

CHAPTER ONE

INTRODUCTION

1.1 Problem Statement

Formation damage refers to the impairment of the permeability of petroleum bearing formations by various adverse processes. Formation damage is an undesirable operational and economic problem that can occur in many phases of field development including drilling, completion, workover, production, stimulation, waterflooding and enhanced oil recovery. In fact, it is among the major causes of a well productivity reduction in petroleum reservoir. Proper understanding of the factors causing these damages is therefore of utmost important. Formation damage indicators include permeability impairment, skin damage and decrease of well performance.

Formation damage can be caused by factor such as fine particle transportation in porous media. Fines enter the formation as suspensions during drilling and completion operations. They can also be generated due to fluid-fluid and fluid-rock interactions. When they enter the formation, they get entrapped in the pore openings, fill and plug the pore spaces of the formation. Sedimentary rocks also contain loosely attached fines. When injection fluids are incompatible with formation fluids, these fines can be released from the rock surface, migrate and plug at pore constrictions. The end result is severe permeability reduction.

It is therefore essential to develop experimental and analytical methods for understanding and preventing and/or controlling formation damage in oil and gas bearing formations (Energy Highlights, 1990). The laboratory experiments are important steps in reaching understanding of the physical basis of formation damage phenomena. From this experimental basis realistic models which allow extrapolation outside the scaleable range may be constructed (Energy

Highlights, 1990). These efforts are necessary to develop and verify accurate mathematical models and computer simulators that can be used for predicting and determining strategies to avoid and/ or mitigate formation damage in petroleum reservoirs (Civan, 1994).

Most of the previous work on formation damage dealt with single-phase flow of fluids or filtrates. Several models have been developed that characterized fines migration in single-phase flow. In reality, multiphase flow is commonly encountered in petroleum bearing formations than single-phase flow. There is scarce information in the literature about formation damage during multiphase flow. The present work seeks to develop a mathematical simulation model for formation damage due to chemical precipitation and particulate processes in the presence of oil and water.

1.2 Objectives

The objectives of this work are as follows:

1. To develop a mathematical model for the simulation of formation damage due to chemical precipitation and particulate processes in two-phase flow of oil and water.
2. To obtain the numerical simulation of the developed model.
3. To compare the results obtained from the simulation with real experimental data available in the literature.

1.3 Limitation of Scope of Research Work

This work considered modelling of formation damage due to chemical precipitation and particulate transportation in two-phase flow of oil and water in petroleum reservoir core. Only linear one dimensional horizontal flow was considered. A mass balance approach was used to macroscopically model the transport and rate processes that can occur during formation damage. The effects of relative permeabilities and capillary pressure were

considered as auxiliary correlations. The variation of permeability with porosity was also considered as parts of auxiliary correlation in the model. Throughout the model formulation, only one type of particulate matter was assumed. Hydrodynamic effect was considered to be the only possible cause of particle release. In addition, only pore surface deposition was modelled. Pore throat blocking effect was not included because of its complexity.

Chapter 2 gives an extensive literature review that includes background information on formation damage. The chapter also contains critical evaluation of some selected models of formation damage.

In Chapter 3, model development and numerical solution is given. Parts of the model formulation include dissolved species mass balance, particulate transportation, liquid mass balance, momentum equation, fluid saturation pressure determination, and fines migration. The solution to the model involves numerical discretisation using finite difference scheme. Thomas algorithm was used for the numerical solution.

Chapter 4 gives the detail results obtained and their discussions. Conclusion and recommendations are discussed in Chapter 5.

CHAPTER TWO

LITERATURE REVIEW

Formation damage is an undesirable operational and economic problem that can occur during various phases of oil and gas recovery from subsurface reservoirs including production, drilling, hydraulic fracturing, and workover operations. Formation damage assessment, control, and remediation are among the most important issues to be resolved for efficient exploitation of hydrocarbon reservoirs. Such damage is caused by various adverse processes, including chemical, physical, biological, and thermal interactions of formation and fluids, and deformation of formation under stress and fluid shear. Formation damage indicators include permeability impairment, skin damage, and decrease of well performance (Civan, 2000).

Formation damage assessment, control, and remediation are among the most important issues to be resolved for efficient exploitation of hydrocarbon reservoirs (Energy Highlights, 1990). Formation damage is caused by physic-chemical, chemical, biological, hydrodynamic, and thermal interactions of porous formation, particles, and fluids and mechanical deformation of formation under stress and fluid shear. These processes are triggered during the drilling, production, workover, and hydraulic fracturing operations.

Formation damage is not necessarily reversible and what gets into porous media does not necessarily come out. This phenomenon is called “the reverse funnel effect” (Porter, 1989). Therefore, it is better to avoid formation than to try to restore it. A verified formation damage model and carefully planned laboratory and field tests can provide scientific guidance and help develop strategies to avoid or minimize formation damage. Properly designed experimental and analytical techniques, and the modelling and simulation approaches can help understanding, diagnosis, evaluation, prevention, remediation, and controlling of formation damage in oil and gas reservoirs (Civan, 2000).

The consequences of formation damage are the reduction of the oil and gas productivity of reservoirs and noneconomic operation. Therefore, it is essential to develop experimental and analytical methods for understanding and preventing and are/or controlling formation damage in oil and gas bearing formations (Energy Highlights, 1990).

2.1 Mechanisms for Formation Damage

The fundamental processes causing damage in petroleum bearing formations are: (1) physico-chemical, (2) chemical, (3) hydrodynamic, (4) thermal, and (5) mechanical. The following seven formation damage mechanisms have been discussed in the literature (Civan, 2000).

1. Fluid-fluid incompatibilities, for example emulsions generated between invading oil based mud filtrate and formation water.
2. Rock-fluid incompatibilities, for example contact of potentially swelling smectite clay or deflocculatable kaoinite clay by non-equilibrium water based fluids with the potential to severely reduce near wellbore permeability.
3. Solids invasion, for example the invasion of weighting agents or drilled solids.
4. Phase trapping/blocking, for example the invasion and entrainment of water based fluids in the wellbore region of a gas well.
5. Chemical adsorption/wettability alteration, for example emulsifier adsorption changing the wettability and fluid flow characteristics of a formation.
6. Fines migration, for example the internal movement of fine particulates within a rock's pore structure resulting in the bridging and plugging of pore throats.

7. Biological activity, for example the introduction of bacterial agents into the formation drilling and the subsequent generation of polysaccharide polymer slimes which reduce permeability.

However, according to Ohen and Civan (1989), the mechanisms by which rock-fluid interactions can lead to permeability damage has been classified into three general categories.

1. Fines Migration: Permeability impairment due to fines migration can be caused by movement of many different types of fines including the non-expanding authigenic clay minerals (smectite, kaolinite and chlorites), the expanding authigenic clay minerals (montmorillonites), quartz, and carbonates. These fines which loosely adhere to the pore wall by physicochemical forces are released due to the drag forces exerted by the flowing fluid. If released in sufficient quantities, the fines may be redeposited or collected downstream at the pore throat.
2. Brine Sensitivity: Some clay minerals are sensitive to composition and ionic strength of the injected water. Smectite and montmorillonites will swell to many times their normal size and kaolinites will disperse. The swollen clay minerals occupy more of the pore space and fines are released by dispersion of the non-expanding clay minerals resulting in a combined effect of permeability alteration due to clay swelling and fines migration.
3. Geochemical Transformation: dissolution, precipitation and chemical reaction can occur when external fluids are injected into a reservoir. An example is the liberation of mobile fines when carbonate cements dissolve in carbon dioxide during CO₂ enhanced recovery processes, a change in pH of an injection fluid may alter the

surface chemistry of clay fines (chemical effect) resulting in a fines migration problem (mechanical effect).

Fines migration and clay swelling have been recognized as the major cause of formation damage observed as permeability impairment. The damage is especially more severe in high clay content, low permeability, and unconsolidated type reservoirs. But Moore (1960) suggested that clay swelling is the dominant permeability reduction mechanism in petroleum reservoirs. In general, there is simultaneous occurrence of fines migration and clay swelling phenomena whose relative contributions to formation damage depend on the mineral composition of the reservoir rock.

Fine particles that may contribute to permeability impairment include clay minerals, quartz, amorphous silica, feldspars, mica, carbonates and barites. As clay minerals are small in size and structurally platy, their surface areas are large and thereby tend to react readily and rapidly with fluids introduced into the porous media (Ohen and Civan, 1989).

2.2 Occurrence of Formation Damage

When fluids are introduced into a porous rock during petroleum exploration and production, its original purpose is to increase the recovery of hydrocarbon. However, due to the incompatibility between injected and indigenous fluids, change of reservoir rock properties can often be expected (Chang, 1992). The following are some of the causes of formation damage during various oil exploitation activities.

2.2.1 Drilling

During drilling, higher pressure is needed in the wellbore to control the formation being penetrated, the pressure differential will result in invasion of mud solids and mud filtrate into reservoir rock near wellbore. Damage from mud solid invasion is strongly influenced by

particle size and pore throat size distribution. The mud filtrate can seep into the porous environment to cause salinity change and induce clay instability. By overpressuring mud filtrate can mobilize the loosely attached fine particles with a shear force.

2.2.2 Production

The temperature and pressure in the reservoirs are constantly changing during the oil and gas production. Organic scale such as paraffin waxes and asphaltenes may deposit out of the crude oil to plug the formation. Inorganic salts such as calcium carbonate and barium sulfate may also precipitate out of the aqueous phase to block flow paths. The large pressure gradient near wellbore often is capable of mobilizing fines residing on the surface of pore wall around the producing wells to cause fines migration.

2.2.3 Waterflooding

Compatibility of the injected water with native reservoir fluids is an important factor that influences the success of a waterflooding program. Ions in the source water may react with ions in the reservoir fluids to form insoluble precipitates. Iron corrosion products from injected pipeline and equipment may generate insoluble compounds, such as iron hydroxide ($\text{Fe}(\text{OH})_2$), after mixing with pore liquid (Krueger, 1986). Pre-filtration of source water is crucial because solids contained in the injected water may induce formation plugging. Sensitive formation clay minerals may swell or be released if water salinity and composition are changed so as to alter the equilibrium state of the clay-pore liquid system.

2.2.4 Stimulation and EOR

Most stimulation and EOR operations involve chemical treatments. Reactions of different kinds occur when chemicals are introduced into formations. Some of the reactions have adverse effect on formation permeability.

During acidizing, acid dissolves iron-containing minerals, such as chlorite, siderite (FeCO_3), and hematite (Fe_2O_3). The ferric ions (Fe^{3+}) may precipitate as low soluble ferric hydroxide ($\text{Fe}(\text{OH})_3$) upon spending of acid (Crowe, 1985). Acid can also dissolve cementing materials, often carbonates, and liberate sand grains to move within the porous structure and cause permeability reduction (Bryant and Buller, 1988; Reed, 1977).

Alkaline flooding may be a cost-effective alternative to micellar-polymer flooding to reduce interfacial tension with the natural surfactant formed in situ (Thornton and Lorenz, 1987). Multivalent ions tend to react with alkali to form precipitates. The precipitates include carbonates, hydroxides, and silicates. The common multivalent ions are calcium and magnesium (Krumrine et al., 1985).

In micellar flooding, sulphonate precipitation occurs when the surfactant contacts reservoir fluid that contains inorganic ions. This precipitation may lead to formation plugging (Somasundaran et al., 1984).

2.3 Modeling Approach

Formation permeability alteration is influenced by the chemistry of pore fluid and mechanical conditions of operations. Generally it is not cost and time efficient to run laboratory tests of formation damage problems since the real reservoir conditions are difficult to achieve in laboratories and techniques for measuring the parameters are still lacking and expensive. A mathematical model has the advantages of flexibility, speed and low cost, and the capability of predicting future behaviour of the reservoir. A mathematical model incorporating common permeability alteration mechanisms, such as clay fines migration, clay swelling, and inorganic scale formation, is developed in the present research. Chemical and mechanical effects are taken into consideration in modelling these damage processes. The model equations describe the generation and consumption of the species, and some empirical

relationships, such as unplugging coefficients and permeability-porosity correlations (Chang, 1992).

There are parameters governing the rate of these interactions and determining the response of output when certain input conditions are applied to the system. From output measurements and known input conditions, providing the model is correctly built, the model can acquire the values of the system parameters that best describe the processes taking place inside the rock material (Chang, 1992).

So many approaches at mathematically modelling permeability impairment due to fines migration and chemical reaction in porous media exist in the literature. The earlier efforts to represent particles movement by mathematical models were directed towards applications in water clarification (Iwasaki, 1937). Prominent among such phenomenological models are those by Gruesbeck and Collins (1982), Khilar and Fogler (1983), Civan and Knapp (1987), wajtonowicz et al. (1987) and Civan et al. (1988). These models have their different shortcoming in predicting formation damage since none of them include a complete description of all the known mechanisms causing permeability impairment (Chang, 1992).

Generally, what is needed is a fundamental, mechanistic and generalized model that completely quantifies the effect external particle invasion, fines generation, migration and retention on the variation of reservoir permeability. The model should provide a complete description of permeability variation resulting from changes in the flow characteristics of petroleum reservoirs during interactions of fluids with the porous rock which is recognized in the petroleum industry as the skin effect. The diversified nature of the various models in the literature is an indication of the complexity of the phenomenon.

The model that set the ground for rock-fluid interaction is that developed by McCune et al. (1975). Acidisation process for petroleum production was modelled. But the model

considered only matrix dissolution. A geochemical model was also developed by Walsh et al. (1984) to simulate reactions flowing fluid in porous media. The model has found wide applicability for underground fluid transport. The model requires detailed information about the nature of the chemical reactions occurring in the formation and particulate motion was not taken into consideration. In the model developed by Ortoleva et al. (1987) geochemical reactions investigated but precipitates formation was left out. Hence, particulate processes were ignored in the model. Arshad (1991) studied selective plugging during petroleum recovery by surfactant injection. Gel-like matter was assumed to be the cause of permeability impairment while pore throat blocking mechanism was not considered. Pore throat blocking mechanism by precipitates was modelled using network analysis by Rege and Fogler (1987 and 1988). However, network models require great deals of computational efforts. Some models are discussed in detail in the following sections.

