INVESTIGATING THE IMPACT OF PRESSURE DEPLETION ON THE ESTIMATION OF RESERVES AND RECOVERY FACTORS IN NATURALLY FRACTURED RESERVOIRS

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By
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INVESTIGATING THE IMPACT OF PRESSURE DEPLETION ON THE ESTIMATION OF RESERVEs AND RECOVERY FACTORS IN NATURALLY FRACTURED RESERVOIRS

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DEDICATION

This research is dedicated to my parents, Mr. and Mrs. Alphonso Q. Kotee, siblings, my late Uncle Austin P. Boulah, the Inter-ministerial Scholarship Committee of Liberia, the Directorate of Technical Cooperation of Africa and to my wife to be Miatta, who contributed to this project in ways she will never know. Last but not the least, to my ex-boss: T. Catfish Brownell, for his encouragement and inspiration in my professional life.

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ABSTRACT

The complexity of naturally fractured reservoirs poses constrains on reservoir engineers in estimating reserves and recovery factor in naturally fractured reservoirs. Naturally fractured reservoirs like all other hydrocarbon reservoirs are subject to uncertainties. This paper analyzes the impact of pressure depletion on the estimation of reserves and recovery factors in naturally fractured reservoirs (NFRs).

Fluids in naturally fractured reservoir are stored both in matrix and fracture pore volumes where the volumetric fraction of the reservoir fluids stored in fracture is indicated by the storage capacity ratio, Chacon and Tiab. Changes in reservoir pore pressure due to production or injection of fluids affect the fracture and matrix pores compressibilities; and storage capacity ratio. The mechanical behavior of matrix and fracture rocks properties due to change in pore pressure has significant impact on reserves estimate and recovery factor. But current analyses for quantifying reserves in naturally fractured reservoirs based on the general material balance equation (GMBE), consider the behavior of naturally fractured reservoirs to be similar to homogeneous reservoirs; i.e., fracture and matrix pore volume compressibilities are assumed to be equal. This wrongful assumption has led to significant errors in estimating reserves and recovery factor, risks optimum reservoir management and the prediction of reservoir future performance. However, in an effort to reduce this significant error, a complete treatment of the material balance model for naturally fractured reservoirs has been developed by Chacon and Tiab [1]. Chacon and Tiab's proposed material balance equation to compute hydrocarbons in place and fractional recovery were developed for both undersaturated and saturated naturally fractured reservoirs (NFRs). The equation was treated in such that fracture and matrix storage capacity ratio was integrated into the equation with a new plotting scheme for better estimation of oil in place.

Chacon and Tiab's proposed material balance equation for naturally fractured reservoirs was adapted in this research. Well test data of volumetric undersaturated and saturated naturally fractured reservoirs were analyzed based on 4 cases of storage capacity ratio. Fracture and matrix pore volume compressibility ratio was neglected for each case and later considered for each case as well. The results were compared and analyzed. Computations shows that in volumetric undersaturated naturally fractured reservoirs, error differences greater than 10% are expected when estimating recovery factors if differences between the fracture and matrix pore compressibility are not taken into account. The percent differences in estimating recovery factors is directly proportional to storage capacity ratio.

In volumetric saturated NFRs, a reservoir with high storage capacity ratio producing at constant rate in line with a reservoir of low storage capacity ratio is expected to deplete it reserve sooner as compare to a reservoir with low storage capacity ratio due to high recovery factor. `

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Above all, I acknowledged the tireless efforts of the Almighty God, the creator of the Universe whom with all my iniquities continues to guide me in my academic endeavors.

ABOUT THE AUTHOR

Aloysius K. Kotee is a Liberian and a graduating candidate in petroleum engineering at the African University of Science and Technology (AUST) Abuja, Nigeria. He obtained his B.Sc. (Dec 2012) degree in Geology from the University of Liberia (UL). Due to his astute academic performance over the years, he benefited several scholarships including the Michel V. Suah scholarship, The Ministry of Lands Mines and Energy of Liberia; and The National Oil Company of Liberia Local scholarship. He is currently a beneficiary of the Directorate of Technical Cooperation in Africa (DTCA) Scholarship at the African University of Science and Technology.

Prior to his enrollment at AUST, he worked with several industrial entities including Trans Africa Trading, Simba Energy Liberia, and BHP Billition Liberia Iron Ore.

He is results-driven, self-reliant - sets aims and targets and leads by example. Based on these attributes, he was charged with the responsibility to supervised assets teams during his work experiences.

At BHP Billiton, where he had excellent experience in Environmental Geology for three years, he served as Environmental Supervisor and was responsible for Land and Biodiversity Management; Water Management, Waste Management and Hydrocarbon Management. Having served in the Metal and Mining sector as Environmental Geologist, he decided to embark on contributing to the emerging oil & gas industry in Liberia with a vision of applying his environmental acknowledge to developing a cleaner, safer, more efficient and cost-effective ways of exploring and producing oil and gas. He is confident that he has the preparations and potentials to prove an asset to Liberia and West Africa at large.

CHAPTER 1-INTRODUCTION

The assumption that fractured reservoirs behave similar to homogeneous reservoirs is not always valid. They added that the assumption leads to wrong estimation of reserves, Chacon, Tiab et al [1].

Naturally fractured reservoirs storage capacity ratio along with fractures and matrix pore volumes compressibilities are key parameters that influence the performance of naturally fractured reservoirs. Ignoring these key parameters at the early development stage, is not optimal reservoir management. Eventually, compressibility ratio and storativity cannot be ignored because the technical and economic performance of the reservoir degrades. The biggest risk is not estimating the storativity early is that such an oversight can severely limit future field-development options.

A material balance equation was proposed for naturally fractured reservoirs in order to quantify hydrocarbons in place and recovery factor taking into account the differences between fracture and matrix pore volume compressibilities and their changes caused by pressure depletion for undersaturated and saturated naturally fractured reservoirs, (Chacon, Tiab et al 2007).

This research shows that ignoring compressibility ratio in naturally fractured reservoirs can lead to increasing errors in reserves estimates and fractional recovery. The errors increases with increasing storativity.

1.1 BACKgROUND

In the last decades, many engineers recognized the need for a comprehensive overview of the subject of NFRs. The complexity associated with naturally fractured formations constrains reservoir engineers to use simplified versions of the Material Balance Equation for determining the initial hydrocarbon in place and predicting reservoir performance.

Nearly all hydrocarbon reservoirs are affected in some way by natural fractures, yet the effect of fractures are often poorly understood and largely underestimated. All naturally fractured reservoirs are not created equally. Some fractures may appear as micro-fissures with an extension of only several micrometers, or as continental fractures with an extension of several thousand kilometers. They may be limited to a single rock formation or layer, or spread through many rock formations or layers.

In Carbonate reservoirs, natural fractures help create secondary porosity and promote communication between reservoir compartments. These formations are believed to have contained significant amounts of the world oil reserves. But large amount of the oil volumes have being left behind because of poor knowledge and/or description of those reservoirs. Natural fractures also occur in siliclastic reservoirs of all types, complicating seemingly straightforward matrix-dominant production behavior. Optimal reservoir management of these reservoirs remains a great challenge for reservoir engineers

This paper seeks to investigate the impact of pressure depilation on reserves estimates and recovery factors in naturally fractured reservoirs using Chacon and Tiab proposed method.

1.3 STATEMENT OF THE PROBLEM

As stated above, significant amount of the World's oil reserves are host to naturally fractured reservoirs. Accurately quantifying these reserves remains a great challenge to petroleum engineers and geoscientists. Hydrocarbons originally in place in naturally fractured reservoirs, are fragmented between fractures and matrix depending on their pore volume compressibilities, storage capacities, and interporosity. But current analyses estimating reserves based on traditional material balance equation (MBE) computations assume that fractured reservoirs behave similar to homogeneous reservoirs, that is, the fracture and matrix pore volume compressibility are equal. This assumption has led to increasing errors in estimating reserves and fractional recovery.

1.4 OBJECTIVES

The research was taken to investigate the following:

- The impact of pressure depletion on estimating of reserves and recovery factors in naturally fractured reservoirs
- Analyzing the effect of storativity in NFRs performance.
- Influence of fractured and matrix pore volumes compressibilities on naturally fractured reservoirs during production and injection.

1.5 MERITS OF THE RESEARCH

There have being lots of work done in characterizing naturally fractured reservoirs. NFRs have being characterized in terms of the nature of fractures and faults as primary pathways for hydrocarbon migration and production in many reservoirs; their nature of acting as channels for water breakthrough and gas coning; and their conductivities in relation to rock stresses. Contrary to one of the key features of naturally fractured reservoirs is the assumption and treatment of naturally fractured reservoirs as homogeneous formations in terms of reserves estimates and recovery factors which of course has led to the underestimation and abandonment of naturally fractured reservoirs.

The advantage of this work is that it tends to better estimate the initial hydrocarbon in place of both saturated and undersaturated naturally fractured reservoirs; considering the storage capacities ratio and interporosities of fractures and matrix by investigating the impact of pressure depletion on naturally fractured reservoirs using the proposed material balanced formulated by Chacon, Tiab et al (SPE 108107).

1.6 RESEARCH TIME TABLE

ACTIVITIES	DATE									
	Mar-14	Apr-14	May-14	Jun-14	Jul-14	Aug-14	Sep-14	Oct-14	Nov-14	Dec-14
Request for a supervisor										
Supervisor Perusal										
Literature review										
Gathering of relevant data										
Methodology										
Results and Analysis										
Conclusion and recommendations										
Review by supervisor and committee members										
Thesis defense										
Final review										
Approval by supervisor and committee members										
Submission of Thesis										

Color key	
	Completed
	Ongoing
	Expected

CHAPTER 2 - OVERVIEW OF NATURALLY FRCATURED RESERVOIRS

2.1 GEOLOGY CLASSIFICATION

From a geologic point of view, fractures are displacement discontinuities mostly found in rocks of upper Earth's crust, which appear as local breaks in the natural sequence of the rock's properties. They are created as a result of mechanical failures of the rocks strength to natural geological stresses, high fluid pressure, drilling activities and even fluid withdrawal, since fluid also partially supports the weight of the overburden rocks.

Naturally fractured rocks can be geologically categorized into three main types based on their porosities frames, Tiab and Donaldson [3].

- Intercrystalline-intergranular, such as the field in Texas, the Elk Basin in Wyoming, and the umm fraud field in Libya;
- Fracture matrix, such as the Spraberry field in Texas, the Kirkuk field in Iraq, the Dukhan field in Qatar, and Masjidi-sulaiman and Haft-Gel fields in Iran; and
- Vugular-solution, such as the Pegasus Ellenburger field and the Canyon Reef field in Texas

The patterns of natural fracture are frequently interpreted on the basis of laboratory-derived fracture patterns corresponding to models of paleostress fields and strain distribution in reservoir at time of fracture.

Stearns and Friedman proposed classification based on stress/strain conditions in laboratory samples and fractures observed in the outcrops and subsurface settings. They generally classified fractures as follows:

- Share fractures exhibit a sense of displacement parallel to the fracture plane. Share fractures are formed when the stresses in the three principal directions are all compressive. They form at an acute angle to maximum principal stress and at obtuse angle to the direction of minimum compressive stress.

- Extension fractures exhibit a sense of displacement perpendicular to and away from the fracture plane. They are formed perpendicular to the minimum stress direction. They are also created as result of stresses in the principal three directions is compressive and occurring in conjunction with share fracture.

- Tension fractures also exhibit a sense of displacement perpendicular to and away from the fracture plane. At least one of the principal stresses has to be tensile in order to form the fracture plane.

In addition to Stearns and Friedman classification of natural fractures, natural fractures have also being classified geologically based on paleostress conditions as follow:

- Tectonic fractures: The orientation, distribution and morphology of these fractures are associated with local tectonic events as in fault and folding. The intensities of fractures associated with faulting is a function of lithology, distance from the fault plane, magnitude of the fault displacement, total strain in the rock mass, and depth of burial.

Fold related fracture systems exhibit complex patterns consistent with the complex stress and strain history associated with the initiation and growth of a fold. They defined in terms of dip and strike of the beds.

