SYNTHESIS OF THE CONVENTIONAL PHENOMENOLOGICAL THEORIES WITH MARGINAL FERMI LIQUID MODEL

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DEDICATION

This research work is dedicated to the most high God, who gives power to the faint and to them who have no might, he gives strength. Glory be to Him for his all round provisions in my life.

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ABSTRACT

The high temperature superconducting cuprates show some non Fermi liquid behaviour in their normal state. Also there is no generally accepted theory of high temperature superconductivity. The BCS theory has failed to explain the superconductive state properties of the cuprate superconductors. Gorter-Casimir two fluid model and London theory has been very useful before the BCS theory. Varma and his co-workers propounded a 'marginal' Fermi liquid theory which explains the normal state properties of the cuprate superconductors.

In this thesis, we have calculated some thermodynamic and electrodynamic properties of the cuprate superconductors. The method involves applying the result of electronic specific heat capacity of cuprates in the normal state obtained by Kuroda and Varma to the Gorter-Casimir two fluid model using the standard variational method. We also applied the two fluid scheme to the London theory and obtained an expression for the magnetic field penetration depth. Our result was compared with other peoples' results and experimental results.

Contents

1 INTRODUCTION
1
1.1 General Background 1
1.2 Classification of Superconductors by their Critical Temperature 4
1.2.1 Low Temperature Superconductors 4
1.2.2 High Temperature Superconductors 4
1.3 Cuprate Superconductors 5
1.3.1 General properties 5
1.3.2 Crystal Structure 6

2 THEORIES OF SUPERCONDUCTIVITY 9
2.1 Landau Fermi Liquid theory 9
2.2 Gorter-Casimir Two Fluid Model 15
2.3 The London Theory 17
2.4 The BCS Theory 20
2.5 Marginal Fermi Liquid Theory 21
2.6 A Phenomenological Marginal Fermi Liquid Theory 22

3 SYNTHESIS OF GORTER-CASIMIR TWO FLUID MODEL WITH MFL MODEL 25

4 SYNTHESIS OF LONDON THEORY WITH MFL MODEL 32

5 DISCUSSION OF RESULTS 36
5.1 Specific heat jump 36
5.2 London Penetration Depth 38
5.3 Conclusion 39

Bibliography 39

CHAPTER 1

INTRODUCTION

1.1 General Background

Superconductivity is a phenomenon characterised by the disappearance of electrical resistance in various metals, alloys and compounds when they are cooled below a certain temperature usually termed the critical temperature, T_c. It is also characterised by the expulsion of the interior magnetic field (called Meissner effect) from the superconductor.

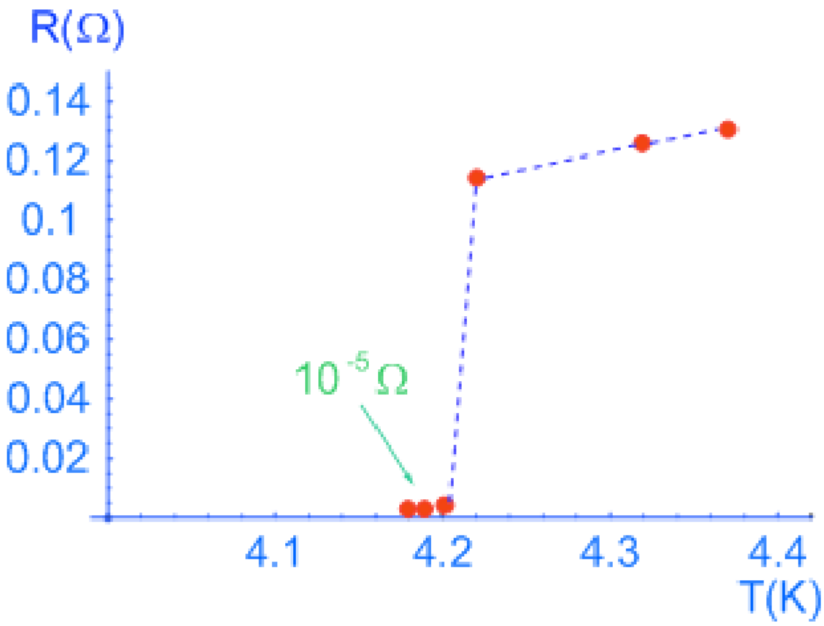

Figure 1.1: Graph of resistance R against temperature

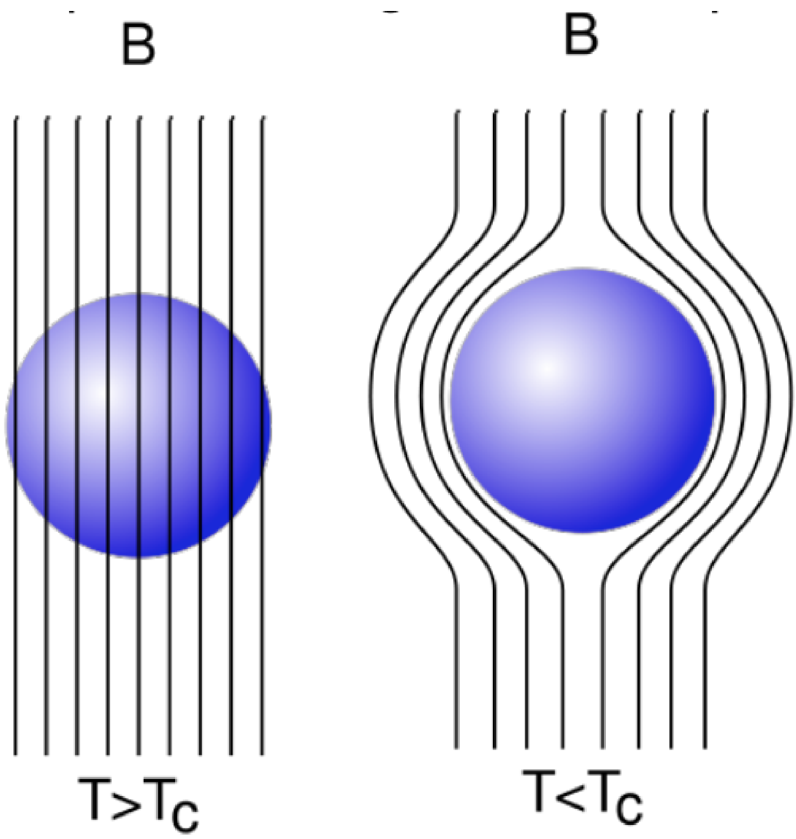

Figure 1.2: Meissner effect

Superconductivity was first observed in mercury by Dutch Physicist, Heike Kamerlingh Onnes of Leiden University in 1911. When he cooled it to the temperature of liquid helium (4.2K), its resistance suddenly disappeared (Solymar, 1993).

In 1933, German researchers, Walter Meissner and Robert Ochsenfeld discovered that a superconducting material will repel a magnetic field.

The first widely accepted microscopic theoretical understanding of superconductivity called BCS theory was advanced in 1957 by American Physicists; john Bardeen, Leon Cooper and Robert Schrieffer.

In 1986, Alex Muller and George Bednorz, researchers at IBM research laboratory in Ruschlikon, Switzerland discovered a critical temperature of 30K when performing measurements of conductivity in ceramic (lanthanum, barium, copper and oxygen) compounds with various concentration of barium. Higher superconducting transition temperature were reached in January 1987 by the group of C. W Chu at the University of Houston in collaboration with the group of M. K Wu at the University of Alabama by replacing yttrium for barium in the Muller and Bednorz molecule. They achieved an incredible critical temperature of 90K [Matsumoto (2010)].

High temperature superconductivity has taken the stage of modern condensed matter theory since its discovery in 1987. High temperature superconducting materials show deviations from the Fermi liquid phenomenology. The BCS theory has not been able to explain the properties of the high Tc cuprates. On a phenomenological level, behaviour close to the optimal doping (ie the regime where the superconducting transition temperature is highest) seems to display 'marginal Fermi liquid' (MFL) behaviour.

Studies of electrodynamic properties provide a clear phenomenological picture, reveal information regarding the pairing state, the energy gap and density of state of the superconductor and give important information on the mechanism of high temperature superconductivity. However, there is no clear concensus about the electrodynamics of cuprates from earlier studies of YBCO and BSCCO [Peil et al (1991)]. This is due in part to their complex microstructural properties: interruption of the conductive Cu-O chains by twin boundaries in YBCO, and the short coherence length which gives rise to a plethora of weak-link phenomena [Halbritter (1990)]. The most important feature observed in experiments is the linear resistivity, which at optimal doping persists in an enormous temperature range from a few kelvin to much above room temperature.

A phenomenological model describing the marginal Fermi liquid behaviour of cuprates have been put forward by Varma and co-workers but its microscopic origin remains highly controversial. To our knowledge, no microscopic theory has so far been able to provide a microscopic explanation for the phenomenon of high temperature superconducting cuprates despite years of efforts and hundreds of papers published on the subject [Chaudhury (1995)].

This research work therefore focuses on the application of the marginal Fermi liquid model to the Gorter-Casimir two fluid model/London theory.

In chapter 2 we look at the general properties of cuprates, the Fermi liquid theory, the BCS theory and some phenomenological theories including Gorter-Casimir two fluid model, London-Pippard theory and the phenomenological marginal Fermi liquid model.

In chapter 3, we apply the expression obtained for the specific heat capacity of a marginal Fermi liquid in the normal phase using a direct approach similar to the Fermi liquid (FL) and phonon-like treatment(chaudhury 1995) to the Gorter-Casimir two fluid model and obtain the normalised specific heat jump and the temperature dependence of the critical field.

In chapter 4 we look at the application of the result of the marginal Fermi liquid calculation to the London-Pippard theory and obtain an expression for the temperature dependence of the penetration depth.

Chapter 5 is the comparison of the results obtained in this thesis with the various results of calculations done by other scientists using various methods and the experimental results. And finally the conclusion.

1.2 Classification of Superconductors by their Critical Temperature

A superconductor is classified as either low temperature superconductor or high temperature superconductor depending on its critical temperature.