2.3.1 Fogler et al. Model

The explanation to this model was given by Chang (1992) which is pertinent by McCune et al. (1975), Fogler et al. (1976), Lund and Fogler (1976), and Hekin and Fogler (1980).

The model was developed primarily to simulate acidization of sandstone cores. During acidization, the acid mixture (HF/HCl) is injected into a sandstone core. The acid mixture dissolves some minerals in the sandstone, such as clays, calcite, and feldspar. In this model, the reaction kinetics between acid and minerals were described mathematically and movement of the permeability front due to the porosity change caused by reaction was predicted.

Assumptions made in this model are: horizontal 1-D flow and domination by convective transport while dispersion of the reactive fluid species was neglected.

The model equations are given below:

The material balance equation for the solute species i in terms of the mole balance is written as

$$\frac{\partial(\phi C_{if})}{\partial t} + \frac{\partial(u C_{if})}{\partial x} = (G_s + G_h)_{if} \quad 2.1$$

Where

C_{if} = concentration of chemical species in fluids (mole/PV)

G_h = rate of generation due to a homogeneous reaction (reaction within the fluid)
(mole/bulk volume/t)

G_s = rate of generation due to a heterogeneous reaction (mole/bulk volume/t)

u = fluid velocity (L/t)

ϕ = porosity (PV/bulk volume)

The mole balance equation for a dissolvable mineral species j in the sandstone can be written as

$$\frac{\partial[(1-\phi)C_{js}]}{\partial t} = R_j \quad 2.2$$

Where

C_{js} = concentration of mineral species j in solid (mole/volume of solid)

R_j = reaction rate of the mineral dissolution (mole/bulk volume/t)

Let M_j be the molecular weight of the mineral j, and ρ_j be its density, then summing all the minerals present in the core

$$\sum_{j=1}^n \frac{C_{js} M_j}{\rho_j} = 1 \quad 2.3$$

Therefore, the total porosity change is the sum of the porosity change caused by each mineral dissolution.

$$\frac{\partial \phi}{\partial t} = - \sum_{j=1}^n \frac{R_j M_j}{\rho_j} \quad 2.4$$

The reaction rate of the mineral (R_j) can be related to the reaction rate of the solute $(G_s + G_h)_{if}$ by a stoichiometric coefficient v_{ij} (Herkin and Fogler, 1980)

$$(G_s + G_h)_{if} = v_{ij} R_j \quad 2.5$$

v_{ij} denotes the consumption (or production) of species I per mole j dissolved.

INITIAL AND BOUNDARY CONDITIONS:

Initially, the sandstone is in equilibrium with the pore fluid and the concentration of the soluble mineral j is constant.

I.C.

$$C_{(HF)_f}(x,0) = 0 \quad \phi = \phi_o, \text{ at } t = 0, 0 \leq x \leq L_x \quad 2.6$$

$$C_{js}(x,0) = C_{jso}, \quad \phi = \phi_o, \text{ at } t = 0, 0 \leq x \leq L_x \quad 2.7$$

A flow of constant concentration HF/HCl is imposed at the left boundary by a constant flow rate when time starts.

B.C. 1:

$$C_{(HF)_f}(0,t) = C_o = \text{constant at } t > 0, x = 0 \quad 2.8$$

B.C.2:

This condition indicates that the concentration of mineral j ahead of the reaction front stays the same as its original value.

$$C_{js}(x, t) = C_{jso}, \text{ for } \phi x / u \geq t \quad 2.9$$

McCune et al. (1975) introduced a set of dimensionless groups to solve these model equations analytically.

2.3.2 Lake et al. Model

The references pertinent to this model are given by Walsh et al. (1982 and 1984), Bryant et al. (1986), and Bhuyan et al. (1990) and explained fully by Chang (1992).

When a reactive fluid is introduced into a porous medium composed of different minerals, some mineral components can be dissolved, transported through the medium, and react with other components in the aqueous solution to form precipitates. A general equilibrium model is developed by taking into account the dissolution and precipitation of minerals during fluid flow through porous media.

Several assumptions were made such as one-dimensional and horizontal flow; the porous medium is homogeneous, isothermal and incompressible; single phase incompressible fluid flow; no flow or migration of solid phase; and so on.

MODEL EQUATIONS:

If j is the total number of aqueous species in the flow stream, and these aqueous species are composed of I elements, the material balance equation for element i is written

$$\text{as } \frac{\partial(\phi C_i)}{\partial t} + u \frac{\partial}{\partial x} \left(\sum_{j=1}^I v_i C_{jf} \right) - \frac{\partial^2}{\partial x^2} \left(\phi \sum_{j=1}^J D_j v_{ij} C_{jf} \right) = 0, \quad I = 1, \dots, I$$

2.10

Where

C_i = total concentration of element i in the system (mole/bulk volume)

C_{jf} = concentration of species j in the aqueous phase (mole/bulk volume)

D_j = dispersion coefficient for aqueous species j (L^2/t)

u = fluid velocity (L/t)

v_{ij} = stoichiometric coefficient, the number of mole of element I in one mole of species j .

If there are N minerals in the solid matrix of the system, that contribute element i to the fluid during chemical reactions. Then,

$$C_i = \sum_{j=1}^J v_{ij} C_{jf} + \sum_{n=1}^N v_{in} C_{ns}, \quad I = 1, \dots, I \quad 2.11$$

Where

C_{ns} = concentration of solid mineral n in the system (mole/bulk volume)

V_{ij} = stoichiometric coefficient, the number of element I in one mole of mineral n .

The first term in Equation 2.11 is the contribution of aqueous species j to the element i in the system, and the second term is the contribution from solid mineral n .

The local porosity is formulated by subtracting the volume fraction occupied by the N minerals from the bulk volume of the system.

$$\phi = 1 - \sum_{n=1}^N \frac{C_{ns} M_n}{\rho_n} \quad 2.12$$

ρ_n is the density (M/L³) and M_n is the molecular weight of the mineral n.

The relationship used between permeability and porosity is

$$\ln \frac{k}{k_o} = \beta(\phi - \phi_o) \quad 2.13$$

β is a parameter which is determined empirically, ϕ_o and k_o are initial porosity and permeability. The overall permeability of the linear bed is given by

$$K = \frac{L_x}{\int_0^{L_x} \frac{dx}{k}} = \frac{k_o}{\int_0^1 e^{\beta(\phi_o - \phi)} dx_D} \quad 2.14$$

INITIAL AND BOUNDARY CONDITIONS:

The total concentration of element I is constant initially, and a constant concentration of element i is injected from the inlet of the system when time starts.

I.C:

$$C_i(x_D, t_D) = C_{io}, \text{ for } x_D > 0, t_D = 0, i = 1, \dots, I \quad 2.15$$

B.C:

$$C_i(0, t_D) = C_{inj}, \text{ for } x_D = 0, t_D > 0, i = 1, \dots, I \quad 2.16$$

In Equations 2.14 through 2.16, the variables are written in dimensionless form.

2.3.3 Ortoleva et al. Model

The detail explanation of this is given by Chang (1992). The references pertinent to the model are given by Chadman et al. (1986) and Ortoleva et al. (1987).

There is a shift in equilibrium in reservoir formation system following the entrance of a foreign fluid. Hence, the infiltration flow of the fluid reacts with solid grains of the medium. The reaction between the fluid and the rock will cause porosity variations, which further alter the permeability of the medium. This necessitates the development of mathematical model describing mass conservation and rate of dissolution/precipitation reaction.

During the development of the model, certain assumptions were made. These are stated as follows:

1. Single mineral in the porous medium and single soluble species.
2. Flow is 1-D and horizontal.
3. Water density is independent of the solute composition. Therefore ρ_w is a constant.

Model Equations:

The general material balance equation for a chemical species is first written in all directions

$$\frac{\partial C_i}{\partial t} = -\nabla \cdot J_i + G_i \quad 2.17$$

Where

C_i = concentration of chemical species i in fluid (mole/rock volume)

G_i = rate of source/sink for chemical species I due to dissolution/precipitation of a grain composing the porous medium (mole/t/rock volume)

J_i = molar flux of species i (mole/t/L²)

To convert the concentration of species i in terms of rock volume to that in terms of fluid volume, the following equation is used

$$C_i = \phi C_{if} \quad 2.18$$

Where

C_{if} = concentration of chemical species i fluid (mole/PV)

ϕ = porosity (PV/rock volume)

Transport of the chemical species is by both diffusion and convection

$$J_i = -\phi D_i(\phi) \nabla C_{if} + \phi C_{if} u \quad 2.19$$

Where

D_i = diffusion coefficient of species i in the porous medium (L²/t)

u = fluid velocity (L/t)

The diffusion coefficient, D_i , depends on the porosity of the porous medium. A phenomenological relation for D_i is (Chadman et al., 1986)

$$D_i(\phi) = \overline{D}_i \phi^m \left(\frac{3}{2} \leq m \leq \frac{5}{2} \right) \quad 2.20$$

Where \overline{D}_i is a constant which represents the diffusion coefficient of species i in water.

Let ϕ_n be the volume fraction of insoluble mineral component in the bulk porous medium, assuming the average volume of a soluble grain is L^3 , and total number of soluble grain contained the matrix is n , therefore

$$nL^3 + \phi + \phi_n = 1 \quad 2.21$$

If R is the rate of volume change of a soluble grain due to the reaction,

$$\frac{\partial L^3}{\partial t} = R \quad 2.22$$

If the effect of nucleation is ignored ($\partial \phi_n / \partial t = 0$), the following equation can be obtained by taking the derivative of Equation 2.21 with respect to time and using 2.22

$$\frac{\partial \phi}{\partial t} = -nR \quad 2.23$$

The negative sign denotes that when the mineral grains get smaller, the porosity of the medium increases.

Assume that the solid dissolves according to the following reaction



Where X_i is the i th solute species in the pore fluid that is, the i th chemical species composing the soluble mineral component. v_i is its stoichiometric coefficient. Let ρ be the density of the soluble mineral component (M/L^3), M be its molecular weight, hence the total generation of species i by the dissolution/precipitation reaction can be written as

$$G_i = -v_i \frac{\rho}{M} nR \quad 2.25$$

The reaction rate constant R can be expressed according to the chemical kinetic rate law. First, the dissolution rate is assumed linearly proportional to the surface area of soluble mineral component contacting with the fluid. Secondly, the rate also depends on the concentration of dissolved ions from the soluble mineral component. Therefore,

$$R = kL^2 \left(\prod_{i=1}^N C_{if}^{vi} - K^{eq} \right) \quad 2.26$$

Where k is the rate coefficient and K^{eq} is the equilibrium constant of reaction 2.24.

$$\phi_f = 1 - \phi_n \quad 2.27$$

ϕ_f is the final porosity if all of the soluble component has been dissolved. From Equation

2.21 and 2.27

$$L^2 = \left(\frac{\phi_f - \phi}{n} \right)^{\frac{1}{3}} \quad 2.28$$

Substitute Equation 2.28 for L^2 in Equation 2.26

$$R = k \left(\frac{\phi_f - \phi}{n} \right)^{\frac{2}{3}} \left(\prod_{i=1}^N C_{if}^{vi} - K^{eq} \right) \quad 2.29$$

After substitution and simplification

$$G_i = -v_i \frac{\rho}{M} n^{1/3} k (\phi_f - \phi)^{2/3} \left(\prod_{i=1}^N C_{if}^{vi} - K^{eq} \right) \quad 2.30$$

The mass conservation equation for water is

$$\frac{\partial}{\partial t} (\phi \rho_w) + \nabla \cdot (\phi \rho_w u) = 0 \quad 2.31$$

Where ρ_w is the density of water (M/L^3).

Darcy's law is applied to relate the flow velocity, u , to pressure gradient and permeability.

$$u = -\frac{k(\phi)}{\mu} \nabla p \quad 2.32$$

In Equation 2.32, k is a function of porosity by the following phenomenological description

$$k(\phi) = \phi^3 / E_2 \left[(1 - \phi)^{2/3} + E_1 (\phi_f - \phi^{2/3}) \right]^2 \quad 2.33$$

Where E_1 and E_2 are constants.

INITIAL AND BOUNDARY CONDITIONS:

The porous medium is assumed to extend from $x = 0$ to $x = L_x$. initially, the porous rock is filled with water which is in equilibrium with respect to the dissolution/precipitation reaction.

I.C:

$$C_{if}(x,0) = \text{constant}, \quad \phi = \phi_o \quad \text{at } t = 0, \quad 0 \leq x \leq L_x \quad 2.34$$

A flow of undersaturated water is imposed at the left boundary by a constant pressure gradient when time starts.

B.C. 1:

$$C_{if}(0,t) = 0, \quad \phi = \phi_f \quad \partial p / \partial x = \text{constant}, \quad \text{at } t > 0, \quad x = 0 \quad 2.35$$

B.C. 2:

$$C_{if}(L_x, t) = C_{if}^{eq}, \quad \phi = \phi_f, \quad \text{at } t > 0, \quad x = L_x \quad 2.36$$

C_{if}^{eq} is the concentration of species i in solution under equilibrium condition.

2.3.4 Arshad's Model (1991)

Water flooding is a widely used secondary oil recovery method. During the process of water injection, the injected fluid tends to flow through low resistance paths, channelling can therefore occur due to the heterogeneity of the reservoir. When this phenomena occurs in an

oil bearing formation, the displacing water will flow through the high permeability zone and bypass the low permeability region. It leaves a significant amount of oil in the low permeability region unrecovered. To alleviate the channelling problem, the injected fluid must be diverted to the low permeability region to create a more uniform flood front. The method used to achieve this is to selectively plug the high permeability region.