- Regional fractures: These fractures systems are characterized by long fractures exhibiting little change in orientation over their length. They show no evidence of offset across the fracture plane and are always perpendicular to the bedding surfaces. A more consistent geometry and relatively larger spacing of regional fractures distinguished it from tectonic fractures.

Many theories have been proposed for the origin of the regional fractures, ranging platen tectonics to cyclic loading/unloading of rocks associated with earth tides.

- Contractional fractures: These types of fractures result from bulk volume reduction of rock which of course leads to desiccation fractures and syneresis fractures, shrinkage due to loss of fluid in subaerial drying and reduction by sub-aqueous or surface dewatering.

Desiccation and syneresis fractures can be either tensile or extension fractures and are initiated by internal body forceses. The fractures tend to be closely spaced and regular and isotropically distributed in three dimensions. Syneresis fractured has been observed in limestone, dolomites, shales, and sandstones. In addition to desiccation and syneresis, thermal contractions and mineral changes in the rock also create fractures in formations, especially in carbonates and clay constituents in sedimentary rocks. In view of the above, the complex stress/strain distribution in reservoir rocks results in the complex fracture pattern. And these patterns have key characteristic that can be used to classify and index natural fracture networks observed in outcrops and subsurface samples.

2.2 Engineering Classification

In addition to geologic classification of naturally fractured reservoir, the engineering aspects deal primarily with quantitative evaluation of naturally fractured reservoirs. This quantification links the geophysical, geologic and engineering disciplines. Some key goals are to estimate hydrocarbons-in-place, predict production rates, and improve ultimate economic recoveries.

Engineering evaluation of naturally fractured reservoirs rely on direct sources of information. The determination of flow units is an important part of the evaluation. Flow unit is defined as a continuous body over a specific reservoir volume that practically possesses consistent petrophysical and fluid properties, which uniquely characterize it static and dynamic communication with the wellbore. Outcrops, drilling history, mud log, conventional and specialized well Logs, seismic information, inflatable packers, and production are all direct information used engineering evaluation of naturally fractured reservoir.

Nelson identified four types of naturally fractured reservoirs, based on the extent to which fractured has altered the porosity and permeability of the reservoir matrix.

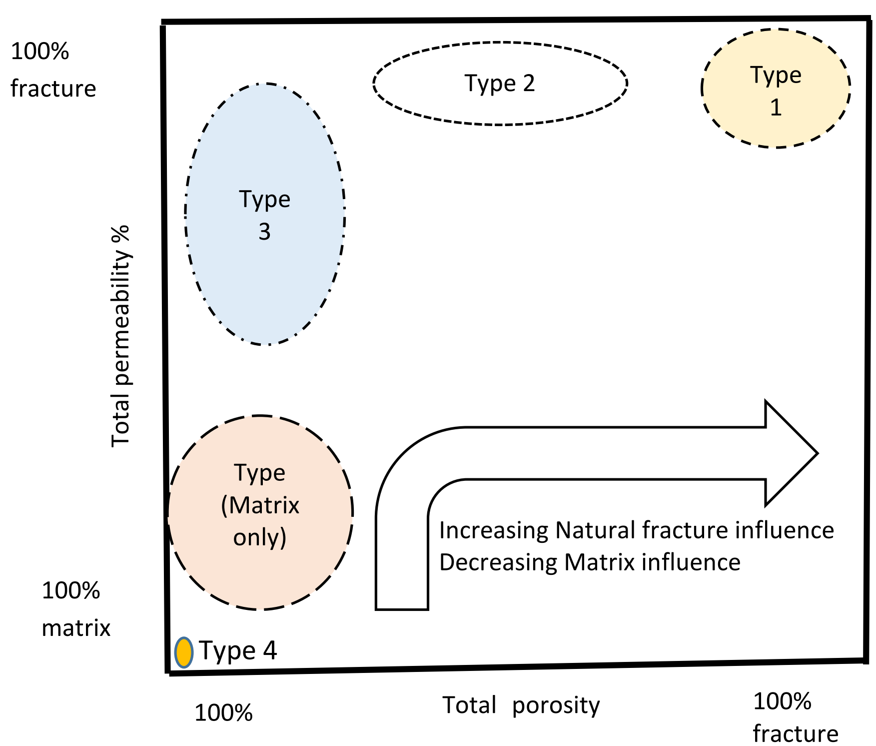
- Type 1: Fractures of type 1 reservoir provide all the reservoir storage capacity and permeability. Examples: Amal field in Libya, the LaPaz and Mara fields in Venezulela, and Pre-Cambrain basement reservoirs in eastern China.

- Type 2: The matrix of type 2 reservoirs is dominantly permeable and exhibit high flow when fractures are added to its previous excellent permeability. Example: Kirkuk field of Iraq.

- Type 3: In type 3 naturally fractured reservoirs, the matrix has negligible permeability but contains most if not all the hydrocarbons. The fractures provide the essential reservoirs permeability, such as in the Monterey fields of California and the Spraberry reservoir of West Texas.

- Type 4: Type 4 reservoirs are often uneconomic to develop and produce. The fractures tend to form barriers to fluid migration and partition formations into relatively small blocks due to mineral filling the fractures. Figure 1 shows a schematic diagram of NFRs types.

Figure 1: A schematic diagram of the four type of NFRs https://www.slb.com/~/media/Files/resources/oilfield_review/ors06/sum06/naturally_fract_reservoirs.pdf



2.3 INDICATORS OF NATURAL FRACTURES

There have being lot of attention drawn to fractures because of their multi roles played in exploration and exploitation of naturally fractured reservoirs. Stearns and Friedman after successfully reviewing the multiple roles played by fractures, they concluded that, fractures could alter the matrix porosity or permeability. If the fractures or connected vugs are filled with secondary minerals they may restrict flow of fluid as in the case of type 4 reservoir. On the other hand, rocks with low matrix porosity, fractures and solution channels increase the pore volume by both increasing porosity and connecting isolated matrix porosity and therefore augment recovery efficiency.

Recognizing fractures in formation at an early stage and estimating its rocks' properties, such as Porosity and permeability, greatly influence the location and number of development wells and, and therefore is of major economic significance. Below are summary of methods proposed by (Aguilera, Saidi, Stearns and Friedman; and Nelson) for detecting fractures.

- Increase penetration rate and Loss of circulating fluids during drilling give clued to the presence of fracture in formations. They must carefully be monitored during drilling.

- Fractures and solution channels in cores provide direct information on the nature of fracture. A special core analysis is recommended for few wells within reservoir parameter to enhance reservoir evaluation.

- In addition to core analysis, well logging measurements based on sonic wave propagation, which are negligibly affected by the borehole conditions are used as fractures indicator. Though, logging tools have being designed to responds differently to various wellbore characteristics, such as lithology, porosity, and fluid saturations, but not to natural fractures. Nonetheless, measurements by the caliper log, density log, or resistivity log, under proper conditions, can be very effective in locating fractured zones. Furthermore, dipmeter data on the FIL (fracture identification log) provide effective methods for fracture detection.

- A graphical approach is also used to evaluate naturally fractured reservoirs. Warren and Root assumed that the formation fluid flows from the matrix to fractures under pseudosteady state and showed a semilog pressure buildup curve indicating a fracture. If the existing fractures dominantly trend in a single direction, the reservoir may appear to have anisotropic permeability. A best result is achieved if pressure interference and pulse tests from enough observation wells are used.

- Natural fractures in a non-deviated borehole can be identified as a high amplitude feature which crosses other bedding planes.

- Down hole televiewers are also used to identify vertical and incline fractures in naturally fractured reservoir.

- A very high production index (500 STB/D/psi)

- A considerable increase in productivity of the well flowing after workover such as artificial stimulation by acidizing is a strong indication of a naturally fractured reservoir.

- Pressure test results that are incompatible with porosity and permeability values obtained from both core analysis and well logging are indicators.
- Lack of precision in seismic recordings and extrapolation from observations outcrops.

CHAPTER 3 –RESEARCH METHODOLOGY

3.1 THE MATERIAL BALANCE EQUATION

Material balance calculations are very well established techniques that apply the law of conservation of matter to petroleum engineering. Since Schilthuis[3] first presented the derivation of the volumetric material balance equation (MBE), several MBE have been presented for single-porosity reservoirs. One of the basic assumptions of conventional MBE is that rock properties, such as porosity and compressibility, are uniform throughout the reservoir. For dual-porosity media, as encountered in naturally fractured reservoirs (NFR), this assumption is no longer valid. Fracture and matrix porosity values change differently with pressure changes since fractures are highly compressible compared to the matrix.

Walsh (1994), developed a generalized material balance equation (GMBE) which is applicable to the full range of conventional reservoir fluids, including volatile-oil and gas-condensates. The general material balance was expressed (reservoir condition) as:

$$N_p B_o + G_p B_g + W_p B_w = \Delta V_{oil} + \Delta V_{gascap} - \Delta V_{water} - \Delta V_{porespace} \qquad (1)$$

The implication of this Tank model is that, since reservoir volume is constant, the net voidage caused by fluid production from inside the reservoir, minus fluid injections and fluid influx, must be made up by expansion of the remaining in-place materials.

This broad definition of the material balance equation can be applied to a variety of reservoir types, be it oil or gas.

The gas produced 'G_p' can be broken into free gas from gas cap and solution gas evolved from oil so that

$$N_pB_o + G_pB_g + W_pB_w = N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w \qquad (2)$$

Therefore,

$$N_p\left[B_o + \left(R_p - R_s\right)B_g\right] + W_pB_w = \Delta V_{oil} + \Delta V_{gascap} - \Delta V_{porespace} \qquad (3)$$

According to Chacon and Tiab, the link between the elastic behavior of the rock and the recovery predictions in the material balance modeling resides in the effective compressibility term and the storage capacity ratio. In view of the above, Chacon and Tiab modified the general volumetric material balance equation using the correct effective compressibilities of the fractured rock.

The general form of the MBE for NFR is given as:

$$\begin{aligned}
&\left[N_f\left(B_t - B_{ti}\right) + \frac{N_f mB_{ti}}{B_{gi}}\left(B_g - B_{gi}\right) + \left(1+m\right)N_fB_{ti}C_f\Delta\overline{P}\right] + \\
&\left[N_m\left(B_t - B_{ti}\right) + \frac{N_m mB_{ti}}{B_{gi}}\left(B_g - B_{gi}\right) + \left(1+m\right)N_mB_{ti}C_{em}\Delta\overline{P}\right] \\
&= N_p\left[B_t + \left(R_p - R_{soi}\right)B_g\right]
\end{aligned} \qquad (4)$$

Where N_f is the original oil in-place in the rock fractures and N_m is the original oil in-place in the matrix, Np is the cumulative produced oil and R_p is the cumulative produced gas-oil ratio, C_{em} is the effective matrix compressibility and C_{ef} is the effective fracture compressibility. Definitions of the remaining variables are given in the nomenclature.

C_{ef} and C_{em} are defined as:

$$C_{e,f} = \frac{C_wS_{wi} + C_{pp.f}}{\left(1 - S_{wi}\right)} \qquad (5)$$

$$C_{e,m} = \frac{C_w S_{wi} + C_{pp.m}}{(1 - S_{wi})} \tag{6}$$

Where,

$$C_{pp(f+m)} = C_{pp,m} + C_{pp.f} \tag{7}$$

$$N = N_f + N_m \tag{8}$$

Equation 4 can be applied to any type of naturally fractured hydrocarbon reservoirs. Volumetric undersaturated and Saturated NFRs were considered in this paper.

