Development of Generalized Well Semi-Analytical Coning Models

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Development of Generalized Well Semi-Analytical Coning Models

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ABSTRACT

Oil deposits are often found in association with a communicating gas or water zone. The production of the oil often leads to the coning of water or gas. This dynamic interaction can be captured by a properly detailed reservoir simulation, which unfortunately may not always be practical. To brigde the gap, researchers over the years have developed both analytical and empirical methods of modelling gas and water coning in oil reservoir. The fundamental questions have always been: what is the critical rate of oil production; what is the breakthrough time if the critical rate is exceeded; and what is the post-breakthrough behaviour?

Using analytically derived line source vertical and horizontal well breakthrough time expressions, a method has been developed to estimate oil critical rate, breakthrough time and post-breakthrough trend for inclined wells. The Post-breakthrough prediction scheme was extended to vertical and horizontal wells. Simplified correlations have also been generated for the easy application of the method without the need of analyzing complex mathematical functions. Within the accuracy of the numerical simulation results, the breakthrough times for the inclined well were consistently and correctly predicted. Literature correlations and numerical simulation comparisons showed that the post-breakthrough production predictions tended to underpredict oil production, but the trends were much more consistent with simulation results than other correlations studied. To the best of the knowledge of the author, this is the first semi-analytical coning model of an inclined well, as well as, the first semi-analytical post-breakthrough trend prediction for vertical and horizontal wells.

DEDICATION

This work is dedicated to Chibuzo Ike, my brother and my dearest friend. May the Lord forever smile on him.

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Chapter One

INTRODUCTION AND PROBLEM DEFINITION

1.1 INTRODUCTION

Quite often, oil deposits are associated with an underlying water aquifer and an overriding gas cap. In many situations, the oil reserve is desired at the surface while the associated fluid is preferred within the reservoir either because they are not valuable at the surface as in the case of produced water or the resources to harness the gas if produced to the surface are not readily available. The reservoir water or gas may also be required for pressure maintenance in production optimization within the reservoir. Whatever may be the intention of prefering to keep the water and/or gas within the reservoir, it is found in practice a difficult goal to achieve due to coning of the unwanted fluid(s). Coning is the tendency of the underlying water in contact with the oil to rise locally towards the producing well due to the greater pressure depletion near the producing well and the viscous drag the production of oil is having on the water-oil interface. The same holds for the gas oil interface in which case the gas projects downwards towards the producing well's perforation against gravitational force arising from gas-oil density difference. The projection is a result of viscous drag on the fluid interface and the local pressure depletion around the well due to oil production.

The production of either water or free gas with the oil will result in the reduction of the rate of oil production and the ultimate recovery of the oil. The reduction of oil production arises from the simple fact that some portion of the well bore that would be transporting oil will have to transport the unwanted fluid. Reduction in recovery arises from pressure depletion and trapping of oil behind the advancing unwanted fluid front. Ordinarily without coning, the unwanted fluid pushes the oil to the well as production progresses but with coning, the unwanted fluid leaves the oil behind, enters the well and may lead to early abandonment of the well.

The production of water has other damaging effect on hydrocarbon production profitability as it increases the spate and damage of corrosion. Corrosive agents like acid anhydride require the presence of water for ionization and chemical activity on metallic materials used in making the production string and other facilities. Obviously, the handling and disposal cost of produced water increases with the rate of coning. Depending on the prevailing environmental policy and the contaminant present in the produced water, this may constitute a huge cost burden. Gas handling, especially in areas with little market for gas, can become very demanding with gas coning. Treatment, pressurization and storage or re-injection may have dear financial implications. The environmentally-damaging alternative to the gas handling problem common in some countries is gas flaring. The latter constitutes enormous economic and environmental hazard.

As can be appreciated, for technical and economic reasons, it is crucial that coning be minimized or delayed. Thus coning minimization or delay is an important aspect of reservoir and production management. Numerous studies[1,2,3,4] have been conducted to understand the initiation and evolution of coning in order to control or minimize the stated negative consequences.

1.2 STATEMENT OF PROBLEM

Work on coning had generally been pursued along the path of preventing or delaying cone generation and evolution, the time to breakthrough if advancement is not checked and the performance of the well after cone breakthrough. A number of empirical and analytical studies have been conducted to model and determine these properties[5]. As will be discussed in detail, in the section on literature review, correlations and models for the determination of critical rate of oil production, time to breakthrough when producing at super-critical rate and the performance of the well after breakthrough have been developed for vertical and horizontal wells. These models may be experimental, empirical or analytical. After breakthrough performance model available are generally not analytical.

Analytical and approximate analytical models have the advantage of being generic and applicable to a varied scenario and they may provide insight into the behaviour of the system that may not have been obvious. Often, they have the additional advantage over other techniques of being relatively cost-effective. However, their accuracy and applicability are often limited by the simplifying assumptions made in their generation and certain mathematical and physical restriction inherent in their formulation. The intended contribution of this work is to develop generalized full range semi-analytical procedures for the computation of critical rate of oil production, cone break-through time for super-critical production and after break-through behaviour and trends. Near analytical solution for after break-through behaviour, to the best of the knowledge of the author, is not available in the literature. Common analytical solutions in coning modelling stop at the time for water breakthrough[3,4].

To the limit of the knowledge of the author, cone development are available for horizontal and vertical wells but not for inclined wells. An inclined or slanted well may be considered a general well trajectory since a vertical well can be obtained from it at one extreme and a horizontal well can be derived at the other. It is sometimes more practical to drill a slanted well than either a vertical or horizontal well. This may be due to some geological or operational conditions such as permeability restriction or the need to sidetrack a particular well path. Horizontal wells, as a rule, perform better than vertical wells in coning mitigation. Developing an expression for inclined well will allow the industry to properly assess the place of inclined well in coning mitigation. A major contribution of this work is to develop expressions for the determination of critical rate of oil production, breakthrough time and after breakthrough trends for inclined wells. Thus, the objective of this research is to develop general procedures for the evaluation of cone development and diphasic production for the three common well configurations: vertical, horizontal and inclined wells without the need of conducting full fledged numerical simulations.

Chapter Two

BASIC CONCEPT AND LITERATURE REVIEW

2.1 THE CONING MECHANISM

Within the reservoir, distinct fluid phases, if mobile, tend to take up positions relative to their density. Gravitational force induces the gas, if present, to occupy the top most part of the reservoir while water goes down the structure and the oil in between. A transition zone may occur at the interface boundary due to the effect of capillary pressure in which sharp fluid interface is only an engineering idealization. Neglecting capillary pressure is a common practice in petroleum reservoir coning modelling[1,3,4]. The state of fluid equilibration within the reservoir is disturbed by oil production. This introduces pressure differential around the producing well that is often referred to as viscous drag. The local pressure differential around the well means that fluid can more easily move towards the producing well. This tendency for local fluid migration towards the producing well is counterbalanced by the tendency of the fluid system to maintain gravitational segregation. The result is that the surface of the fluid interface is tensed and deformed into some characteristic bell shape[6].

As long as production continues under circumstances and rate that allows for appreciable pressure differential around the well, the cone continues to grow towards the well. The height of the cone above or below the original or prevailing fluid interface for the case of water or gas coning respectively, determines the net gravitational pull on the interface. At some certain point, the amount of gravitational force may effectively balance the viscous force and the growth of the cone is halted. At this stage, the cone is said to be stable as it does not progress with time. When it does progress towards the well with time, the cone is said to be unstable. Unstable cone may eventually break into the well and multiphase production will ensue. The time period between production commencements from original condition to when the unwanted fluid cones into the well is referred to as the breakthrough time[6].

If the production rate is such that the height of the developed cone is just about to break into the well before the cone becomes stable, the rate is called the critical oil production rate. It is theoretically the maximum production rate possible without cone breakthrough. In many situations, the critical rate is uneconomic and is often exceeded. Rates above the critical are termed super-critical. At such a rate, the cone at some time gets to the perforated interval. The determination of this time of breakthrough had been the subject of an appreciable amount of paper as it promotes production and surface facility optimization[3,4]. Surface facilities utilization is dependent on the type and quantity of the produced fluid and hence on the time of cone breakthrough.

Another important consideration in coning analysis is the after breakthrough performance. For wells producing super-critically, the cone breakthrough is inevitable. But the important question that must be asked and addressed is the performance of the well after the production of free-gas or water commences. This has a considerable impact on the productivity of the well and the cost and feasibility of unwanted fluid handling and disposal.

2.2 RESERVOIR AND ENGINEERING FACTORS IN CONING

Water and gas coning is essentially a vertical movement against gravity which arises as a result of the well drawdown on the reservoir fluids and thus coning is a strong function of vertical permeability. Especially for vertical wells, flow to the well is primarily a function of the horizontal permeability. Consequently, the ratio of the vertical to horizontal permeability will have appreciable influence on coning. A reservoir with relatively low vertical permeability when compared with the horizontal permeability will have lower coning tendency than one that have relatively high vertical permeability.

Vertical fractures can greatly increase the ease of coning. In naturally fractured reservoirs such as fractured limestone and reef system with highly permeable vertical fractures, water or gas coning can prove a severe challenge. This is especially true if the reservoir has limited matrix permeability and thus the entire water movement is through the fractures as much as the oil flow[5]. As a rule, reservoirs with low permeability tend to have greater coning problem because of the greater pressure drawdown required for a given rate of production. This relationship stems from Darcy's law where rate of production is a direct function of permeability and pressure drop. The low permeability translates to low well productivity, all things being equal, and to maintain a given production rate under this condition necessitates an increase in the pressure gradient. This of course will increase the threat of coning around the well.

The same argument holds for the relevance of horizontal well technology and well stimulation in the control of coning. Pegging production at critical rate is not very popular in many instances but the use of long horizontal well can ensure that the drawdown per unit length remains low while the rate of production is favourably high. Successful well stimulation provides minimal pressure drop for an appreciable rate of production.

The mechanism of coning is essentially the same for gas and water. On the one hand, it can be observed that the greater density contrast between gas and oil as compared to that between oil and water implies that gas will have a lower tendency to cone than water. This is based on the fact that gravitational force acts against the fluid interface advancement to the producing well and this force is directly proportional to gas or water density difference with respect to oil. So based on density, gas has less coning tendency. At the same time, it must also be recognized that the advancement of the cone is a function of the coning fluid's mobility and since gas has a much lower viscosity than water, it tends to advance more rapidly to the well than does water. This later observation counters the effect of the density difference and thus the coning trend in the two fluids tends to be comparable. A consequence of this is that for oil sandwiched between gas and water, a preferred point of completion would be at the centre of the reservoir. “From the practical standpoint, however, many wells are perforated closer to water-oil contact than to the gas-oil contact.”[5]

It is always desirable that wells are completed as far as possible from the fluid contact. When either gas or water is present, the well is preferably completed at the bottom or top of the oil respectively. When both are present (i.e. gas and water zones) the point of completion may be preferably based on the relative strength or activity of the unwanted fluid. If a large and active gas cap is present, for example, the completion should be made closer to the water interface and vice versa.

Since coning is linked to viscous drag arising from production, it can be said that reducing the rate of production in order to reduce the viscous force acting on the fluid interface will allow gravity with time to restore the fluid to it former minimum potential energy level. This could be observed in practice especially when the reservoir’s permeability is appreciably high and sufficient time is allowed for stabilization. However, in some instance, re-stabilization may not be easy to occur especially for low permeability reservoirs. In a general note, once cone breakthrough has occurred, the relative permeability to oil around the well may have been permanently altered and thus re-stabilization may not bring complete succour. Cone re-stabilization with shut-in can be particularly challenging with gas coning.

2.3 TYPICAL CONING SOLUTION PROCEDURES

Water and gas coning is one of the fundamental petroleum engineering problem since oil is very often found below a gas zone, or above a water zone or sandwiched between these two zones. Petroleum engineers over the years have consequently been developing schemes to properly understand the underlying principles of the phenomenon in order to effectively manage it. Simplified correlations that relate coning tendencies to reservoir and fluid properties have always been utilized. Some early rather sophisticated attempts to analyse water and gas coning employed physical analogs. A review of approaches and techniques in confronting the coning challenge will now be presented.

Walter Karplus in 1956 published an electrical analog study of water coning in petroleum reservoirs[7]. The stated objectives of the work is to define and quantify how coning tendencies are related to well and reservoirs parameters such as depth of penetration of a vertical well, the thickness of the oil zone, the constant production rate, the permeability of the reservoirs, the viscosity of the oil and the densities of the oil and water. To achieve this, steady state flow was assumed and zero capillary pressure. The homogeneous radial diffusivity equation was discretized using finite difference. The resulting algebraic expression was compared to the equation describing the voltage distribution in a resistance network. The electrical potential was stated to be proportional to the fluid flow velocity potential and the electrical resistivity was related to the inverse of the mobility.

This procedure required the adjustment of the lower boundary resistors by trial and error as the water cone developed and thus was intensely laborious. An analog computer was installed to simulate the fluid interface boundary. The detailed electronic design and operation made this procedure particularly difficult and expensive.

Chaperon I.[1] developed analytical expressions for the determination of oil critical rate. The formulation focused on the presence of a gas cap but the author stated that the presence of water aquifer can be evaluated in the reverse order with appropriate density difference specified. The horizontal well expression was developed assuming the well to be located at the bottom of the oil zone. The derivation was first made assuming isotropic and homogeneous formation and later anisotropy was incorporated. The formulations covered both steady state and pseudosteady state conditions. The adopted flow potential parameter effectively modelled radial flow close to the well and at the same time linear flow far away from the well.

The generation of the critical rate equation was based on two premises: a balance between the viscous flow potential difference and the gravity potential difference; and the excess of the buoyancy forces over the viscous forces. The former related the cone height to the dimensionless rate while the later ensured that the critical cone was stable. The critical rate was expressed per unit length of the horizontal well. It was concluded that the critical rate of horizontal wells was partially independent of anisotropy but was greatly influenced by the lateral boundaries. The critical rate, however, increased with increase in horizontal permeability and reservoir oil thickness.

The formulation for vertical well assumed that the perforation of the well was so small that it could be represented by a point source. The completion was made at the bottom of the oil zone at a distance farthest from the gas interface. The bottom oil zone was bounded by an impermeable zone. Flow was thereby pseudo-radial except close to the perforation where it was assumed hemispherical. Flow restriction within a narrow interval of the point source well was modelled using infinite row of image wells. The formulation was for steady state but could be extended to the pseudo-steady state with modification of the external boundary length. The critical rate was defined as the maximum rate that satisfies the stability condition.

The formulation can be extended to anisotropic cases with appropriate change in variables that must affect all the three principal coordinates unlike the formulation of horizontal wells which only affect two principal coordinates: the x and z. The calculated value of the critical rate was said to be affected by anisotropy as well as the drainage radius. Critical rate for vertical wells increases when vertical permeability decreases. The trend is reversed for horizontal wells. The author presented a simple procedure for determining the improvement in critical rate that can be obtained from drilling a horizontal well rather than a vertical well. The improvement ratio was found to be dependent on the formation anisotropy: it decreases with increase in anisotropy. By and large, the critical rate of horizontal well was reported to be greater than that of the vertical well as expected but this difference, according to the author, closes out when the vertical permeability decreases.

Hoyland L. A. et al[2] attempted to develop empirical correlation and analytical expression for the determination of critical rate. The analytical solution was reported as an extension of the classic work of Muskat and Wychoff, “An Approximate Theory of Water Coning in Oil Production” Trans., AIME (1935). In their model, the reservoir fluid was considered single phase and the well infinitely conducting. The steady state solution of the diffusivity equation was used to develop a dimensionless expression for the well critical rate.

The authors reported an extensive simulation run to develop an empirical correlation for oil critical rate under water coning. A standard, three-phase, black oil numerical model implementing finite difference formulation was built. The model was operated above bubble point and capillary pressure was neglected. The reservoir was considered homogeneous and anisotropic in one case and isotropic in the other. To simulate a constant pressure water oil contact, the bottom grids were assigned infinite porosity and permeability. The effect of different reservoir and well properties were investigated separately by varying one parameter at a time while keeping others constant. The critical rate was reported to be a linear function of oil permeability, oil-water density difference, oil viscosity and oil FVF and a non-linear function of well penetration ratio, radial extent, total oil thickness and permeability ratio.

It was reported that the isotropic case yielded a single generalized equation while the anisotropic case proved to be more challenging. A graphical representation of the critical rate trend for the anisotropic case was made in lieu of a single mathematical expression. It was reported that consideration of the influence of the cone shape in an analytical solution greatly improves the accuracy of the solution in predicting critical rate of oil production under the influence of water coning.

Karcher B. J. et al[8] reviewed several of the mathematical expressions used in predicting the behaviour of horizontal wells and the assumptions that were made in their development so as to promote their right use. Among the several well performance parameters considered, coning tendencies and after breakthrough behaviours were examined using numerical simulations. The authors opined that the behaviour of water coning is dependent on the water-oil interface characteristics. They identified three basic boundary conditions. The first they referred to as constant interface elevation which they describe as the condition that exist when an isolated well with a relatively small rate of production compared with the available oil is bounded by strong lateral aquifers. The second was termed free surface boundary condition; the rate of production is relatively high in this case compared to the available oil and the vertical movement of the fluid interface is patent. The third was constant potential boundary in which the production of several oil wells causes an even movement of the fluid interface.

The authors stated that critical rate of oil production is dependent on a number of parameters: drainage radius, well penetration, mobility ratio and Reynolds number (ratio of viscous to gravity forces) all defined dimensionlessly. The authors pointed out that at low values of the drainage radius when compared to the height of the oil interval, the result of critical rate computation may be unrepresentative. According to the authors, critical rate for horizontal wells can be about three times that of a vertical well. The sweep efficiency is even more remarkable for the horizontal well when compared with the vertical.

Simulation studies of after breakthrough behaviours for horizontal and vertical wells were aimed at understanding the effects of drainage radius, rate and mobility ratio. For extreme mobility ratios, the final recoveries of vertical and horizontal wells after breakthrough were said to be comparable. On the effect of rate of production on performance, it was reported that at unfavourable mobility ratio, the horizontal wells' recovery were reduced. According to the study, drainage radius tended to be inversely related to recoveries.

Papatzacos et al[4] presented a horizontal well model for the calculation of the time of cone breakthrough and also the optimal depth for the drilling of the well between two active fluid contacts sandwiching the oil zone. The governing equation was solved semi-analytically neglecting the effect of capillary pressure i.e. sharp fluid interfaces were assumed. The authors assumed gravity equilibrium which required only the solution of the diffusivity equation for oil and opined that their formulation was valid for low rates of production. With that, it was assumed that the mobility ratio of the system is always unity: individual mobilities of the gas and water do not influence the solution, only their densities do. They also reported a solution in which constant pressure at the moveable fluid interface was assumed rather than gravity equilibrium. An infinitely long line sink horizontal well of uniform and constant flux was assumed with no fixed boundaries and thus implying infinite-acting flow. Complete oil displacement of the incompressible fluid with zero capillary pressure was assumed.