The model considers nucleation and size growth of the precipitates, deposition of precipitating particles, entrainment of the particles, dissolution of the precipitated particles, and surfactant adsorption.

The assumptions made in the development of this model include:

1. Flow is 1-D, horizontal, and isothermal in a homogeneous and isotropic porous medium.
2. The effects of pore structure in the porous medium are not considered.
3. All fluids are incompressible.
4. Bulk fluid velocity and bulk density are constant.
5. Pore throat blocking by single particle is not considered.

MODEL EQUATIONS

The mass balance equation in terms of the total surfactant concentration is

$$\frac{\partial C^T}{\partial t_D} + \frac{\partial C}{\partial x_d} + \frac{\partial C_{p,b}}{\partial x_d} + \frac{D}{uL_x} \frac{\partial^2 C}{\partial x_d^2} = 0 \quad 2.37$$

Where

C = concentration of surfactant in solution ($\mu\text{mole/L}^3$ of solution)

$C_{p,d}$ = concentration of surfactant in the form of suspended precipitates ($\mu\text{mole/L}^3$ of solution)

C^T = the total surfactant concentration ($\mu\text{mole/L}^3$ of solution)

u = fluid velocity (L/t)

The total surfactant concentration C^T is the sum of surfactant in all forms

$$C^T = C_m + C_M + \bar{C} + C_{p,b} + C_{p,d} \quad 2.38$$

Where

C_m = concentration of monomer ($\mu\text{mole/L}^3$ of solution)

C_M = concentration of micelle ($\mu\text{mole/L}^3$ of solution)

$C_{p,b}$ = concentration of suspended precipitates ($\mu\text{mole/L}^3$ of solution)

\bar{C} = concentration of adsorbed surfactant ($\mu\text{mole/L}^3$ of solution)

The surfactant concentration in aqueous form C is the sum of the monomer concentration (C_m) and the micelle concentration (C_M).

$$C = C_m + C_M \quad 2.39$$

X_d and t_d are dimensionless distance and dimensionless time respectively.

The surfactant adsorption isotherm was also considered in the model.

INITIAL AND BOUNDARY CONDITIONS:

I.C.

$$C^T_{DPC} = C^T_{SDS} = 0, \quad \text{at } t = 0, \quad 0 \leq x \leq L_x \quad 2.40$$

B.C. for DPC

$$C^T_{DPC} = C_{DPC,inj} = \text{constant}, \quad \text{at } 0 < t < \text{time for certain PV DPC injected and } x = 0 \quad 2.41$$

$$C^T_{DPC} = 0, \quad t > \text{time for certain PV DPC}, \quad x = 0 \quad 2.42$$

B.C. for SDS

$$C^T_{SDS} = 0, \quad t < \text{time to start SDS injection},$$

$$x = 0 < t < \text{time for certain PV SDS injected}, \quad x = 0 \quad 2.43$$

$$C^T_{SDS} = C_{SDS,inj} = \text{constant}, \quad \text{time to start SDS injection} < t < \text{time for}$$

$$\text{certain PV SDS injected}, \quad x = 0 \quad 2.44$$

$$C^T_{SDS} = 0, \quad t > \text{time to certain PV SDS injected}, \quad x = 0 \quad 2.45$$

The above reviewed models of formation damage missed some particulate processes such as pore throat blocking and bridging, and particle re-entrainment from pore wall. Clay swelling is also missing in these models. These processes play important roles in permeability reduction phenomena. Therefore, description of formation damage cannot be complete without considering these processes.

Chang and Civan (1992) have developed permeability reduction model that dealt with the said processes for single phase flow. Their model also incorporated inorganic scale formation due to geochemical reactions in porous media.

Liu (1994) developed a similar model to that of Chang and Civan (1992). His model covers two-phase flow in porous reservoir formation. The model did not consider geochemical reactions. It also does not consider clay swelling.

2.3.5 Chang and Civan Model

The references pertinent to the model are given by Chang and Civan (1992).

Mole Balance Equation:

Two types of species in the flowing suspension are considered: the dissolved species and the particulate matter. The mole balance of dissolved species is given by the following transient state, convection-diffusion equation

$$\frac{\partial(\phi C)}{\partial t} + \frac{\partial}{\partial x} \left[\left(u C - D \frac{\partial C}{\partial x} \right) \right] + \sigma = 0 \quad 0 \leq x \leq L, \quad t > 0 \quad 2.46$$

INITIAL AND BOUNDARY CONDITIONS:

$$C = C_o, \quad 0 \leq x \leq L, \quad t = 0 \quad 2.47$$

$$u C - D \frac{\partial C}{\partial x} = u_{inj} C_{inj} \quad 2.48$$

$$\frac{\partial C}{\partial x} = 0, \quad x = L, \quad t > 0 \quad 2.49$$

t and x denote the time and distance variables, respectively. L is the core length; u is the flowing solution volumetric flux. C is the mole concentration of a dissolved species in the

solution. σ is the rate of loss of species in the solution per unit bulk volume of porous medium due to the rock-fluid interactions. D is the dispersivity coefficient. ϕ is the porosity of the medium.

Neglecting the dispersion effect, the transport equation for particulate matter in the flowing solution is written as

$$\frac{\partial(\phi C_p)}{\partial t} + \frac{\partial(f_p u C_p)}{\partial x} + \sigma = 0 \quad 0 \leq x \leq L, \quad t > 0 \quad 2.50$$

Subject to the following initial and boundary conditions

$$C_p = C_{po}, \quad x = 0, \quad t > 0 \quad 2.51$$

$$f_p u C_p = f_{pinj} u_{inj} C_{inj}, \quad x = 0, \quad t > 0 \quad 2.52$$

f_p is a porous media particle transport efficiency factor representing the fraction of pore throats allowing particle transport due to the screening effect of porous matrix.

The liquid mass balance equation for the flowing solution is given by

$$-\frac{\partial(\rho_l u)}{\partial x} + w = \frac{\partial(\rho_l \phi)}{\partial t} \quad 2.53$$

Subject to the following initial and boundary conditions

$$u = u_o \quad 0 \leq x \leq L, \quad t = 0 \quad 2.54$$

$$u = u_{inj}, \quad x = 0, \quad t > 0$$

w is the mass rate of liquid loss per unit bulk volume of porous media. ρ_l is the density.

Momentum Balance Equation

Formation damage modelling often deals with flow through narrow paths because the pores and pore throats become narrower during particle deposition and pore throat plugging. Then, the fluid accelerates when it passes through the narrow passages. Therefore, the Darcy equation, which represents the linear relationship between pressure gradients and fluid flux, may no longer be valid in describing the real flow behaviour. Hence, the Forcheimer equation below is utilized as the momentum balance equation:

$$-\left(\frac{\partial p}{\partial x}\right) = (\mu/K)u + (1.013 \times 10^6)^{-1} \beta \rho u^2 \quad 2.55$$

μ is the fluid viscosity. K is the permeability of the porous medium. β is a non-Darcy coefficient. The empirical correlation for this coefficient is given by Geertsman (1974) as

$$\beta = 50 / (\phi^{5.5} K^{0.5}) \quad 2.56$$

For convenience, Equation 2.55 can be rewritten as

$$-\left(\frac{\partial p}{\partial x}\right) = (\mu/K)u N_{nd}$$

2.57

N_{nd} is the non-Darcy number given by

$$N_{nd} = 1 + (1.013 \times 10^6)^{-1} Ku\rho\beta / \mu \quad 2.58$$

After substitution and simplification we have

$$\frac{\partial}{\partial x} \left(\rho \frac{K}{N_{nd}} \frac{\partial p}{\partial x} \right) + \dot{w} = \frac{\partial(\phi\rho)}{\partial t}, \quad 0 \leq x \leq L, \quad t > 0 \quad 2.59$$

The permeability alteration equation was expressed in terms of porous media particle transport efficiency factor. The particle transport efficiency factor was expressed as a function of pore throat size distribution.

In the model equations four types of chemical reactions were considered in a geological formation. These are:

1. Dissolution and precipitation of alumino-silicate minerals
2. Dissolution and precipitation formed by metal ions and some ionic compounds.
3. Cation exchange on the surface of solid matrix.
4. Crystal nucleation and growth for the formation of precipitates.

The rate equations of these chemical processes were also considered to be part of the model.

2.3.6 Liu's Model

The references pertinent to this are given by Liu (1994). The following assumptions were made during model development.

1. The formation was assumed to be isotropic.
2. Processes were assumed to be isothermal.
3. One-dimensional and horizontal flow was assumed, where effects due to gravity were considered negligible.
4. Single-phase flow of mud filtrate occurs in the external filter cake.
5. Darcy flow through the filter cake was assumed.
6. Two-phase flow of both water and oil occurs in the formation.

7. Formation damage due to chemical reactions and organic deposits was not considered.

The model formulation procedure:

The same modelling procedure was followed as in the case of Chang and Civan model (1992). Two phase flow and transport of mineral salts were considered but geochemical reaction was not treated.

CHAPTER THREE

MODEL DEVELOPMENT AND NUMERICAL SOLUTION

3.1 Model Development

Several causes of petroleum reservoir formation damage have been discussed in the literature. In this chapter a mathematical model to account for various mechanisms causing permeability impairment in petroleum reservoirs is developed considering two-phase flow. The mechanisms considered include fines migration and insoluble scale deposition caused by chemical precipitation reaction. Transport equations of fluid phase flow and particulate matter based on general macroscopic continuity and momentum balance equations are derived. Different fluid-rock interactions are modelled by kinetic laws or empirical relations. Also a correlation relating particle retention to permeability alteration is included to establish a complete formation damage model.

3.1.1 Model Assumptions

Fluid and particulate flow through porous media is governed by the properties of particles, fluids and formations, and flow geometry. The following were the assumptions made for the model formulation:

1. Isotropic formation.
2. Isothermal processes throughout the system.
3. One-dimensional and horizontal flow was considered.
4. Two-phase flow of water and oil occurs in the reservoir.
5. There is no particle transfer at oil-water interface.
6. Solid particles are uniformly suspended in an incompressible fluid.

7. Variations of capillary pressure, relative permeabilities and dispersion coefficient are not considered during formation damage.
8. Single species of Particle is considered whose wettability is not altered during formation damage.
9. Interfacial drag force between water and oil phase is neglected.

3.1.2 Mole Balance Equation

Mass balance equation is one of the most essential parts of fluid flow through porous media. To keep track of a reacting specie concentration, mole balance is usually used. Mole balance can be obtained from mass balance by simply including the specie molecular weight.

Two types of species in the flowing suspension are considered: the dissolved species and the particulate matter. The dissolved specie considered here are divalent ions such as magnesium (Mg^{2+}) and calcium ion (Ca^{2+}) while the particulate species are the insoluble products of precipitation reaction such as calcite deposits. This liquid suspension is assumed to be flowing in one dimensional direction into the porous media. Fig. 3.1 in Section 3.1.6 depict the process.

The amount of a component in a solution is determined by the convection-diffusion process. During convection transport the component is moved by the bulk liquid while transportation by molecular concentration gradient occurs by diffusion.

Mole Balance of Dissolved Species

The mole balance of dissolved species is given by Cheng and Civan (1992) and modified here as:

$$\frac{\partial(\phi S_l C_l)}{\partial t} + \frac{\partial}{\partial x} \left[\left(u_l C_l - \phi S_l D_l \frac{\partial C_l}{\partial x} \right) \right] + \phi R_l = 0 \quad l = o, w \quad 3.1$$

Where u is the superficial velocity (cm/s), C_1 is the concentration of dissolved specie in solution (mol/cm³). ϕ is the porosity, S_l is the saturation of phase 1, D_l is the effective dispersion coefficient (cm²/sec). t and x are the time and distance variables respectively. R_l is the rate of loss of species in the solution per unit bulk volume of the porous media (mol/cm³sec).

The initial and boundary conditions of Equation (3.1) are:

$$C_1 = C_{1o}, \quad 0 \leq x \leq L, \quad t = 0 \quad 3.2a$$

$$u_1 C_1 - D_l \frac{\partial C_1}{\partial x} = u_{1,inj} C_{1,inj} \quad x = 0, \quad t > 0 \quad 3.2b$$

$$\frac{\partial C_1}{\partial x} = 0, \quad x = L, \quad t > 0 \quad 3.2c$$

Particulate Transportation

For the case of particle transport the diffusion term is neglected. As it was shown by Herzig et al (1970), particle dispersion is negligible when its diameter is larger than one micron.

Therefore, the continuity equation for particle transport p is given by

$$\frac{\partial(\phi S_l C_p)}{\partial t} + \frac{\partial(u_l C_p)}{\partial x} + R_{p,l} = 0 \quad l = o, w \quad 3.3$$

The following initial and boundary conditions hold for Equation 3.3:

$$C_p = C_{po}, \quad 0 \leq x \leq L \quad t = 0 \quad 3.4a$$

$$u_l C_p = U_{in} C_{p,in} \quad x = 0, t > 0 \quad 3.4b$$

Where C_p is the mass concentration of particle (g/cm³). $R_{p,l}$ is the net rate of loss of particle in fluid phase 1 (g/cm³ s).

Liquid Mass Balance Equation

The liquid mass balance equation for the two phase flow of water (w) and oil (o) is given by

$$\frac{\partial(\rho_l u_l)}{\partial x} + \frac{\partial(\rho_l S_l \phi)}{\partial t} = 0 \quad l = o, w \quad 3.5$$

Equation 5 is subject to the following initial and boundary conditions:

$$u = u_o, \quad 0 \leq x \leq L, \quad t = 0 \quad 3.6a$$

$$u = u_{inj}, \quad x = 0, \quad t > 0 \quad 3.6b$$

ρ_l is the density (gm/cm³) of fluid phase l containing particulate matter, u_l is the superficial velocity (cm/sec) of fluid phase l. In equation 3.5, the rate of fluid loss is assumed negligible.