3.2 Material Balance equation for Undersaturated Naturally Fractured Reservoirs

Assumptions

- ◦ P > Pb
- ◦ No original or final gas cap, M=0
- ◦ No water influx or production

Equation 4 was reduced to,

$$N_f\left[(B_t - B_{ti}) + B_{ti} c_{e,f} \Delta\overline{P}\right] + N_m\left[(B_t - B_{ti}) + B_{ti} C_{e,m} \Delta\overline{P}\right] = N_p\left[B_t + (R_p - R_{soi}) B_g\right] \tag{9}$$

Where

$$B_t = B_o + B_g\left(R_{soi} - R_{so}\right) \tag{10}$$

For pressure above bubble point pressure, the solution gas-oil ratio R_s remains. Where, B_t=B_o

Equation 6 again reduced to oil reservoir below the bubble when they have not reach critical gas saturation, and no free gas is being produced.

$$N_p B_o = N_f\left[\left(B_o - B_{oi}\right) + B_{oi}C_{ef}\Delta\overline{P}\right] + N_m\left[\left(B_o - B_{oi}\right) + B_{oi}C_{em}\Delta\overline{P}\right] \tag{11}$$

3.3 GRAPHICAL SOLUTION OF THE MATERIAL BALANCE EQUATION

The MBE was rearranged by Penuela and others to obtain variable groups that are plotted to result in a straight line They presented the material balance equation in a compacted form Chacon and Tiab [1],

$$F = N_p\left[B_o + \left(R_p\right)B_g\right] \tag{12}$$

$$E_{of} = B_o - B_{oi} + \left(R_{si} - R_s\right)B_g + C_{e.f}\Delta\overline{P_{oi}}B_{oi} \tag{13}$$

$$E_{om} = B_o - B_{oi} + \left(R_{si} - R_s\right)B_g + C_{e.m}\Delta\overline{P_{oi}}B_{oi} \tag{14}$$

Where Eo_m represents the net expansion of the original oil phase in the matrix system and Eo_f is the net expansion of the original oil-phase in the fracture network. Diagnostic plots can be constructed as Havlena and Odeh proposed, Chacon and Tiab [1]. The rearranged MBE can be written as follows

$$\frac{F}{E_{om}} = N_m + N_f\,\frac{E_{of}}{E_{om}} \tag{15}$$

A plot of $\frac{F}{E_{om}}$ versus $\frac{E_{of}}{E_{om}}$ leads to a straight line with a y-intercept N_m and slope N_f as represented in the Figure 2.

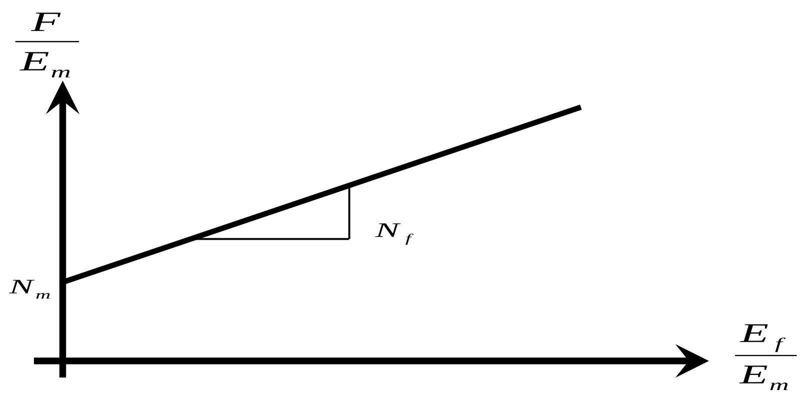


Figure 2: Material balance plotting scheme proposed by Penuelaet al.3

3.4 THE MATERIAL BALANCE EQUATION AS FUNCTION OF THE STORAGE CAPACITY FOR UNDERSATURATED AND SATURATED AND NATURALLY FRACTURED RESERVOIRS

Using the general definition of the storage capacity ratio introduced by Warren and Root. Chacon and Tiab developed and integrated storage capacity into the compacted material balance equation resulting into a straight line passing through the origin with a slope N_f.

Warran and Root defined the storage capacity ratio as;

$$\omega = \frac{(\phi C_t)_f}{(\phi C_t)_{f+m}} \tag{16}$$

According to Chacon and Tiab.

$$\omega = \frac{(\phi C_t)_f}{(\phi C_t)_{f+m}} \approx \frac{N_f}{N}, \tag{17}$$

Substituting into equation and rearranging becomes,

$$F = N\left[\omega_i E_{of} + (1-\omega)E_{om}\right] \quad (18)$$

The plot of $\omega_i E_{of} + (1-\omega_i)E_{om}$ vestures F yield a straight line as presented in figure 3

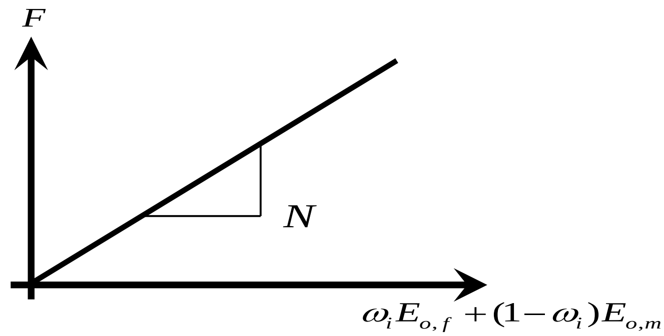


Figure 3: Material balance as function of storage capacity ratio for a volumetric undersaturated NFR.

The advantage of Chacon and Tiab new plotting method over Penuela et al is that, it requires less production data and only one regression parameter is needed to obtain a good estimates of the total original hydrocarbon in place.

The initial oil in place in the fractures was given as,

$$N_f = \omega_i N \quad (19)$$

Recall that,

$$N = N_f + N_m \quad (20)$$

Where,

$$N_m = (1-\omega)N \tag{21}$$

The fractional recovery can be estimated by combining equations as:

$$\frac{N_p}{N} = \frac{(B_t - B_{ti}) + B_{ti}C_e\Delta\overline{P}}{B_t + (R_P - R_{soi})B_g} \tag{22}$$

Blow the bubble point, the cumulative gas-oil ratio at any point pressure is:

$$R_p = \frac{\sum \Delta N_P R}{N_p} \tag{23}$$

3.5 Saturated Naturally Fractured Reservoirs

Assumptions

- ◦ P ≤ Pb
- ◦ No original gas cap
- ◦ No water influx or production

The following terms were derived from the general material balance equation to be:

$$E_o = B_t - B_{ti} \tag{24}$$

$$E_g = B_g - B_{gi} \tag{25}$$

$$E_m = E_o + \frac{mB_{ti}}{B_{gi}} E_g + (1-m) B_{ti} C_{em} \Delta \overline{P} \tag{26}$$

Where:

E_o = Expansion of the oil, rb/STB.

E_g = Expansion of the gas, rb/STB.

E_f = Expansion of oil, gas and pore volume inside the fractures, rb/STB.

E_m = Expansion of oil, gas and pore volume inside the matrix, rb/STB.

The general material balance equation for saturated NFRs was compacted to as:

$$F = N_f E_f + N_m E_m + W_e \tag{27}$$

Writing equation in terms of storage capacity ratio for a volumetric saturated NFRs we have,

$$F = N\left[\omega_i E_f + (1-\omega_i) E_m\right] \tag{28}$$

The plot of F as the y-coordinate and $\omega_i E_f + (1-\omega) E_m$ as the x-coordinate would also yield in a straight line passing through the origin with slope N, as shown in figure 4.

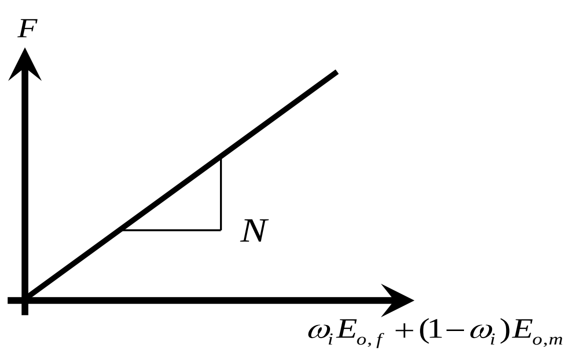


Figure 4: Material balance as function of storage capacity ratio for a volumetric saturated NFR.

In order to test the accuracy of the model, hypothetical cases were designed for volumetric undersaturated and saturated naturally fractured reservoirs using Craft and Hawkins data presented in an example for a homogeneous saturated reservoir in which the effects of considering or not considering the compressibilities in the computation of recovery were analyzed, Chacon, Tiab et al [1]. They used a storage capacity ration of 0.01 with the following assumption:

a) Negligible compressibility
b) Fractures and matrix pore volumes compressibilities were considered equal
c) Fractures and matrix pore volume compressibility ratio was considered to be 25.
d) Fractures and matrix pore volume compressibility ratio was considered to be 50.
e) Fractures and matrix pore volume compressibility ratio was considered to be 75.
f) Fractures and matrix pore volume compressibility ratio was considered to be 100.

CHAPTER 4 - APPLICATIONS

4.0 VOLUMETRIC UNDERSATURATED NATURALLY FRACTURED RESERVOIRS

To test the accuracy of Chacon and Tiab proposed Material Balance equation (MBE) in investigating the impact of pressure depletion on naturally fractured reservoirs, four (4) hypothetical cases of oil production form a NFR were designed. Case 1 models a situation where the storage capacity ratios was assumed to be 0.05 While case 2, 3 and 4 storage capacities were assumed to be 0.1, 0.2 and 0.4.

All four cases were analyzed in terms of the following fractures and matrix pore volumes compressibility ratio.

g) Negligible compressibility (GMBE assumption)

h) Fractures and matrix pore volumes compressibilities were considered equal

i) Fractures and matrix pore volume compressibility ratio was considered to be 25.

j) Fractures and matrix pore volume compressibility ratio was considered to be 50.

k) Fractures and matrix pore volume compressibility ratio was considered to be 75.

l) Fractures and matrix pore volume compressibility ratio was considered to be 100.

Petrophysical and fluid properties used for all cases are given in Tables 1 and Fig 3. Production data were obtained from Craft and Hawkins presented example for a homogeneous undersaturated reservoir in which the effects of considering or not considering the compressibilities in the computation of recovery were analyzed. This study uses the example of a homogeneous undersaturated reservoir in a naturally fractured reservoir

Given the following reservoir and fluid properties:

P_i = 4000 psia cpp,m = 5x10-6 psi-1

P_b = 2500 psia φ = 10%

S_w = 30% ω = 0.01

c_w = 3x10-6 psi-1

Table 2: Undersaturated Volumetric NFR PVT Data

Pressure, psi	***R_{soi}, SCF/STB***	***B_g, rb/SCF***	***B_t, rb/STB***
400	1000	0.00083	1.3000
2500	1000	0.00133	1.3200
2300	920	0.00144	1.3952
2250	900	0.00148	1.4180
2200	880	0.00151	1.4410

4.1 Summary of Results and Analyses

Using the above reservoir and fluid properties, the impact of pressure depletion on oil in- place and recovery factor were analyzed based on Chacon and Tiab proposed NFR material balance equation.

Case 1: Storage capacity ratios was assumed to be 0.05

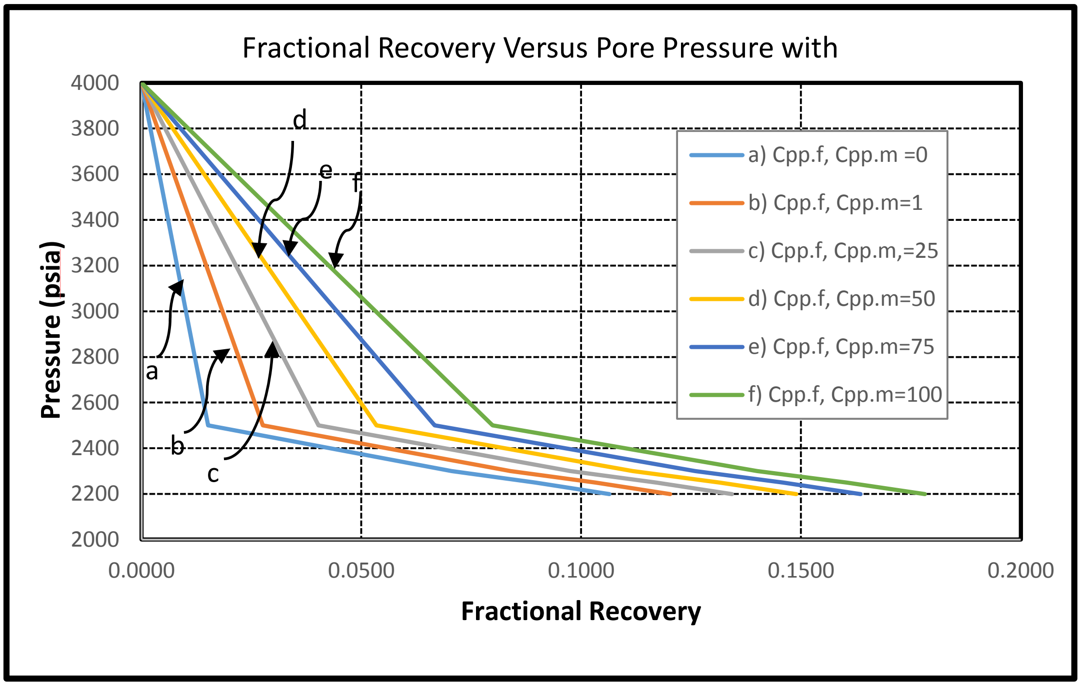


Figure 7. Sensitivity analysis on recovery versus pore pressure in an undersaturated NFR with storage capacity of 0.05.