Their semi-analytical solutions yielded numerical expressions for the optimal well placement as a function of the dimensionless rate and density difference for a sandwiched reservoir and the time to breakthrough. The density difference or constrast for the sandwiched oil reservoir with gas cap and water aquifer was defined as

$$\varphi = \frac{\rho_w - \rho_o}{\rho_o - \rho_g} \qquad 2.1$$

For a unit value of the density contrast, the optimal well placement was found to be at the middle of the oil interval. This implies that the two-cone solution with unit density contrast and doubled height and rate is equivalent to placing the horizontal well at an impervious top or bottom of the oil zone. A distinct formulation for only gas or water in communication with the oil was also developed. As expected, the breakthrough time is a function of the distance of the well from the single fluid interface and the rate of oil production. It was reported that the assumption of vertical equilibrium is equivalent to a moving constant-pressure boundary at large dimensionless flow rates. The results were curved fitted to allow for easy application.

The results of their work were validated with a numerical simulation using commercial black oil simulator. A uniform grid was employed in the oil zone with regular coarsening in the water and/or gas zone. The uniform grid in the oil zone was said to have been adopted to properly define the movement of the fluid interface. Breakthrough times were identified from plots of WOR and/or GOR and were said to be marked by increase in the fluid ratios. Citing numerical dispersion, the authors stated that the simulation breakthrough time is bound to be early. They reported that the vertical to horizontal permeability ratio had no effect on the dimensionless breakthrough time as a function of the dimensionless rate. This indicates that the dimensionless quantities were properly formulated to eliminate anisotropic effect. The result of the simulation was reported to have been consistently less than the analytical development but the trend was generally the same. At low rates, the results of the analytical solution came very close to the numerical results. The authors explained this outcome by stating that the assumption of vertical equilibrium is more valid at low rates. The presence of residual oil was also reported to contribute to disparity in the simulation results and the analytical solution. Generally, the values of the gas and/or water viscosity were reported to influence the accuracy of the analytical solution when compared with that of the simulation. The simulation result indicated that as the rate of oil production increased, the optimal well placement got closer to the original WOC. Actual field data of breakthrough time was reported to be comparable to the result of the analytical formulation.

Coning modelling had generally assumed zero capillary effects. A work by Russell T. J. et al[9] considered the effect of capillary pressure in the development and evolution of gas or water cone. Their analytical method looked at the simultaneous flow of two phases within the reservoir and the rate of oil production necessary for minimum unwanted fluid production. The authors noted that the common industry practice of partially completing the vertical well away from the fluid contact brings a mixed fortune. The distance from the fluid interface may indeed provide a large clearance that the unwanted fluid must traverse before it gets to the perforation, but the partial completion increases the pressure gradient around the well. The flow under this setting is spherical rather than radial and this actually amplifies the coning challenge.

The stated aim of the work was to develop analytical expressions that allowed for the modelling of both single and simultaneous two phase flow that incorporated the effect of capillary pressure and relative permeability. The mathematical development assumed steady state Darcy flow and homogeneous reservoir properties. Fluids were considered immiscible and incompressible with constant viscosity. At most, two-phase flow was allowed and the well's trajectory was vertical with diffused flow due to capillary pressure. Vertical equilibrium was assumed and this was interpreted to imply that the pressure gradient in the vertical direction was hydrostatic. The saturation profile was determined as a function of relative distance from the vertical well. Critical rate of oil production was defined as the maximum oil rate beyond which the other phase becomes mobile. The formulation attempted to describe the behaviour of saturation and relative permeability influence around the wellbore rather than the arrival of a coning water or gas front as is conventionally studied. The ratio of buoyancy forces to capillary forces, termed bond number, was identified as a key determinate of the reservoir coning characteristics.

De Souza, A.L.S. et al[10] developed empirical correlations for the determination of gas and water cone behaviour in horizontal wells undergoing multiphase flow. The authors identified and generated dimensionless parameters from the general fluid flow equation for anisotropic reservoirs. The following assumptions were made: homogeneous reservoir, constant viscosity, zero capillary pressure and diphasic flow under constant total production. Strong aquifer support was modelled at the bottom of the oil zone while the other boundaries were no flow condition. Employing numerous numerical simulations, they were able to develop correlations for breakthrough time, maximum oil rate and post-breakthrough behaviour. A linear relation between the breakthrough time and the dimensionless interface distance from the well was first developed. The separate effects of other dimensionless parameters were then assessed and their combined effects incorporated as a correction term. A similar approach was adopted for the post-breakthrough behaviour correlation. They also developed guidelines for the required size and pattern of grid blocks for reservoir simulation attempting to study coning characteristics. They advanced a method of obtaining relatively accurate simulation results from quick, coarse grid cells by matching it to some "pseudofunctions". The authors intended their correlations to be useful in a preliminary evaluation of coning pattern before a fully fledged simulation experiment.

Ozkan E. and Raghavan R.[3] presented approximate analytical expressions for the determination of cone breakthrough time. They assumed a homogeneous and anisotropic reservoir with the oil-water mobility ratio being unity. Fluid density difference was incorporated in the modelling but capillary pressure was neglected. Gravity equilibrium was assumed. The resulting equation with appropriate boundary conditions was solved analytically to a form that was said to require simple numerical integration. The development was originally for point source configuration but was integrated for horizontal and vertical wells with the midpoints situated at the initial point source. For specific cases, simplified correlations were generated for quick evaluation of breakthrough time and the critical rate of oil production. The authors remarked that their solution incorporated an active aquifer and is bound to give results that appear very pessimistic when compared to other works that had assumed a dead aquifer. This work will be reviewed further as it presents an adequate basis for the application of the basic concept advanced in this current work.

Mathematical formulations certainly are not the only solution proffered to the coning challenge. A number of innovative approaches involving the use of polymers, selective well perforation, down hole water sink, etc. have been attempted with notable results. The solution to the problem of water and gas coning, for example, was studied with specific interest in the role of polymer gels in selectively reducing the relative permeability of water and/or gas in favour of oil by Kantzas A. et al[11]. The evaluation was done via computer simulation. The gel placement and its effect was modelled by alteration in the absolute or relative permeability of affected blocks around the well.

The degrees of alteration were arrived at by conducting a core analysis in which the specific gels were tested on actual reservoir core samples. A radial reservoir model was employed for the assessment using a commercial reservoir simulator. Homogeneous and heterogeneous sandstones and carbonate were evaluated in the work.

The authors reported that a gel barrier that extends to about 4 m around the well have negligible impact on mitigating coning except when the horizontal permeability is low. If the barrier extended to about 10 m around the well, gas and water coning were appreciably retarded but this increased the time required to produce an equivalent amount of oil assuming no barrier was placed. Installing the barrier after the production process was reported to produce no beneficial result and this is to be expected: permeability once impaired by the flow of unwanted phase can hardly be completely restored to its previous state.

It can be appreciated from this review that analytical and semi-analytical techniques in coning analyses are very common approaches in the oil and gas industry. Their popularity may be attributed to their cost-effectiveness and generality. Their applications, however, have been restricted to breakthrough and critical rate formulations. Hence, the development of semi-analytical procedures for the calculation of post-breakthrough trend in vertical and horizontal wells would constitute a major contribution to the industry. Inclined wells are hardly mentioned in the literature with regard to coning modelling. Yet, inclined wells are not uncommon in the field and they obviously do have the coning challenge like other well types. This research seeks to bridge this important knowledge gab in the industry by developing semi-analytical procedures for the calculation of inclined well oil critical rate, breakthrough time and after breakthrough trend.

Chapter three

METHODOLOGY

3.1 PREAMBLE

The major questions that are associated with the coning of unwanted fluid in the course of oil production can be categorized into three: what is the maximum stable monophasic or critical rate of oil production, what is the breakthrough time for supercritical production rate, and what is the trend of production after breakthrough? The answers to these questions are found to be dependent on particular well configurations: vertical, horizontal or slanted. Answers to these questions have been pursued empirically and analytically as described in the previous chapter. This study intends to proffer an approximate analytical solution to the three basic coning challenges. These implies the generation of generalized expressions and procedures for the determination of critical rate of oil production, breakthrough time and after breakthrough behaviour prediction for the three basic well configurations.

To provide the three fundamental answers for the three common well configurations, the use of analytical generalized break-through time expressions for vertical and horizontal wells has been proposed in this research. Such a solution should appreciably be a line source solution generated from a single point source solution as a kernel. This provides the opportunity to be able to resolve the slanted well into vertical and horizontal components. The break-through time formulation of Ozkan and Raghavan[3] were found to have the desired properties for such a development. Consequently, their development will be reproduced here to emphasize their assumptions and properties. It must be emphasised that the proposal in this work is a general one and it is therefore applicable to any other suitably formulated complementary vertical and horizontal break-through time expressions.

3.2 MATHEMATICAL FORMULATION OF HORIZONTAL AND VERTICAL WELL CONING MODEL AFTER OZKAN AND RAGHAVAN[3]

A steady state radial expression for the flow of fluid in the reservoir due to the presence of an oil reserve with active aquifer support was presented as follows for oil and water respectively assuming homogenous and anisotropic reservoir conditions.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_o}{\mu_o}\frac{\partial \Delta P_o}{\partial r}\right]+\frac{\partial}{\partial z}\left[\frac{k_o}{\mu_o}\frac{\partial \Delta P_o}{\partial z}\right]=0 \quad 3.1$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_w}{\mu_w}\frac{\partial \Delta P_w}{\partial r}\right]+\frac{\partial}{\partial z}\left[\frac{k_w}{\mu_w}\frac{\partial \Delta P_w}{\partial z}\right]=0 \quad 3.2$$

where $\Delta P = P_i - P$.

It was assumed that the mobilities of the two phases were identical.

Subtracting equation 3.2 from 3.1 results in:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_o}{\mu_o}\frac{\partial}{\partial r}(\Delta P_o-\Delta P_w)\right]+\frac{\partial}{\partial z}\left[\frac{k_o}{\mu_o}\frac{\partial}{\partial z}(\Delta P_o-\Delta P_w)\right]=0 \quad 3.3$$

The gravity term for the water phase given by $\Delta P_w = \rho_w g z$, was added and subtracted from the phase pressure difference for the terms in the inner most paranthesis.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_o}{\mu_o}\frac{\partial}{\partial r}(\Delta P_o-\rho_w g z)\right]+\frac{\partial}{\partial z}(\Delta P_o-\rho_w g z)=0 \quad 3.4$$

The following definition (3.5) was introduced into equation 3.4, and the expression took the form given in 3.6 below

$$\varphi=\frac{1}{\mu}[\Delta P_o-\rho_w g z] \quad 3.5$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r k_r\frac{\partial\varphi}{\partial r}\right]+\frac{\partial}{\partial z}\left[k_z\frac{\partial\varphi}{\partial z}\right]=0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_r}{k_z}\frac{\partial\varphi}{\partial r}\right]+\frac{\partial^2\varphi}{\partial z^2}=0 \quad 3.6$$

The velocity of the water interface was expressed in the form of Darcy law as follows

$$\partial z = v\times\partial t=\frac{k_z}{f}\frac{\partial\varphi}{\partial z}\partial t \quad 3.7$$

The following dimensionless parameters were introduced into equations 3.6 and 3.7

$$\varphi_D = \frac{\varphi\mu_o}{\Delta\rho g h} \quad 3.8$$

$$z_D = \frac{z}{h} \quad 3.9$$

$$t_D = \frac{k_z \Delta\rho g t}{f \mu_o h} \quad 3.10$$

And in field units

$$t_D = \frac{k_z \Delta\rho t}{364.6 f \mu_o h} \quad 3.11$$

$f = \emptyset(1 - S_{wc} - S_{or})$ indicates the net path taken by the aquifer water in its coning towards the producing well.

Using the dimensionless terms, 3.6 and 3.7 were written as follows

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left[r_D \frac{\partial \varphi_D}{\partial r_D}\right] + \frac{\partial^2 \varphi_D}{\partial r_D^2} = 0 \quad 3.12$$

$$\partial z_D = \frac{\partial \varphi_D}{\partial z_D} \partial t_D \quad 3.13$$

The following boundary conditions were imposed

$$\varphi_D(r_D, z_D = 0) = 0 \quad 3.14$$

$$\frac{\partial \varphi_D}{\partial z_D}(r_D, z_D = 1) = -1 \quad 3.15$$

$$\frac{\partial \varphi_D}{\partial z_D}(r_D = r_{eD}, z_D) = 0 \quad 3.16$$

$$\lim_{\epsilon \to 0}\left(\lim_{r_D \to 0} \frac{1}{\epsilon} \int_{z_{wD}+\frac{\epsilon}{2}}^{z_{wD}+\frac{\epsilon}{2}} r_D \frac{\partial \varphi_D}{\partial r_D} dz_{wD}\right) = -q_D \quad 3.17$$

Equation 3.11 may be written as follows

$$\frac{\partial^2 \varphi_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial \varphi_D}{\partial r_D} + \frac{\partial^2 \varphi_D}{\partial z_D^2} = 0 \quad 3.18$$

The solution of equation 3.18 with the stated boundary conditions gave the following analytical horizontal and vertical well potential terms respectively.

$$\varphi_D(r_D = 0, z_D \neq z_{wD}) = \frac{q_D}{2L_D} ln \frac{tan\frac{\pi}{4}(z_{wD}+z_D)}{tan\frac{\pi}{4}|z_{wD}-z_D|} - F_L + F_{BH} - z_D \quad 3.19a$$

$$F_L = \frac{2q_D}{L_D}\sum_{n=1}^{\infty} \frac{sin\epsilon_n z_D sin\epsilon_n z_{wD}}{\epsilon_n} Ki_1(\epsilon_n L_D) \quad 3.19b$$

$$Ki_1 = \frac{\pi}{2} - \int_0^x K_0(u)\, du$$ 3.19c

$$F_{BH} = \frac{2q_D}{L_D} \sum_{n=1}^{\infty} \frac{sin\epsilon_n z_D sin\epsilon_n z_{wD}}{\epsilon_n} \frac{K_1(\epsilon_n r_{eD})}{I_1(\epsilon_n r_{eD})} \int_0^{\epsilon_n L_D} I_0(u) du$$ 3.19d

$$\varphi_D(r_D = 0, z_D \neq z_{wD}) = \frac{q_D}{2L_D} ln \frac{gamma\left(\frac{3-z_D+b}{4}\right) gamma\left(\frac{1-z_D-b}{4}\right)}{gamma\left(\frac{3-z_D-b}{4}\right) gamma\left(\frac{1-z_D+b}{4}\right)} + \frac{q_D}{b} ln \frac{tan\frac{\pi}{4}(1-z_D-b)}{tan\frac{\pi}{4}(1-z_D+b)} + F_{BV} - z_D$$

3.20a

$$F_{BV} = \frac{2q_D}{b} \sum_{n=1}^{\infty} \frac{sin\epsilon_n z_D sin\epsilon_n b}{\epsilon_n} \frac{K_1(\epsilon_n r_{eD})}{I_1(\epsilon_n r_{eD})}$$ 3.20b

Where:

The dimensionless oil production rate is given by

$$q_D = \frac{q\mu_o B_o}{2\pi k_r h^2 \Delta\rho g}$$ 3.21

And in field units

$$q_D = \frac{325.7 q\mu_o B_o}{k_r h^2 \Delta\rho}$$ 3.22

The dimensionless horizontal well length

$$L_D = \frac{L}{2h} \sqrt{{k_z}/{k_r}}$$ 3.23

Vertical well penetration ratio

$$b = {h_w}/{h}$$ 3.24

And $\epsilon_n = (2n-1)\pi/2$ 3.25

The differentiation of the dimensionless potential terms with respect to the dimensionless vertical coordinate results in:

For the horizontal well

$$\frac{\partial\varphi_{Dh}}{\partial z_D} = \frac{q_D}{2L_D}\left(\frac{{\pi}/{4} sec^2 {\pi}/{4}(z_{wD}+z_D)}{tan{\pi}/{4}(z_{wD}+z_D)} + \frac{{\pi}/{4} sec^2 {\pi}/{4}|z_{wD}-z_D|}{tan{\pi}/{4}|z_{wD}-z_D|}\right) - \frac{\partial F_L}{\partial z_D} + \frac{\partial F_{BH}}{\partial z_D} - 1$$ 3.26a

$$\frac{\partial F_L}{\partial z_D} = \frac{2q_D}{L_D} \sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD} \left(\frac{\pi}{2} - \int_0^{\epsilon_n L_D} K_0(u) du\right)$$ 3.26b

$$\frac{\partial F_{BH}}{\partial z_D} = \frac{2q_D}{L_D}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n z_{wD} \frac{K_1(\epsilon_n r_{eD})}{I_1(\epsilon_n r_{eD})}\int_0^{\epsilon_n L_D} I_0(u)du \qquad 3.26c$$

For the vertical well

$$\frac{\partial \varphi_{DV}}{\partial z_D} =$$

$$\frac{q_D}{b}\left(-\frac{1}{4}psi\left(\frac{3-z_D+b}{4}\right) - \frac{1}{4}psi\left(\frac{1-z_D-b}{4}\right) + \frac{1}{4}psi\left(\frac{3-z_D-b}{4}\right) + \frac{1}{4}psi\left(\frac{1-z_D+b}{4}\right)\right) +$$

$$\frac{q_D}{2b}\left(\frac{-\pi/_4 sec^{2\pi}/_4(1-z_D-b)}{tan^{\pi}/_4(1-z_D-b)} + \frac{\pi/_4 sec^{2\pi}/_4(1-z_D+b)}{tan^{\pi}/_4(1-z_D+b)}\right) + \frac{\partial F_{BV}}{\partial z_D} - 1 \qquad 3.27a$$

$$\frac{\partial F_{BV}}{\partial z_D} = \frac{2q_D}{b}\sum_{n=1}^{\infty} cos\epsilon_n z_D sin\epsilon_n\, b \frac{K_1(\epsilon_n r_{eD})}{I_1(\epsilon_n r_{eD})} \qquad 3.27b$$

Ozkan and Raghavan analyzed the potential term to give the time function for the progress of the oil-water interface due to the production of oil at a constant rate.

$$t_D = \int_0^{zD} \frac{dz'_D}{\left(\partial\varphi_D/_{\partial z'_D}\right)_{r_D=0}} \qquad 3.28$$

This integration was implemented numerically over *Scilab*. The breakthrough time is the time taken to reach the well's external radius for the horizontal well and the perforation for the vertical well. Like Ref. 3, a dimensionless radius value of 0.002 was used in the determination of the breakthrough time in developing correlations since the well radius was not a significant parameter in the determination of the breakthrough time. In the prediction of post-breakthrough trend for horizontal and inclined wells as proposed further on, however, the value of the dimensionless well radius is very important.

The critical rate is obtained as the maximum rate that leads to infinite breakthrough time. It is taken that when the well is produced at its critical rate, assuming negligible depletion, the free gas or water never gets to the perforation as the gravitational force balances the viscous force and the cone ceases to advance[5]. After sufficient depletion, the fluid contact changes and the critical rate is bound to change as well.

3.3 SLANTED WELL FORMULATION

Considering a slanted well of θ inclination angle to the vertical and length l_w.

