} = \bar{P}_{D2}'\big|_{y_D=0,h_{y_D}} = \bar{P}_{Df}'\big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[\bar{P}_{D1} - \bar{P}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \bar{P}_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-1-14)$$

$$\left[\bar{P}_{Df} - \bar{P}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \bar{P}_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-1-15)$$

(I) Finite cosine Fourier transform of (D-1-9) through (D-1-15) w.r.t. (z_D)

$$\frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial {y_D}^2} - (k^2 + s)\, \dot{\bar{P}}_{D1} = -\frac{4\pi}{s\mathrm{h}_{zD}} \cos(kz_{WD})\, \delta(x_D - x_{WD})\delta(y_D - y_{WD}) \quad x_D > 0 \quad (D-1-16)$$

$$\frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {x_D}^2} + v_{ry} \frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial {y_D}^2} - (k^2 v_{rz} + \eta_D s)\, \dot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (D-1-17)$$

$$v_y \frac{\partial^2 \dot{\bar{P}}_{Df}}{\partial {x_D}^2} + \frac{1}{F_{CD}} \left[\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} = \left(k^2 v_z + \eta_{Df} s\right) \dot{\bar{P}}_{Df} \qquad (D-1-18)$$

(D-1-16), (D-1-17), (D-1-18) were gotten using (D-1-13)

<u>BC</u>: $\dot{\bar{P}}_{D1}(x_D \to \infty, y_D, k, s) = \dot{\bar{P}}_{D2}(x_D \to -\infty, y_D, k, s) = 0 \qquad (D-1-19)$

<u>OTHER CONDITIONS</u>

$$\left.\dot{\bar{P}}_{D1}{}'\right|_{y_D=0,h_{y_D}} = \left.\dot{\bar{P}}_{D2}{}'\right|_{y_D=0,h_{y_D}} = \left.\dot{\bar{P}}_{Df}{}'\right|_{y_D=0,h_{y_D}} = 0 \quad \forall \quad x_D > 0, x_D < 0, x_D = 0 \quad (D-1-20)$$

$$\left[\dot{\bar{P}}_{D1} - \dot{\bar{P}}_{Df} \right]_{x_D=0} = w_D M_1 \left.\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (D-1-21)$$

$$\left[\dot{\bar{P}}_{Df} - \dot{\bar{P}}_{D2} \right]_{x_D=0} = w_D M_2 \left.\frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (D-1-22)$$

(J) Finite cosine Fourier transform from (D-1-16) through to (D-1-22) w.r.t.(y_D)

$$\frac{\partial^2 \ddot{\bar{P}}_{D1}}{\partial {x_D}^2} - (l^2 + k^2 + s)\, \ddot{\bar{P}}_{D1} = -\frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\, \delta(x_D - x_{WD}) \quad x_D > 0 \qquad (D-1-23)$$

$$\frac{\partial^2 \ddot{\bar{P}}_{D2}}{\partial {x_D}^2} - (l^2 v_{ry} + k^2 v_{rz} + \eta_D s)\, \ddot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (D-1-24)$$

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right) \ddot{\bar{P}}_{Df} = \frac{1}{F_{CD}} \left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} \qquad (D-1-25)$$

(D-1-23), (D-1-24), (D-1-25) were gotten using (D-1-20)

BC: $\ddot{\bar{P}}_{D1}(x_D \to \infty, l, k, s) = \ddot{\bar{P}}_{D2}(x_D \to -\infty, l, k, s) = 0$ $\qquad (D-1-26)$

OTHER CONDITIONS

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{Df} \right]_{x_D=0} = w_D M_1 \left.\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (D-1-27)$$

$$\left[\ddot{\bar{P}}_{Df} - \ddot{\bar{P}}_{D2} \right]_{x_D=0} = w_D M_2 \left.\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (D-1-28)$$

(D) Eliminate $\ddot{\bar{P}}_{Df}$ from the pair of equations (D-1-25) & (D-1-28), (D-1-27) & (D-1-28)

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\ddot{\bar{P}}_{D2} + w_D M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (D-1-29)$$

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} + M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (D-1-30)$$

but (D-1-23) is still non-homogenous in x_D on the R.H.S so,

(E)Laplace transform w.r.t.(x_D) of (D-1-23)

$$\ddot{\bar{\bar{P}}}_{D1}\left[ś^2 - (l^2 + k^2 + s)\right] - \ddot{\bar{P}}'_{D1}(x_D = 0) - ś\,\ddot{\bar{P}}_{D1}(x_D = 0) = -\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{sh_{yD}h_{zD}} e^{-śx_{WD}}$$

$$\ddot{\bar{\bar{P}}}_{D1}(ś, l, k, s) = \frac{ś\,\ddot{\bar{P}}_{D1}(0, l, k, s)}{ś^2 - (l^2 + k^2 + s)} + \frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)}{ś^2 - (l^2 + k^2 + s)} - \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\,e^{-śx_{WD}}}{sh_{yD}h_{zD}[ś^2 - (l^2 + k^2 + s)]}$$

$$(D-1-31)$$

(F)Inverse Laplace transform of (D-1-31) w.r.t. ($ś$)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \ddot{\bar{P}}_{D1}(0, l, k, s)\cosh(Qx_D) + \frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)\sinh(Qx_D)}{Q}$$
$$-\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{sh_{yD}h_{zD}}\frac{\sinh Q(x_D - x_{WD})}{Q} \qquad (D-1-32)$$

where $Q = \sqrt{l^2 + k^2 + s}$

Applying (D-1-26) in (D-1-32)

$$\ddot{\bar{P}}_{D1}(0, l, k, s) + \frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)}{Q} = \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\,e^{-Qx_{WD}}}{sh_{yD}h_{zD}Q} \qquad (D-1-33)$$

(G) Consider general solution of (D-1-24) Region II honoring respective B.C

$$\ddot{\bar{P}}_{D2}(x_D, l, k, s) = A_1 e^{Q_1 x_D} + A_2 e^{-Q_1 x_D} \qquad where\ Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + \eta_D s}$$

using $(D-1-26)$, as $x_D \to -\infty$ and dividing both sides by $e^{-Q_1 x_D}$

$$1 = A_2 + 0, \qquad so\ that\ \ddot{\bar{P}}_{D2}(x_D, l, k, s) = A_1 e^{Q_1 x_D} \qquad (D-1-34)$$

(H) Substitute (D-1-34) into (D-1-29)

If $Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$; $\qquad Q_2 A_1 (1 + w_D M_2 Q_1) = \frac{1}{F_{CD}} \left[\ddot{\bar{P}}'_{D1} \Big|_{x_D=0} - k_{rx} A_1 Q_1 \right]$

$$\ddot{\bar{P}}'_{D1} \Big|_{x_D=0} = A_1 [k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)] = A_1 C_p{}' \qquad (D-1-35)$$

where $C_p{}' = k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)$ $\qquad (D-1-36)$

(III) Substitute (D-1-35) & (D-1-34) into (D-1-30)

$$\ddot{\bar{P}}_{D1} \Big|_{x_D=0} - A_1 = w_D \left[M_1 A_1 C_p{}' + M_2 A_1 Q_1 \right]$$

$$\ddot{\bar{P}}_{D1} \Big|_{x_D=0} = A_1 \left[1 + w_D (M_1 C_p{}' + M_2 Q_1) \right] = A_1 C_p \qquad (D-1-37)$$

where $C_p = 1 + w_D (M_1 C_p{}' + M_2 Q_1)$ $\qquad (D-1-38)$

(J) Solve (D-1-33), (D-1-35), (D-1-37) simultaneously

$$\frac{A_1 C_p{}'}{Q} + A_1 C_p = \frac{8\pi \cos(k z_{WD}) \cos(l y_{WD})\, e^{-Q x_{WD}}}{s \mathrm{h}_{yD} \mathrm{h}_{zD} Q}$$

$$So \qquad A_1 = \frac{8\pi \cos(k z_{WD}) \cos(l y_{WD})\, e^{-Q x_{WD}}}{s \mathrm{h}_{yD} \mathrm{h}_{zD} (Q C_p + C_p{}')} \qquad (D-1-39)$$

(K) Substitute (D-1-35) & (D-1-37) into (D-1-32)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = A_1 C_p \cosh(Qx_D) + \frac{A_1 C_p{}' \sinh(Qx_D)}{Q} - \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \frac{\sinh Q(x_D - x_{WD})}{Q}$$

Substituting for A₁,

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})\, e^{-Qx_{WD}}}{2s\mathrm{h}_{yD}\mathrm{h}_{zD}\left(QC_p + C_p{}'\right)} [C_p(e^{Qx_D} + e^{-Qx_D}) + \frac{C_p{}'}{Q}(e^{Qx_D} - e^{-Qx_D})] - \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})\, e^{-Qx_{WD}}}{2s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} [\left(e^{Q(x_D - x_{WD})} - e^{-Q(x_D - x_{WD})}\right)]$$

Collecting like terms,

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{4\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} [e^{Q(x_D - x_{WD})} \frac{\left(QC_p + C_p{}'\right)}{\left(QC_p + C_p{}'\right)} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p - C_p{}'\right)} + \left(e^{-Q(x_D - x_{WD})} - e^{Q(x_D - x_{WD})}\right)]$$

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{4\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} \left[e^{-Q(x_D - x_{WD})} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right] \qquad (D - 1 - 40)$$

(L) Considering near wellbore specific Skin factor (S_w) and pressure at wellbore,

$$from\ Darcy's\ law \quad \frac{\partial \Delta P}{\partial x} = \frac{q\mu}{2\pi r_W L k_{x1}}$$

$$Also \quad \Delta P_s = \frac{q\mu S}{2\pi L k_{xz}}\ where\ k_{xz} = \sqrt{k_{x1}k_{z1}}$$

$$After\ relevant\ subtitution, \quad \Delta P_s = r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial \Delta P}{\partial x}$$

$$Applying\ dimensionless\ transformation, \quad P_{Ds} = \frac{1}{L} r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial P_D}{\partial x_D}$$

$$So \quad P_{WD} = P_D - P_{Ds}\ at\ x_{D=}x_{WD}\ where\ S_w = r_{WD} S\sqrt{\frac{k_{x1}}{k_{z1}}}$$

$$After\ taking\ relevant\ double\ fourier\ and\ laplace\ transform,$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \left[\ddot{\bar{P}}_{D1}(x_D, l, k, s) - S_W \frac{\partial \ddot{\bar{P}}_{D1}(x_D, l, k, s)}{\partial x_D}\right]_{x_{D=}x_{WD}} \quad (D-1-41)$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \frac{4\pi\cos(kz_{WD})\cos(ly_{WD})(1+\mathrm{QS_w})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}Q}\left[1 + e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{}'\right)}{\left(QC_p + C_p{}'\right)}\right] \quad (D-1-42)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{WD}(x_{WD}, y_D, k, s)_{with\ skin} = \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}\Big|_{m=0}\ and\ \mathcal{F}_m = \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}$$

$$\mathcal{F}_0^m = \frac{4\pi\cos(kz_{WD})(1+\mathrm{RS_w})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}R}\left[1 + e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{}'\right)}{\left(RD_p + D_p{}'\right)}\right]$$

$$\dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}$$

$$= \frac{4\pi\cos(kz_{WD})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}\left[\begin{array}{c} \frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p{}'\right)}{\left(RD_p+D_p{}'\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})\cos(ly_{WD})\cos(ly_D)}{Q}\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p{}'\right)}{\left(QC_p+C_p{}'\right)}\right\}\end{array}\right] \quad (D-1-43)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}\Big|_{n=0} \quad and\ \ \mathcal{F}_n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}$$

$$\mathcal{F}_0^n = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}\left[\begin{array}{c} \frac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{2\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\frac{\left(\sqrt{\mathrm{s}}E_p-E_p{}'\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p{}'\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{}'\right)}{\left(TF_p+F_p{}'\right)}\right\}\end{array}\right]$$

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{4\pi}{\mathrm{sh}_{yD}\mathrm{h}_{zD}}(\alpha+\beta) \quad (D-1-44)$$

$$\alpha = \left[\begin{array}{c} \frac{\left(1+\sqrt{\mathrm{s}}\mathrm{S_w}\right)}{4\sqrt{\mathrm{s}}}\left\{1+e^{-2\sqrt{\mathrm{s}}x_{WD}}\frac{\left(\sqrt{\mathrm{s}}E_p-E_p{}'\right)}{\left(\sqrt{\mathrm{s}}E_p+E_p{}'\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{}'\right)}{\left(TF_p+F_p{}'\right)}\right\}\end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p{}'\right)}{\left(RD_p+D_p{}'\right)}\right\} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p{}'\right)}{\left(QC_p+C_p{}'\right)}\right\}\end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

D – 2 Naturally Fractured Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2} + \frac{\partial^2 P_{D1}}{\partial {y_D}^2} + \frac{\partial^2 P_{D1}}{\partial {z_D}^2} + 2\pi\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD})$$

$$= \omega_1 \frac{\partial P_{D1}}{\partial t_D} + (1 - \omega_1)\frac{\partial P_{D_{m_1}}}{\partial t_D} \qquad x_D > 0 \qquad (D-2-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2} = \eta_D \frac{\partial P_{D2}}{\partial t_D} + \eta_{Dm}\frac{\partial P_{D_{m_2}}}{\partial t_D} \qquad x_D < 0 \qquad (D-2-2)$$

$$v_y \frac{\partial^2 P_{Df}}{\partial {y_D}^2} + v_z \frac{\partial^2 P_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D} - k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (D-2-3a)$$

$$(1-\omega_1)\frac{\partial P_{Dm1}}{\partial t_D} = \lambda_1\left(P_{D1} - P_{Dm1}\right) \qquad (D-2-3b)$$

$$\omega_t(1-\omega_2)\frac{\partial P_{Dm2}}{\partial t_D} = \lambda_2\left(P_{D2} - P_{Dm2}\right) \qquad (D-2-3c)$$

IC: $P_{D1}(x_D, y_D, z_D, 0) = P_{D2}(x_D, y_D, z_D, 0) = P_{Df}(y_D, z_D, 0) = 0$ $\qquad (D-2-4)$

BC: $P_{D1}(x_D \to \infty, y_D, z_D, t_D) = P_{D2}(x_D \to -\infty, y_D, z_D, t_D) = 0$ $\qquad (D-2-5)$

OTHER CONDITIONS

$$P_{D1}'|_{z_D=0,h_{z_D}} = P_{D2}'|_{z_D=0,h_{z_D}} = P_{Df}'\big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (D-2-6)$$

$$P_{D1}'|_{y_D=0,h_{y_D}} = P_{D2}'|_{y_D=0,h_{y_D}} = P_{Df}'\big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[P_{D1} - P_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial P_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-2-7)$$

$$\left[P_{Df} - P_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial P_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-2-8)$$

(K) Laplace transform of (D-2-1), (D-1-2), (D-1-3a), (D-1-3b), (D-1-3c), (D-1-5), (D-1-6), (D-1-7),(D-1-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {y_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {z_D}^2} + \frac{2\pi}{s}\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = sf_1(s)\bar{P}_{D1} \qquad x_D > 0$$

$$(D-2-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial {z_D}^2} = sf_2(s)\bar{P}_{D2} \qquad x_D < 0 \qquad (D-2-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 \bar{P}_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}s\bar{P}_{Df} \qquad (D-2-11)$$

(D-2-9), (D-2-10), (D-2-11) were gotten using (D-2-3b), (D-2-3c), (D-2-4)

<u>BC</u>: $\bar{P}_{D1}(x_D \to \infty, y_D, z_D, s) = \bar{P}_{D2}(x_D \to -\infty, y_D, z_D, s) = 0 \qquad (D-2-12)$

<u>OTHER CONDITIONS</u>

$$\left.\bar{P}_{D1}'\right|_{z_D=0,h_{z_D}} = \left.\bar{P}_{D2}'\right|_{z_D=0,h_{z_D}} = \left.\bar{P}_{Df}'\right|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (D-2-13)$$

$$\left.\bar{P}_{D1}'\right|_{y_D=0,h_{y_D}} = \left.\bar{P}_{D2}'\right|_{y_D=0,h_{y_D}} = \left.\bar{P}_{Df}'\right|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[\bar{P}_{D1} - \bar{P}_{Df}\right]_{x_D=0} = w_D M_1 \left.\frac{\partial \bar{P}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (D-2-14)$$

$$\left[\bar{P}_{Df} - \bar{P}_{D2}\right]_{x_D=0} = w_D M_2 \left.\frac{\partial \bar{P}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (D-2-15)$$

(L) Finite cosine Fourier transform of (D-2-9) through (D-2-15) w.r.t. (z_D)

$$\frac{\partial^2 \bar{\dot{P}}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{\dot{P}}_{D1}}{\partial {y_D}^2} - \left(k^2 + sf_1(s)\right) \bar{\dot{P}}_{D1} = -\frac{4\pi}{\mathrm{sh}_{zD}} \cos(kz_{WD})\, \delta(x_D - x_{WD})\delta(y_D - y_{WD}) \quad x_D > 0$$

$$(D-2-16)$$

$$\frac{\partial^2 \bar{\dot{P}}_{D2}}{\partial {x_D}^2} + v_{ry} \frac{\partial^2 \bar{\dot{P}}_{D2}}{\partial {y_D}^2} - (k^2 v_{rz} + sf_2(s))\, \bar{\dot{P}}_{D2} = 0 \qquad x_D < 0 \qquad (D-2-17)$$

$$v_y \frac{\partial^2 \bar{\dot{P}}_{Df}}{\partial {x_D}^2} + \frac{1}{F_{CD}} \left[\frac{\partial \bar{\dot{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \bar{\dot{P}}_{D2}}{\partial x_D} \right]_{x_D=0} = \left(k^2 v_z + \eta_{Df} s\right) \bar{\dot{P}}_{Df} \qquad (D-2-18)$$

(D-2-16), (D-2-17), (D-2-18) were gotten using (D-2-13)

<u>BC</u>: $\bar{\dot{P}}_{D1}(x_D \to \infty, y_D, k, s) = \bar{\dot{P}}_{D2}(x_D \to -\infty, y_D, k, s) = 0 \qquad (D-2-19)$

<u>OTHER CONDITIONS</u>

$$\bar{\dot{P}}_{D1}{}' \Big|_{y_D=0,h_{y_D}} = \bar{\dot{P}}_{D2}{}' \Big|_{y_D=0,h_{y_D}} = \bar{\dot{P}}_{Df}{}' \Big|_{y_D=0,h_{y_D}} = 0 \quad \forall \quad x_D > 0, x_D < 0, x_D = 0 \qquad (D-2-20)$$

$$\left[\bar{\dot{P}}_{D1} - \bar{\dot{P}}_{Df} \right]_{x_D=0} = w_D M_1 \frac{\partial \bar{\dot{P}}_{D1}}{\partial x_D} \Bigg|_{x_D=0} \qquad (D-2-21)$$

$$\left[\bar{\dot{P}}_{Df} - \bar{\dot{P}}_{D2} \right]_{x_D=0} = w_D M_2 \frac{\partial \bar{\dot{P}}_{D2}}{\partial x_D} \Bigg|_{x_D=0} \qquad (D-2-22)$$

(M) Finite cosine fourier transform from (D-2-16) through to (D-2-22) w.r.t.(y_D)

$$\frac{\partial^2 \ddot{\bar{P}}_{D1}}{\partial {x_D}^2} - (l^2 + k^2 + sf_1(s))\,\ddot{\bar{P}}_{D1} = -\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\delta(x_D - x_{WD}) \qquad x_D > 0 \qquad (D-2-23)$$

$$\frac{\partial^2 \ddot{\bar{P}}_{D2}}{\partial {x_D}^2} - (l^2 v_{ry} + k^2 v_{rz} + sf_2(s))\,\ddot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (D-2-24)$$

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\ddot{\bar{P}}_{Df} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (D-2-25)$$

(D-2-23), (D-2-24), (D-2-25) were gotten using (D-2-20)

<u>BC</u>: $\ddot{\bar{P}}_{D1}(x_D \to \infty, l, k, s) = \ddot{\bar{P}}_{D2}(x_D \to -\infty, l, k, s) = 0$ $\qquad (D-2-26)$

<u>OTHER CONDITIONS</u>

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-2-27)$$

$$\left[\ddot{\bar{P}}_{Df} - \ddot{\bar{P}}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (D-2-28)$$

(D) Eliminate $\ddot{\bar{P}}_{Df}$ from the pair of equations (D-2-25) & (D-2-28), (D-2-27) & (D-2-28)

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\ddot{\bar{P}}_{D2} + w_D M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (D-2-29)$$

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} + M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (D-2-30)$$

but (D-2-23) is still non-homogenous in x_D on the R.H.S so,

(E)Laplace transform w.r.t.(x_D) of (D-2-23)

$$\ddot{\bar{\bar{P}}}_{D1}\left[\acute{s}^2 - \left(l^2 + k^2 + s f_1(s)\right)\right] - \ddot{\bar{P}}'_{D1}(x_D = 0) - \acute{s}\,\ddot{\bar{P}}_{D1}(x_D = 0) = -\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{s h_{yD} h_{zD}} e^{-\acute{s} x_{WD}}$$

$$\ddot{\bar{\bar{P}}}_{D1}(\acute{s}, l, k, s) = \frac{\acute{s}\,\ddot{\bar{P}}_{D1}(0,l,k,s)}{\acute{s}^2 - \left(l^2 + k^2 + s f_1(s)\right)} + \frac{\ddot{\bar{P}}'_{D1}(0,l,k,s)}{\acute{s}^2 - \left(l^2 + k^2 + s f_1(s)\right)} - \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\, e^{-\acute{s} x_{WD}}}{s h_{yD} h_{zD}\left[\acute{s}^2 - (l^2 + k^2 + s f_1(s))\right]} \qquad (D-2-31)$$

(F)Inverse Laplace transform of (D-2-31) w.r.t. ($\acute{s}$)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \ddot{\bar{P}}_{D1}(0,l,k,s)\cosh(Qx_D) + \frac{\ddot{\bar{P}}'_{D1}(0,l,k,s)\sinh(Qx_D)}{Q} - \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{s h_{yD} h_{zD}} \frac{\sinh Q(x_D - x_{WD})}{Q} \qquad (D-2-32)$$

where $Q = \sqrt{l^2 + k^2 + s f_1(s)}$

Applying (D-2-26) in (D-2-32)

$$\ddot{\bar{P}}_{D1}(0,l,k,s) + \frac{\ddot{\bar{P}}'_{D1}(0,l,k,s)}{Q} = \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\, e^{-Qx_{WD}}}{s h_{yD} h_{zD} Q} \qquad (D-2-33)$$

(G) Consider general solution of (D-2-24) Region II honoring respective B.C

$$\ddot{\bar{P}}_{D2}(x_D, l, k, s) = A_1 e^{Q_1 x_D} + A_2 e^{-Q_1 x_D} \qquad \textit{where} \text{ where } Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + s f_2(s)}$$

using $(\mathrm{D}-2-26)$, as $x_D \to -\infty$ and dividing both sides by $\mathrm{e}^{-\mathrm{Q_1 x_D}}$

$$0 = A_2 + 0, \qquad \textit{so that } \ddot{\bar{P}}_{D2}(x_D, l, k, s) = A_1 e^{Q_1 x_D} \qquad (D-2-34)$$

(H) Substitute (D-2-34) into (D-2-29)