The saturations add up to one as:

$$S_o + S_w = 1 \quad 3.7$$

3.1.3 Momentum Balance Equation

Darcy's Law is usually used as the momentum balance equation for fluid flow in porous media. Considering the possibility of the generation of inertial effects by rapid flow due to the narrowing of pores during formation damage, the pressure gradient for the two phase flow of oil and water is given by the extended Forchheimer equation as

$$-\frac{\partial p_l}{\partial x} = \frac{\mu_l}{K k_{rl}} u_l + \frac{\beta \rho_l}{1.0133 \times 10^6 k_{rl}} u_l^2 \quad l = o, w \quad 3.8$$

Where K is the absolute permeability of the porous media in Darcies, p_l , μ_l and k_{rl} are the pressure (atm), viscosity (cp) and relative permeability of fluid phase l. β (cm⁻¹) is the non-Darcy flow coefficient.

Rearranging Equation 8 in the form of Darcy Equation according to Liu (1994) yields

$$u_l = -f_{nd,l} \frac{Kk_{r,l}}{\mu_l} \frac{\partial p_l}{\partial x} \quad l = o, w \quad 3.9$$

Where $f_{nd,l}$ is the non-Darcy flow factor for the fluid phase l.

$$f_{nd,l} = \left(1 + \frac{K\beta\rho_l u_l}{1.0133 \times 10^6 \mu_l} \right)^{-1} \quad l = o, w \quad 3.10$$

β is given by Geertsman as

$$\beta = \frac{50}{\phi^{5.5} K^{0.5}} \quad 3.11$$

3.1.4 Determination of Fluid Saturations and Pressures

Substitute Equation 9 into Equation (3.5) to obtain

$$\frac{\partial(\rho_l S_l \phi)}{\partial t} - \frac{\partial}{\partial x} \left(\rho_l f_{nd,l} \frac{Kk_{r,l}}{\mu_l} \frac{\partial p_l}{\partial x} \right) = 0 \quad l = o, w \quad 3.12$$

The following boundary and initial conditions apply

$$p_l = p_o, \quad 0 \leq x \leq L \quad t = 0 \quad 3.13a^*$$

$$p_l = p_{inj}, \quad x = 0, \quad t > 0 \quad 3.13b^*$$

$$p = p_{out} \quad x = L, \quad t > 0 \quad 3.13c^*$$

The capillary pressure and relative permeability data are usually not readily available. The following correlation for relative permeability and saturation is used in this study.

$$k_{rl} = k_{rl}^o \left(\frac{S_l - S_{lr}}{1 - S_{or} - S_{wr}} \right)^{n_l} \quad l = o, w \quad 3.14$$

Where k_{rl}^o is the end-point relative permeability of phase l, S_{lr} is the irreducible saturation of phase l and n_l is the empirical constant for phase l.

The capillary pressure is defined as the difference between the non-wetting and wetting phase pressures according to

$$P_{cow} = P_o - P_w \quad 3.15$$

In a given porous medium, the capillary pressure is a function of fluid saturation and can be given by a Leverett J-function (Collins, 1976). In the present study, the capillary pressure-saturation curve will be represented by a three-constant hyperbolic function (Donaldson et al., 1991).

$$P_{cow} = \frac{x + y S_w}{1 + z S_w} \quad 3.16$$

Where x, y and z are constants determined by fitting experimental data to the correlation. The correlation offers some advantages in that it is readily differentiated, integrated or extrapolated for some mathematical applications. It also finds application in smoothening laboratory data (Liu, 1994).

The composite density for each fluid phase varies with concentration of particulate materials suspended in the phase. Liu (1994) gave an equation for this density which is modified here for a single particle in the flowing phase as

$$\rho_l = \rho_l^o + \left(1 - \rho_l^o / \rho_p \right) C_{p,l} \quad l = o, w \quad 3.17$$

Where ρ_l^o is the density (gm/cm³) of pure oil or water, ρ_p the density of the particle and $C_{p,l}$ is the mass concentration of the particle in gm/cm³.

Also the viscosity of fluid containing suspended particles was found to depend on the shape and concentration of the particles and is given by Einstein equation as follows Heimenz (1986):

$$\mu_l / \mu_l^o = 1 + 2.5C_{p,l} / \rho_p \quad l = o, w \quad 3.18$$

Where μ_l^o is the viscosity (cp) of pure oil or water.

3.1.5 Fines Migration

In this work, only a single type of particulate matter is assumed to be injected with the flowing liquid into the core. Insoluble solid scale is formed as a result of chemical precipitation. Particles can also be mobilised and entrained within the porous formation. The mean size of these in situ particles is always smaller than the mean diameter of the pores (Khiler and Fogler, 1987).

Porous media are usually thought to be made up of interconnected pores joined together by pore throats. Basically, two types of particle invasion in porous media have been identified: pore surface deposition and pore throat blocking (Tien and Paytakes, 1979, Gruesbeck and Collins, 1982 and Ohen and Civan, 1993). Pore surface deposition is relatively uniform deposition of particles on pore body while pore throat blocking is the plugging of pore throat by a single particle or by several minute particles that form bridge at the pore throats.

In view of Equation 3.3, particle deposition and mobilisation will be considered as the possible cause of its loss from each phase. Pore throat blockage will not be considered here because of its complexity. Also, particle transfer at fluid-fluid interface will be neglected.

Therefore, the rate of particle loss from each fluid phase is given as

$$R_{p,l} = \frac{\partial \sigma_{p,l}}{\partial t} \quad l = o, w \quad 3.19$$

Where $\sigma_{p,l}$ is the mass of particle specie available on the pore body in contact with the fluid phase l per unit bulk volume of porous media.

Fines Mobilisation from Pore Bodies

Hydrodynamic effect will be considered here as the only possible cause of particle release from pore bodies. The equation proposed by Liu (1994) is adopted and modified here for hydrodynamically induced mobilisation of in situ fines in sand stone formations. This is given as follows:

$$\frac{\partial \sigma_{fp,w}}{\partial t} = -k_{hp,w} \sigma_{fp,w} (u_w - u_{wc}) \quad 3.20$$

$$k_{hp,w} = 0 \quad \text{for} \quad u_w \leq u_{wc}$$

Subject to the following condition

$$\sigma_{fp,w} = \sigma_{fpo} \quad \text{for } t=0$$

Where $k_{hp,w}$ is a rate coefficient for hydrodynamically induced particle mobilisation and u_{wc} is the critical superficial velocity of water phase above which insitu particles can be released by hydrodynamic forces (Gruesbeck and Collins, 1982). $\sigma_{fp,w}$ is mass of fine particle available on the pore body per unit bulk volume of porous media.

Particle Capture

As mentioned earlier, particle deposition at pore bodies only will be considered here. Pore body deposition can occur in both oil and water phases (Liu, 1994). Also, as reported by Gruesbeck and Collins (1982) that for surface deposition there exist a critical velocity below which only particle retention takes place and above which both fine particle retention and entrainment occurs. A modification of Gruesbeck and Collins (1982) model for pore body deposition in two phase flow was suggested by Liu (1994) as follows:

$$\frac{\partial \sigma_{p,l}}{\partial t} = k_{d,p,l} u_l C_{p,l} - k_{e,p,l} \sigma_{p,l} (u_l - u_{lc}) \quad l = o, w \quad 3.21$$

$$k_{e,p,l} = 0 \quad \text{if} \quad u_l \leq u_{lc}$$

Initial condition:

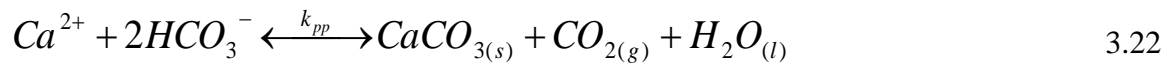
$$\sigma_{p,l} = \sigma_{p,l,0} \quad t=0$$

Where $k_{d,p,l}$ and $k_{e,p,l}$ are the rate constants for particle deposition and entrainment respectively in fluid phase l. u_{lc} is the critical velocity of fluid phase l.

3.1.6 Chemical Reaction (Precipitation)

As mentioned earlier, divalent ions are assumed to be dissolved in the fluid flowing in porous media. Amaefule et al. (1988) explain that induced scaling occurs by mixing of formation brine with extraneous incompatible fluids invading the reservoir formation during drilling, cementing, completion, and workover operations. The same may occur by injection of fluids for enhanced recovery purposes. Any increase of the dissolved calcium (Ca^{2+}) ion concentration caused by these operations is compensated by calcium carbonate (CaCO_3) precipitation, following Le Chatelier's cause-and-effect principle. In this work attention will

be given to the precipitation of calcite as a possible cause of formation damage. When there is high concentration of Ca^{2+} in the formation brine (as it is in the case of carbonate reservoirs) chemical slug (such as bicarbonates) introduced into the formation will mix with the formation brine and cause precipitation. The equation for the reaction that is assumed to be going on in the porous media is given by



$CaCO_3$ can exist as a particulate substance in the fluid as cementing material or as deposits on the pore wall. Ca^{2+} is believed to be dissolved in the formation fluid. Its source may include dissolutions of cemented minerals, suspended crystals or deposited crystal grains. In the present model, dissolution processes will not be considered because it is a very slow process and dissolution cannot occur within the time scale of flow in the near wellbore (Nordstrong and Munoz, 1994).

The diagram below depicts what is happening before and after the chemical reaction has taken place in the porous media:

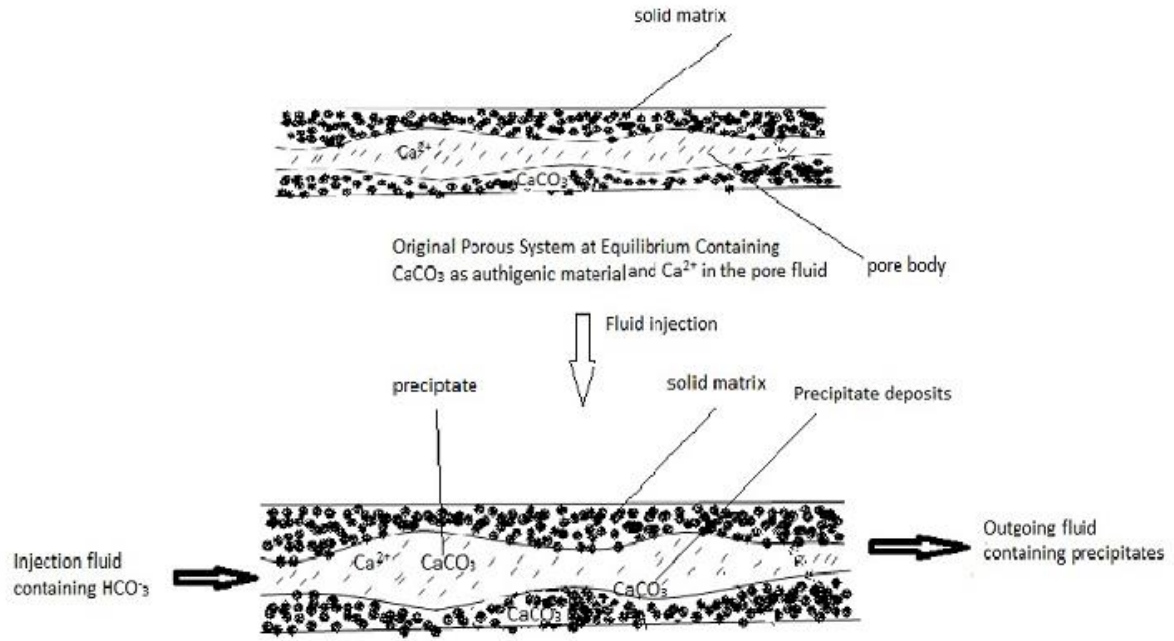


Fig. 3.1 Changes as a result of Fluid Injection in Porous Media

The rate of consumption of HCO_3^- in the reaction is given by

$$\frac{dC_{HCO_3^-}}{dt} = R_l = k_{dis} C_{CaCO_3} P_{CO_2} C_{H_2O} - k_{pp} C_{Ca^{2+}} C_{HCO_3^-}^2 \quad 3.23$$

The rate of calcite precipitation is given by

$$\frac{dC_{CaCO_3}}{dt} = -\frac{1}{2} R_l = \frac{1}{2} (k_{pp} C_{Ca^{2+}} C_{HCO_3^-}^2 - k_{dis} C_{CaCO_3} P_{CO_2} C_{H_2O}) \quad 3.24$$

Where k_{pp} and k_{dis} refer to the reaction rate constants for the precipitation and dissolution respectively, C_i is the concentration of specie i where $i=Ca^{2+}, HCO_3^-, CaCO_3, CO_2, H_2O$

If the rate of dissolution is slower than the rate of precipitation, that is, $k_{pp} \gg k_{dis}$

$$-R_l = \frac{dC_{CaCO_3}}{dt} = \frac{1}{2} k_{pp} C_{Ca^{2+}} C_{HCO_3^-}^2 \quad 3.25a$$

Because Ca^{2+} is in abundance compared to HCO_3^- , its (Ca^{2+}) concentration can be taken as a constant. Therefore, the rate becomes second order with respect to HCO_3^- concentration

$$-R_l = \frac{dC_{\text{CaCO}_3}}{dt} = \frac{1}{2}k_{pp}C_{\text{HCO}_3^-}^2 \quad 3.25b$$

With reference to Equations 3.1 and 3.3, Equation 3.25b can be written as

$$R_l = \frac{dC_p}{dt} = -\frac{1}{2}k_{pp}C_l^2 \quad 3.26$$

3.2 Permeability Alteration

Formation damage is usually characterised by permeability impairment. The effective porosity-permeability relationship is expressed by the power law function given by Civan et al. (1990).

$$\frac{k}{k_i} = \left(\frac{\phi}{\phi_i} \right)^3 \quad 3.27$$

K_i and ϕ_i refer to initial porosity and permeability respectively.