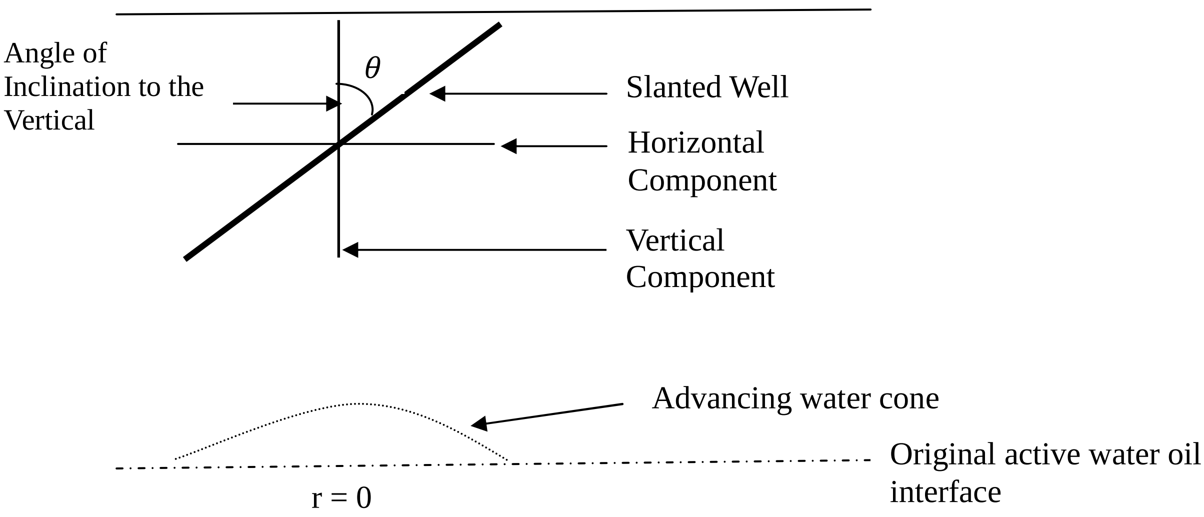

Fig 3.1 Slanted Well Coning Model

The slanted well is considered a line source/sink and its associated vectors can therefore be resolved into perpendicular components using simple plan angle relationships. Adopting the dimensionless slanted well procedure of Abbaszadeh M. and Hegemean P. (1990)[12], the dimensionless inclination angle and well length can be expressed as:

$$tan\theta_D = \frac{l_{whD}}{l_{wvD}} = \frac{\frac{l_{wh}}{h}\sqrt{k_z/k_r}}{\frac{l_{wv}}{h}} = \frac{l_{wh}}{l_{wv}}\sqrt{k_z/k_r} = tan\theta\sqrt{k_z/k_r}$$

$$\therefore \theta_D = tan^{-1}\left(tan\theta\sqrt{k_z/k_r}\right) \quad 3.29$$

Where l_{wh} and l_{wv} are the horizontal and vertical components of the slanted well.

$$L_{wD}^2 = l_{whD}^2 + l_{wvD}^2 = \left(\frac{l_w sin\theta}{h}\sqrt{k_z/k_r}\right)^2 + \left(\frac{l_w cos\theta}{h}\right)^2 = \left(\frac{l_w}{h}\right)^2\left[\left(k_z/k_r\, sin^2\theta\right) + cos^2\theta\right]$$

$$L_{wD} = \frac{l_w}{h}\sqrt{k_z/k_r\, sin^2\theta + cos^2\theta} \quad 3.30$$

The formulation for the slanted well was developed assuming that the volumetric flow to the well can be resolved into vertical and horizontal components and their resultant viscous drag effect is a sum of the effects due to the vertical and horizontal components. Consequently, the potential term for the slanted well is given by

$$\varphi_{Ds} = \varphi_{Dv} + \varphi_{Dh} \quad 3.31$$

The time formulation is therefore given as

$$t_D = \int_0^{zD} \frac{dz'_D}{\left(\partial\varphi_{Dh}/\partial z'_D + \partial\varphi_{Dv}/\partial z'_D\right)_{r_D=0}} \quad 3.32$$

To obtain the resolved potential parameters, both the dimensionless length of the slanted well and its production rate was resolved into vertical and horizontal components as follows:

$$b = L_{wD}cos\theta_D$$

$$L_D = L_{wD}sin\theta_D \quad 3.33$$

$$q_{Dv} = q_D cos\theta_D$$

$$q_{Dh} = q_D sin\theta_D$$

The breakthrough time for the slanted well is the time required for the cone height to get to the tips of the vertical component, *b,* of the resolved well length. Just like for the purely vertical and horizontal wells, the critical rate for the inclined well is the maximum rate at which the breakthrough time tends to infinity. Practically, at this rate, the computed breakthrough time becomes excessively large in comparison to previous rates and any attempt to exceed this rate may result in computational error.

3.4 POST BREAKTHROUGH BEHAVIOUR

From the designation of the potential term, Ozkan and Raghavan (1990)[3] intended the formulations to be applicable before the advancing fluid front reaches the well position or perforation. Attempting to exceed this limit introduces some erratic behaviour to the formulation which implies that after breakthrough formulation cannot be directly generated.

3.4.1 FUNDAMENTAL CONCEPT

To develop an after breakthrough analytical formulation for a producing well, the effect of the vertical well penetration ratio was considered in the development of vertical viscous drag that could lead to water zone coning into the oil well. Water coning is discussed but the principle apply equally to gas coning with the appropriate density difference and direction specification. If the vertical well completely penetrates the pay zone, the flow is essentially radial and if laminar, each layer tends to flow to the well without interfering with what is below or above. If the pay zone is only partially penetrated, then the effect of the vertical flow becomes significant as the fluid below the perforation will tend to flow upward to make it into the perforation and creating a viscous disturbance in the process.

Water breakthrough is attained when the water zone gets to the well's perforation. As production continues, some portion of the oil production is taken over by the water phase in what is called water coning. This reduction in oil production, all things being equal, can be assumed to be the direct consequence of the reduction of the perforation interval open to oil flow. Hence, after water breakthrough, the oil production progressively reduces as the perforation open to oil production reduces. The order of this process may be reversed. If the decrease in the length of the perforation open to oil production is gradually made in steps and the drop in oil production computed from the initial oil production and the ratio of oil perforation diminution, then it seems feasible that the water coning tendencies can be obtained.

3.4.2 VERTICAL WELL POST-BREAKTHROUGH FORMULATION

Following the rate and flow area reduction concept for coning modelling developed in the previous section, an after breakthrough formulation for a vertical well can be developed from breakthrough time formulations. The iterative process is applicable to other well configurations but it is best demonstrated using the vertical well because of its continous simple vertical clearance for the rise of the cone. To illustrate the process, the length of the vertical perforation open to flow was divided into *n* equal intervals of length *m*.

$$m = b/n \qquad 3.34$$

At each progress level, *n*, the effective well length was obtained by subtracting an interval perforation length.

$$b_{n+1} = b_n - m \qquad 3.35$$

And the corresponding oil production rate was obtained as follows

$$q_{Dv_{n+1}} = q_{Dv} \times {}^{b_{n+1}}/_{b} \qquad 3.36$$

The dimensionless time required for the fluid interface to get to the shrunken well perforation is obtained by applying the breakthrough time formulation (equations 3.28 and 3.27) which actually determines the time for the fluid interface to get to the specified well perforation having incorporated the current perforation length and oil production rate. This time value is assumed to be representative of the after breakthrough time needed for the oil production to drop to its current value. The water production is obtained by simple difference between the original production rate and the current rate of oil production. The procedure is repeated for other growth levels.

The steps involved can be summarized as follows:

1. Determine penetration ratio and divide it into a convenient number of equal segments
2. Obtain the dimensionless liquid production rate which is the initial oil rate before coning using 3.22
3. Starting from the initial penetration, calculate the breakthrough time which is the beginning of the after breakthrough production using 3.27 and 3.28, integrating from the base of the oil zone to the dimensionless well penetration i.e. 0 to 1 - b.
4. Reduce the penetration ratio by a value of one segment obtained in step 1.
5. Calculate the effective oil dimensionless rate as given in 3.36
6. Determine the time required for the interface to get to the shrunken well penetration at the reduced dimensionless rate by implementing 3.27 and 3.28, integrating from zero to a value of (1-b) i.e. the distance from the original fluid interface to the tip of the updated perforation. Real time can be obtained from the dimensionless value using 3.11 appropriately.
7. Tranform the current rate into real rate to give the oil rate using equation 3.22. Water production is the difference between the current oil rate and the initial production value.

3.4.3 HORIZONTAL WELL POST-BREAKTHROUGH FORMULATION

For the horizontal well, the process is a little more involved. Breakthrough occurs when the fluid front gets to the surface of the horizontal well. As production continues at the same total dimensionless fluid rate, q_D, the rate of oil production reduces at the expense of the production of unwanted fluid: water or gas. This reduction in volumetric oil rate can be related to the reduction in the surface open to oil production. Once again, the description is for water coning but the trend of gas coning is simply in the reversed direction with the inclusion of the appropriate density difference. The movement of the fluid front is vertical and therefore the decrease in the surface open to oil can be viewed on a vertical cross section of the horizontal well.

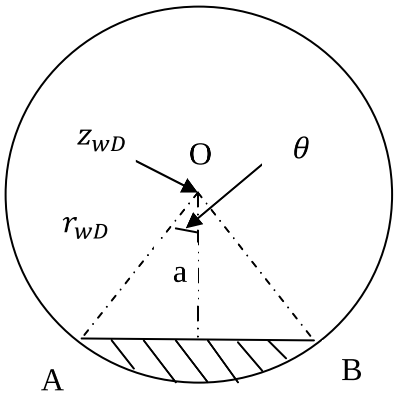

Fig. 3.2 Cross sectional view of a horizontal well after breakthrough; cone front below the original well centre

The hatched segment is assumed to be under the advancing unwanted fluid front while the other free segment indicate the area open to oil production at that given instance of the cone evolution. If it is assumed that the fluid interface had moved a distance, *h*, vertically over the cross-sectional area, then its distance from the well's centre, *O*, is given by

$$a = r_{wD} - h \qquad 3.37$$

From Ref. 10, the dimensionless well radius is given as

$$r_{wD} = \frac{1}{2h} r_w \left(\left({}^{k_r}/_{k_z} \right)^{0.25} + \left({}^{k_z}/_{k_r} \right)^{0.25} \right) \qquad 3.38$$

Half the angle substended at the centre by the advancing fluid front, θ , is given by

$$\theta = cos^{-1}\left({}^{a}/_{r_{wD}} \right) \qquad 3.39$$

$Area\ of\ segment\ under\ the\ fluid\ front = Area\ of\ sector\ AOB - Area\ of\ triangle\ AOB =$

$\frac{2\theta}{360}\pi r_{wD}^2 - \frac{1}{2}(2a^2 tan\theta) = \frac{\theta}{180}\pi r_{wD}^2 - a^2 tan\theta$

The area open to oil production is given by

$$Oil\ Area = \pi r_{wD}^2 - Area\ of\ Segment\ behind\ front \qquad 3.40$$

The reduced oil production due to the progress of the cone front is given by

$$Oil\ Production = \frac{Oil\ Area}{\pi r_{wD}^2} \times q_D \qquad 3.41$$

The decrease of the effective well area due to the incursion of the cone front requires a change in the height of the centre of the well and the external surface of the well. These two parameters are required to compute the dimensionless time needed for the cone height to progress to the current well position. The assumed upward movement, h, of the cone before reaching the well's original centre implies a rise in the well centre of *0.5h* i.e.

$$z_{wD_{n+1}} = z_{wD_n} + 0.5h \qquad 3.42$$

The lower surface of the horizontal well at this instance is at

$$z_{wD} + r_{wD} - h \qquad 3.43$$

The values of 3.41, 3.42 and 3.43 are inputed into the expression for the calculation of breakthrough time (3.28 and 3.26). The calculated time is the dimensionless time required to produce the well right from zero cone height to the current oil rate and cone height.

The cone height can be increased again by *h* and the entire process repeated to calculate the dimensionless time required for the new cone advancement and the drop in the oil rate. On crossing the original centre of the well by a value of *c*, the computation is slightly modified.

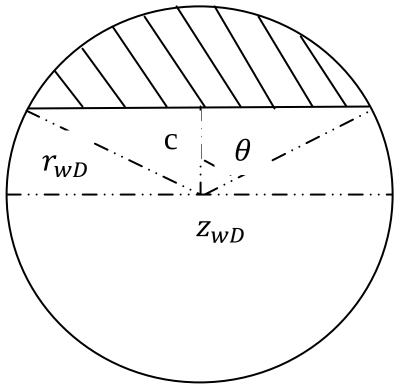

Fig 3.3 Cross sectional view of a horizontal well after breakthrough; cone front is above the original well centre's vertical height, z_{wD}.

Half the substended angle at the original centre is now given by

$$\theta = cos^{-1}\left({c}/{r_{wD}}\right) \quad 3.44$$

The area open to oil production is now reduced to just the segment above the cone front.

$$Oil\ Area = \frac{\theta}{180}\pi r_{wD}^2 - c^2 tan\theta \quad 3.45$$

Oil production is still given be 3.41 and the well vertical centre by 3.42 where the increment value is *c,* the advance of the cone above the original well centre.

The current well lower surface is given by $z_{wD} + c$ 3.46

The calculation of the dimensionless time is now made with the updated parameters using 3.28 and 3.26. Once again, this estimates the dimensionless time required to produce the well to that particular condition.

The steps can be summarized as follows

1. Compute dimensionless initial rate, well radius and length using 3.22, 3.38 and 3.23 respectively
2. Divide the dimensionless well radius into convenient equal segments

3. Reduce the diameter by one segment value assuming that the invading cone had swamped that segment.
4. Calculate the distance to the well centre using equation 3.37
5. Calculate the substended angle using equation 3.39
6. Calculate the area open to oil production using equation 3.40
7. Determine the reduced oil production using 3.41
8. Obtain the effective well centre using 3.42 and the current well outer surface using 3.43
9. Implement 3.26 and 3.28, integrating from the original base of the oil zone to the current well lower surface to obtain the time to attain the current conditions i.e from 0 to $(z_{wD} + r_{wD} - h)$.
10. Transform into real values using equations 3.11 and 3.22 appropriately to obtain the time and oil rate. The water production is the difference between the current oil rate and the original rate.
11. The process is repeated while sequentially reducing the radius.
12. On crossing the original well centre, the subtended angle is given by equation 3.44
13. The oil area is obtained from equation 3.45 and the oil rate obtained from equation 3.41
14. The well's lower surface is now given by equation 3.46
15. Time is calculated as in step 8 above and the oil and water rate obtained as in step 9 above.

3.4.4 SLANTED WELL POST-BREAKTHROUGH FORMULATION

The handling of the after breakthrough trends for slanted wells involves the combination of the effects of vertical and horizontal components. The process can be broken down into three stages. During the first stage, the cone front progresses along the lower half of the vertical component. The dimensionless time for each cone step is calculated from the inclined well breakthrough time formulation given in equation 3.32 and using the vertical and horizontal potential functions derivative given in 3.27 and 3.26 respectively. At each computation step, the height of the cone front is increased along the vertical component of the inclined well. The oil rate is reduced as in the vertical well after breakthrough production description made above. The current inclined well rate is arrived at by recombining the horizontal component rate (which remains constant during this stage since the cone height has not reached it) and the vertical component rate (which has been reduced due to the progress of the cone height) using Pythagora's theorem.

$$q_D = \sqrt{q_{Dv}^2 + q_{Dh}^2} \qquad 3.47$$

The second stage starts when the advancing cone front reaches the lower surface of the horizontal component. The effective well centre for the horizontal component is given by

$$z_{wD} = 1 - (0.5L_{wD}cos\theta_D) \qquad 3.48$$

And the effective horizontal component lower surface is obtained by substracting the radius from the well centre.

Here the horizontal and vertical rate components are reducing simultaneously. The vertical and horizontal after breakthrough trends discussed previously are used appropriately. The vertical penetration ratio, b, reduces as follows

$$b_n = 0.5L_{wD}cos\theta_D + r_{wD} - h \qquad 3.49$$

Where *h* is the unit cone growth step.

The vertical component rate reduction is given by

$$q_{Dv_n} = q_{Dv} \times {}^{b_n}/_{L_{wD}cos\theta_D} \qquad 3.50$$

The second stage is better splitted into two halves as was done for the simpler pure horizontal well.

At the third stage, the horizontal component has been swamped under the advancing cone and so it ceases to contribute towards the production of oil. Consequently, the trend is exactly identical to that of a simple vertical well and the vertical well analysis can be use. The penetration ratio reduction is now given by

$$b_n = 0.5L_{wD}cos\theta_D - r_{wD} - h \quad 3.51$$

The oil rate reduces as in equation 3.50 above and the time progress is obtained using the vertical time function given by equation 3.28 implementing equation 3.27.

The inclined well post-breakthrough development is outlined below.

1. Compute dimensionless rate, well radius and length using equations 3.22, 3.38 and 3.30 respectively.
2. Determine the vertical and horizontal components of the rate and well length using equation 3.33.
3. Determine the horizontal component vertical height using equation 3.48. The lower surface is obtained by substracting the dimensionless radius from the component vertical height
4. During the first stage, which is operative while the fluid interface is yet to get to the horizontal component's lower surface, half the vertical component length minus the well radius ($0.5L_{wD}cos\theta_D - r_{wD}$) is divided into convenient equal segments.
5. The vertical component is reduced by a value of the segment obtained in 4.
6. The vertical component rate is reduced as given in equation 3.50
7. Using equations 3.22, 3.26 and 3.27 the dimensionless time is obtained employing the current values of the resolved rate and length.
8. The dimensionless oil rate is obtained from equation 3.47
9. Real time and oil rate are obtained using equations 3.11 and 3.22 respectively. The water rate is the difference between the initial production rate and the current oil rate.

10. The vertical component is reduced again by a value of the segment obtained in 4. Steps 6 to 9 are repeated. The process stops when the value of the penetration ratio is equal to half its initial value plus the well radius $(0.5L_{wD}cos\theta_D + r_{wD})$ or the interface reaches the lower surface computed in step 3.
11. At this point, the second stage begins. For the length of the well diameter the horizontal component is treated as a horizontal well and the vertical component as a vertical well. The diameter is divided into equal segments of length, *h.*
12. The vertical component is reduced as given in 3.49 and the horizontal vertical height is updated as given in 3.42.
13. The subtended angle in the horizontal component is computed from 3.37 with the distance to the well centre given by 3.37
14. The oil area is computed from 3.40 which is used to obtain the horizontal component's dimensionless oil rate from 3.41. The vertical component rate is obtained from 3.50
15. The updated values are used in the computation of the dimensionless time using 3.32 and implementing 3.26 and 3.27 appropriately.
16. The current dimensionless rate is determined employing 3.47
17. Real values can be obtained as demonstrated in the vertical and horizontal well outline above.
18. Steps 12 to 17 is repeated until the horizontal component original centre $\left(z_{wD} = 1 - (0.5L_{wD}cos\theta_D)\right)$ is reached.
19. The horizontal component substended angle is now given be 3.44 and its oil area by 3.45.
20. The oil rate for the horizontal and vertical is still given by 3.41 and 3.50 respectively.
21. The dimensionless time is calculated using 3.32, 3.26 and 3.27 and current rate given by 3.47. Real values are obtained as before.

22. Steps 19 to 21 are repeated until the horizontal component vanishes and the penetration ratio is $b = 0.5L_{wD}cos\theta_D - r_{wD}$.
23. At this point, the exact simple procedure developed for the vertical well can be used since only the vertical rate contribute to oil production.
24. Divide the remaining penetration ratio given in step 23 into convenient equal intervals and apply steps 4 to 7 of the vertical well post-breakthrough computation steps outlined above.

Chapter Four

RESULT AND ANALYSIS

Implementing the methodology set out in the previous chapter allows for the computation of critical rate, breakthrough time and after breakthrough trends for vertical, horizontal and inclined wells. The computation requires the use of a computational software. *Scilab,*a free source mathematical computational software, was used in the calculation. Writing a simple program to carry out the computation ensures that several computation can be done within reasonable time. Scilab codes developed for major computations are presented in Appendix C.