If $Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$; $\qquad Q_2 A_1 (1 + w_D M_2 Q_1) = \frac{1}{F_{CD}} \left[\ddot{\bar{P}}'_{D1} \Big|_{x_D=0} - k_{rx} A_1 Q_1 \right]$

$$\ddot{\bar{P}}'_{D1} \Big|_{x_D=0} = A_1 [k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)] = A_1 C_p{}' \qquad (D-2-35)$$

where $C_p{}' = k_{rx} Q_1 + F_{CD} Q_2 (1 + w_D M_2 Q_1)$ $\qquad (D-2-36)$

(I) Substitute (D-2-35) & (D-2-34) into (D-2-30)

$$\ddot{\bar{P}}_{D1} \Big|_{x_D=0} - A_1 = w_D \left[M_1 A_1 C_p{}' + M_2 A_1 Q_1 \right]$$

$$\ddot{\bar{P}}_{D1} \Big|_{x_D=0} = A_1 \left[1 + w_D (M_1 C_p{}' + M_2 Q_1) \right] = A_1 C_p \qquad (D-2-37)$$

where $C_p = 1 + w_D (M_1 C_p{}' + M_2 Q_1)$ $\qquad (D-2-38)$

(J) Solve (D-2-33), (D-2-35), (D-2-37) simultaneously

$$\frac{A_1 C_p{}'}{Q} + A_1 C_p = \frac{8\pi \cos(k z_{WD}) \cos(l y_{WD})\, e^{-Q x_{WD}}}{s \mathrm{h}_{yD} \mathrm{h}_{zD} Q}$$

$$\textit{So} \qquad A_1 = \frac{8\pi \cos(k z_{WD}) \cos(l y_{WD})\, e^{-Q x_{WD}}}{\mathrm{s h}_{yD} \mathrm{h}_{zD} (Q C_p + C_p{}')} \qquad (D-2-39)$$

(K) Substitute (D-2-35) & (D-2-37) into (D-2-32)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = A_1 C_p \cosh(Qx_D) + \frac{A_1 C_p{'} \sinh(Qx_D)}{Q} - \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \frac{\sinh Q(x_D - x_{WD})}{Q}$$

Substituting for A_1,

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})\, e^{-Qx_{WD}}}{2s\mathrm{h}_{yD}\mathrm{h}_{zD}\left(QC_p + C_p{'}\right)} [C_p(e^{Qx_D} + e^{-Qx_D}) + \frac{C_p{'}}{Q}(e^{Qx_D} - e^{-Qx_D})] - \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})\, e^{-Qx_{WD}}}{2s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} [\left(e^{Q(x_D - x_{WD})} - e^{-Q(x_D - x_{WD})}\right)]$$

Collecting like terms,

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{4\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} [e^{Q(x_D - x_{WD})} \frac{\left(QC_p + C_p{'}\right)}{\left(QC_p + C_p{'}\right)} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{'}\right)}{\left(QC_p - C_p{'}\right)} + \left(e^{-Q(x_D - x_{WD})} - e^{Q(x_D - x_{WD})}\right)]$$

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{4\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}Q} \left[e^{-Q(x_D - x_{WD})} + e^{-Q(x_D + x_{WD})} \frac{\left(QC_p - C_p{'}\right)}{\left(QC_p + C_p{'}\right)}\right] \qquad (D-2-40)$$

(L) Considering near wellbore specific Skin factor (S_w) and pressure at wellbore,

$$from\ Darcy's\ law \quad \frac{\partial \Delta P}{\partial x} = \frac{q\mu}{2\pi r_W L k_{x1}}$$

$$Also \quad \Delta P_s = \frac{q\mu S}{2\pi L k_{xz}}\ where\ k_{xz} = \sqrt{k_{x1}k_{z1}}$$

$$After\ relevant\ subtitution, \quad \Delta P_s = r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial \Delta P}{\partial x}$$

$$Applying\ dimensionless\ transformation, \quad P_{Ds} = \frac{1}{L} r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial P_D}{\partial x_D}$$

$$So \quad P_{WD} \ = \ P_D - \ P_{Ds}\ at\ x_D = x_{WD}\ where\ S_w = \ r_{WD} S\sqrt{\frac{k_{x1}}{k_{z1}}}$$

$$After\ taking\ relevant\ double\ fourier\ and\ laplace\ transform,$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \left[\ddot{\bar{P}}_{D1}(x_D, l, k, s) - S_w\frac{\partial \ddot{\bar{P}}_{D1}(x_D, l, k, s)}{\partial x_D}\right]_{x_D = x_{WD}} \qquad (D-2-41)$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \frac{4\pi\cos(kz_{WD})\cos(ly_{WD})(1 + \mathrm{QS_w})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}Q}\left[1 + e^{-2Qx_{WD}}\frac{\left(QC_p - C_p{'}\right)}{\left(QC_p + C_p{'}\right)}\right] \qquad (D-2-42)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{WD}(x_{WD}, y_D, k, s)_{with\ skin} \ = \ \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}\Big|_{m=0}\ and\ \mathcal{F}_m = \ \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}$$

$$\mathcal{F}_0^m = \frac{4\pi\cos(kz_{WD})\,(1 + \mathrm{RS_w})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}R}\left[1 + e^{-2Rx_{WD}}\frac{\left(RD_p - D_p{'}\right)}{\left(RD_p + D_p{'}\right)}\right]$$

$$\bar{\dot{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin}$$

$$= \frac{4\pi\cos(kz_{WD})}{sh_{yD}h_{zD}}\left[\begin{array}{c} \frac{(1+\mathrm{RS_w})}{2R}\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p{'}\right)}{\left(RD_p+D_p{'}\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p{'}\right)}{\left(QC_p+C_p{'}\right)}\right\}\end{array}\right] \quad (D-2-43)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\bar{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin} = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \bar{\dot{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin}\Big|_{n=0} \quad and\ \ \mathcal{F}_n = \bar{\dot{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin}$$

$$\mathcal{F}_0^n = \frac{4\pi}{sh_{yD}h_{zD}}\left[\begin{array}{c} \frac{\left(1+\sqrt{sf_1(s)}\mathrm{S_w}\right)}{2\sqrt{sf_1(s)}}\left\{1+e^{-2\sqrt{sf_1(s)}x_{WD}}\frac{\left(\sqrt{sf_1(s)}E_p-E_p{'}\right)}{\left(\sqrt{sf_1(s)}E_p+E_p{'}\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{'}\right)}{\left(TF_p+F_p{'}\right)}\right\}\end{array}\right]$$

$$\bar{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin} = \frac{4\pi}{sh_{yD}h_{zD}}(\alpha+\beta) \quad (D-2-44)$$

$$\alpha = \left[\begin{array}{c} \frac{\left(1+\sqrt{sf_1(s)}\mathrm{S_w}\right)}{4\sqrt{sf_1(s)}}\left\{1+e^{-2\sqrt{sf_1(s)}x_{WD}}\frac{\left(\sqrt{sf_1(s)}E_p-E_p{'}\right)}{\left(\sqrt{sf_1(s)}E_p+E_p{'}\right)}\right\} \\ +\sum_{m=1}^{\infty}\frac{(1+\mathrm{TS_w})}{2T}\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Tx_{WD}}\frac{\left(TF_p-F_p{'}\right)}{\left(TF_p+F_p{'}\right)}\right\}\end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{n=1}^{\infty}\frac{(1+\mathrm{RS_w})}{2R}\cos(kz_{WD})\cos(kz_D)\left\{1+e^{-2Rx_{WD}}\frac{\left(RD_p-D_p{'}\right)}{\left(RD_p+D_p{'}\right)}\right\} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(1+\mathrm{QS_w})}{Q}\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)\left\{1+e^{-2Qx_{WD}}\frac{\left(QC_p-C_p{'}\right)}{\left(QC_p+C_p{'}\right)}\right\}\end{array}\right]$$

$$\alpha = \mathcal{F}_{00}+\mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n}+\mathcal{F}_{mn}$$

APPENDIX E – Point Source Solution for Producer in Reservoir Finite in Three Directions

The summation of series used in this section was adapted from Gradshteyn et al. (2000).

E-1 Clastic Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2} + \frac{\partial^2 P_{D1}}{\partial {y_D}^2} + \frac{\partial^2 P_{D1}}{\partial {z_D}^2} + 2\pi\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = \frac{\partial P_{D1}}{\partial t_D} \quad x_D > 0 \quad (E-1-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2} = \eta_D \frac{\partial P_{D2}}{\partial t_D} \qquad x_D < 0 \qquad (E-1-2)$$

$$v_y \frac{\partial^2 P_{Df}}{\partial {y_D}^2} + v_z \frac{\partial^2 P_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D} - k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (E-1-3)$$

<u>IC</u>: $$P_{D1}(x_D, y_D, z_D, 0) = P_{D2}(x_D, y_D, z_D, 0) = P_{Df}(y_D, z_D, 0) = 0 \qquad (E-1-4)$$

<u>OTHER CONDITIONS</u>

$$P_{D1}'|_{x_D=h_{x_D}} = P_{D2}'|_{x_D=-h_{x_D}} = 0 \qquad (E-1-5)$$

$$P_{D1}'|_{z_D=0,h_{z_D}} = P_{D2}'|_{z_D=0,h_{z_D}} = P_{Df}'\Big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (E-1-6)$$

$$P_{D1}'|_{y_D=0,h_{y_D}} = P_{D2}'|_{y_D=0,h_{y_D}} = P_{Df}'\Big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[P_{D1} - P_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial P_{D1}}{\partial x_D}\Bigg|_{x_D=0} \qquad (E-1-7)$$

$$\left[P_{Df} - P_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial P_{D2}}{\partial x_D}\Bigg|_{x_D=0} \qquad (E-1-8)$$

(N) Laplace transform of (E-1-1), (E-1-2), (E-1-3), (E-1-5), (E-1-6), (E-1-7), (E-1-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {y_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {z_D}^2} + \frac{2\pi}{s}\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = s\bar{P}_{D1} \qquad x_D > 0 \; (E-1-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial {z_D}^2} = \eta_D s\bar{P}_{D2} \qquad x_D < 0 \qquad (E-1-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 \bar{P}_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}s\bar{P}_{Df} \qquad (E-1-11)$$

(E-1-9), (E-1-10), (E-1-11) were gotten using (E-1-4)

<u>OTHER CONDITIONS</u>

$$\bar{P}_{D1}{}'\Big|_{x_D=h_{xD}} = \bar{P}_{D2}{}'\Big|_{x_D=-h_{x_D}} = 0 \qquad (E-1-12)$$

$$\bar{P}_{D1}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{D2}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{Df}{}'\Big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (E-1-13)$$

$$\bar{P}_{D1}{}'\Big|_{y_D=0,h_{y_D}} = \bar{P}_{D2}{}'\Big|_{y_D=0,h_{y_D}} = \bar{P}_{Df}{}'\Big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[\bar{P}_{D1} - \bar{P}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \bar{P}_{D1}}{\partial x_D}\Big|_{x_D=0} \qquad (E-1-14)$$

$$\left[\bar{P}_{Df} - \bar{P}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \bar{P}_{D2}}{\partial x_D}\Big|_{x_D=0} \qquad (E-1-15)$$

(O) Finite cosine fourier transform of (E-1-9) through (E-1-15) w.r.t. (z_D)

$$\frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial x_D{}^2} + \frac{\partial^2 \dot{\bar{P}}_{D1}}{\partial y_D{}^2} - (k^2 + s)\,\dot{\bar{P}}_{D1} = -\frac{4\pi}{s\mathrm{h}_{zD}}\cos(kz_{WD})\,\delta(x_D - x_{WD})\delta(y_D - y_{WD}) \;\; x_D > 0 \quad (E-1-16)$$

$$\frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial x_D{}^2} + v_{ry}\frac{\partial^2 \dot{\bar{P}}_{D2}}{\partial y_D{}^2} - (k^2 v_{rz} + \eta_D s)\,\dot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (E-1-17)$$

$$v_y\frac{\partial^2 \dot{\bar{P}}_{Df}}{\partial x_D{}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \left(k^2 v_z + \eta_{Df} s\right)\dot{\bar{P}}_{Df} \qquad (E-1-18)$$

(E-1-16), (E-1-17), (E-1-18) were gotten using (E-1-13)

<u>OTHER CONDITIONS</u>

$$\left.\dot{\bar{P}}_{D1}{}'\right|_{x_D=h_{xD}} = \left.\dot{\bar{P}}_{D2}{}'\right|_{x_D=-h_{xD}} = 0 \qquad (E-1-19)$$

$$\left.\dot{\bar{P}}_{D1}{}'\right|_{y_D=0,h_{y_D}} = \left.\dot{\bar{P}}_{D2}{}'\right|_{y_D=0,h_{y_D}} = \left.\dot{\bar{P}}_{Df}{}'\right|_{y_D=0,h_{y_D}} = 0 \quad \forall \quad x_D > 0, x_D < 0, x_D = 0 \qquad (E-1-20)$$

$$\left[\dot{\bar{P}}_{D1} - \dot{\bar{P}}_{Df}\right]_{x_D=0} = w_D M_1 \left.\frac{\partial \dot{\bar{P}}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (E-1-21)$$

$$\left[\dot{\bar{P}}_{Df} - \dot{\bar{P}}_{D2}\right]_{x_D=0} = w_D M_2 \left.\frac{\partial \dot{\bar{P}}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (E-1-22)$$

(P) Finite cosine fourier transform from (E-1-16) through to (E-1-22) w.r.t.(y_D)

$$\frac{\partial^2 \ddot{\bar{P}}_{D1}}{\partial x_D{}^2} - (l^2 + k^2 + s)\,\ddot{\bar{P}}_{D1} = -\frac{8\pi \cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\delta(x_D - x_{WD}) \quad x_D > 0 \qquad (E-1-23)$$

$$\frac{\partial^2 \ddot{\bar{P}}_{D2}}{\partial x_D{}^2} - (l^2 v_{ry} + k^2 v_{rz} + \eta_D s)\,\ddot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (E-1-24)$$

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\ddot{\bar{P}}_{Df} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (E-1-25)$$

(E-1-23), (E-1-24), (E-1-25) were gotten using (E-1-20)

<u>OTHER CONDITIONS</u>

$$\ddot{\bar{P}}_{D1}{}'\Big|_{x_D=h_{xD}} = \ddot{\bar{P}}_{D2}{}'\Big|_{x_D=-h_{xD}} = 0 \qquad (E-1-26)$$

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-1-27)$$

$$\left[\ddot{\bar{P}}_{Df} - \ddot{\bar{P}}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-1-28)$$

(D) Eliminate $\ddot{\overline{P}}_{Df}$ from the pair of equations (E-1-25) & (E-1-28), (E-1-27) & (E-1-28)

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\ddot{\overline{P}}_{D2} + w_D M_2 \frac{\partial \ddot{\overline{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\overline{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\overline{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (E-1-29)$$

$$\left[\ddot{\overline{P}}_{D1} - \ddot{\overline{P}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \ddot{\overline{P}}_{D1}}{\partial x_D} + M_2 \frac{\partial \ddot{\overline{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (E-1-30)$$

but (E-1-23) is still non-homogenous in x_D on the R.H.S so,

(E)Finite cosine Fourier transform w.r.t.(x_D) of (E-1-23) applying (E-1-26)

$$-\dddot{\overline{P}}_{D1}[\omega^2 + (l^2 + k^2 + s)] - \frac{2}{\mathrm{h}_{xD}}\ddot{\overline{P}}'_{D1}(x_D = 0) = -\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}}$$

$$\dddot{\overline{P}}_{D1}(\omega, l, k, s) = \frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}[\omega^2 + (l^2 + k^2 + s)]} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\overline{P}}'_{D1}(0, l, k, s)}{[\omega^2 + (l^2 + k^2 + s)]} \qquad (E-1-31)$$

(F)Inverse Finite cosine Fourier transform (E-1-31) w.r.t. (ω)

$$\ddot{\overline{P}}_{D1}(x_D, l, k, s) = \frac{1}{2}\left\{\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}(l^2 + k^2 + s)} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\overline{P}}'_{D1}(0, l, k, s)}{(l^2 + k^2 + s)}\right\}$$

$$+\sum_{p=1}^{\infty}\cos(\omega x_D)\left\{\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}[\omega^2 + (l^2 + k^2 + s)]} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\overline{P}}'_{D1}(0, l, k, s)}{[\omega^2 + (l^2 + k^2 + s)]}\right\} \qquad (E-1-32)$$

where $Q = \sqrt{l^2 + k^2 + s}$

Setting $x_D = 0$ in (E-1-32)

$$\ddot{\overline{P}}_{D1}(0, l, k, s) = -\ddot{\overline{P}}'_{D1}(0, l, k, s)\frac{2}{\mathrm{h}_{xD}}\left\{\frac{1}{2(l^2 + k^2 + s)} + \sum_{p=1}^{\infty}\frac{1}{[\omega^2 + (l^2 + k^2 + s)]}\right\}$$

$$+\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}}\left\{\frac{1}{2(l^2 + k^2 + s)} + \sum_{p=1}^{\infty}\frac{\cos(\omega x_{WD})}{[\omega^2 + (l^2 + k^2 + s)]}\right\} \qquad (E-1-33)$$

(G) Consider general solution of (E-1-24) Region II honoring respective B.C

$$\ddot{\bar{P}}_{D2}(x_D,l,k,s) = A_1 e^{Q_1 x_D} + A_2 e^{-Q_1 x_D} \qquad where\ \ Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + \eta_D s}$$

using (E − 1 − 26), $\left.\ddot{\bar{P}}_{D2}{}'\right|_{x_D=-h_{xD}} = 0$

$$0 = Q_1 A_1 e^{-Q_1 h_{xD}} - Q_1 A_2 e^{Q_1 h_{xD}}, \qquad A_2 = A_1 e^{-2Q_1 h_{xD}}$$

$$\ddot{\bar{P}}_{D2}(x_D,l,k,s) = A_1 \left\{ e^{Q_1 x_D} + e^{-Q_1 (x_D + 2h_{xD})} \right\} \qquad (E-1-34)$$

(H) Substitute (E-1-34) into (E-1-29)

If $Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$;

$$\left.\ddot{\bar{P}}_{D1}'\right|_{x_D=0} = A_1 C_p{}' \qquad (E-1-35)$$

where $C_p{}' = Q_1 \left(1 - e^{-2Q_1 h_{xD}}\right)(w_D M_2 Q_1 F_{CD} + k_{rx}) + Q_2 F_{CD} \left(1 + e^{-2Q_1 h_{xD}}\right)$ $\qquad (E-1-36)$

(IV) Substitute (E-1-35) & (E-1-34) into (E-1-30)

$$\left.\ddot{\bar{P}}_{D1}\right|_{x_D=0} = A_1 C_p \qquad (E-1-37)$$

where $C_p = e^{-2Q_1 h_{xD}}(1 - w_D M_2 Q_1) + 1 + w_D (M_1 C_p{}' + M_2 Q_1)$ $\qquad (E-1-38)$

(J) Substitute (E-1-35), (E-1-37) simultaneously into (E-1-33)

$$A_1 C_p = -A_1 C_p{}' \frac{2}{h_{xD}} \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2 + (l^2+k^2+s)]} \right\}$$
$$+ \frac{16\pi \cos(k z_{WD}) \cos(l y_{WD})}{s h_{xD} h_{yD} h_{zD}} \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2 + (l^2+k^2+s)]} \right\}$$

$$A_1 = \left[\frac{\frac{16\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2+(l^2+k^2+s)]} \right\}}{C_p + C_p{}' \frac{2}{\mathrm{h}_{xD}} \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2+(l^2+k^2+s)]} \right\}} \right]$$

$$If \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2+(l^2+k^2+s)]} \right\} = \frac{h_{xD} \cosh Q(h_{xD} - x_{WD})}{2Q \sinh Q(h_{xD})}$$

$$and \left\{ \frac{1}{2(l^2+k^2+s)} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2+(l^2+k^2+s)]} \right\} = \frac{h_{xD} \coth(Qh_{xD})}{2Q}$$

$$A_1 = \left[\frac{16\pi \cos(kz_{WD}) \cos(ly_{WD}) \cosh Q(h_{xD} - x_{WD})}{2s\mathrm{h}_{yD}\mathrm{h}_{zD} \{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})\}} \right] \qquad (E-1-39)$$

(K) Substitute (E-1-39) into (E-1-32)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{16\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \times$$

$$\left\{ \begin{array}{c} \frac{1}{2(l^2+k^2+s)} \left[1 - \frac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \right] \\ + \sum_{p=1}^{\infty} \frac{\cos(\omega x_D)}{[\omega^2+(l^2+k^2+s)]} \left[\cos(\omega x_{WD}) - \frac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \right] \end{array} \right\} \qquad (E-1-40)$$

(L) Considering near wellbore specific Skin factor (S_w) and pressure at wellbore,

$$from\ Darcy's\ law \quad \frac{\partial \Delta P}{\partial x} = \frac{q\mu}{2\pi r_W L k_{x1}}$$

$$Also \quad \Delta P_s = \frac{q\mu S}{2\pi L k_{xz}}\ where\ k_{xz} = \sqrt{k_{x1}k_{z1}}$$

$$After\ relevant\ subtitution, \quad \Delta P_s = r_W S \sqrt{\frac{k_{x1}}{k_{z1}}} \frac{\partial \Delta P}{\partial x}$$

$$Applying\ dimensionless\ transformation, \quad P_{Ds} = \frac{1}{L} r_W S \sqrt{\frac{k_{x1}}{k_{z1}}} \frac{\partial P_D}{\partial x_D}$$

$$So \quad P_{WD} = P_D - P_{Ds}\ at\ x_D = x_{WD}\ where\ S_w = r_{WD} S \sqrt{\frac{k_{x1}}{k_{z1}}}$$

$$After\ taking\ relevant\ double\ fourier\ and\ laplace\ transform,$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \left[\ddot{\bar{P}}_{D1}(x_D, l, k, s) - S_w \frac{\partial \ddot{\bar{P}}_{D1}(x_D, l, k, s)}{\partial x_D} \right]_{x_D = x_{WD}} \qquad (E-1-41)$$

$$\left[-S_w \frac{\partial \ddot{\bar{P}}_{D1}(x_D, l, k, s)}{\partial x_D} \right]_{x_D = x_{WD}} = \frac{S_w 16\pi \cos(kz_{WD}) \cos(ly_{WD})}{s h_{xD} h_{yD} h_{zD}} *$$

$$\sum_{p=1}^{\infty} \left\{ \begin{array}{c} \dfrac{\omega \sin(\omega x_{wD}) \cos(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + s)]} \\ - \dfrac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{Q C_p \sinh Q(h_{xD}) + C_p{}' \cosh(Q h_{xD})} \dfrac{\omega \sin(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + s)]} \end{array} \right\}$$

$$If \left\{ \sum_{p=1}^{\infty} \frac{\omega \sin(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + s)]} \right\} = \frac{h_{xD} \sinh Q(h_{xD} - x_{WD})}{2 \sinh Q(h_{xD})}$$

$$and \left\{ \sum_{p=1}^{\infty} \frac{\omega \sin(\omega x_{wD}) \cos(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + s)]} \right\} = \frac{h_{xD} \sinh Q(h_{xD} - 2x_{WD})}{4 \sinh Q(h_{xD})}$$

$$\left[-S_w \frac{\partial \ddot{\bar{P}}_{D1}(x_D,l,k,s)}{\partial x_D}\right]_{x_D=x_{WD}} =$$

$$\frac{S_w 16\pi\cos(kz_{WD})\cos(ly_{WD})}{sh_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}4\sinh Q(h_{xD})}\sum_{p=1}^{\infty}\left\{\begin{array}{c} h_{xD}\sinh Q(h_{xD}-2x_{WD}) \\ -\dfrac{h_{xD}C_p{}'\sinh 2Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})}\end{array}\right\}$$

$$\left[\ddot{\bar{P}}_{D1}(x_D,l,k,s)\right]_{x_D=x_{WD}} =$$

$$\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{sh_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}}\left\{\sum_{p=1}^{\infty}\frac{\cos^2(\omega x_{WD})}{[\omega^2+(l^2+k^2+s)]}+\frac{1}{2(l^2+k^2+s)}\right.$$
$$\left.-\frac{h_{xD}C_p{}'[1+\cosh 2Q(h_{xD}-x_{WD})]}{4\,\mathrm{Q}\sinh Q(h_{xD})\,[QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})]}\right\}$$

$$\ddot{\bar{P}}_{WD}(x_{WD},l,k,s)_{with\ skin} =$$

$$\frac{4\pi\cos(kz_{WD})\cos(ly_{WD})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}\,\mathrm{Q}\sinh Q(h_{xD})}\left\{\cosh Q(h_{xD}-2x_{WD})+S_w\mathrm{Q}\sinh Q(h_{xD}-2x_{WD})+\cosh(Qh_{xD})\right.$$
$$\left.-\frac{C_p{}'[S_w\mathrm{Q}\sinh 2Q(h_{xD}-x_{WD})+1+\cosh 2Q(h_{xD}-x_{WD})]}{QC_p\sinh Q(h_{xD})+C_p{}'\cosh Q(h_{xD})}\right\} \qquad (E-1-42)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{WD}(x_{WD},y_D,k,s)_{with\ skin} = \frac{\mathcal{F}_0^m}{2}+\sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{WD}(x_{WD},l,k,s)_{with\ skin}\Big|_{m=0}$ $and\ \mathcal{F}_m = \ddot{\bar{P}}_{WD}(x_{WD},l,k,s)_{with\ skin}$

$$\mathcal{F}_0^m = \frac{4\pi\cos(kz_{WD})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}\,\mathrm{R}\sinh R(h_{xD})}\left\{\cosh R(h_{xD}-2x_{WD})+S_w\mathrm{R}\sinh R(h_{xD}-2x_{WD})+\cosh(Rh_{xD})\right.$$
$$\left.-\frac{D_p{}'[S_w\mathrm{R}\sinh 2R(h_{xD}-x_{WD})+1+\cosh 2R(h_{xD}-x_{WD})]}{RD_p\sinh R(h_{xD})+D_p{}'\cosh R(h_{xD})}\right\}$$

$$\dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin} = \frac{4\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \times$$

$$\left[\begin{array}{l} \dfrac{1}{2\,\mathrm{R}\sinh R(h_{xD})}\left\{\begin{array}{l}\cosh R(h_{xD}-2x_{WD}) + S_w \mathrm{R}\sinh R(h_{xD}-2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w \mathrm{R}\sinh 2R(h_{xD}-x_{WD}) + 1 + \cosh 2R(h_{xD}-x_{WD})]}{RD_p\sinh R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{array}\right\} \\ + \displaystyle\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})}\left\{\begin{array}{l}\cosh Q(h_{xD}-2x_{WD}) + S_w \mathrm{Q}\sinh Q(h_{xD}-2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w \mathrm{Q}\sinh 2Q(h_{xD}-x_{WD}) + 1 + \cosh 2Q(h_{xD}-x_{WD})]}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{array}\right\}\end{array}\right]$$