1.2.1 Low Temperature Superconductors

They are superconductors whose critical temperature is below 77K. This means that they cannot be cooled below their critical temperature using liquid helium at atmospheric pressure. Most low temperature superconductors are conventional superconductors (ie they display superconductivity as described by the BCS theory or its extensions).

Table 1.1: Some low temperature superconductors and their critical temperature

Superconductor	Critical Temperature (K)	Critical Field (Gauss)
Lead (Pb)	7.196	800
Tin (Sn)	3.72	310
Aluminium (Al)	1.175	110

1.2.2 High Temperature Superconductors

These are materials that can be cooled to their critical temperature T_c using liquid nitrogen. This is because their critical temperature is above the boiling point of nitrogen at atmospheric pressure.

An example of high temperature superconductor is the so called 1-2-3 compound which contains one part Yttrium, two parts barium, three parts copper, and seven parts oxygen (1Y 2Ba 3Cu 7O). The class of materials in which the proportion of the various elements changes slightly with respect to the 1-2-3 compounds mentioned above is called $YBCO$.

1.3 Cuprate Superconductors

1.3.1 General properties

Cuprate superconductors subdivided into several classes:

- The $La_{2-x}M_xCuO_4$ (LMCO) type.
- The $YBa_2Cu_3O_{6+x}$ (YBCO) type.
- The Bi, Ti, and Hg-type with the general formula
  $A_mM_2Ca_{n-1}Cu_nO_x$
  where $A = Bi, Tl, Hg$ and $M = Ba, Sr$

The value of T_c strongly depends on the concentration of oxygen and other doping ions and on various types of disorder such as impurities etc. As structural studies show, the cuprate superconductors with a general chemical formula $A - mM_2R - n - 1Cu_nO_{2n+m+2}$ we have a layered structure: $n(CuO_2)$-layers interleaving with n-1 R-layers define the active conducting block, while $[(MO)(AO)_m(MO)]$-layers form the charge reservoir block. The physical properties and the superconducting T_c are strongly influenced by the concentration of charge carriers, which is regulated by variation of the charge-reservoir block composition.

Generally, the superconducting transition temperature, T_c for copper oxide superconductor has a parabolic dependence on the concentration of the charge carriers P with a maximum at the optimal doping P_{opt}.

Studies of the thermodynamic properties of copper-oxide superconductors evidenced in a number of peculiarities of the temperature dependence of the critical magnetic fields, the penetration depth λ_L and a small correlation ϵ. This results in a very large value of the Ginzburg-Landau parameter $k = \frac{\lambda_L}{\epsilon} >> 1$, which means that the cuprates are strong type II superconducters with an extremely large upper critical magnetic field H_{c2}.

1.3.2 Crystal Structure

The structure of a cuprate, that is the relative positions of the atoms in a periodic arrangement in a crystal, is called a *perovskite*.

Table 1.2: Crystal system and the lattice parameters of typical high temperature cuprate superconductors

Superconductor	Crystal system	a(nm)	b(nm)	(nm)
$L_{a-2}Ba_xC_uO_4$	Tetragonal	0.3790	-	1.323
$Nd_{2-x}C_{2x}C_uO_4$	Tetragonal	0.3945	-	1.217
$YBa_2C_{u3}C_uO_4$	Orthorhombic	0.3823	0.3887	1.168
$B_{12}Sr_2Ca_{o-8}C_{u2}O_8$	Orthorhombic	0.5404	0.5415	3.708
$(B_1, Pb)_2Sr_2Ca_2C_u3O_{10}$	Orthorhombic	0.5404	0.5415	3.708
$H_gBa_2CaC_{u2}O_{0+x}$	Tetragonal	0.3858	-	1.266

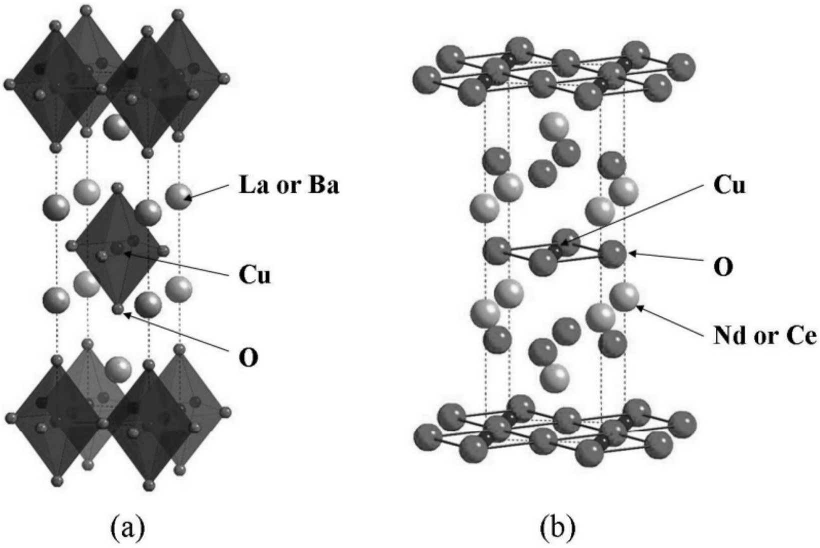

Figure 1.3: Crystal structures of (a) $La_{2-x}Ba_xCuO_4$ and (b) $Nd_{2-x}Ce-xCuO_4$ superconductors

$La_{2-x}Ba_xCuO_4$, in which some of the La^{3+} ions are substituted by Ba^{2+} has a structure referred to as the K_2NiF_4 structure. This material has a layer structure in which the (La, Ba) layers and CuO_6 octahedrons are stacked alternatively, and the Cu_2 layers are formed parallel to the bottom face. The carriers in this material are holes. Sr^{2+} ions can be used instead of Ba^{2+} ions.

As for the $Nd_{2-x}Ce-xCuO_4$ in which Nd^{3+} ions are substituted by Ce^{4+}, the carriers are electrons. Its structure consists of the planar CuO_2 layers in which the apical oxygen atoms above and below the Cu atoms are eliminated [Matsumoto (2010)].

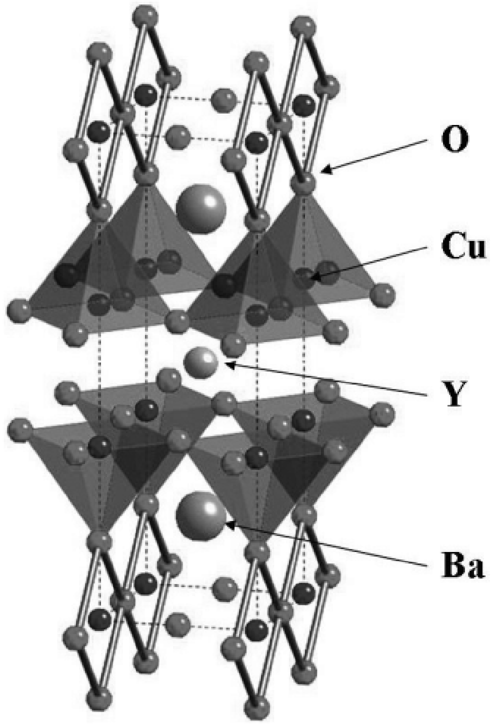

Figure 1.4: Crystal structure of $YBa_2Cu_3O_{7-x}$ superconductor showing two adjoining pyramid-type CuO_2 planes and CuO chains in the unit cell.

$YBa_2Cu_3O_{7-x}$ (YBCO or Y123) with a T_c of upto 93K, has layers of Y atoms sandwiched between the two adjoining pyramid type CuO_2 layers. Moreover, there is a triple periodic structure in which Y and Ba atoms are in line as Ba-Y-Ba and there are also one-dimensional chains of Cu-O-Cu in the direction of the b-axis of the crystal. This is shown in figure (1.4) [Ibid].

CHAPTER 2

THEORIES OF SUPERCONDUCTIVITY

2.1 Landau Fermi Liquid theory

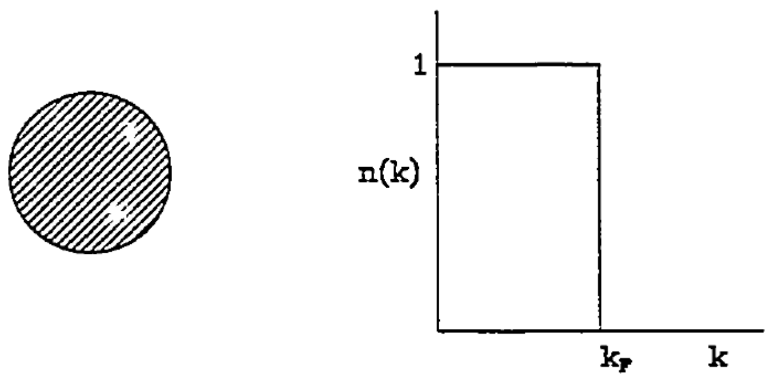

Figure 2.1: Free Fermi gas at $T = 0K$. On the left is the momentum states of the electron in momentum space with all states occupied up to the radius of the sphere given as k_F. On the right is the corresponding momentum occupation function $n(k)$.

Because of the interaction of the conduction electrons with each other through their electrostatic interaction, the electrons suffer collisions. Furthermore, a moving electron causes an inertial reaction in the surrounding electron gas, thereby increasing the effective mass of the electron. The effects of the electron-electron interactions are usually described within the framework of Landau theory of Fermi liquid. The object of the theory is to give a unified account of the effect of interactions. The landau Fermi liquid theory has been successful in explaining systems such as metals, nuclear matter, and liquid He-3

(Schofield 1999).

According to Kittel(1976), a Fermi gas is a system of non interacting fermions; the same system with interactions is a Fermi liquid.

In metals, conduction electrons although crowded together, travel long distances between collisions with each other. Two factors are responsible for the long mean free path:
-The exclusion principle
-the screening of the coulomb interaction between two electrons.
Figure (2.2) is collision $1 + 2 \longrightarrow 3 + 4$ between an electron in the excited orbital 1 (lying outside a filled Fermi sphere) and an electron in the filled orbital 2 in the Fermi sea. All energies are taken with reference to the Fermi level μ taken as zero of energy. This E_1 will be positive and E_2 will be negative.