3.3 Porosity Variation

In order to predict the local porosity in Equation 3.27, the total porosity variation must be determined. The variation is as a result of processes considered earlier including particle deposition, entrainment and precipitation. Therefore, the total porosity variation is the summation of these rate processes. In the present work, the methodology presented by Chang and Civan (1992) for total porosity change is adopted and given below:

Porosity Change due to Particle Deposition

$$\left. \frac{\partial \phi}{\partial t} \right|_{d,pp} = -k_{dep} C_{p,l} \phi u_l - k_{pp} C_l^2 \quad 3.28$$

$$k_{dep} = k_d \frac{1}{\rho_p} \quad 3.29$$

$$k_{pp} = \frac{1}{2} k_{pp} \frac{M_p}{\rho_p} \quad 3.3$$

Porosity Change due to Particle Entrainment

$$\left. \frac{\partial \phi}{\partial t} \right|_{ent} = k_{ent} \sigma_{p,l} (u_l - u_{lc}) \quad 3.31$$

$$k_{ent} = k_{e,p,l} \frac{1}{\rho_p} \quad 3.32$$

Porosity Change due to fines Mobilisation

$$\left. \frac{\partial \phi}{\partial t} \right|_{fm} = k_{fm} \sigma_{fpw} (u_w - u_{wc}) \quad 3.33$$

$$k_{fm} = -k_{hp,w} \frac{1}{\rho_p} \quad 3.34$$

Where M_p and ρ_p are the molecular weight and density (g/cm^3) of the solid particles respectively.

The subscript d,pp , ent and fm denote deposition, entrainment and fines migration respectively.

Total Porosity Change

The overall porosity change due to particle deposition, entrainment and fines migration is therefore given as follows:

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial t} \Big|_{d,pp} + \frac{\partial \phi}{\partial t} \Big|_{ent} + \frac{\partial \phi}{\partial t} \Big|_{fm} \quad 3.35$$

3.4 Numerical Solution

Finite difference numerical scheme was used to solve the set of non- linear partial differential equations developed in the model. Density variation of fluids containing particulate suspensions was dealt with first. Central difference in space and implicit in time was used except at the outlet point. Implicit method for particulate transport equation and the IMPES (implicit pressure and explicit saturation) method for pressure-saturation equation were also used. A new porosity is calculated using initial conditions. Flow velocity is calculated next with the updated porosity in the liquid material balance. Rate equations were solved explicitly.

3.4.1 Porosity Change

Using explicit finite difference approximation to Equation (3.35), the discretised form of porosity variation equation is given as follows:

$$\phi_j^{n+1} = \phi_j^n + \Delta t \left(\frac{d\phi}{dt} \Big|_{d,pp} + \frac{d\phi}{dt} \Big|_{ent} + \frac{d\phi}{dt} \Big|_{fm} \right)^n \quad 3.36$$

The various changes are discretised from the equations developed in Section 3.3. From Equation 3.28, the porosity change due to deposition is approximated by

$$\left. \frac{d\phi}{dt} \right|_{d,pp}^n = -k_{dep} C_{p,l}^n \phi u_l^n - k_{pp} (C_l^n)^2 \quad 3.37$$

Similarly, the porosity change due to entrainment from Equation 3.31 is

$$\left. \frac{d\phi}{dt} \right|_{ent}^n = k_{ent} \sigma_{p,l}^n (u_l^n - u_{lc}) \quad 3.38$$

From Equation 33,

$$\left. \frac{d\phi}{dt} \right|_{fm}^n = k_{fm} \sigma_{fpw}^n (u_w^n - u_{wc}) \quad 3.39$$

3.4.2 Permeability Change

From Equation 3.27

$$k^{n+1} = k_i \left(\frac{\phi^n}{\phi_i} \right)^3 \quad 3.40$$

3.4.3 Fluid Density Variation

The fluid density of each phase changes during formation damage due to the change in particle concentration.

Expanding Equation 3.5

$$\frac{\partial(\phi S_l)}{\partial t} + \frac{\partial u_l}{\partial x} + \Omega_{\rho,l} = 0 \quad l = o, w \quad 3.41$$

Where

$$\Omega_{\rho,l} = \frac{1}{\rho_l} \left(\phi S_l \frac{\partial \rho_l}{\partial t} + u_l \frac{\partial \rho_l}{\partial x} \right) \quad l = o, w \quad 3.42$$

Differentiating the pressure-saturation Equation 3.12 and simplifying,

$$\frac{\partial(\phi S_l)}{\partial t} - \frac{\partial}{\partial x} \left(f_{nd,l} \frac{Kkr_l}{\mu_l} \frac{\partial \rho_l}{\partial x} \right) + \Omega_{\rho,l} = 0 \quad l = o, w \quad 0 \leq x \leq L \quad 3.43$$

From Equation 3.3 of particle transport in oil and water phases

$$u_l \frac{\partial C_{p,l}}{\partial x} + \phi S_l \frac{\partial C_{p,l}}{\partial t} = -R_{p,l} + C_{p,l} \Omega_{\rho,l} \quad l = o, w \quad 3.44$$

The convective-diffusive-reactive equation in Equation 3.1 is thus given as

$$\phi S_l \frac{\partial C_l}{\partial t} + u_l \frac{\partial C_l}{\partial x} - \frac{\partial}{\partial x} \left(\phi S_l D \frac{\partial C_l}{\partial x} \right) = -\phi R_l + C_l \Omega_{\rho,l} \quad l = o, w \quad 3.45$$

3.4.5 Finite Difference Scheme

Grid Size

Grid sizes are chosen to equal and represented by Δx where

$$\Delta x = \frac{L}{N-1} \quad 3.46$$

L- length of the core

N- number of divisions

Time Step

If t^n and t^{n+1} are the new and old time respectively and the time step size is Δt

$$t^{n+1} = t^n + \Delta t \quad 3.47$$

Equation 3.43 can be written for each phase. For oil phase

$$\frac{\partial(\phi S_o)}{\partial t} - \frac{\partial}{\partial x} \left(f_{nd,o} \frac{Kkr_o}{\mu_o} \frac{\partial p_o}{\partial x} \right) + \Omega_{\rho,o} = 0 \quad 3.48$$

For water phase

$$\frac{\partial(\phi S_w)}{\partial t} - \frac{\partial}{\partial x} \left(f_{nd,w} \frac{Kkr_w}{\mu_w} \frac{\partial p_w}{\partial x} \right) + \Omega_{\rho,w} = 0 \quad 3.49$$

Now, if we define mobility ratio as

$$\lambda_l = \frac{f_{nd,l} Kkr_l}{\mu_l} \quad l = o, w \quad 3.50$$

Equations 3.48 and 3.49 becomes

$$\frac{\partial(\phi S_o)}{\partial t} - \frac{\partial}{\partial x} \left(\lambda_o \frac{\partial p_o}{\partial x} \right) + \Omega_{\rho,o} = 0 \quad 3.51$$

$$\frac{\partial(\phi S_w)}{\partial t} - \frac{\partial}{\partial x} \left(\lambda_w \frac{\partial p_w}{\partial x} \right) + \Omega_{\rho,w} = 0 \quad 3.52$$

The IMPES method is applied where the pressure equation is solved implicitly and the saturation equation is solved explicitly.

Add Equations 3.51 and 3.52

$$\frac{\partial}{\partial t} [\phi(S_o + S_w)] - \frac{\partial}{\partial x} \left(\lambda_o \frac{\partial p_o}{\partial x} + \lambda_w \frac{\partial p_w}{\partial x} \right) + \Omega_{\rho,o} + \Omega_{\rho,w} = 0 \quad 3.53$$

Recall Equation 7 for saturation and Equation 3.15 for capillary pressure

$$p_o = p_{cow} + p_w \quad 3.54$$

Substitute Equations 3.7 and 3.54 into Equation 3.53

$$\frac{\partial}{\partial x} \left[(\lambda_o + \lambda_w) \frac{\partial p_w}{\partial x} \right] = \frac{\partial \phi}{\partial t} - \frac{\partial}{\partial x} \left(\lambda_o \frac{\partial p_{cow}}{\partial x} \right) + \Omega_{\rho,o} + \Omega_{\rho,w} \quad 3.55$$

A central difference approximation is applied to the second derivative of Equation 3.55

$$\frac{\partial}{\partial x} \left[(\lambda_o + \lambda_w) \frac{\partial p_w}{\partial x} \right] = \frac{(\lambda_o + \lambda_w)^n \frac{\partial p_w}{\partial x} \Big|_{j+\frac{1}{2}}^{n+1} - (\lambda_o + \lambda_w)^n \frac{\partial p_w}{\partial x} \Big|_{j-\frac{1}{2}}^{n+1}}{\Delta x} \quad 3.56$$

Therefore,

$$\frac{\partial}{\partial x} \left[(\lambda_o + \lambda_w) \frac{\partial p_w}{\partial x} \right] = \frac{1}{\Delta x} \left[(\lambda_o + \lambda_w)^n \frac{p^{n+1}_{wj+1} - p^{n+1}_{wj}}{\Delta x} - (\lambda_o + \lambda_w)^n \frac{p^{n+1}_{wj} - p^{n+1}_{wj-1}}{\Delta x} \right]$$

For $j=2, \dots, N-1$ 3.57

Constant Pressure Condition:

The inner boundary condition is given as

$$p^{n+1}_{w1} = p_{inj} \quad 3.58$$

The outer boundary condition is also given as

$$p^{n+1}_{wN} = p_{out} \quad 3.59$$

The inner boundary condition can also be specified as a constant velocity

$$-(\lambda_o + \lambda_w)^n_1 \frac{p^{n+1}_{w2} - p^{n+1}_{w1}}{\Delta x} = u_{inj} + \lambda^n_{o1} \frac{p^n_{cow2} - p^n_{cow1}}{\Delta x} \quad 3.60$$

From the definition of mobility ratio in Equation 3.50 and with reference to Equation 3.57

$$(\lambda_l)_{j+\frac{1}{2}} = (f_{nd,l})_{j+\frac{1}{2}} \frac{K_{j+\frac{1}{2}}(kr_l)_{j+\frac{1}{2}}}{(\mu_l)_{j+\frac{1}{2}}} \quad 3.61$$

The parameters $(f_{nd,l})_{j+1/2}$, $K_{j+1/2}$ and $(\mu_l)_{j+1/2}$ are mean values at the grid points j and $j+1$ for $j=1, \dots, N-1$. Therefore, $(\lambda_l)_{j+\frac{1}{2}}$ should be computed as an average value. Evaluating the absolute permeability, non-Darcy factor and viscosity as arithmetic averages, we have

$$K_{j+\frac{1}{2}} = \frac{K_j + K_{j+1}}{2} \quad 3.62$$

One- point upstream weighting method is applied to the relative permeability

$$(kr_l)_{j+\frac{1}{2}} = (kr_l)_j \quad 3.63$$

$$(\mu_l)_{j+\frac{1}{2}} = \frac{(\mu_l)_j + (\mu_l)_{j+1}}{2} \quad 3.64$$

For $(\lambda_l)_{j-\frac{1}{2}}$, the arithmetic averages are computed at $j-1$ and j .

The terms on the right hand side of Equation 3.55 are evaluated explicitly at the old time level.

$$(\Omega_{\rho,l})^n_j = \frac{1}{\rho^n_{lj}} \left(\phi^n_j S^n_{lj} \frac{\rho^n_{lj} - \rho^{n-1}_{lj}}{\Delta t} + u^n_{lj} \frac{\rho^n_{lj+1} - \rho^n_{lj-1}}{2\Delta x} \right) \quad 3.65$$

For $j=2, \dots, N-1$

But for $j=1$, the density derivative is replaced by a forward difference approximation

$$(\Omega_{\rho,l})^n_1 = \frac{1}{\rho^n_{l1}} \left(\phi^n_1 S^n_{l1} \frac{\rho^n_{l1} - \rho^{n-1}_{l1}}{\Delta t} + u^n_{l1} \frac{\rho^n_{l2} - \rho^n_{l1}}{\Delta x} \right) \quad 3.66$$

For $j=N$, the spatial derivative of the density is replaced by a backward difference

$$\left(\Omega_{\rho,l}\right)^n_N = \frac{1}{\rho^{n}_{IN}} \left(\phi^n_N S^n_{IN} \frac{\rho^{n}_{IN} - \rho^{n-1}_{IN}}{\Delta t} + u^n_{IN} \frac{\rho^{n}_{IN} - \rho^{n}_{IN-1}}{\Delta x} \right) \quad 3.67$$

The capillary pressure in 3.55 which is a second- order derivative is approximated by a central difference as

$$\frac{\partial}{\partial x} \left(\lambda_o \frac{\partial p_{cow}}{\partial x} \right)^n_j = \frac{1}{\Delta x} \left[(\lambda_o)^n_{j+\frac{1}{2}} \frac{\partial p_{cow}}{\partial x} \Big|_{j+\frac{1}{2}} - (\lambda_o)^n_{j-\frac{1}{2}} \frac{\partial p_{cow}}{\partial x} \Big|_{j-\frac{1}{2}} \right] \quad 3.68$$

$$\frac{\partial}{\partial x} \left(\lambda_o \frac{\partial p_{cow}}{\partial x} \right)^n_j = \frac{1}{\Delta x} \left[(\lambda_o)^n_{j+\frac{1}{2}} \frac{(p_{cow})^n_{j+1} - (p_{cow})^n_j}{\Delta x} \right] - (\lambda_o)^n_{j-\frac{1}{2}} \frac{(p_{cow})^n_j - (p_{cow})^n_{j-1}}{\Delta x} \quad 3.69$$