$(E - 1 - 43)$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}\Big|_{n=0}\ \ and\ \ \mathcal{F}_n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\,skin}$$

$\mathcal{F}_0^n =$

$$\left[\begin{array}{l} \dfrac{1}{2\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{l}\cosh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD}) + S_w\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD}) + \cosh(\sqrt{s}h_{xD}) \\ -\dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\sinh 2\sqrt{\mathrm{s}}(h_{xD}-x_{WD}) + 1 + \cosh 2\sqrt{\mathrm{s}}(h_{xD}-x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\} \\ + \displaystyle\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{T\sinh T(h_{xD})}\left\{\begin{array}{l}\cosh\mathrm{T}(h_{xD}-2x_{WD}) + S_w\mathrm{T}\sinh\mathrm{T}(h_{xD}-2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w\mathrm{T}\sinh 2\mathrm{T}(h_{xD}-x_{WD}) + 1 + \cosh 2\mathrm{T}(h_{xD}-x_{WD})]}{TF_p\sinh\mathrm{T}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{array}\right\}\end{array}\right]$$

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\,skin} = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (E - 1 - 44)$$

$$\mathcal{F}_{00} = \left[\dfrac{1}{4\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{l}\cosh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD}) + S_w\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD}) + \cosh(\sqrt{s}h_{xD}) \\ -\dfrac{E_p{}'\left[S_w\sqrt{\mathrm{s}}\sinh 2\sqrt{\mathrm{s}}(h_{xD}-x_{WD}) + 1 + \cosh 2\sqrt{\mathrm{s}}(h_{xD}-x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})} \begin{Bmatrix} \cosh\mathrm{T}(h_{xD}-2x_{WD}) + S_w \mathrm{T}\sinh\mathrm{T}(h_{xD}-2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w \mathrm{T}\sinh 2\mathrm{T}(h_{xD}-x_{WD}) + 1 + \cosh 2\mathrm{T}(h_{xD}-x_{WD})]}{TF_p \sinh\mathrm{T}(h_{xD}) + F_p{}'\cosh T(h_{xD})} \end{Bmatrix}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty} \frac{\cos(kz_{WD})\cos(kz_D)}{2R\sinh R(h_{xD})} \begin{Bmatrix} \cosh R(h_{xD}-2x_{WD}) + S_w \mathrm{R}\sinh R(h_{xD}-2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w \mathrm{R}\sinh 2R(h_{xD}-x_{WD}) + 1 + \cosh 2R(h_{xD}-x_{WD})]}{RD_p \sinh R(h_{xD}) + D_p{}'\cosh R(h_{xD})} \end{Bmatrix}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{matrix} \displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})} \times \\ \begin{Bmatrix} \cosh Q(h_{xD}-2x_{WD}) + S_w \mathrm{Q}\sinh Q(h_{xD}-2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w \mathrm{Q}\sinh 2Q(h_{xD}-x_{WD}) + 1 + \cosh 2Q(h_{xD}-x_{WD})]}{QC_p \sinh Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})} \end{Bmatrix} \end{matrix}\right]$$

E –2 Naturally Fractured Reservoirs

$$\frac{\partial^2 P_{D1}}{\partial {x_D}^2} + \frac{\partial^2 P_{D1}}{\partial {y_D}^2} + \frac{\partial^2 P_{D1}}{\partial {z_D}^2} + 2\pi\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD})$$

$$= \omega_1 \frac{\partial P_{D1}}{\partial t_D} + (1-\omega_1)\frac{\partial P_{D_{m_1}}}{\partial t_D} \qquad x_D > 0 \qquad (E-2-1)$$

$$\frac{\partial^2 P_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 P_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 P_{D2}}{\partial {z_D}^2} = \eta_D \frac{\partial P_{D2}}{\partial t_D} + \eta_{Dm}\frac{\partial P_{D_{m_2}}}{\partial t_D} \qquad x_D < 0 \qquad (E-2-2)$$

$$v_y \frac{\partial^2 P_{Df}}{\partial {y_D}^2} + v_z \frac{\partial^2 P_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial P_{D1}}{\partial x_D} - k_{rx}\frac{\partial P_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}\frac{\partial P_{Df}}{\partial t_D} \qquad (E-2-3a)$$

$$(1-\omega_1)\frac{\partial P_{Dm1}}{\partial t_D} = \lambda_1(P_{D1} - P_{Dm1}) \qquad (E-2-3b)$$

$$\omega_t(1-\omega_2)\frac{\partial P_{Dm2}}{\partial t_D} = \lambda_2(P_{D2} - P_{Dm2}) \qquad (E-2-3c)$$

<u>IC</u>: $P_{D1}(x_D, y_D, z_D, 0) = P_{D2}(x_D, y_D, z_D, 0) = P_{Df}(y_D, z_D, 0) = 0$ $\qquad (E-2-4)$

<u>OTHER CONDITIONS</u>

$$P_{D1}'|_{x_D=h_{x_D}} = P_{D2}'|_{x_D=-h_{x_D}} = 0 \qquad (E-2-5)$$

$$P_{D1}'|_{z_D=0,h_{z_D}} = P_{D2}'|_{z_D=0,h_{z_D}} = P_{Df}'\big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (E-2-6)$$

$$P_{D1}'|_{y_D=0,h_{y_D}} = P_{D2}'|_{y_D=0,h_{y_D}} = P_{Df}'\big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$[P_{D1} - P_{Df}]_{x_D=0} = w_D M_1 \frac{\partial P_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-2-7)$$

$$[P_{Df} - P_{D2}]_{x_D=0} = w_D M_2 \frac{\partial P_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-2-8)$$

(Q) Laplace transform of (E-2-1), (E-2-2), (E-2-3a), (E-2- 3b), (E-2- 3c), (E-2-5), (E-2-6), (E-2-7), (E-2-8) w.r.t.(t_D)

$$\frac{\partial^2 \bar{P}_{D1}}{\partial {x_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {y_D}^2} + \frac{\partial^2 \bar{P}_{D1}}{\partial {z_D}^2} + \frac{4\pi}{s}\delta(x_D - x_{WD})\delta(y_D - y_{WD})\delta(z_D - z_{WD}) = sf_1(s)\bar{P}_{D1} \qquad x_D > 0 \qquad (E-2-9)$$

$$\frac{\partial^2 \bar{P}_{D2}}{\partial {x_D}^2} + v_{ry}\frac{\partial^2 \bar{P}_{D2}}{\partial {y_D}^2} + v_{rz}\frac{\partial^2 \bar{P}_{D2}}{\partial {z_D}^2} = sf_2(s)\bar{P}_{D2} \qquad x_D < 0 \qquad (E-2-10)$$

$$v_y\frac{\partial^2 \bar{P}_{Df}}{\partial {y_D}^2} + v_z\frac{\partial^2 \bar{P}_{Df}}{\partial {z_D}^2} + \frac{1}{F_{CD}}\left[\frac{\partial \bar{P}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \bar{P}_{D2}}{\partial x_D}\right]_{x_D=0} = \eta_{Df}s\bar{P}_{Df} \qquad (E-2-11)$$

(E-2-9), (E-2-10), (E-2-11) were gotten using (E-2-4)

<u>OTHER CONDITIONS</u>

$$\bar{P}_{D1}{}'\Big|_{x_D=h_{xD}} = \bar{P}_{D2}{}'\Big|_{x_D=-h_{x_D}} = 0 \qquad (E-2-12)$$

$$\bar{P}_{D1}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{D2}{}'\Big|_{z_D=0,h_{z_D}} = \bar{P}_{Df}{}'\Big|_{z_D=0,h_{z_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0 \qquad (E-2-13)$$

$$\bar{P}_{D1}{}'\Big|_{y_D=0,h_{y_D}} = \bar{P}_{D2}{}'\Big|_{y_D=0,h_{y_D}} = \bar{P}_{Df}{}'\Big|_{y_D=0,h_{y_D}} = 0 \qquad \forall\, x_D > 0, x_D < 0, x_D = 0$$

$$\left[\bar{P}_{D1} - \bar{P}_{Df}\right]_{x_D=0} = w_D M_1 \frac{\partial \bar{P}_{D1}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-2-14)$$

$$\left[\bar{P}_{Df} - \bar{P}_{D2}\right]_{x_D=0} = w_D M_2 \frac{\partial \bar{P}_{D2}}{\partial x_D}\bigg|_{x_D=0} \qquad (E-2-15)$$

(R) Finite cosine fourier transform of (E-2-9) through (E-2-15) w.r.t. (z_D)

$$\frac{\partial^2 \dot{\bar{\bar{P}}}_{D1}}{\partial {x_D}^2}+\frac{\partial^2 \dot{\bar{\bar{P}}}_{D1}}{\partial {y_D}^2}-\left(k^2+sf_1(s)\right)\dot{\bar{\bar{P}}}_{D1}=-\frac{4\pi}{\mathrm{sh}_{zD}}\cos(kz_{WD})\,\delta(x_D-x_{WD})\delta(y_D-y_{WD})\quad x_D>0$$

$$(E-2-16)$$

$$\frac{\partial^2 \dot{\bar{\bar{P}}}_{D2}}{\partial {x_D}^2}+v_{ry}\frac{\partial^2 \dot{\bar{\bar{P}}}_{D2}}{\partial {y_D}^2}-\left(k^2v_{rz}+sf_2(s)\right)\dot{\bar{\bar{P}}}_{D2}=0 \qquad x_D<0 \qquad (E-2-17)$$

$$v_y\frac{\partial^2 \dot{\bar{\bar{P}}}_{Df}}{\partial {x_D}^2}+\frac{1}{F_{CD}}\left[\frac{\partial \dot{\bar{\bar{P}}}_{D1}}{\partial x_D}-k_{rx}\frac{\partial \dot{\bar{\bar{P}}}_{D2}}{\partial x_D}\right]_{x_D=0}=\left(k^2v_z+\eta_{Df}s\right)\dot{\bar{\bar{P}}}_{Df} \qquad (E-2-18)$$

(E-2-16), (E-2-17), (E-2-18) were gotten using (E-2-13)

<u>OTHER CONDITIONS</u>

$$\left.\dot{\bar{\bar{P}}}_{D1}{}'\right|_{x_D=h_{xD}}=\left.\dot{\bar{\bar{P}}}_{D2}{}'\right|_{x_D=-h_{xD}}=0 \qquad (E-2-19)$$

$$\left.\dot{\bar{\bar{P}}}_{D1}{}'\right|_{y_D=0,h_{y_D}}=\left.\dot{\bar{\bar{P}}}_{D2}{}'\right|_{y_D=0,h_{y_D}}=\left.\dot{\bar{\bar{P}}}_{Df}{}'\right|_{y_D=0,h_{y_D}}=0 \quad \forall \quad x_D>0, x_D<0, x_D=0 \qquad (E-2-20)$$

$$\left[\dot{\bar{\bar{P}}}_{D1}-\dot{\bar{\bar{P}}}_{Df}\right]_{x_D=0}=w_DM_1\left.\frac{\partial \dot{\bar{\bar{P}}}_{D1}}{\partial x_D}\right|_{x_D=0} \qquad (E-2-21)$$

$$\left[\dot{\bar{\bar{P}}}_{Df}-\dot{\bar{\bar{P}}}_{D2}\right]_{x_D=0}=w_DM_2\left.\frac{\partial \dot{\bar{\bar{P}}}_{D2}}{\partial x_D}\right|_{x_D=0} \qquad (E-2-22)$$

(S) Finite cosine fourier transform from (E-2-16) through to (E-2-22) w.r.t.(y_D)

$$\frac{\partial^2 \ddot{\bar{P}}_{D1}}{\partial {x_D}^2} - (l^2 + k^2 + sf_1(s))\, \ddot{\bar{P}}_{D1} = -\frac{8\pi \cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \delta(x_D - x_{WD}) \quad x_D > 0 \qquad (E-2-23)$$

$$\frac{\partial^2 \ddot{\bar{P}}_{D2}}{\partial {x_D}^2} - (l^2 v_{ry} + k^2 v_{rz} + sf_2(s))\, \ddot{\bar{P}}_{D2} = 0 \qquad x_D < 0 \qquad (E-2-24)$$

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right) \ddot{\bar{P}}_{Df} = \frac{1}{F_{CD}} \left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx} \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D} \right]_{x_D=0} \qquad (E-2-25)$$

(E-2-23), (E-2-24), (E-2-25) were gotten using (E-2-20)

<u>OTHER CONDITIONS</u>

$$\left. \ddot{\bar{P}}_{D1}{}' \right|_{x_D=h_{xD}} = \left. \ddot{\bar{P}}_{D2}{}' \right|_{x_D=-h_{xD}} = 0 \qquad (E-2-26)$$

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{Df} \right]_{x_D=0} = w_D M_1 \left. \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} \right|_{x_D=0} \qquad (E-2-27)$$

$$\left[\ddot{\bar{P}}_{Df} - \ddot{\bar{P}}_{D2} \right]_{x_D=0} = w_D M_2 \left. \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D} \right|_{x_D=0} \qquad (E-2-28)$$

(D) Eliminate $\ddot{\bar{P}}_{Df}$ from the pair of equations (E-2-25) & (E-2-28), (E-2-27) & (E-2-28)

$$\left(l^2 v_y + k^2 v_z + \eta_{Df} s\right)\left[\ddot{\bar{P}}_{D2} + w_D M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} = \frac{1}{F_{CD}}\left[\frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} - k_{rx}\frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (E-2-29)$$

$$\left[\ddot{\bar{P}}_{D1} - \ddot{\bar{P}}_{D2}\right]_{x_D=0} = w_D\left[M_1 \frac{\partial \ddot{\bar{P}}_{D1}}{\partial x_D} + M_2 \frac{\partial \ddot{\bar{P}}_{D2}}{\partial x_D}\right]_{x_D=0} \qquad (E-2-30)$$

but Eq. (E-2-23) is still non-homogenous in x_D on the R.H.S so,

(E)Finite cosine Fourier transform w.r.t.(x_D) of Eq. (E-2-23) applying Eq. (E-2-26)

$$-\dddot{\bar{P}}_{D1}\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right] - \frac{2}{\mathrm{h}_{xD}}\ddot{\bar{P}}'_{D1}(x_D = 0) = -\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}}$$

$$\dddot{\bar{P}}_{D1}(\omega, l, k, s) = \frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)}{\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]} \qquad (E-2-31)$$

(F)Inverse Finite cosine Fourier transforms Eq. (E-2-31) w.r.t. (ω)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{1}{2}\left\{\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}\left(l^2 + k^2 + sf_1(s)\right)} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)}{\left(l^2 + k^2 + sf_1(s)\right)}\right\}$$

$$+\sum_{p=1}^{\infty}\cos(\omega x_D)\left\{\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cos(\omega x_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]} - \frac{2}{\mathrm{h}_{xD}}\frac{\ddot{\bar{P}}'_{D1}(0, l, k, s)}{\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]}\right\}$$

$$(E-2-32)$$

$$\text{where } Q = \sqrt{l^2 + k^2 + sf_1(s)}$$

Setting $x_D = 0$ in (E-2-32)

$$\ddot{\bar{P}}_{D1}(0, l, k, s) = -\ddot{\bar{P}}'_{D1}(0, l, k, s)\frac{2}{\mathrm{h}_{xD}}\left\{\frac{1}{2\left(l^2 + k^2 + sf_1(s)\right)} + \sum_{p=1}^{\infty}\frac{1}{\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]}\right\}$$

$$+\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}}\left\{\frac{1}{2\left(l^2 + k^2 + sf_1(s)\right)} + \sum_{p=1}^{\infty}\frac{\cos(\omega x_{WD})}{\left[\omega^2 + \left(l^2 + k^2 + sf_1(s)\right)\right]}\right\} \qquad (E-2-33)$$

(G) Consider general solution of (E-2-24) Region II honoring respective B.C

$$\ddot{\bar{P}}_{D2}\,(x_D, l, k, s) = A_1 e^{Q_1 x_D} + A_2 e^{-Q_1 x_D} \qquad \text{where } Q_1 = \sqrt{l^2 v_{ry} + k^2 v_{rz} + s f_2(s)}$$

using (E − 2 − 26), $\left.\ddot{\bar{P}}_{D2}{}'\right|_{x_D=-h_{xD}} = 0$

$$0 = Q_1 A_1 e^{-Q_1 h_{xD}} - Q_1 A_2 e^{Q_1 h_{xD}}, \qquad A_2 = A_1 e^{-2Q_1 h_{xD}}$$

$$so\ that\ \ddot{\bar{P}}_{D2}\,(x_D, l, k, s) = A_1 \left\{e^{Q_1 x_D} + e^{-Q_1(x_D + 2h_{xD})}\right\} \qquad (E-2-34)$$

(H) Substitute (E-2-34) into (E-2-29)

If $Q_2 = l^2 v_y + k^2 v_z + \eta_{Df} s$;

$$\left.\ddot{\bar{P}}'_{D1}\right|_{x_D=0} = A_1 C_p{}' \qquad (E-2-35)$$

where $C_p{}' = Q_1\left(1 - e^{-2Q_1 h_{xD}}\right)(w_D M_2 Q_1 F_{CD} + k_{rx}) + Q_2 F_{CD}\left(1 + e^{-2Q_1 h_{xD}}\right)$ (E − 2 − 36)

(V) Substitute (E-2-35) & (E-2-34) into (E-2-30)

$$\left.\ddot{\bar{P}}_{D1}\right|_{x_D=0} = A_1 C_p \qquad (E-2-37)$$

where $C_p = e^{-2Q_1 h_{xD}}(1 - w_D M_2 Q_1) + 1 + w_D(M_1 C_p{}' + M_2 Q_1)$ (E − 2 − 38)

(J) Substitute (E-2-35), (E-2-37) simultaneously into (E-2-33)

$$A_1 C_p = -A_1 C_p{}' \frac{2}{\mathrm{h}_{xD}} \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\}$$

$$+ \frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\}$$

$$A_1 = \left[\frac{\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\}}{C_p + C_p{}' \frac{2}{\mathrm{h}_{xD}} \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\}} \right]$$

$$If \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{\cos(\omega x_{WD})}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\} = \frac{h_{xD}\cosh Q(h_{xD}-x_{WD})}{2Q\sinh Q(h_{xD})}$$

$$and \left\{ \frac{1}{2(l^2+k^2+sf_1(s))} + \sum_{p=1}^{\infty} \frac{1}{[\omega^2+(l^2+k^2+sf_1(s))]} \right\} = \frac{h_{xD}\coth(Qh_{xD})}{2Q}$$

$$A_1 = \left[\frac{16\pi\cos(kz_{WD})\cos(ly_{WD})\cosh Q(h_{xD}-x_{WD})}{2s\mathrm{h}_{yD}\mathrm{h}_{zD}\{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})\}} \right] \qquad (E-2-39)$$

(K) Substitute (E-2-39) into (E-2-32)

$$\ddot{\bar{P}}_{D1}(x_D,l,k,s) = \frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} *$$

$$\left\{ \begin{array}{c} \frac{1}{2(l^2+k^2+sf_1(s))} \left[1 - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})} \right] \\ + \sum_{p=1}^{\infty} \frac{\cos(\omega x_D)}{[\omega^2+(l^2+k^2+sf_1(s))]} \left[\cos(\omega x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})} \right] \end{array} \right\} \qquad (E-2-40)$$