4.1 BREAKTHROUGH TIME

The time for an advancing cone to reach the producing well perforation is obviously a strong function of the rate of fluid production. This relationship was explored for the three well trajectories studied.

4.1.1 VERTICAL WELL

The dimensionless breakthrough time trend for the vertical well as a function of the dimensionless rate and penetration ratio is given in Fig. 4.1 below. From Fig. 4.1, it can be seen that as the dimensionless oil production rate increases, the breakthrough time reduces since the higher fluid withdrawal implies greater viscous drag on the fluid interface and therefore greater velocity of the cone height towards the producing perforation. The figure also indicates that as the well penetration ratio increases, the breakthrough time declines. This is in correspondence with the physics of the reservoir since a shorter distance would have to be transversed by the advancing cone front for a completion that gets closer to the fluid interface. This effect is notably more pronounced at large dimensionless rates. At much lower dimensionless rate, the effect of the penetration ratio on breakthrough time diminishes. This trend can be explained by noting that at such low rates, the effect of spherical versus radial flow becomes important. For small penetration ratio, the well would have to transverse a longer distance but the limited entry increases the pressure gradient around the well and this increases the viscous drag. This increase in viscous drag due to the spherical flow tends to cancel out the benefit of the larger distance between the well and the fluid interface. For larger penetration ratio, the flow tends to be radial and exerts less drag on the fluid interface. This reduction in viscous force works in the opposite direction with the shorter distance between the well and the fluid contact.

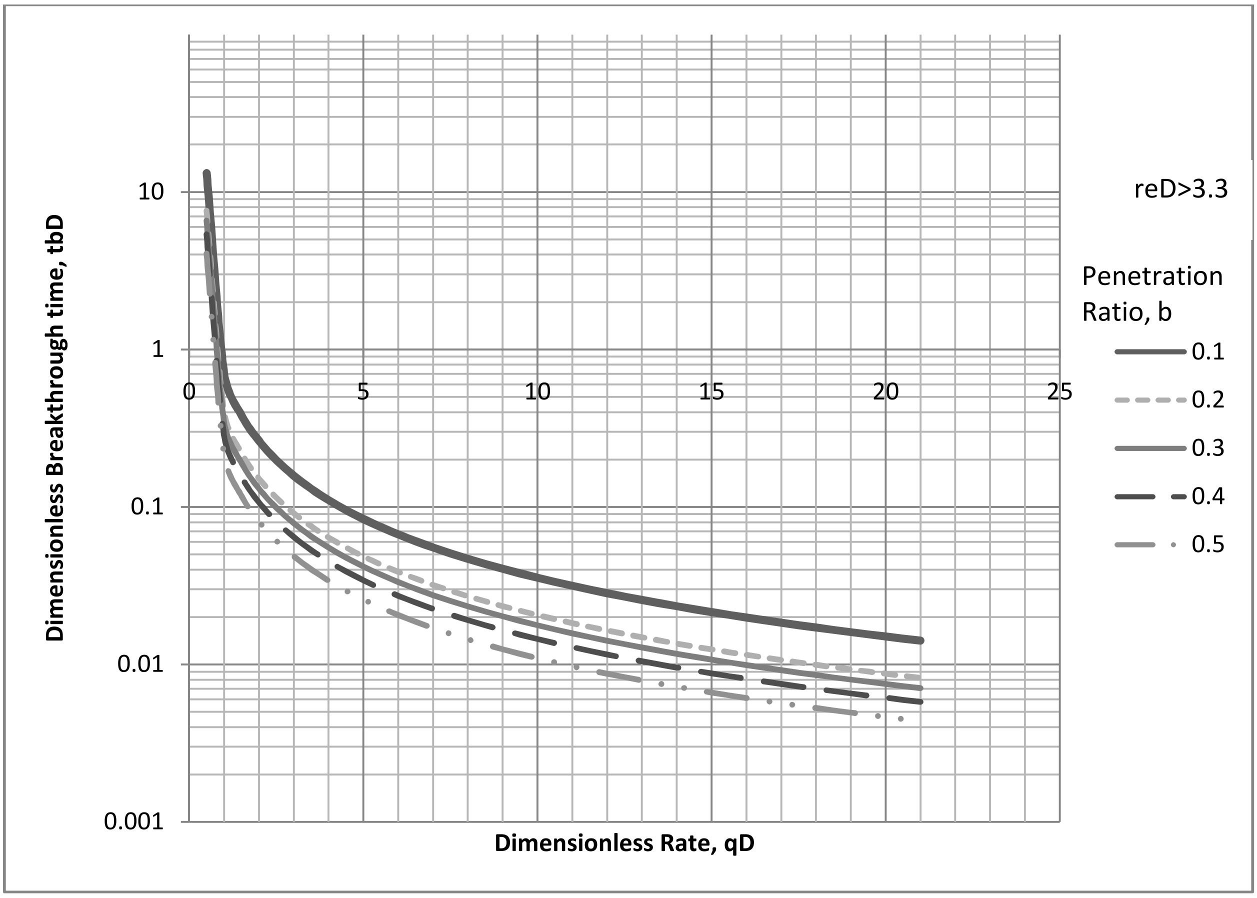

Fig 4.1 Breakthrough time trend for vertical well

Fig. 4.1 was generated from the mathematical expression 4.1 which is an empirical correlation developed from the breakthrough time procedure discussed in chapter three.

$$t_{bD} = \frac{0.3789}{q_D^{1.2383}} EXP\left(0.1378/_{q_D^4}\right) * (2.257b^3 - 3.223b^2 - 0.084b + 1.066) \qquad 4.1$$

The above correlation in 4.1 is a modification of the correlation presented by Ozkan and Raghavan (1990) for the simplified calculation of breakthrough time. Their correlation has the limitation of only been applicable to a condition in which the vertical well perforation is far smaller than the pay zone thickness. The current form of the correlation permits the convenient determination of breakthrough time for all practical penetration ratios i.e. it is valid for small or large perforation ratios. This current correlation, as was the correlation of Ozkan and Raghavan (1990), was developed for dimensionless reservoir radius, reD>3.3.

4.1.2 HORIZONTAL WELL

The breakthrough trend of a horizontal well as a function of dimensionless production rate and well length is presented in Fig 4.2 below. The figure shows an inverse relation for dimensionless breakthrough time and oil production rate and a direct relation for the dimensionless well length as expected.

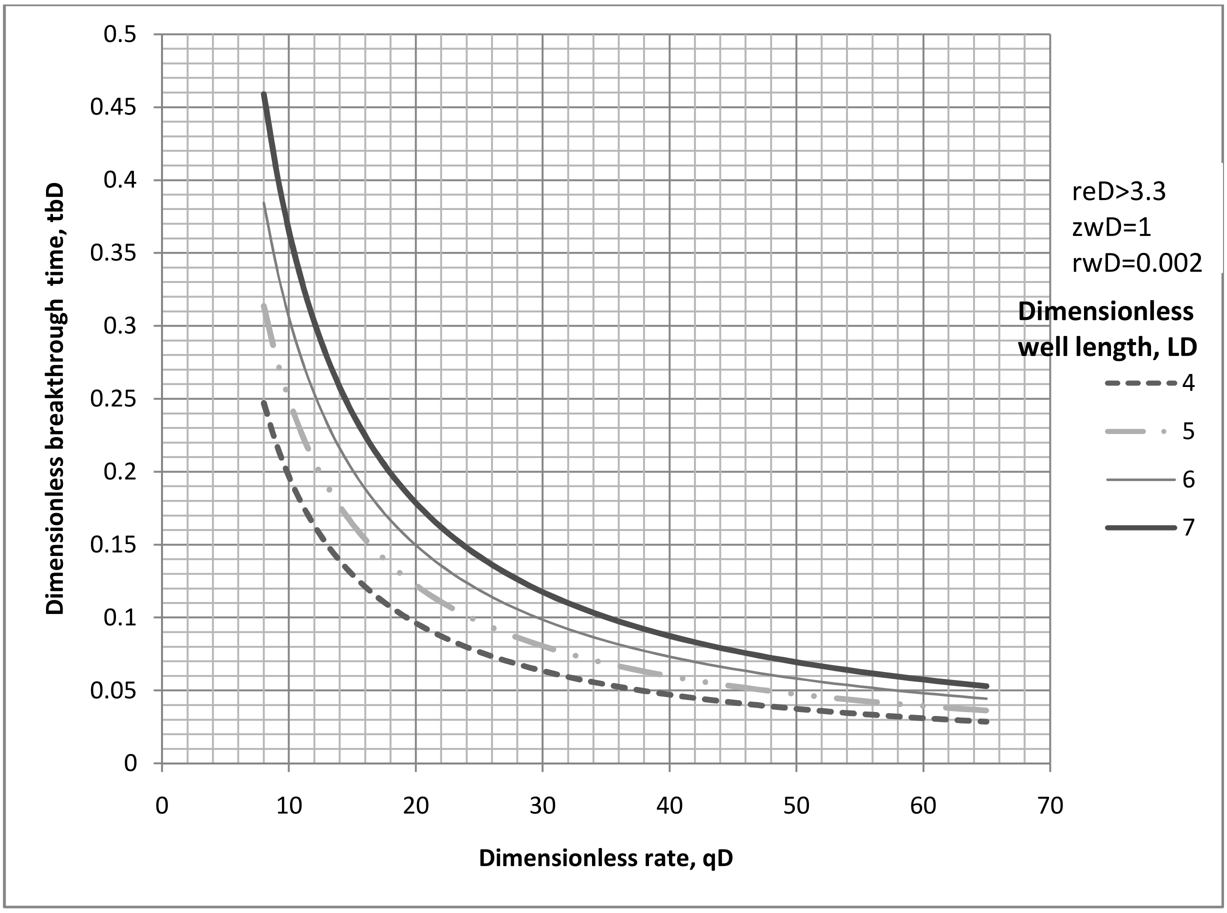

Fig 4.2 Breakthrough trend for horizontal wells

As production rate increases, the breakthrough time decreases due to greater viscous drag and as the well length increases, the effective pressure gradient required for a given production rate reduces and so does the viscous drag. This results in an increase in the time taken by the cone to get to the well. Fig. 4.2 straightens out on log-log indicating a log-log relation for the breakthrough time and the dimensionless production rate.

Fig. 4.2 can be generated from the mathematical expression 4.2.

$$t_{bDz_{wD=1}} = 16.69455(0.001L_D^2 + 0.025L_D + 0.01)q_D^{-1.03} \qquad 4.2$$

The dimensionless breakthrough time calculated from equation 4.2 is restricted for a well completion height, zwD = 1 and rwD=0.002. To extend the correlation to well position other than zwD = 1, correlation 4.3 was generated.

$$\frac{t_{bD}}{t_{bD_{zwD=1}}} = -3.247z_{wD}^4 + 5.172z_{wD}^3 - 1.39z_{wD}^2 + 0.501z_{wD} - 0.032 \qquad 4.3$$

4.1.3 INCLINED WELL

The breakthrough time trend for inclined wells as a function of dimensionless rate and inclination angle is presented in Fig 4.3. The trend is very similar to that of the horizontal well. The dimensionless breakthrough time reduces with increase in dimensionless production rate and increases with increase in the inclination angle. The inclination angle measures how far the well is deviated from the vertical. For high inclination angle, the well is closer to the horizontal position than to the vertical. In such a condition, the well has more clearance from the fluid contact. This translates to a large distance that must be transversed before the advancing cone gets to the well's perforation. Hence, the breakthrough time is expectedly greater than when the inclination is smaller, and thus the well is closer to the vertical orientation, which reduces its distance from the fluid interface.

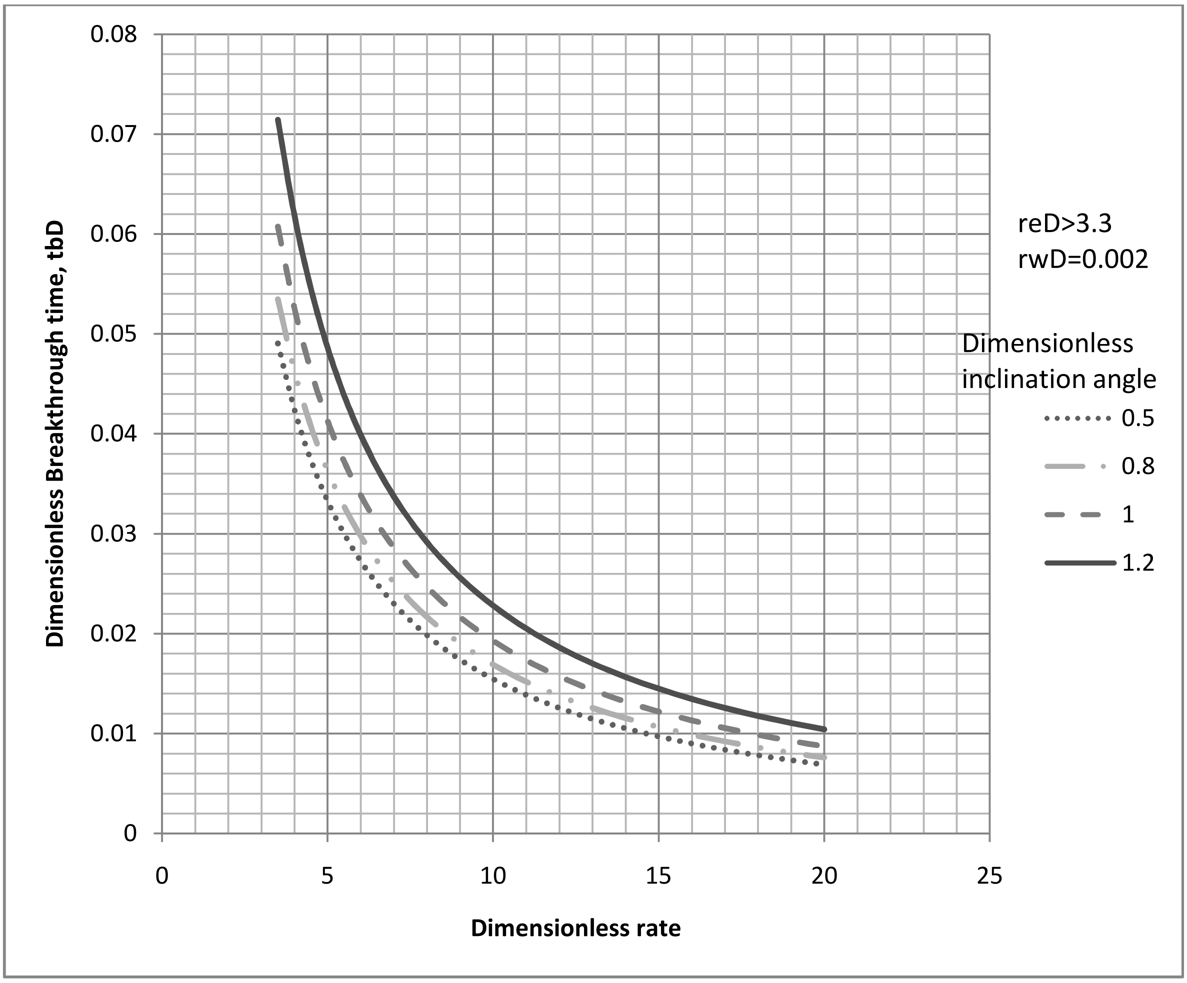

Fig 4.3 Breakthrough time trend for Inclined wells

Simplified correlation for the calculation of breakthrough time for inclined wells is given in equation 4.4 and 4.5.

$$t_{bD}^{*} = 0.232(0.005L_{wD}^{4} - 0.011L_{wD}^{3} - 0.008L_{wD}^{2} + 0.007L_{wD} + 0.033)(0.023\theta_{D}^{2} - 0.022\theta_{D} + 0.032)q_{D}^{-1.06} \quad 4.4$$

$$t_{bD} = 891.5t_{bD}^{*} - 0.001 \quad 4.5$$

For $L_{wD} < 1.2$ and $qD < 25$.

4.2 CRITICAL RATE OF OIL PRODUCTION

The critical rate has been defined as the maximum stable oil production rate under a given reservoir and well conditions such that no free water or gas is produced, if present[6]. Practically, it is expected that the critical rate should lead to an infinite breakthrough time since the developed cone remains stable and stationary while the applicable conditions remain unchanged. The authors of Ref. 3 presented simplified analytical expressions for the calculation of critical oil production rate for vertical and horizontal wells. The correlations were tested and found to be substantially representative and are therefore reproduced in equations 4.6 and 4.7 below.

For horizontal well

$$q_{cD} = (1.0194 - 0.1021z_{wD} - 0.2807z_{wD}^2)z_{wD}L_D \qquad 4.6$$

For vertical well

$$q_{cD} = 0.546 - 0.021b - 0.525b^2 \qquad 4.7$$

Analytical approach for the calculation of inclined well critical rate of oil production follows the same procedure as for vertical and horizontal well. For a given dimensionless well length, angle of inclination and reservoir properties, different dimensionless rate are inserted into the breakthrough time expression presented in chapter three. The rate that gives a breakthrough time much more greater than other rates and having no other practical higher rate at that given reservoir and well conditions is considered to be the critical oil rate. This trial and error approach can sometimes be very inconvenient which underscores the value of a simplified correlation for estimation of inclined well's critical rate of oil production like the ones presented in 4.6 and 4.7 for vertical and horizontal wells. The correlations for inclined wells are given in 4.8 and 4.9 below.

$$q_{cD}^* = (0.214\theta_D^4 - 0.661\theta_D^3 + 1.137\theta_D^2 - 0.844\theta_D + 0.939)(-0.07L_{wD}^4 + 0.033L_{wD}^3 + 0.03L_{wD}^2 + 0.001L_{wD} + 0.802) \qquad 4.8$$

$$q_{cD} = 1.247q_{cD}^* - 0.003 \qquad 4.9$$

4.3 AFTER BREAKTHROUGH BEHAVIOUR

When the fluid gets to the well perforation, breakthrough is said to have occurred. At this point the rate of fluid production is shared between the production of oil and the production of the coning phase. A typical after breakthrough history was computed as developed in chapter three for Dataset 1 presented in Appendix B and depicted in figure 4.4.