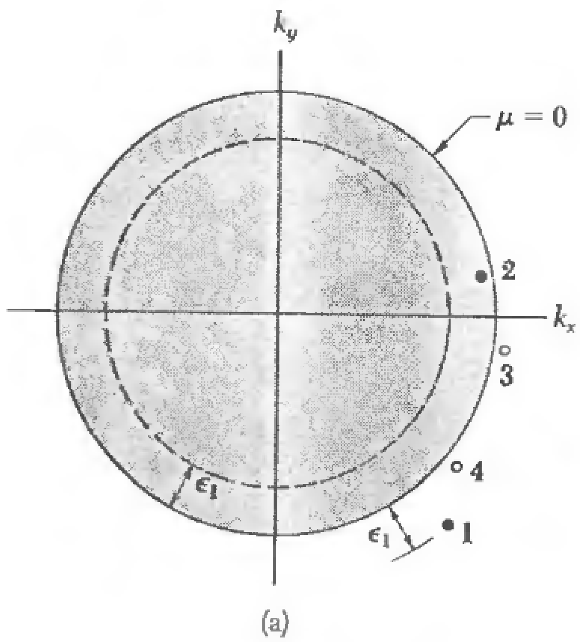

Figure 2.2: The electrons in initial orbitals 1 and 2 collide and occupy orbitals 3 and 4 if they were initially vacant. Energy and momentum are conserved.

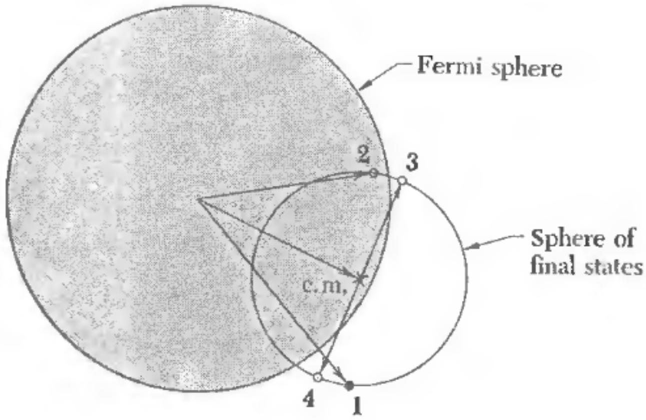

Figure 2.3: All pairs of orbitals 3 and 4 conserve energy and momentum if they lie on opposite ends of a diameter of a small sphere. the small sphere was drawn from the center of mass to pass through 1 and 2 but not all pairs of point 3,4 are allowed by Pauli exclusion principle for both 3,4 must lie outside the Fermi sphere.

Because of the exclusion principle, the orbitals 3 and 4 of the electrons after collision must lie outside the Fermi sphere, all orbitals within the sphere being already occupied. Conservation of energy requires that $|E_2| < E_1$, otherwise $E_3 + E_4 = E_1 + E_2$ would not be positive. This means that collisions are possible only if the orbital 2 lies within a shell of thickness E_1 within the Fermi surface as shown in figure (2.2). It is shown in in figure (2.3) that even if orbital 3,4 at the opposite ends of a diameter of a circle satisfy the conservation laws, collision can only occur if both orbitals 3,4 lie outside the Fermi sea.

When E_1 is exactly E_F, conservation of energy can only be satisfied if E_1, E_3 and E_4 are also all exactly E_F and there is no phase space for the process. Consequently, the life time of an electron at $T = 0K$ is infinite.

At finite T, there is now a smearing of the distribution about E_F on the scale of K_BT and the choice of E_2 will be within the range of K_BT. The final scattered E_3 will also vary within K_BT but E_4 will now be fixed by the conservation law and hence the scattering goes as $(K_BT)^2$. Likewise, if the energies of the particles can vary within an energy shell of the Fermi surface, a similar argument for $E_1 > E_F$ will lead to frequency dependence of the scattering rate varying as w^2,
where

$$w = E_1 - E_F$$

The scattering rate τ is given as

$$\frac{1}{\tau} = aw^2 + bT^2 \tag{2.1.1}$$

In general, because of the coulomb interactions, electrons with momentum $k < k_F$ will spend some time outside the Fermi sea as shown in figure 2. This causes a smearing in the momentum occupation distribution with state just above k^F now occupied and some just below k_F unoccupied. However, there will still remain a finite jump at k_F called spectral weight which we will denote as $z_k(w)$. For the non interacting free Fermi gas, $z_k(w) = 1$ and due to correlation, $z_k(w) < 1$ and varies as $(1 + \lambda)^{-1}$, where λ is a measure of the strength of the correlations in the system.

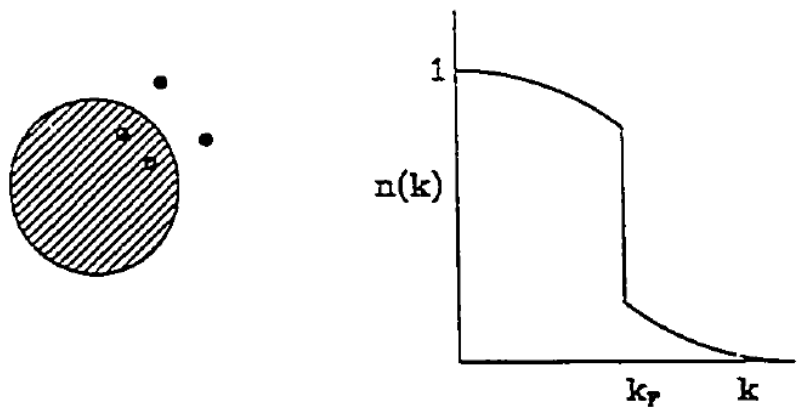

Figure 2.4: An interacting sea of electrons at $T = 0$. On the left is the momentum states of the electrons in momentum space, where some electrons occupy some states outside the free Fermi sea leaving holes behind. On the right is the momentum occupation function $n(k)$ which is smeared around the Fermi surface at k_F.

To describe the complicated interacting many body system is a very difficult problem, but fortunately all that needs to be known for most properties of interest is information about elementary excitations of the system.

In Landau's theory, the ground state can be very complicated, but if an extra particle of energy E_k with $k > k_F$ is added just above the Fermi surface, the excitation can be described in terms of the energy measured with respect to the Fermi surface. Therefore low-lying excited states of a complicated ground state can be mapped back to the elementary excitations of the non-interacting system on-one-to one correspondence. The excitations of the real system (full) system called quasiparticles can be described in terms of the plane wave states of the non-interacting system with an effective mass $m^* = m(1 + \lambda)$. These quasiparticles still have between them a residual interaction. If one decomposes this quasiparticle state, one would find that it is a superposition of all the elementary bare electron-hole pair states of the non interacting system. The net result is that through this mapping, we can revert back to describing the system in terms of single particle states but with renormalized quantities like effective mass.

The correlation energies of a metal are of the order of eV, whereas the condensation energy of the superconducting state is of the order of meV. For superconductivity to occur in the presence of of such an overwhelming correlation energy, one uses Fermi liquid theory to fold the correlation energy into a renormalized mass of the quasiparticles so that superconductivity occurs due to weak interaction between long lived quasiparticles in the presence of a filled Fermi sea of quasiparticles.

For a non-interacting sea of electrons, an added particle representing an excitation of momentum $\bar{k}$ has a well-defined energy E_k and an infinite lifetime. In the fully interacting system, the interaction can shift the single particle energies E_k and cause scattering between quasiparticles. This causes the quasiparticle to have a finite lifetime for occupation of s state$(\bar{k}, \sigma)$ [Nicol (1991)]. The shift in energy is given in terms of the many body self energy $\Sigma(\bar{E}_k)$.

$$E_k = \epsilon + \Sigma(\bar{E}_k) \tag{2.1.2}$$

where $\bar{E}_k$ is a complex function.
For the quasiparticle picture to remain valid, the damping must be small and hence the imaginary part must be small with respect to the real part. If

$$\bar{E}_k = Re\bar{E}_k - \frac{i\hbar}{2\tau(\bar{E}_k)} \tag{2.1.3}$$

where $\frac{1}{\tau(\bar{E}_k)}$ is the damping rate, then τ can be evaluated at the real value of $\bar{E}_k$.
Hence, in general, we can write the excitation energy $E_k = Re\bar{E}_k$ as

$$E_k = \epsilon_k + Re\Sigma(\bar{k}, E_k + i\Gamma_k) \tag{2.1.4}$$

and the lifetime as

$$\frac{\hbar}{2\tau_k} = -\Gamma_k = Im\Sigma(\bar{k}, E_k + i\Gamma_k) \approx Im\Sigma(\bar{k}, E_k + i0^+) \tag{2.1.5}$$

To discuss the smearing in the momentum distribution which indicates the existence of a Fermi surface, we define the spectral weight $z_k(w)$.
To do this,we must examine how the single particle Green's function is changed in the presence of interactions. In a fully interacting system, the single particle Green's function is Given by the Dyson's equation as

$$G(\bar{k}, \omega) = G^o(\bar{k}, \omega) + G^o(\bar{k}, \omega)\Sigma(\bar{k}, \omega)G(\bar{k}, \omega) \tag{2.1.6}$$

or

$$G(\bar{k},\omega) = \frac{1}{G^{o-1}(\bar{k},\omega) - \Sigma(\bar{k},\omega)} \tag{2.1.7}$$

where the single particle Green's function in the non interacting case $G^o(\bar{k},\omega)$ is given as

$$G^o(\bar{k},\omega) = \frac{1}{\omega - \epsilon_k = i0^+} \tag{2.1.8}$$

where the single particle energies are located at the poles of the Green's function $w = \epsilon_k$.
The fully renormalized single particle Green's function is

$$G(\bar{k},\omega) = \frac{1}{\omega - \epsilon_k - \Sigma(\bar{k},\omega)} \tag{2.1.9}$$

where the pole of the Green's function in this case occur at the quasiparticle energies

$$E_k = Re\omega = \epsilon_k + Re[\Sigma(\bar{k}, E_k + i\Gamma_k)] \tag{2.1.10}$$

and are shifted away from the real axis into the imaginary plane by the amount of the quasiparticle damping $Im[w]$

$$Im[\omega] = \frac{\hbar}{2\tau_k} = Im\Sigma(\bar{k}, E_k + i\Gamma_k) \tag{2.1.11}$$