For the time-dependent porosity in Equation 3.55

$$\left(\frac{\partial \phi}{\partial t} \right)^n_j = \frac{\phi^n_j - \phi^{n-1}_j}{\Delta t} \quad 3.70$$

For $j=1, \dots, N$

The water saturation is obtained explicitly by re-writing Equation 3.52

$$\phi \frac{\partial S_w}{\partial t} + S_w \frac{\partial \phi}{\partial t} = \left(\frac{\partial}{\partial x} \lambda_w \frac{\partial p_w}{\partial x} \right) - \Omega_{\rho,w} \quad 3.71$$

Now discretizing Equation 3.71 will lead to

$$\phi^n_j \frac{S^{n+1}_{wj} - S^n_{wj}}{\Delta t} + S^{n+1}_{wj} \frac{\phi^n_j - \phi^{n+1}_j}{\Delta t} = \left(\frac{\partial}{\partial x} \lambda_w \frac{\partial p_w}{\partial x} \right)^n_j - \left(\Omega_{\rho,w}\right)^n_j \quad 3.72$$

For $j=1, 2, \dots, N$

Also, approximating equation by central difference

$$(u_l)^{n+1}_j = (-\lambda_l)^{n+1}_j \frac{(p_l)^{n+1}_{j+1} - (p_l)^{n+1}_{j-1}}{2\Delta x} \quad l = o, w \quad 3.73$$

For j=1, forward difference approximation is applied

$$(u_l)^{n+1}_1 = (-\lambda_l)^{n+1}_1 \frac{(p_l)^{n+1}_2 - (p_l)^{n+1}_1}{\Delta x} \quad l = o, w \quad 3.74$$

For N=1, the velocity is approximated by backward difference

$$l = o, w \quad 3.75$$

For the particle transportation, with reference to Equation 3.44

$$\begin{aligned} (u_l)^{n+1}_N &= (-\lambda_l)^{n+1}_N \frac{(p_l)^{n+1}_N - (p_l)^{n+1}_{N-1}}{\Delta x} \\ (u_l)^{n+1}_j &= \frac{(C_{p,l})^{n+1}_{j+1} - (C_{p,l})^{n+1}_{j-1}}{2\Delta x} + \phi^n_j (S_l)^{n+1}_j \frac{(C_{p,l})^{n+1}_j - (C_{p,l})^n_j}{\Delta t} \quad l = o, w \quad 3.76 \\ &= -(R_{p,l})^n_j + (C_{p,l})^n_j (\Omega_{\rho,l})^n_j \end{aligned}$$

For j=2,....., N-1

But for j=1, and considering no-flux condition, that is,

$$\frac{\partial C_{p,l}}{\partial x} = 0 \quad 3.77$$

$$\phi^n_1 S^{n+1}_{l1} \frac{(C_{p,l})^{n+1}_1 - (C_{p,l})^n_1}{\Delta t} = -(R_{p,l})^n_1 + (C_{p,l} \Omega_{\rho,l})^n_1 \quad 3.78$$

For j=N, using the forward difference approximation to Equation 3.44

$$\begin{aligned}
& (u_l)^{n+1} \frac{(C_{p,l})^{n+1}_N - (C_{p,l})^{n+1}_{N-1}}{\Delta x} + \phi^n_N (S_l)^{n+1}_N \frac{(C_{p,l})^{n+1}_N - (C_{p,l})^n_N}{\Delta t} \\
& = -(R_{p,l})^n_N + (C_{p,l})^n_N (\Omega_{\rho,l})^n_N
\end{aligned} \tag{3.79}$$

For the solution to the concentration of dissolved species, Equation 1 is discretised as follows.

From Equation 3.45

$$\begin{aligned}
& \phi_j S_{lj} \frac{C^{n+1}_{lj} - C^n_{lj}}{\Delta t} + u_{lj} \frac{C^{n+1}_{lj+1} - C^{n+1}_{lj-1}}{2\Delta x} \\
& - \frac{(\phi S_l D)_{j+\frac{1}{2}} (C^{n+1}_{lj+1} - C^{n+1}_{lj}) - (\phi S_l D)_{j-\frac{1}{2}} (C^{n+1}_{lj} - C^{n+1}_{lj-1})}{(\Delta x)^2} = (C_l \Omega_{\rho,l})^n_j
\end{aligned} \quad l = o, w \tag{3.80}$$

Inner boundary condition:

$$u^{n+1}_{l1} C^{n+1}_{l1} - \phi_l S_{l1} D \frac{C^{n+1}_{l2} - C^{n+1}_{l1}}{\Delta x} = u_{linj} C_{linj} \tag{3.81}$$

Outer boundary condition:

For the outer boundary condition (j=N), let

$$C_{N+1} = C_{N-1} \tag{3.82}$$

Then,

$$(\phi S_l)_N \frac{C^{n+1}_{lN} - C^n_{lN}}{\Delta t} - 2(\phi S_l D)_N \frac{C^{n+1}_{lN-1} - C^{n+1}_{lN}}{(\Delta x)^2} = (C_l \Omega_{\rho,l})^n_N \tag{3.83}$$

Rate Equations:

For the rate of deposition and entrainment, using Equation 3.21

$$\left(\frac{d\sigma_{p,l}}{dt}\right)^{n+1}_j = k_{d,p,l}(u_l C_{p,l})^{n+1}_j - k_{e,p,l}(\sigma_{p,l})^n_j (u^{n+1}_{lj} - u_{lc}) \quad 3.84$$

The quantity of particles deposited on pore bodies is given by

$$(\sigma_{p,l})^{n+1}_j = (\sigma_{p,l})^n_j + \Delta t \left(\frac{d\sigma_{p,l}}{dt}\right)^{n+1}_j \quad 3.85$$

For the rate of precipitation

$$\left(\frac{dC_{p,l}}{dt}\right)^{n+1}_j = -\frac{1}{2} k_{pp} (C^{n+1}_{lj})^2 \quad 3.86$$

Amount of precipitate formed

$$(C_{p,l})^{n+1}_j = (C_{p,l})^n_j + \Delta t \left(\frac{dC_{p,l}}{dt}\right)^{n+1}_j \quad 3.87$$

Rate of fines mobilisation is also approximated in a similar way as

$$\left(\frac{\partial\sigma_{fp,w}}{\partial t}\right)^{n+1}_j = -k_{hp,w}(\sigma_{fp,w})^n_j (u^{n+1}_{wj} - u_{wc}) \quad 3.88$$

Amount of fines mobilised

$$(\sigma_{f,pw})^{n+1}_j = (\sigma_{f,pw})^n_j + \Delta t \left(\frac{\partial\sigma_{fp,w}}{\partial t}\right)^{n+1}_j \quad 3.89$$

The simulation procedure is summarised as follows:

1. Rock and fluid properties values are initialised.
2. Grid is defined
3. Pressure equation is solved using Thomas algorithm.

4. Oil and water saturations are calculated.
5. Superficial velocities are computed.
6. Dissolved species concentrations are calculated.
7. Particle transport equation is solved using Thomas algorithm.
8. The equations are solved.
9. Porosity and permeability variations are calculated.
10. The time is updated and steps 3-9 are repeated.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

The first simulation involved formation damage due to in situ fines migration in two-phase flow as conducted by Sarker (1988). As reported by Liu (1994), Saker's experiment involved oil flooding to establish connate water saturation. The core was displaced with fresh water which caused formation damage due to in-situ fines migration. Although, in the present model, the injection fluid contains bicarbonates, Liu's work was still used for validation because of unavailability of established literature data. The simulation was carried out using the model parameter values as given in Table 4.1. Fig. 4.1 shows the pressure drop and pore volume injected. The pore volume injected was calculated from the porosity variation and core dimensions. A matching trend to some extent was obtained between the present model and that of Liu (1994). There is quite deviation between the two models as the volume of injecting fluid reaches about 0.8 cm³.

Table 4.1: Input Model Parameters for Fines Migration

Parameter	Value
σ_{fp,w_o} (gm/Cm3)	0.0188
D (mole/sec)	1.0×10^{-3}
$k_{hp,w}$ (cm ⁻¹)	1×10^{-4}

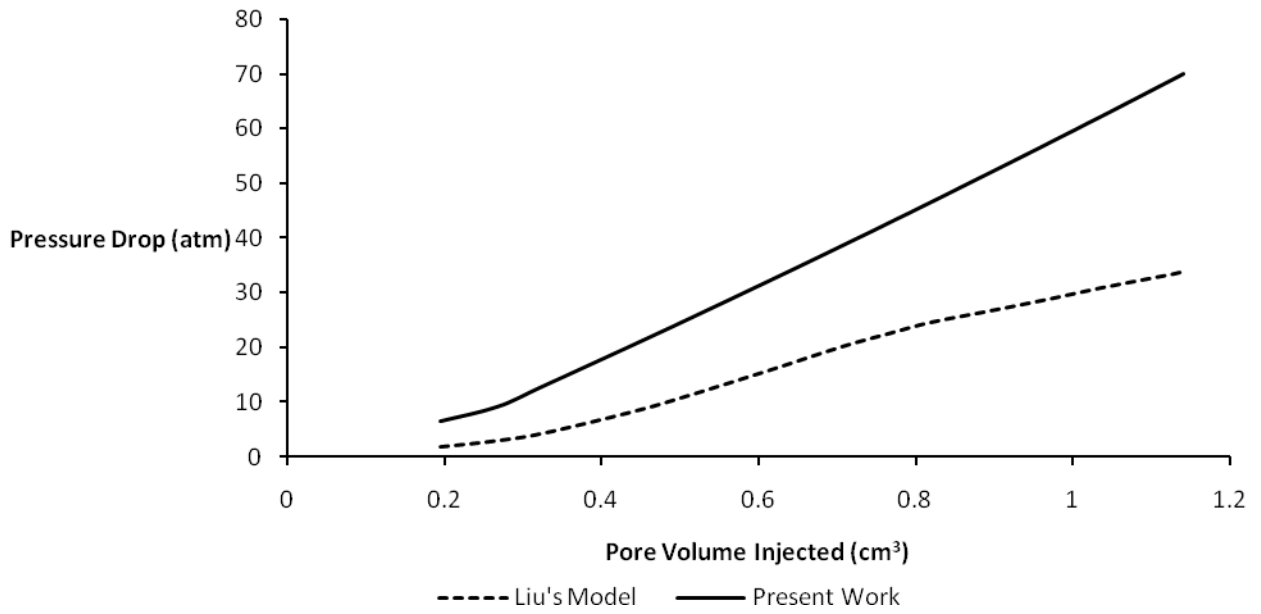


Fig. 4.1: Pressure Drop across Damaged Core for Two-Phase Flow

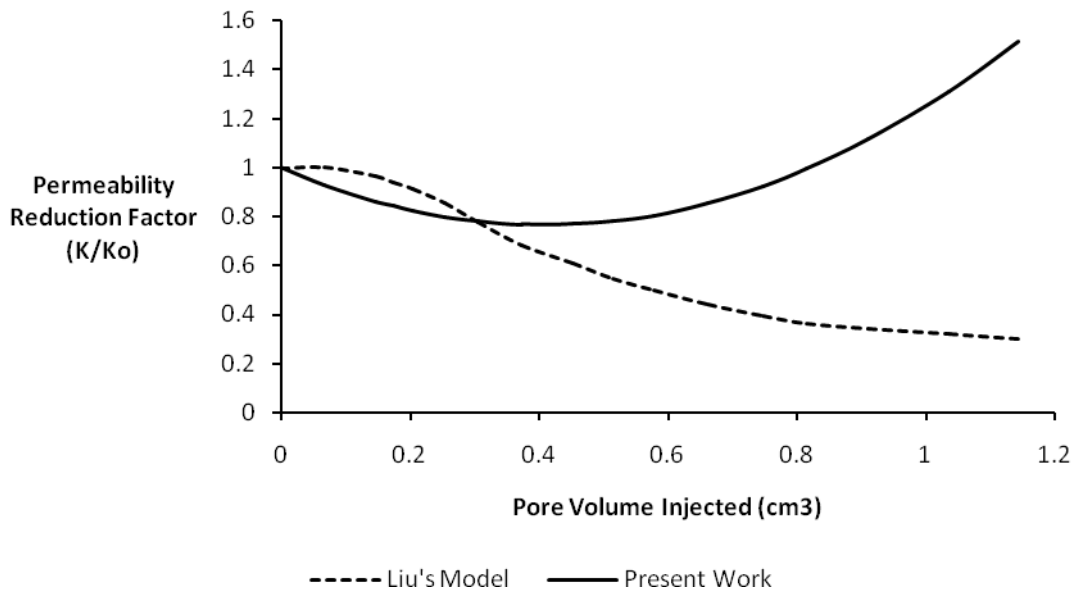


Fig. 4.2: Permeability Reduction of Damaged Core for Two-Phase Flow

The effect of formation damage on permeability is shown in Fig. 4.2. It shows that simulation results match Liu's model at the beginning of the injection. A large deviation in the two trends occurs at injection volume of about 0.35 cm^3 . For the case of Liu, the permeability keeps on decreasing until it reaches a particular value where it remains barely constant. For the present work, the permeability as can be seen in the figure decreases initially as well due to the effect of formation damage. But an abnormal behaviour is observed in the results where the permeability that is supposed to be decreasing due to impairment is seeing to be increasing. There are two possible reasons to this anomalous observation. The first is that pore throat blocking which is an important factor when considering permeability impairment in porous media was neglected in the modelling procedure. Secondly, only hydrodynamic effect was considered for particle release. The effect of critical salt concentration was not included. Another possible explanation to this deviation is that Liu's simulation was based on fresh water injection while in this work bicarbonate injection was considered.

In Fig. 4.3, the concentration of effluent fines is predicted. There is also an agreement between the two models at early stage of injection. The same reason for permeability deviation could be applied for this deviation at the later stage of injection.