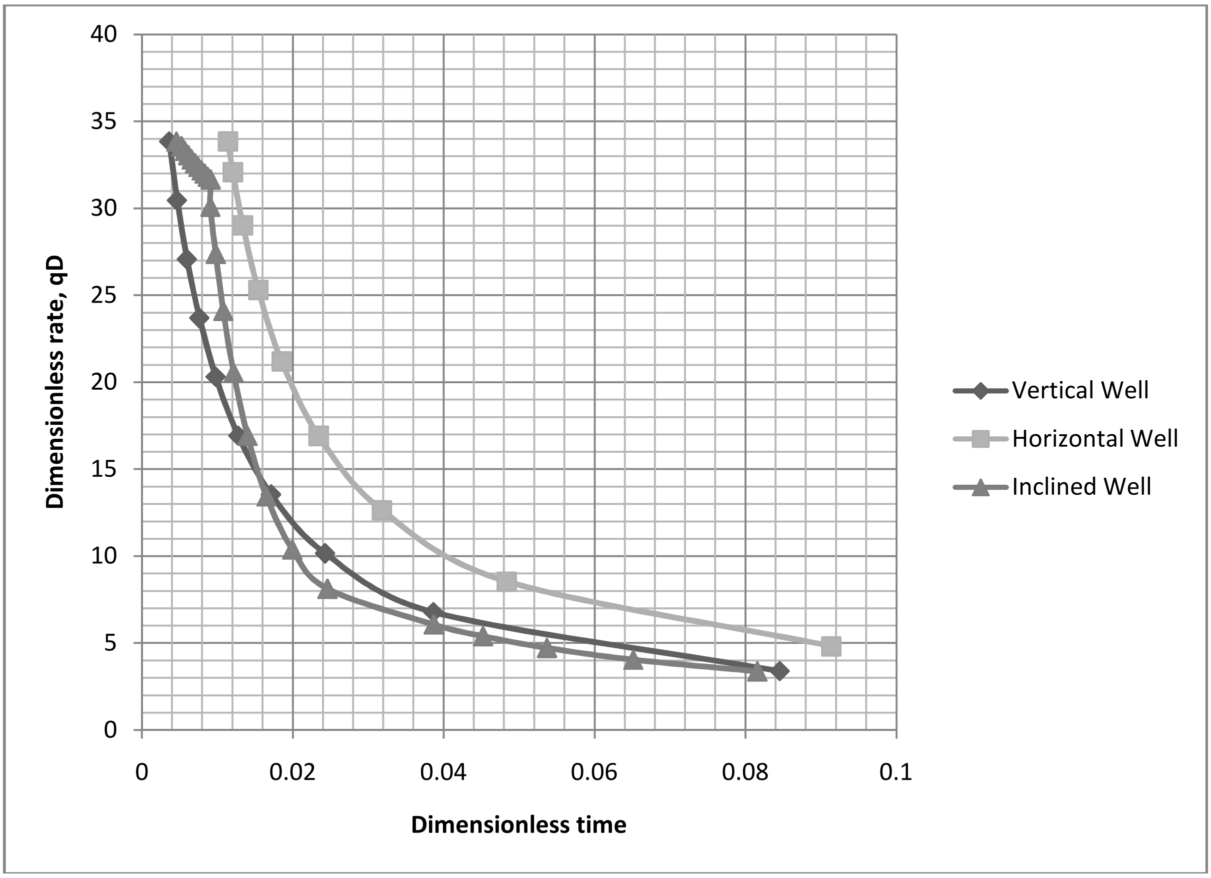

Fig 4.4 After breakthrough trends

The graph of the breakthrough trends (Fig. 4.4) indicates that the horizontal well clearly maintained a higher performance for the entire length of the production: it recorded maximum breakthrough time and had the largest rate at the end of the production period. The inclined well corresponded closely with the rate of production of the horizontal well at the earlier stage but fell behind the rate of the vertical well at a later time (see Fig. 4.4) while approaching the rate of the vertical well towards the end of the production. The rather rough course of the inclined well trend is a result of the different models used to represent its different post-breakthrough stages as developed in chapter three.

4.4 MODEL RESULT COMPARISON

4.4.1 LITERATURE MODEL

As noted in chapter one, the petroleum industry has a number of critical rate and after breakthrough models, both analytically and empirically. Authors of Ref 3 compared the critical rate and breakthrough time results obtained from the horizontal and after breakthrough models discussed in this work and there is little purpose in repeating that here. In this research, the author could not find any correlations or expressions for inclined wells in the literature. Hence, such comparison to a literature based model could not be made. The after breakthrough trends for vertical wells developed in this work were compared to the literature correlations of Sobocinski and Cornelius and the Bournazel and Jeanson as presented in the book, *Horizontal Well Technology*[5]. The comparisons were done using Dataset 2 (presented in Appendix B), adopted from Ref. 5.

4.4.2 NUMERICAL SIMULATION

Inclined well breakthrough time predictions using the analytical model developed in this work were compared with simulation results. Post-breakthrough predictions comparison to numerical simulation results for all the well types studied was done.

A simple two phase numerical reservoir model was built to compare its performance with the analytical solution developed in this study. A radial coordinate of 10 by 1 by 29 grid system for the radial, angular and vertical directions was adopted. The oil pay zone of 42 feet was modelled by twenty one (21) grid layers of 2 ft each to provide a refined vertical griding while the other remaining 8 layers were for the water aquifer with varying thickness ranging from 8 to 200 ft from top to bottom of the aquifer. Uniform permeability of 37 md and 3.7 md in the horizontal and vertical directions, respectively, were modeled throughout the reservoir. The layer thicknesses and porosity, however, were varied especially for the water zone where comparatively large values were allocated to create the effect of an active water aquifer. To further ensure steady state flow, a horizontal water injection well was placed at the bottom of the aquifer for voidage replacement. The injector was constrained to inject water at the corresponding production rate in the oil zone. This scheme was adopted from the work of De Souza, A.L.S. et al (1998)[10]. The oil was considered dead while two cases of relative permeabilty specification were studied. A two-phase modified relative permeability table adapted from Eclipse 100 Chap Test Data (Revised July 1990) from Eclipse Dataset was one case and a two-point relative permeability function to mimic the analytical development was the other. The data are reproduced in Appendix B. Variable liquid production rates were specified for the single well draining the reservoir. In each production case, the average reservoir pressure remained fairly constant throughout the simulation.

4.4.3 RESULTS AND DISCUSSIONS

The results of the results comparison will be presented in this section. The following legend would be adopted in the figures.

Legend

- This Study – refers to this research work
- 2 – Sobocinski and Cornelius Breakthrough time (Mobility ratio = 3.27) /Kuo and Desbrisay post-breakthrough prediction
- 3 – Sobocinski and Cornelius Breakthrough time (Mobility ratio = 1) /Kuo and Desbrisay post-breakthrough prediction
- 4 – Bournazel and Jeanson breakthrough time (Mobility ratio = 3.27) /Kuo and Desbrisay post-breakthrough prediction
- 5 - Bournazel and Jeanson breakthrough time (Mobility ratio = 1) /Kuo and Desbrisay post-

breakthrough prediction

- 6 – Numerical Simulation with detailed permeability specification
- 7 - Numerical Simulation with straightline permeability specification

The semi-analytical formulation developed in this research assumed unit mobility ratio. Consequently, it was considered appropriate to investigate the effect of including and not including mobility ratio in literature models in relation to this current work.

The inclined well breakthrough time predictions comparison to numerical simulations is given in Fig. 4.5.

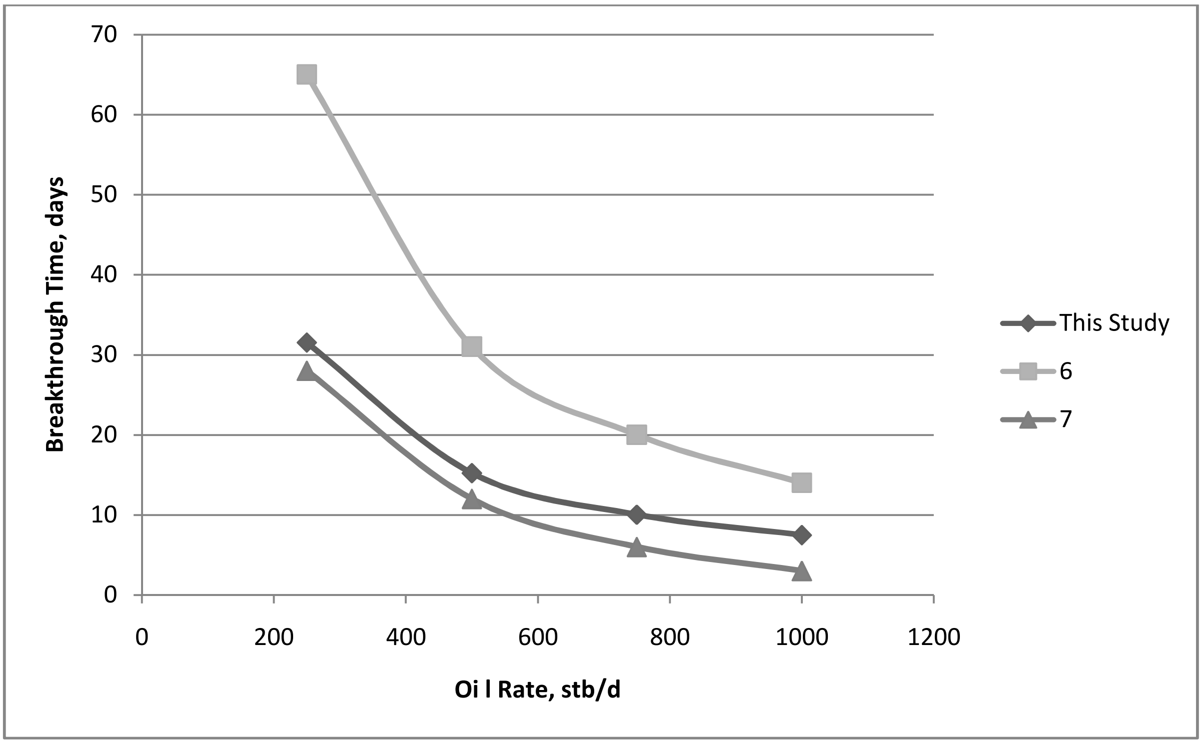

Fig. 4.5 Breakthrough time prediction comparison for inclined wells

From Fig. 4.5, it can be seen that the predictions made by the semi-analytical model developed in this work can be considered consistent with simulation results. The semi-analytical predictions were closer to the result of the straightline permeability simulation. This can be easily attributed to the assumption of single value permeability made in the semi-analytical development. It is interesting to note that the trends of the breakthrough predictions were very consistent and the difference in predictions diminished with increase in production rate. The relative agreement of the predictions at higher rates may be explained by noting that at higher rates, the semi-analytical model implicit assumption that the water phase takes the place of the oil without flowing[4] becomes more valid. And at higher flow rates, the effect of the relative permeability specification difference tended to diminish.

The post-breakthrough prediction comparison for vertical well are given in Figures 4.6 – 4.10.

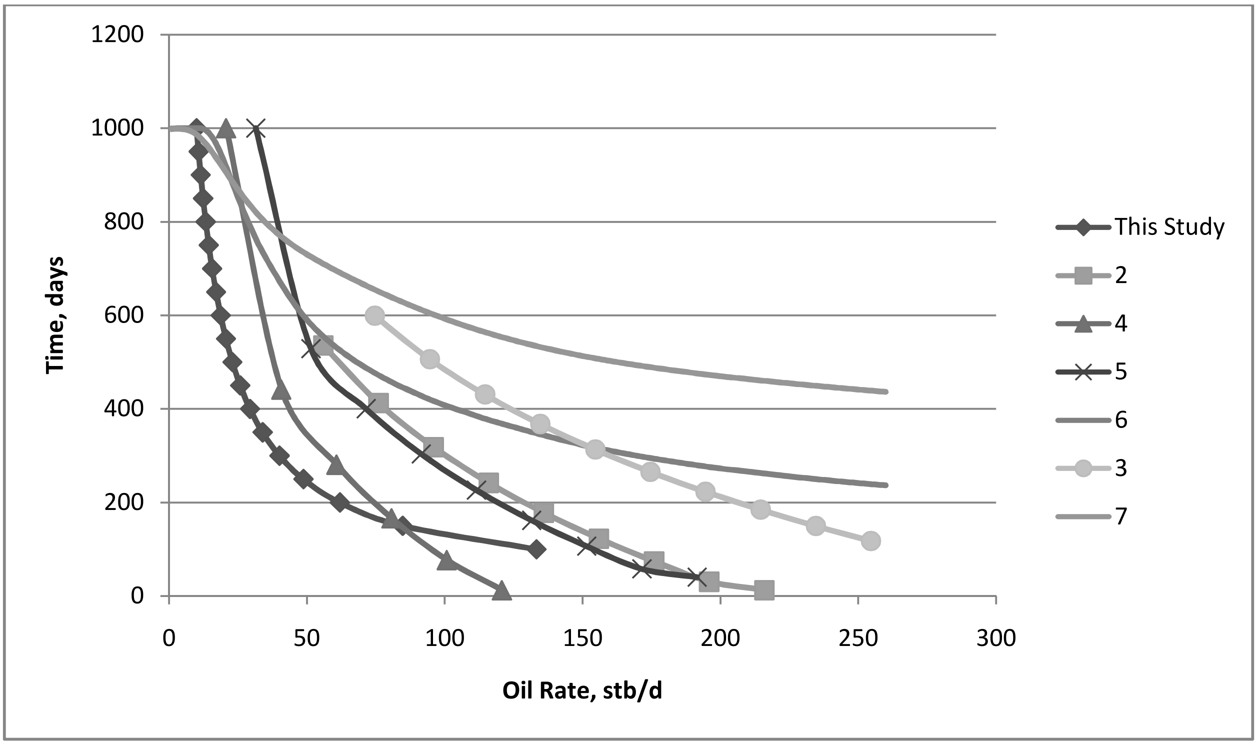

Fig. 4.6 Vertical well post-breakthrough prediction comparison (liquid rate 1000 b/d; perforation length 10 ft)

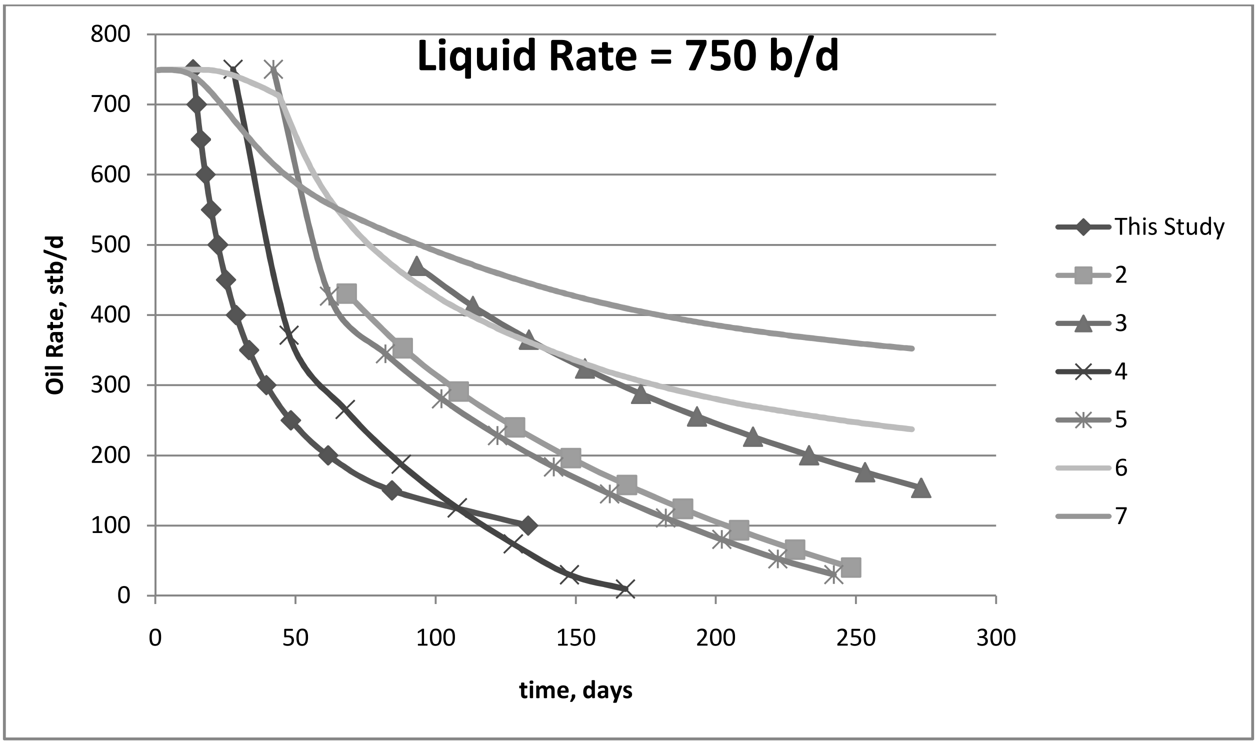

Fig. 4.7 Vertical well post-breakthrough prediction comparison (liquid rate 750 b/d; perforation length 10 ft)

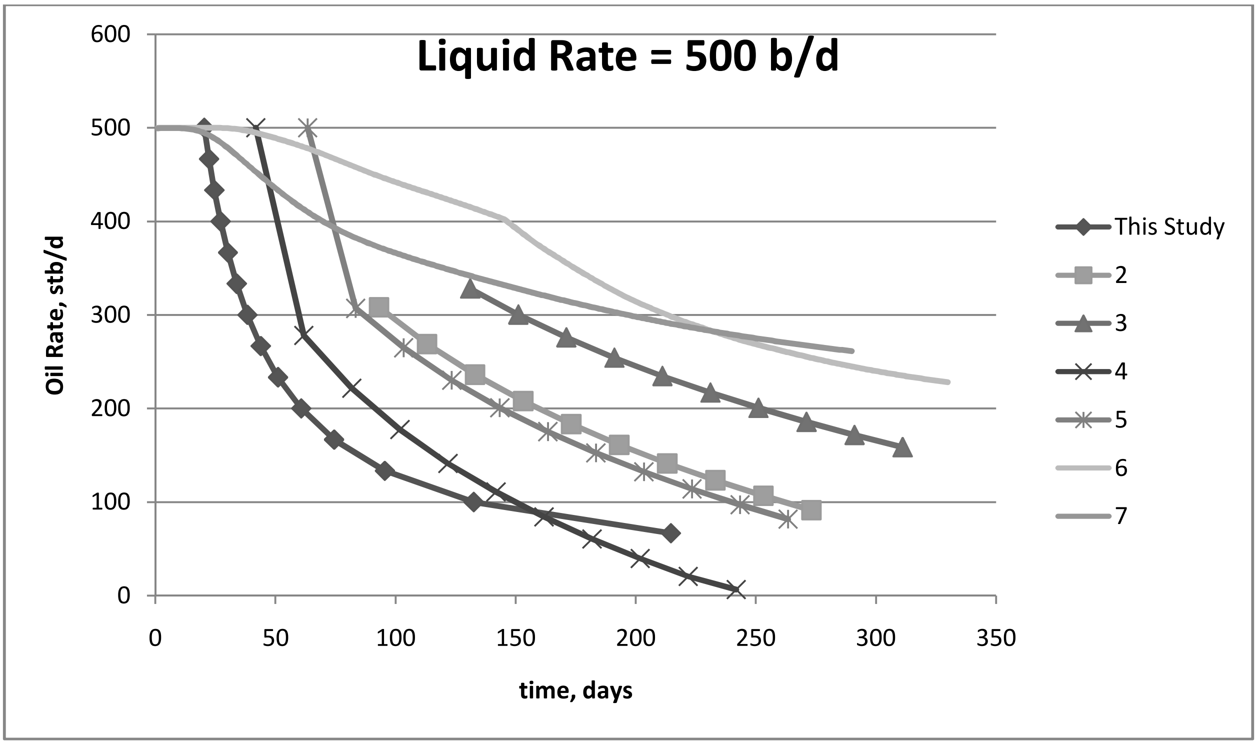

Fig. 4.8 Vertical well post-breakthrough prediction comparison (liquid rate 500 b/d; perforation length 10 ft)