Case 2: Storage capacity ratios was assumed to be 0.1

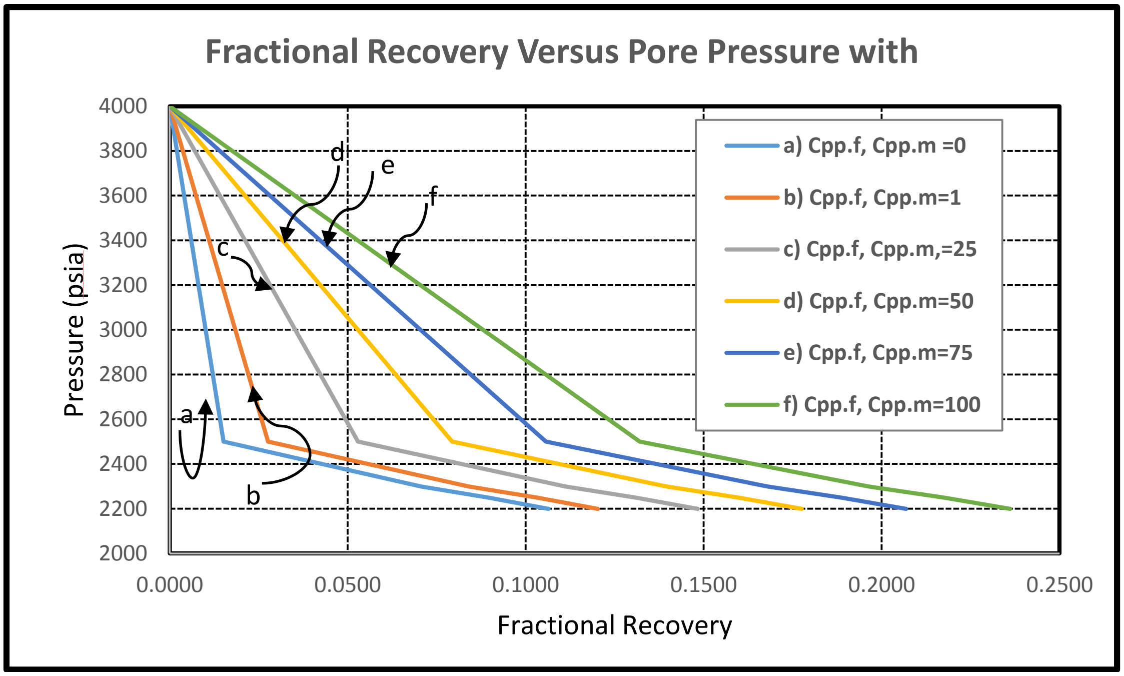


Figure 8. Sensitivity analysis on recovery versus pore pressure in an undersaturated NFR with storage capacity of 0.1.

Case 3: Storage capacity ratios was assumed to be 0.2

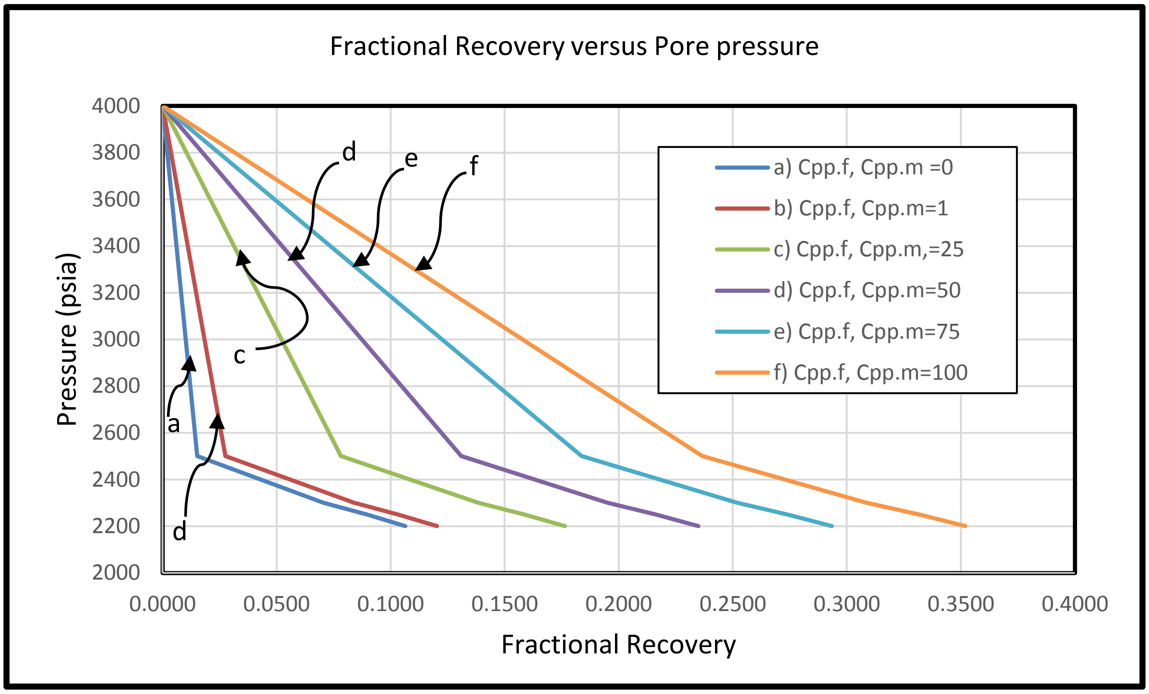


Figure 9. Sensitivity analysis on recovery versus pore pressure in an undersaturated NFR with storage capacity of 0.2

Case 4: Storage capacity ratios was assumed to be 0.4

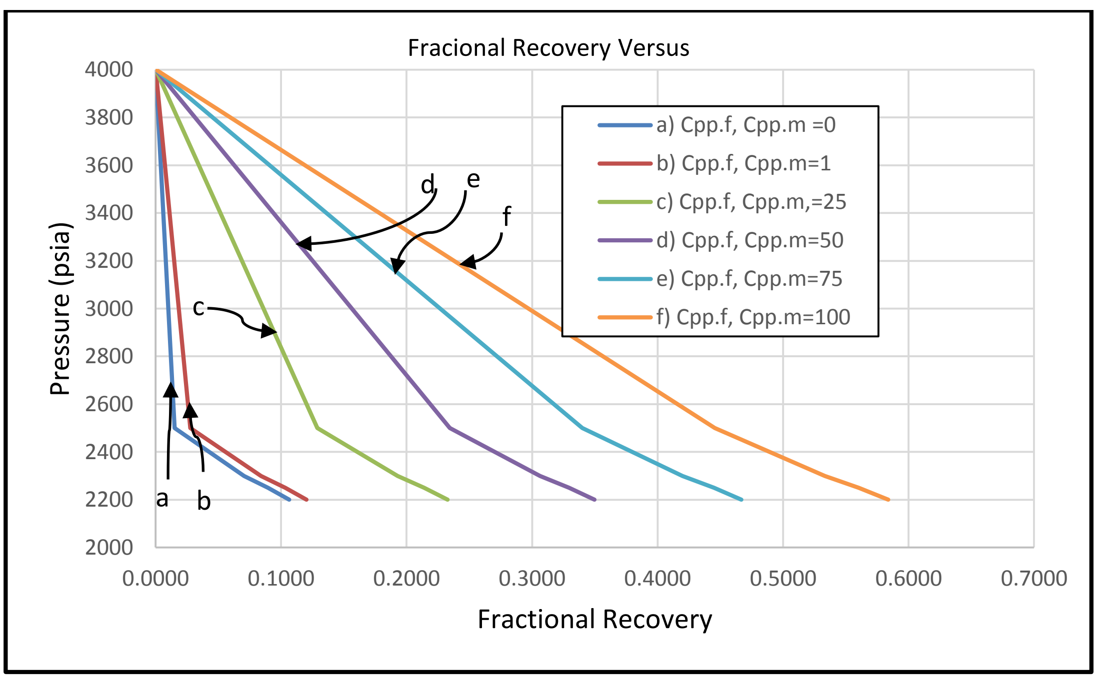


Figure 10. Sensitivity analysis on recovery versus pore pressure in an undersaturated NFR with storage capacity of 0.4

Computations for all cases (case 1, 2, 3 and 4) are presented in tables of Appendix A.

Figure 7 shows significant difference in estimating recovery factor if differences between the fractures pore volume, and matrix pore volume compressibilities are not taken into account in the computations. 26.17% to 67.41% differences in recovery factors are seen in a volumetric undersaturated NFRs with fracture and matrix pores compressibility ratio >1 and with a storage capacity (ω) of 0.05. Figure 8 also shows significant percent differences ranging from 39.37% to 95.36%; 29.3% difference over figure 7.

This phenomenon is seen in figure 9 and 10. The recovery factor is seen to be directly proportional to storativity if and only if fractures and matrix compressibility ratio is factored in. The higher the storage capacity ratio, the increase in recovery factor. Summary of the phenomenon is clearly presented in figure 11. The sharp increase in recovery factor at the bubble point pressure (2500 psia.) on figure 5, 6, 7, and 8 denote a typical production characteristics of a solution-Gas Drive Reservoirs.

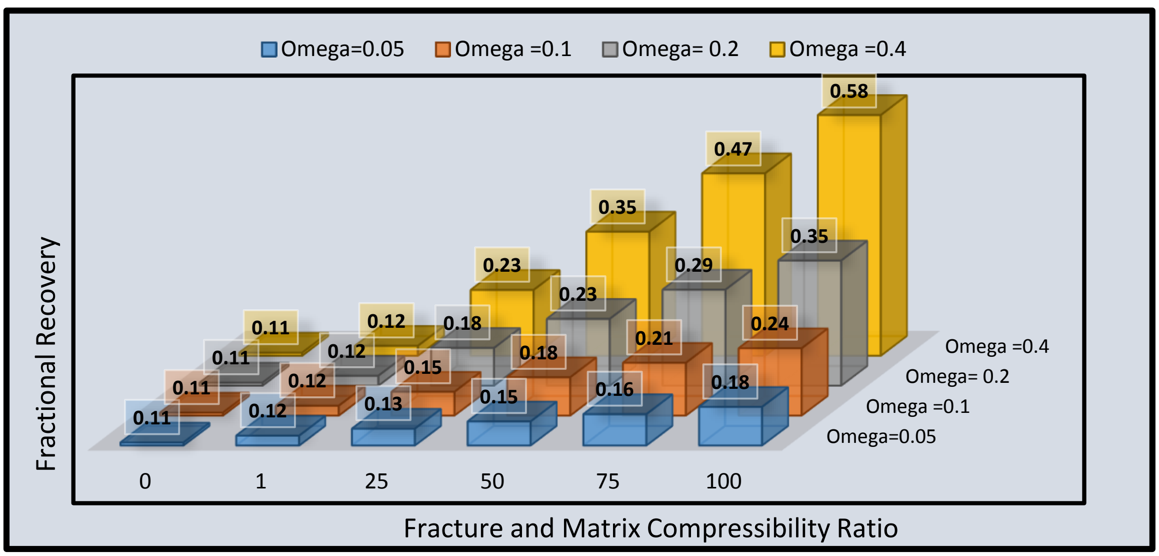


Figure 11: Summary analysis of recovery factor with respect to storage capacity and fractures and matrix pore compressibility ratio

Summary of computations for all cases are found in Tables of Appendix A.