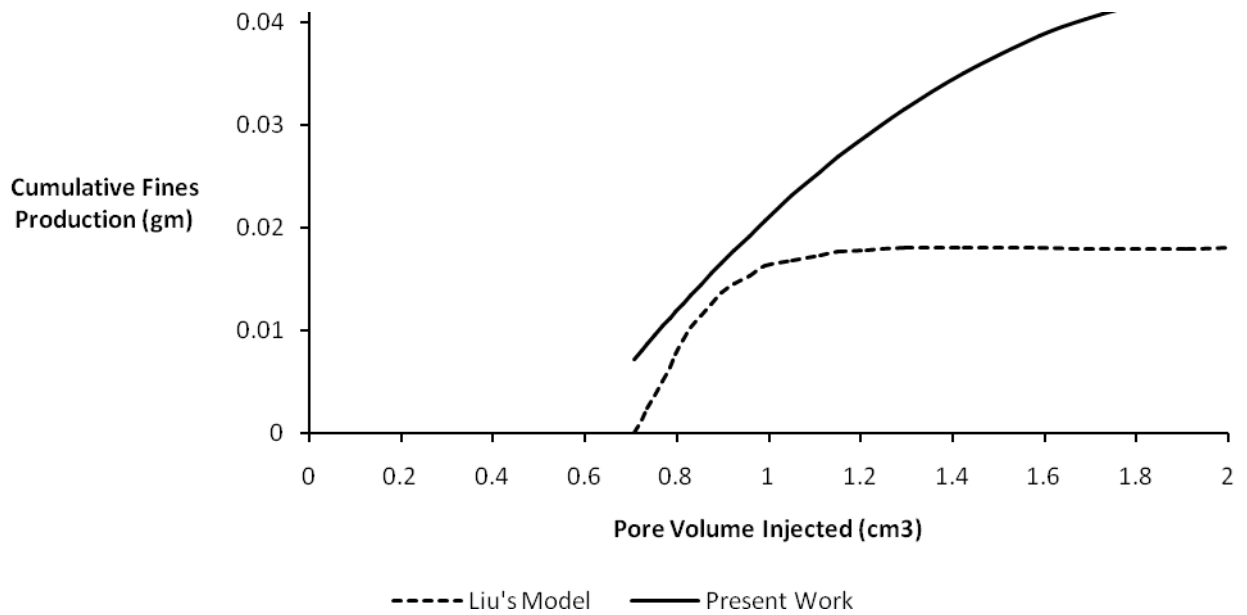


Fig. 4.3: Cumulative Fines Production during Two-Phase Flow of a Damaged Core

The model was further tested for chemical precipitation reaction. The work of Chang and Civan (1992) is used for the model validation. The validity of their work was demonstrated by using experimental data obtained by Read and Ringen (1982). The data involves the sea water injection plan for a North Sea oil reservoir. Chang and Civan (1992) model considers formation damage by inorganic chemical precipitation but for single-phase flow. Despite the fact that their model treated only single-phase flow, it is used here for the scarcity of literature data for the case of two-phase flow. The parameters used for this simulation are taken from the chemical analyses by Read and Ringen (1982). The properties of the core are given in Table 4.2. The estimated model parameters are given in Table 4.3. The outlet concentration of CaCO₃ precipitates is given in Fig. 4.4. A common trend is observed to some extent between the two models. The amount of CaCO₃ precipitates formed during the chemical reaction indicates the process of permeability reduction by solid deposition. The simulated porosity and permeability are respectively given in Fig. 4.5 and Fig. 4.6 and comparison is also shown with literature work.

Table 4.2: Properties of the Rock and Injection Rate by Read and Ringen (1982)

Property	Unit	Value
Core length	cm	10.16
Core diameter	cm	3.81
Porosity	%	34
Permeability	Darcy	3
Injection rate	cm ³ /sec	0.0965

Table 4.3: Estimated Rate Constants for CaCO₃ by Chang and Civan (1992) for the Case Study of Read and Ringen's Data

Type of Rate Constant	Unit	Value
Precipitation	cm ³ /(mol.sec)	1.0039X10 ⁻¹
Deposition	cm ⁻¹	1.8232X10 ⁻⁴
Entrainment	cm ⁻¹	2.1642X10 ⁻³

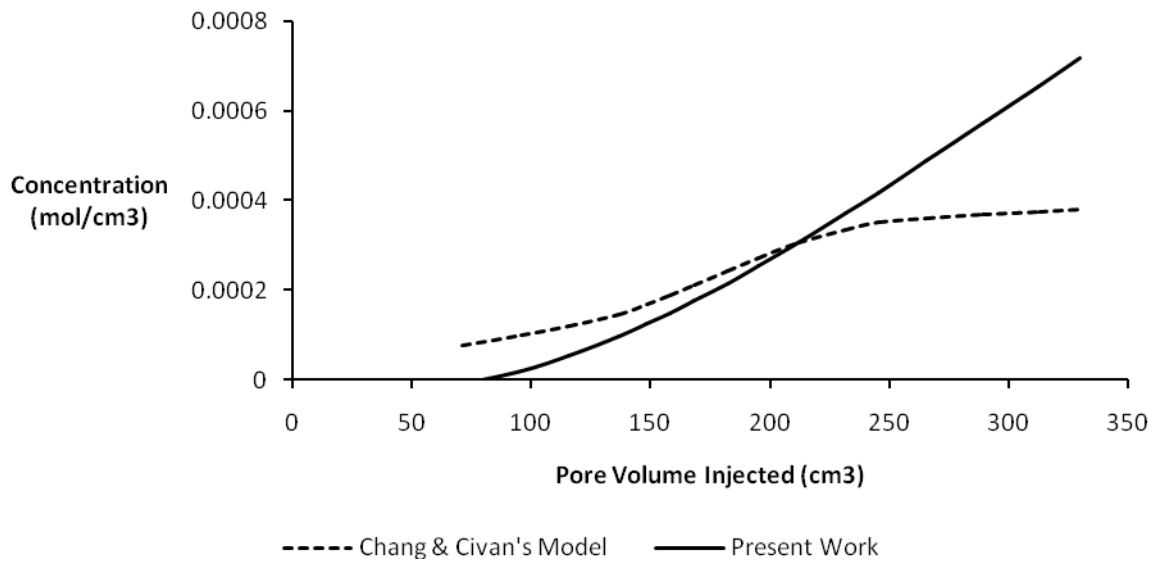


Fig. 4.4: CaCO₃ Precipitates Concentration for the Solution Using Read and Ringen's Data

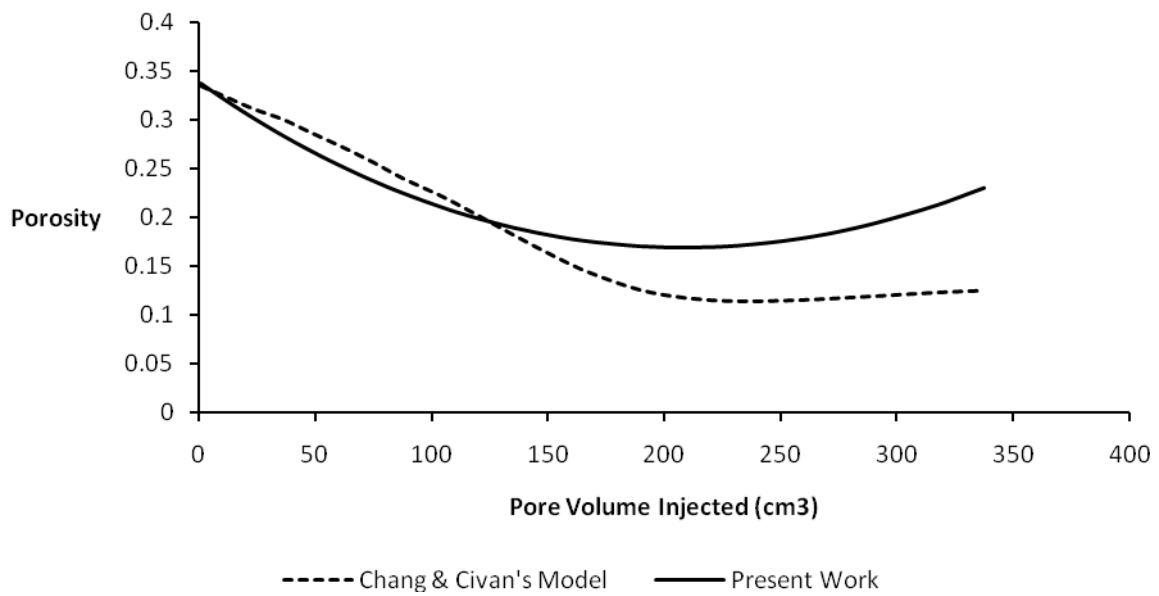


Fig. 4.5: Porosity Reduction of the Porous Media for the Solution Using Read and Ringen's

Data

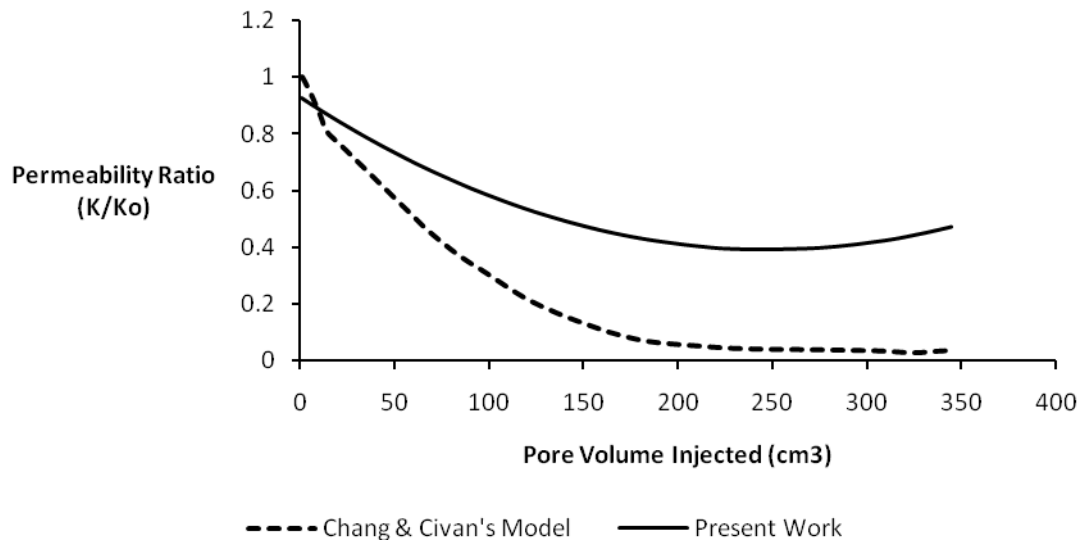


Fig. 4.6: Permeability Reduction of the Porous Media for the Solution Using Read and Ringen's Data

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this work a detailed model for petroleum formation damage due to chemical precipitation and fines migration during two-phase flow was developed. Only calcium carbonate was assumed to be the particulate matter that is formed as a result of the reaction. It is also the only fine attachment that can be dislodged from the solid rock structure. Hydrodynamic force was assumed to be the only factor causing fines mobilisation. For the permeability impairment, only pore body deposition was modelled. Permeability reduction caused by pore throat blocking was not taken into consideration. In general, a macroscopic approach to material and momentum balance was applied to the fluid and particle during the model development. Chemical reaction balance was also taken for the geochemical reaction. Different fluid-rock interactions are modelled by kinetic laws or empirical relations as was done in various literatures. A correlation relating permeability and porosity change was adopted to complete the model.

A numerical method was used for the solution to the model. The model was discretised using finite difference scheme. Thomas algorithm was used as the numerical method for the solution. The simulation was performed using Visual Basic programming language. The simulation results were compared using available models in the literature. Although exact modelling phenomena have not been found in the literature, a reasonable match was found for the various cases considered. A major cause of the mismatch between the present work and literature models is omission of pore throat blocking process in the model development. Never the less, the model was found to be a useful tool for predicting formation damage caused by inorganic deposition and fines migration during two phase flow of oil and water. In

fact, it can be said with high degree of confidence that this is the first type of formation damage model that incorporated geochemical reactions and fines migration during two phase flow.

5.2 Recommendations

On the basis of findings from this research work, the following further studies are recommended to be carried out.

1. Pore throat blocking which is an essential part of permeability impairment mechanism should be considered in the model development. The effect of salt concentration and pH on particle mobilisation and capture should also be incorporated.
2. The model can be extended to include other types of geochemical reactions such as dissolution and precipitation of alumina-silicate minerals, cation exchange, and crystal growth and nucleation. The effects of organic precipitation should be incorporated too.
3. The model should be modified to include different types of particles. Effect of particle wettability is highly recommended for consideration.
4. The model was developed for one-dimension. Future model should be developed to include reservoir multi-dimensional structure.
5. A parameter estimation technique should be used to determine the model parameters. A well designed laboratory experiment is recommended for the measurement of the system parameters. The results obtained from this experiment are to be compared with the estimated values. Sensitivity analysis of the system parameters should also be carried out with the aim of establishing the most sensitive system parameters which are then ranked.

REFERENCES

Amaefule, J. O., Kersey, D. G., Norman, D. L., & Shannon, P. M., "Advances in Formation Assessment and Control Strategies" CIM Paper No. 88-39-65., Proceedings of the 39th Annual Technical Meeting of Petroleum Society of CIM and Canadian Gas Processors Association, June 12-16, 1988, Calgary, Alberta, pp. 16.

Arshad, S. A., "A Study of Surfactant Precipitation in Porous Media with Applications in Surfactant-Assisted Enhanced Oil Recovery Processes", PhD dissertation, U. of Oklahoma, Norman, OK (1991).

Bhuyan, D., Lake, L. W., and Pope, G. A.: "Mathematical Modelling of High-pH Chemical Flooding", SPE Reservoir Engineering (May 1990) 213-220.