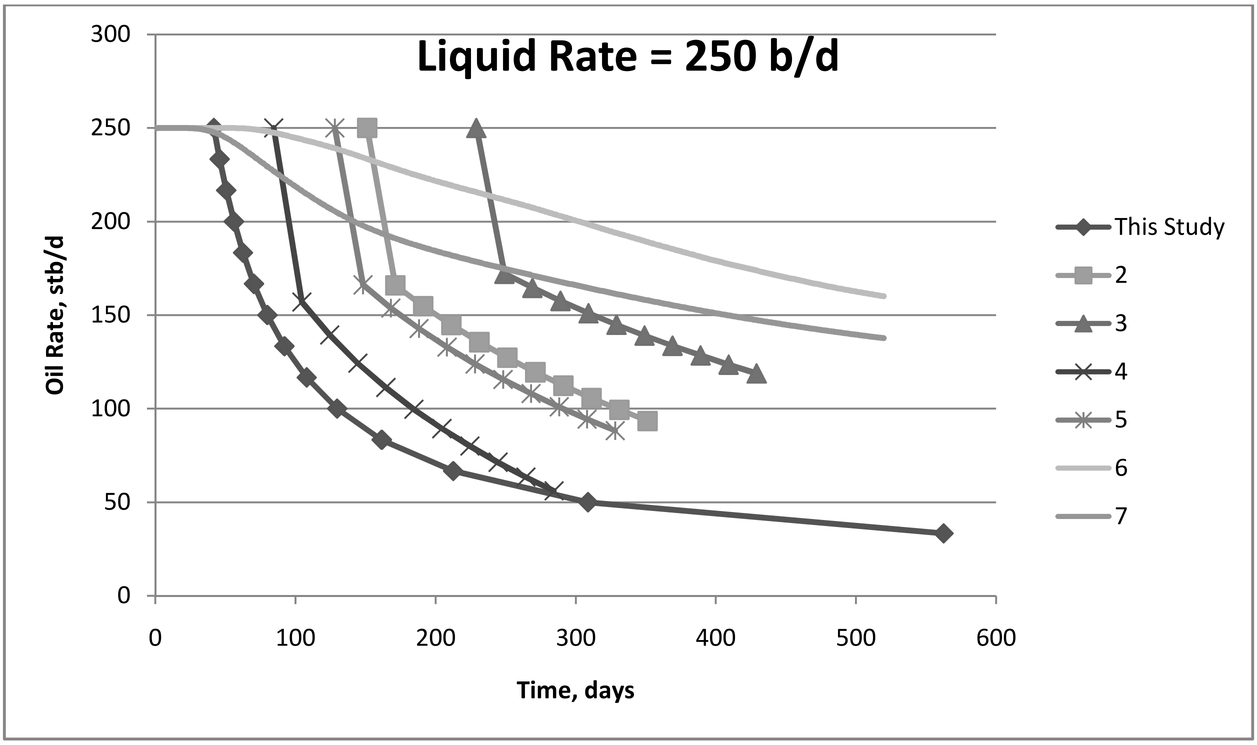

Fig. 4.9 Vertical well post-breakthrough prediction comparison (liquid rate 250 b/d; perforation length 10 ft)

Vertical well predictions shown in Fig. 4.6 – 4.7 reveal great differences in the predicted post-breakthrough oil production with time. The simulation results consistently gave higher oil production and the analytical development in this work gave the lowest values except at late times. The lower oil rate prediction by this semi-analytical model may be seen as the result of the simplifying assumptions made in its formulation considering the physical complexity of multiphase fluid flow. The assumption that phase flow is dependent only the area open to flow may be very important in this regard. Fluid flow at the well perforation is seemingly more complex. The exact nature of this interaction with respect to the dimensionless scheme adopted in this work is yet to be fully understood.

Observation of the trends may reveal that the difference between the semi-analytical formulation and numerical simulation results reduces as the total allowable liquid rate increases. Once again, the assumption that the water gets to the perforation without flowing probably becomes more realistic at higher rates as advanced in the discussion of Fig. 4.5. The general trend of the semi-analytical solutions corresponded more with the simulation results than the other correlations studied. This property is very important as it implies that the predictions of this study's semi-analytical model is physically more consistent than the predictions of the analysed literature correlations. The consistency in this study's prediction implies that it can be used, at least, to predict production 'worst-case' scenario and with suitable scaling may be accomodated for other scenarios. The latter application may be in developing physically consistent post-breakthrough correlation using the analytical predictions as backbone or framework.

Horizontal well post-breakthrough prediction comparisons are presented in Fig. 4.10 – 4.11.

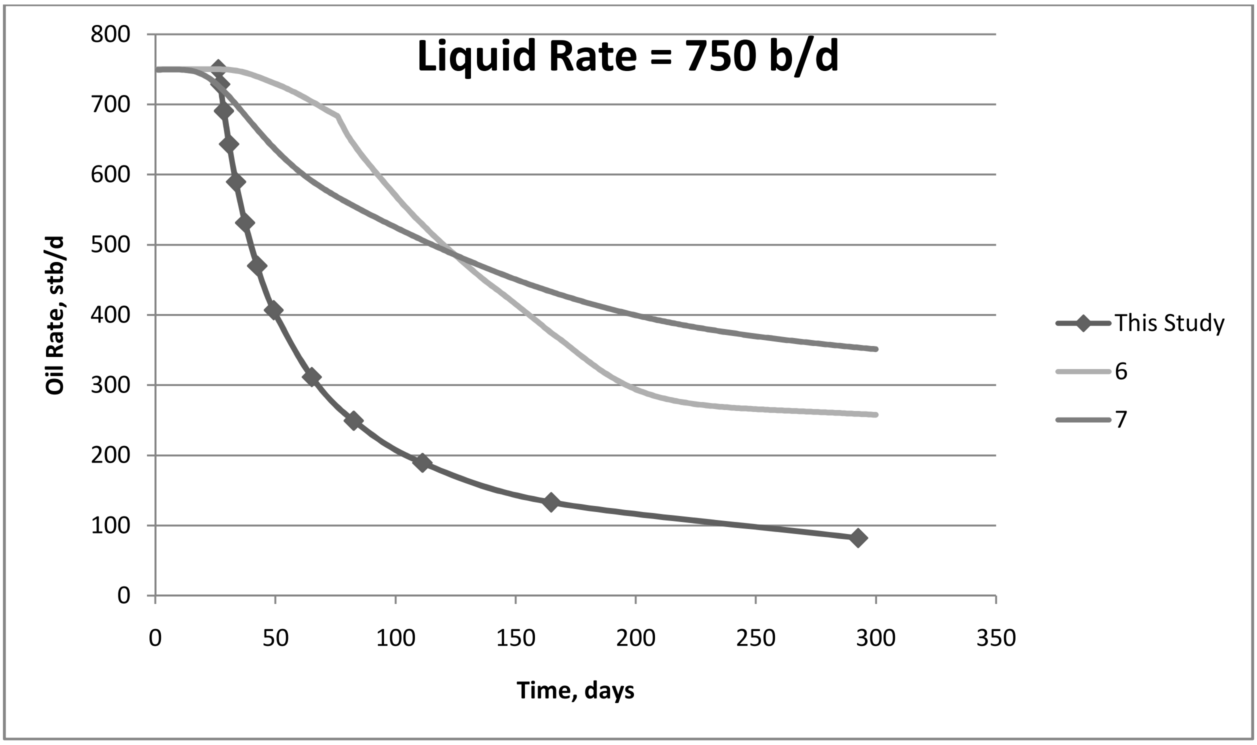

Fig. 4.10 Horizontal well post-breakthrough prediction comparison (liquid rate 750 b/d; length 109.21ft)

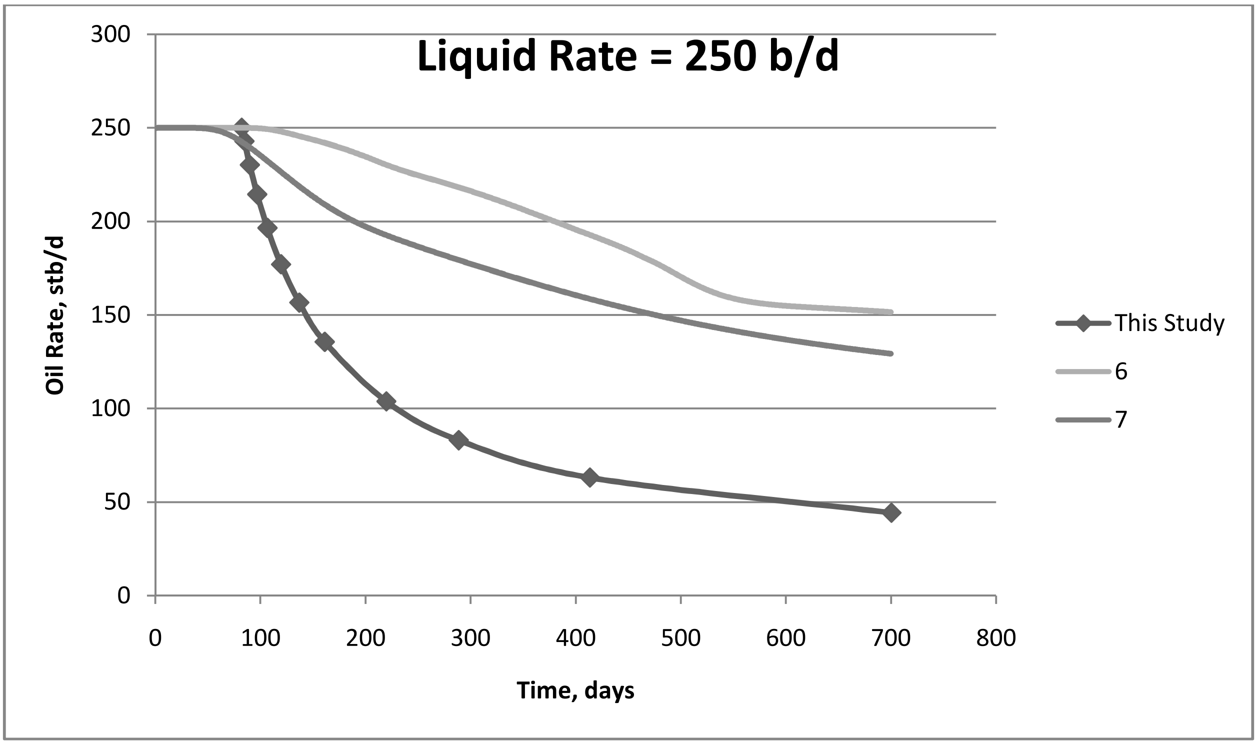

Fig. 4.11 Horizontal well post-breakthrough prediction comparison (liquid rate 250 b/d; length 109.21ft)

The horizontal well comparison shows similar trend as that of the vertical well. The basic trends in predictions are generally consistent but there was great difference in actual values predicted. The difference also tended to close in as the rate of liquid production increases.

The post breakthrough predictions for an inclined well is given in Fig. 4.12 – 4.14.

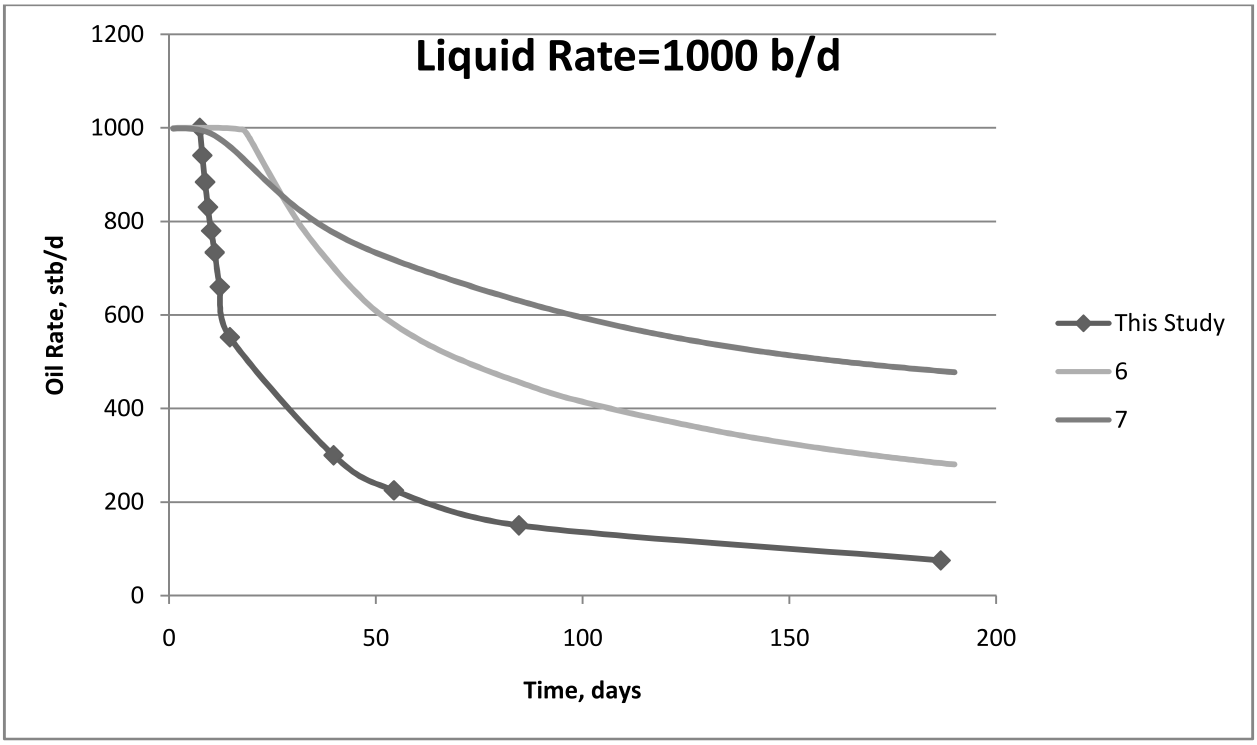

Fig. 4.12 Inclined well post-breakthrough prediction comparison (liquid rate 1000 b/d; perforated length 25.44 ft; inclination to the vertical 66.86^{o})

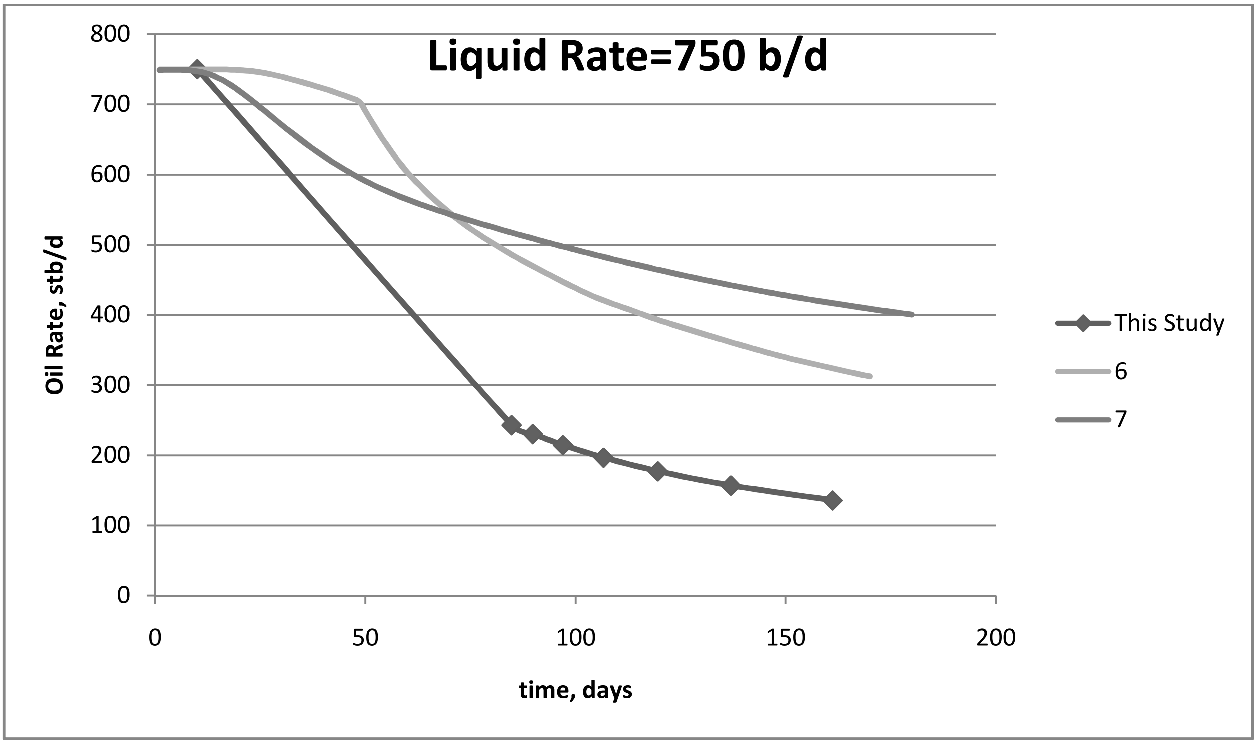

Fig. 4.13 Inclined well post-breakthrough prediction comparison (liquid rate 750 b/d; perforated length 25.44 ft; inclination to the vertical 66.86^{o})

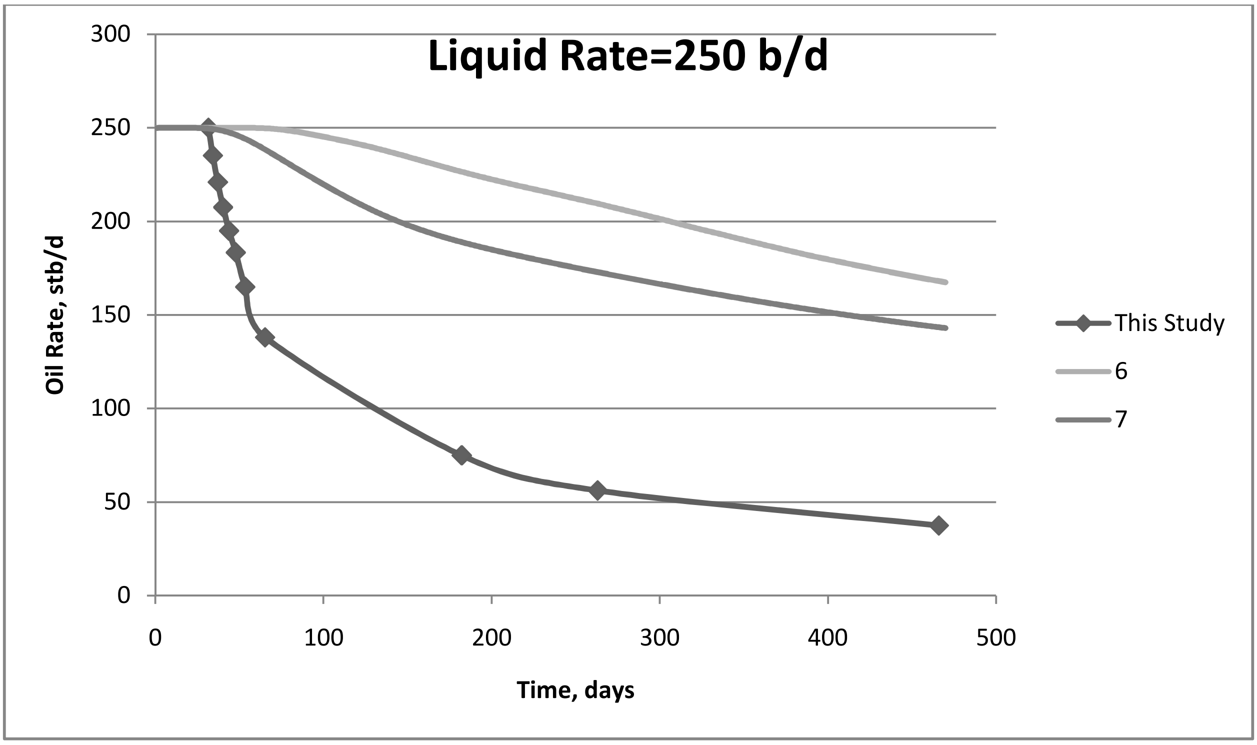

Fig. 4.14 Inclined well post-breakthrough prediction comparison (liquid rate 1000 b/d) (perforated length 25.44 ft; inclination to the vertical 66.86^o)

What was said about the other two well types applies essentially to the inclined well.