Near the Fermi surface, the damping is small and so we can Taylor expand the denominator of equation (2.1.9) about w.

$$\omega - \epsilon_k - \Sigma(\bar{k},\omega) = \omega - \epsilon_k - \Sigma_1(\bar{k},\omega) - i\Sigma_2(\bar{k},\omega) \tag{2.1.12}$$

where

$$\Sigma_1(\bar{k},\omega) = Re\Sigma(\bar{k},\omega) \tag{2.1.13}$$

and

$$\Sigma_2(\bar{k},\omega) = Im\Sigma(\bar{k},\omega) \tag{2.1.14}$$

$$\omega - \epsilon_k - \Sigma_1(\bar{k},\omega) - i\Sigma_2(\bar{k},\omega) \approx \omega - \epsilon_k - \left[\Sigma_1(\bar{k},0) + \omega\frac{\partial\Sigma_1}{\partial w}\bigg|_{w=0}\right] - i\Sigma_2(\bar{k},w) \tag{2.1.15}$$

$$\approx \omega\left(1 - \frac{\partial\Sigma_1}{\partial\Sigma\omega}\bigg|_{\omega=0}\right) - [\epsilon_k + \Sigma_1(\bar{k},0)] - i\Sigma_2(\bar{k},\omega) \tag{2.1.16}$$

Then the pole occurs at ω_o given by

$$w_o = E_k - \frac{i\hbar}{2\tau_k} \tag{2.1.17}$$

where

$$E_k = \left(1 - \frac{\partial \Sigma_1}{\partial \omega}\bigg|_{\omega=0}\right)^{-1} [\epsilon_k + \Sigma_1(\bar{k}, 0)] \tag{2.1.18}$$

and

$$\frac{\hbar}{2\tau_k} = -\left(1 - \frac{\partial \Sigma_1}{\partial \omega}\bigg|_{\omega=0}\right)^{-1} \Sigma_2(\bar{k}, E_k) \tag{2.1.19}$$

For low temperatures in problems the electron-phonon problem, $\Sigma_1(\bar{k}, 0)$ is small but $\frac{\partial \Sigma_1}{\partial \omega}$ can be large (~ 1). This latter quantity is called the mass enhancement parameter λ_k

$$\lambda_k = \frac{\partial \Sigma_1}{\partial \omega}\bigg|_{\omega=0} \tag{2.1.20}$$

The quasiparticle residue or spectral weight is defined to be z_k, where

$$z_k^{-1}(\omega = 0) = 1 - \frac{\partial \Sigma_1}{\partial \omega}\bigg|_{w=0} = 1 + \lambda_k \tag{2.1.21}$$

$z_k(0)$ is a measure of the size of the discontinuity in the momentum distribution at k_F. The fact that it is non zero defines a Fermi surface and ensures that Fermi liquid theory holds.
z_k is also a measure of how much plane wave mixtureis left in the single particle Green's function:

$$G(\bar{k}, \omega) = \frac{1}{\omega - \epsilon_k - \Sigma(\bar{k}, \omega)} = \frac{z_k}{\omega - E_k + i\Gamma_k} + G^{incoh} \tag{2.1.22}$$

where G^{incoh} is the incoherent part of the Green's function.

2.2 Gorter-Casimir Two Fluid Model

The Gorter-Casimir two fluid model was first formulated by Gorter and Casimir in 1934 [Gorter and Casimir (1934)a,b]. The two fluid model is based on two fundamental assumptions:

- The ground state of the system is formed by super-electrons. This is the superconducting state and the most condensate state is at zero temperature.

- The order parameter associated to the condensate is related to the number of super-electrons and is dependent on the temperature.

Gorter- Casimir model is an “ad hoc“ model, ie there is no physical basis for the assumed expression for the free energy but it provides a fairly accurate representation of the experimental results.
Let x represent the fraction of electrons in the normal fluid and $1-x$, the ones in the superfluid. Gorter and Casimir assumed the following expression for the free energy of the electrons:

$$F(x,T) = x^{\frac{1}{2}} f_n(T) + (1-x) f_s(T) \tag{2.2.1}$$

where

$$f_n(T) = -\frac{\gamma}{2} T^2 \tag{2.2.2}$$

and

$$f_s(T) = -\beta \tag{2.2.3}$$

γ is Sommerfeld constant and it is proportional to single electron density of states at the Fermi surface. Hence it is proportional to the effective mass of normal electrons. The free energy for the electrons in a normal metal is just $f_n(T)$ whereas $f_s(T)$ gives the condensation energy associated to the superfluid.

Minimizing the free energy $F(x,T)$ with respect to x (ie $\frac{\partial F(x,T)}{\partial x} = 0$), one finds the fraction of normal electrons at a temperature T.

$$\frac{\partial F(x,T)}{\partial x} = \frac{1}{2} x^{-\frac{1}{2}} f_n(T) - f_s(T) = 0$$

$$x = \frac{\gamma^2}{16\beta^2} T^4 \tag{2.2.4}$$

At critical temperature T_c, $x = 1$. Therefore

$$T_c^2 = \frac{4\beta}{\gamma} \tag{2.2.5}$$

Supstituting (2.25) into (2.2.4) gives

$$x = \left(\frac{T}{T_c}\right)^4 \tag{2.2.6}$$

Substituting (2.2.6) into (2.2.1) and eliminating γ gives

$$F_s = -\beta \left[1 + \left(\frac{T}{T_c}\right)^4\right] \tag{2.2.7}$$

At $T = 0$, $x = 0$ and we have

$$F_n = -2\beta \left(\frac{T}{T_c}\right)^2 \tag{2.2.8}$$

and

$$F_n(T) - F_s(T) = \beta \left(1 - \left(\frac{T}{T_c}\right)^2\right)^2 \tag{2.2.9}$$

From the thermodynamic relation, it can be shown that

$$\frac{H_c^2(T)}{8\pi} = F_N(T) - F_s(T) \tag{2.2.10}$$

where $\frac{H_c^2(T)}{8\pi}$ is the stabilization energy density of the pure superconducting state and we have

$$H_c(T) = H_o \left(1 - \left(\frac{T}{T_c}\right)^2\right) \tag{2.2.11}$$

where $H_c(T)$ is the critical magnetic field and H_o is the the critical magnetic field at $T = 0K$.

The Gorter-Casimir model is important in predicting:

- The form of the variation of the magnetic field as a function of temperature.

- The variation of the penetration depth λ_L as a function of temperature.

2.3 The London Theory

The brothers H and F London in 1935 [London and London (1935)] gave a phenomenological description of the basic facts of superconductivity by proposing a scheme based on a two fluid type concept with super fluid and normal fluid densities n_s and n_n associated with velocities v_s and v_n respectively.
The densities satisfy

$$n = n_s + n_n$$

where n is the average electron number per unit volume.
They pointed out that if the electrons encountered no resistance, an applied electric field E would accelerate them steadily (Shoenberg, 1960).
That is

$$m\frac{d\bar{v}}{dt} = -eE$$

where m and e are their mass and charge respectively.

$$\bar{J}_s = en_s\bar{v}_s \tag{2.3.1}$$

where $\bar{J}_s$ is the supercurrent density.

$$\frac{\partial \bar{J}_s}{\partial t} = -en_s\frac{d\bar{v}_s}{dt} \tag{2.3.2}$$

But

$$\frac{d\bar{v}_s}{dt} = -\frac{eE}{m} \tag{2.3.3}$$

Substituting equation (2.3.2) into (2.3.3) gives

$$\frac{\partial \bar{J}_s}{\partial t} = \frac{n_s e^2 E}{m} \tag{2.3.4}$$

Equation (2.3.4) is the first London equation. Taking the curl of equation (5) gives

$$\nabla \times \frac{\partial \bar{J}_s}{\partial t} = \frac{n_s e^2}{m}\nabla \times E \tag{2.3.5}$$

From Faraday's law

$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \tag{2.3.6}$$

Applying equation (2.3.6) into equation (2.3.5) gives

$$\nabla \times \frac{\partial \bar{J}_s}{\partial t} = -\frac{n_s e^2}{mc}\frac{\partial B}{\partial t} \tag{2.3.7}$$

and we get

$$\nabla \times \bar{J}_s = -\frac{n_s e^2}{mc} B \tag{2.3.8}$$

Equation (2.3.8) is called the second London equation.
Ampere's law says that

$$\nabla \times B = \frac{4\pi}{c}\bar{J} + \frac{1}{c}\frac{\partial E}{\partial t}$$

Considering only the superfluid part, ie J_s and neglecting the displacement current $\frac{\partial E}{\partial t}$, we have

$$\nabla \times B = \frac{4\pi}{c}\bar{J}_s$$

$$\nabla \times \nabla \times B = \frac{4\pi}{c}\nabla \times \bar{J}_s$$

Using the vector identity

$$\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A,$$

we have

$$\nabla \times (\nabla \times B) = \nabla(\nabla . B) - \nabla^2 B$$

But from Maxwell's laws of electromagnetism,

$$\nabla . B = 0$$

Therefore, we write

$$-\nabla^2 B = \frac{4\pi}{c}\nabla \times \bar{J}_s$$

From London second equation,

$$\nabla^2 B = \frac{4\pi}{c}\frac{n_s e^2}{mc} B \tag{2.3.9}$$

$$\nabla^2 B = \frac{1}{\lambda_L} B \tag{2.3.10}$$

where

$$\lambda_L = \left(\frac{mc^2}{4\pi n_s e^2} \right)^{\frac{1}{2}} \tag{2.3.11}$$

λ_L is the London penetration depth.
The central point of the London theory is that the supercurrent is always determined by the local magnetic field. london theory addresses two main issues:

- The physical interpretation of transport properties (electronic properties) such as the Meissner effect.