4.2 VOLUMETRIC SATURATED NATURALLY FRACTURED RESERVOIRS

In continuation of the investigation of pressure depletion on naturally fractured reservoir, four (4) hypothetical cases of oil production were also designed using Chacon and Tiab proposed NFRs material balance equation. Case 1 models a situation where the storage capacity ratios was assumed to be 0.05; while case 2, 3 and 4 storage capacities were assumed to be 0.1, 0.2 and 0.4.

All four cases were analyzed in terms of the following fractures and matrix pore volumes compressibility ratio.

a) Negligible compressibility (GMBE assumption)
b) Fractures and matrix pore volumes compressibilities were considered equal
c) Fractures and matrix pore volume compressibility ratio was considered to be 25.
d) Fractures and matrix pore volume compressibility ratio was considered to be 50.
e) Fractures and matrix pore volume compressibility ratio was considered to be 75.
f) Fractures and matrix pore volume compressibility ratio was considered to be 100

Craft and Hawkins presented an example for a homogeneous saturated reservoir in which the Compressibilities were neglected in the computation of hydrocarbons in place. This analysis extends Craft and Hawkins example to a naturally fractured reservoir, and analyzes the effect of compressibilities in the estimation of hydrocarbons originally in place. In order to analyze the compressibility effects, the following was assumed: no water drive and no water production.

Reservoir Rock and fluid Properties

Volume of bulk oil zone = 112000 ac-ft

Volume of bulk gas zone = 19600 ac-ft

Initial reservoir pressure = 2710 psia

Initial FVF, B_{ti} = 1.34 rb/STB

Pore volume matrix compressibility, cpp,m = 3.5x10-6 psi-1

Connate water saturation, Swi = 0.2

Gas volume factor at 2000 psia, B_g = 0.001510 rb/SCF

Storage capacity ratio of the NFR, ω = 0.01

Initial gas volume factor, B_{gi} = 0.001116 rb/SCF

Initial dissolved GOR, R_{soi} = 562 SCF/STB

Oil produced during the interval, Np = 20 MM STB

Reservoir pressure at the end of the interval = 2000 psia

Average produced GOR, Rp = 700 SCF/STB

Two phase FVF, B_t = 1.4954 rb/STB

FVF of the water, Bw = 1.028 rb/STB

4.4 SUMMARY OF RESULTS AND ANALYSES

Using the above Reservoir data obtained from Chacon and Tiab examples of volumetric saturated (SPE 108107), the investigation of pressure depletion on oil reserves estimates and recovery factor in saturated naturally fractured reservoir was performed using Chacon and Tiab proposed material balance equation. Appendix- summarizes the results for each case and shows a comparison with volumetric estimate.

Case 1: Storage capacity ratio was assumed to be 0.05

Table4: Differences in OOIP and fractional recovery estimations in NFR with storage capacity of 0.05

Cases of Compressibilities	***Original Oil in Place, N (STB)***	***Fractional Recovery***	***Difference in original oil in place, %***	***Difference in fractional recovery, %***
Cpp,f/Cpp,m = 0	143060926.3	0.1398	0	0
$C_{pp,f}/C_{pp,m}$ = 1	139700681.4	0.1432	2.3488	2.4053
$C_{pp,f}/C_{pp,m}$ = 25	136418308	0.1466	4.6432	4.8693
$C_{pp,f}/C_{pp,m}$ = 50	133159268.5	0.1502	6.9213	7.4360
$C_{pp,f}/C_{pp,m}$ = 75	130052312.8	0.1538	9.0931	10.0026
$C_{pp,f}/C_{pp,m}$ = 100	127087038.3	0.1574	11.1658	12.5693

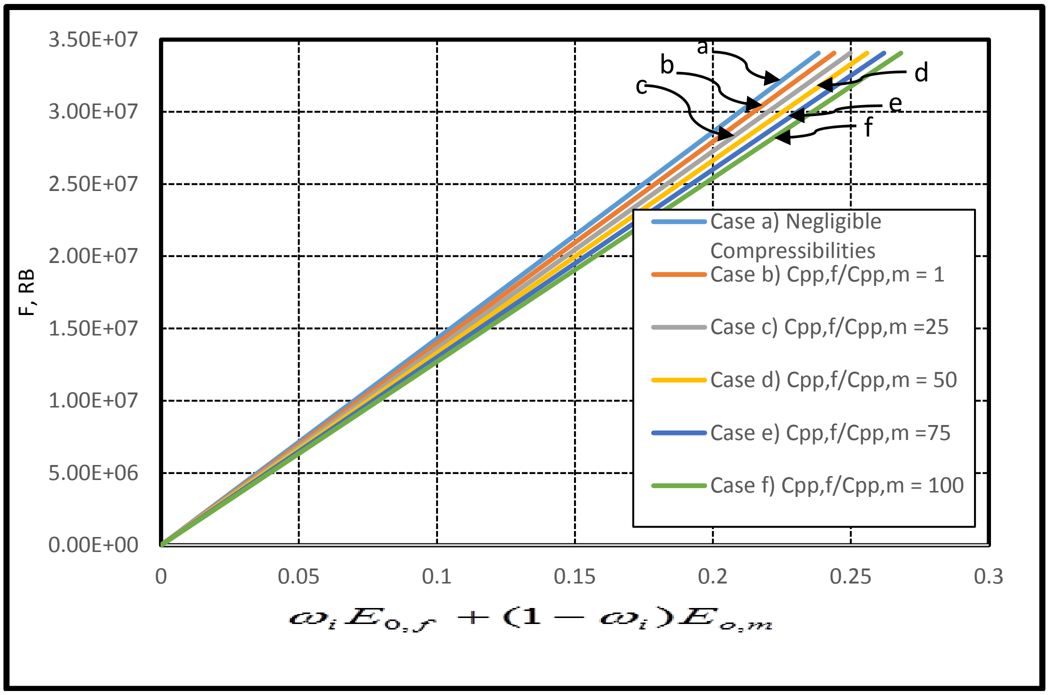


Figure 12: Material balance as function of storage capacity ratio for a volumetric saturated NFR with storage capacity of 0.05

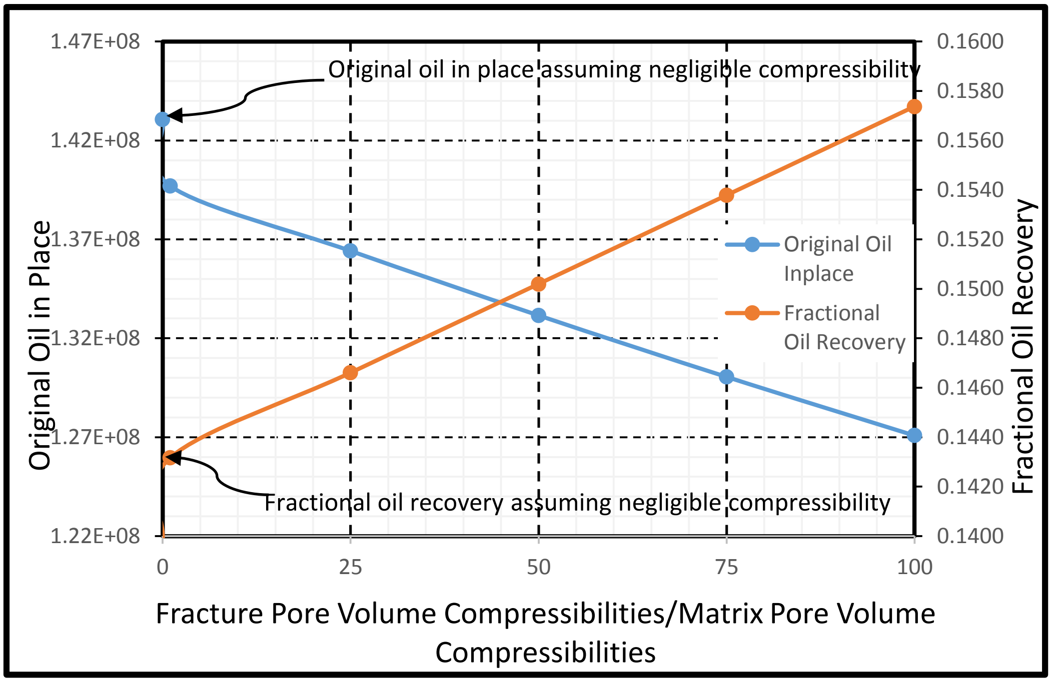


Figure 13: summary of results for original oil in place and fractional recovery with storage capacity of 0.05

Case 2: Storage capacity ratios was assumed to be 0.1

Table5: Differences in OOIP and fractional recovery estimations in NFR with storage capacity of 0.1

Cases of Compressibillities	Original Oil in Place,N (STB)	Fractional Recovery	Difference in original oil in place, %	Difference in fractional recovery, %
Negligible (Cpp,f/Cpp,m = 0)	143060926.3	0.1398	0	0
$C_{pp,f}/C_{pp,m}$ = 1	139700681.4	0.1432	2.3488	2.4053
$C_{pp,f}/C_{pp,m}$ = 25	133286637.4	0.1501	6.8323	7.3333
$C_{pp,f}/C_{pp,m}$ = 50	127203050.6	0.1572	11.0847	12.4666
$C_{pp,f}/C_{pp,m}$ = 75	121650567.7	0.1644	14.9659	17.5999
$C_{pp,f}/C_{pp,m}$ = 100	116562548.5	0.1716	18.5224	22.7332

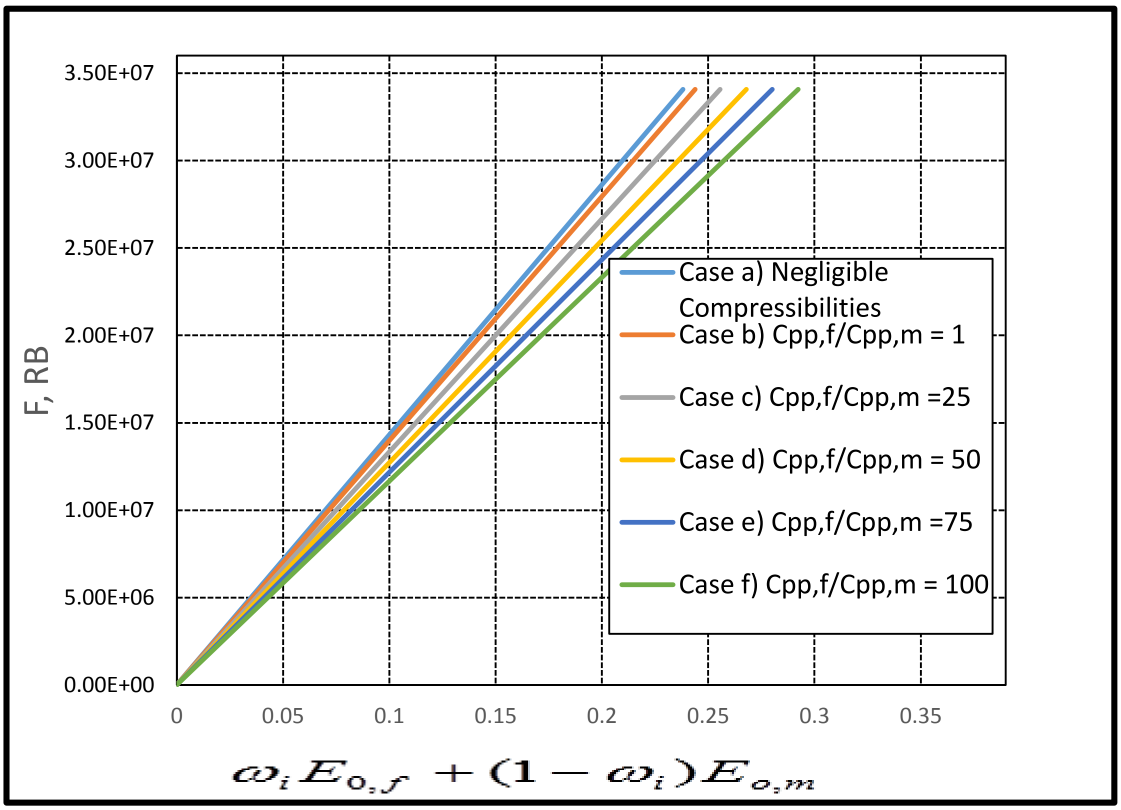


Figure 14: Material balance as function of storage capacity ratio for a volumetric saturated NFR with storage capacity of 0.1