Briant, S. L. And Buller, D. C., "Formation Damage from Acid Treatments" paper SPE 17597 presented at the SPE International Meeting on Petroleum, Tianjin, China, Nov. 1-4, 1988.

Chadman, J., Hoff, D., Merino, E., Ortoleva, P., and Sen, A.: "Reactive Infiltration Instabilities" IAM J. Applied Math. (1986) 36, 207-221.

Chang, F. and Civan, F. "Predictability of Formation Damage by Modeling Chemical and Mechanical Processes", paper SPE 23793 presented at the SPE International Symposium on Formation Damage Control held in Lafayette, Louisiana, Feb. 26-27, 1992. Pp. 1-7.

Civan, F. "Predictability of Formation Damage: An Assessment Study and Generalised Models, Final Report, U.S. DOE Contract No. DE-AC22-90-BC14658, April 1994.

Civan, F. and Knapp, R. M. "Effect of Clay Swelling and Fines Migration on Formation Permeability", paper SPE 16235 presented at the SPE Production Operation Symposium, Oklahoma City, OK. Mar. 8-10, 1987.

Civan, F., Knapp, R. M., and Ohen, H. A. "Alteration of Permeability by Fine Particle Processes" J. of Petroleum Science and Engineering, (1989) 3, 65-79.

Collins, R. E., "Flow of Fluids through Porous Materials" PennWell Books, Tulsa, Oklahoma, 1976, pp. 31.

Crowe, C. W., "Evaluation of Agents for Preventing Precipitation of Ferric Hydroxide from Spent Treating Acid" JPT (Apr. 1985) 691-695.

Energy Highlights, "Formation Damage Control in Petroleum Reservoirs" article provided by F. Civan, The University of Oklahoma Energy Centre, Vol. 1, No. 2, p. 5, summer 1990.

Fogler, H. S., Lund, K., and McCune, C. C., "The Flow and Reaction of HCl/HF Acid Mixtures in Porous Sandstone Cores" SPEJ (October 1976) 248-260; Trans., AIME, 261.

Geertsman, J., "Estimating the Coefficient of Inertial Resistance in Fluid Flow Through Porous Media, "SPEJ (October 1974) 445-450.

Gruesbeck, C. and Collins, R. E. "Entrainment and Deposition of Fines Particles in Porous Media" SPEJ (Dec. 1982a), 847-856.

Gruesbeck, C. and Collins, R. E. "Particle Transport Through Perforations" SPEJ (Dec. 1982b), 857-865.

Hekim, Y. and Fogler, H. S. "On the Movement of Multiple Reaction Zones in Porous Media" AIChE J. (1980) **26**, No. 3, 403-411.

Herzig, J. P., Leclerc, D. M. and LeGoff, P., “Flow of Suspensions through Porous Media – Application to Deep Bed Filtration”, *Industrial and Engineering Chemistry*, Vol. 62, No. 5, 1970, pp. 8-35.

Hiemenz, P. C., “Principles of Colloid and Surface Chemistry” Marcel Dekker, Inc., New York and Basel, 1986, pp. 62-64.

Iwasaki, T., “Some Notes on Sand Filtration” *Journal of American Water Works Association*, Vol. 29 (1937) p. 1591-1602.

Khilar, K. C. and Fogler, H. S., “Water Sensitivity of Sandstones” *Society of Petroleum Engineers Journal*, Feb. 1983, pp. 55-64.

Krueger, R. F., “Overview of Formation Damage and Well Productivity in Well Oilfield Operations”, *Journal of Petroleum Technology*, Feb. 1986, pp. 131-152.

Krumrine, P. H., Mayer, E. H., and Brock, G. F., “Scale Formation During Alkaline Flooding” *JPT* (Aug. 1985) 1466-1474.

Liu, Xinghui “ Modeling of Formation Damage Due to Particulate Processes in Two-Phase Flow Systems” PhD dissertation, U. Of Oklahoma, Norman, OK (1994).

Lund, K. and Fogler, H. S., “Acidisation – V, The Prediction of The Movement of Acid and Permeability Fronts in Sandstone” *Chem. Eng. Sci.* (1976) 31, 381-392.

McCune, C. C., Fogler, H. S., Lund, K., Cunningham, J. R., and Ault, J. W., “A New Model of The Physical and Chemical Changes in Sandstone During Acidising” *SPEJ* (October 1975) 361-370.

Moore, J. E., “Clay Mineralogy Problems in Oil Recovery” *Petroleum Engineer*, Vol. 32, 78 (1960)

Nordstrom, D. K., and Munoz, J. L., "Geochemical Thermodynamics, 2nd ed., Blackwell Scientific Publications, Boston, 1994.

Ohen, H. A. And Civan, F., "Formation Damage in Petroleum Reservoirs II: Parameters Estimation and Simulation Studies" Submitted to Society of Petroleum Engineers for Publication (July, 1989).

Ortoleva, P., Merino, E., Moore C. and Chadman, J., "Geochemical Self-Organisation I: Reactive Transport Feedbacks and Modeling Approach" Amer. J. Of Sci. I1987) 287, 979-1007.

Porter, K. E., "An Overview of Formation Damage", JPT, Vol. 41, No. 8, 1989, pp. 780-786.

Reed, M. G., "Formation Permeability Damage by Mica Alteration and Carbonate Dissolution", JPT (Sept. 1977) 1056-1060.

Rege, S. D. And Fogler, H. S., "Network Model for Straining Dominated Particle Entrapment in Porous Media", Chem. Eng. Sci., (1987) 42, No. 7, 1553-1564.

Rege, S. D. and Fogler, H. S., "A Network Model for Deep Bed Filtration of Solid Particles and Emulsion Drops" AIChE J. (1988) 34, No. 11, 1761-1772.

Sarker, A. K., "An Experimental Investigation of Fines Migration in Two Phase Flow", M.S. Thesis, University of Texas at Austin, 1988.

Somasundaran, P., Celic, M., Goyal, A., and Manev, E., "The Role of Surfactant Precipitation and Redissolution in the Adsorption of Sulfonate on Minerals", SPEJ (Apri. 1984) 233-239.

Tien, C. and Payatakes, A. C., "Advances in Deep Bed Filtration". AIChE Journal, Vol. 25, No. 5, 1979, pp. 737-759.

Thornton, S. D., and Lorenz, P. B., "Role of Silicate and Aluminate Ions in the Reaction of Sodium Hydroxide with Reservoir Minerals", paper SPE 16277 presented at the SPE International Symposium on Oilfield Chemistry, San Antonio, TX, Feb. 4-6, 1987.

Walsh, M. P., Lake, L. W., and Schechter, R. S., " A Description of Chemical Precipitation Mechanisms and Their Role in Formation During Stimulation by Hydrofluoric Acid" JPT (September 1982) 2097-2112.

APPENDIX

Source Code for the Numerical Simulation

```
Public Class frmPressure

    Dim PW() As Single
    Dim Y() As Single
    Dim G() As Single
    Dim a() As Single
    Dim b() As Single
    Dim c() As Single
    Dim d() As Single

    Dim N As Integer
    Dim Fndo As Single
    Dim Fndw As Single
    Dim K As Single
    Dim L As Single

    Dim deltaT As Single
    Dim deltaX As Single
    Dim deltaPc As Single

    Dim EE As Single
    Dim FF As Single
    Dim GG As Single
    Dim MM As Single
    Dim NN As Single
    Dim OO As Single
    Dim QQ As Single
```

```
Dim MUo As Single
Dim MUw As Single
Dim Ko As Single
Dim PHI() As Single
```

```
Dim So() As Single
Dim Sw() As Single
Dim Cpo As Single
Dim Cpw As Single
Dim Uo As Single
Dim Uw As Single
Dim Co As Single
Dim Cw As Single
Dim SIGMApo As Single
Dim SIGMApw As Single
Dim SIGMAfpw As Single
Dim Pcow() As Single
Dim RHOo As Single
Dim RHOw As Single
```

```
Dim Krw As Double
Dim Kro As Double
Dim Mw As Single
Dim Mo As Single
```

```
Dim LAMBDAw As Single
Dim LAMBDAo As Single
```

```
Dim rnd As New Random 'Random number generator
```

```
Private Sub btnCompute_Click(ByVal sender As System.Object, ByVal e As  
System.EventArgs) Handles btnCompute.Click
```

```
Dim J As Integer
```

```
InitValues()
```

```
For J = 1 To (N - 1)
```

```
Pcow(J) = (1 + (0.12 * Sw(J))) / 1 + (8.5 * Sw(J))
```

```
EE = -0.00018232 * Cpo * PHI(J) * Uo - (0.10039 * (Co ^ 2))
```

```
FF = 0.0021642 * SIGMApo * (Uo - 1)
```

```
GG = 0.09764 * SIGMAfpw * (Uw - 0.1)
```

```
PHI(J + 1) = PHI(J) + deltaT * (EE + FF + GG)
```

```
K = 0.496 * (PHI(J + 1) / 0.218) ^ 3
```

```
MUo = 8 * (1 + Cpo)
```

```
MUw = 1 + Cpw
```

```
Kro = 13.047 * (So(J) - 0.3) ^ 3.25
```

```
Krw = 0.117 * (Sw(J) - 0.27) ^ 1.75
```

```
LAMBDAw = Fndw * ((K * Krw) / MUw)
```

```
LAMBDAo = Fndo * ((K * Kro) / MUo)
```

```
deltaX = L / (N - 1)
```

MM = (PHI(J) - 0) / deltaT 'The formula is (phi at n - phi at n-1)/deltaT
but we are assuming we are at the first run so phi at n-1 is zero

NN = 1 / RHOo * (PHI(J) * So(J) * ((RHOo - 0) / deltaT) + Uo * (RHOo - 0)
/ 2 * deltaX)

OO = 1 / RHOw * (PHI(J) * Sw(J) * ((RHOw - 0) / deltaT) + Uw * (RHOw - 0)
/ 2 * deltaX)

Sw(J + 1) = (Sw(J) * (PHI(J) / deltaT) - NN) / (-1 * PHI(J) / deltaT) +
(PHI(N) / deltaT)

So(J + 1) = 1 - Sw(J + 1)

So(J - 1) = 1 - So(J - 1)

Pcow(J - 1) = (1 + (0.12 * So(J + 1))) / 1 + (8.5 * So(J + 1))

QQ = LAMBDAo * (Pcow(J + 1) - Pcow(J)) - LAMBDAo * (Pcow(J) - Pcow(J - 1))

a(J) = LAMBDAw + LAMBDAo

c(J) = LAMBDAw + LAMBDAo

b(J) = -1 * (a(J) + c(J))

d(J) = (deltaX ^ 2) * (MM + NN + OO) ^ 2 - QQ

If J = 1 Then

Y(J) = c(J) / b(J)

G(J) = d(J) / b(J)

Else

Y(J) = c(J) / (b(J) - a(J) * Y(J - 1))

G(J) = (d(J) - a(J) * G(J - 1)) / (b(J) - a(J) * Y(J - 1))

End If

Next J

```
For J = (N - 1) To 2 Step -1
```

```
    PW(J) = G(J) - (Y(J) * PW(J + 1))
```

```
Next J
```

```
Tabulate()
```

```
End Sub
```

```
Private Sub Tabulate()
```

```
    Dim I As Integer
```

```
    Grid.Rows.Clear()
```

```
    For I = 1 To N
```

```
        Grid.Rows.Add()
```

```
        Grid.Rows(I - 1).Cells("J").Value = I
```

```
        Grid.Rows(I - 1).Cells("G").Value = G(I)
```

```
        Grid.Rows(I - 1).Cells("W").Value = Y(I)
```

```
        Grid.Rows(I - 1).Cells("PW").Value = PW(I)
```

```
    Next
```

```
End Sub
```

```
Private Sub InitValues()
```



```
If txtN.Text.Trim = "" Then Exit Sub
If txtPW1.Text.Trim = "" Then Exit Sub
If txtPWn.Text.Trim = "" Then Exit Sub
If txtDeltaT.Text.Trim = "" Then Exit Sub
If txtL.Text.Trim = "" Then Exit Sub
```

```
N = txtN.Text.Trim
```

```
ReDim PW(N)
ReDim Y(N)
ReDim G(N)
ReDim a(N)
ReDim b(N)
ReDim c(N)
ReDim d(N)
ReDim So(N)
ReDim Sw(N)
ReDim PHI(N)
ReDim Pcow(N)
```

```
Mw = txtL.Text.Trim
```

```
L = txtL.Text.Trim
```

```
deltaT = txtDeltaT.Text.Trim
```

```
PW(N) = txtPWn.Text.Trim
```

```
PW(1) = txtPW1.Text.Trim
```

```
a(N) = 0
```

```
b(N) = 1
```

```
c(N) = 0
```

```
d(N) = 480
```

```
Ko = 0.496
PHI(1) = 0.218
So(1) = 0.8
Sw(1) = 0.3
Pcow(0) = 0
Cpo = 0
Cpw = 0
Uo = 0.008927
Uw = Uo
Co = 0.0000275
Cw = 0.0000275
SIGMApo = 0.025
SIGMApw = 0.092
SIGMAfpw = 0.013
RHOo = 0.8 + (0.68 * Cpo)
RHOw = 1 + (0.6 * Cpw)
Fndw = 0.95
Fndo = 0.85
```

End Sub

```
Private Sub frmPressure_Load(ByVal sender As Object, ByVal e As System.EventArgs)
```

Handles Me.Load

```
Grid.Columns.Add("J", "J")
Grid.Columns.Add("G", "G")
Grid.Columns.Add("W", "W")
Grid.Columns.Add("PW", "PW")
```

End Sub

```
Private Sub btnClose_Click(ByVal sender As System.Object, ByVal e As
System.EventArgs) Handles btnClose.Click
    Application.Exit()
End Sub
End Class
```