- How supercurrents is transported in a superconductor.

2.4 The BCS Theory

In 1957, John Bardeen, in collaboration with Leon Cooper and his doctoral student John Robert Schrieffer, proposed the standard theory of superconductivity known as the BCS theory (named from their initials).

BCS theory, the first widely accepted microscopic theoretical understanding of superconductivity, explains conventional superconductivity. BCS theory views superconductivity as a macroscopic quantum mechanical effect. It proposes that electrons with opposite spin can become paired, forming Cooper pairs. In many superconductors, the attractive interaction between electrons (necessary for pairing) is brought about indirectly by the interaction between the electrons and the vibrating crystal lattice (the phonons). Roughly speaking the picture is the following:

An electron moving through a conductor will attract nearby positive charges in the lattice. This deformation of the lattice causes another electron, with opposite "spin", to move into the region of higher positive charge density. The two electrons are then held together with a certain binding energy. If this binding energy is higher than the energy provided by kicks from oscillating atoms in the conductor (which is true at low temperatures), then the electron pair will stick together and resist all kicks, thus not experiencing resistance.

Bardeen, Cooper and Schrieffer proposed a wave function for a superconductor which incorporated the idea of Cooper pairing. Using this wave function, they were able to successfully describe the features of the superconducting state. Their theory requires one parameter and once this is specified, one is able to calculate all the superconducting properties of the material. However, the theory is not able to describe all materials. In particular, the BCS theory makes prediction for the ratios of various quantities which are completely independent of any material parameters. This is due to simplifying approximations made with respect to the nature of electron-phonon interaction. The BCS theory treats the effective electron-electron interaction as a constant for energy transfers less than θ_c and zero for energy transfer greater than θ_c. In reality such interaction is not a constant and reflects various details of the material, such as the lattice structure, the electronic structure and the coupling strength between the electrons and the phonons [Peter J. W (1990)].

The BCS theory has a parameter g defined as

$$g = N(o)V_{eff} \tag{2.4.1}$$

where V_{eff} is the effective attractive interaction between Cooper pair From BCS equation for T_c one gets

$$T_c = 1.13\theta_c \exp\left(-\frac{1}{g}\right) \tag{2.4.2}$$

θ_c is the Debye frequency or a characteristic phonon energy, of the Fermi surface and it is equivalent to the characteristic frequency in the marginal Fermi liquid theory. In the weak coupling regime, $0 < g < 0.3$

2.5 Marginal Fermi Liquid Theory

In general, the unusual normal state properties of the high temperature superconducting copper-oxide compounds seem to point to a scattering rate that is linear in frequency ω and linear in temperature, T. This indicates that these materials could not be described by the conventional Fermi liquid picture. Recent data from angle resolved photoemission experiments, which are capable of measuring the Fermi surface of a material, showed that indeed these materials have a Fermi surface. This gave support to the idea of a marginal Fermi liquid. This is a theory that yields a Fermi surface in the weakest possible sense of the definition but otherwise does not make the same predictions as the Fermi liquid theory [Nicol (1991)].

Recall that the quasiparticle residue or the spectral weight function $z_k(w)$ which characterises a jump at k_F and hence indicates the presence of a Fermi surface when $z_k(w = 0)$ is non zero, depends on the real part of the self energy. If

$$\Sigma_1(\bar{k}, w) \sim w \ln\left|\frac{w}{w_c}\right|$$

where w_c is a high energy cut off. Then

$$\frac{\partial \Sigma_1(\bar{k}, w)}{\partial w} = \ln\left|\frac{w}{w_c}\right| + 1$$

or

$$\left.\frac{\partial \Sigma_1}{\partial w}\right|_{w=E_k} = \ln\left|\frac{E_k}{w_c}\right| + 1 \tag{2.5.1}$$

and hence

$$z_k^{-1} = 1 - \left.\frac{\partial \Sigma_1}{\partial w}\right|_{w=E_k} = -\ln\left|\frac{E_k}{w_c}\right|$$

$$z_k^{-1} = \ln\left|\frac{w_c}{E_k}\right|$$

$$z_k = \frac{1}{\ln\left|\frac{w_c}{E_k}\right|} \tag{2.5.2}$$

Now when the quasiparticle is on the Fermi surface, $E_k = 0$ and $z_k \to 0$. Hence, the jump in the distribution tends to zero but in a very weak way (ie logarithmically), and thus a Fermi surface just barely remains in the weakest sense.

The linear w scattering rate experimentally observed in the high temperature copper-oxide superconductor will arise if Σ_2 is equal to

$$\frac{1}{2\tau_k} = \Sigma_2 \propto x$$

where $x = |w|$ or T.

Table 2.1 shows the contrast between Fermi liquid theory and marginal Fermi liquid theory.

Table 2.1: Contrast between Fermi liquid theory and marginal Fermi liquid theory

Fermi Liquid Theory	Marginal Fermi Liquid Theory
$Re\sum \sim w$	$R_e\sum \sim \ln(\frac{x}{w_c}),\ \ with\ \ x = \max(\lvert w\rvert, T)$
$Im\sum \sim w^2$	$Im\sum \sim x\ ,\ with\ \ x = \max(\lvert w\rvert, T)$
$T(w \to 0) \to \infty$	$T(w \to 0) \to \infty$
$Z_k = \frac{1}{(1+\lambda)\leq 1}$	$Z_k\ (\ln w)^{-1} \to 0\ \ as\ \ w \to 0$

It is important to note that there are still well defined quasiparticles with an infinite lifetime at the Fermi surface, but that the self energy function is quite different and will give rise to different physics.

2.6 A Phenomenological Marginal Fermi Liquid Theory

Varma et al(1989) postulated that in the copper oxide system, there are charge and spin density fluctuations of the electronic system. These fluctuations lead to a polarisability of the electronic medium that would renormalize the electron through the self energy.
Their proposal for this polarisability is as follows:

$$ImP(\bar{q}, w)\begin{cases} -N(0)\frac{w}{T}, & for \quad |w|<T \\ -N(0)signw & for \quad |w|>T \end{cases} \tag{2.6.1}$$

where N(0) is the single particle density of states at the Fermi energy. The form of this polarisability is postulated to come from the vertex corrections in the particle-hole susceptibility shown in figure (2.5).
The self energy that arises from applying the Feyman rules to figure (2.5) is given as

$$\Sigma(\bar{q}, w) \sim g^2 N^2(0)\left(w \ln \frac{x}{w_c} - i\frac{\pi}{2}x\right)$$

where

$$x = max(|w|, T).$$

The parameter w_c is taken to be some high energy cut off on the polarisability and g is a coupling constant at the vertex of the electron interaction with the polarisation bubble. We have

$$z_k \sim \frac{1}{\ln |\frac{w_c}{E_k}|}$$

and the quasiparticle life time is still infinite at the Fermi surface ie

$$\frac{1}{2\tau_k} = \frac{\pi}{2} E_k$$

which as $E_k \to 0$, $\tau_k \to \infty$.
This form of the scattering rate $\frac{1}{\tau}$ which has the form

$$\frac{1}{\tau} = aw + bT$$

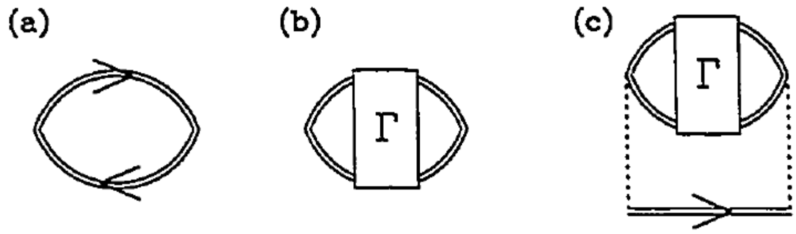

Figure 2.5: The Feyman diagram for the particle hole susceptibility $\chi(\bar{q}, w)$ (a) without vertex corrections (b) with vertex corrections (c) the electron self energy due to this susceptibility, where the dashed lines represent a coupling g.

Kuroda and Varma (1990) calculated the specific heat of the marginal Fermi liquid in the normal phase using a Fermi liquid-like formula in the presence of electron-boson coupling constant. They obtained the electronic specific heat C_v of the marginal Fermi liquid given as

$$C_v = N(0)\left(3 + 2\ln\frac{\theta_c}{T}\right)T \tag{2.6.2}$$

where N(0) is the bare single electron density of states at the Fermi surface and θ_c is the characteristic frequency in the marginal Fermi liquid theory.