(L) Considering near wellbore specific Skin factor (S_w) and pressure at wellbore,

$$from\ Darcy's\ law \quad \frac{\partial \Delta P}{\partial x} = \frac{q\mu}{2\pi r_W L k_{x1}}$$

$$Also \quad \Delta P_s = \frac{q\mu S}{2\pi L k_{xz}}\ where\ k_{xz} = \sqrt{k_{x1}k_{z1}}$$

$$After\ relevant\ subtitution, \quad \Delta P_s = r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial \Delta P}{\partial x}$$

$$Applying\ dimensionless\ transformation, \quad P_{Ds} = \frac{1}{L} r_W S\sqrt{\frac{k_{x1}}{k_{z1}}}\frac{\partial P_D}{\partial x_D}$$

$$So \quad P_{WD} = P_D - P_{Ds}\ at\ x_D = x_{WD}\ where\ S_w = r_{WD} S\sqrt{\frac{k_{x1}}{k_{z1}}}$$

$$After\ taking\ relevant\ double\ fourier\ and\ laplace\ transform,$$

$$\ddot{\overline{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} = \left[\ddot{\overline{P}}_{D1}(x_D, l, k, s) - S_w \frac{\partial \ddot{\overline{P}}_{D1}(x_D, l, k, s)}{\partial x_D}\right]_{x_D = x_{WD}} \qquad (E-2-41)$$

$$\left[-S_w \frac{\partial \ddot{\overline{P}}_{D1}(x_D, l, k, s)}{\partial x_D}\right]_{x_D = x_{WD}} = -\frac{S_w 16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} *$$

$$\sum_{p=1}^{\infty}\left\{\begin{array}{c}\dfrac{\omega\sin(\omega x_{wD})\cos(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + sf_1(s))]} \\ -\dfrac{C_p{}'\cosh Q(h_{xD} - x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})}\dfrac{\omega\sin(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + sf_1(s))]}\end{array}\right\}$$

$$If\ \left\{\sum_{p=1}^{\infty}\frac{\omega\sin(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + sf_1(s))]}\right\} = \frac{h_{xD}\sinh Q(h_{xD} - x_{WD})}{2\sinh Q(h_{xD})}$$

$$and\ \left\{\sum_{p=1}^{\infty}\frac{\omega\sin(\omega x_{wD})\cos(\omega x_{wD})}{[\omega^2 + (l^2 + k^2 + sf_1(s))]}\right\} = \frac{\mathrm{h}_{xD}\sinh Q(h_{xD} - 2x_{WD})}{4\sinh Q(h_{xD})}$$

$$\left[-S_w \frac{\partial \ddot{\bar{P}}_{D1}(x_D, l, k, s)}{\partial x_D}\right]_{x_D = x_{WD}} =$$

$$\frac{S_w 16\pi \cos(kz_{WD}) \cos(ly_{WD})}{sh_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD} 4 \sinh Q(h_{xD})} \sum_{p=1}^{\infty} \left\{ \begin{array}{c} h_{xD} \sinh Q(h_{xD} - 2x_{WD}) \\ -\dfrac{h_{xD} C_p{}' \sinh 2Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \end{array} \right\}$$

$$\left[\ddot{\bar{P}}_{D1}(x_D, l, k, s)\right]_{x_D = x_{WD}} =$$

$$\frac{16\pi \cos(kz_{WD}) \cos(ly_{WD})}{sh_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \left\{ \sum_{p=1}^{\infty} \frac{\cos^2(\omega x_{WD})}{[\omega^2 + (l^2 + k^2 + s)]} + \frac{1}{2(l^2 + k^2 + s)]} - \frac{h_{xD} C_p{}' [1 + \cosh 2Q(h_{xD} - x_{WD})]}{4\, \mathrm{Q}\sinh Q(h_{xD})\, [QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})]} \right\}$$

$$\ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin} =$$

$$\frac{4\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\, \mathrm{Q}\sinh Q(h_{xD})} \left\{ \cosh Q(h_{xD} - 2x_{WD}) + S_w \mathrm{Q}\sinh Q(h_{xD} - 2x_{WD}) + \cosh(Qh_{xD}) - \frac{C_p{}'[S_w \mathrm{Q}\sinh 2Q(h_{xD} - x_{WD}) + 1 + \cosh 2Q(h_{xD} - x_{WD})]}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh Q(h_{xD})} \right\} \quad (E-2-42)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{WD}(x_{WD}, y_D, k, s)_{with\ skin} = \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty} \mathcal{F}_m \cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}\Big|_{m=0} \ and\ \mathcal{F}_m = \ddot{\bar{P}}_{WD}(x_{WD}, l, k, s)_{with\ skin}$$

$$\mathcal{F}_0^m = \frac{4\pi \cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\, \mathrm{R}\sinh R(h_{xD})} \left\{ \cosh R(h_{xD} - 2x_{WD}) + S_w \mathrm{R}\sinh R(h_{xD} - 2x_{WD}) + \cosh(Rh_{xD}) - \frac{D_p{}'[S_w \mathrm{R}\sinh 2R(h_{xD} - x_{WD}) + 1 + \cosh 2R(h_{xD} - x_{WD})]}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh R(h_{xD})} \right\}$$

$$\dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin} = \frac{4\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \; *$$

$$\left[\begin{array}{l} \dfrac{1}{2\,\mathrm{Rsinh}R(h_{xD})}\left\{\begin{array}{c}\mathrm{cosh}R(h_{xD}-2x_{WD}) + S_w\mathrm{Rsinh}R(h_{xD}-2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{}'[S_w\mathrm{Rsinh2}R(h_{xD}-x_{WD}) + 1 + \mathrm{cosh2}R(h_{xD}-x_{WD})]}{RD_p\,\mathrm{sinh}R(h_{xD}) + D_p{}'\cosh R(h_{xD})}\end{array}\right\} \\ + \displaystyle\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})}\left\{\begin{array}{c}\mathrm{cosh}Q(h_{xD}-2x_{WD}) + S_w\mathrm{Qsinh}Q(h_{xD}-2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{}'[S_w\mathrm{Qsinh2}Q(h_{xD}-x_{WD}) + 1 + \mathrm{cosh2}Q(h_{xD}-x_{WD})]}{QC_p\,\mathrm{sinh}Q(h_{xD}) + C_p{}'\cosh Q(h_{xD})}\end{array}\right\}\end{array}\right]$$

$$(E-2-43)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin} = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin}\Big|_{n=0}\ \ and\ \ \mathcal{F}_n = \dot{\overline{P}}_{WD}\,(x_{WD}, y_D, k, s)_{with\ skin}$$

$$\mathcal{F}_0^n =$$

$$\left[\begin{array}{l} \qquad\qquad\dfrac{1}{2\sqrt{sf_1(s)}\mathrm{sinh}\sqrt{sf_1(s)}(h_{xD})}\; * \\ \left\{\begin{array}{c}\mathrm{cosh}\sqrt{sf_1(s)}(h_{xD}-2x_{WD}) + S_w\sqrt{sf_1(s)}\mathrm{sinh}\sqrt{sf_1(s)}(h_{xD}-2x_{WD}) + \cosh\left(\sqrt{sf_1(s)}h_{xD}\right) \\ -\dfrac{E_p{}'\left[S_w\sqrt{sf_1(s)}\mathrm{sinh2}\sqrt{sf_1(s)}(h_{xD}-x_{WD}) + 1 + \mathrm{cosh2}\sqrt{sf_1(s)}(h_{xD}-x_{WD})\right]}{\sqrt{sf_1(s)}E_p\,\mathrm{sinh}\sqrt{sf_1(s)}(h_{xD}) + E_p{}'\cosh\sqrt{sf_1(s)}(h_{xD})}\end{array}\right\} \\ + \displaystyle\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)}{T\sinh T(h_{xD})}\left\{\begin{array}{c}\mathrm{coshT}(h_{xD}-2x_{WD}) + S_w\mathrm{TsinhT}(h_{xD}-2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{}'[S_w\mathrm{Tsinh2T}(h_{xD}-x_{WD}) + 1 + \mathrm{cosh2T}(h_{xD}-x_{WD})]}{TF_p\,\mathrm{sinhT}(h_{xD}) + F_p{}'\cosh T(h_{xD})}\end{array}\right\}\end{array}\right]$$

$$\overline{P}_{WD}\,(x_{WD}, y_D, z_D, s)_{with\ skin} = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (E-2-44)$$

$$\mathcal{F}_{00} = \left[\frac{1}{4\sqrt{sf_1(s)}\sinh\sqrt{sf_1(s)}(h_{xD})} * \left\{ \begin{matrix} \cosh\sqrt{sf_1(s)}(h_{xD}-2x_{WD}) + S_w\sqrt{sf_1(s)}\sinh\sqrt{sf_1(s)}(h_{xD}-2x_{WD}) + \cosh\left(\sqrt{sf_1(s)}h_{xD}\right) \\ -\dfrac{E_p{'}\left[S_w\sqrt{sf_1(s)}\sinh 2\sqrt{sf_1(s)}(h_{xD}-x_{WD}) + 1 + \cosh 2\sqrt{sf_1(s)}(h_{xD}-x_{WD})\right]}{\sqrt{sf_1(s)}E_p\sinh\sqrt{sf_1(s)}(h_{xD}) + E_p{'}\cosh\sqrt{sf_1(s)}(h_{xD})} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)}{2T\sinh T(h_{xD})} \left\{ \begin{matrix} \cosh T(h_{xD}-2x_{WD}) + S_w T\sinh T(h_{xD}-2x_{WD}) + \cosh(Th_{xD}) \\ -\dfrac{F_p{'}[S_w T\sinh 2T(h_{xD}-x_{WD}) + 1 + \cosh 2T(h_{xD}-x_{WD})]}{TF_p\sinh T(h_{xD}) + F_p{'}\cosh T(h_{xD})} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty} \frac{\cos(kz_{WD})\cos(kz_D)}{2R\sinh R(h_{xD})} \left\{ \begin{matrix} \cosh R(h_{xD}-2x_{WD}) + S_w R\sinh R(h_{xD}-2x_{WD}) + \cosh(Rh_{xD}) \\ -\dfrac{D_p{'}[S_w R\sinh 2R(h_{xD}-x_{WD}) + 1 + \cosh 2R(h_{xD}-x_{WD})]}{RD_p\sinh R(h_{xD}) + D_p{'}\cosh R(h_{xD})} \end{matrix} \right\} \right]$$

$$\mathcal{F}_{mn} = \left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\cos(kz_{WD})\cos(kz_D)\cos(ly_{WD})\cos(ly_D)}{Q\sinh Q(h_{xD})} \left\{ \begin{matrix} \cosh Q(h_{xD}-2x_{WD}) + S_w Q\sinh Q(h_{xD}-2x_{WD}) + \cosh(Qh_{xD}) \\ -\dfrac{C_p{'}[S_w Q\sinh 2Q(h_{xD}-x_{WD}) + 1 + \cosh 2Q(h_{xD}-x_{WD})]}{QC_p\sinh Q(h_{xD}) + C_p{'}\cosh Q(h_{xD})} \end{matrix} \right\} \right]$$

APPENDIX F – Point Source Solution for Observer in Reservoir Infinite in Two Directions

F –1 Clastic Reservoir

Steps (A) to (K) is same as in Appendix C-1

(L) Substituting for A_1 in (C-1-33)

$$\hat{\dot{\overline{P}}}_{D2}(x_D,\omega,k,s) = \frac{2\sqrt{2\pi}\cos(kz_{WD})\,e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD}(QC_p + C_p^{'})} e^{Q_1(x_D - x_{WD})} \qquad (F-1-40)$$

(M) Inverse Infinite Fourier Transform w.r.t. (ω)

$$\dot{\overline{P}}_{D2}(x_D,y_D,k,s) = \frac{2\cos(kz_{WD})}{\mathrm{sh}_{zD}} \int_{-\infty}^{\infty} \frac{e^{Q_1(x_D-x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \qquad (F-1-41)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{D2}(x_D,y_D,z_D,s) \ = \ \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \left.\dot{\overline{P}}_{D2}(x_D,y_D,k,s)\right|_{n=0} \ and\ \mathcal{F}_n = \dot{\overline{P}}_{D2}(x_D,y_D,k,s)$$

$$\mathcal{F}_0^n = \frac{2}{\mathrm{sh}_{zD}} \int_{-\infty}^{\infty} \frac{e^{R_1(x_D-x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_D-y_{WD})} d\omega$$

$$\overline{P}_{D2}(x_D,y_D,k,s)$$

$$= \frac{1}{\mathrm{sh}_{zD}} \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{e^{R_1(x_D-x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_D-y_{WD})} d\omega \\ \displaystyle + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} 2\cos(kz_D)\cos(kz_{WD}) \frac{e^{Q_1(x_D-x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_D-y_{WD})} d\omega \end{array} \right] \qquad (F-1-42)$$

F –2 Naturally Fractured Reservoirs

Steps (A) to (K) is same as in Appendix C-2

(L) Substituting for A_1 in (F-2-33)

$$\hat{\dot{\overline{P}}}_{D2}(x_D, \omega, k, s) = \frac{2\sqrt{2\pi}\cos(kz_{WD})\, e^{-i\omega y_{WD}}}{\mathrm{sh}_{zD}(QC_p + C_p^{'})} e^{Q_1(x_D - x_{WD})} \qquad (F-2-40)$$

(M) Inverse Infinite Fourier Transform w.r.t. (ω)

$$\dot{\overline{P}}_{D2}(x_D, y_D, k, s) = \frac{2\cos(kz_{WD})}{\mathrm{sh}_{zD}} \int_{-\infty}^{\infty} \frac{e^{Q_1(x_D - x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \qquad (F-2-41)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_{D2}(x_D, y_D, k, s)\Big|_{n=0}\ and\ \mathcal{F}_n = \dot{\overline{P}}_{D2}(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n = \frac{2}{\mathrm{sh}_{zD}} \int_{-\infty}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega$$

$$\overline{P}_{D2}(x_D, y_D, k, s)$$

$$= \frac{1}{\mathrm{sh}_{zD}} \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{RD_p + D_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \\ \displaystyle + \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} 2\cos(kz_D)\cos(kz_{WD}) \frac{e^{Q_1(x_D - x_{WD})}}{QC_p + C_p^{'}} e^{i\omega(y_D - y_{WD})} d\omega \end{array} \right] \qquad (F-2-42)$$

APPENDIX G – Point Source Solution for Observer in Reservoir Finite in Two Directions

G –1 Clastic Reservoir

Steps (A) to (K) is same as in Appendix D-1

(L) Substituting for A_1 in (D-1-34)

$$\ddot{\overline{P}}_{D2}\,(x_D, l, k, s) = \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}(QC_p + C_p^{'})}e^{Q_1(x_D - x_{WD})} \qquad (G-1-41)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s) \;=\; \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty} \mathcal{F}_m \cos(ly_D)$$

$$where\; \mathcal{F}_0^m = \;\ddot{\overline{P}}_{D2}\,(x_D, l, k, s)\Big|_{m=0} \;\; and \;\; \mathcal{F}_m = \;\ddot{\overline{P}}_{D2}\,(x_D, l, k, s)$$

$$\mathcal{F}_0^m = \frac{8\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\left(RD_p + D_p^{'}\right)}e^{R_1(x_D - x_{WD})}$$

$$\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s) = \frac{4\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\left[\frac{e^{R_1(x_D - x_{WD})}}{\left(RD_p + D_p^{'}\right)} + \sum_{m=1}^{\infty}\frac{2e^{Q_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(QC_p + C_p^{'}\right)}\right]$$

$$(G-1-42)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;=\; \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\; \mathcal{F}_0^n = \;\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)\Big|_{n=0} \;\; and \;\; \mathcal{F}_n = \;\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n = \frac{4\pi}{sh_{yD}h_{zD}}\left[\frac{e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{\left(\sqrt{s}E_p + E_p{}'\right)} + \sum_{m=1}^{\infty}\frac{2e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(TF_p + F_p{}'\right)}\right]$$

$$\overline{P}_{D2}(x_D, y_D, z_D, s) = \frac{4\pi}{sh_{yD}h_{zD}}(\alpha + \beta) \qquad (G-1-43)$$

$$\alpha = \left[\frac{e^{\sqrt{s\eta_D}(x_D - x_{WD})}}{2\left(\sqrt{s}E_p + E_p{}'\right)} + \sum_{m=1}^{\infty}\frac{e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(TF_p + F_p{}'\right)}\right]$$

$$\beta = \left[\begin{array}{c}\displaystyle\sum_{n=1}^{\infty}\frac{e^{R_1(x_D - x_{WD})}}{\left(RD_p + D_p{}'\right)}\cos(kz_D)\cos(kz_{WD}) \\ \displaystyle+\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2e^{Q_1(x_D - x_{WD})}}{\left(QC_p + C_p{}'\right)}\cos(kz_D)\cos(kz_{WD})\cos(ly_{WD})\cos(ly_D)\end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

G –2 Naturally Fractured Reservoirs

Steps (A) to (K) is same as in Appendix D-2

(L) Substituting for A_1 in (G-2-34)

$$\ddot{\bar{P}}_{D2}\,(x_D, l, k, s) = \frac{8\pi \cos(kz_{WD}) \cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}(QC_p + C_p^{'})} e^{Q_1(x_D - x_{WD})} \qquad (G-2-41)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s) \;=\; \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty} \mathcal{F}_m \cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \;\ddot{\bar{P}}_{D2}\,(x_D, l, k, s)\Big|_{m=0} \;\; and\;\; \mathcal{F}_m = \;\ddot{\bar{P}}_{D2}\,(x_D, l, k, s)$$

$$\mathcal{F}_0^m = \frac{8\pi \cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\left(RD_p + D_p^{'}\right)} e^{R_1(x_D - x_{WD})}$$

$$\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s) = \frac{4\pi \cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \left[\frac{e^{R_1(x_D - x_{WD})}}{\left(RD_p + D_p^{'}\right)} + \sum_{m=1}^{\infty} \frac{2e^{Q_1(x_D - x_{WD})} \cos(ly_{WD}) \cos(ly_D)}{\left(QC_p + C_p^{'}\right)}\right] \qquad (G-2-42)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\bar{P}_{D2}\,(x_D, y_D, z_D, s) \;=\; \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_{WD})$$

$$where\ \mathcal{F}_0^n = \;\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s)\Big|_{n=0} \;\; and\;\; \mathcal{F}_n = \;\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \left[\frac{e^{\sqrt{sf_2(s)}(x_D - x_{WD})}}{\left(\sqrt{sf_1(s)}E_p + E_p^{'}\right)} + \sum_{m=1}^{\infty} \frac{2e^{T_1(x_D - x_{WD})} \cos(ly_{WD}) \cos(ly_D)}{\left(TF_p + F_p^{'}\right)}\right]$$

$$\overline{P}_{D2}\left(x_D, y_D, z_D, s\right) = \frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\alpha + \beta) \qquad (G-2-43)$$

$$\alpha = \left[\frac{e^{\sqrt{sf_2(s)}(x_D - x_{WD})}}{2\left(\sqrt{sf_1(s)}E_p + E_p{'}\right)} + \sum_{m=1}^{\infty} \frac{e^{T_1(x_D - x_{WD})}\cos(ly_{WD})\cos(ly_D)}{\left(TF_p + F_p{'}\right)}\right]$$

$$\beta = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty} \frac{e^{R_1(x_D - x_{WD})}}{\left(RD_p + D_p{'}\right)}\cos(kz_D)\cos(kz_{WD}) \\ \displaystyle + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2e^{Q_1(x_D - x_{WD})}}{\left(QC_p + C_p{'}\right)}\cos(kz_D)\cos(kz_{WD})\cos(ly_{WD})\cos(ly_D) \end{array}\right]$$

$$\alpha = \mathcal{F}_{00} + \mathcal{F}_{m0}; \qquad \beta = \mathcal{F}_{0n} + \mathcal{F}_{mn}$$

APPENDIX H – Point Source Solution for Observer in Reservoir Finite in Three Directions

H –1 Clastic Reservoir

Steps (A) to (K) is same as in Appendix E-1

(L) Substituting for A_1 in (E-1-34)

$$\ddot{\bar{P}}_{D2}\,(x_D, l, k, s) =$$

$$\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\cosh Q(h_{xD}-x_{WD})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}\{QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})\}}\left\{e^{Q_1x_D}+e^{-Q_1(x_D+2h_{xD})}\right\} \qquad (H-1-41)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s) \;=\; \frac{\mathcal{F}_0^m}{2}+\sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{D2}\,(x_D, l, k, s)\Big|_{m=0}\ \ and\ \ \mathcal{F}_m = \ddot{\bar{P}}_{D2}\,(x_D, l, k, s)$$

$$\mathcal{F}_0^m = \frac{8\pi\cos(kz_{WD})\cosh R(h_{xD}-x_{WD})}{\mathrm{sh}_{yD}\mathrm{h}_{zD}\{RD_p\sinh R(h_{xD})+D_p{}'\cosh(Rh_{xD})\}}\left[e^{R_1x_D}+e^{-R_1(x_D+2h_{xD})}\right]$$

$$\bar{P}_{D2}\,(x_D, y_D, k, s) = \frac{4\pi\cos(kz_{WD})}{sh_{yD}\mathrm{h}_{zD}}\times$$

$$\left[\begin{array}{l} \dfrac{\cosh R(h_{xD}-x_{WD})}{RD_p\sinh R(h_{xD})+D_p{}'\cosh(Rh_{xD})}\left[e^{R_1x_D}+e^{-R_1(x_D+2h_{xD})}\right] \\ +\displaystyle\sum_{m=1}^{\infty}\dfrac{2\cos(ly_{WD})\cos(ly_D)\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})}\left[e^{Q_1x_D}+e^{-Q_1(x_D+2h_{xD})}\right]\end{array}\right] \qquad (H-1-42)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;=\; \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \;\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)\Big|_{n=0}\;\; and\;\; \mathcal{F}_n = \;\dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n =$$

$$\frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}}E_p \sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}' \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)}\left[e^{\sqrt{\mathrm{s}\eta_D}x_D} + e^{-\sqrt{\mathrm{s}\eta_D}(x_D+2h_{xD})}\right]$$
$$+\sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})}\,\left[e^{T_1 x_D} + e^{-T_1(x_D+2h_{xD})}\right]$$

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;= \frac{4\pi}{\mathrm{s}h_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (H-1-43)$$

$$\mathcal{F}_{00} = \frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{2\left[\sqrt{\mathrm{s}}E_p \sinh\sqrt{\mathrm{s}}(h_{xD}) + E_p{}' \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)\right]}\left[e^{\sqrt{\mathrm{s}\eta_D}x_D} + e^{-\sqrt{\mathrm{s}\eta_D}(x_D+2h_{xD})}\right]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})}\,\left[e^{T_1 x_D} + e^{-T_1(x_D+2h_{xD})}\right]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{\cos(kz_D)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh(Rh_{xD})}\left[e^{R_1 x_D} + e^{-R_1(x_D+2h_{xD})}\right]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cos(kz_D)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \times \left[e^{Q_1 x_D} + e^{-Q_1(x_D+2h_{xD})}\right]$$

H –2 Naturally Fractured Reservoirs

Steps (A) to (K) is same as in Appendix E-2

(L) Substituting for A_1 in (34)

$$\ddot{\bar{P}}_{D2}\,(x_D, l, k, s) =$$

$$\frac{8\pi\cos(kz_{WD})\cos(ly_{WD})\cosh Q(h_{xD}-x_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\{QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})\}}\{e^{Q_1x_D}+e^{-Q_1(x_D+2h_{xD})}\} \qquad (H-2-41)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_{D2}\,(x_D, y_D, k, s) = \frac{\mathcal{F}_0^m}{2}+\sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{D2}\,(x_D, l, k, s)\Big|_{m=0}\ and\ \mathcal{F}_m = \ddot{\bar{P}}_{D2}\,(x_D, l, k, s)$$

$$\mathcal{F}_0^m = \frac{8\pi\cos(kz_{WD})\cosh R(h_{xD}-x_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}\{RD_p\sinh R(h_{xD})+D_p{}'\cosh(Rh_{xD})\}}[e^{R_1x_D}+e^{-R_1(x_D+2h_{xD})}]$$

$$\dot{\bar{P}}_{WD}\,(x_{WD}, y_{WD}, k, s)_{with\ skin} = \frac{4\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}\times$$

$$\left[\begin{array}{l}\dfrac{\cosh R(h_{xD}-x_{WD})}{RD_p\sinh R(h_{xD})+D_p{}'\cosh(Rh_{xD})}[e^{R_1x_D}+e^{-R_1(x_D+2h_{xD})}]\\ +\displaystyle\sum_{m=1}^{\infty}\frac{2\cos(ly_{WD})\cos(ly_D)\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD})+C_p{}'\cosh(Qh_{xD})}\ [e^{Q_1x_D}+e^{-Q_1(x_D+2h_{xD})}]\end{array}\right] \qquad (H-2-42)$$

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;=\; \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty} \mathcal{F}_n \cos(kz_D)$$

$$where\; \mathcal{F}_0^n \;=\; \dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)\Big|_{n=0} \;\; and\;\; \mathcal{F}_n \;=\; \dot{\overline{P}}_{D2}\,(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n =$$

$$\frac{\cosh\sqrt{sf_1(s)}(h_{xD} - x_{WD})}{\sqrt{sf_1(s)}E_p \sinh\sqrt{sf_1(s)}(h_{xD}) + E_p{}' \cosh\left(\sqrt{sf_1(s)}h_{xD}\right)}\left[e^{\sqrt{sf_2(s)}x_D} + e^{-\sqrt{sf_2(s)}(x_D + 2h_{xD})}\right] + \sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})}\left[e^{T_1 x_D} + e^{-T_1(x_D + 2h_{xD})}\right]$$

$$\overline{P}_{D2}\,(x_D, y_D, z_D, s) \;=\frac{4\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (H-2-43)$$

$$\mathcal{F}_{00} = \frac{\cosh\sqrt{sf_1(s)}(h_{xD} - x_{WD})}{2\left[\sqrt{sf_1(s)}E_p \sinh\sqrt{sf_1(s)}(h_{xD}) + E_p{}' \cosh\left(\sqrt{sf_1(s)}h_{xD}\right)\right]}\left[e^{\sqrt{sf_2(s)}x_D} + e^{-\sqrt{sf_2(s)}(x_D + 2h_{xD})}\right]$$

$$\mathcal{F}_{m0} = \sum_{m=1}^{\infty} \frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_{WD})}{TF_p \sinh T(h_{xD}) + F_p{}' \cosh(Th_{xD})}\left[e^{T_1 x_D} + e^{-T_1(x_D + 2h_{xD})}\right]$$

$$\mathcal{F}_{0n} = \sum_{n=1}^{\infty} \frac{\cos(kz_D)\cos(kz_{WD})\cosh R(h_{xD} - x_{WD})}{RD_p \sinh R(h_{xD}) + D_p{}' \cosh(Rh_{xD})}\left[e^{R_1 x_D} + e^{-R_1(x_D + 2h_{xD})}\right]$$

$$\mathcal{F}_{mn} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos(ly_{WD})\cos(ly_D)\cos(kz_D)\cos(kz_{WD})\cosh Q(h_{xD} - x_{WD})}{QC_p \sinh Q(h_{xD}) + C_p{}' \cosh(Qh_{xD})} \times \left[e^{Q_1 x_D} + e^{-Q_1(x_D + 2h_{xD})}\right]$$

APPENDIX I – Well Productivity Indices under Closed Boundary Conditions

The method used in obtaining the well performances below was adapted from Babu et al. (1988).