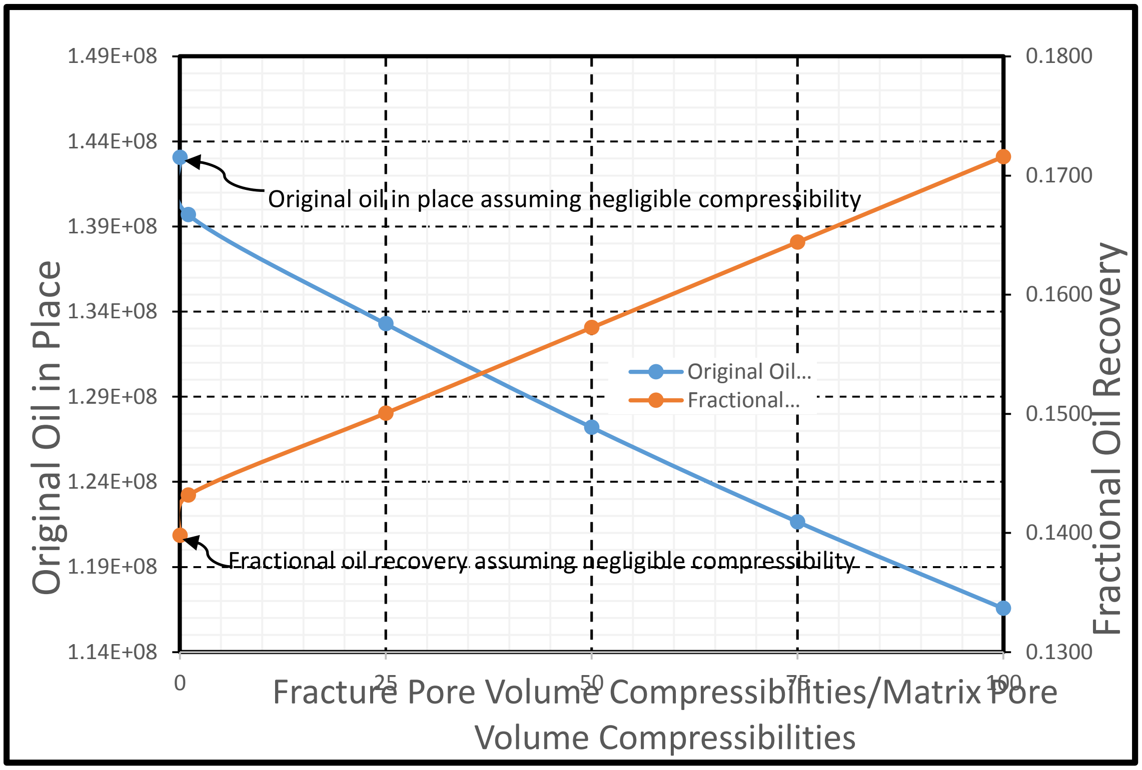


Figure 15: summary of results for original oil in place and fractional recovery with storage capacity of 0.1

Case 3: Storage capacity ratios was assumed to be 0.2

Table 6: Differences in OOIP and fractional recovery estimations in NFR with storage capacity of 0.2

Cases of Compressibilities	***Original Oil in Place, N (STB)***	***Fractional Recovery***	***Difference in original oil in place, %***	***Difference in fractional recovery, %***
Cpp,f/Cpp,m = 0	143060926.3	0.1398	0	0
$C_{pp,f}/C_{pp,m}$ = 1	139700681.4	0.1432	2.3488	2.4053
$C_{pp,f}/C_{pp,m}$ = 25	127435712	0.1569	10.9221	12.2613
$C_{pp,f}/C_{pp,m}$ = 50	116757883.8	0.1713	18.3859	22.5279
$C_{pp,f}/C_{pp,m}$ = 75	107731103.2	0.1856	24.6956	32.7945
$C_{pp,f}/C_{pp,m}$ = 100	99999914.08	0.2000	30.0998	43.0610

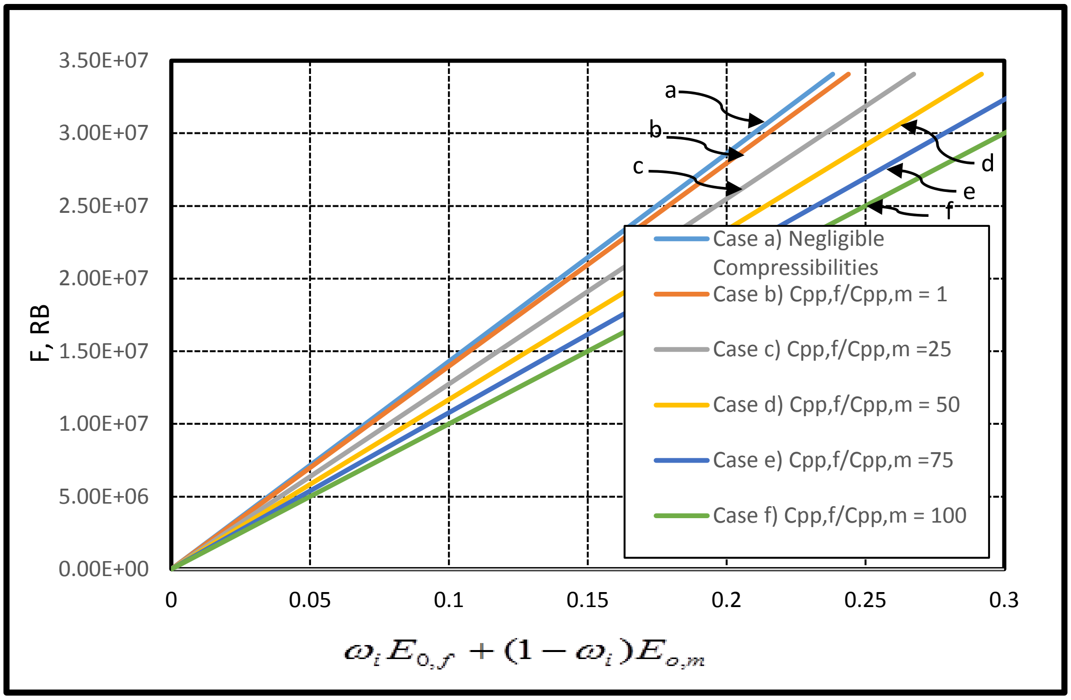


Figure 16: Material balance as function of storage capacity ratio for a volumetric saturated NFR with storage capacity of 0.2

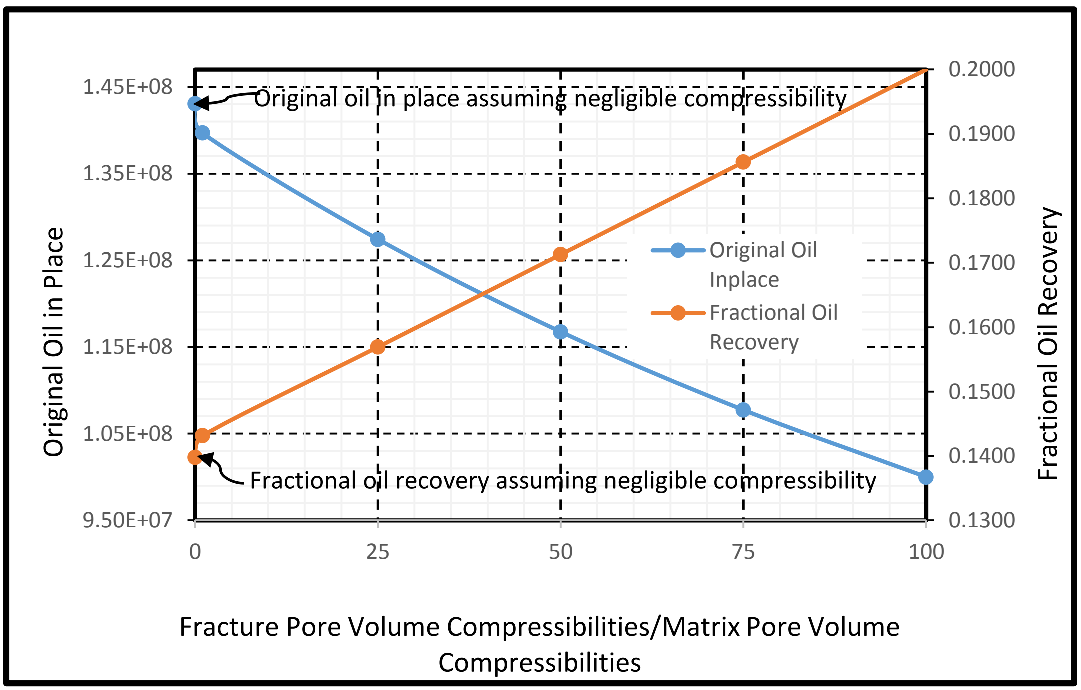


Figure 17: summary of results for original oil in place and fractional recovery

Case 4: Storage capacity ratios was assumed to be 0.4

Table 7: Differences in OOIP and fractional recovery estimations in NFR with storage capacity of 0.4

Cases of Compressibilities	*Original Oil in Place, N (STB)*	*Fractional Recovery*	*Difference in original oil in place, %*	*Difference in fractional recovery, %*
Cpp,f/Cpp,m = 0	143060926.3	0.1398	0	0
$C_{pp,f}/C_{pp,m}$ = 1	139700681.4	0.1432	2.3488	2.4053
$C_{pp,f}/C_{pp,m}$ = 25	117150525.1	0.1707	18.1114	22.1172
$C_{pp,f}/C_{pp,m}$ = 50	100287795.3	0.1994	29.8985	42.6504
$C_{pp,f}/C_{pp,m}$ = 75	87668700.22	0.2281	38.7193	63.1836
$C_{pp,f}/C_{pp,m}$ = 100	77870364.12	0.2568	45.5684	83.7168

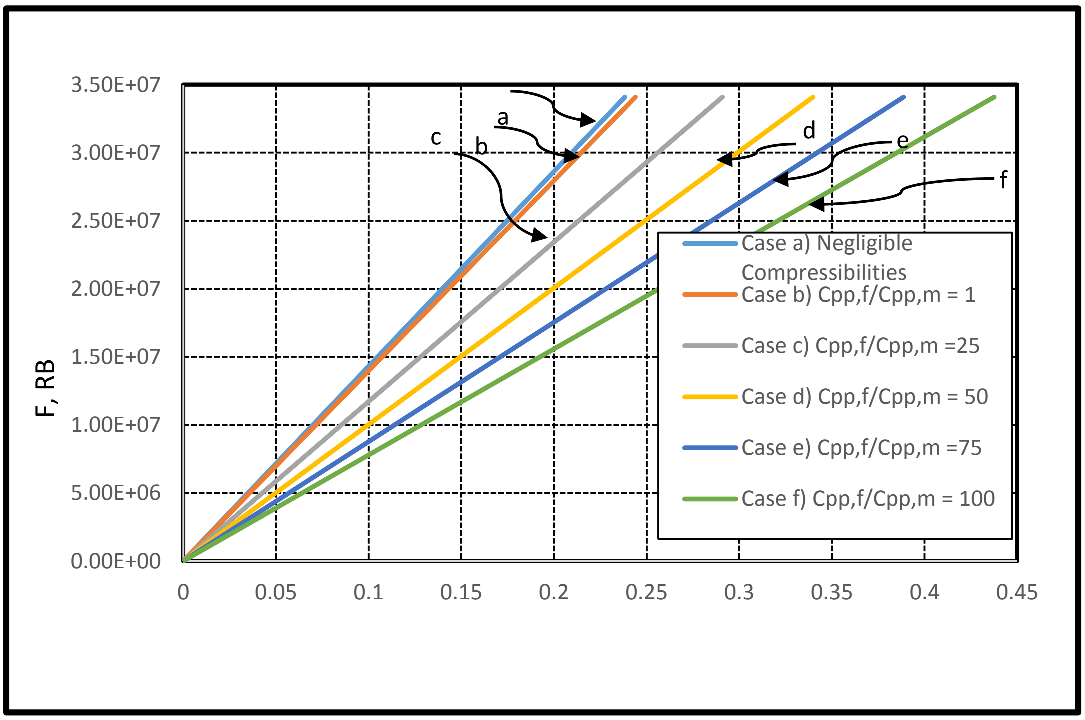


Figure 18: Material balance as function of storage capacity ratio for a volumetric saturated NFR with storage capacity of 0.2