CHAPTER 3

SYNTHESIS OF GORTER-CASIMIR TWO FLUID MODEL WITH MFL MODEL

The free energy of electrons in a material is given by

$$f_n = U - T\int_0^T \frac{C_v(T)}{T}dT \qquad where \qquad U = \int_0^T C_v(T)dT \tag{3.0.1}$$

U is the internal energy of the electrons in the system.
We then have that the free energy for the electrons in the normal 'marginal' Fermi liquid is given as

$$f_n = -N(0)\left(3 + \ln\frac{\theta_c}{T}\right)T^2 \tag{3.0.2}$$

The expression for the free energy of all the electrons in the 'marginal' Fermi liquid superconductor is now given as

$$F(x,T) = -x^{\frac{1}{2}}N(0)\left(3 + \ln\frac{\theta_c}{T}\right)T^2 + (1-x)(-\beta) \tag{3.0.3}$$

Minimizing $F(x,T)$ with respect to x gives

$$\frac{\partial F(x,T)}{\partial x} = -\frac{1}{2}x^{-\frac{1}{2}}N(0)\left(3 + \ln\frac{\theta_c}{T}\right)T^2 + \beta = 0 \tag{3.0.4}$$

This gives

$$x^{\frac{1}{2}} = \frac{N(0)}{2\beta}\left(3 + \ln\frac{\theta_c}{T}\right)T^2 \tag{3.0.5}$$

At critical temperature T_c, $x = 1$ and we have that

$$\frac{\beta}{N(0)} = \frac{1}{2}\left(3 + \ln\frac{\theta_c}{T_c}\right)T_c^2 \tag{3.0.6}$$

Substituting equation (3.0.6) into equation (3.0.5), we have

$$x^{\frac{1}{2}} = \frac{\left(3 + \ln\frac{\theta_c}{T}\right)}{\left(3 + \ln\frac{\theta_c}{T_c}\right)}\frac{T^2}{{T_c}^2} \tag{3.0.7}$$

and

$$x = \left(\frac{3 + \ln\frac{\theta_c}{T}}{3 + \ln\frac{\theta_c}{T_c}}\right)^2\left(\frac{T}{T_c}\right)^4 \tag{3.0.8}$$

where x represents the fraction of electrons in the normal fluid of the 'marginal' Fermi liquid.
Substituting the expression for x in equation (3.0.8) into (3.0.3) gives

$$F_s(T) = -\left(\frac{3 + \ln\frac{\theta_c}{T}}{3 + \ln\frac{\theta_c}{T_c}}\right)\frac{T^4}{{T_c}^2}N(0)\left(3 + \ln\frac{\theta_c}{T_c}\right) + \left[1 - \left(\frac{3 + \ln\frac{\theta_c}{T}}{3 + \ln\frac{\theta_c}{T_c}}\right)\frac{T^2}{{T_c}^2}\right][-\beta] \tag{3.0.9}$$

Since

$$\beta = \frac{1}{2}N(0)\left(3 + \ln\frac{\theta_c}{T_c}\right)T_c^2 \tag{3.0.10}$$

We have that

$$F_s(T) = -\frac{1}{2}N(0)\frac{\left(3 + \ln\frac{\theta_c}{T}\right)^2}{\left(3 + \ln\frac{\theta_c}{T_c}\right)}\frac{T^4}{{T_c}^2} - \frac{1}{2}N(0)\left(3 + \ln\frac{\theta_c}{T_c}\right)T_c^2 \tag{3.0.11}$$

Re-arranging equation (3.0.11) gives

$$F_s(T) = -\frac{1}{2}N(0)\left[\frac{\left(3 + \ln\frac{\theta_c}{T}\right)^2}{\left(3 + \ln\frac{\theta_c}{T_c}\right)}\frac{T^4}{{T_c}^2} + \left(3 + \ln\frac{\theta_c}{T_c}\right)T_c^2\right] \tag{3.0.12}$$

$F_N(T)$ is given as

$$F_N(T) = -N(0)\left(3 + \ln\frac{\theta_c}{T}\right)T^2 \tag{3.0.13}$$

Recall that

$$C_v^s = -T\frac{\partial^2 F_s(T)}{\partial T^2}$$

and we get

$$C_v^s = N(0)\frac{T^3}{T_c^2}\frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\left[6\left(3+\ln\frac{\theta_c}{T}\right)^2 - 7\left(3+\ln\frac{\theta_c}{T}\right)+1\right] \qquad (3.0.14)$$

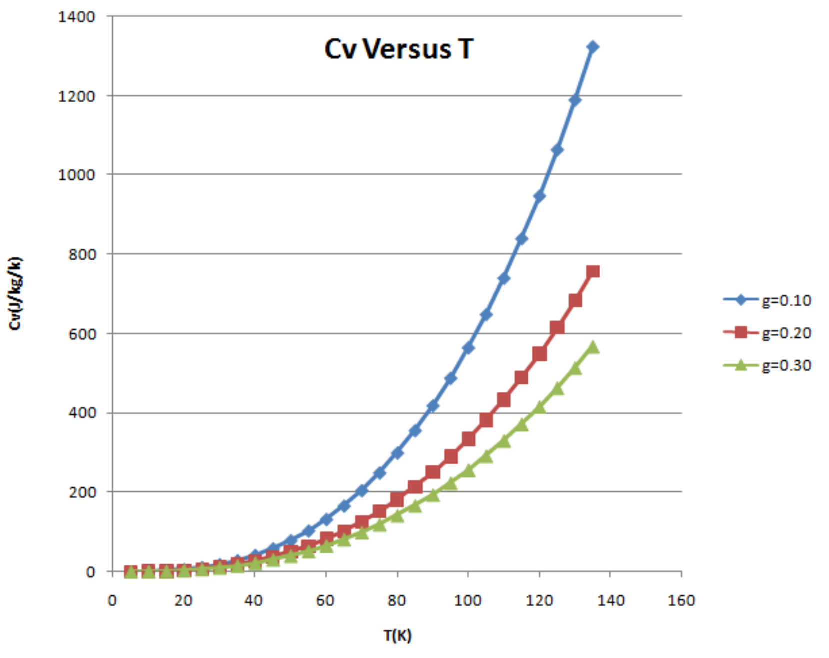

Figure 3.1: Graph of specific heat in the superconducting phase C_v against temperature T.

Also

$$C_v^N = -T\frac{\partial^2 F_N(T)}{\partial T^2} = N(0)\left(3+2\ln\frac{\theta_c}{T}\right)T \qquad (3.0.15)$$

The difference between the specific heat capacity in the superconducting state C_v^s and the specific heat capacity in the normal state C_v^N, called the specific heat jump $\triangle C_v$ is given as

$$\triangle C_v = C_v^s - C_v^N$$

Substituting for C_v^s and C_v^N gives

$$\triangle C_v = N(0)\frac{T^3}{T_c^2}\frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\left[6\left(3+\ln\frac{\theta_c}{T}\right)^2 - 7\left(3+\ln\frac{\theta_c}{T}\right)+1\right] - N(0)\left(3+\ln\frac{\theta_c}{T}\right)T$$

At the critical temperature $\triangle C_v$ is given as

$$\triangle C_v|_{T=T_c} = N(0)T_c \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\left[6\left(3+\ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\right] - N(0)\left(3+\ln\frac{\theta_c}{T_c}\right)T_c \tag{3.0.16}$$

The ratio of the two types of specific heat is given as

$$\frac{C_v^s}{C_v^N} = \left(\frac{T}{T_c}\right)^2 \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)\left(3+2\ln\frac{\theta_c}{T}\right)}\left[6\left(3+\ln\frac{\theta_c}{T}\right)^2 - 7\left(3+\ln\frac{\theta_c}{T}\right)+1\right]$$

At critical temperature T_c, the ratio of the two types of specific heat is given as

$$\left.\frac{C_v^s}{C_v^N}\right|_{T=T_c} = \frac{1}{\left(3+2\ln\frac{\theta_c}{T_c}\right)}\left[6\left(3+\ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\right] \tag{3.0.17}$$

The normalized specific heat jump $\frac{\triangle C_v}{C_v^N}$ is

$$\frac{\triangle C_v}{C_v^N} = \left(\frac{T}{T_c}\right)^2 \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)\left(3+2\ln\frac{\theta_c}{T}\right)}\left[6\left(3+\ln\frac{\theta_c}{T}\right)^2 - 7\left(3+\ln\frac{\theta_c}{T}\right)+1\right] - 1$$

The normalized specific heat jump at the transition temperature is given as

$$R = \left.\frac{\triangle C_v}{C_v^N}\right|_{T=T_c}$$

and we have

$$R = \frac{1}{\left(3+2\ln\frac{\theta_c}{T_c}\right)}\left[6\left(3+\ln\frac{\theta_c}{T_c}\right) - 7 + \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\right] - 1 \tag{3.0.18}$$

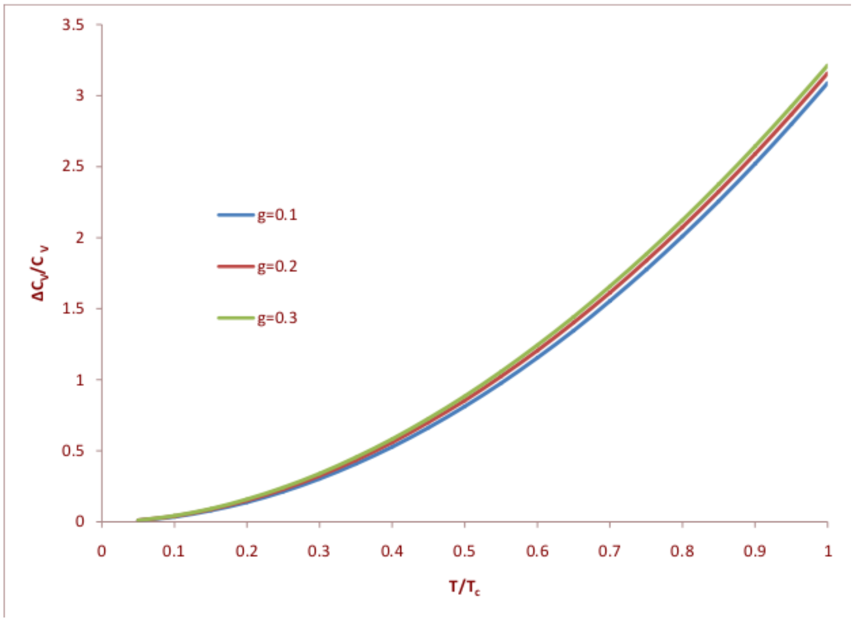

Figure 3.2: Graph of normalized specific heat jump in the superconducting phase C_v against temperature $\frac{T}{T_c}$

The slope of the specific heat jump at the transition temperature is given as

$$\left.\frac{d(\triangle C_v)}{dT}\right|_{T=T_c} = N(0)\left[18\left(3+\ln\frac{\theta_c}{T_c}\right) - 33 + \frac{10}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\right] - N(0)\left(1+2\ln\frac{\theta_c}{T_c}\right) \tag{3.0.19}$$