I –1 Clastic Reservoir

Steps (A) to (K) is same as in Appendix E-1

- **Horizontal Wells**

(L)

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{16\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{xD}\mathrm{h}_{yD}\mathrm{h}_{zD}} \times \left\{ \begin{array}{c} \frac{1}{2(l^2+k^2+s)}\left[1 - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})}\right] \\ +\sum_{p=1}^{\infty}\frac{\cos(\omega x_D)}{[\omega^2+(l^2+k^2+s)]}\left[\cos(\omega x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})}\right] \end{array} \right\} \quad (\mathrm{I}-1-40)$$

$$\ddot{\bar{P}}_{D1}(x_D, l, k, s) = \frac{8\pi\cos(kz_{WD})\cos(ly_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \times \frac{\cosh Q(h_{xD}-x_D)}{Q\sinh Q(h_{xD})}\left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh Q(h_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \quad (\mathrm{I}-1-41)$$

(M) Inverse Fourier Cosine Transform w.r.t. (l)

$$\dot{\bar{P}}_D(x_D, y_D, k, s) = \frac{\mathcal{F}_0^m}{2} + \sum_{m=1}^{\infty}\mathcal{F}_m\cos(ly_D)$$

$$where\ \mathcal{F}_0^m = \ddot{\bar{P}}_{D1}(x_D, l, k, s)\Big|_{m=0}\ \ and\ \ \mathcal{F}_m = \ddot{\bar{P}}_{D1}(x_D, l, k, s)$$

$$\mathcal{F}_0^m = \frac{8\pi\cos(kz_{WD})}{s\mathrm{h}_{yD}\mathrm{h}_{zD}} \times \frac{\cosh R(h_{xD}-x_D)}{R\sinh R(h_{xD})}\left\{\cosh R(x_{WD}) - \frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh R(h_{xD}) + D_p{}'\cosh(Rh_{xD})}\right\}$$

$$\dot{\overline{P}}_D(x_D, y_D, k, s) = \frac{8\pi\cos(kz_{WD})}{sh_{yD}h_{zD}} \times$$

$$\left[\begin{array}{l} \frac{\cosh R(h_{xD}-x_D)}{2R\sinh R(h_{xD})}\left\{\cosh R(x_{WD}) - \frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD}) + D_p{}'\cosh(Rh_{xD})}\right\} \\ + \sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)\cosh Q(h_{xD}-x_D)}{Q\sinh(Qh_{xD})}\left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \end{array}\right]$$

(I – 1 – 42)

(N) Inverse Fourier Cosine Transform w.r.t. (k)

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{\mathcal{F}_0^n}{2} + \sum_{n=1}^{\infty}\mathcal{F}_n\cos(kz_D)$$

$$where\ \mathcal{F}_0^n = \dot{\overline{P}}_D(x_D, y_D, k, s)\Big|_{n=0} \quad and\ \mathcal{F}_n = \dot{\overline{P}}_D(x_D, y_D, k, s)$$

$$\mathcal{F}_0^n =$$

$$\left[\begin{array}{l} \frac{\cosh\sqrt{s}(h_{xD}-x_D)}{2\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{\cosh\sqrt{s}(x_{WD}) - \frac{E_p{}'\cosh\sqrt{s}(h_{xD}-x_{WD})}{\sqrt{s}E_p\sinh\left(\sqrt{s}h_{xD}\right) + E_p{}'\cosh\left(\sqrt{s}h_{xD}\right)}\right\} \\ + \sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD}-x_D)}{T\sinh T(h_{xD})}\left\{\cosh(Tx_{WD}) - \frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD}) + F_p{}'\cosh(Th_{xD})}\right\} \end{array}\right]$$

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{8\pi}{sh_{yD}h_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad \text{(I – 1 – 43)}$$

$$\mathcal{F}_{00} = \left[\frac{\cosh\sqrt{s}(h_{xD}-x_D)}{4\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{\cosh\sqrt{s}(x_{WD}) - \frac{E_p{}'\cosh\sqrt{s}(h_{xD}-x_{WD})}{\sqrt{s}E_p\sinh\left(\sqrt{s}h_{xD}\right) + E_p{}'\cosh\left(\sqrt{s}h_{xD}\right)}\right\}\right]$$

$$\mathcal{F}_{m0} =$$

$$\left[\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD}-x_D)}{2T\sinh T(h_{xD})}\left\{\cosh(Tx_{WD}) - \frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD}) + F_p{}'\cosh(Th_{xD})}\right\}\right]$$

$\mathcal{F}_{0n} =$

$$\left[\sum_{n=1}^{\infty}\frac{\cos(kz_{WD})\cos(kz_D)\cosh R(h_{xD}-x_D)}{2R\sinh R(h_{xD})}\left\{\cosh R(x_{WD})-\frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD})+D_p{}'\cosh(Rh_{xD})}\right\}\right]$$

$\mathcal{F}_{mn} =$

$$\left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\cos(ly_{WD})\cos(ly_D)\cos(kz_{WD})\cos(kz_D)\cosh Q(h_{xD}-x_D)}{Q\sinh(Qh_{xD})}\left\{\cosh Q(x_{WD})-\frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD})+C_p{}'\cosh(Qh_{xD})}\right\}\right]$$

(O) Converting the point source solution to line source solution by integrating the RHS of (I-1-43) w.r.t. (L_{yD}) from Y_{WD} - $L_{yD}\sin\theta^{/}$ to Y_{WD} + $L_{yD}\sin\theta^{/}$

Converting the anisotropic system to an equivalent isotropic system

$$\overline{P}_D(x_D,y_D,z_D,s) = \frac{8\pi}{s h_{yD} h_{zD} \sin\theta}(\mathcal{F}_{00}+\mathcal{F}_{m0}+\mathcal{F}_{0n}+\mathcal{F}_{mn}) \qquad \text{(I – 1 – 44)}$$

$$\mathcal{F}_{00} = \left[\frac{L_{yD}\cosh\sqrt{s}(h_{xD}-x_D)}{2\sqrt{s}\,s\sinh\sqrt{s}(h_{xD})}\left\{\cosh\sqrt{s}(x_{WD})-\frac{E_p{}'\cosh\sqrt{s}(h_{xD}-x_{WD})}{\sqrt{s}E_p\sinh(\sqrt{s}h_{xD})+E_p{}'\cosh(\sqrt{s}h_{xD})}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty}\frac{\sin l(L_{yD})\cos^2(ly_{WD})\cosh T(h_{xD}-x_D)}{Tl\sinh T(h_{xD})}\left\{\cosh(Tx_{WD})-\frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD})+F_p{}'\cosh(Th_{xD})}\right\}\right]$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty}\frac{L_{yD}\cos(kz_{WD})\cos(kz_D)\cosh R(h_{xD}-x_D)}{R\sinh R(h_{xD})}\left\{\cosh R(x_{WD})-\frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD})+D_p{}'\cosh(Rh_{xD})}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})\cos(kz_{WD})\cos(kz_D)\cosh Q(h_{xD}-x_D)}{Ql\sinh(Qh_{xD})} \\ \left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \end{array} \right]$$

(P) Calculating $\bar{P}_{D_{avg}}$

$$\bar{P}_{D_{avg}} = \frac{\int \bar{P}_D(x_D, y_D, k, s)\,dV}{\int dV} \qquad \text{(I – 1 – 45)}$$

$$\bar{P}_{D_{avg}}(x_D, y_D, z_D, s) \;= \frac{8\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad \text{(I – 1 – 46)}$$

$$\mathcal{F}_{00} = \left[\frac{L_{yD}}{2s\mathrm{h}_{xD}}\left\{\cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD}-x_{WD})}{\sqrt{\mathrm{s}}E_p\sinh\left(\sqrt{\mathrm{s}}h_{xD}\right) + E_p{}'\cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c} \sum_{m=1}^{\infty}\frac{1}{T\mathrm{h}_{xD}}\frac{\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{Tl} \\ \left\{\cosh(Tx_{WD}) - \frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD}) + F_p{}'\cosh(Th_{xD})}\right\} \end{array} \right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c} \sum_{n=1}^{\infty}\frac{L_{yD}\sin(kh_{zD})}{Rk\mathrm{h}_{xD}\mathrm{h}_{zD}}\frac{\cos(kz_{WD})}{R} \\ \left\{\cosh R(x_{WD}) - \frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD}) + D_p{}'\cosh(Rh_{xD})}\right\} \end{array} \right] = 0$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\sin l\left(L_{yD}\right)\sin(kh_{zD})}{Qk\mathrm{h}_{xD}\mathrm{h}_{zD}}\frac{\cos^2(\,ly_{WD})\cos(kz_{WD})}{Ql} \\ \left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \end{array} \right] = 0$$

$$\bar{P}_{D\,avg}(x_D, y_D, z_D, s) \;= \frac{8\pi}{\mathrm{s}\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0}) \qquad (\mathrm{I-1-47})$$

$$\mathcal{F}_{00} = \left[\frac{L_{yD}}{2\mathrm{sh}_{xD}}\left\{\cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}}E_p \sinh\left(\sqrt{\mathrm{s}}h_{xD}\right) + E_p{}' \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c} \displaystyle\sum_{m=1}^{\infty} \frac{1}{T\mathrm{h}_{xD}} \frac{\sin l\left(L_{yD}\right) \cos^2(\,l y_{WD})}{Tl} \\ \left\{\cosh(Tx_{WD}) - \dfrac{F_p{}' \cosh T(h_{xD} - x_{WD})}{TF_p \sinh(Th_{xD}) + F_p{}' \cosh(Th_{xD})}\right\} \end{array}\right]$$

$$Recall\ that\ \bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{2\pi L\sqrt{k_{y_1}k_{z_1}}(\bar{P}_{avg} - \bar{P}_{wf}\,)}{\mu_1 q}$$

$$\bar{P}_D\ (x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{8\pi}{\mathrm{s}\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (\mathrm{I-1-48})$$

$$\mathcal{F}_{00} = \left[\frac{L_{yD}}{2\sqrt{\mathrm{s}}}\left\{\frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_D)}{\sinh\sqrt{\mathrm{s}}(h_{xD})} - \frac{1}{\sqrt{\mathrm{s}}\mathrm{h}_{xD}}\right\}\left\{\cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}}E_p \sinh\left(\sqrt{\mathrm{s}}h_{xD}\right) + E_p{}' \cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c} \displaystyle\sum_{m=1}^{\infty} \frac{\sin l\left(L_{yD}\right) \cos^2(\,l y_{WD})}{Tl}\left\{\frac{\cosh T(h_{xD} - x_D)}{\sinh T(h_{xD})} - \frac{1}{T\mathrm{h}_{xD}}\right\} \\ \left\{\cosh(Tx_{WD}) - \dfrac{F_p{}' \cosh T(h_{xD} - x_{WD})}{TF_p \sinh(Th_{xD}) + F_p{}' \cosh(Th_{xD})}\right\} \end{array}\right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty} \frac{L_{yD} \cos(kz_{WD}) \cos(kz_D) \cosh R(h_{xD} - x_D)}{R \sinh R(h_{xD})} \\ \left\{\cosh R(x_{WD}) - \dfrac{D_p{}' \cosh R(h_{xD} - x_{WD})}{RD_p \sinh(Rh_{xD}) + D_p{}' \cosh(Rh_{xD})}\right\} \end{array}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2 \sin l\left(L_{yD}\right) \cos^2(\,l y_{WD}) \cos(kz_{WD}) \cos(kz_D) \cosh Q(h_{xD} - x_D)}{Ql \sinh(Qh_{xD})} \\ \left\{\cosh Q(x_{WD}) - \dfrac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh(Qh_{xD}) + C_p{}' \cosh(Qh_{xD})}\right\} \end{array}\right]$$

$At\ long\ time, s \to 0$

$\mathcal{F}_{00} \to 0$

$$\mathcal{F}_{m0} \to \left[\sum_{m=1}^{\infty} \frac{\sin l(L_{yD}) \cos^2(\ ly_{WD})}{l^2} \left\{\frac{\cosh l(h_{xD} - x_D)}{\sinh l(h_{xD})} - \frac{1}{l\mathrm{h}_{xD}}\right\} \times \left\{\cosh(lx_{WD}) - \frac{G_p{}' \cosh l(h_{xD} - x_{WD})}{lG_p \sinh(lh_{xD}) + G_p{}' \cosh(lh_{xD})}\right\}\right]$$

$where\ G_p{}' = F_p{}'|_{s\to 0}\ and\ G_p = F_p|_{s\to 0}$

$$\mathcal{F}_{0n} \to \left[\sum_{n=1}^{\infty} \frac{L_{yD} \cos(kz_{WD}) \cos(kz_D) \cosh k(h_{xD} - x_D)}{k \sinh\mathrm{k}(h_{xD})} \left\{\cosh\mathrm{k}(x_{WD}) - \frac{B_p{}' \cosh k(h_{xD} - x_{WD})}{kB_p \sinh(kh_{xD}) + B_p{}' \cosh(kh_{xD})}\right\}\right]$$

$where\ B_p{}' = D_p{}'|_{s\to 0}\ and\ B_p = D_p|_{s\to 0}$

$$\mathcal{F}_{mn} \to \left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2 \sin l(L_{yD}) \cos^2(\ ly_{WD}) \cos(kz_{WD}) \cos(kz_D) \cosh U(h_{xD} - x_D)}{Ul \sinh(Uh_{xD})} \left\{\cosh U(x_{WD}) - \frac{A_p{}' \cosh U(h_{xD} - x_{WD})}{UA_p \sinh(Uh_{xD}) + A_p{}' \cosh(Uh_{xD})}\right\}\right]$$

$where\ \ U = Q|_{s\to 0}\ and\ A_p{}' = C_p{}'|_{s\to 0}\ and\ A_p = C_p|_{s\to 0}$

$After\ Laplace\ Inversion$

$$P_D(x_D, y_D, z_D, t \to \infty) - P_{D\,avg}(x_D, y_D, z_D, t \to \infty) = \frac{8\pi}{\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta} (\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$P_D - P_{D\,avg} = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}}(P_{avg} - P_{wf}\ )}{\mu_1 q}$$

$$Generally, Productivity\ Index(PI) = \frac{q}{P_{avg} - P_{wf}} = \frac{sin\theta \mathrm{h}_{yD} \mathrm{h}_{zD} L\sqrt{k_{y_1} k_{z_1}}}{4\mu_1(\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn} + S)} \qquad (\mathrm{I}-1-49)$$

$For\ PI\ at\ wellbore, set\ x_D = x_{WD}\ in\ (\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$

- **Hydraulically Fractured Vertical Wells**

Steps (A) to (N) is same for Horizontal Wells above

(O) Converting the point source solution to plane source solution by integrating the RHS of (I-1-43) w.r.t. (L_{fD} and L_{zD}) from Y_{WD} - $L_{fD}\sin\theta^{/}$ to Y_{WD} + $L_{fD}\sin\theta^{/}$ and Z_{WD} – L_{zD} to Z_{WD} + L_{zD} respectively

Converting the anisotropic system to an equivalent isotropic system

Taking into account the effect of partial penetration of the well in this system,

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{8\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD} sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{4\pi}{s\mathrm{h}_{yD} L_{zD} sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (\mathrm{I}-1-50)$$

$$\mathcal{F}_{00} = \left[\frac{L_{fD} L_{zD} \cosh\sqrt{s}(h_{xD} - x_D)}{\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{\cosh\sqrt{s}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s}E_p \sinh\left(\sqrt{s}h_{xD}\right) + E_p{}' \cosh\left(\sqrt{s}h_{xD}\right)}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{l}\displaystyle\sum_{m=1}^{\infty} \frac{2L_{zD} \sin l\left(L_{fD}\right) \cos^2(\,ly_{WD}) \cosh T(h_{xD} - x_D)}{Tl \sinh T(h_{xD})} \\ \left\{\cosh(Tx_{WD}) - \dfrac{F_p{}' \cosh T(h_{xD} - x_{WD})}{TF_p \sinh(Th_{xD}) + F_p{}' \cosh(Th_{xD})}\right\}\end{array}\right]$$

$$\mathcal{F}_{0n}=\left[\sum_{n=1}^{\infty}\frac{2L_{fD}\sin k(L_{zD})\cos^2(kz_{WD})\cosh R(h_{xD}-x_D)}{Rk\sinh R(h_{xD})}\left\{\cosh R(x_{WD})-\frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD})+D_p{}'\cosh(Rh_{xD})}\right\}\right]$$

$$\mathcal{F}_{mn}=\left[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4\sin l(L_{fD})\sin k(L_{zD})\cos^2(ly_{WD})\cos^2(kz_{WD})\cosh Q(h_{xD}-x_D)}{Qkl\sinh(Qh_{xD})}\left\{\cosh Q(x_{WD})-\frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD})+C_p{}'\cosh(Qh_{xD})}\right\}\right]$$

(P) Calculating $\bar{P}_{D\,avg}$

$$\bar{P}_{D\,avg}=\frac{\int\bar{P}_D(x_D,y_D,k,s)\,dV}{\int dV}\qquad(\mathrm{I}-1-51)$$

$$\bar{P}_{D\,avg}(x_D,y_D,z_D,s)=\frac{4\pi}{\mathrm{sh}_{yD}L_{zD}sin\theta}(\mathcal{F}_{00}+\mathcal{F}_{m0}+\mathcal{F}_{0n}+\mathcal{F}_{mn})\qquad(\mathrm{I}-1-52)$$

$$\mathcal{F}_{00}=\left[\frac{L_{fD}L_{zD}}{\mathrm{sh}_{xD}}\left\{\cosh\sqrt{\mathrm{s}}(x_{WD})-\frac{E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD}-x_{WD})}{\sqrt{\mathrm{s}}E_p\sinh(\sqrt{\mathrm{s}}h_{xD})+E_p{}'\cosh(\sqrt{\mathrm{s}}h_{xD})}\right\}\right]$$

$$\mathcal{F}_{m0}=\left[\sum_{m=1}^{\infty}\frac{1}{\mathrm{Th}_{xD}}\frac{2L_{zD}\sin l(L_{fD})\cos^2(ly_{WD})}{Tl}\left\{\cosh(Tx_{WD})-\frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD})+F_p{}'\cosh(Th_{xD})}\right\}\right]$$

$$\mathcal{F}_{0n}=\left[\sum_{n=1}^{\infty}\frac{1}{R\mathrm{h}_{xD}}\frac{2L_{fD}\sin k(L_{zD})\cos^2(kz_{WD})}{Rk}\left\{\cosh R(x_{WD})-\frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD})+D_p{}'\cosh(Rh_{xD})}\right\}\right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{Q\mathrm{h}_{xD}} \frac{4\sin l\left(L_{fD}\right)\sin k(L_{zD})\cos^2(\,ly_{WD})\cos^2(\,kz_{WD})}{Qkl} \\ \left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \end{array} \right]$$

$$Recall\ that\ \bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{2\pi L\sqrt{k_{y_1}k_{z_1}}(\bar{P}_{avg} - \bar{P}_{wf}\,)}{\mu_1 q}$$

$$\bar{P}_D\ (x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{4\pi}{\mathrm{sh}_{yD}L_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (\mathrm{I}-1-53)$$

$$\mathcal{F}_{00} = \left[\begin{array}{c} \frac{L_{fD}L_{zD}}{\sqrt{\mathrm{s}}}\left\{\frac{\cosh\sqrt{\mathrm{s}}(h_{xD}-x_D)}{\sinh\sqrt{\mathrm{s}}(h_{xD})} - \frac{1}{\sqrt{\mathrm{s}}\mathrm{h}_{xD}}\right\} \\ \left\{\cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD}-x_{WD})}{\sqrt{\mathrm{s}}E_p\sinh\left(\sqrt{\mathrm{s}}h_{xD}\right) + E_p{}'\cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)}\right\} \end{array} \right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c} \sum_{m=1}^{\infty} \frac{2L_{zD}\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})}{Tl}\left\{\frac{\cosh T(h_{xD}-x_D)}{\sinh T(h_{xD})} - \frac{1}{T\mathrm{h}_{xD}}\right\} \\ \left\{\cosh(Tx_{WD}) - \frac{F_p{}'\cosh T(h_{xD}-x_{WD})}{TF_p\sinh(Th_{xD}) + F_p{}'\cosh(Th_{xD})}\right\} \end{array} \right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{2L_{fD}\sin k(L_{zD})\cos^2(\,kz_{WD})}{Rk}\left\{\frac{\cosh R(h_{xD}-x_D)}{\sinh R(h_{xD})} - \frac{1}{R\mathrm{h}_{xD}}\right\} \\ \left\{\cosh R(x_{WD}) - \frac{D_p{}'\cosh R(h_{xD}-x_{WD})}{RD_p\sinh(Rh_{xD}) + D_p{}'\cosh(Rh_{xD})}\right\} \end{array} \right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4\sin l\left(L_{fD}\right)\sin k(L_{zD})\cos^2(\,ly_{WD})\cos^2(\,kz_{WD})}{Qkl}\left\{\frac{\cosh Q(h_{xD}-x_D)}{\sinh Q(h_{xD})} - \frac{1}{Q\mathrm{h}_{xD}}\right\} \\ \left\{\cosh Q(x_{WD}) - \frac{C_p{}'\cosh Q(h_{xD}-x_{WD})}{QC_p\sinh(Qh_{xD}) + C_p{}'\cosh(Qh_{xD})}\right\} \end{array} \right]$$