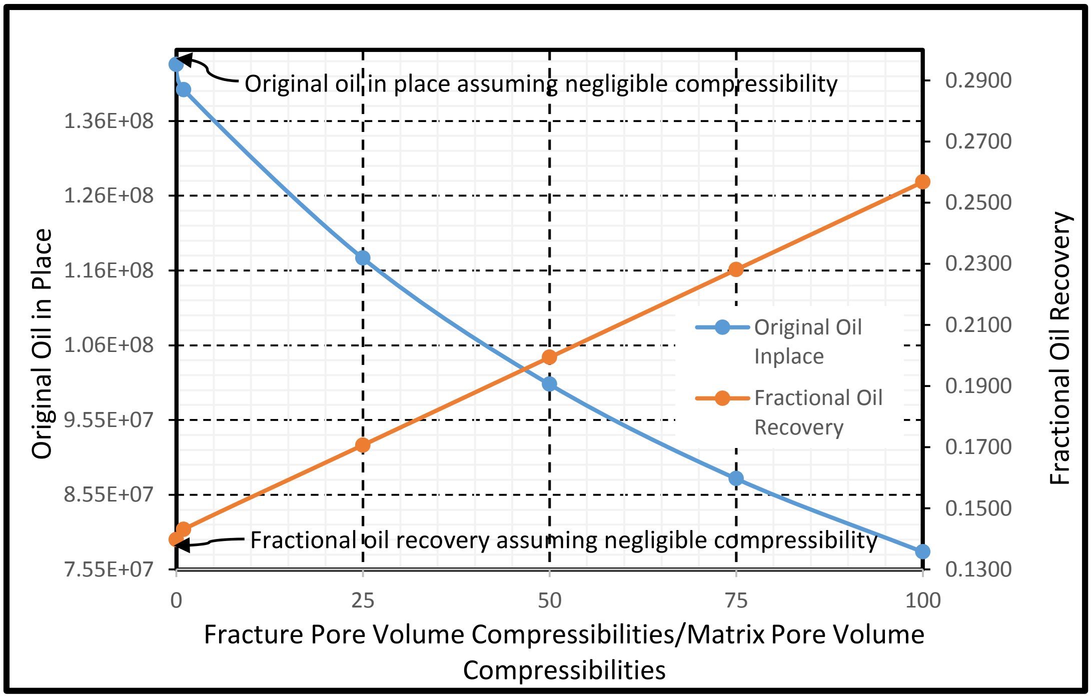


Figure 19: summary of results for original oil in place and fractional recovery

Cases investigated in this paper show that errors in estimating reserves and recovery factor is expected to be > 5% if fracture and matrix pore volume compressibility ratio is neglected in the computation of reserves and recovery factor. It was observed that errors in estimating reserves and recovery factor increase with increasing storage capacity ratio in NFRs.

Table 4 of case one shows 6% and 7% errors in estimating oil in place and recovery factor. Table 5 of case 2 shows errors of 10.7% and 12.5%; while table 6 and 7 show 17% and 22%; errors of 26% and 42% were also observed in case 4.

Summary of computations for all cases are found in Tables of Appendix B.

CHAPTER 5 - CONCLUSIONS

The research shows that Chacon and Tiab proposed equation can be used to evaluate the influence of fracture and matrix compressibility ratio on oil in-place and recovery factor computations in naturally fractured reservoirs. Early estimation of the reservoir storativity, will have larger impact on oil in-place estimates.

In volumetric undersaturated naturally fractured reservoirs, recovery factor is directly proportional to storage capacity ratio of naturally fractured reservoirs.

Errors in reserves estimates and recovery factor in volumetric saturated NFRs increases with increasing storage capacity ratio if compressibilities of fractured and matrix pores are neglected. In volumetric saturated NFRs, a reservoir with high storage capacity ratio producing at constant rate in line with a reservoir of low storage capacity ratio is expected to deplete it reserve sooner as compare to a reservoir with low storage capacity ratio due to high recovery factor. `

Finally, the synthetic cases showed that Chacon and Tiab proposed equation is simple to use and will help the reservoir engineer to obtain an accurate simultaneous estimation of recovery factor and oil stored both in the matrix and the fracture systems in a naturally fractured formation.

5.1 RECOMMENDATION

The research was limited to volumetric naturally fractured reservoirs; however, further analysis can be done considering naturally fractured reservoirs with active water drive.

The effect of interporosity on reserves estimates and recover factor in naturally fractured reservoir can also be investigated in future work.

NOMENCLATURE:

B_{gi} = initial gas formation volume factor, rb/SCF.

B_0 = oil formation volume factor, rb/STB.

B_w = water formation volume factor, rb/STB.

c = compressibility, 1/psi.

c_e = effective compressibility, 1/psi.

c_{pp} = isothermal pore volume compressibility due to changes in pore pressure, 1/psi.

c_w = water isothermal compressibility, psi^{-1}.

$c_{e,f}$ = effective compressibility of the fracture system, also known as the rock expansion term due to changes in rock and water compressibility of the fracture rock system, psi^{-1}.

$c_{e,m}$= effective compressibility of the matrix system, also known as the rock expansion term due to changes in rock and water compressibility of the matrix, psi^{-1}.

$c_{e,(f+m)}$ = effective compressibility of the fractured rock system, also known as the rock expansion term due to changes in rock and water compressibility matrix + fracture, psi^{-1}.

$c_{pp,(f+m)}$ = fracture rock system (matrix + fracture) isothermal pore compressibility, psi^{-1}.

$c_{pp,f}$ = fracture isothermal pore compressibility, psi^{-1}.

cpp,m = matrix isothermal pore compressibility, psi^{-1}.

Δp = change in average reservoir pressure ≈ change in effective stress, psi.

$E_{o,f}$ = expansion of the initial amount of oil contained inside the fractures, bbl/STB.

$E_{o,m}$ = expansion of the initial amount of oil contained inside the matrix, bbl/STB.

F = amount of oil produced, RB.

m = ratio of the initial gas cap volume to the initial oil volume.

N = initial reservoir oil, STB.

N_f = initial reservoir oil in the fractures, STB.

N_m = initial reservoir oil in the matrix, STB.

N_p = cumulative produced oil, STB.

p = pressure, psi.

p_i = initial pressure, psi.

R_{soi} = initial solution gas-oil ratio, SCF/STB.

R_p = cumulative produced gas-oil ratio, SCF/STB.

R_{so} = solution gas-oil ratio, SCF/STB.

S = saturation, fraction.

S_{wi} = initial water saturation.

W_p = cumulative produced water, STB.

W_e = water influx into reservoir, bbl.

Greek Symbols

Δ = change, drop.

ω = storage capacity ratio, dimensionless.

φ = porosity, dimensionless.

Subscripts

c = confining.

e = effective.

f = fracture.

f+m = total NFR system (fracture + matrix).

g = gas.

i = initial.

m = matrix.

o = oil.

p = pore space.

t = total.

w = water.

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APPENDIX A: SUMMARY OF COMPUTATIONS FOR THE UNDERSATURATED RESERVOIR

Table A 1.1: Summary of results for all assumption in case 1 (Storativity of 0.05).

Fractional Recovery						
Pressure (psia)	***Case a) Cpp,f/Cpp,m=0, Neglegible Compressibility***	***Case b) Cpp,f/Cpp,m=1***	***Case c) Cpp,f/Cpp,m=25***	***Case d) Cpp,f/Cpp,m=50***	***Case e) Cpp,f/Cpp,m=75***	***Case a) Cpp,f/Cpp,m=100***
4000	0.000	0.000	0.000	0.000	0.000	0.000
2500	0.015	0.028	0.040	0.053	0.067	0.080
2300	0.071	0.084	0.098	0.112	0.126	0.140
2250	0.090	0.103	0.117	0.132	0.146	0.160
2200	0.106	0.120	0.134	0.149	0.164	0.178

Table A.1.2. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=0.

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.020	0.020	0.020	1000	0.015
2300	1700	0.095	0.095	0.095	960	0.071
2250	1750	0.118	0.118	0.118	910	0.090
2200	1800	0.141	0.141	0.141	890	0.106

Table A.1.3. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=1.

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.036	0.036	0.036	1000	0.028
2300	1700	0.114	0.114	0.114	960	0.084
2250	1750	0.137	0.137	0.137	910	0.103
2200	1800	0.161	0.161	0.161	890	0.120

Table A.1.4. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=25.

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.371	0.036	0.053	1000	0.040
2300	1700	0.493	0.114	0.133	960	0.098
2250	1750	0.527	0.137	0.157	910	0.117
2200	1800	0.562	0.161	0.181	890	0.134

Table A.1.5. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=50

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.719	0.036	0.071	1000	0.053
2300	1700	0.887	0.114	0.153	960	0.112
2250	1750	0.933	0.137	0.177	910	0.132
2200	1800	0.980	0.161	0.202	890	0.149

Table A.1.6. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=75

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.067	0.036	0.088	1000	0.067
2300	1700	1.282	0.114	0.172	960	0.126
2250	1750	1.340	0.137	0.197	910	0.146
2200	1800	1.398	0.161	0.223	890	0.164

Table A.1.6. Computations summary for case 1: $c_{pp,f}/c_{pp,m}$=100

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.415	0.036	0.105	1000	0.080
2300	1700	1.677	0.114	0.192	960	0.140
2250	1750	1.746	0.137	0.218	910	0.160
2200	1800	1.815	0.161	0.243	890	0.178

Table A 2.1: Summary of results for all assumption in case 2 (Storativity of 0.1).

Fractional Recovery						
Pressure (psia)	***Case a) Cpp,f/Cpp,m=0, Neglegible Compressibility***	***Case b) Cpp,f/Cpp,m=1***	***Case c) Cpp,f/Cpp,m=25***	***Case d) Cpp,f/Cpp,m=50***	***Case e) Cpp,f/Cpp,m=75***	***Case a) Cpp,f/Cpp,m=100***
4000	0	0	0	0	0	0
2500	0.015	0.028	0.053	0.079	0.106	0.132
2300	0.071	0.084	0.111	0.140	0.168	0.196
2250	0.090	0.103	0.131	0.160	0.189	0.218
2200	0.106	0.120	0.148	0.178	0.207	0.236

Table A.2.2. Computations summary for case 2: $c_{pp,f}/c_{pp,m}$= 0

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.020	0.020	0.020	1000	0.015
2300	1700	0.095	0.095	0.095	960	0.071
2250	1750	0.118	0.118	0.118	910	0.090
2200	1800	0.141	0.141	0.141	890	0.106

Table A.2.3. Computations summary for case 2: $c_{pp,f}/c_{pp,m}$= 1

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.036	0.036	0.036	1000	0.028
2300	1700	0.114	0.114	0.114	960	0.084
2250	1750	0.137	0.137	0.137	910	0.103
2200	1800	0.161	0.161	0.161	890	0.120

Table A.2.4. Computations summary for case 2: $c_{pp,f}/c_{pp,m}$= 25

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.371	0.036	0.070	1000	0.053
2300	1700	0.493	0.114	0.152	960	0.111
2250	1750	0.527	0.137	0.176	910	0.131
2200	1800	0.562	0.161	0.201	890	0.148

Table A.2.5. Computations summary for case 2: $c_{pp,f}/c_{pp,m}$= 50

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.719	0.036	0.105	1000	0.079
2300	1700	0.887	0.114	0.191	960	0.140
2250	1750	0.933	0.137	0.217	910	0.160
2200	1800	0.980	0.161	0.243	890	0.178

Table A.2.6. Computations summary for case 2: $c_{pp,f} / c_{pp,m} = 75$

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.067	0.036	0.140	1000	0.106
2300	1700	1.282	0.114	0.231	960	0.168
2250	1750	1.340	0.137	0.257	910	0.189
2200	1800	1.398	0.161	0.284	890	0.207

Table A.2.7. Computations summary for case 2: $c_{pp,f} / c_{pp,m} = 100$

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.415	0.036	0.174	1000	0.132
2300	1700	1.677	0.114	0.270	960	0.196
2250	1750	1.746	0.137	0.298	910	0.218
2200	1800	1.815	0.161	0.326	890	0.236

Table A 3.1: Summary of results for all assumption in case 3 (Storativity of 0.2).