The normalized slope of the specific heat jump is given as

$$\frac{\frac{d(\triangle C_v)}{dT}}{\frac{dC_v^N}{dT}} = \left(\frac{T}{T_c}\right)^2 \frac{1}{\left(3+\ln\frac{\theta_c}{T_c}\right)\left(1+2\ln\frac{\theta_c}{T}\right)}\left[18\left(3+\ln\frac{\theta_c}{T}\right)^2 - 33\left(3+\ln\frac{\theta_c}{T}\right) + 10\right] - 1$$

At transition temperature, the normalized slope of the specific heat jump is given as

$$D = \left.\frac{\frac{d(\triangle C_v)}{dT}}{\frac{dC_v^N}{dT}}\right|_{T=T_c}$$

$$D = \frac{1}{\left(1+2\ln\frac{\theta_c}{T_c}\right)}\left[18\left(3+\ln\frac{\theta_c}{T_c}\right) - 33 + \frac{10}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\right] - 1 \tag{3.0.20}$$

To show how the critical magnetic field H_c depends on the temperature T, we recall the thermodynamic relation

$$\frac{H_c{}^2(T)}{8\pi} = F_n(T) - F_s(T) \tag{3.0.21}$$

where $\frac{H_c(T)}{8\pi}$ is the stabilization energy density of the pure superconducting state. Sustituting the values of F_s and F_N into equation (3.0.21) above gives

$$\frac{H_c{}^2(T)}{8\pi} = -N(0)\left[\left(3+\ln\frac{\theta_c}{T}\right)T^2 - \frac{1}{2}\frac{\left(3+\ln\frac{\theta_c}{T}\right)^2}{\left(3+\ln\frac{\theta_c}{T_c}\right)}\frac{T^4}{T_c{}^2} - \frac{1}{2}\left(3+\ln\frac{\theta_c}{T_c}\right)T_c^2\right] \tag{3.0.22}$$

Substituting β for $\frac{1}{2}N(0)\left(3+\ln\frac{\theta_c}{T_c}\right)T_c^2$ into equation (3.0.22) above, we have that

$$\frac{H_c{}^2(T)}{8\pi} = -\beta\left[2\left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)\left(\frac{T}{T_c}\right)^2 - \left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)^2\left(\frac{T}{T_c}\right)^4 - 1\right] \tag{3.0.23}$$

Factorising equation (3.0.23) gives

$$\frac{H_c{}^2(T)}{8\pi} = \beta\left[1 - \left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)\left(\frac{T}{T_c}\right)^2\right]^2 \tag{3.0.24}$$

At $T = 0$, $H_c(T) = H_1$ and we have

$$\frac{H_1^2}{8\pi} = \beta \tag{3.0.25}$$

Substituting equation (3.0.25) into (3.0.24) gives

$$\frac{H_c{}^2(T)}{8\pi} = \frac{H_1^2}{8\pi}\left[1 - \left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)\left(\frac{T}{T_c}\right)^2\right]^2 \tag{3.0.26}$$

and

$$H_c(T) = H_1\left[1 - \left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)\left(\frac{T}{T_c}\right)^2\right] \tag{3.0.27}$$

where

$$H_1 = (8\pi\beta)^{\frac{1}{2}}$$

Equation (3.0.27) is the expression for the temperature dependence of the critical magnetic field for the marginal Fermi liquids.

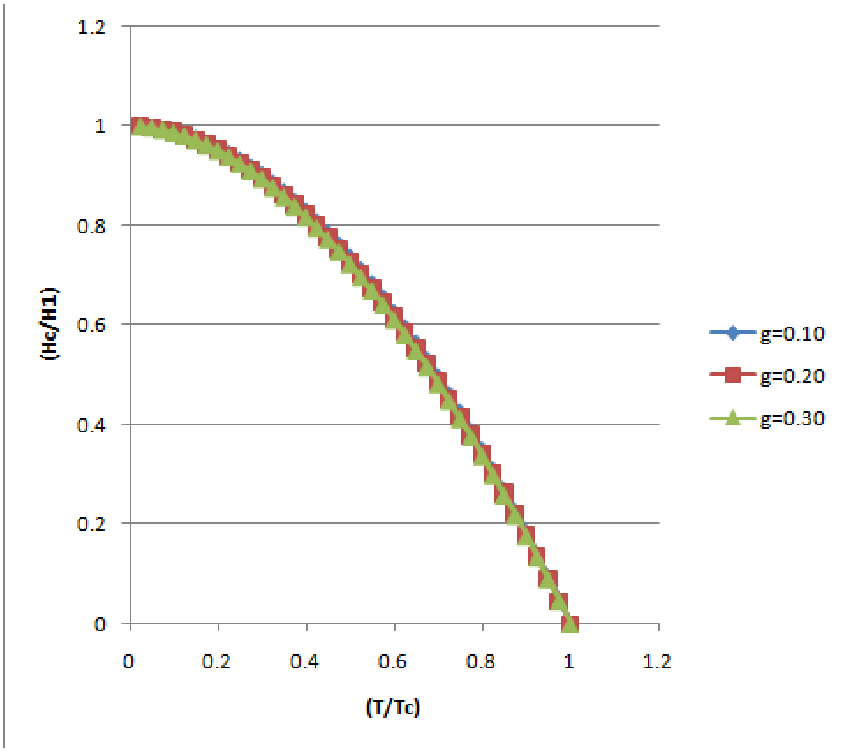

Figure 3.3: Graph $\frac{H_c}{H_1}$ against $\frac{T_c}{T}$.

CHAPTER 4

SYNTHESIS OF LONDON THEORY WITH MFL MODEL

The zero frequency penetration depth is a measure of the distance scale on which a static magnetic field will penetrate into a superconductor. Although the superconductor has the property that it excludes all the magnetic flux, at the superconducting surface screening current are produced to provide the diamagnetism and it is in the surface layer that the field may still penetrate [Nicol, (1991)].

From the two fluid model,

$$\frac{n_s}{n} = 1 - x \tag{4.0.1}$$

Substituting the value of x in equation (3.0.8) into (4.0.1), we have

$$n_s = n\left[1 - \left(\frac{3 + \ln\frac{\theta_c}{T}}{3 + \ln\frac{\theta_c}{T_c}}\right)^2 \left(\frac{T}{T_c}\right)^4\right] \tag{4.0.2}$$

where n_s and n retain their meaning.
From Londons' equations,

$$\lambda_L(T) = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{\frac{1}{2}} \tag{4.0.3}$$

where c is a constant and m and e are the mass and charge of electron respectively.
At $T = 0$, $n_s = n$ and the penetration depth $\lambda_L(T) = \lambda_L(0)$ and we get

$$\lambda_L(0) = \left(\frac{mc^2}{4\pi ne^2}\right)^{\frac{1}{2}} \tag{4.0.4}$$

Substituting for n_s in equation (4.0.2) into equation (4.0.3) gives

$$\lambda_L(T) = \left(\frac{mc^2}{4\pi ne^2}\right)^{\frac{1}{2}} \left[\frac{1}{1-\left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)^2\left(\frac{T}{T_c}\right)^4}\right]^{\frac{1}{2}} \tag{4.0.5}$$

or

$$\lambda_L(T) = \frac{\lambda_L(0)}{\left[1-\left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)^2\left(\frac{T}{T_c}\right)^4\right]^{\frac{1}{2}}} \tag{4.0.6}$$

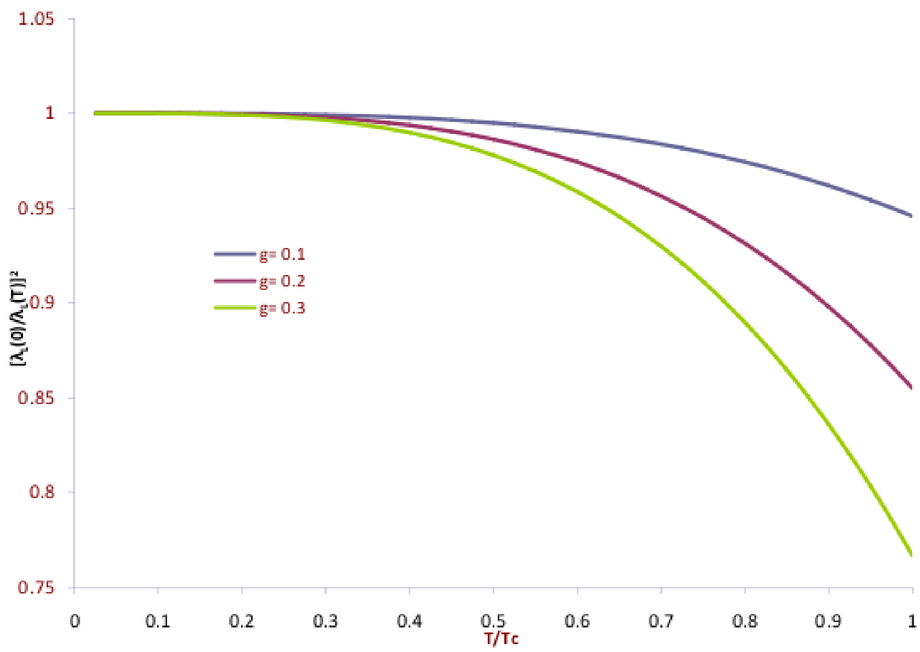

Figure 4.1: Graph of $\left[\frac{\lambda_L(0)}{\lambda_L(T)}\right]^2$ against $\frac{T}{T_c}$

Solving London equation for a semi-infinite plane superconductor whose doundary coincides with $y = 0$ in an external magnetic field H_1 oriented along the z axis and taking into account the symetry of the system, we can write the problem as

$$\frac{d^2H(y)}{dy^2} - \frac{1}{\lambda_L^2}H(y) = 0 \tag{4.0.7}$$

with the boundary conditions

$$H(0) = H_1 \qquad and \qquad H(\infty) = 0$$

The solution to equation (4.0.7) is

$$H(y) = H_1 \exp\left(-\frac{y}{\lambda_L}\right) \tag{4.0.8}$$

where

$$\lambda_L(T) = \left(\frac{mc^2}{4\pi ne^2}\right)^{\frac{1}{2}} \left[\frac{1}{1 - \left(\frac{3+\ln\frac{\theta_c}{T}}{3+\ln\frac{\theta_c}{T_c}}\right)^2 \left(\frac{T}{T_c}\right)^4}\right]^{\frac{1}{2}}$$

This means that for the semi-infinite plane, the field $H(y)$ would decay exponentially from its free space value H_1.