$At\ long\ time, s \to 0$

$$\bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) \to \frac{4\pi}{\mathrm{s}\mathrm{h}_{yD} L_{zD} sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\mathcal{F}_{00} \to 0$$

$$\mathcal{F}_{m0} \to \left[\begin{array}{c} \displaystyle\sum_{m=1}^{\infty} \frac{2L_{zD} \sin l\left(L_{fD}\right) \cos^2(\,l y_{WD})}{l^2} \left\{\frac{\cosh l(h_{xD} - x_D)}{\sinh l(h_{xD})} - \frac{1}{l\mathrm{h}_{xD}}\right\} \times \\ \left\{\cosh(l x_{WD}) - \frac{G_p{}' \cosh l(h_{xD} - x_{WD})}{lG_p \sinh(l h_{xD}) + G_p{}' \cosh(l h_{xD})}\right\} \end{array}\right]$$

$where\ G_p{}' = F_p{}'|_{s\to 0}\ and\ G_p = F_p|_{s\to 0}$

$$\mathcal{F}_{0n} \to \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty} \frac{2L_{fD} \sin k(L_{zD}) \cos^2(\,k z_{WD})}{k^2} \left\{\frac{\cosh \mathrm{k}(h_{xD} - x_D)}{\sinh k(h_{xD})} - \frac{1}{k\mathrm{h}_{xD}}\right\} \times \\ \left\{\cosh \mathrm{k}(x_{WD}) - \frac{B_p{}' \cosh k(h_{xD} - x_{WD})}{kB_p \sinh(k h_{xD}) + B_p{}' \cosh(k h_{xD})}\right\} \end{array}\right]$$

$where\ B_p{}' = D_p{}'|_{s\to 0}\ and\ B_p = D_p|_{s\to 0}$

$$\mathcal{F}_{mn} \to \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{4 \sin l\left(L_{fD}\right) \sin k(L_{zD}) \cos^2(\,l y_{WD}) \cos^2(\,k z_{WD})}{Ukl} \left\{\frac{\cosh \mathrm{U}(h_{xD} - x_D)}{\sinh(U h_{xD})} - \frac{1}{U\mathrm{h}_{xD}}\right\} \\ \left\{\cosh U(x_{WD}) - \frac{A_p{}' \cosh U(h_{xD} - x_{WD})}{UA_p \sinh(U h_{xD}) + A_p{}' \cosh(U h_{xD})}\right\} \end{array}\right]$$

$where\ \ U = Q|_{s\to 0}\ and\ A_p{}' = C_p{}'|_{s\to 0}\ and\ A_p = C_p|_{s\to 0}$

$After\ Laplace\ Inversion$

$$P_D(x_D, y_D, z_D, t \to \infty) - P_{D\,avg}(x_D, y_D, z_D, t \to \infty) = \frac{4\pi}{\mathrm{h}_{yD} L_{zD} sin\theta}(\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad \text{(I – 1 – 54)}$$

$$P_D - P_{D\,avg} = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}}(P_{avg} - P_{wf}\,)}{\mu_1 q}$$

$$Generally, Productivity\ Index(PI) = \frac{q}{P_{avg} - P_{wf}} = \frac{sin\theta \mathrm{h}_{yD} \mathrm{L}_{zD} L\sqrt{k_{y_1} k_{z_1}}}{2\mu_1(\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn} + S)} \qquad (\mathrm{I}-1-55)$$

$For\ PI\ at\ wellbore, set\ x_D = x_{WD}\ in\ (\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$

- Vertical Wells

Steps (A) to (N) is same for Horizontal Wells above

(O) Converting the point source solution to line source solution by integrating the RHS of (I-1-43) w.r.t. (L_{zD}) from Z_{wD}-L_{zD} to Z_{wD} + L_{zD}

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{\mathrm{h}_{zD}}{2L_{zD}} \times \frac{8\pi}{s\mathrm{h}_{yD}\mathrm{h}_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\overline{P}_D(x_D, y_D, z_D, s) = \frac{4\pi}{s\mathrm{h}_{yD}L_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (\mathrm{I}-1-56)$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}\cosh\sqrt{\mathrm{s}}(h_{xD} - x_D)}{2\sqrt{\mathrm{s}}\mathrm{s}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}}E_p\sinh(\sqrt{\mathrm{s}}h_{xD}) + E_p{}'\cosh(\sqrt{\mathrm{s}}h_{xD})}\right\}\right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c}\displaystyle\sum_{m=1}^{\infty}\frac{L_{zD}\cos(ly_{WD})\cos(ly_D)\cosh T(h_{xD} - x_D)}{T\sinh T(h_{xD})} \\ \left\{\cosh(Tx_{WD}) - \dfrac{F_p{}'\cosh T(h_{xD} - x_{WD})}{TF_p\sinh(Th_{xD}) + F_p{}'\cosh(Th_{xD})}\right\}\end{array}\right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c}\displaystyle\sum_{n=1}^{\infty}\frac{\sin k(L_{zD})\cos^2(kz_{WD})\cosh R(h_{xD} - x_D)}{Rk\sinh R(h_{xD})} \\ \left\{\cosh R(x_{WD}) - \dfrac{D_p{}'\cosh R(h_{xD} - x_{WD})}{RD_p\sinh(Rh_{xD}) + D_p{}'\cosh(Rh_{xD})}\right\}\end{array}\right]$$

$$\mathcal{F}_{mn} = \left[\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2 \sin k(L_{zD}) \cos(ly_{WD}) \cos(ly_D) \cos^2(kz_{WD}) \cosh Q(h_{xD} - x_D)}{Qk \sinh(Qh_{xD})} \left\{ \cosh Q(x_{WD}) - \frac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh(Qh_{xD}) + C_p{}' \cosh(Qh_{xD})} \right\} \right]$$

(P) Calculating $\bar{P}_{D_{avg}}$

$$\bar{P}_{D_{avg}} = \frac{\int \bar{P}_D(x_D, y_D, k, s)\, dV}{\int dV} \qquad \text{(I – 1 – 57)}$$

$$\bar{P}_{D_{avg}}(x_D, y_D, z_D, s) = \frac{4\pi}{s h_{yD} L_{zD}} (\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad \text{(I – 1 – 58)}$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}}{2 s h_{xD}} \left\{ \cosh\sqrt{s}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s} E_p \sinh(\sqrt{s} h_{xD}) + E_p{}' \cosh(\sqrt{s} h_{xD})} \right\} \right]$$

$$\mathcal{F}_{m0} = \left[\sum_{m=1}^{\infty} \frac{L_{zD}}{T h_{yD} h_{xD}} \frac{\sin l(h_{yD}) \cos^2(ly_{WD})}{Tl} \left\{ \cosh(Tx_{WD}) - \frac{F_p{}' \cosh T(h_{xD} - x_{WD})}{TF_p \sinh(Th_{xD}) + F_p{}' \cosh(Th_{xD})} \right\} \right] = 0$$

$$\mathcal{F}_{0n} = \left[\sum_{n=1}^{\infty} \frac{\sin k(L_{zD}) \cos^2(kz_{WD})}{R h_{xD}} \frac{1}{Rk} \left\{ \cosh R(x_{WD}) - \frac{D_p{}' \cosh R(h_{xD} - x_{WD})}{RD_p \sinh(Rh_{xD}) + D_p{}' \cosh(Rh_{xD})} \right\} \right]$$

$$\mathcal{F}_{mn} = \left[\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2 \sin k(L_{zD}) \sin(lh_{yD}) \cos^2(ly_{WD})}{Qk h_{yD} h_{xD}} \frac{\cos^2(kz_{WD})}{Ql} \left\{ \cosh Q(x_{WD}) - \frac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh(Qh_{xD}) + C_p{}' \cosh(Qh_{xD})} \right\} \right] = 0$$

$$\bar{P}_{D_{avg}}(x_D, y_D, z_D, s) = \frac{4\pi}{s h_{yD} L_{zD}} (\mathcal{F}_{00} + \mathcal{F}_{0n}) \qquad \text{(I – 1 – 59)}$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}}{2 s h_{xD}} \left\{ \cosh\sqrt{s}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{s}(h_{xD} - x_{WD})}{\sqrt{s} E_p \sinh(\sqrt{s} h_{xD}) + E_p{}' \cosh(\sqrt{s} h_{xD})} \right\} \right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{\sin k(L_{zD}) \cos^2(kz_{WD})}{R\mathrm{h}_{xD}} \frac{1}{Rk} \\ \left\{ \cosh R(x_{WD}) - \frac{D_p{}' \cosh R(h_{xD} - x_{WD})}{RD_p \sinh(Rh_{xD}) + D_p{}' \cosh(Rh_{xD})} \right\} \end{array} \right]$$

$$Recall\ that\ \bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}}(\bar{P}_{avg} - \bar{P}_{wf})}{\mu_1 q}$$

$$\bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) = \frac{4\pi}{s\mathrm{h}_{yD} L_{zD}} (\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (\mathrm{I} - 1 - 60)$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}}{2\sqrt{\mathrm{s}}} \left\{ \frac{\cosh\sqrt{\mathrm{s}}(h_{xD} - x_D)}{\sinh\sqrt{\mathrm{s}}(h_{xD})} - \frac{1}{\sqrt{\mathrm{s}}\mathrm{h}_{xD}} \right\} \left\{ \cosh\sqrt{\mathrm{s}}(x_{WD}) - \frac{E_p{}' \cosh\sqrt{\mathrm{s}}(h_{xD} - x_{WD})}{\sqrt{\mathrm{s}} E_p \sinh(\sqrt{s} h_{xD}) + E_p{}' \cosh(\sqrt{s} h_{xD})} \right\} \right]$$

$$\mathcal{F}_{m0} = \left[\begin{array}{c} \sum_{m=1}^{\infty} \frac{L_{zD} \cos(ly_{WD}) \cos(ly_D) \cosh T(h_{xD} - x_D)}{T \sinh T(h_{xD})} \\ \left\{ \cosh(Tx_{WD}) - \frac{F_p{}' \cosh T(h_{xD} - x_{WD})}{TF_p \sinh(Th_{xD}) + F_p{}' \cosh(Th_{xD})} \right\} \end{array} \right]$$

$$\mathcal{F}_{0n} = \left[\begin{array}{c} \sum_{n=1}^{\infty} \frac{\sin k(L_{zD}) \cos^2(kz_{WD})}{Rk} \left\{ \frac{\cosh R(h_{xD} - x_D)}{\sinh R(h_{xD})} - \frac{1}{R\mathrm{h}_{xD}} \right\} \\ \left\{ \cosh R(x_{WD}) - \frac{D_p{}' \cosh R(h_{xD} - x_{WD})}{RD_p \sinh(Rh_{xD}) + D_p{}' \cosh(Rh_{xD})} \right\} \end{array} \right]$$

$$\mathcal{F}_{mn} = \left[\begin{array}{c} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2 \sin k(L_{zD}) \cos(ly_{WD}) \cos(ly_D) \cos^2(kz_{WD}) \cosh Q(h_{xD} - x_D)}{Qk \sinh(Qh_{xD})} \\ \left\{ \cosh Q(x_{WD}) - \frac{C_p{}' \cosh Q(h_{xD} - x_{WD})}{QC_p \sinh(Qh_{xD}) + C_p{}' \cosh(Qh_{xD})} \right\} \end{array} \right]$$

$At\ long\ time, s \to 0$

$$\bar{P}_D(x_D, y_D, z_D, s) - \bar{P}_{D\,avg}(x_D, y_D, z_D, s) \to \frac{4\pi}{s\mathrm{h}_{yD} L_{zD}} (\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$$

$$\mathcal{F}_{00} \rightarrow 0$$

$$\mathcal{F}_{m0} \rightarrow \left[\sum_{m=1}^{\infty} \frac{L_{zD} \cos(ly_{WD}) \cos(ly_D) \cosh l(h_{xD} - x_D)}{l \sinh l(h_{xD})} \times \left\{ \cosh(lx_{WD}) - \frac{G_p{}' \cosh l(h_{xD} - x_{WD})}{lG_p \sinh(lh_{xD}) + G_p{}' \cosh(lh_{xD})} \right\} \right]$$

$$where\ G_p{}' = F_p{}'|_{s\rightarrow 0}\ and\ G_p = F_p|_{s\rightarrow 0}$$

$$\mathcal{F}_{0n} \rightarrow \left[\sum_{n=1}^{\infty} \frac{\sin k(L_{zD}) \cos^2(kz_{WD})}{k^2} \left\{ \frac{\cosh k(h_{xD} - x_D)}{\sinh k(h_{xD})} - \frac{1}{k\mathrm{h}_{xD}} \right\} \left\{ \cosh k(x_{WD}) - \frac{B_p{}' \cosh k(h_{xD} - x_{WD})}{kB_p \sinh(kh_{xD}) + B_p{}' \cosh(kh_{xD})} \right\} \right]$$

$$where\ B_p{}' = D_p{}'|_{s\rightarrow 0}\ and\ B_p = D_p|_{s\rightarrow 0}$$

$$\mathcal{F}_{mn} \rightarrow \left[\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{2 \sin k(L_{zD}) \cos(ly_{WD}) \cos(ly_D) \cos^2(kz_{WD}) \cosh U(h_{xD} - x_D)}{Uk \sinh(Uh_{xD})} \left\{ \cosh U(x_{WD}) - \frac{A_p{}' \cosh U(h_{xD} - x_{WD})}{UA_p \sinh(Uh_{xD}) + A_p{}' \cosh(Uh_{xD})} \right\} \right]$$

$$where\ \ U = Q|_{s\rightarrow 0}\ and\ A_p{}' = C_p{}'|_{s\rightarrow 0}\ and\ A_p = C_p|_{s\rightarrow 0}$$

$After\ Laplace\ Inversion$

$$P_D(x_D, y_D, z_D, t \rightarrow \infty) - P_{D_{avg}}(x_D, y_D, z_D, t \rightarrow \infty) = \frac{4\pi}{s\mathrm{h}_{yD} L_{zD}} (\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad \text{(I – 1 – 61)}$$

$$P_D - P_{D_{avg}} = \frac{2\pi L\sqrt{k_{y_1} k_{z_1}} (P_{avg} - P_{wf})}{\mu_1 q}$$

$$Generally, Productivity\ Index(PI) = \frac{q}{P_{avg} - P_{wf}} = \frac{h_{yD} L_{zD} L\sqrt{k_{y_1} k_{z_1}}}{2\mu_1(\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn} + S)} \qquad \text{(I – 1 – 62)}$$

$For\ PI\ at\ wellbore, set\ x_D = x_{WD}\ in\ (\mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn})$

I-2: Naturally Fractured Reservoirs

Everything is the same as section I-1 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

APPENDIX J – Pseudo-Skin Pressure of Wells

J –1: Clastic Reservoir

From the line and plane source Wellbore Pressure Equations already derived,

- Horizontal Wells

(i) Infinite in Two Directions

$At\ long\ time, s \to 0$

$$\overline{P}_{WD}\left(x_{WD}, y_{WD}, z_{WD}, s \to 0\right) = \frac{2}{\mathrm{s}h_{zD} sin\theta}(\alpha + \beta) \qquad (J-1-1)$$

$$\alpha = \begin{bmatrix} \displaystyle\int_{-\infty}^{\infty} \frac{\sin\left(\omega L_{yD}\right)}{2\omega^2}\left\{e^{-2\omega x_{WD}} \frac{\left(\omega D_p - D_p{}'\right)}{\left(\omega D_p + D_p{}'\right)}\right\} d\omega \\ \displaystyle+\sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{\sin\left(\omega L_{yD}\right)\cos^2(k z_{WD})}{\omega\sqrt{\omega^2 + k^2}}\left\{e^{-2x_{WD}\sqrt{\omega^2+k^2}} \frac{\left(C_p\sqrt{\omega^2 + k^2} - C_p{}'\right)}{\left(C_p\sqrt{\omega^2 + k^2} + C_p{}'\right)}\right\} d\omega \end{bmatrix}$$

$$\beta = \left[\sum_{n=1}^{\infty} \int_{-\infty}^{\infty} \frac{\sin\left(\omega L_{yD}\right)\cos^2(k z_{WD})}{\omega\sqrt{\omega^2 + k^2}} d\omega\right]$$

$$\bar{P}_{SD}\left(x_{WD}, y_{WD}, z_{WD}, s \to 0\right) = \frac{2\beta}{s h_{zD} sin\theta}$$

$$P_{SD}\left(x_{WD}, y_{WD}, z_{WD}\right) = \frac{2\beta}{h_{zD} sin\theta} \qquad (J-1-2)$$

(ii) Finite in Two Directions

$At\ long\ time, s \to 0$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{4\pi}{sh_{yD}h_{zD}sin\theta}(\alpha + \beta) \qquad (J-1-3)$$

$$\alpha = \left[\begin{array}{c} \frac{L_{yD}}{2\sqrt{\mathrm{s}}}\left\{1 + \frac{\left(\sqrt{\mathrm{s}}E_p - E_p{}'\right)}{\left(\sqrt{\mathrm{s}}E_p + E_p{}'\right)}\right\} + \sum_{m=1}^{\infty}\frac{\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l^2}e^{-2lx_{WD}}\frac{\left(lF_p - F_p{}'\right)}{\left(lF_p + F_p{}'\right)} \\ + \sum_{n=1}^{\infty}\frac{L_{yD}\cos^2(\,kz_{WD})}{k}e^{-2kx_{WD}}\frac{\left(kD_p - D_p{}'\right)}{\left(kD_p + D_p{}'\right)} \\ + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,kz_{WD})\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l\sqrt{l^2+k^2}}e^{-2x_{WD}\sqrt{l^2+k^2}}\frac{\left(C_p\sqrt{l^2+k^2} - C_p{}'\right)}{\left(C_p\sqrt{l^2+k^2} + C_p{}'\right)} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{m=1}^{\infty}\frac{\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l^2} + \sum_{n=1}^{\infty}\frac{L_{yD}\cos^2(\,kz_{WD})}{k} \\ + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,kz_{WD})\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l\sqrt{l^2+k^2}} \end{array}\right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{4\pi\beta}{sh_{yD}h_{zD}sin\theta}$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{4\pi\beta}{h_{yD}h_{zD}sin\theta} \qquad (J-1-4)$$

(iii) **Finite in Three Directions**

$At\ long\ time, s \to 0$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{4\pi}{sh_{yD}h_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (J-1-5)$$

$$\mathcal{F}_{00}=\left[\frac{L_{yD}}{2\sqrt{\mathrm{s}}\sinh\sqrt{\mathrm{s}}(h_{xD})}\left\{\begin{array}{c}\cosh\sqrt{\mathrm{s}}(h_{xD}-2x_{WD})+\cosh\left(\sqrt{\mathrm{s}}h_{xD}\right)\\ -\dfrac{E_p{}'\left[1+\cosh2\sqrt{\mathrm{s}}(h_{xD}-x_{WD})\right]}{\sqrt{\mathrm{s}}E_p\sinh\sqrt{\mathrm{s}}(h_{xD})+E_p{}'\cosh\sqrt{\mathrm{s}}(h_{xD})}\end{array}\right\}\right]$$

$$\mathcal{F}_{m01}=\left[\sum_{m=1}^{\infty}\frac{\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l^2\sinh l(h_{xD})}\{\cosh l(h_{xD}-2x_{WD})+\cosh(lh_{xD})\}\right]$$

$$\mathcal{F}_{m02}=\left[-\sum_{m=1}^{\infty}\frac{\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l^2\sinh l(h_{xD})}\frac{F_p{}'[1+\cosh2l(h_{xD}-x_{WD})]}{lF_p\sinh l(h_{xD})+F_p{}'\cosh l(h_{xD})}\right]$$

$$\mathcal{F}_{0n1}=\left[\sum_{n=1}^{\infty}\frac{L_{yD}\cos^2(\,kz_{WD})}{k\sinh k(h_{xD})}\{\cosh k(h_{xD}-2x_{WD})+\cosh(kh_{xD})\}\right]$$

$$\mathcal{F}_{0n2}=\left[-\sum_{n=1}^{\infty}\frac{L_{yD}\cos^2(\,kz_{WD})}{k\sinh k(h_{xD})}\frac{D_p{}'[1+\cosh2k(h_{xD}-x_{WD})]}{kD_p\sinh k(h_{xD})+D_p{}'\cosh k(h_{xD})]}\right]$$

$$\mathcal{F}_{mn1}=\left[\begin{array}{c}\displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,kz_{WD})\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right)}\times\\ \left\{\cosh(h_{xD}-2x_{WD})(\sqrt{l^2+k^2})+\cosh\left(h_{xD}\sqrt{l^2+k^2}\right)\right\}\end{array}\right]$$

$$\mathcal{F}_{mn2}=\left[\begin{array}{c}\displaystyle-\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,kz_{WD})\sin l\left(L_{yD}\right)\cos^2(\,ly_{WD})}{l\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right)}\times\\ \left\{\dfrac{C_p{}'\left[1+\cosh2(h_{xD}-x_{WD})(\sqrt{l^2+k^2})\right]}{C_p\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right)+C_p{}'\cosh\left(h_{xD}\sqrt{l^2+k^2}\right)}\right\}\end{array}\right]$$

$$\bar{P}_{SD}(x_{WD},y_{WD},z_{WD},s\to0)=\frac{4\pi}{sh_{yD}h_{zD}sin\theta}(\mathcal{F}_{m01}+\mathcal{F}_{0n1}+\mathcal{F}_{mn1})$$

$$P_{SD}(x_{WD},y_{WD},z_{WD})=\frac{4\pi}{h_{yD}h_{zD}sin\theta}(\mathcal{F}_{m01}+\mathcal{F}_{0n1}+\mathcal{F}_{mn1})\qquad(J-1-6)$$

- **Hydraulically Fractured Vertical Wells**

(i) **Infinite in Two Directions**

$At\ long\ time, s \rightarrow 0$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{2}{sL_{zD}sin\theta}(\alpha + \beta) \qquad (J-1-7)$$

$$\alpha = \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{L_{zD}\sin\left(\omega L_{fD}\right)}{2\omega^2}\left\{e^{-2\omega x_{WD}}\frac{\left(\omega D_p - D_p{'}\right)}{\left(\omega D_p + D_p{'}\right)}\right\}d\omega \\ \displaystyle +\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{\sin\left(\omega L_{fD}\right)\sin(kL_{zD})\cos^2(\,kz_{WD})}{\omega k\sqrt{\omega^2+k^2}}\left\{e^{-2x_{WD}\sqrt{\omega^2+k^2}}\frac{\left(C_p\sqrt{\omega^2+k^2}-C_p{'}\right)}{\left(C_p\sqrt{\omega^2+k^2}+C_p{'}\right)}\right\}d\omega \end{array}\right]$$

$$\beta = \left[\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{\sin\left(\omega L_{yD}\right)\sin(kL_{zD})\cos^2(\,kz_{WD})}{\omega k\sqrt{\omega^2+k^2}}d\omega\right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{2\beta}{sL_{zD}sin\theta}$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{2\beta}{L_{zD}sin\theta} \qquad (J-1-8)$$