Fractional Recovery						
Pressure (psia)	***Case a) Cpp,f/Cpp,m=0, Neglegible Compressibility***	***Case b) Cpp,f/Cpp,m=1***	***Case c) Cpp,f/Cpp,m=25***	***Case d) Cpp,f/Cpp,m=50***	***Case e) Cpp,f/Cpp,m=75***	***Case a) Cpp,f/Cpp,m=100***
4000	0	0	0	0	0	0
2500	0.015	0.028	0.078	0.131	0.184	0.237
2300	0.071	0.084	0.138	0.195	0.252	0.309
2250	0.090	0.103	0.159	0.216	0.274	0.332
2200	0.106	0.120	0.176	0.235	0.293	0.352

Table A.3.2. Computations summary for case 3: $c_{pp,f} / c_{pp,m} = 0$

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.020	0.020	0.020	1000	0.015
2300	1700	0.095	0.095	0.095	960	0.071
2250	1750	0.118	0.118	0.118	910	0.090
2200	1800	0.141	0.141	0.141	890	0.106

Table A.3.3. Computations summary for case 3: $c_{pp,f}/c_{pp,m}$= 1

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.036	0.036	0.036	1000	0.028
2300	1700	0.114	0.114	0.114	960	0.084
2250	1750	0.137	0.137	0.137	910	0.103
2200	1800	0.161	0.161	0.161	890	0.120

Table A.3.4. Computations summary for case 3: $c_{pp,f}/c_{pp,m}$= 25

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.371	0.036	0.103	1000	0.078
2300	1700	0.493	0.114	0.190	960	0.138
2250	1750	0.527	0.137	0.215	910	0.159
2200	1800	0.562	0.161	0.241	890	0.176

Table A.3.5. Computations summary for case 3: $c_{pp,f}/c_{pp,m}$= 50

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.719	0.036	0.173	1000	0.131
2300	1700	0.887	0.114	0.269	960	0.195
2250	1750	0.933	0.137	0.296	910	0.216
2200	1800	0.980	0.161	0.325	890	0.235

Table A.3.6. Computations summary for case 3: $c_{pp,f}/c_{pp,m}$= 75

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.067	0.036	0.243	1000	0.184
2300	1700	1.282	0.114	0.347	960	0.252
2250	1750	1.340	0.137	0.378	910	0.274
2200	1800	1.398	0.161	0.408	890	0.293

Table A.3.7. Computations summary for case 3: $c_{pp,f}/c_{pp,m}$= 100

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.415	0.036	0.312	1000	0.237
2300	1700	1.677	0.114	0.426	960	0.309
2250	1750	1.746	0.137	0.459	910	0.332
2200	1800	1.815	0.161	0.492	890	0.352

Table A 4.1: Summary of results for all assumption in case 4 (Storativity of 0.4).

Fractional Recovery						
Pressure (psia)	***Case a) Cpp,f/Cpp,m=0, Neglegible Compressibility***	***Case b) Cpp,f/Cpp,m=1***	***Case c) Cpp,f/Cpp,m=25***	***Case d) Cpp,f/Cpp,m=50***	***Case e) Cpp,f/Cpp,m=75***	***Case a) Cpp,f/Cpp,m=100***
4000	0	0	0	0	0	0
2500	0.015	0.028	0.129	0.234	0.340	0.445
2300	0.071	0.084	0.193	0.306	0.420	0.533
2250	0.090	0.103	0.214	0.330	0.445	0.560
2200	0.106	0.120	0.233	0.350	0.467	0.584

Table A.4.2. Computations summary for case 4: $c_{pp,f}$ /$c_{pp,m}$= 0

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.020	0.020	0.020	1000	0.015
2300	1700	0.095	0.095	0.095	960	0.071
2250	1750	0.118	0.118	0.118	910	0.090
2200	1800	0.141	0.141	0.141	890	0.106

Table A.4.3. Computations summary for case 4: $c_{pp,f}$ /$c_{pp,m}$= 1

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.036	0.036	0.036	1000	0.028
2300	1700	0.114	0.114	0.114	960	0.084
2250	1750	0.137	0.137	0.137	910	0.103
2200	1800	0.161	0.161	0.161	890	0.120

Table A.4.4. Computations summary for case 4: $c_{pp,f}/c_{pp,m}$= 25

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.371	0.036	0.170	1000	0.129
2300	1700	0.493	0.114	0.265	960	0.193
2250	1750	0.527	0.137	0.293	910	0.214
2200	1800	0.562	0.161	0.321	890	0.233

Table A.4.5. Computations summary for case 4: $c_{pp,f}/c_{pp,m}$= 50

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	0.719	0.036	0.309	1000	0.234
2300	1700	0.887	0.114	0.423	960	0.306
2250	1750	0.933	0.137	0.456	910	0.330
2200	1800	0.980	0.161	0.488	890	0.350

Table A.4.6. Computations summary for case 4: $c_{pp,f}/c_{pp,m}$= 75

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.067	0.036	0.449	1000	0.340
2300	1700	1.282	0.114	0.581	960	0.420
2250	1750	1.340	0.137	0.618	910	0.445
2200	1800	1.398	0.161	0.655	890	0.467

Table A.4.7. Computations summary for case 4: $c_{pp,f}/c_{pp,m}$= 100

Pressure (psia)	Pressure Depletion(ΔP) psia	Eo.f rb/STB	$E_{o.m}$ rb/STB	$\omega E_{of} + (1-\omega_i)E_{om}$ Rb/STB	Rso(avg) SCF/STB	Np/N
4000	0	0	0	0		0
2500	1500	1.415	0.036	0.588	1000	0.445
2300	1700	1.677	0.114	0.739	960	0.533
2250	1750	1.746	0.137	0.781	910	0.560
2200	1800	1.815	0.161	0.823	890	0.584

APPENDIX B: SUMMARY OF COMPUTATIONS FOR THE SATURATED RESERVOIR EXAMPLE

Table B.1. Summary of results Case 1 (Storativity of 0.05)

Cases of Compressibillities	***$C_{pp.f}$ (psi-1)***	***$C_{e.f}$ (psi-1)***	***$E_{o.f}$ rb/STB***	***Eo.m rb/STB***	***X-AXIS Rb/STB***	***Original Oil in Place,N (STB)***	***Fractional Recovery***
Cpp,f/Cpp,m = 0	0.0	0.0	0.2382	0.2382	0.2382	1.43E+08	0.1398
$C_{pp,f}/C_{pp,m}$ = 1	3.50E-06	5.13E-06	0.2439	0.2439	0.2439	1.40E+08	0.1432
$C_{pp,f}/C_{pp,m}$ = 25	8.75E-05	1.10E-04	0.3613	0.2439	0.2498	1.36E+08	0.1466
$C_{pp,f}/C_{pp,m}$ = 50	1.75E-04	2.20E-04	0.4836	0.2439	0.2559	1.33E+08	0.1502
$C_{pp,f}/C_{pp,m}$ = 75	2.63E-04	3.29E-04	0.6058	0.2439	0.2620	1.30E+08	0.1538
$C_{pp,f}/C_{pp,m}$ = 100	3.50E-04	4.38E-04	0.7281	0.2439	0.2681	1.27E+08	0.1574

Table B.2. Summary of results Case 2 (Storativity of 0.1)

Cases of Compressibillities	***$C_{pp.f}$ (psi-1)***	***$C_{e.f}$ (psi-1)***	***$E_{o.f}$ rb/STB***	***Eo.m rb/STB***	***X-AXIS Rb/STB***	***Original Oil in Place,N (STB)***	***Fractional Recovery***
Cpp,f/Cpp,m = 0	0.0	0.0	0.2382	0.2382	0.2382	1.43E+08	0.1398
$C_{pp,f}/C_{pp,m}$ = 1	3.50E-06	5.13E-06	0.2439	0.2439	0.2439	1.40E+08	0.1432
$C_{pp,f}/C_{pp,m}$ = 25	8.75E-05	1.10E-04	0.3613	0.2439	0.2557	1.33E+08	0.1501
$C_{pp,f}/C_{pp,m}$ = 50	1.75E-04	2.20E-04	0.4836	0.2439	0.2679	1.27E+08	0.1572
$C_{pp,f}/C_{pp,m}$ = 75	2.63E-04	3.29E-04	0.6058	0.2439	0.2801	1.22E+08	0.1644
$C_{pp,f}/C_{pp,m}$ = 100	3.50E-04	4.38E-04	0.7281	0.2439	0.2923	1.17E+08	0.1716

Table B.3. Summary of results Case 3 (Storativity of 0.2)

Cases of Compressibillities	***$C_{pp.f}$ (psi-1)***	***$C_{e.f}$ (psi-1)***	***$E_{o.f}$ rb/STB***	***Eo.m rb/STB***	***X-AXIS Rb/STB***	***Original Oil in Place,N (STB)***	***Fractional Recovery***
Cpp,f/Cpp,m = 0	0.0	0.0	0.2382	0.2382	0.2382	1.43E+08	0.1398
$C_{pp,f}/C_{pp,m}$ = 1	3.50E-06	5.13E-06	0.2439	0.2439	0.2439	1.40E+08	0.1432
$C_{pp,f}/C_{pp,m}$ = 25	8.75E-05	1.10E-04	0.3613	0.2439	0.2674	1.27E+08	0.1569
$C_{pp,f}/C_{pp,m}$ = 50	1.75E-04	2.20E-04	0.4836	0.2439	0.2918	1.17E+08	0.1713
$C_{pp,f}/C_{pp,m}$ = 75	2.63E-04	3.29E-04	0.6058	0.2439	0.3163	1.08E+08	0.1856
$C_{pp,f}/C_{pp,m}$ = 100	3.50E-04	4.38E-04	0.7281	0.2439	0.3408	1.00E+08	0.2000

Table B.4. Summary of results Case 4 (Storativity of 0.4)

Cases of Compressibillities	***$C_{pp.f}$ (psi-1)***	***$C_{e.f}$ (psi-1)***	***$E_{o.f}$ rb/STB***	***Eo.m rb/STB***	***X-AXIS Rb/STB***	***Original Oil in Place,N (STB)***	***Fractional Recovery***
Cpp,f/Cpp,m = 0	0.0	0.0	0.2382	0.2382	0.2382	1.43E+08	0.1398
$C_{pp,f}/C_{pp,m}$ = 1	3.50E-06	5.13E-06	0.2439	0.2439	0.2439	1.40E+08	0.1432
$C_{pp,f}/C_{pp,m}$ = 25	8.75E-05	1.10E-04	0.3613	0.2439	0.2909	1.17E+08	0.1707
$C_{pp,f}/C_{pp,m}$ = 50	1.75E-04	2.20E-04	0.4836	0.2439	0.3398	1.00E+08	0.1994
$C_{pp,f}/C_{pp,m}$ = 75	2.63E-04	3.29E-04	0.6058	0.2439	0.3887	8.77E+07	0.2281
$C_{pp,f}/C_{pp,m}$ = 100	3.50E-04	4.38E-04	0.7281	0.2439	0.4376	7.79E+07	0.2568