In the application of Drude model to the ac-electrical conductivity of metals, we get a frequency dependent complex dielectric constant $\epsilon(\omega)$ given as

$$\epsilon(\omega) = 1 + 4\pi i\frac{\sigma(\omega)}{\omega} \tag{4.0.9}$$

where $\sigma(\omega)$ is a frequency dependent conductivity given as

$$\sigma(\omega) = \frac{\sigma_o}{1 - i\omega t} \tag{4.0.10}$$

and σ_o is given as

$$\sigma_o = \frac{ne^2\tau}{m} \tag{4.0.11}$$

τ is the relaxation time or time between collision of an electron [Ashkroft and Mermin (1976)].
We have from equations (4.0.10) and (4.0.11) that

$$\epsilon(\omega) = \frac{1 - 4\pi ne^2}{m\omega^2} \tag{4.0.12}$$

or

$$\epsilon(\omega) = 1 - \left(\frac{\omega_p^2}{\omega^2}\right) \tag{4.0.13}$$

where

$$\omega_p^2 = \frac{4\pi ne^2}{m} \tag{4.0.14}$$

ω_p^2 is the plasma frequency.

But from equation (4.0.4) we know that $\lambda_L(0)$ is given as

$$\lambda_L(0) = \left(\frac{mc^2}{4\pi n e^2} \right)^{\frac{1}{2}}$$

and we have that

$$\omega_p^2 = \left(\frac{c}{\lambda_L(0)} \right)^2 \tag{4.0.15}$$

c is the speed of light. The London penetration depth $\lambda_L(T)$ can be related to the free electron plasma frequency w_p^2 as follows:

$$\lambda_L(T) = \frac{c}{(w_p^2)^{\frac{1}{2}}} \frac{1}{\left[1 - \left(\frac{3+\ln \frac{\theta_c}{T}}{3+\ln \frac{\theta_c}{T_c}} \right)^2 \left(\frac{T}{T_c} \right)^4 \right]^{\frac{1}{2}}} \tag{4.0.16}$$

CHAPTER 5

DISCUSSION OF RESULTS

5.1 Specific heat jump

In our model, we have incorporated the marginal Fermi liquid theory into the Gorter-Casimir two fluid model. Due to the polarizability of the electronic medium coming from the charge and spin density fluctuation of the electronic system, there is an extra term to the bare electron density of state. So one should expect that the specific heat jump would be enhanced over the value that one would observe in a material where there is no such fluctuations. Specific heat measurements give information on the electron-phonon coupling strength. BCS theory and its subsequent refinements based on the Eliashberg equations show that high critical temperatures in phonon mediated superconductor are favoured by high phonon frequencies and by large density of states at the Fermi level.

The quantity of interest is the difference between the specific heats of the normal and superconducting states. The normalized specific heat jump varies from 1.43 in the BCS weak coupling limit.

At low temperatures, the lattice contribution to the total specific heat is small and can be accurately subtracted. The normal specific heat can be obtained by applying a magnetic field of sufficent strength to cause the sample to be normal.

In the oxide superconductors, there are difficulties associated with these measurements. Because the critical temperatures of these oxide materials are relatively high, the lattice contribution to the total specific heat is quite large compared to the electronic contribution. An additional complication is that it is only possible to get normal state data close to critical temperature as the critical fields are quite large and difficult to obtain in the laboratory.

Figure (5.1) is the experimental results for YBCO. Comparing with figure 3.1, observe that at low temperatures, there is an upturn in the specific heat rather than the expected exponential decay. However, there is still a linear term but there is no concensus yet on its origin. Analysis of the experimental data is usually done by assuming that the BCS relation $\frac{\triangle C}{\gamma T_c} = 1.43$ holds. However it is pointed out by Beckman et al that γ extracted by this analysis is not in good agreement with values from high T_c magnetization experiments and band structure calculations.

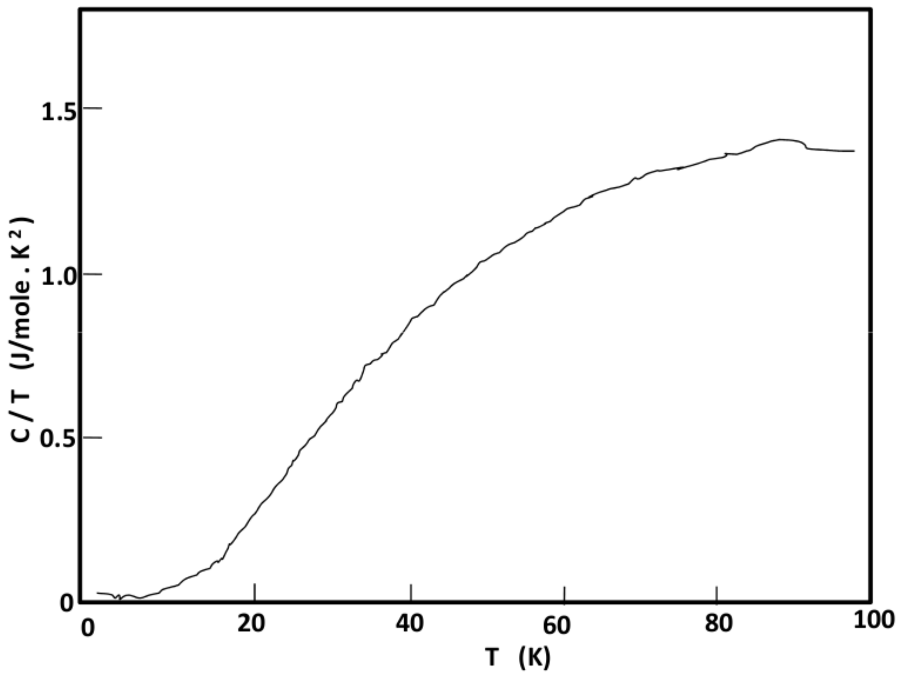

Figure 5.1: Experimental result for the specific heat of $YBa_2Cu_3O_7$.

Loram and Mirza have used differential calorimetry on YBCO samples and report a normalized specific heat jump of 4.1. Philips at al have reported a value of 4.8.

From various observations, it would seem that there is a strong evidence for the specific heat jump to be large in the high T_c materials. This large value of the normalized specific heat jump is consistent with the result of ~ 3.02 in the model of synthesizing the Gorter-Casimir two fluid model with marginal Fermi liquid theory as done in the BCS weak coupling regime this thesis.

5.2 London Penetration Depth

In the second part of this thesis, we calculated the magnetic field penetration depth by applying the two fluid scheme to London theory. The main aim was to investigate the effect of the charge and spin density fluctuations of the electronic system in the copper oxide materials. This we have done within the scope of the BCS weak coupling theory.

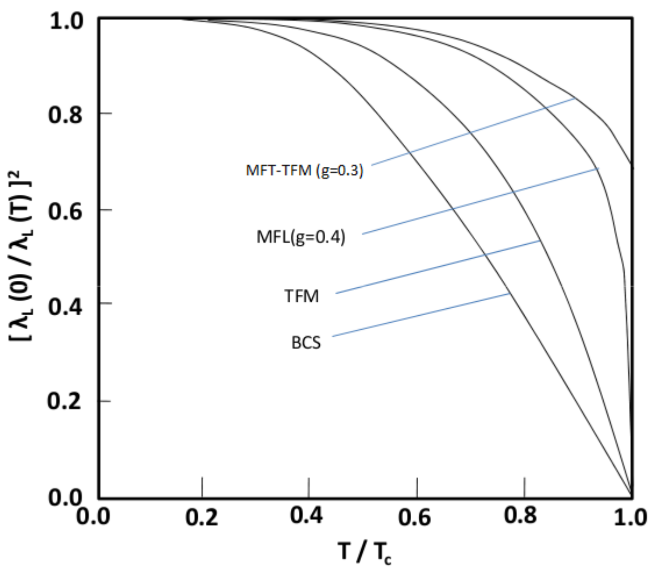

Figure 5.2: Comparism of the results of penetration depth from the BCS, TFM, MFL, and our calculation (MFL-TFM)

In figure (5.2) we have compared various results of the London penetration depth for the cuprate superconductor. The BCS weak coupling, the Gorter-Casimir two fluid model (TFM), the marginal Fermi liquid model (MFL) as done in the strong coupling regime by Nicol et al (1990) and the synthesis of the MFL theory with London theory within the Two Fluid scheme (MFT-TFM) calculated in this thesis.

Note, however, that there is currently no consensus on the precise shape of $\frac{\lambda_L^2(0)}{\lambda_L^2(T)}$ in YBCO [Mao et al (1995)]

The London penetration depth from our result is close to the result of other results. If we extend our calculation to the BCS strong coupling regime, we hope to get a result closer to the experimental result.

We also calculated the electrodynamic property in particular magnetic field penetration depth and related the london penetration depth $\lambda_L(T)$ to the free elecron plasma frequency w_p^2.It is seen that the London penetration depth varies inversely with $(w_p^2)^{\frac{1}{2}}$.

5.3 Conclusion

In this thesis, we have applied the result of the marginal Fermi liquid theory to the Gorter-Casimir two fluid model and London theory and used our result to calculate some thermodynamic properties like the specific heat jump and the temperature dependence of the critical magnetic field. The results of our calculation is closer to the experimental result than that of each of the theory independently.
From our calculations, it is clear that increase in the free electron plasma frequency will lead to a derease in the London penetration depth. This is seen in equation (4.0.16).

In this thesis, we have only modified the normal fluid part of the Gorter-Casimir two fluid model. A more accurate result can be obtained by modifying the super fluid part. One method of doing this is a scheme based on many body formalism which can lead to the free energy of the full superconducting phase for a marginal Fermi liquid superconductor. From this one can in principle subtract the normal fluid and extract the superfluid contribution.

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