(ii) **Finite in Two Directions**

$At\ long\ time, s \rightarrow 0$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{2\pi}{sh_{yD}L_{zD}sin\theta}(\alpha + \beta) \qquad (J-1-9)$$

$$
\alpha = \left[\begin{array}{c} \frac{L_{zD}L_{fD}}{\sqrt{s}}\left\{1+\frac{\left(\sqrt{s}E_p - E_p{'}\right)}{\left(\sqrt{s}E_p + E_p{'}\right)}\right\} + \sum_{m=1}^{\infty}\frac{2L_{zD}\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})}{l^2}e^{-2lx_{WD}}\frac{\left(lF_p - F_p{'}\right)}{\left(lF_p + F_p{'}\right)} \\ +\sum_{n=1}^{\infty}\frac{2L_{fD}\sin k(L_{zD})\cos^2(\,kz_{WD})}{k^2}e^{-2kx_{WD}}\frac{\left(kD_p - D_p{'}\right)}{\left(kD_p + D_p{'}\right)} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4\cos^2(\,kz_{WD})\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})}{kl\sqrt{l^2+k^2}}e^{-2x_{WD}\sqrt{l^2+k^2}}\frac{\left(C_p\sqrt{l^2+k^2} - C_p{'}\right)}{\left(C_p\sqrt{l^2+k^2} + C_p{'}\right)} \end{array} \right]
$$

$$
\beta = \left[\begin{array}{c} \sum_{m=1}^{\infty}\frac{2L_{zD}\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})}{l^2} + \sum_{n=1}^{\infty}\frac{2L_{fD}\sin k(L_{zD})\cos^2(\,kz_{WD})}{k^2} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4\cos^2(\,kz_{WD})\sin k(L_{zD})\sin l\left(L_{fD}\right)\cos^2(\,ly_{WD})}{kl\sqrt{l^2+k^2}} \end{array} \right]
$$

$$
\overline{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi\beta}{sh_{yD}L_{zD}sin\theta}
$$

$$
P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{2\pi\beta}{h_{yD}L_{zD}sin\theta} \qquad (J-1-10)
$$

(iii) **Finite in Three Directions**

$At\ long\ time, s \to 0$

$$
\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi}{sh_{yD}L_{zD}sin\theta}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (J-1-11)
$$

$$
\mathcal{F}_{00} = \left[\frac{L_{zD}L_{fD}}{\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{ \begin{array}{c} \cosh\sqrt{s}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{s}h_{xD}\right) \\ -\frac{E_p{'}\left[1+\cosh 2\sqrt{s}(h_{xD} - x_{WD})\right]}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{'}\cosh\sqrt{s}(h_{xD})} \end{array} \right\}\right]
$$

$$\mathcal{F}_{m01} = \left[\sum_{m=1}^{\infty} \frac{2L_{zD} \sin l\left(L_{fD}\right) \cos^2(\, l y_{WD})}{l^2 \sinh l(h_{xD})} \{\cosh l(h_{xD} - 2x_{WD}) + \cosh(l h_{xD})\} \right]$$

$$\mathcal{F}_{m02} = \left[-\sum_{m=1}^{\infty} \frac{2L_{zD} \sin l\left(L_{fD}\right) \cos^2(\, l y_{WD})}{l^2 \sinh l(h_{xD})} \frac{F_p{}^{'}[1 + \cosh 2l(h_{xD} - x_{WD})]}{l F_p \sinh l(h_{xD}) + F_p{}^{'} \cosh l(h_{xD})} \right]$$

$$\mathcal{F}_{0n1} = \left[\sum_{n=1}^{\infty} \frac{2L_{fD} \sin k(L_{zD}) \cos^2(\, k z_{WD})}{k^2 \sinh k(h_{xD})} \{\cosh k(h_{xD} - 2x_{WD}) + \cosh(k h_{xD})\} \right]$$

$$\mathcal{F}_{0n2} = \left[-\sum_{n=1}^{\infty} \frac{2L_{fD} \sin k(L_{zD}) \cos^2(\, k z_{WD})}{k^2 \sinh k(h_{xD})} \frac{D_p{}^{'}[1 + \cosh 2k(h_{xD} - x_{WD})]}{k D_p \sinh k(h_{xD}) + D_p{}^{'} \cosh k(h_{xD})]} \right]$$

$$\mathcal{F}_{mn1} = \left[\begin{array}{l} \displaystyle\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4 \cos^2(\, k z_{WD}) \sin k(L_{zD}) \sin l\left(L_{yD}\right) \cos^2(\, l y_{WD})}{k l \sqrt{l^2 + k^2} \sinh\left(h_{xD} \sqrt{l^2 + k^2}\right)} \times \\ \quad \left\{ \cosh(h_{xD} - 2x_{WD})(\sqrt{l^2 + k^2}) + \cosh\left(h_{xD} \sqrt{l^2 + k^2}\right) \right\} \end{array} \right]$$

$$\mathcal{F}_{mn2} = \left[\begin{array}{l} \displaystyle -\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{4 \cos^2(\, k z_{WD}) \sin k(L_{zD}) \sin l\left(L_{yD}\right) \cos^2(\, l y_{WD})}{k l \sqrt{l^2 + k^2} \sinh\left(h_{xD} \sqrt{l^2 + k^2}\right)} \times \\ \displaystyle \left\{ \frac{C_p{}^{'} \left[1 + \cosh 2(h_{xD} - x_{WD})(\sqrt{l^2 + k^2})\right]}{C_p \sqrt{l^2 + k^2} \sinh\left(h_{xD} \sqrt{l^2 + k^2}\right) + C_p{}^{'} \cosh\left(h_{xD} \sqrt{l^2 + k^2}\right)} \right\} \end{array} \right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi}{s h_{yD} L_{zD} sin\theta} (\mathcal{F}_{m01} + \mathcal{F}_{0n1} + \mathcal{F}_{mn1})$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{2\pi}{h_{yD} L_{zD} sin\theta} (\mathcal{F}_{m01} + \mathcal{F}_{0n1} + \mathcal{F}_{mn1}) \qquad (J - 1 - 12)$$

- **Vertical Wells**

(i) **Infinite in Two Directions**

$$At\ long\ time, s \rightarrow 0$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{1}{sL_{zD}}(\alpha + \beta) \qquad (J-1-13)$$

$$\alpha = \left[\begin{array}{c} \displaystyle\int_{-\infty}^{\infty} \frac{L_{zD}}{2\omega}\left\{e^{-2\omega x_{WD}}\,\frac{\left(\omega D_p - D_p{'}\right)}{\left(\omega D_p + D_p{'}\right)}\right\}d\omega \\ \displaystyle+\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{\sin(kL_{zD})\cos^2(kz_{WD})}{k\sqrt{\omega^2+k^2}}\left\{e^{-2x_{WD}\sqrt{\omega^2+k^2}}\,\frac{\left(C_p\sqrt{\omega^2+k^2} - C_p{'}\right)}{\left(C_p\sqrt{\omega^2+k^2} + C_p{'}\right)}\right\}d\omega \end{array}\right]$$

$$\beta = \left[\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}\frac{\sin(kL_{zD})\cos^2(kz_{WD})}{k\sqrt{\omega^2+k^2}}\,d\omega\right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{\beta}{sL_{zD}}$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{\beta}{L_{zD}} \qquad (J-1-14)$$

(ii) **Finite in Two Directions**

$$At\ long\ time, s \rightarrow 0$$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \rightarrow 0) = \frac{2\pi}{sh_{yD}L_{zD}}(\alpha + \beta) \qquad (J-1-15)$$

$$\alpha = \left[\begin{array}{c} \frac{L_{zD}}{2\sqrt{s}}\left\{1+\frac{\left(\sqrt{s}E_p - E_p{}'\right)}{\left(\sqrt{s}E_p + E_p{}'\right)}\right\} + \sum_{m=1}^{\infty}\frac{L_{zD}\cos^2(\,l y_{WD})}{l}e^{-2l x_{WD}}\frac{\left(lF_p - F_p{}'\right)}{\left(lF_p + F_p{}'\right)} \\ +\sum_{n=1}^{\infty}\frac{\sin k(L_{zD})\cos^2(\,k z_{WD})}{k^2}e^{-2k x_{WD}}\frac{\left(kD_p - D_p{}'\right)}{\left(kD_p + D_p{}'\right)} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,k z_{WD})\sin k(L_{zD})\cos^2(\,l y_{WD})}{k\sqrt{l^2+k^2}}e^{-2x_{WD}\sqrt{l^2+k^2}}\frac{\left(C_p\sqrt{l^2+k^2} - C_p{}'\right)}{\left(C_p\sqrt{l^2+k^2} + C_p{}'\right)} \end{array}\right]$$

$$\beta = \left[\begin{array}{c} \sum_{m=1}^{\infty}\frac{L_{zD}\cos^2(\,l y_{WD})}{l} + \sum_{n=1}^{\infty}\frac{\sin k(L_{zD})\cos^2(\,k z_{WD})}{k^2} \\ +\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{2\cos^2(\,k z_{WD})\sin k(L_{zD})\cos^2(\,l y_{WD})}{k\sqrt{l^2+k^2}} \end{array}\right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi\beta}{s h_{yD} L_{zD}}$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{2\pi\beta}{h_{yD} L_{zD}} \qquad (J-1-16)$$

(iii) **Finite in Three Directions**

$At\ long\ time, s \to 0$

$$\overline{P}_{WD}\,(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi}{s h_{yD} L_{zD}}(\mathcal{F}_{00} + \mathcal{F}_{m0} + \mathcal{F}_{0n} + \mathcal{F}_{mn}) \qquad (J-1-17)$$

$$\mathcal{F}_{00} = \left[\frac{L_{zD}}{2\sqrt{s}\sinh\sqrt{s}(h_{xD})}\left\{\begin{array}{c} \cosh\sqrt{s}(h_{xD} - 2x_{WD}) + \cosh\left(\sqrt{s}h_{xD}\right) \\ -\frac{E_p{}'\left[1+\cosh 2\sqrt{s}(h_{xD} - x_{WD})\right]}{\sqrt{s}E_p\sinh\sqrt{s}(h_{xD}) + E_p{}'\cosh\sqrt{s}(h_{xD})} \end{array}\right\}\right]$$

$$\mathcal{F}_{m01} = \left[\sum_{m=1}^{\infty} \frac{L_{zD}\cos^2(\,l y_{WD})}{l\sinh l(h_{xD})}\{\cosh l(h_{xD} - 2x_{WD}) + \cosh(l h_{xD})\}\right]$$

$$\mathcal{F}_{m02} = \left[-\sum_{m=1}^{\infty} \frac{L_{zD}\cos^2(\,l y_{WD})}{l\sinh l(h_{xD})} \frac{F_p{}'[1 + \cosh 2l(h_{xD} - x_{WD})]}{lF_p \sinh l(h_{xD}) + F_p{}' \cosh l(h_{xD})}\right]$$

$$\mathcal{F}_{0n1} = \left[\sum_{n=1}^{\infty} \frac{\sin k(L_{zD})\cos^2(\,k z_{WD})}{k^2 \sinh k(h_{xD})}\{\cosh k(h_{xD} - 2x_{WD}) + \cosh(k h_{xD})\}\right]$$

$$\mathcal{F}_{0n2} = \left[-\sum_{n=1}^{\infty} \frac{\sin k(L_{zD})\cos^2(\,k z_{WD})}{k^2 \sinh k(h_{xD})} \frac{D_p{}'[1 + \cosh 2k(h_{xD} - x_{WD})]}{kD_p \sinh k(h_{xD}) + D_p{}' \cosh k(h_{xD})]}\right]$$

$$\mathcal{F}_{mn1} = \left[\begin{array}{c} \displaystyle\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos^2(\,k z_{WD})\sin k(L_{zD})\cos^2(\,l y_{WD})}{k\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right)} \times \\ \left\{\cosh(h_{xD} - 2x_{WD})(\sqrt{l^2+k^2}) + \cosh\left(h_{xD}\sqrt{l^2+k^2}\right)\right\} \end{array}\right]$$

$$\mathcal{F}_{mn2} = \left[\begin{array}{c} \displaystyle -\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{2\cos^2(\,k z_{WD})\sin k(L_{zD})\cos^2(\,l y_{WD})}{k\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right)} \times \\ \left\{\dfrac{C_p{}'\left[1 + \cosh 2(h_{xD} - x_{WD})(\sqrt{l^2+k^2})\right]}{C_p\sqrt{l^2+k^2}\sinh\left(h_{xD}\sqrt{l^2+k^2}\right) + C_p{}'\cosh\left(h_{xD}\sqrt{l^2+k^2}\right)}\right\} \end{array}\right]$$

$$\bar{P}_{SD}(x_{WD}, y_{WD}, z_{WD}, s \to 0) = \frac{2\pi}{s h_{yD} L_{zD}}(\mathcal{F}_{m01} + \mathcal{F}_{0n1} + \mathcal{F}_{mn1})$$

$$P_{SD}(x_{WD}, y_{WD}, z_{WD}) = \frac{2\pi}{h_{yD} L_{zD}}(\mathcal{F}_{m01} + \mathcal{F}_{0n1} + \mathcal{F}_{mn1}) \qquad (J-1-18)$$

J –2 Naturally Fractured Reservoirs

Everything is the same as section J-1 except replacing s with $sf_1(s)$ and $\eta_D s$ with $sf_2(s)$

APPENDIX K – Data for Type Curve Generation

The rock and fluid properties used to generate the type curves were adapted from Yildiz et al. (1997) and are given below.

q, 3000 STB/D

B_o , 1.5 RBBL/STB

μ, cp 1.5

φ, fraction 0.1

C_t , 30x10-6 psi^{-1}

h, 60 ft

r_w, 0.354 ft

L, 1000 ft

	Low Areal Anisotropy	High Areal Anisotropy
kx, md	100	100
ky, md	50	1
kz, md	10	1

APPENDIX L – Simulation Data

The following data was fed into the Eclipse 2010.1 for pressure drawdown

q, 5000 STB/D

B_o, 1.5 RBBL/STB

μ, cp 1.0

φ, fraction 0.2

C_t , 2.8x10-6 psi^{-1}

h, 100 ft

r_w, 0.3 ft

L, 385 ft

Θ, 45°

k_{x1},md 404.5 k_{y1},md 404.5 k_{z1},md 40.45

k_{x2},md 404.5 k_{y2},md 404.5 k_{z2},md 40.45

Table L.1: Eclipse Pressure data for Fig. 3.6.1

t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi
0.00018	34.73958	0.0002	34.63974	0.00022	34.48487	0.00024	34.33185	0.00026	34.15049
0.000181	34.73358	0.000201	34.62167	0.000221	34.47562	0.000241	34.3302	0.000261	34.15186
0.000182	34.72517	0.000202	34.60234	0.000222	34.47634	0.000242	34.30325	0.000262	34.12557
0.000183	34.71477	0.000203	34.61124	0.000223	34.46422	0.000243	34.28693	0.000263	34.12401
0.000184	34.72972	0.000204	34.60829	0.000224	34.45111	0.000244	34.2818	0.000264	34.12139
0.000185	34.71477	0.000205	34.58314	0.000225	34.44711	0.000245	34.27512	0.000265	34.09225
0.000186	34.68867	0.000206	34.60794	0.000226	34.43096	0.000246	34.25542	0.000266	34.10125
0.000187	34.71619	0.000207	34.59953	0.000227	34.43561	0.000247	34.25896	0.000267	34.09551
0.000188	34.69481	0.000208	34.54836	0.000228	34.42781	0.000248	34.27322	0.000268	34.08931
0.000189	34.68136	0.000209	34.56843	0.000229	34.41824	0.000249	34.24952	0.000269	34.08225
0.00019	34.70311	0.00021	34.55508	0.00023	34.39626	0.00025	34.23751	0.00027	34.0334
0.000191	34.68526	0.000211	34.56173	0.000231	34.39571	0.000251	34.22376	0.000271	34.03741
0.000192	34.68451	0.000212	34.55553	0.000232	34.40541	0.000252	34.2085	0.000272	34.06766
0.000193	34.6821	0.000213	34.52713	0.000233	34.39051	0.000253	34.21792	0.000273	34.02935
0.000194	34.66745	0.000214	34.52892	0.000234	34.36266	0.000254	34.20118	0.000274	34.01719
0.000195	34.66095	0.000215	34.51802	0.000235	34.3332	0.000255	34.18276	0.000275	34.04453
0.000196	34.6426	0.000216	34.51652	0.000236	34.3379	0.000256	34.1883	0.000276	34.03041
0.000197	34.62245	0.000217	34.51362	0.000237	34.37705	0.000257	34.19386	0.000277	34.01582
0.000198	34.6399	0.000218	34.50912	0.000238	34.343	0.000258	34.17218	0.000278	33.99934
0.000199	34.64555	0.000219	34.48122	0.000239	34.3083	0.000259	34.14881	0.000279	33.98173

t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi	t, hr	t ΔP/, psi
0.00028	33.97799	0.00032	33.68017	0.00036	33.40758	0.0004	33.1601	0.00044	32.97791
0.000281	33.95901	0.000321	33.68842	0.000361	33.35668	0.000401	33.1422	0.000441	33.00889
0.000282	33.9808	0.000322	33.64902	0.000362	33.41281	0.000402	33.16487	0.000442	32.97329
0.000283	33.95933	0.000323	33.64069	0.000363	33.41361	0.000403	33.14687	0.000443	32.98116
0.000284	33.93814	0.000324	33.64713	0.000364	33.36033	0.000404	33.14845	0.000444	32.94446
0.000285	33.95791	0.000325	33.63702	0.000365	33.36128	0.000405	33.169	0.000445	32.93019
0.000286	33.93355	0.000326	33.62697	0.000366	33.37931	0.000406	33.14972	0.000446	32.98184
0.000287	33.90853	0.000327	33.61579	0.000367	33.34141	0.000407	33.15037	0.000447	32.9434
0.000288	33.89764	0.000328	33.60333	0.000368	33.32256	0.000408	33.10925	0.000448	32.92786
0.000289	33.89974	0.000329	33.60692	0.000369	33.37637	0.000409	33.1082	0.000449	32.95686
0.00029	33.9007	0.00033	33.59402	0.00037	33.33657	0.00041	33.14865	0.00045	32.96264
0.000291	33.8866	0.000331	33.58002	0.000371	33.33411	0.000411	33.10616	0.000451	32.9677
0.000292	33.90134	0.000332	33.59818	0.000372	33.33111	0.000412	33.08347	0.000452	32.95089
0.000293	33.87064	0.000333	33.61608	0.000373	33.3278	0.000413	33.1222	0.000453	32.91066
0.000294	33.83903	0.000334	33.58367	0.000374	33.32297	0.000414	33.09935	0.000454	32.91486
0.000295	33.8953	0.000335	33.53352	0.000375	33.28107	0.000415	33.0756	0.000455	32.9188
0.000296	33.8624	0.000336	33.56628	0.000376	33.27601	0.000416	33.11347	0.000456	32.94624
0.000297	33.82843	0.000337	33.56478	0.000377	33.3082	0.000417	33.08857	0.000457	32.92696
0.000298	33.86727	0.000338	33.52972	0.000378	33.32032	0.000418	33.064	0.000458	32.88426
0.000299	33.83139	0.000339	33.52712	0.000379	33.27597	0.000419	33.1011	0.000459	32.9329
0.0003	33.79525	0.00034	33.54066	0.00038	33.26906	0.00042	33.07472	0.00046	32.88999
0.000301	33.80233	0.000341	33.55408	0.000381	33.2996	0.000421	33.02717	0.000461	32.86949
0.000302	33.80847	0.000342	33.53333	0.000382	33.29076	0.000422	33.06395	0.000462	32.94046
0.000303	33.79963	0.000343	33.49392	0.000383	33.22497	0.000423	33.0574	0.000463	32.89576
0.000304	33.8051	0.000344	33.50531	0.000384	33.23552	0.000424	33.0293	0.000464	32.85139
0.000305	33.80889	0.000345	33.51671	0.000385	33.2834	0.000425	33.10737	0.000465	32.92219
0.000306	33.76657	0.000346	33.49308	0.000386	33.25335	0.000426	33.07901	0.000466	32.87575
0.000307	33.77013	0.000347	33.46778	0.000387	33.2047	0.000427	32.9858	0.000467	32.90016
0.000308	33.77218	0.000348	33.47731	0.000388	33.25197	0.000428	32.9985	0.000468	32.90045
0.000309	33.75799	0.000349	33.48651	0.000389	33.23967	0.000429	33.0546	0.000469	32.85369
0.00031	33.74314	0.00035	33.44256	0.00039	33.2277	0.00043	33.02416	0.00047	32.87605
0.000311	33.71243	0.000351	33.45008	0.000391	33.2154	0.000431	33.03606	0.000471	32.82846
0.000312	33.71167	0.000352	33.43968	0.000392	33.22242	0.000432	33.06915	0.000472	32.85155
0.000313	33.74119	0.000353	33.44691	0.000393	33.20807	0.000433	32.97289	0.000473	32.92099
0.000314	33.72319	0.000354	33.43526	0.000394	33.1745	0.000434	33.00621	0.000474	32.87135
0.000315	33.72093	0.000355	33.40528	0.000395	33.2196	0.000435	33.03786	0.000475	32.82269
0.000316	33.71713	0.000356	33.42818	0.000396	33.20465	0.000436	33.0049	0.000476	32.89195
0.000317	33.66492	0.000357	33.43311	0.000397	33.18897	0.000437	33.01539	0.000477	32.84106
0.000318	33.69194	0.000358	33.41936	0.000398	33.19297	0.000438	32.98149	0.000478	32.8384
0.000319	33.68649	0.000359	33.44051	0.000399	33.1769	0.000439	32.94666	0.000479	32.85939