4.5 EXAMPLE CALCULATIONS USING CORRELATIONS

To perform the coning computation presented in the chapter three of this research report, simple computer program may be written implementing the developed algorithm or the Scilab code provided in the appendix can be used directly or modified appropriately. But when the full analytical expression cannot be used, the correlations presented earlier in this chapter can be used as would be illustrated in this section. It is important that the assumptions made in developing the correlations be applicable to a given case before use. The calculation of the breakthrough and critical rate are rather straight forward: calculating the appropriate dimensionless parameters and inputing into the appropriate correlation. The result is then reconverted into real parameters using the dimensionless definition. The after breakthrough computation requires more repetitive steps. This example calculation are based on Dataset 2 presented in Appendix B except when otherwise indicated.

Vertical Well

Vertical well length = 42 ft; Well rate = 500 stb

Critical Rate

$$q_{cD} = 0.546 - 0.021b - 0.525b^2$$

$$b = \frac{h_p}{h_o} = \frac{42}{84} = 0.5$$

$$q_{cD} = 0.40425$$

$$q_o = \frac{q_{cD} k_r h_o^2 \Delta\rho}{325.7 \mu_o B_o} = 45.2 \text{ stb}$$

Breakthrough Time

$$t_{bD} = \frac{0.3789}{q_D^{1.2383}} \text{EXP}\left(0.1378 / q_D^4\right) * (2.257b^3 - 3.223b^2 - 0.084b + 1.066)$$

$$q_D = \frac{325.7 q \mu_o B_o}{k_r h^2 \Delta\rho} = 4.4719$$

$$t_{bD} = 0.065068$$

$$t = \frac{364.6 t_D f \mu_o h}{k_z \Delta\rho} = 215.49 \text{ days}$$

The value obtained using Sobocinski and Cornelius method[5] is 402.2 days.

Using Bournazel and Jeanson Method[5], the breakthrough time is 222.8 days.

Using the original correlation of Ref 3, the breakthrough time is 196.4 days.

Sobocinski and Cornelius formulation and the method of Bournazel and Jeason incorporated the unfavourable mobility ratio of 3.27; their predicted breakthrough time is larger if unit mobility ratio assumption were made.

Post-breakthrough Computation

For ease of calculation, the penetration ratio would be divided into 5 equal intervals of length = 0.5/5 = 0.1 each

First post-breakthrough step

1. bn = 0.5 – 0.1 = 0.4

2. $q_{Dn} = \left({}^{b_n}/_{b} \right) \times q_D = \left({}^{0.4}/_{0.5} \right) \times 4.4719 = 3.5775$

3. Using b = 0.4 and qD = 3.5775, we substitute in the breakthrough time expression to get the time value at this current condition

   $$t_{bD} = \frac{0.3789}{q_D^{1.2383}} EXP \left({}^{0.1378}/_{q_D^4} \right) * (2.257b^3 - 3.223b^2 - 0.084b + 1.066) = 0.098429$$

4. Converting rate and time to real values

   $t = \frac{364.6 t_D f \mu_o h}{k_z \Delta\rho} = 325.97 \text{ days}$

   $q_o = \frac{q_{cD} k_r h_o^2 \Delta\rho}{325.7 \mu_o B_o} = 400 \text{ stb}$

   qw = 500 – 400 = 100 stb

   So at the 325.97 day, the oil rate had reduced to 400 stb and the water rate had become 100 stb

The second and subsequent post-breakthrough step follow similar process

Horizontal Well

Well length = 560 ft, completed 70 ft from the oil-water contact; Well rate = 1000 stb

Critical Rate

$$q_{cD} = (1.0194 - 0.1021z_{wD} - 0.2807z_{wD}^2)z_{wD}L_D$$

$$z_{wD} = {}^{70}\!/_{84} = 0.83$$

$$L_D = \frac{L}{2h}\sqrt{{}^{k_z}\!/_{k_r}} = \frac{560}{2\times 84}\sqrt{{}^{3.5}\!/_{35}} = 1.05409$$

$$q_{cD} = 0.648544$$

$$q_o = \frac{q_{cD}k_r h_o^2 \Delta\rho}{325.7\mu_o B_o} = 72.51 \text{ stb}$$

Chaperon method[1] gave a critical rate of 39.74 stb/d

Breakthrough Time

$$q_D = \frac{325.7q\mu_o B_o}{k_r h^2 \Delta\rho} = 8.943744$$

$$t_{bDz_{wD=1}} = 16.69455(0.001L_D^2 + 0.025L_D + 0.01)q_D^{-1.03} = 0.065481$$

$$\frac{t_{bD}}{t_{bD_{zwD=1}}} = -3.247z_{wD}^4 + 5.172z_{wD}^3 - 1.39z_{wD}^2 + 0.501z_{wD} - 0.032 = 0.84257$$

$$t_{bD} = 0.065481 \times 0.84257 = 0.05517$$

$$t = \frac{364.6t_D f\mu_o h}{k_z \Delta\rho} = 182.71 \text{ days}$$

The breakthrough time using Papatzacos'[4] method is 346.89 days.

The breakthrough time using the correlation of Ref. 3 is 168.7 days.

It should be noted that the correlations of Ref. 3 and that of Papatzacos et al assumed that the horizontal well was completed at the top of the oil zone.

Post Breakthrough Trend

$$r_{wD} = \frac{1}{2h}r_w\left(\left({}^{k_r}\!/_{k_z}\right)^{0.25} + \left({}^{k_z}\!/_{k_r}\right)^{0.25}\right) = \frac{0.29}{2\times 84}\left(\left({}^{35}\!/_{3.5}\right)^{0.25} + \left({}^{3.5}\!/_{35}\right)^{0.25}\right) = 0.00404$$

Dividing the diameter into five segments of length (0.00404/5 =) 0.000808 for ease of computation.

First post-breakthrough step

1. For a fluid advancement of one segment, the distance of the interface from the well center is given by

   $a = r_{wD} - 0.000808 = 0.003232$

2. Half angle substended at the centre,

   $\theta = \cos^{-1}\left({}^{a}/_{r_{wD}}\right) = 36.87^{o}$

3. Area of segment under the fluid front $= \frac{\theta}{180}\pi r_{wD}^{2} - a^{2}\tan\theta = 2.669 \times 10^{-6}$

   Oil Area $= \pi r_{wD}^{2} -$ Area of Segment behind front $= 4.8607 \times 10^{-5}$

4. Reduced oil production

   Oil Production $= \frac{\text{Oil Area}}{\pi r_{wD}^{2}} \times q_{D} = \frac{4.8607\times10^{-5}}{5.1276\times10^{-5}} \times 8.943744 = 8.4782$

5. Well centre rises as given below

   $z_{wD_{n+1}} = z_{wD_{n}} + 0.5h = 0.83 + 0.5 \times 0.000808 = 0.830404$

6. $t_{bDz_{wD=1}} = 16.69455(0.001L_{D}^{2} + 0.025L_{D} + 0.01)q_{D}^{-1.03} = 0.069188$

   $\frac{t_{bD}}{t_{bD_{zwD=1}}} = -3.247z_{wD}^{4} + 5.172z_{wD}^{3} - 1.39z_{wD}^{2} + 0.501z_{wD} - 0.032 = 0.84316$

   $t_{bD} = 0.069188 \times 0.84316 = 0.058336$

7. Converting dimenesionless time and rate to actual time and rate

   t = 193.19 days

   qo = 947.94 stb

8. Water production = 1000 – 947.94 = 52.06 bbl

Similar procedure is followed for other fluid interface advancement.

Slanted Well

Well length is 42 ft completed at the top of the oil zone and an inclination of 50° to the vertical

Well rate 1000 stb/d

Critical Rate

Converting angle to radian = 0.87266

$$\theta_D = \tan^{-1}\left(\tan\theta\sqrt{k_z/k_r}\right) = \tan^{-1}\left(\tan 0.87266\sqrt{3.5/35}\right) = 0.118615$$

$$L_{wD} = \frac{l_w}{h}\sqrt{k_z/k_r \sin^2\theta + \cos^2\theta} = \frac{42}{84}\sqrt{3.5/35 \sin^2 0.87266 + \cos^2 0.87266} = 0.34346$$

$$q^*_{cD} = (0.214\theta_D^4 - 0.661\theta_D^3 + 1.137\theta_D^2 - 0.844\theta_D + 0.939)(-0.07L_{wD}^4 + 0.033L_{wD}^3 +$$
$$0.03L_{wD}^2 + 0.001L_{wD} + 0.802) = 0.68839$$

$$q_{cD} = 1.247q^*_{cD} - 0.003 = 0.85543$$

qo = 95.646 stb

Breakthrough Time

qD = 8.943744

$$t^*_{bD} = 0.232(0.005L_{wD}^4 - 0.011L_{wD}^3 - 0.008L_{wD}^2 + 0.007L_{wD} + 0.033)(0.023\theta_D^2 - 0.022\theta_D$$
$$+ 0.032)q_D^{-1.06} = 2.30354 \times 10^{-5}$$

$$t_{bD} = 891.5t^*_{bD} - 0.001 = 0.019536$$

t = 64.70 days

Post Breakthrough Trend

The inclined well can be represented as below.

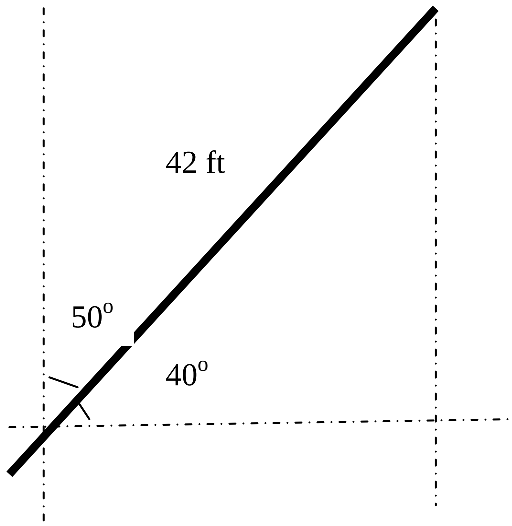

Fig 4.15 Inclined well representation for post-breakthrough trend computation

The vertical height of the inclined well, h = 42 x sin 40 = 27.0 ft

The vertical advancement of the cone front can be captured in convenient steps of, say, 10.

Segment length = 27/10 = 2.7 ft

First post-breakthrough step

For one segment advancement of the cone front, the well rate and length is given by

1. $h_{n+1} = h_n - 2.7 = 27 - 2.7 = 24.3 \text{ ft}$

2. $l_{wn+1} = \frac{h_{n+1}}{\sin 40} = \frac{24.3}{\sin 40} = 37.8 \text{ ft}$

3. $L_{wD} = \frac{l_w}{h}\sqrt{{}^{k_z}/_{k_r} \sin^2\theta + \cos^2\theta} = \frac{37.8}{84}\sqrt{{}^{3.5}/_{35} \sin^2 0.87266 + \cos^2 0.87266} =$

   0.309114

4. $q_{Dn+1} = \frac{0.309114}{0.34346} \times 8.943744 = 8.04937$

5. $t^*_{bD} = 0.232(0.005L^4_{wD} - 0.011L^3_{wD} - 0.008L^2_{wD} + 0.007L_{wD} + 0.033)(0.023\theta^2_D - 0.022\theta_D + 0.032)q_D^{-1.06} = 2.57842 \times 10^{-5}$

   $t_{bD} = 891.5t^*_{bD} - 0.001 = 0.021987$

6. Converting to real values

   t = 72.81 days

   qo = 900 stb

7. Water rate = 1000 – 900 = 100 bbl

Chapter Five

SUMMARY, CONCLUSIONS AND FUTURE DIRECTION

5.1 SUMMARY

The stated objective of this study is to develop a general procedure for the calculation of the critical rate for oil production in the presence of an active water aquifer or gas cap, the determination of the time of breakthrough if a rate higher than the critical is maintained and the after breakthrough behaviour when supercritical production rate is allowed. It was stated that the industry lacked the complete set of approximate analytical approaches to cone evolution modelling for vertical, horizontal and inclined wells. Adhering to that target, a procedure for calculating the stated parameters for all three well configurations using fundamental analytical line source solutions for vertical and horizontal wells as developed in Ref. 3 was generated. Other suitable horizontal and vertical analytical model pair may be used as the solution kernel. This procedure provides a complete suite of semi-analytical solutions for the determination of critical rate, breakthrough time and after breakthrough trend for vertical, horizontal and inclined wells.

Simplified correlations for easy application of the procedure were generated. The correlations were developed to have as little restriction as possible in their application and this motivated the elimination of some previous restriction made in the generation of similar correlations. The developed correlations can be used for quick estimation of well and reservoir behaviour before embarking on a comprehensive numerical simulation as some authors have suggested[3]. They will also be useful in checking the results of more complex numerical simulations.

5.2 CONCLUSIONS

1. A semi-analytical procedure for computing inclined well breakthrough time have been developed.

2. The formulation was used in calculating critical rate of oil production for inclind well following a procedure demonstrated by Ozkan and Raghavan[3] for vertical and horizontal wells.
3. A novel approach in semi-analytically generating post-breakthrough prediction for inclined wells was presented.
4. This approach was extended to vertical and horizontal wells.
5. Simplified correlations for the determination of inclined well critical rate of oil production, breakthrough time for vertical, horizontal and inclined wells were generated from the semi-analytical formulation developed in this work.
6. Simulation comparison showed that the inclined well breakthrough time prediction was consistent and accurate.
7. Simulation and literature comparison showed that the post-breakthrough predictions made by the technique advanced in this research tends to underpredict oil production. But, importantly, the basic trend of the predicted oil rate decline were consistent with simulation results. It outperform the literature correlations employed in this research in this regard.
8. A possible reason for the high water prediction is the assumption that phase flow is solely dependent on the area open to the flow of a particular phase; the implicit assumption that the water advances without flowing through tortuous path or overcoming some resistance. Using other analytical line source solution would provide some avenue of analysing these explanations.

5.3 FURTHER STUDIES

The natural extention of this work will be to employ other line source analytical solution pairs other than what was used in this work. It will be interesting to note how the method perform with analytical solutions that could relax the assumption of unit mobility ratio and steady state flow. These two assumptions may not always be attained in practice and their inclusion reduces the accuracy of the procedure for common practical well conditions. Effort geared towards developing a correcting correlation for mobility ratio could extend the utility of the current research. More research on the incorporation of other factors affecting phase flow at the perforation could substantially improve on this work. The solution of the fundamental diffusivity equation for other reservoir conditions such as simultaneous active water aquifer and gas cap and pseudosteady state flow for weak pressure support will increase the practicality of the procedure.

The post-breakthrough predictions from this study could be used as backbone in generating consistent post-breakthrough correlations.

The results of this study will be very useful in furthering the work reported in Ref. 13. In that paper, a simple procedure for the calculation of maximum efficient rate for rim oil reservoir development and reserve potential evaluation were advanced for vertical well because its complete analytical description (critical rate, breakthrough time and post-breakthrough trends) were readily available in simplified expressions. With the development of simplified descriptions for horizontal and inclined wells, those wells may be employed in the sort of comparative analysis developed in the paper (Ref. 13) for rim oil appraisal and development.

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APPENDIX A

NOMENCLATURE

a,c	post-breakthrough fluid interface distance from well centre, ft
b	vertical well penetration ratio
B	formation volume factor, rb/stb
f	microscopic displacement efficiency, fraction
g	accelaration due to gravity
h	height, ft; horizontal component
I	modified bessel function of the first kind
k	permeability, md
K	modified bessel function of the second kind
L	horizontal well length
Lw	inclined well length
O	well centre
P	pressure, psi
q	fluid flow rate
r	radial coordinate; radius,ft
S	fluid saturation, fraction
t	time, days
v	velocity, ft/s; vertical component
z	vertical coordinate, ft
θ	inclination angle, degree
μ	viscosity, cp
Φ	velocity potential – oil, psi
φ	velocity potential – interface, d-1

ρ density,

$\emptyset$ porosity, fraction

SUBSCRIPTS

b breakthrough

D dimensionless

e external

o oil

or irreducible oil

w water, well

wc connate water

APPENDIX B

DATA SETS AND RELATIVE PERMEABILITY SPECIFICATIONS

Table B.1 Data Set 1 (Adapted from Ref. 3)

Initial oil zone thickness	42 ft
Water density	1.095 g/cm^3
Oil density	0.861 g/cm^3
Oil formation volume factor	1.102
Oil viscosity	1.44 g/cm^3
Horizontal permeabilty	37 md
Vertical permeability	3.7 md
Wellbore radius	0.29 ft
Drainage radius	1053 ft
Porosity	0.164
Residual oil saturation	0.337
Connate water saturation	0.288
Production rate	1000 stb/d
Vertical well perforated interval	24 ft
Horizontal well length	560 ft
Mobility ratio	3.27

Table B.2 Data Set 2 (Adapted from Ref. 5)

Initial oil zone thickness	84 ft
Water density	1.095 g/cm^3
Oil density	0.861 g/cm^3
Oil formation volume factor	1.102
Oil viscosity	1.44 g/cm^3
Horizontal permeabilty	35 md
Vertical permeability	3.5 md
Wellbore radius	0.29 ft
Drainage radius	1053 ft
Porosity	0.164
Residual oil saturation	0.337
Connate water saturation	0.288
Production rate	1000 stb/d
Vertical well perforated interval	24 ft
Horizontal well length	560 ft
Mobility ratio	3.27

Table B.3 Detailed Relative Permeability Specification

Water Relative Permeability

Water Saturation	Relative Permeabiliy
0.22	0
0.3	0.07
0.4	0.15
0.5	0.24
0.6	0.33
0.8	0.65
0.9	0.83
1	1

Oil Relative Permeability

Oil Saturation	Relative Permeabiliy
0	0
0.2	0
0.38	0.00432
0.4	0.0048
0.48	0.5288
0.5	0.0649
0.58	0.11298
0.6	0.125
0.68	0.345
0.7	0.4
0.74	0.7
0.78	1