t, hr	t $\Delta P^{/}$, psi	t, hr	t $\Delta P^{/}$, psi	t, hr	t $\Delta P^{/}$, psi	t, hr	t $\Delta P^{/}$, psi	t, hr	t $\Delta P^{/}$, psi
0.00048	32.80814	0.00052	32.73374	0.00056	32.6759	0.0006	32.58057	0.00064	32.57575
0.000481	32.80396	0.000521	32.77049	0.000561	32.67877	0.000601	32.57486	0.000641	32.56249
0.000482	32.84821	0.000522	32.75531	0.000562	32.65287	0.000602	32.62825	0.000642	32.51684
0.000483	32.84399	0.000523	32.74054	0.000563	32.62539	0.000603	32.59185	0.000643	32.47135
0.000484	32.83964	0.000524	32.69817	0.000564	32.6273	0.000604	32.58554	0.000644	32.5541
0.000485	32.80986	0.000525	32.68089	0.000565	32.62845	0.000605	32.57879	0.000645	32.60537
0.000486	32.80496	0.000526	32.74324	0.000566	32.60129	0.000606	32.54205	0.000646	32.55897
0.000487	32.79939	0.000527	32.75276	0.000567	32.65889	0.000607	32.59673	0.000647	32.54379
0.000488	32.81833	0.000528	32.68296	0.000568	32.65959	0.000608	32.58937	0.000648	32.59425
0.000489	32.83579	0.000529	32.66539	0.000569	32.57593	0.000609	32.58119	0.000649	32.54705
0.00049	32.78086	0.00053	32.72724	0.00057	32.63347	0.00061	32.60414	0.00065	32.53224
0.000491	32.79856	0.000531	32.73667	0.000571	32.68929	0.000611	32.59665	0.000651	32.51699
0.000492	32.79145	0.000532	32.66554	0.000572	32.60359	0.000612	32.5888	0.000652	32.53475
0.000493	32.78508	0.000533	32.67275	0.000573	32.60339	0.000613	32.54989	0.000653	32.58543
0.000494	32.80228	0.000534	32.73389	0.000574	32.66045	0.000614	32.60319	0.000654	32.50436
0.000495	32.81826	0.000535	32.71479	0.000575	32.63095	0.000615	32.56466	0.000655	32.55329
0.000496	32.83506	0.000536	32.64216	0.000576	32.63014	0.000616	32.55657	0.000656	32.60279
0.000497	32.82639	0.000537	32.64946	0.000577	32.62987	0.000617	32.60815	0.000657	32.55415
0.000498	32.74305	0.000538	32.71108	0.000578	32.59987	0.000618	32.50639	0.000658	32.50495
0.000499	32.75926	0.000539	32.69087	0.000579	32.65545	0.000619	32.59019	0.000659	32.52139
0.0005	32.80074	0.00054	32.69674	0.00058	32.6247	0.00062	32.61165	0.00066	32.57064
0.000501	32.79108	0.000541	32.70315	0.000581	32.53559	0.000621	32.57135	0.000661	32.58797
0.000502	32.78019	0.000542	32.68245	0.000582	32.62094	0.000622	32.59239	0.000662	32.57113
0.000503	32.71969	0.000543	32.66099	0.000583	32.67685	0.000623	32.55237	0.000663	32.51995
0.000504	32.70946	0.000544	32.66694	0.000584	32.58793	0.000624	32.51097	0.000664	32.50244
0.000505	32.79966	0.000545	32.67246	0.000585	32.55587	0.000625	32.59375	0.000665	32.55149
0.000506	32.81364	0.000546	32.65164	0.000586	32.61064	0.000626	32.6456	0.000666	32.6005
0.000507	32.7264	0.000547	32.71127	0.000587	32.60739	0.000627	32.50949	0.000667	32.48255
0.000508	32.74133	0.000548	32.71519	0.000588	32.57505	0.000628	32.49874	0.000668	32.53247
0.000509	32.75484	0.000549	32.63759	0.000589	32.63045	0.000629	32.58205	0.000669	32.61427
0.00051	32.74186	0.00055	32.64241	0.00059	32.62659	0.00063	32.60328	0.00067	32.52814
0.000511	32.75469	0.000551	32.70165	0.000591	32.62289	0.000631	32.56017	0.000671	32.50975
0.000512	32.74204	0.000552	32.67799	0.000592	32.59012	0.000632	32.54769	0.000672	32.491
0.000513	32.75486	0.000553	32.59889	0.000593	32.58623	0.000633	32.53579	0.000673	32.53929
0.000514	32.76731	0.000554	32.65913	0.000594	32.6401	0.000634	32.58755	0.000674	32.58754
0.000515	32.75458	0.000555	32.71794	0.000595	32.60559	0.000635	32.54335	0.000675	32.53459
0.000516	32.74082	0.000556	32.63695	0.000596	32.60089	0.000636	32.53124	0.000676	32.51643
0.000517	32.75159	0.000557	32.63989	0.000597	32.56605	0.000637	32.58209	0.000677	32.56452
0.000518	32.71141	0.000558	32.64259	0.000598	32.561	0.000638	32.57062	0.000678	32.50964
0.000519	32.67086	0.000559	32.61745	0.000599	32.61519	0.000639	32.62173	0.000679	32.48989

t, hr	$t\,\Delta P'$, psi	t, hr	$t\,\Delta P'$, psi	t, hr	$t\,\Delta P'$, psi	t, hr	$t\,\Delta P'$, psi	t, hr	$t\,\Delta P'$, psi
0.00068	32.5378	0.00072	32.50764	0.00076	32.56698	0.0008	32.51953	0.00084	32.59184
0.000681	32.58575	0.000721	32.55289	0.000761	32.60937	0.000801	32.56048	0.000841	32.63048
0.000682	32.56504	0.000722	32.52661	0.000762	32.57523	0.000802	32.60099	0.000842	32.58498
0.000683	32.51049	0.000723	32.46352	0.000763	32.50328	0.000803	32.56155	0.000843	32.53965
0.000684	32.55907	0.000724	32.5437	0.000764	32.54645	0.000804	32.60183	0.000844	32.57922
0.000685	32.53823	0.000725	32.58838	0.000765	32.58865	0.000805	32.52158	0.000845	32.57536
0.000686	32.4819	0.000726	32.52439	0.000766	32.51643	0.000806	32.56333	0.000846	32.61303
0.000687	32.52919	0.000727	32.56955	0.000767	32.52038	0.000807	32.60353	0.000847	32.69388
0.000688	32.61089	0.000728	32.5048	0.000768	32.52551	0.000808	32.56208	0.000848	32.60529
0.000689	32.55479	0.000729	32.47679	0.000769	32.56792	0.000809	32.56188	0.000849	32.51654
0.00069	32.49885	0.00073	32.63167	0.00077	32.5323	0.00081	32.60223	0.00085	32.59723
0.000691	32.54693	0.000731	32.56657	0.000771	32.53588	0.000811	32.60195	0.000851	32.63548
0.000692	32.52456	0.000732	32.50065	0.000772	32.61673	0.000812	32.5204	0.000852	32.63226
0.000693	32.50139	0.000733	32.47165	0.000773	32.58165	0.000813	32.51968	0.000853	32.58532
0.000694	32.54829	0.000734	32.55273	0.000774	32.50785	0.000814	32.60142	0.000854	32.62274
0.000695	32.56055	0.000735	32.55998	0.000775	32.55066	0.000815	32.64136	0.000855	32.66058
0.000696	32.5378	0.000736	32.4943	0.000776	32.55381	0.000816	32.55814	0.000856	32.61328
0.000697	32.51469	0.000737	32.57623	0.000777	32.51708	0.000817	32.51644	0.000857	32.65154
0.000698	32.52649	0.000738	32.54636	0.000778	32.59815	0.000818	32.55608	0.000858	32.5608
0.000699	32.60907	0.000739	32.58968	0.000779	32.60085	0.000819	32.59588	0.000859	32.59987
0.0007	32.55063	0.00074	32.59653	0.00078	32.56473	0.00082	32.63585	0.00086	32.68056
0.000701	32.49125	0.000741	32.45555	0.000781	32.52818	0.000821	32.55332	0.000861	32.58848
0.000702	32.57248	0.000742	32.49955	0.000782	32.53115	0.000822	32.5931	0.000862	32.62659
0.000703	32.54849	0.000743	32.54298	0.000783	32.57352	0.000823	32.67268	0.000863	32.66434
0.000704	32.4894	0.000744	32.54974	0.000784	32.53666	0.000824	32.58883	0.000864	32.57238
0.000705	32.50035	0.000745	32.59436	0.000785	32.57718	0.000825	32.50474	0.000865	32.52368
0.000706	32.58164	0.000746	32.48898	0.000786	32.53998	0.000826	32.58569	0.000866	32.64815
0.000707	32.55786	0.000747	32.53184	0.000787	32.58175	0.000827	32.62468	0.000867	32.68662
0.000708	32.49786	0.000748	32.61238	0.000788	32.62295	0.000828	32.54008	0.000868	32.63746
0.000709	32.54295	0.000749	32.54368	0.000789	32.54588	0.000829	32.58036	0.000869	32.58708
0.00071	32.58885	0.00075	32.47475	0.00079	32.46858	0.00083	32.61982	0.00087	32.62478
0.000711	32.52779	0.000751	32.55585	0.000791	32.55047	0.000831	32.57494	0.000871	32.61864
0.000712	32.50254	0.000752	32.59971	0.000792	32.59148	0.000832	32.61418	0.000872	32.56909
0.000713	32.54825	0.000753	32.53026	0.000793	32.51285	0.000833	32.65318	0.000873	32.60618
0.000714	32.48772	0.000754	32.49698	0.000794	32.59343	0.000834	32.5675	0.000874	32.64363
0.000715	32.49746	0.000755	32.54045	0.000795	32.63438	0.000835	32.60664	0.000875	32.72566
0.000716	32.54183	0.000756	32.58335	0.000796	32.51613	0.000836	32.56173	0.000876	32.63168
0.000717	32.55148	0.000757	32.51278	0.000797	32.59735	0.000837	32.60197	0.000877	32.62434
0.000718	32.59705	0.000758	32.55583	0.000798	32.63882	0.000838	32.64062	0.000878	32.66118
0.000719	32.57045	0.000759	32.52285	0.000799	32.55995	0.000839	32.55304	0.000879	32.61058
0.00088	32.55974	0.000935	32.63118	0.00099	32.76878	0.188	36.01011	0.408	47.39232

t, hr	t ΔP$^{/}$, psi	t, hr	t ΔP$^{/}$, psi	t, hr	t ΔP$^{/}$, psi	t, hr	t ΔP$^{/}$, psi	t, hr	t ΔP$^{/}$, psi
0.000887	32.59678	0.000942	32.68724	0.000997	32.70134	0.216	37.43129	0.436	48.46534
0.000888	32.63383	0.000943	32.67458	0.000998	32.78413	0.22	37.66927	0.44	48.67025
0.000889	32.67044	0.000944	32.71032	0.000999	32.81749	0.224	37.84886	0.444	48.9442
0.00089	32.57482	0.000945	32.79216	0.001	32.68453	0.228	38.05295	0.448	49.11797
0.000891	32.61126	0.000946	32.68399	0.012	36.91237	0.232	38.25035	0.452	49.2208
0.000892	32.64688	0.000947	32.57664	0.016	35.57974	0.236	38.49145	0.456	49.47201
0.000893	32.72818	0.000948	32.75323	0.02	33.30935	0.24	38.66924	0.46	49.74119
0.000894	32.63064	0.000949	32.78748	0.024	31.65054	0.244	38.86259	0.464	49.60614
0.000895	32.66754	0.00095	32.67984	0.028	30.31394	0.248	39.06654	0.468	50.02564
0.000896	32.70348	0.000951	32.76283	0.032	29.62868	0.252	39.318	0.472	50.58068
0.000897	32.60568	0.000952	32.65407	0.036	29.25294	0.256	39.49126	0.476	50.52436
0.000898	32.68786	0.000953	32.68778	0.04	29.09208	0.26	39.66221	0.48	50.68425
0.000899	32.58943	0.000954	32.76943	0.044	29.05011	0.264	39.87868	0.484	50.87125
0.0009	32.62499	0.000955	32.61304	0.048	29.22458	0.268	40.13894	0.488	51.07781
0.000901	32.66078	0.000956	32.64729	0.052	29.23028	0.272	40.30815	0.492	51.3375
0.000902	32.69733	0.000957	32.72908	0.056	29.42926	0.276	40.46016	0.496	51.21177
0.000903	32.77864	0.000958	32.76328	0.06	29.53451	0.28	40.69038	0.5	51.67652
0.000904	32.67939	0.000959	32.75046	0.064	29.71705	0.284	40.95131	0.504	52.17042
0.000905	32.58055	0.00096	32.68877	0.068	29.85876	0.288	41.11808	0.508	52.09218
0.000906	32.61675	0.000961	32.77004	0.072	30.06464	0.292	41.26084	0.512	52.27811
0.000907	32.65158	0.000962	32.75563	0.076	30.31761	0.296	41.49991	0.516	52.50668
0.000908	32.59704	0.000963	32.64538	0.08	30.50276	0.3	41.75369	0.52	52.69789
0.000909	32.72394	0.000964	32.72769	0.084	30.73892	0.304	41.92571	0.524	52.94728
0.00091	32.75958	0.000965	32.76154	0.088	30.911	0.308	42.06538	0.528	53.09888
0.000911	32.56778	0.000966	32.74708	0.092	31.13641	0.312	42.29746	0.532	53.25684
0.000912	32.60383	0.000967	32.68516	0.096	31.30366	0.316	42.55385	0.536	53.48883
0.000913	32.68612	0.000968	32.7676	0.1	31.53232	0.32	42.72957	0.54	53.76076
0.000914	32.72212	0.000969	32.80034	0.104	31.71372	0.324	42.86468	0.544	53.56344
0.000915	32.71078	0.00097	32.68884	0.108	31.929	0.328	43.10667	0.548	54.06243
0.000916	32.65513	0.000971	32.67378	0.112	32.10048	0.332	43.3587	0.552	54.66236
0.000917	32.69068	0.000972	32.70753	0.116	32.32471	0.336	43.26382	0.556	54.554
0.000918	32.68064	0.000973	32.78994	0.12	32.50892	0.34	43.67375	0.56	54.71468
0.000919	32.62424	0.000974	32.72712	0.124	32.72754	0.344	44.1579	0.564	54.90892
0.00092	32.56768	0.000975	32.76075	0.128	32.89931	0.348	44.14712	0.568	55.12032
0.000921	32.74237	0.000976	32.74433	0.132	33.12227	0.352	44.30709	0.572	55.38814
0.000922	32.77762	0.000977	32.68038	0.136	33.31271	0.356	44.51698	0.576	55.1885
0.000923	32.67404	0.000978	32.71389	0.14	33.53705	0.36	44.70693	0.58	55.71047
0.000924	32.70944	0.000979	32.74744	0.144	33.71589	0.364	44.94906	0.584	56.31305
0.000925	32.65208	0.00098	32.78063	0.148	33.94159	0.368	45.10623	0.588	56.17209
0.000926	32.68758	0.000981	32.76508	0.152	34.13059	0.372	45.29063	0.592	56.32449
0.000927	32.67644	0.000982	32.74949	0.156	34.35777	0.376	45.50487	0.596	56.54936
0.000928	32.62008	0.000983	32.78294	0.16	34.5415	0.38	45.75159	0.6	56.75617
0.000929	32.65496	0.000984	32.76678	0.164	34.76135	0.384	45.92047	0.604	57.02209
0.00093	32.68923	0.000985	32.70168	0.168	34.95486	0.388	46.06448	0.608	57.18512
0.000931	32.72428	0.000986	32.73685	0.172	35.18048	0.392	46.29046	0.612	57.34066
0.000932	32.61974	0.000987	32.77004	0.176	35.3637	0.396	46.55388	0.616	57.57643
0.000933	32.70164	0.000988	32.80144	0.18	35.58893	0.4	46.41637	0.62	57.84233

t, hr	$t\ \Delta P^{/}$, psi	t, hr	$t\ \Delta P^{/}$, psi	t, hr	$t\ \Delta P^{/}$, psi	t, hr	$t\ \Delta P^{/}$, psi	t, hr	$t\ \Delta P^{/}$, psi
0.628	58.15425	0.824	68.5124	1.02	79.168	1.216	89.87461	1.412	100.488
0.632	58.84083	0.828	68.81	1.024	79.36955	1.22	90.14596	1.416	101.4506
0.636	58.64317	0.832	68.55099	1.028	79.59722	1.224	90.35866	1.42	100.9823
0.64	58.82791	0.836	69.16603	1.032	79.80908	1.228	90.64516	1.424	101.1726
0.644	59.0181	0.84	69.85177	1.036	80.11132	1.232	90.79597	1.428	101.3548
0.648	59.22323	0.844	69.63288	1.04	80.2582	1.236	90.95749	1.432	101.598
0.652	59.52343	0.848	69.81883	1.044	80.3915	1.24	91.2287	1.436	101.9395
0.656	59.69132	0.852	70.04808	1.048	80.6929	1.244	91.53076	1.44	102.075
0.66	59.80006	0.856	70.24212	1.052	81.0075	1.248	91.70891	1.444	102.2031
0.664	60.03894	0.86	70.54506	1.056	80.52811	1.252	91.74326	1.448	102.4797
0.668	60.32411	0.864	70.7149	1.06	81.29677	1.256	92.06226	1.452	102.8408
0.672	60.07835	0.868	70.86519	1.064	82.21413	1.26	92.35968	1.456	102.279
0.676	60.6456	0.872	71.09712	1.068	81.77169	1.264	91.91949	1.46	103.1192
0.68	61.33855	0.876	71.40857	1.072	81.97829	1.268	92.68266	1.464	104.0479
0.684	61.13558	0.88	71.0867	1.076	82.23639	1.272	93.58305	1.468	103.5335
0.688	61.30627	0.884	71.74465	1.08	82.42789	1.276	93.18235	1.472	103.7747
0.692	61.52244	0.888	72.51051	1.084	82.73873	1.28	93.3575	1.476	104.0053
0.696	61.74033	0.892	72.18806	1.088	82.90195	1.284	93.59015	1.48	104.1893
0.7	61.99708	0.896	72.40525	1.092	83.0099	1.288	93.77875	1.484	104.5323
0.704	61.761	0.9	72.65618	1.096	83.30683	1.292	94.07671	1.488	104.659
0.708	62.36515	0.904	72.8477	1.1	83.65838	1.296	93.69736	1.492	104.8148
0.712	62.98817	0.908	73.15425	1.104	83.15288	1.3	94.50025	1.496	105.1098
0.716	62.77847	0.912	72.8055	1.108	83.93898	1.304	95.3015	1.5	105.3807
0.72	62.98788	0.916	73.50555	1.112	84.82521	1.308	94.88175	1.504	104.7297
0.724	63.21793	0.92	74.26114	1.116	84.42874	1.312	95.08455	1.508	105.7169
0.728	63.40514	0.924	73.93541	1.12	84.62781	1.316	95.31283	1.512	106.7232
0.732	63.69693	0.928	74.13256	1.124	84.83718	1.32	95.54887	1.516	106.085
0.736	63.8713	0.932	74.37694	1.128	85.09043	1.324	95.84634	1.52	106.3212
0.74	64.02275	0.936	74.60729	1.132	85.39868	1.328	95.97818	1.524	106.5495
0.744	64.25925	0.94	74.90984	1.136	85.52293	1.332	96.13887	1.528	106.7675
0.748	64.53793	0.944	75.04383	1.14	85.65908	1.336	96.37289	1.532	107.1282
0.752	64.24287	0.948	75.19193	1.144	85.95419	1.34	96.66734	1.536	107.2079
0.756	64.86374	0.952	75.47864	1.148	86.29113	1.344	96.14119	1.54	107.3413
0.76	65.59149	0.956	75.76708	1.152	85.77482	1.348	97.06334	1.544	107.6137
0.764	65.35384	0.96	75.34148	1.156	86.57797	1.352	97.94833	1.548	107.9377
0.768	65.52591	0.964	76.09213	1.16	87.50202	1.356	97.41468	1.552	108.1322
0.772	65.72951	0.968	76.91368	1.164	87.02667	1.36	97.6438	1.556	108.0878
0.776	65.97642	0.972	76.56052	1.168	87.26015	1.364	97.90424	1.56	108.4372
0.78	66.25511	0.976	76.78312	1.172	87.49293	1.368	98.15104	1.564	108.8168
0.784	66.41057	0.98	76.98222	1.176	87.71411	1.372	98.44424	1.568	108.1871
0.788	66.53992	0.984	77.20414	1.18	88.01651	1.376	98.60429	1.572	109.1035
0.792	66.802	0.988	77.52518	1.184	88.15899	1.38	98.7607	1.576	110.0987
0.796	67.1262	0.992	77.68196	1.188	88.31627	1.384	98.98758	1.58	109.5164
0.8	66.81361	0.996	77.79091	1.192	88.60732	1.388	99.27883	1.584	109.7277
0.804	67.40311	1	78.06012	1.196	88.91097	1.392	99.48994	1.588	109.9121
0.808	68.14981	1.004	78.37835	1.2	88.35921	1.396	99.52014	1.592	110.1255
0.812	67.90689	1.008	77.94904	1.204	89.22554	1.4	99.85264	1.596	110.506

APPENDIX M – Field Data

Field: AX **Well: AX-15** **Reservoir:** G1234Y

Type of Test: FG/BU/SG **Date of Test:** 21/07/09 **Company:** OTMAD

Date of Production Test: 21/07/09 **Wireline Company**: SAFEDRILL

Well Type: Horizontal

Oil Production Rate (during test or prior to shut-in), STB/D = 4747

Producing GOR (SCF/STB) = 689

BS&W (%) = 0

Table M.1: Reservoir, Fluid and Well Data for AX-15

Porosity ($\phi_1 = \phi_2 = \phi_f$),%	=	28	Oil Viscosity ($\mu_1 = \mu_2 = \mu_f$), cp	=	0.212
Oil compressibility, C_t,1/psi	=	30.1E-6	B_O, RB/STB	=	1.496
Drainage thickness, h, ft	=	137.2	Slotted liner interval, ft	=	1806.1
Max Deviation (degrees)	=	0.5	Wellbore radius, ft	=	0.35
Reservoir Width (ft)	=	2500	Wellbore gradient (psi/ft)	=	0.387
Oil Specific Gravity(^{O}API)	=	27.0	Reservoir gradient (psi/ft)	=	0.357

Table M.2: Summary of 3D semi-analytical input values used in Fig. 3.6.3

Parameter	**3D Semi-analytical Solution Match Values**
	Type curve
K_h, md (horizontal permeability)	**5287**
K_v, md (vertical permeability)	**131**
S (Skin Factor)	**25**
C_s (STB/psi)	**0.0154**
L, ft (well length)	**2000**
Well location, z_w, (ft) (from bottom)	**64.99**

Field: AX **Well: AX-20** **Reservoir:** G5678Y

Type of Test: FG/BU/SG **Date of Test:** 11/05/09 **Company:** OTMAD

Date of Production Test: 09/05/09 **Wireline Company**: SAFEDRILL

Well Type: Horizontal

Oil Production Rate (during test or prior to shut-in), STB/D = 4654

Producing GOR (SCF/STB) = 689

BS&W (%) = 0

Table M.3: Reservoir, Fluid and Well Data for AX-20

Porosity ($\phi_1 = \phi_2 = \phi_f$),%	=	28	Oil Viscosity ($\mu_1 = \mu_2 = \mu_f$), cp	=	0.360
Oil compressibility, Ct,1/psi	=	8.0E-6	Bo, RB/STB	=	1.390
Net Drainage thickness, h, ft	=	169.54	Slotted liner interval, ft	=	8293.4
Max Deviation (degrees)	=	0.5	Wellbore radius, ft	=	0.35
Reservoir Width (ft)	=	1700	Wellbore gradient (psi/ft)	=	0.356
Oil Specific Gravity(^{0}API)	=	40.4	Reservoir gradient (psi/ft)	=	0.356

Table M.4: Summary of 3D semi-analytical input values used in Fig. 3.6.4

Parameter	**3D Semi-analytical Solution Match Values**
	Type curve
K_h, md (horizontal permeability) (**K_{x2}/K_h=10**)	**686.96**
K_v, md (vertical permeability)	**42.2**
S (Skin Factor)	**20**
C_s(STB/psi)	**0.0000044**
L, ft(well length)	**2000**
Well location, z_w, (ft) (from bottom)	**84.77**

Field: AX **Well: AX-23** **Reservoir:** G1357Y

Type of Test: FG/BU/SG **Date of Test:** 11/05/02 **Company:** OTMAD

Date of Production Test: 09/05/02 **Wireline Company**: SAFEDRILL

Well Type: Horizontal

Oil Production Rate (during test or prior to shut-in), STB/D = 3794

Producing GOR (SCF/STB) = 915

BS&W (%) = 0

Table M.5: Reservoir, Fluid and Well Data for AX-23

Porosity (ϕ_1= ϕ_2= ϕ_f),%	=	28	Oil Viscosity (μ_1= μ_2= μ_f), cp	=	0.316
Oil compressibility, C_t,1/psi	=	21.0E-6	B_o, RB/STB	=	1.629
Drainage thickness, h, ft	=	98.7	Slotted liner interval, ft	=	1626.6
Max Deviation (degrees)	=	0.5	Wellbore radius, ft	=	0.35
Reservoir Width (ft)	=	1950	Wellbore gradient (psi/ft)	=	0.226
Oil Specific Gravity(^{O}API)	=	41.7	Reservoir gradient (psi/ft)	=	0.354

Table M.6: Summary of 3D semi-analytical input values used in Fig. 3.6.5

Parameter	**3D Semi-analytical Solution Match Values**
	Type curve
K_h, md (horizontal permeability) (**K_{x2}/K_h=10**)	**3244.96**
K_v, md (vertical permeability)	**160.87**
S (Skin Factor)	**10**
C_s(STB/psi)	**0.000008**
L, ft(well length)	**1626**
Well location, z_w, (ft) (from bottom)	**49.4**