Table B.4 Simplified Relative Permeability Specification

Water Relative Permeability

Water Saturation	Relative Permeabiliy
0.288	0
0.663	1

Oil Relative Permeabiltiy

Oil Saturation	Relative Permeabiliy
0.337	0
0.712	1

APPENDIX C

SCILAB CODES FOR THE CALCULATION OF CONING PARAMETERS

C.1 VERTICAL WELL

C.1.1 BREAKTHROUGH TIME CODES

```
function q2=DFBV(zD)
//Data input
ho = 42;
pw = 1.095;
po = 0.861;
Bo = 1.102;
uo = 1.44;
kr = 37;
kz = 3.7;
rw = 0.29;
re = 1053;
M = 3.27;
poro = 0.164;
sor = 0.337;
swc = 0.288;
q = 1000;
hw=10.84;
re = 1053;

b = hw/ho;
reD = (re/ho)*(kz/kr)^0.5;
qD = (325.7*q*uo*Bo)/(kr*ho*ho*(pw-po));
ans2 = 0; p = 1; term2 = 1;
while( ans2 + term2 ~= ans2 )
ep=(2*p-1)*%pi/2;
term2 = cos(ep*zD)*sin(ep*b)*besselk(1,ep*reD)/besseli(1,ep*reD);
ans2 = ans2 + term2;
p = p + 1;
end
A1 = (%pi/4)*(sec((%pi/4)*(1-zD+b)))^2;
B1 = (%pi/4)*(sec((%pi/4)*(1-zD-b)))^2;
a4 = tan((%pi/4)*(1-zD+b));
b4 = tan((%pi/4)*(1-zD-b));
GDV = (qD/b)*(-0.25*dlgamma((3-zD+b)/4)-0.25*dlgamma((1-zD-
b)/4)+0.25*dlgamma((3-zD-b)/4)+0.25*dlgamma((1-
zD+b)/4))+(qD/(2*b))*((A1/a4)-(B1/b4))+((2*qD)/b)*ans2-1;
q2 = 1/GDV
endfunction
```

tD = integrate(‘DFBV(zD)’,’zD’,0,1-b)

C.1.2 POST-BREAKTHROUGH TREND

```
function [time, oilRate]=BThv(n)
//Input data as above
time=ones(1:n-2);
oilRate=ones(1:n-2);
b=hw/ho;
m=b/n;
qD = (325.7*q*uo*Bo)/(kr*ho^2*(pw-po));
term=0;//k=1;
for k=1:n-2
b=b-m;
qDv=qD*(b/(hw/ho));
term=integrate('DFBVbt2(zD,b,qDv)','zD',0,1-b);
time(k)=term
oilRate(k)=qDv
end
endfunction

function q2=DFBVbt2(zD, b, qDv)
//b = hw/ho;
reD = (re/ho)*(kz/kr)^0.5;
//qD = (325.7*q*uo*Bo)/(kr*ho*ho*(pw-po));
ans2 = 0; p = 1; term2 = 1;
while( ans2 + term2 ~= ans2 )
ep=(2*p-1)*%pi/2;
term2 = cos(ep*zD)*sin(ep*b)*besselk(1,ep*reD)/besseli(1,ep*reD);
ans2 = ans2 + term2;
p = p + 1;
end
A1 = (%pi/4)*(sec((%pi/4)*(1-zD+b)))^2;
B1 = (%pi/4)*(sec((%pi/4)*(1-zD-b)))^2;
a4 = tan((%pi/4)*(1-zD+b));
b4 = tan((%pi/4)*(1-zD-b));
GDV = (qDv/b)*(-0.25*dlgamma((3-zD+b)/4)-0.25*dlgamma((1-zD-
b)/4)+0.25*dlgamma((3-zD-b)/4)+0.25*dlgamma((1-
zD+b)/4))+(qDv/(2*b))*((A1/a4)-(B1/b4))+((2*qDv)/b)*ans2-1;
q2 = 1/GDV
endfunction
```

C.2 HORIZONTAL WELL

C.2.1 BREAKTHROUGH TIME

```
function q=DFL(zD)

//Specify input parameters
LD = (lw/ho)*(kz/kr)^0.5;
rwD=0.5*rw*((kr/kz)^0.25+(kz/kr)^0.25)/ho;
zwD = 1-rwD;
reD = (re/ho)*(kz/kr)^0.5;

qD = (325.7*qo*uo*Bo)/(kr*ho*ho*(pw-po));
ans = 0; n = 1; term = 1;
while( ans + term ~= ans )
en=(2*n-1)*%pi/2;
```

```
term = cos(en*zD)*sin(en*zwD)*(%pi/2-integrate('besselk(0,u)','u',0,en*LD));
ans = ans + term;
n = n + 1;
end
ans1 = 0; m = 1; term1 = 1;
while( ans1 + term1 ~= ans1 )
em=(2*m-1)*%pi/2;
term1 =
cos(em*zD)*sin(em*zwD)*(besselk(1,em*reD)/besseli(1,em*reD))*(integrate('besseli(0,u)','u',0,em*LD));
ans1 = ans1 + term1;
m = m + 1;
end
a2=(%pi/4)*(zwD+zD);
b2=(%pi/4)*abs(zwD-zD);
A=(%pi/4)*sec(a2)*sec(a2);
B=(%pi/4)*sec(b2)*sec(b2);
a3=tan((%pi/4)*(zwD+zD))
b3=tan((%pi/4)*abs(zwD-zD))
q = 1/((qD/(2*LD))*((A/a3)+(B/b3))-((2*qD)/LD)*ans+((2*qD)/LD)*ans1-1);
endfunction
```

tD = integrate('DFL(zD)','zD',0,zwD-rwD)

C.2.2 POST BREAKTHROUGH TREND

```
function [time, oilRate]=bthwell(n)

// Input data
time=ones(1:n-2);
oilRate=ones(1:n-2);
rwD=0.5*rw*((kr/kz)^0.25+(kz/kr)^0.25)/ho;
//For wells completed on top of the oil zone
zwD=1-rwD;
//If completed at a positon order than top, the specification position
//must be specified
m=n/2;
p=rwD/m;
qDh = (325.7*q*uo*Bo)/(kr*ho^2*(pw-po));
term=0;
term2=0;
for k=1:m
zwD=zwD+0.5*p;
h=k*p;
a=rwD-h;
thet=acosd(a/rwD);
seg=((thet/180)*%pi*rwD^2)-(a*rwD*sind(thet));
oilArea=%pi*rwD^2-seg;
qD=(oilArea/(%pi*rwD^2))*qDh;
term=integrate('DFLbt(zD,zwD,qD)','zD',0,(zwD+h-rwD));
time(k)=term
oilRate(k)=qD
end
for l=m+1:n-2;
c=(l-m)*p;
zwD=zwD+0.5*p;
```

```
thet2=acosd(c/rwD);
seg2=((thet2/180)*%pi*rwD^2)-(c^2*tand(thet2));
//oilArea2=(0.5*%pi*rwD^2)+seg2;
qD=(seg2/(%pi*rwD^2))*qDh;
term2=integrate('DFLbt(zD,zwD,qD)','zD',0,zwD+c);
time(l)=term2
oilRate(l)=qD;
end
endfunction

function q=DFLbt(zD, zwD, qD)
//zwD = 0.8;
reD = (re/ho)*(kz/kr)^0.5;
//qD = (325.7*qo*uo*Bo)/(kr*ho*ho*(pw-po));
ans = 0; n = 1; term = 1;
while( ans + term ~= ans )
en=(2*n-1)*%pi/2;
term = cos(en*zD)*sin(en*zwD)*(%pi/2-integrate('besselk(0,u)','u',0,en*LD));
ans = ans + term;
n = n + 1;
end
ans1 = 0; m = 1; term1 = 1;
while( ans1 + term1 ~= ans1 )
em=(2*m-1)*%pi/2;
term1 =
cos(em*zD)*sin(em*zwD)*(besselk(1,em*reD)/besseli(1,em*reD))*(integrate('bes
seli(0,u)','u',0,em*LD));
ans1 = ans1 + term1;
m = m + 1;
end
a2=(%pi/4)*(zwD+zD);
b2=(%pi/4)*abs(zwD-zD);
A=(%pi/4)*sec(a2)*sec(a2);
B=(%pi/4)*sec(b2)*sec(b2);
a3=tan((%pi/4)*(zwD+zD))
b3=tan((%pi/4)*abs(zwD-zD))
q = 1/((qD/(2*LD))*((A/a3)+(B/b3))-((2*qD)/LD)*ans+((2*qD)/LD)*ans1-1);
endfunction
```

C.3 INCLINED WELL

C.3.1 BREAKTHROUGTH TIME

```
function q3=Incline(zD)
//Input data
reD = (re/ho)*(kz/kr)^0.5;
qD = (325.7*qo*uo*Bo)/(kr*ho*ho*(pw-po));
Lw = 29.095;
theta=(21.87/180)*%pi;
thetaD=atan(((kz/kr)^0.5)*tan(theta));
LwD=(Lw/ho)*((kz/kr)*(sin(theta))^2+(cos(theta))^2)^0.5;
zwD = 1-(0.5*LwD*cos(thetaD));
qDv = qD*cos(thetaD);
b = LwD*cos(thetaD);
LD = LwD*sin(thetaD);
qDh = qD*sin(thetaD);
```

```
ans2 = 0; p = 1; term2 = 1;
while( ans2 + term2 ~= ans2 )
ep=(2*p-1)*%pi/2;
term2 = cos(ep*zD)*sin(ep*b)*besselk(1,ep*reD)/besseli(1,ep*reD);
ans2 = ans2 + term2;
p = p + 1;
end
A1 = (%pi/4)*(sec((%pi/4)*(1-zD+b)))^2;
B1 = (%pi/4)*(sec((%pi/4)*(1-zD-b)))^2;
a4 = tan((%pi/4)*(1-zD+b));
b4 = tan((%pi/4)*(1-zD-b));
GDV = (qDv/b)*(-0.25*dlgamma((3-zD+b)/4)-0.25*dlgamma((1-zD-
b)/4)+0.25*dlgamma((3-zD-b)/4)+0.25*dlgamma((1-
zD+b)/4))+(qDv/(2*b))*((A1/a4)-(B1/b4))+((2*qDv)/b)*ans2-1;
ans = 0; n = 1; term = 1;
while( ans + term ~= ans )
en=(2*n-1)*%pi/2;
term = cos(en*zD)*sin(en*zwD)*(%pi/2-integrate('besselk(0,u)','u',0,en*LD));
ans = ans + term;
n = n + 1;
end
ans1 = 0; m = 1; term1 = 1;
while( ans1 + term1 ~= ans1 )
em=(2*m-1)*%pi/2;
term1 =
cos(em*zD)*sin(em*zwD)*(besselk(1,em*reD)/besseli(1,em*reD))*(integrate('bes
seli(0,u)','u',0,em*LD));
ans1 = ans1 + term1;
m = m + 1;
end
a2=(%pi/4)*(zwD+zD);
b2=(%pi/4)*abs(zwD-zD);
A=(%pi/4)*sec(a2)*sec(a2);
B=(%pi/4)*sec(b2)*sec(b2);
a3=tan((%pi/4)*(zwD+zD))
b3=tan((%pi/4)*abs(zwD-zD))
DFL = (qDh/(2*LD))*((A/a3)+(B/b3))-((2*qDh)/LD)*ans+((2*qDh)/LD)*ans1-1;
q3 = 1/(GDV+DFL)
endfunction
```

tD = integrate(‘Incline(zD)’, ‘zD’,0, 1-LwD*cos(thetaD))

B.3.2 POST BREAKTHROUGH TREND

```
function [time, oilRate]=Inclinewell(n)
//Input data
rwD=0.5*rw*((kr/kz)^0.25+(kz/kr)^0.25)/ho;
time=ones(1:3*n-2);
oilRate=ones(1:3*n-2);
//First stage
Lw = 29.095;
theta=(21.87/180)*%pi;
thetaD=atan(((kz/kr)^0.5)*tan(theta));
LwD=(Lw/ho)*((kz/kr)*(sin(theta))^2+(cos(theta))^2)^0.5;
zwD = 1-(0.5*LwD*cos(thetaD));
b = LwD*cos(thetaD);
```

```
m=(0.5*LwD*cos(thetaD)-rwD)/n;
qD = (325.7*q*uo*Bo)/(kr*ho^2*(pw-po));
qDvi = qD*cos(thetaD);
qDhi = qD*sin(thetaD);
term=0;
for k=1:n
b=b-m;
qDv=qDvi*(b/(LwD*cos(thetaD)));
term=integrate('Inclinebt(zD,b,qDv)','zD',0,1-b);
time(k)=term
oilRate(k)=(qDv^2+qDhi^2)^0.5
end
//Second stage
r=n/2;
p=rwD/r;
term=0;
term2=0;
for i=1:r
h=i*p;
zwD=zwD+0.5*p;
a=rwD-h;
thet=acosd(a/rwD);
seg=((thet/180)*%pi*rwD^2)-(a*rwD*sind(thet));
oilArea=%pi*rwD^2-seg;
qDh=(oilArea/(%pi*rwD^2))*qDhi;
b=0.5*LwD*cos(thetaD)+rwD-h;
qDv=qDvi*(b/(LwD*cos(thetaD)));
term=integrate('Inclinebt2(zD,b,qDv,qDh,zwD)','zD',0,1-b);
time(i+n)=term
oilRate(i+n)=(qDh^2+qDv^2)^0.5
end
for l=r+1:n-2
c=(l-r)*p;
zwD=zwD+0.5*p;
thet2=acosd(c/rwD);
seg2=((thet2/180)*%pi*rwD^2)-(c^2*tand(thet2));
qDh=(seg2/(%pi*rwD^2))*qDhi;
b=0.5*LwD*cos(thetaD)-c;
qDv=qDvi*(b/(LwD*cos(thetaD)));
term2=integrate('Inclinebt2(zD,b,qDv,qDh,zwD)','zD',0,1-b);
time(l+n)=term2
oilRate(l+n)=(qDh^2+qDv^2)^0.5
end
//Third stage
b = 0.5*LwD*cos(thetaD)-rwD;
m=b/n;
term=0;
for k=1:n-1
b=b-m;
qDv=qDvi*(b/(LwD*cos(thetaD)));
term=integrate('DFBVbt2(zD,b,qDv)','zD',0,1-b);
time(k+2*n-2)=term
oilRate(k+2*n-2)=qDv
end
endfunction

function q3=Inclinebt(zD, b, qDv)
```

```
re = 1053;
reD = (re/ho)*(kz/kr)^0.5;
qD = (325.7*qo*uo*Bo)/(kr*ho*ho*(pw-po));
Lw = 29.095;
theta=(21.87/180)*%pi;
thetaD=atan(((kz/kr)^0.5)*tan(theta));
LwD=(Lw/ho)*((kz/kr)*(sin(theta))^2+(cos(theta))^2)^0.5;
//qDv = qD*cos(thetaD);
zwD = 1-(0.5*LwD*cos(thetaD));
LD = LwD*sin(thetaD);
qDh = qD*sin(thetaD);
ans2 = 0; p = 1; term2 = 1;
while( ans2 + term2 ~= ans2 )
ep=(2*p-1)*%pi/2;
term2 = cos(ep*zD)*sin(ep*b)*besselk(1,ep*reD)/besseli(1,ep*reD);
ans2 = ans2 + term2;
p = p + 1;
end
A1 = (%pi/4)*(sec((%pi/4)*(1-zD+b)))^2;
B1 = (%pi/4)*(sec((%pi/4)*(1-zD-b)))^2;
a4 = tan((%pi/4)*(1-zD+b));
b4 = tan((%pi/4)*(1-zD-b));
GDV = (qDv/b)*(-0.25*dlgamma((3-zD+b)/4)-0.25*dlgamma((1-zD-
b)/4)+0.25*dlgamma((3-zD-b)/4)+0.25*dlgamma((1-
zD+b)/4))+(qDv/(2*b))*((A1/a4)-(B1/b4))+((2*qDv)/b)*ans2-1;
ans = 0; n = 1; term = 1;
while( ans + term ~= ans )
en=(2*n-1)*%pi/2;
term = cos(en*zD)*sin(en*zwD)*(%pi/2-integrate('besselk(0,u)','u',0,en*LD));
ans = ans + term;
n = n + 1;
end
ans1 = 0; m = 1; term1 = 1;
while( ans1 + term1 ~= ans1 )
em=(2*m-1)*%pi/2;
term1 =
cos(em*zD)*sin(em*zwD)*(besselk(1,em*reD)/besseli(1,em*reD))*(integrate('bes
seli(0,u)','u',0,em*LD));
ans1 = ans1 + term1;
m = m + 1;
end
a2=(%pi/4)*(zwD+zD);
b2=(%pi/4)*abs(zwD-zD);
A=(%pi/4)*sec(a2)*sec(a2);
B=(%pi/4)*sec(b2)*sec(b2);
a3=tan((%pi/4)*(zwD+zD))
b3=tan((%pi/4)*abs(zwD-zD))
DFL = (qDh/(2*LD))*((A/a3)+(B/b3))-((2*qDh)/LD)*ans+((2*qDh)/LD)*ans1-1;
q3 = 1/(GDV+DFL)
endfunction

function q3=Inclinebt2(zD, b, qDv, qDh, zwD)
//Input data
//zwD = 1-(0.5*LwD*cos(thetaD));
reD = (re/ho)*(kz/kr)^0.5;
qD = (325.7*qo*uo*Bo)/(kr*ho*ho*(pw-po));
Lw = 29.095;
theta=(21.87/180)*%pi;
```

```
thetaD=atan(((kz/kr)^0.5)*tan(theta));
LwD=(Lw/ho)*((kz/kr)*(sin(theta))^2+(cos(theta))^2)^0.5;
//qDv = qD*cos(thetaD);
//b = LwD*cos(thetaD);
LD = LwD*sin(thetaD);
//qDh = qD*sin(thetaD);
ans2 = 0; p = 1; term2 = 1;
while( ans2 + term2 ~= ans2 )
ep=(2*p-1)*%pi/2;
term2 = cos(ep*zD)*sin(ep*b)*besselk(1,ep*reD)/besseli(1,ep*reD);
ans2 = ans2 + term2;
p = p + 1;
end
A1 = (%pi/4)*(sec((%pi/4)*(1-zD+b)))^2;
B1 = (%pi/4)*(sec((%pi/4)*(1-zD-b)))^2;
a4 = tan((%pi/4)*(1-zD+b));
b4 = tan((%pi/4)*(1-zD-b));
GDV = (qDv/b)*(-0.25*dlgamma((3-zD+b)/4)-0.25*dlgamma((1-zD-
b)/4)+0.25*dlgamma((3-zD-b)/4)+0.25*dlgamma((1-
zD+b)/4))+(qDv/(2*b))*((A1/a4)-(B1/b4))+((2*qDv)/b)*ans2-1;
ans = 0; n = 1; term = 1;
while( ans + term ~= ans )
en=(2*n-1)*%pi/2;
term = cos(en*zD)*sin(en*zwD)*(%pi/2-integrate('besselk(0,u)','u',0,en*LD));
ans = ans + term;
n = n + 1;
end
ans1 = 0; m = 1; term1 = 1;
while( ans1 + term1 ~= ans1 )
em=(2*m-1)*%pi/2;
term1 =
cos(em*zD)*sin(em*zwD)*(besselk(1,em*reD)/besseli(1,em*reD))*(integrate('bes
seli(0,u)','u',0,em*LD));
ans1 = ans1 + term1;
m = m + 1;
end
a2=(%pi/4)*(zwD+zD);
b2=(%pi/4)*abs(zwD-zD);
A=(%pi/4)*sec(a2)*sec(a2);
B=(%pi/4)*sec(b2)*sec(b2);
a3=tan((%pi/4)*(zwD+zD))
b3=tan((%pi/4)*abs(zwD-zD))
DFL = (qDh/(2*LD))*((A/a3)+(B/b3))-((2*qDh)/LD)*ans+((2*qDh)/LD)*ans1-1;
q3 = 1/(GDV+DFL)
endfunction
```